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On the Measurement of Local Surface Friction on a Flat Plate by means of Preston Tubes

By STAFF OF AERODYNAMICS DIVISION, N.P.L.

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On the Measurement of Local Surface Friction on a Flat Plate by means of Preston Tubes

By Staff of Aerodynamics Division, N.P.L.

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Summary. Measurements of local surface friction were made by Preston's method using a round pitot attached to the surface ('Preston tube') and were compared with the values given by differentiation of overall measurements of surface friction by the wake-traverse method and with determinations by means of surface pitots of the type used by Stanton. The results establish that the calibration curve for Preston tubes on a flat plate is similar to that obtained in pipe flow but that, for a given Preston-tube reading, the local surface friction is about 11 per cent higher on the flat plate than in a circular pipe. The calibration formula for the flat plate is

$$\log_{10}\frac{\tau_0 d^2}{4\rho\nu^2} = \bar{2}\cdot 647 + 0\cdot 875\log_{10}\frac{(P-p)d^2}{4\rho\nu^2},$$

compared with Preston's pipe formula

$$\log_{10}\frac{\tau_0 d^2}{4\rho\nu^2} = \bar{2}\cdot 604 + 0\cdot 875\log_{10}\frac{(P-p)d^2}{4\rho\nu^2}.$$

Corresponding differences were observed in the constants of the semi-logarithmic inner law for the boundary-layer velocity profile, of almost the right amount to account for the calibration differences.

1. *Introduction.* In Ref. 1, J. H. Preston proposed a novel method for the measurement of local surface friction in turbulent flow by means of round Pitot tubes attached to the surface. By assuming that there was a region in the boundary layer near the surface where the conditions were a function only of the surface friction, the physical properties of the fluid and a representative

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† This report combines the results of the following three A.R.C. papers:

1. 'Measurements of the turbulent surface-friction drag of a flat plate in two-dimensional flow' by R. C. Pankhurst, E. F. Relf and W. S. Walker. A.R.C. 17,024. August, 1954.
2. 'The use of Pitot tubes to measure skin friction on a flat plate' by E. F. Relf, R. C. Pankhurst and W. S. Walker. A.R.C. 17,025. August, 1954.
3. 'Calibration of Preston tubes on a flat plate, using measurements of local skin friction' by P. Bradshaw and N. Gregory. A.R.C. 20,199. May, 1958.

length, he obtained a universal non-dimensional relation between the surface friction and the difference between the pressure in the Pitot tube and the static pressure on the surface. He conducted experiments in a long pipe, so that he could conveniently get the surface friction from the pressure drop along the pipe, and showed that over a considerable range of Reynolds number and pitot diameter his results were well represented by the formula

$$\log_{10} \frac{\tau_0 d^2}{4\rho\nu^2} = \bar{2}\cdot 604 + \frac{7}{8} \log_{10} \frac{(P - p)d^2}{4\rho\nu^2}, \tag{1}$$

where τ_0 is the shear stress at the wall, d is the diameter of the pitot and $(P - p)$ is the pitot pressure difference.

A main object of the present investigations was to establish the corresponding calibration curve for a flat plate, which arguments based on the assumption of complete universality of the logarithmic velocity profile would have led one to expect to be the same as for a pipe. Two distinct series of experiments were made. The first series employed a plate of 10-ft chord in the 13 ft × 9 ft Wind Tunnel at the National Physical Laboratory, values of the local surface friction being obtained by differentiation of the overall surface friction as determined by the wake-traverse method. The second series used another plate (of 6-ft chord) in another wind tunnel (the 9 ft × 7 ft 'South' Tunnel) and determined the local surface friction more directly by means of surface tubes (Stanton tubes) which were calibrated in turbulent channel flow. Measurements of boundary-layer velocity profiles were included.

2. *Experiments in* 13 *ft* × 9 *ft Wind Tunnel.* 2.1. *Apparatus.* The tests in the 13 ft × 9 ft Wind Tunnel were done on a smooth flat plate of 10-ft chord, mounted vertically to span the tunnel. The plate was $\frac{3}{4}$ in. thick, with a straight taper over a length of 4 in. down to an edge of $\frac{1}{32}$-in. radius at the nose and the tail. The middle 2 ft of span (*see* Fig. 1) was very carefully constructed of laminated pine; the required high standard of surface flatness was achieved by rubbing down after applying three coats of Phenoglaze lacquer and repeating this process three times before applying a final finishing coat and burnishing. The outer portions consisted of laminated board and were attached to the middle portion by ten flush-fitting metal straps 6 in. long and 2 in. wide. The model was kept rigid by bracing wires, all of which were attached outside the middle (test) section.

The drag was determined using a pitot comb in the wake, and the wind speed was obtained from a pitot-static tube placed half-way along the plate and half-way between it and one of the tunnel walls. All the pressures were recorded on a multitube alcohol manometer whose inclination was adjusted to suit each wind speed.

The intensity of turbulence in this tunnel (longitudinal component) increases from 0·10 per cent at 60 ft/sec to 0·16 per cent at 240 ft/sec. These levels of turbulence would not be expected to affect surface-friction drag, particularly with the boundary layer turbulent right from the leading edge.

2.2. *Range of Tests.* Except for one measurement with natural transition, all the tests were made with a transition wire on each surface $\frac{1}{2}$ in. from the leading edge. The wire diameter was chosen on the basis of the criterion given in Ref. 2, and the china-clay technique was used to confirm that transition did in fact occur at the wire. The wake-traverse measurements were made with the comb 2 in. downstream of the trailing edge and at 2 ft from the trailing edge. At some wind speeds observations were taken with more than one wire size.

The range of wind speed extended from 60 ft/sec to 210 ft/sec, giving Reynolds numbers (based on chord) from 4×10^6 to 13×10^6.

Checks at both 100 ft/sec and at 200 ft/sec showed that there was no detectable difference in the readings when the wake comb was moved up $10\frac{1}{2}$ in. so as to be in line with the straps which fastened the middle 2 ft of span to the outer portions. That there was no measurable spanwise variation over this distance confirmed that there were no appreciable 'edge effects' from the ends of the 2 ft of test section and no interference from the bracing wires, and supports the contention that the flow conditions in the experiment were as nearly two-dimensional as it was possible to make them.

2.3. *Determination of Overall Surface Friction.* The profile drag was calculated from the Jones formula[3]

$$C_D = \int 2\sqrt{(g - p)}(1 - \sqrt{g})\, d(y/c)\,, \tag{2}$$

where

$$g = 1 - \frac{H_0 - H}{H_0 - P_0}\,,$$

$$p = \frac{P - P_0}{H_0 - P_0}\,.$$

H and P denote respectively the total pressure and static pressure in the wake, y represents distance across stream, c is the chord, and H_0 and P_0 refer to the undisturbed flow. The usual correction[4] was applied for the displacement of the effective centre of the Pitot tubes of the wake comb. $(H_0 - H)$ was given by the pitot comb itself, as its outermost tubes were well outside the wake; $(H_0 - P_0)$ was given by the reference pitot-static tube placed half-way along the plate and half-way between it and one of the tunnel walls; and $(P - P_0)$ was obtained by subtracting from the reading of the static side of this instrument the readings of static pressure tubes incorporated in the pitot comb. This procedure uses the observations to give directly the best possible estimate of what the drag would have been in free-air conditions. It does not involve any estimate of the actual drag in the constrained flow in the tunnel, and it allows for blockage effects automatically. No correction is needed for 'horizontal buoyancy' drag (due to longitudinal static pressure gradient) as this is not included in the analysis which leads to equation (2). The only source of uncertainty in the application of this formula is that arising from using for P_0 the value opposite the centre of the plate, but the error due to this can only be small. However, the values obtained from the farther downstream position of the wake comb are to be preferred to those obtained closer to the trailing edge, as the value of p decreases with distance downstream and varies less across the wake.

The normal-pressure drag was estimated from pressure-plotting measurements on the wedge-shaped nose and tail portions of the model. It amounted to about 2 per cent of the drag given by equation (2). The surface-friction drag C_f for either surface of the plate was therefore taken to be half that given by equation (2) after being reduced by 0·00014 to allow for the normal-pressure drag.

Reynolds numbers were calculated from the pitot-static readings opposite the centre of the plate and the value of air density appropriate to the temperature and pressure measured in the working-section of the tunnel.

2.4. *Results for Overall Surface Friction.* The values of C_f are set out in the following Table and are shown plotted in Fig. 2.

Measured Values of Surface-Friction Coefficients

Comb position:		2 in. from T.E.		2 ft from T.E.	
Nominal wind-speed (ft/sec)	Wire diameter (in.)	$R \times 10^{-6}$	$C_f \times 10^3$	$R \times 10^{-6}$	$C_f \times 10^3$
60	0·020	3·85	3·34	3·96*	3·43*
100	None†	6·4	[2·63]		
	0·012	6·44	3·08	6·44*	3·12*
	0·020	6·44	3·09	6·45	3·14
110	0·012			6·94*	3·09*
120	0·012	7·75	2·98	7·56	3·07
130	0·012			8·14*	3·01*
150	0·012	9·69	2·91	9·38*	2·95*
180	0·0065			11·32*	2·86*
	0·012	11·53	2·83	11·18	2·87
200	0·0065			12·54*	2·85*
	0·012	12·59	2·77	12·55	2·84
	0·020	12·83	2·80	12·61	2·86
210	0·012	13·28	2·80	13·05	2·79

* Preferred values.

† *i.e.*, natural transition. This was about 26 in. from the leading edge.

(*a*) *Comparison with Previous Determinations.* The new results are seen to lie close to the well-known Schoenherr curve. For ease of subsequent numerical computation, however, it was convenient to represent them approximately by an equation of the same form as the Prandtl-Schlichting equation, but with the constant factor (0·455) reduced to 0·442.

(*b*) *Effect of Comb Position.* The values of C_f obtained with the comb 2 ft from the trailing edge are consistently about $1\frac{1}{2}$ per cent higher than at 2 in. from the trailing edge and are believed to be the more reliable set of readings.

(*c*) *Effect of Wire Size.* The results for the three wire sizes show that the greatest drag reduction due to reducing the wire diameter was considerably less than 1 per cent, and the contribution of the wire to the measured drag is believed to be negligible.

(*d*) *Preferred Values.* For the reasons already indicated, the most reliable results are believed to be those obtained with the farther downstream position of the comb and with the smallest wire size which precipitates transition at each wind speed. These particular determinations are marked with an asterisk in the Table and are indicated by the letter P in Fig. 2; their probable error is believed to be less than 1 per cent. They lie close to the Schoenherr curve.

(*e*) *Natural Transition.* Lastly, the one measurement with natural transition gives a value of C_f which is almost exactly that calculated by the method of Squire and Young[5], using the observed position of transition.

2.5. *Values Deduced for Local Surface Friction.* As stated above, the overall surface-friction coefficient (C_f) could be represented approximately by an equation of the Prandtl-Schlichting form:

$$C_f = A\,(\log_{10} R_x)^{-2\cdot58}\,. \tag{3}$$

From this formula and the relation

$$c_f = \frac{d}{dx}(xC_f)\,,$$

the local friction coefficient c_f is found to be

$$c_f = C_f\left(1 - \frac{1\cdot12}{\log_{10} R_x}\right). \tag{4}$$

2.6. *Preston Tubes in Turbulent Boundary Layers.* It can easily be shown that Preston's two arguments $\tau_0 d^2/4\rho\nu^2$ and $(P - p)d^2/4\rho\nu^2$ in equation (1) are the same as $c_f R^2/8$ and $c_p R^2/8$ where c_p is the pitot pressure difference divided by $\frac{1}{2}\rho U^2$, and R is a Reynolds number based on the pitot diameter and the free-stream speed U. It is also convenient to re-write equation (1) in the alternative form

$$c_f = 0\cdot0521 c_p^{\,7/8} R^{\,-1/4}\,. \tag{1a}$$

For most of the work on Preston tubes in the 13 ft $\times$ 9 ft Wind Tunnel, two sizes of pitot were used, of diameter 0·125 in. and 0·0483 in., with a few observations using a 0·0364-in. tube, chiefly for the laminar-flow case. The tests covered positions on the plate ranging from 1 ft to 9 ft back and a speed range of 100 to 200 ft/sec. The Reynolds number (R_x) was thus varied from about 6×10^5 to just over 10^7. The readings were taken on an inclined tube multimanometer, on which were included the two sides of a standard pitot-static tube, placed half-way along the flat plate and half-way between it and one of the tunnel walls. The quantity c_p could therefore be obtained directly as the ratio of two lengths measured on the manometer, and the actual speed of the tunnel only entered in the determination of R_x and thence c_f. The static pressure at the datum hole in the tunnel wall and the total pressure well upstream were also recorded.

Turbulence was stimulated by wires 0·012 in. in diameter placed half an inch from the leading edge of the plate. It had been established in the pitot traverse drag experiments, by the china-clay technique, that such a wire was effective at speeds of 100 ft/sec and above, and that at 200 ft/sec the reduction of the wire diameter to 0·0065 in. produced a negligible drag reduction.

As it was desired not to deface the plate in any way the static pressure on the surface was first obtained by means of two very small static tubes placed on the surface, one on each side of the Preston tube. It soon became evident that this process was not entirely satisfactory, as there were small unaccountable errors in the readings. The two tubes did not give identical results when used at the same place, nor could the results be exactly repeated if a tube were removed and replaced. Accordingly, after most of the observations had been taken, static holes were drilled in the plate along the median line where the Preston tube had been and a series of readings taken. The inclusion of the 'speed' pitot-static and the hole in the tunnel wall in all the observations made it easy to relate these static hole readings to the earlier experiments with the Preston tubes. The results of this part of the investigation are exhibited in Figs. 3 and 4. In Fig. 3 the common logarithms of the two relevant parameters are plotted against each other and the points fall well on a line parallel to that found by Preston, but decidedly above it; the equation of this line is:

$$\log_{10}\frac{\tau_0 d^2}{4\rho\nu^2} = \bar{2}\cdot 647 + \frac{7}{8}\log_{10}\frac{(P-p)d^2}{4\rho\nu^2} \tag{5}$$

or

$$c_f = 0\cdot 0576\, c_p{}^{7/8}R^{-1/4}\,. \tag{5a}$$

The points at 9 ft back, and to a much lesser degree, those at 7 ft back, fall slightly below the line. No explanation of this is offered, but it was noticed that if the static pressure on the plate is plotted against distance back, the curve indicates constant static pressure up to 5 ft back and then a progressive fall, reaching about $0\cdot 03 \times \frac{1}{2}\rho U^2$ at 9 ft back. Calculations by the Squire-Young method, however, show that this pressure gradient is not large enough to account for the above effect.

The equation (5) above was used to calculate the local surface-friction coefficient c_f from the observed pitot pressure c_p, just as if the method were being used, as Preston suggested, to measure an unknown surface friction. The results are given in Fig. 4, in which the line of local surface friction deduced from equations (3) and (4), with $A = 0\cdot 442$, is added for comparison. The points of course lie evenly about this line, except those at 9 ft back, since they were used to determine equation (5), but the scatter is much better seen in this plotting than in the insensitive double logarithmic plotting of Fig. 3. The ordinate scale is a very open one and the results are surprisingly good in the range of log R_x from about $6\cdot 2$ to $6\cdot 8$. In using the method to measure an unknown surface friction (but whose order is known), one could always select a pitot diameter such that the parameter $(\tau_0 d^2)/(4\rho\nu^2)$ would lie in the range it occupies near log $R_x = 6\cdot 5$ in the above tests.

Preston was exceedingly careful to make his tubes with precisely the same ratio of internal to external diameter, and to grind the ends perfectly square and free from burrs. It was thought worth while to see whether the ratio of internal and external diameters was an important factor or not. The original pair of tubes had this ratio very close to $0\cdot 60$, as used by Preston, so two other tubes were made with the ratio approximately $0\cdot 55$ and $0\cdot 70$ and tested at 5 ft from the leading edge of the plate. The results obtained were indistinguishable from those with the $0\cdot 60$ tube, so that it would appear that the ratio of diameters is not at all critical and any value near $0\cdot 60$ will do. The effect of change of shape of the end of the tube, or of any burrs on the edge, was not examined, but might well be appreciable.

2.7. *Preston Tubes in Laminar Boundary Layers.* A few observations were taken with a smaller pitot ($0\cdot 0364$ in.) in the laminar layer present at the front part of the plate when the turbulence stimulator was removed. The extent of laminar flow was observed by the china-clay technique over a range of speeds and the results are presented in the Table below:

Speed (ft/sec)	60	100	120	150	180	200
Distance of transition from L.E. (in.)	52	30–31	26–27	20	18	13–15
Reynolds number at transition (R_x)	$1 \cdot 64 \times 10^6$	$1 \cdot 62$	$1 \cdot 68$	$1 \cdot 59$	$1 \cdot 71$	$1 \cdot 48$

The precise location of transition was not well defined but the approximate constancy of the Reynolds number at transition is gratifying. At least the observations are sufficient to show when the Preston tube would be expected to be in the laminar region and when it would not.

The results are exhibited in Fig. 5, in the same form as those of Fig. 3. It is seen that with the tube 1 ft back, where it would be expected always to be in laminar flow in the speed range 60 to 200 ft/sec, the points lie very well on a line whose slope is one half. The first three points at 2 ft back lie on a parallel line a little lower, and then diverge. At 3 ft back only the first point (60 ft/sec) is near these lines. It should be made clear that in calculating the ordinates the local laminar surface friction was used in all cases, i.e., $\frac{1}{2}C_f$.

If it is assumed that the velocity gradient in a laminar layer is constant out to beyond the diameter of the Preston tube, it is possible to deduce an approximate theory for the behaviour of the tube. Thus, if the effective centre of the tube is at a distance Kd from the surface, the velocity there will be approximately $Kd(du/dy)$.

We have the relations:

$$\mu \frac{du}{dy} = \tau_0 = \tfrac{1}{2}\rho U^2 c_f$$

$$c_p = \frac{u^2}{U^2} = \left(Kd\frac{du}{dy}\right)^2 \Big/ U^2 .$$

From these it can easily be shown that

$$\log_{10} \frac{c_f R^2}{8} = \tfrac{1}{2}\left(\log_{10} \frac{c_p R^2}{8} - \log_{10} 2K^2\right)$$

or, in Preston's notation:

$$\log_{10} \frac{\tau_0 d^2}{4\rho \nu^2} = \tfrac{1}{2}\left(\log_{10} \frac{(P - p)d^2}{4\rho\nu^2} - \log_{10} 2K^2\right)$$

or, more simply, $c_f = 2\sqrt{c_p}/KR$. If $K = \frac{1}{2}$, i.e., the effective centre is at the centre of the tube, this becomes

$$\log_{10} \frac{\tau_0 d^2}{4\rho\nu^2} = \tfrac{1}{2}\left(\log_{10} \frac{(P - p)d^2}{4\rho\nu^2} + \log_{10} 2\right)$$

as given by Preston in his paper, and shown dotted in Fig. 5. With the line found from the points at 1 ft back on the flat plate the value of $-\log_{10} 2K^2$ is about $0 \cdot 10$ and this leads to $K = 0 \cdot 63$. This may be compared with the value $0 \cdot 68$ given by Young and Maas for a tube *in a free stream* with a transverse velocity gradient. The line through the three points at 2 ft back gives $\log_{10} 2K^2 \simeq 0$, so that $K \doteq 0 \cdot 71$. It would thus appear that the displacement of the effective centre for a tube resting on the surface is at least of the same order as that found in a free stream.

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If a similar reasoning is applied to the turbulent boundary layer the result is absurd, for the gradient at the surface is so steep that, even at the centre of a pitot of this size, the velocity, if the gradient remained constant that far, would be several times the free-stream velocity.

2.8. *Observations with Free Transition.* It was thought interesting to calculate the local surface friction, assuming the boundary layer to be fully turbulent (*i.e.*, using equation (5)) at points behind the free transition front. The results at 2 ft and 3 ft from the leading edge of the plate are plotted in Figs. 6a and 6b together with two curves marked A and B. The curve A is the local surface friction obtained from equations (3) and (4) with $A = 0 \cdot 442$ assuming that turbulence starts at the leading edge of the plate, the assumption that Prandtl made when he first considered transition problems[6]. The curve B is obtained by supposing the turbulence to start at the transition point as though this were the leading edge, that is, neglecting entirely the presence of the laminar part in front of the transition point. It will be seen that when the Preston tube is well behind the transition front, the value of c_f obtained agrees closely with curve B, and that only when the tube is close behind the transition point does one get a lower value, as would be expected, because the tube is then in the region of change from laminar to turbulent conditions. When this change is complete it appears that the conditions in the turbulent layer are closely those that would be found on a plate with a completely turbulent boundary layer and with its leading edge at the transition point and not those found by assuming that turbulent flow starts at the actual leading edge. A similar conclusion is reached if the local surface friction behind some assumed transition point is calculated by Squire and Young's method[5].

2.9. *Conclusions from* 13 *ft* $\times$ 9 *ft Wind-Tunnel Experiments.* It is concluded from the experiments described above that there is a relation for turbulent flow on a flat plate which has the same order of accuracy as that found by Preston in a pipe, but which is not identical with his relation. There is no apparent reason why it should be identical, for the conditions are different and this difference must be reflected in the boundary-layer velocity profiles. In the pipe there is no change of friction and none in boundary-layer thickness as one moves along the pipe, whereas on the flat plate both these quantities change with distance along the plate. It would appear, therefore, that if the Preston-tube method is used to measure turbulent surface friction it would be better to use the relation (5) found in this report if the flow in question is expected to resemble that on a flat plate more closely than that in a pipe. Equations (1a) and (5a), which are alternative forms of equations (1) and (5), show at once that for a given c_p reading the flat-plate results give a surface friction some 11 per cent higher than that given by Preston's pipe formula.

The tests in a laminar layer confirm well the slope of one-half deduced on the assumption that the velocity gradient is constant over the diameter of the Pitot tube, and suggest that the effective centre of the tube lies between $0 \cdot 6$ and $0 \cdot 7$ of the diameter from the point of contact. It is not so easy to get values in the transition region on the flat plate, for there is no way of finding the local surface friction, as there is in a pipe, unless the force on an element of the surface were measured. In the turbulent region well behind natural transition, the Preston tube readings indicate that the conditions are closely those that would occur if the part of the plate in front of the transition point were not there at all.

3. *Experiments in* 9 *ft* $\times$ 7 *ft Tunnel.* Because it was not immediately clear why the pipe and flat-plate Preston-tube calibrations should differ so widely, further experiments, using another method of surface-friction measurement, were planned. It was suggested by Fage that a comparison

should be made between the readings of Preston tubes and Stanton tubes mounted in a circular pipe and on a flat plate. Stanton tubes, very small flat Pitot tubes mounted on the surface, had previously been used by Stanton, Marshall and Bryant[7] and Fage and Falkner[8]. It was thought that comparison of the correlations of Preston-tube and Stanton-tube readings in the pipe and on the plate would provide evidence concerning the differing calibrations found in the earlier experiments, without the controversy attendant upon an actual determination of surface friction. At an early stage it was decided to substitute a two-dimensional channel for the circular pipe, so that both working surfaces should be flat, a great experimental advantage. It was felt that the closest comparison between plate and channel flow would be obtained if the height of the channel were about twice the thickness of the boundary layer on the plate, so that the two velocity profiles should be nearly congruent: unfortunately the mass flow of the driving pumps available restricted the possible duct aspect ratio to only 6:1. This was not sufficient to obtain truly two-dimensional flow (*see* Fig. 9), and accordingly it was decided to use the Stanton tubes as surface-friction measuring instruments as was done in Ref. 8, calibrating them in turbulent flow in the rectangular duct. This method of using small pitots to calibrate larger ones does not beg the question, as it is well established that the slope of a friction-layer velocity profile at the surface is related to the wall shear stress by $(\partial u/\partial y)_0 = \tau_0/\mu$, quite irrespective of any universal relations that may exist outside the linear layer.

3.1. *Apparatus.* The rectangular duct is shown in Fig. 7. A 2-mm diameter trip wire was fitted inside the bellmouth, to ensure that the channel flow was fully developed by the end of the entry length, which was 25 ft (150 channel heights) long. The Preston and Stanton tubes, and also pitot traverse gear, were mounted on polished brass discs, which fitted into the cast-iron floor of the working section. The section of the floor carrying the discs could be traversed laterally, and the discs could be yawed through $\pm$ 20 deg. The Stanton tubes could also be mounted on $\frac{1}{2}$ in. diameter plugs fitting either into holes cut in the side wall of the channel or into one of the brass discs. Thus the distribution of surface friction round the duct perimeter could be investigated.

The flat plate used in the 9 ft $\times$ 7 ft Tunnel spanned the 7-ft dimension of the tunnel and was 6 ft in chord, and was fitted with a trip wire of approximately 0·02 in. diameter $\frac{1}{2}$ in. from the leading edge. It was verified by the china-clay method and by the use of a stethoscope that the boundary layer remained entirely turbulent down to the lowest tunnel speed used. The brass discs carrying the instruments fitted into a steel plate inserted into the plywood plate so that the instruments stood 5 ft from the leading edge, which was taken as the origin of the boundary layer for calculations of R_x. The Preston tubes, and, more particularly, the delicate Stanton tubes could therefore be moved from duct to plate while mounted permanently on the surface of the discs, thus avoiding difficulties in setting up the instruments. The longitudinal component of turbulent intensity in this tunnel is about 0·25 per cent.

Three Preston tubes were used, having diameters of 0·047 in., $0{\cdot}082_5$ in. and 0·124 in. The ratios of internal to external diameter were all about 0·6, but in view of the conclusions mentioned above, that small deviations from the ratio of 0·600 used by Preston did not alter the calibration, these ratios were not checked.

Several Stanton tubes were used. The earlier specimens were steel wedges about 0·2 in. square, with a height at the rear up to 0·03 in., screwed to the surface. The gaps were not measured but were nominally less than 0·005 in. Later, pieces of razor blades were soldered to the surface, giving gaps of about 0·003 to 0·004 in. The results from the various tubes agreed to within the likely accuracy of measurement.

The Stanton-tube pressure differences, which ranged up to 0·5 in. water, and the Preston-tube pressures which ranged up to 3 in. water, were measured on an inclined U-tube alcohol manometer. Some runs were made using a Chattock gauge to measure the Stanton-tube pressures, and these agreed with the other runs, with little apparent difference in accuracy.

The Stanton-tube calibrations were found to be sensitive to dust, and changed slightly from time to time, so that in all cases the runs in the tunnel were made immediately before, or after, the runs in the duct, and care was taken in checking readings to ensure that the tubes did not become partially blocked during a run: some runs were discarded when this had obviously happened. The need to keep the time for a run as short as possible was the chief reason for using a U-tube instead of the, nominally, more sensitive Chattock gauge.

3.2. *Results.* The variation of local surface-friction coefficient c_f with Reynolds number $R_x = U_x/\nu$ is shown in Fig. 8, together with a set, chosen at random, of Stanton-tube observations and a curve showing the change in c_f which would be produced by a change of 0·0025 in. water (0·01 in. on the inclined manometer) in the Stanton-tube pressure, this being the accuracy to which the readings were recorded. Owing to the large range of shear stress which had to be covered, the results at the lower Reynolds numbers show considerable scatter (the two ringed points almost certainly resulted from a mis-reading of the U-tube). In order to avoid this scatter, it would be necessary to use Stanton tubes of larger size for the lower speeds, which would be permissible as the sub-layer then becomes thicker. Alternatively, R_x could be altered by changing x instead of U, but this was not practicable in the present experiment. The discrepancy of about 3 per cent between these surface friction results and those deduced from the wake traverse measurements is not to be regarded seriously. It might be a genuine consequence of leading-edge conditions altering the origin of the boundary layer, but it is certainly within the likely error of either experiment.

Further details of the calibration and use of Stanton tubes to measure surface friction are given in Ref. 9, but it may be of interest to note, in Fig. 9, the distribution of surface friction round the perimeter of the rectangular duct. This was found by placing the Stanton tubes at various points on the walls and floor as previously described. It is seen that the surface friction at the mid-point of the floor is greater (by about 14 per cent) than the average value corresponding to the observed pressure gradient. Clearly, the Stanton tube measures local and not average surface friction.

The Preston-tube calibration is shown in Fig. 10. It agrees quite closely with the values given by equation (5), except that there are slight discrepancies at the lower tunnel speeds, probably due to the inaccuracy of c_f measurement at these speeds. It was noted, however, that the position of effective centre of the Preston tubes varied slightly with tunnel speed (*see* Fig. 11), and variations were also discerned in Fig. 3, so that these results may be genuine. The present experiment did not detect any consistent variation of the semi-logarithmic law with tunnel speed, so that the effects if real are probably instrumental in origin. The changes in effective centre position are plotted in Fig. 11. It may be noted that the considerable scatter corresponds to errors in measurement of distance from surface of only one or two thousandths of an inch. The effective centres are slightly nearer the surface than the 0·62 × diameter suggested by Preston for pipe flow: a check on one of Preston's readings for the wind tunnel gives a value of only $0 \cdot 5d$.

One of the velocity traverses (that at the lowest tunnel speed) from which the tube effective centres were found is shown in Fig. 12. The semi-logarithmic law derived from these profiles and the surface-friction measurements is approximately

$$\frac{u}{u_\tau} = 4\cdot 9 \log_{10} \frac{u_\tau y}{\nu} + 5\cdot 9$$

and this law, together with Preston's law for the circular pipe,

$$\frac{u}{u_\tau} = 5\cdot 5 \log_{10} \frac{u_\tau y}{\nu} + 5\cdot 8$$

is also shown in Fig. 12. The rate of growth of momentum thickness obtained from these profiles indicated an average c_f of $0\cdot 00298$ when substituted in the von Kármán momentum integral equation (neglecting turbulent terms) and this agrees very well with the directly measured values. The same apparatus was later used in the 13 ft × 9 ft Wind Tunnel and the results were accurately repeated.

The graph of Fig. 13 shows a comparison between the yaw sensitivity of a Preston tube, and of a Pitot tube in a free stream. Though the sensitivity will depend on the length/diameter ratio (about 8:1 in the case shown) it seems that the Preston tube is a little more sensitive to yaw than an ordinary Pitot tube, and so care should be taken to align the Preston tube along the flow direction to within ± 5 deg, say. As the calibration is restricted to two-dimensional flow this should be easy enough.

3.3. *Conclusions from 9 ft × 7 ft Tunnel Experiments.* These experiments on a flat plate give the relation

$$\log_{10} \frac{\tau_0 d^2}{4\rho\nu^2} = \bar{2}\cdot 650 + 0\cdot 875 \log_{10} \frac{(P - p)d^2}{4\rho\nu^2}$$

for the calibration of circular Preston tubes with a ratio of internal to external diameter near $0\cdot 6$ in the range $1\cdot 7 \times 10^6 < R_x < 6 \times 10^6$. This is practically the same as equation (5), thus confirming the conclusions drawn from the wake-traverse measurements, and incidentally justifying the method of finding local c_f by wake traverse and differentiation.

The incidental measurements of local surface friction by the Stanton tube agree quite well with the Prandtl-Schlichting law in the range of Reynolds number tested. The boundary-layer velocity profiles, together with these surface-friction measurements, indicate a semi-logarithmic inner law

$$\frac{u}{u_\tau} = 4\cdot 9 \log_{10} \frac{u_\tau y}{\nu} + 5\cdot 9$$

which is tentatively suggested as typical of flat-plate boundary layers. The difference between the Preston-tube calibrations for a pipe and a plate can be explained by the difference between this law and the law

$$\frac{u}{u_\tau} = 5\cdot 5 \log_{10} \frac{u_\tau y}{\nu} + 5\cdot 8$$

for a circular pipe, and possibly by a slight shift in the position of the 'effective centre' of the tubes. The velocity profiles and surface-friction measurements satisfy the von Kármán momentum integral equations, providing a further check on the accuracy of the observations.

4. *Discussion.* Although these results apparently establish the difference in inner law, and therefore in Preston-tube calibration, between circular pipe and flat plate, the reasons are not entirely clear.

The 'inner law' of turbulent flow near a wall is derived from the hypothesis that near the wall

$$\frac{u}{u_\tau} = f_1\left(\frac{u_\tau y}{\nu}\right), \frac{\tau}{\tau_0} = g_1\left(\frac{u_\tau y}{\nu}\right)$$

and that near the outer edge of the layer

$$\frac{U - u}{u_\tau} = f_2\left(\frac{y}{\delta}\right), \frac{\tau}{\tau_0} = g_2\left(\frac{y}{\delta}\right).$$

If these relations overlap in some part of the flow, then it may be shown that the functions f and g must be either constants or logarithms in this overlap region. Experiment shows that

$$\frac{u}{u_\tau} = A \log \frac{u_\tau y}{\nu} + B \text{ and } \frac{\tau}{\tau_0} = 1,$$

but there is still some doubt about the value of the constants. It has been said that A at least should be a universal constant for all wall flows, but it is clear that the outer flow differs considerably from case to case, and it is therefore reasonable to expect the possibility of differences in the overlap region, even though all flows are undoubtedly similar very near the walls, so that the 'universal' logarithmic profile may be only a first approximation, differing slightly between various flows.

The differences between pipe and boundary-layer flow may be set out as follows:

(1) Intermittent nature of turbulent flow in the outer part of the boundary layer, as opposed to the pipe in which the flow is continuously turbulent. Schubauer and others have shown that the fraction of the time for which the flow is turbulent changes, in a Gaussian curve, from unity at $0{\cdot}4\delta$ to zero at $1{\cdot}2\delta$. δ is the distance at which $u = U_\infty$. This region does not include the inner-law layer, and therefore the effect on the inner law must be a secondary one caused by differences in the outer law. It seems most likely that this feature is responsible for most of the apparent difference between the inner laws.

(2) Growth of boundary layer, leading to non-zero mean velocities normal to the surface. This might show up as a dependence of inner law on pressure gradient. No conclusive evidence of this has yet been found. It is also possible that a vertical component of velocity ahead of the Preston tube might be responsible for the observed differences in effective centre position.

(3) Effect of pressure gradient in pipe. It is possible that this might lead to differences in Preston-tube calibrations owing to the changes of pressure in the stagnation region just ahead of the tube, but this is unlikely if $dp/dx \, . \, d/(P - p)$ is small, which it is in all practical cases.

(4) Effect of spanwise surface curvature. This might well affect the mean velocity profile if Townsend's suggestion of the control of the flow by large eddies is correct, and it was hoped to check this point in two-dimensional channel flow: this was not possible with the low-aspect-ratio channel. Any direct effect of curvature on the Preston tubes would show up as a dependence of calibration on d/r and has not been noticed.

(5) There is, of course, the possibility that the turbulent boundary layers used in the N.P.L. experiments were not in equilibrium. Trip wires were used near the leading edges of both plates to precipitate transition, but unless a trip wire is grossly oversize the distance, measured in multiples of the boundary-layer thickness, required to reach equilibrium should not be of a greater order than in the case of natural transition farther downstream, and in the latter case the effective boundary-layer thickness is much greater, so that Ul/ν, where l is the length of the transition region, may well

be much less. In any case, if the boundary layer is not in equilibrium after a delay represented by $R_x = 6 \times 10^6$ it is unlikely that experimental methods based on any assumption of equilibrium will be of use in practice.

The possibility of an almost identical 11 to 12 per cent error in c_f in both sets of experiments is not a very likely one, though observations of surface friction are notoriously difficult. It should be emphasised that both $c_f - R_x$ curves are in the range given by other experiments, though admittedly near the upper boundary, and that very few experimental observations lie outside a region of width about 7 per cent of c_f in the interval $10^6 < R_x < 10^7$, so that the use of any one of the well-known c_f laws, instead of a direct measurement of c_f, would still have shown a difference between the pipe and plate calibrations, though not necessarily such a large one.

The present position with regard to Preston tubes and turbulent surface friction therefore seems to be that Preston's original suggestion that surface pitots could be used for local surface-friction measurements is amply confirmed, but that the calibrations for pipe and boundary-layer flow differ by, at the very least, 5 per cent (taking the lower boundary of the c_f curves mentioned above as correct) and almost certainly by about 11 per cent as shown in these experiments. An extension to boundary layers in a pressure gradient has not yet been made, but it is hoped to do this shortly with the aid of the Stanton-tube method. In the meantime it is reasonable to assume that the present calibration holds without too much error in all two-dimensional cases not too near separations.

A full explanation of the reasons for this difference in calibration, and the apparent differences in velocity profile, is not yet available, and must await further investigation of the whole turbulent boundary-layer problem. The conclusions of this report, being based entirely on experimental observation, are incomplete in the sense that all empirical knowledge is incomplete, but they provide justification for the wide application of Preston's simple and convenient method of measuring turbulent surface friction.

Note. After the preparation of this report, details were published in Ref. 10 of an American calibration of Preston tubes. The calibration

$$\log_{10} \frac{\tau_0 d^2}{4\rho\nu^2} = \bar{2}\cdot634 + 0\cdot877 \log_{10} \frac{(P-p)d^2}{4\rho\nu^2}$$

agrees with that of the present report to within about 1 per cent, and the present data are fitted slightly better by the American calibration in which the slope of the line, as well as the intercept, has been changed from the value found by Preston in a pipe.

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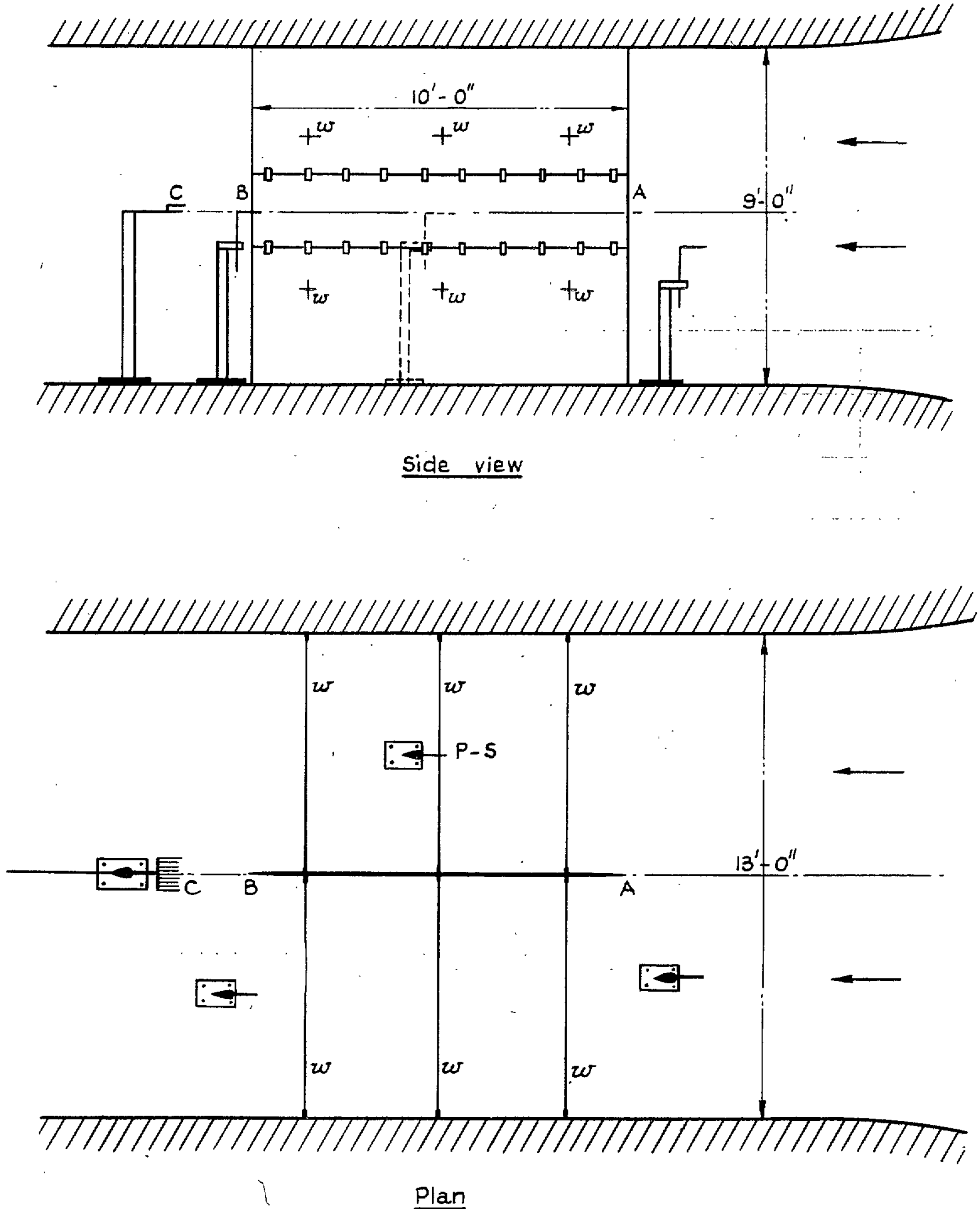

AB: plate

C: Pitot comb

w: bracing wires for plate

P-S: reference Pitot-static tube. Auxiliary tubes also shown.

FIG. 1. Arrangement of plate in the 13 ft × 9 ft Wind Tunnel.

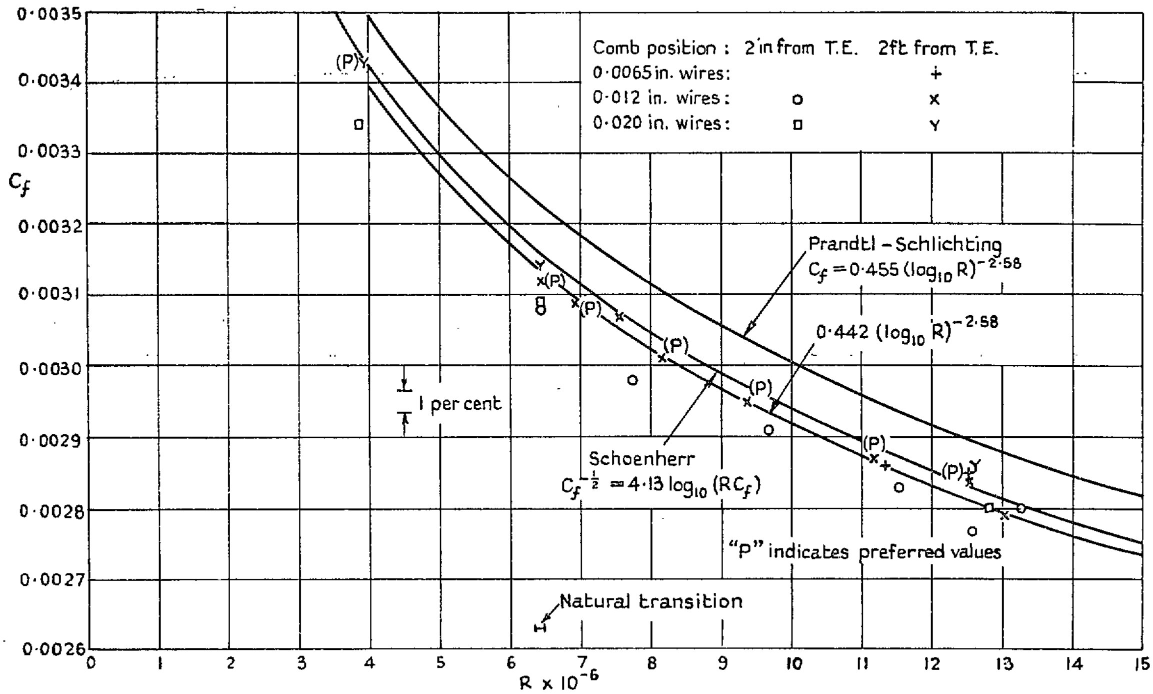

FIG. 2. Comparison of results with Schoenherr and Prandtl-Schlichting curves.

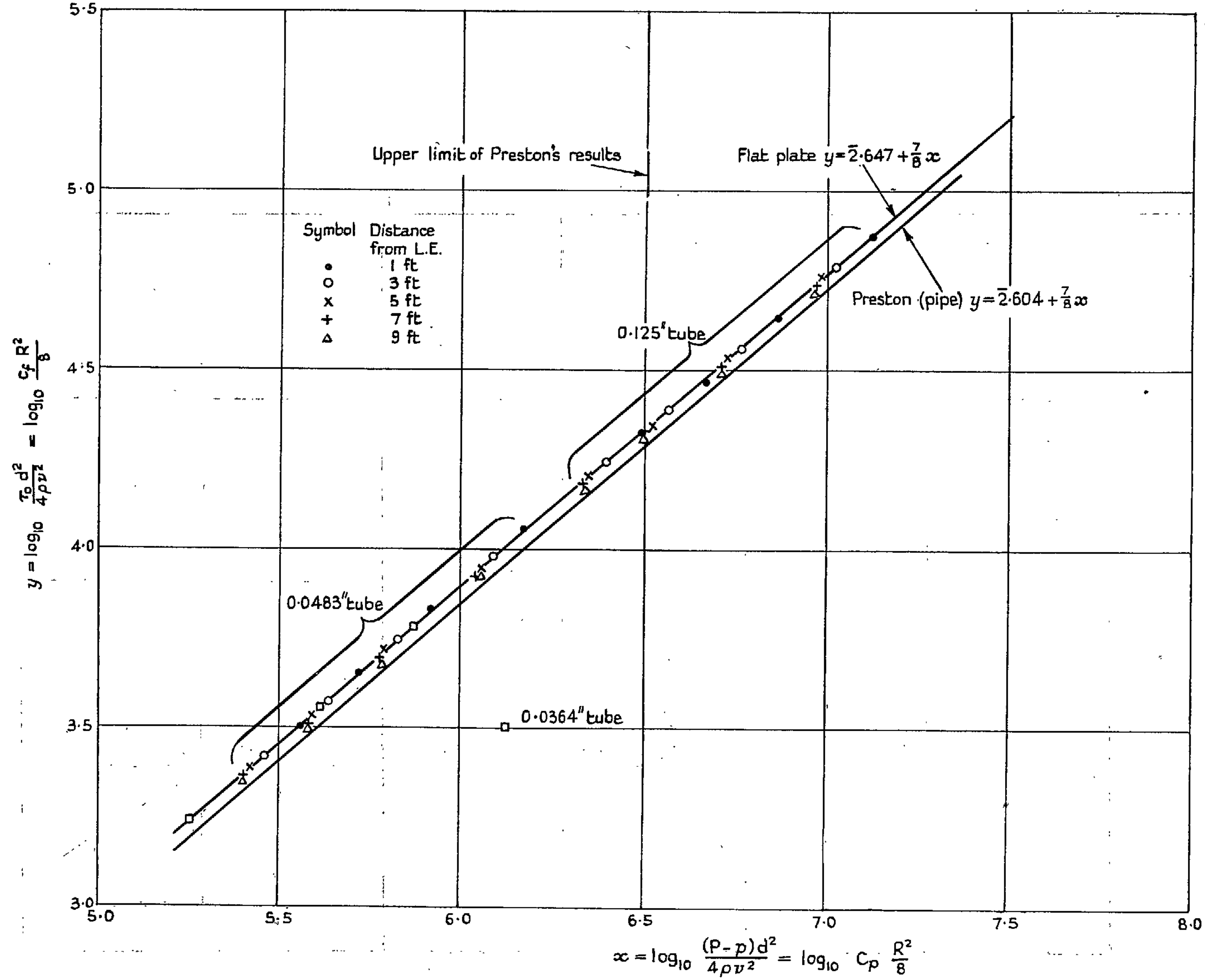

FIG. 3. Turbulent flow: Relation between the shear stress at the surface and the pressure difference recorded by the Preston tube.

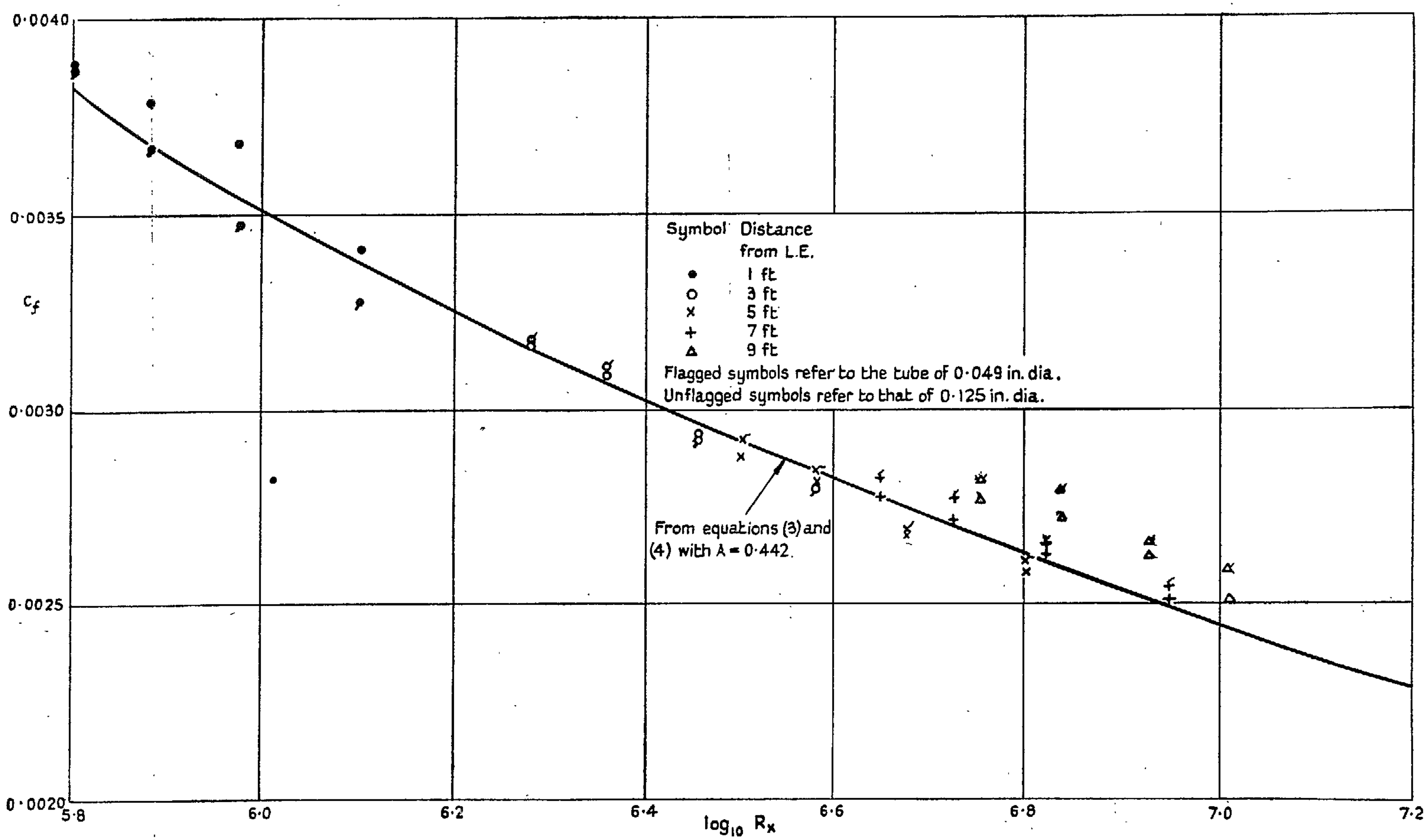

FIG. 4. Local turbulent surface friction as a function of Reynolds number.

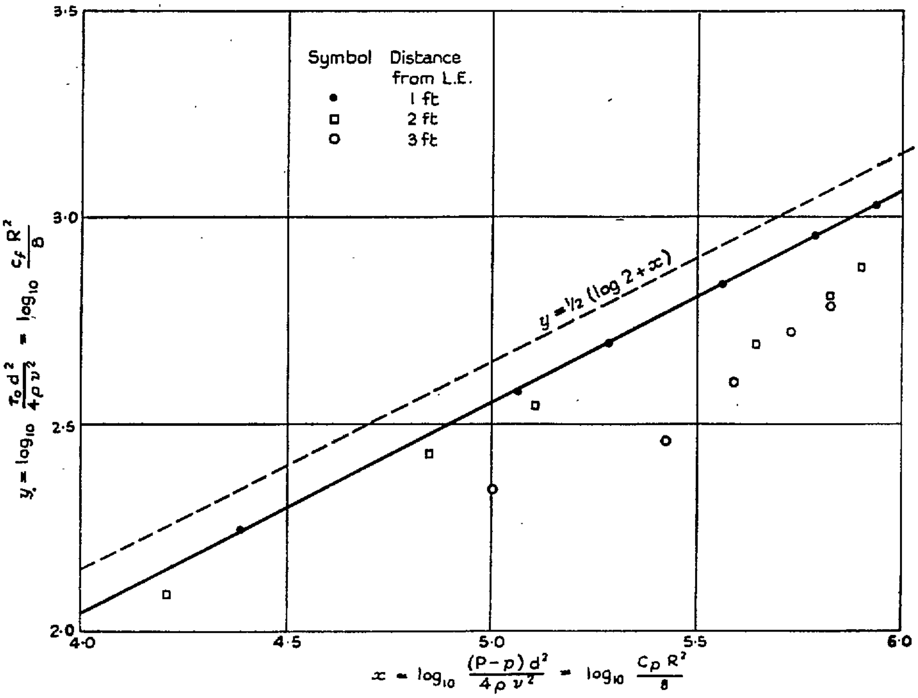

FIG. 5. Results with laminar flow.

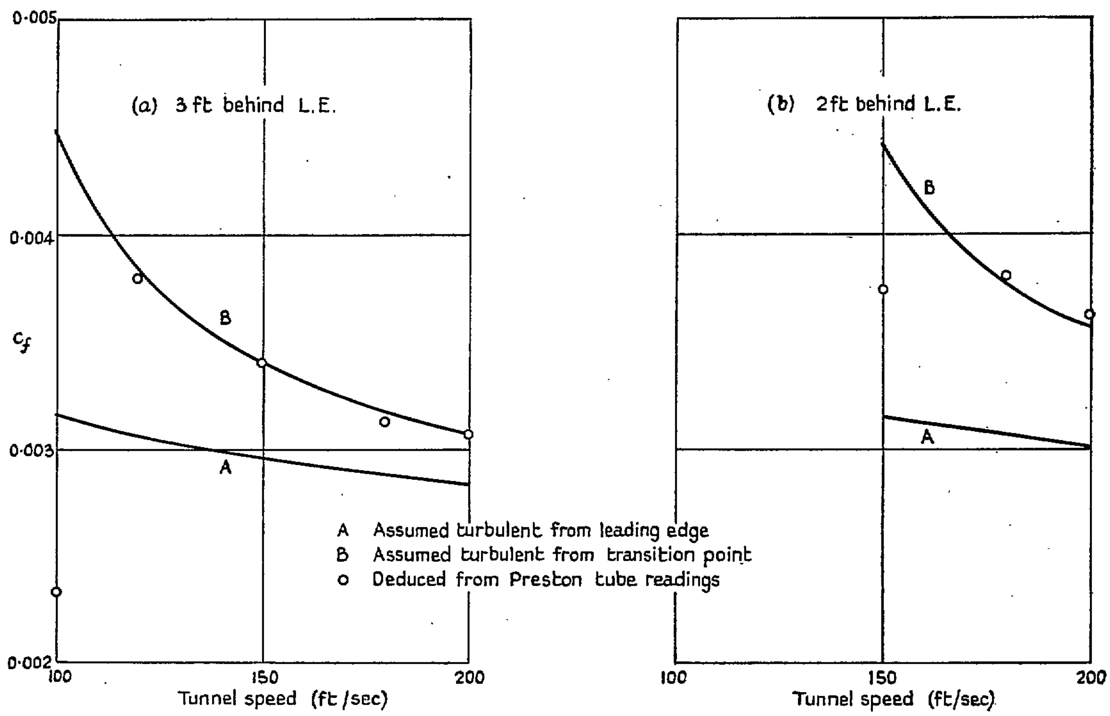

FIG. 6. Results with natural transition.

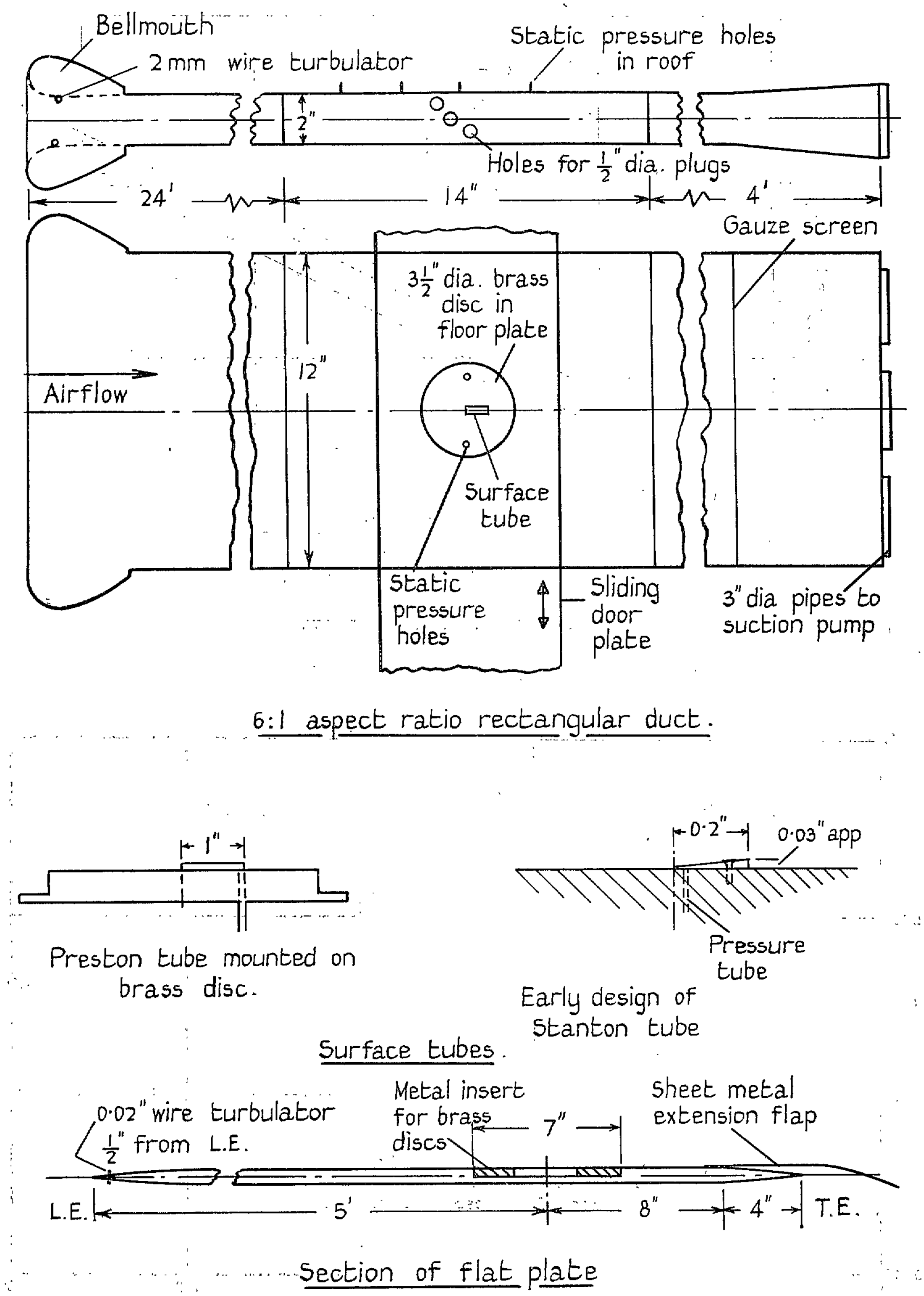

FIG. 7. Details of apparatus used in 9 ft × 7 ft wind tunnel.

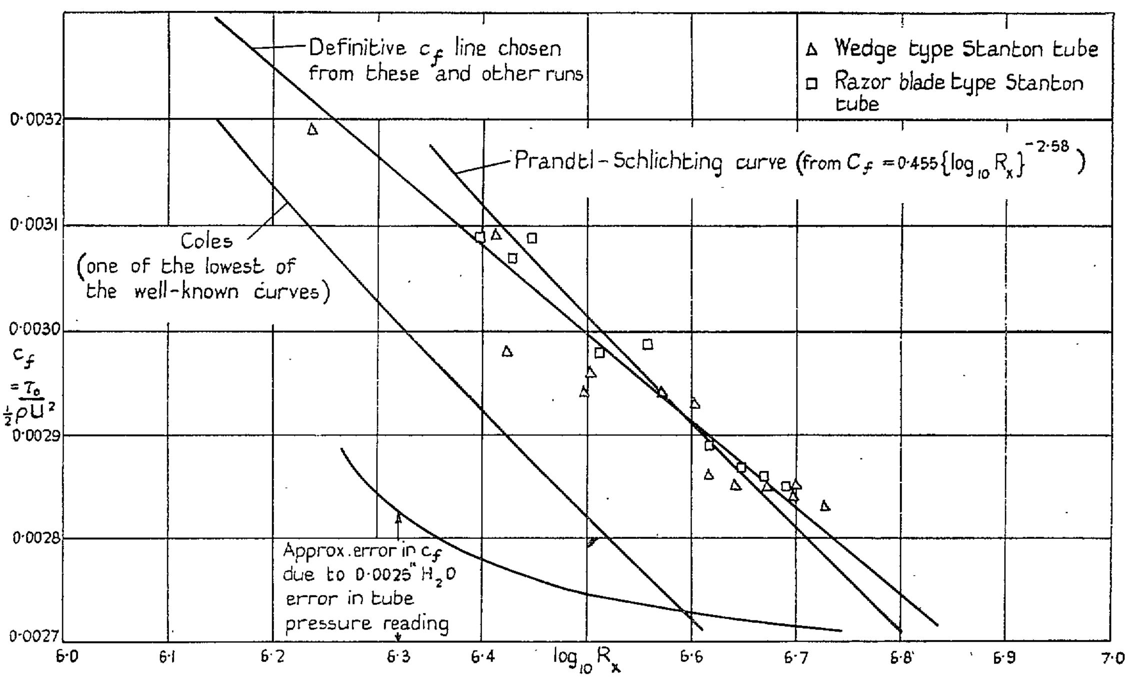

FIG. 8. Variation of local surface-friction coefficient with Reynolds number.

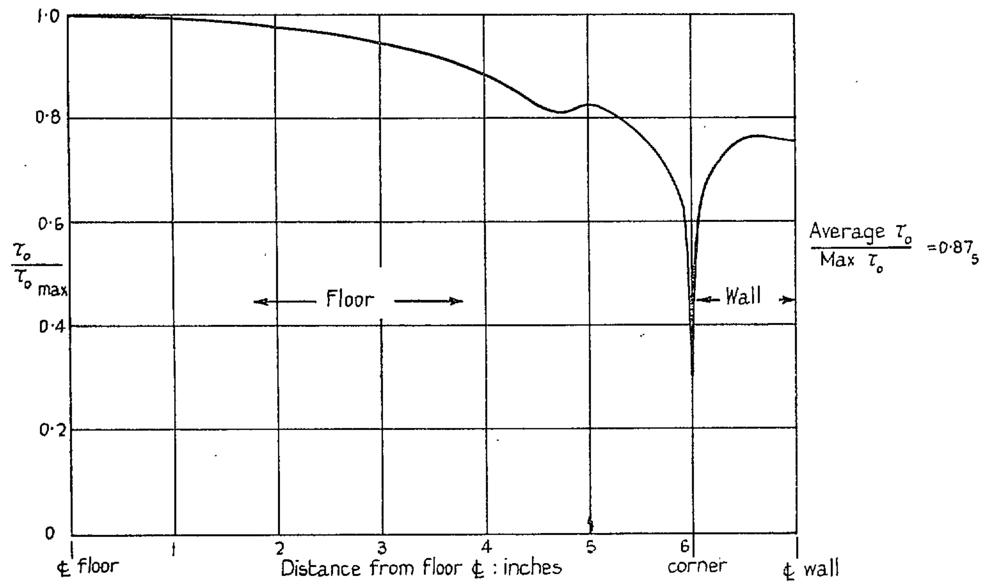

FIG. 9. Variation of surface friction round duct perimeter.

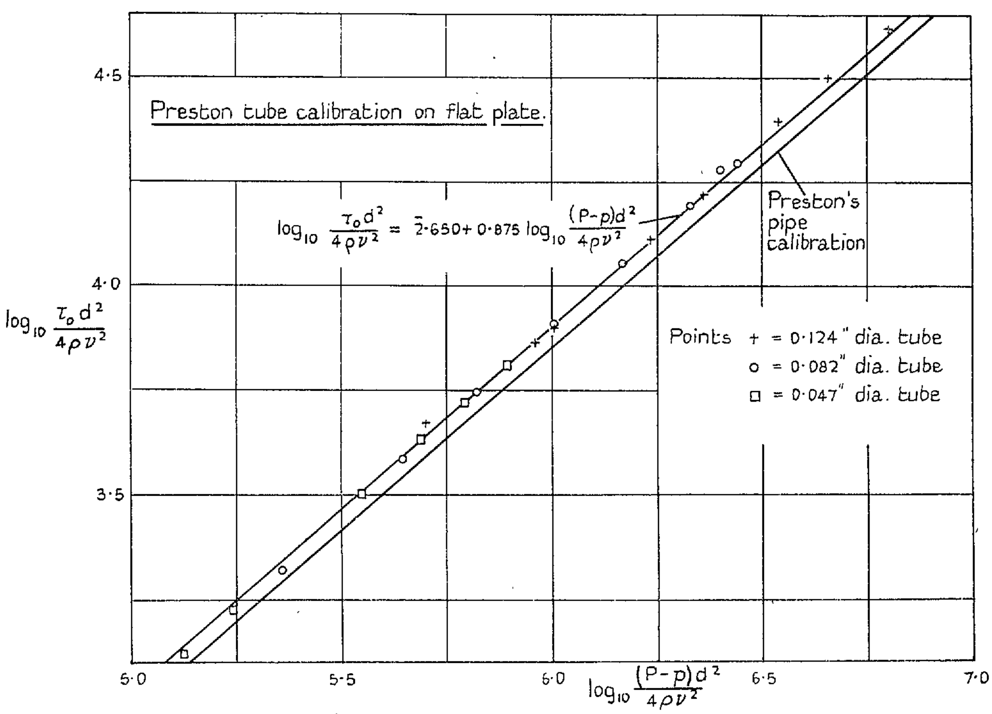

FIG. 10. Preston-tube calibrations.

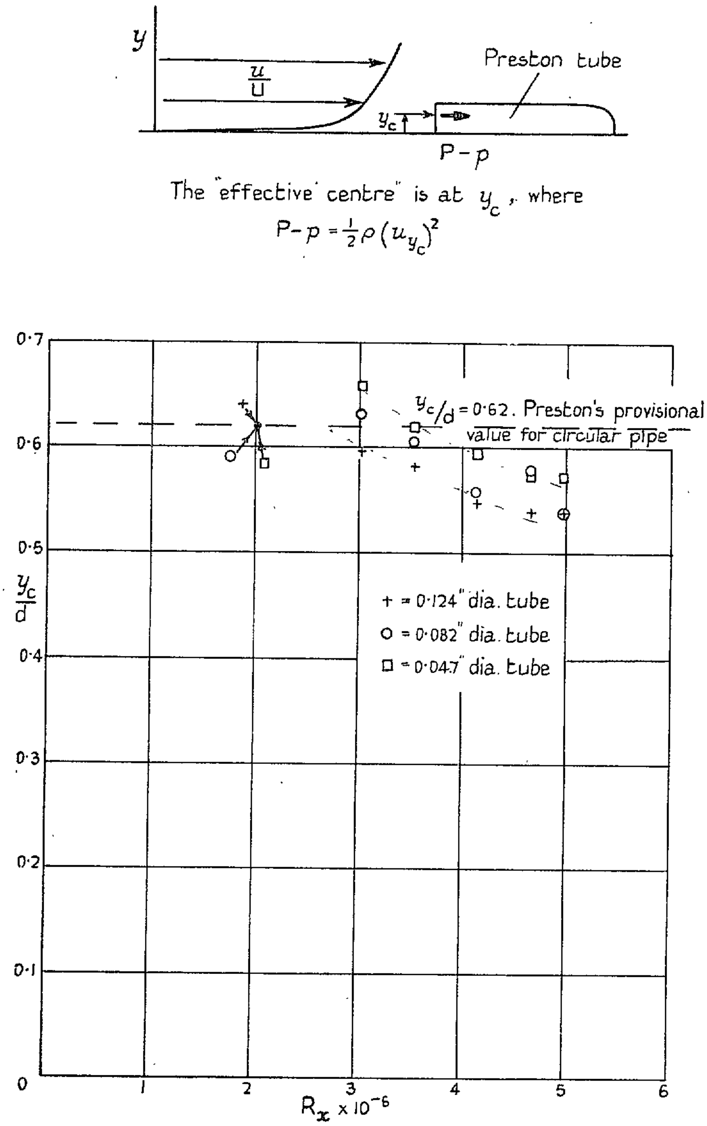

FIG. 11. Heights of Preston-tube 'effective centre' above surface.

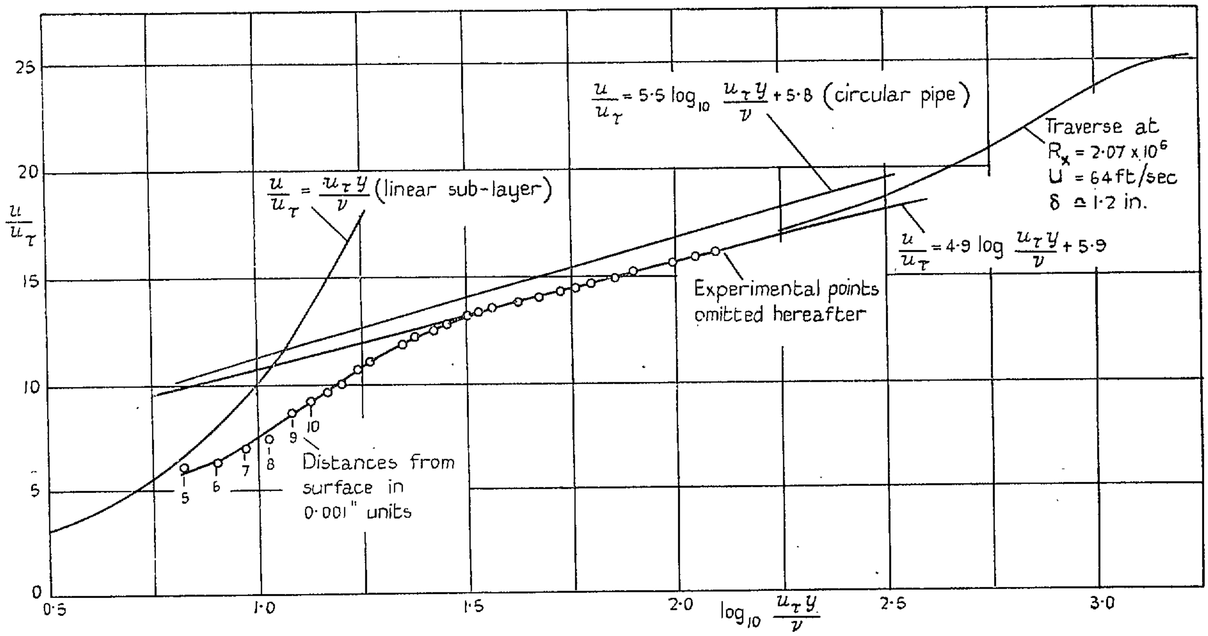

FIG. 12. Specimen velocity traverse.

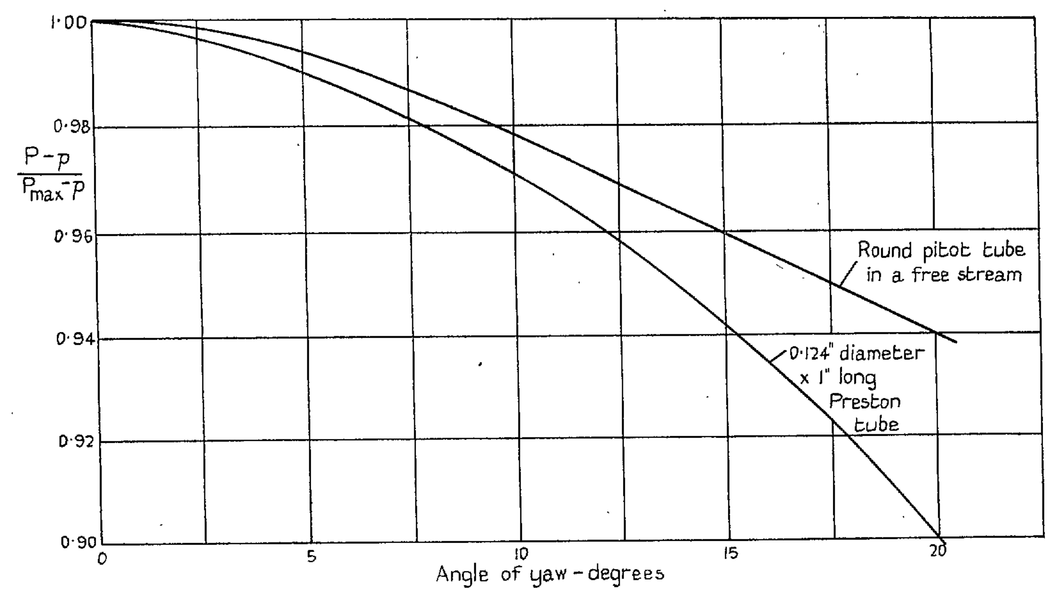

FIG. 13. Yaw sensitivity of a Preston tube.

(80318) Wt. 66/991 K.5 3/61 Hw.

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