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Summary. The author's method for compressible laminar boundary layers, in which a linear viscosity-temperature relationship and a Prandtl number of unity are assumed, is used to investigate some cases in which the wall temperature is not uniform. It is shown that the effect of wall-temperature distribution does not make any essential difference to the method. For example, according to the basic method separation occurs when the pressure-gradient parameter reaches a certain value. The values of this parameter for a wall-temperature distribution $T_w(x)$ are derived from the values for uniform temperature $T_w(0)$ by direct multiplication by $T_w(x)/T_w(0)$.

An alternative and longer, but theoretically more acceptable, expression is derived for the boundary-layer momentum thickness. This expression is obtained by using a kinetic-energy integral equation, in which the momentum equation is multiplied by the local velocity before being integrated across the boundary layer. Accordingly the assumed approximate temperature profile, which is least accurate near to the wall, affects the solution of the momentum equation only through integrals in which small contributions come from near to the wall.

The method is used to examine the problem of heat transfer through a laminar boundary layer with zero pressure gradient, when the wall temperature is a polynomial in x. The predicted heat transfer agrees well with the known exact value when the wall temperature does not vary too rapidly in space. In particular, for wall temperatures like x^n, it is about 40 per cent too high when $n = \frac{1}{2}$, 20 per cent too high when $n = 1$, almost exact when $n = 2$, 20 per cent too low when $n = 4$ and 40 per cent too low when $n = 10$.

The method is then used to consider a problem of cooling by radiation at high Mach number when a uniform stream is assumed and the wall temperature determined by the balance between heat transfer to the wall and radiation from it. Whereas an accurate earlier solution for the wall temperature had required the solution of a non-linear integral equation, the present method yields a simple algebraic equation. Nevertheless, the error in the predicted wall temperature varies from zero at the leading edge to only 18 per cent infinitely far downstream.

Some consideration is then given to the effects of non-uniform wall temperature on boundary-layer separation. Some examples are worked out, illustrating the delay in separation when the wall temperature decreases from the leading edge, as is the case in problems of cooling by radiation. Further, estimates are made of the cooling required to completely suppress separation. These results have been calculated by both the basic and lengthier methods, and indicate that the simpler basic method will be adequate in many practical cases.

Finally, since the assumed approximate temperature profile is obtained by downstream integration from the pressure minimum, and is severely in error near to a forward stagnation point, it is indicated that a method due to Cohen and Reshotko should give accurate results for flow upstream of the pressure minimum. It is shown in this paper that even in the case of variable wall temperature, the differential equation arising in their method may be integrated by two simple quadratures in regions of favourable pressure gradient. Accordingly their method is recommended for use upstream of the pressure minimum, where a convenient join can be made with the present method, which should give the more accurate results downstream.

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1. *Introduction.* Comparatively little theoretical work has been done on heat transfer from bodies whose temperature is non-uniform. Mathematically the problem is the extremely complex one of simultaneously integrating two non-linear partial differential equations, the momentum and energy equations. In incompressible flow the velocity profile is unaffected by small temperature differences, so that the two equations become effectively independent. In such a case the momentum equation is a non-linear partial differential equation for the velocity; when this is solved there remains the energy equation, a linear partial differential equation for the temperature.

Fage and Falkner[1] (1931) have integrated the energy equation for incompressible flow in the cases when the velocity u_1 outside the boundary layer is proportional to some power of the distance x from the leading edge, that is,

$$u_1(x) = U_1(x/c)^{m}, \tag{1.1}$$

and the temperature of the wall $T_w(x)$ differs from its value at the leading edge by another arbitrary power of x, that is,

$$T_w(x) = T_w(0) + \theta(x/c)^{\beta}\,. \tag{1.2}$$

Fage and Falkner assumed that in the energy equation the velocity field (u, v) could be replaced by its asymptotic form near the wall $y = 0$.

Subsequently Chapman and Rubesin[2] (1949) considered the problem of compressible flow with a uniform main stream velocity, when the wall temperature is expressible as a polynomial in x. They assumed a linear relationship between viscosity μ and absolute temperature T, and integrated the energy equation numerically for a Prandtl number $\sigma = 0{\cdot}72$. Their results constitute an exact solution of the energy equation.

Lighthill[3] (1950) used an alternative approximation for the velocity, which also is asymptotically exact as the surface is approached. He thus obtained an otherwise analytical solution which generalizes the results of the preceding papers. He obtained a formula for the heat-transfer rate which is valid at low Mach number for arbitrary wall temperature and pressure gradient, and is also valid at high Mach number for arbitrary wall temperature but zero pressure gradient. After establishing the good agreement between his solution and the earlier exact solutions of Ref. 2, Lighthill goes on to determine the temperature distribution along a surface at which heat transfer to the surface is entirely balanced by radiation from it, as will be approximately true in many practical cases at high Mach numbers. This calculation involves the solution of a non-linear integral equation, and indicates temperatures higher near the nose, and lower downstream, than are found by assuming uniform wall temperature and averaging the heat-transfer balance.

In the present paper use is made of an earlier idea of the author[4] (1958), in which the total temperature

$$T_H = T + \frac{u^2}{2JC_p} \tag{1.3}$$

is approximated as a quadratic function of the velocity u, which satisfies the temperature boundary conditions and has the correct shape in the outer part of the boundary layer. The resulting temperature profile is an exact integral of the energy equation when $\sigma = 1$ provided (i) there is zero heat transfer (arbitrary pressure gradient being permitted), or (ii) the pressure gradient is zero and the wall takes any uniform temperature. The differential equation from which the shape of the outer part of the temperature profile is determined is shown to be independent of the wall-temperature distribution. Its solution is therefore exactly as in the case of uniform wall temperature. The resulting approximation to the temperature profile is thus determined once and for all in terms of the local velocity profile and the local values of the temperature at the wall and at the edge of the boundary layer.

Subject to this temperature profile, the momentum integral equation is used to derive a general approximate solution of the laminar boundary-layer equations. The solution is formally the same as for the case of uniform temperature, so that the momentum thickness is derived by means of two quadratures, after which the displacement thickness and skin-friction follow as multiples of prescribed universal functions. It is found that for flow past a wall with a sharp leading edge the wall-temperature distribution does not affect the momentum thickness except through its value at the leading edge.

An alternative integral formula for the momentum thickness is then derived by using the 'kinetic-energy' integral equation, obtained by multiplying the momentum equation by the local velocity before integrating across the boundary layer. The result is very similar to that obtained by the momentum integral method, but the integrand of the first quadrature is a little lengthier. The result, however, is theoretically more acceptable, as the approximate temperature profile enters the analysis through integrals in which only small contributions arise from the region near the wall where the approximation is least valid.

The heat transfer implied by the assumed temperature profile is examined for the case when the pressure gradient is zero and the wall temperature is expressed as a polynomial. The agreement with exact theory is within 26 per cent for wall temperatures varying like x^5 or less rapidly. The paper then deals with a flow with zero pressure gradient, in which the wall temperature is to be determined by the balance between heat transfer to the wall and radiation from it. The solution of this problem yields a very simple algebraic equation. The error in the predicted wall temperature is always less than 18 per cent, this maximum error being attained asymptotically far from the leading edge.

Some consideration is then given to the effects of variable wall temperature on the separation of a compressible laminar boundary layer. Further, an estimate is made of the cooling required to prevent separation of a given boundary layer. These calculations have been carried out both by the simpler and by the lengthier method, and indicate that for many practical purposes the simpler method will be adequate.

Finally, it is pointed out that the approximate temperature profile was derived on the basis of an integration downstream from the pressure minimum, regardless of conditions at the forward stagnation point. Accordingly, it cannot possibly be expected that the method will in general give good results near the forward stagnation point. It follows that the solution upstream of the pressure minimum must be obtained by some other method, and an accurate procedure for this region is outlined. It is also shown how a convenient join with the present method may be made at the pressure minimum.

2. *Integration of Momentum and Thermal-Energy Equations.* In an earlier paper (Curle[4] (1958)) the author developed an approximate method for integrating the compressible laminar boundary-layer equations when $\mu \propto T$ and $\sigma = 1$. The transformed co-ordinate, Y, normal to the wall, was introduced with

$$Y = \int_0^y \left(\frac{\nu_s}{\nu}\right)^{1/2} dy = \left(\frac{p}{p_s}\right)^{1/2} \int_0^y \frac{T}{T_s}\, dy\,, \tag{2.1}$$

where s refers to a standard reference condition, and the stream function ψ was written as

$$\psi(x, y) = \left(\frac{p}{p_s}\right)^{1/2} \chi(x, Y). \tag{2.2}$$

Then it was shown, following Howarth[5] (1948), that the momentum equation of the boundary layer reduced to

$$\frac{\partial^2\chi}{\partial x \partial Y}\frac{\partial \chi}{\partial Y} - \frac{\partial^2\chi}{\partial Y^2}\frac{\partial\chi}{\partial x} = G u_1 \frac{du_1}{dx} + \nu_s \frac{\partial^3\chi}{\partial Y^3}, \tag{2.3}$$

where

$$G = \frac{T}{T_1} - \frac{\gamma}{2a_1^2}\chi\frac{\partial^2\chi}{\partial Y^2}. \tag{2.4}$$

By assuming that the temperature distribution could be approximated as

$$T_H = T_w + (T_z - T_w)\frac{u}{u_1} + KT_1\left(\frac{u}{u_1} - \frac{u^2}{u_1^2}\right), \tag{2.5}$$

where

$$T_z = T_1\left\{1 + \frac{\gamma-1}{2}M_1^2\right\} \tag{2.6}$$

is the wall temperature appropriate to zero heat transfer, and K is to be determined, it was shown that the solution of (2.3) could be reduced approximately to the following:

$$G_1(x) = \exp\left\{2\int_0^x\left(\frac{T_w}{T_1} + 2 + K - \frac{1}{2}M_1^2\right)\frac{u_1'}{u_1}\,dx\right\}, \tag{2.7}$$

$$\delta_2^2 = \frac{0{\cdot}45\nu_1}{G_1}\int_0^x \frac{G_1}{u_1}\,dx, \tag{2.8}$$

$$m' = -\frac{u_1'\delta_2^2}{\nu_1}\frac{T_w}{T_1}, \tag{2.9}$$

$$\left(\frac{\partial u}{\partial y}\right)_w = \frac{u_1}{\nu_w^{1/2}}\left(\frac{\delta_2^2}{\nu_1}\right)^{-1/2} l'(m'), \tag{2.10}$$

$$\delta_1 = \delta_2\left\{\frac{T_w}{T_1}H'(m') + K + \frac{\gamma-1}{2}M_1^2\right\}. \tag{2.11}$$

The temperature profile (2.5) satisfies the temperature boundary conditions, and KT_1 may be chosen so that the temperature profile is correct in the outer part of the boundary layer. To determine KT_1 we therefore write

$$P_1 = \left\{\frac{\partial T}{\partial(u/u_1)}\right\}_1, \tag{2.12}$$

where the ordinary differential equation which P_1 satisfies has been derived by Gadd[6] (1952). Since he made no use of the boundary conditions at the wall it follows that this equation must hold equally in the case of non-uniform wall temperature, considered here. The integration therefore proceeds exactly as in Ref. 4, so that

$$\frac{Q_1 - 2T_z}{Q_1} = A\,\frac{1 + \frac{\gamma-1}{2}M_0^2\xi}{1 - \xi}, \tag{2.13}$$

where

$$Q_1 = P_1 + \frac{u_1^2}{JC_p} = \left\{ \frac{\partial T_H}{\partial(u/u_1)} \right\}_1 , \tag{2.14}$$

$$\xi = 1 - \frac{u_1^2}{u_0^2} , \tag{2.15}$$

suffix zero denotes main-stream values at the pressure minimum or at the leading edge (if sharp), and A is an arbitrary constant of integration.

The constant A will be chosen so that the correct temperature profile results at a sharp leading edge, where $\xi = 0$. The condition at such a leading edge is

$$Q_1(0) = T_z - T_w(0) , \tag{2.16}$$

since the temperature profile must be that appropriate to zero pressure gradient. Thus (2.13) becomes

$$\frac{Q_1 - 2T_z}{Q_1} = \frac{T_w(0) + T_z}{T_w(0) - T_z} \frac{1 + \frac{\gamma - 1}{2} M_0^2 \xi}{1 - \xi} , \tag{2.17}$$

which may alternatively be written as

$$Q_1 = \frac{\{T_z - T_w(0)\}(1 - \xi)}{1 + \frac{1}{2}\xi\left\{\frac{T_w(0)}{T_0} - 1 + \frac{\gamma - 1}{2} M_0^2\right\}} . \tag{2.18}$$

Now by (2.5) and (2.14) the approximate value Q_1 is

$$Q_1 = T_z - T_w - KT_1 , \tag{2.19}$$

and upon equating (2.18) and (2.19) we find that

$$T_w + KT_1 = T_z - \frac{\{T_z - T_w(0)\}(1 - \xi)}{1 + \frac{1}{2}\xi\left\{\frac{T_w(0)}{T_0} - 1 + \frac{\gamma - 1}{2} M_0^2\right\}}$$

$$= T_w(0) + \frac{\{T_z - T_w(0)\}\xi}{1 + \theta_0(\xi - 1)} , \tag{2.20}$$

where

$$\theta_0 = \frac{T_w(0)/T_0 - 1 + \frac{1}{2}(\gamma - 1) M_0^2}{T_w(0)/T_0 + 1 + \frac{1}{2}(\gamma - 1) M_0^2} . \tag{2.21}$$

It follows from (2.7) that the function G_1 does not depend upon the wall temperature $T_w(x)$, but only upon its value $T_w(0)$ at the leading edge. Hence if the solution of a particular boundary-layer problem with uniform wall temperature $T_w(0)$ has been computed, the solution of the corresponding problem with arbitrary wall temperature $T_w(x)$ can be obtained with very little extra effort. In particular, it follows from (2.7) that the momentum thickness δ_2 is independent of wall temperature, save for the dependence upon the value at the sharp leading edge.

3. *The Kinetic-Energy Integral Equation.* Equation (2.8), a simple integral expression for the boundary-layer momentum thickness, is a generalization of Thwaites's result (Ref. 7 (1949)) for incompressible flow

$$\delta_2^2 = \frac{0 \cdot 45\nu}{u_1^6} \int_0^x u_1^5 \, dx , \tag{3.1}$$

and assumes that a certain function $L(m)$ is a linear function of m approximately. A more convincing indication of the general validity of (3.1) was given by Truckenbrodt[8] (1952), who made use of the so-called energy integral equation, due to Leibenson[9] (1935). This equation, obtained by multiplying the momentum equation by the local velocity u and then integrating across the boundary layer, will in this paper be called the kinetic-energy integral equation, to distinguish it from the thermal-energy integral equation. It can be shown for incompressible flow that

$$\delta_2^2 = \frac{4\nu}{h_1^2 u_1^6} \int_0^x h_1 h_2 u_1^5 \, dx \,, \tag{3.2}$$

where

$$h_1 = \frac{\delta_3}{\delta_2} = \frac{\displaystyle\int_0^\infty \frac{u}{u_1}\left(1 - \frac{u^2}{u_1^2}\right) dy}{\displaystyle\int_0^\infty \frac{u}{u_1}\left(1 - \frac{u}{u_1}\right) dy} \,, \tag{3.3}$$

and

$$h_2 = \frac{\delta_2}{u_1^2} \int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy \,. \tag{3.4}$$

Truckenbrodt suggested that h_1 and h_2 might be treated as constants. This is a reasonable approximation, since h_1 equals $1 \cdot 62$ at a stagnation point, $1 \cdot 57$ for a Blasius layer and $1 \cdot 52$ for a typical separation profile, the corresponding values of h_2 being respectively $0 \cdot 204$, $0 \cdot 173$ and $0 \cdot 157$. The reason for the approximate constancy of these values of h_1 is that the main changes in a developing boundary-layer profile occur near the wall, whereas the two integrals in (1.6) are not vitally affected by the shape near the wall. Similarly there is a tendency for decreases in $\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy$, as separation is approached, to be partially balanced by the increase in δ_2, so that h_2 does not vary very much. It is clear, then, that the approximate constancy of h_1 and h_2 is a consequence of the general shape of boundary-layer profiles, and so one could reasonably expect it to hold for any boundary layer. With h_1 and h_2 constant, (3.2) becomes

$$\delta_2^2 = \frac{4 h_2 \nu}{h_1 u_1^6} \int_0^x u_1^5 \, dx \,. \tag{3.5}$$

On the grounds that the major part of the integral in (3.2) would arise near to where u_1 is a maximum, we choose h_1 and h_2 to be close to the values for a Blasius layer. Thus, $h_1 = 1 \cdot 58$ and $h_2 = 0 \cdot 179$ leads to Thwaites's result (3.1).

To develop a compressible flow result, analogous to (3.2), we multiply (2.3) by

$$u = \frac{\partial \chi}{\partial Y} \,, \tag{3.6}$$

and integrate from $Y = 0$ to $Y = \infty$. Then some algebra, similar to that for the incompressible case, leads to the result

$$\frac{d}{dx}\{u_1^3 \delta_3'\} = 2\nu_s \int_0^\infty \left(\frac{\partial u}{\partial Y}\right)^2 dY - 2u_1 u_1' \int_0^\infty (G - 1) u \, dY \,, \tag{3.7}$$

where

$$\delta_3' = \int_0^\infty \frac{u}{u_1}\left(1 - \frac{u^2}{u_1^2}\right) dY \tag{3.8}$$

is a kind of 'energy thickness', measured in the transformed normal co-ordinate.

We define

$$D' = \int_0^\infty \mu_s \left(\frac{\partial u}{\partial Y}\right)^2 dY , \tag{3.9}$$

so that (3.7) becomes

$$\frac{d}{dx}(u_1^3 \, \delta_3') = \frac{2D'}{\rho_s} - 2u_1 u_1' \int_0^\infty (G - 1) u \, dY , \tag{3.10}$$

which may be compared with the incompressible form (Leibenson[9] (1935))

$$\frac{d}{dx}(u_1^3 \delta_3) = \frac{2D}{\rho} . \tag{3.11}$$

Now from (2.4), by making use of the approximate temperature profile (2.5), we find

$$G - 1 = \left(\frac{T_w}{T_1} - 1\right)\left(1 - \frac{u}{u_1}\right) + \left(K + \frac{\gamma - 1}{2} M_1^2\right)\left(\frac{u}{u_1} - \frac{u^2}{u_1^2}\right) - \frac{\gamma}{2a_1^2} \chi \frac{\partial^2 \chi}{\partial Y^2} , \tag{3.12}$$

so that

$$\int_0^\infty (G - 1) u \, dY = u_1\left(\frac{T_w}{T_1} - 1\right)\delta_2' + u_1\left(K + \frac{\gamma - 1}{2} M_1^2\right)(\delta_3' - \delta_2') - \frac{\gamma}{4a_1^2}\int_0^\infty \chi d\left\{\left(\frac{\partial \chi}{\partial Y}\right)^2\right\} ,$$

and after some algebra this yields

$$\int_0^\infty (G - 1) u \, dY = u_1 \delta_3' \left\{K + \frac{\gamma - 2}{4} M_1^2\right\} - u_1 \delta_2' \left\{K + \frac{T_z - T_w}{T_1}\right\} . \tag{3.13}$$

We now substitute from (3.13) into (3.10), put

$$\frac{\delta_3'}{\delta_2'} = h_1 , \quad \frac{\delta_2' D'}{\mu_s u_1^2} = h_2 , \tag{3.14}$$

and multiply by $2h_1 u_1^3 \delta_2'$, whence, after some manipulation, it follows that

$$\frac{d}{dx}\{h_1^2 u_1^6 \delta_2'^2\} = 4h_1 h_2 \nu_s u_1^5 - 4h_1 u_1^5 u_1' \delta_2'^2 \left\{h_1\left(K + \frac{\gamma - 2}{4} M_1^2\right) - \left(K + \frac{T_z - T_w}{T_1}\right)\right\} ,$$

which may alternatively be written

$$\begin{aligned} \frac{d}{dx}\{h_1^2 \delta_2'^2\} &= \frac{4h_1 h_2 \nu_s}{u_1} - \frac{4h_1^2 \delta_2'^2 u_1'}{u_1}\left\{K + \frac{\gamma - 2}{4} M_1^2 + \frac{3}{2} - \frac{1}{h_1}\left(K + \frac{T_z - T_w}{T_1}\right)\right\} , \\ &= \frac{4h_1 h_2 \nu_s}{u_1} - h_1^2 \delta_2'^2 g(x) , \end{aligned} \tag{3.15}$$

where

$$g(x) = 4\frac{u_1'}{u_1}\left\{K + \frac{\gamma - 2}{4} M_1^2 + \frac{3}{2} - \frac{1}{h_1}\left(K + \frac{T_z - T_w}{T_1}\right)\right\} . \tag{3.16}$$

This equation has an integrating factor

$$G_2(x) = \exp\left\{\int_0^x g(x) \, dx\right\} , \tag{3.17}$$

and the solution may be written

$$\delta_2'^2 = \frac{4\nu_s}{h_1^2 G_2}\int_0^x \frac{h_1 h_2 G_2}{u_1} dx . \tag{3.18}$$

Now no approximations have been made in deriving this result, except in using the temperature profile (2.5) to evaluate $\int_0^\infty \left(\frac{T}{T_1} - 1\right) u \, dY$. This temperature profile is correct at the wall and in the outer part of the boundary layer. Only in the inner part of the boundary layer should it be in error. We note that because of the factor u in the integrand the contribution from this region will be fairly small, and the results accordingly more reliable than those obtained by the procedure of Ref. 4, where integrals of the form $\int_0^\infty \left(\frac{T}{T_1} - 1\right) dY$ arose. We now make the further approximation that h_1 and h_2 may be taken as constants. The validity of this approximation, as was discussed earlier, rests upon the general shape of boundary-layer velocity profiles, and holds for a very wide range of shapes. As for the incompressible case, with

$$h_1 = 1\cdot58, \quad h_2 = 0\cdot179\,, \tag{3.19}$$

which are very close to the values for a Blasius layer, (3.18) becomes

$$\delta_2'^2 = \frac{0\cdot45\nu_s}{G_2}\int_0^x \frac{G_2}{u_1}\, dx\,, \tag{3.20}$$

and hence (Ref. 4)

$$\delta_2^2 = \frac{0\cdot45\nu_1}{G_2}\int_0^x \frac{G_2}{u_1}\, dx\,. \tag{3.21}$$

The result (3.21) differs from that of Ref. 4 only in that $G_2(x)$ replaces the function $G_1(x)$, defined by (2.7). It is easy to see that G_1 and G_2 would be identical if h_1 were equal to 2. Accordingly, we write

$$\frac{1}{h_1} = \frac{1}{2} + \epsilon\,, \tag{3.22}$$

where, with the value $h_1 = 1\cdot58$ adopted above,

$$\epsilon = 0\cdot133\,, \tag{3.23}$$

and (3.16) becomes

$$g(x) = 2\frac{u_1'}{u_1}\left\{\left(\frac{T_w}{T_1} + 2 + K - \frac{1}{2}M_1^2\right) - 2\epsilon\left(K + \frac{T_z - T_w}{T_1}\right)\right\}\,, \tag{3.24}$$

where the first terms are those appearing in G_1.

It may be noted that the additional terms are zero when there is zero heat transfer, since it can be seen from (2.5) that K is then zero. When there is not very much heat transfer, G_2 will not differ greatly from G_1, especially since the extra terms have a relatively small multiplying factor. It would then seem possible, since G_2 appears in the numerator of the integrand of (3.21), and in the denominator outside the integral, that the predicted values of δ_2, using G_1 or G_2, would vary even less. It is only when there is considerable heat transfer that the results should differ significantly, and the slightly lengthier procedure using G_2 would be preferable. In such circumstances the momentum thickness δ_2 will depend upon the wall-temperature distribution. It will be found later, however (Section 6), that for many practical purposes the simpler form should be adequate.

4. *Results for Polynomial Temperature Distribution.* Chapman and Rubesin[2] (1949) considered the case of a uniform main stream, that is

$$u_1(x) = u_0\,, \tag{4.1}$$

and a temperature distribution at the wall

$$T_w(x) = T_z + \sum a_n x^n\,, \tag{4.2}$$

where, however, n does not necessarily take only integral values. The series $\Sigma\, a_n x^n$ is the difference between the actual wall temperature and that appropriate to zero heat transfer. Their solution indicates that the temperature gradient at the wall is

$$\left(\frac{\partial T}{\partial y}\right)_w = -\,0\cdot 296\left(\frac{u_1}{\nu_w x}\right)^{1/2} \sum a_n b_n x^n\,, \tag{4.3}$$

where

$$\left.\begin{aligned} b_0 &= 1\cdot 00 \\ b_1 &= 1\cdot 65 \\ b_2 &= 2\cdot 02 \\ b_3 &= 2\cdot 29 \\ b_4 &= 2\cdot 52 \\ b_5 &= 2\cdot 70 \\ b_{10} &= 3\cdot 40 \end{aligned}\right\} . \tag{4.4}$$

These results were calculated for a Prandtl number of $\sigma = 0\cdot 72$, but the work of Lighthill[3] (1950) indicates that the effects of Prandtl number may be largely accounted for by replacing the multiple $0\cdot 296$ by $0\cdot 332\sigma^{1/3}$, the b_n being essentially independent of σ.

For the case of a uniform mainstream it is known that the skin-friction is formally equal to its value in incompressible flow,

$$\left(\frac{\partial u}{\partial y}\right)_w = 0\cdot 332 u_1\left(\frac{u_1}{\nu_w x}\right)^{1/2} . \tag{4.5}$$

This corresponds to the fact that the solution of the momentum equation, (2.7) to (2.11), is simply $m' = 0$. For the mainstream velocity (4.1), ξ, defined by (2.15), is zero. Hence the approximate temperature profile, given by (2.5) and (2.20), becomes

$$T_H = T_w + (T_z - T_w)\frac{u}{u_1} + \{T_w(0) - T_w\}\left(\frac{u}{u_1} - \frac{u^2}{u_1^2}\right) , \tag{4.6}$$

from which we deduce that the temperature gradient at the wall is

$$\begin{aligned} \left(\frac{\partial T}{\partial y}\right)_w &= \frac{1}{u_1}\left(\frac{\partial u}{\partial y}\right)_w \{T_z + T_w(0) - 2T_w\} \\ &= 0\cdot 332\left(\frac{u_1}{\nu_w x}\right)^{1/2}\{T_z + T_w(0) - 2T_w\} . \end{aligned} \tag{4.7}$$

If we put $T_w(x)$ equal to the value (4.2) assumed by Chapman and Rubesin, this yields

$$\left(\frac{\partial T}{\partial y}\right)_w = -\,0\cdot 332\left(\frac{u_1}{\nu_w x}\right)^{1/2}\left\{2\sum a_n x^n - a_0\right\} . \tag{4.8}$$

We compare this expression with the exact value (4.3), remembering that when $\sigma = 1$, as is so in the present work, the factor of $0\cdot 296$ must be changed to $0\cdot 332$. The approximate solution is equivalent to putting b_0 equal to unity, and all succeeding b_n equal to two.

In view of the remarkable simplicity of the present method this is quite a good approximation to (3.4) for not too rapidly varying wall temperatures. If the wall temperature is proportional to x^N, which varies very rapidly near $x = 0$ when N is small, the predicted heat transfer tends to twice the exact value in the limit as $N \to 0$. At the other extreme for large values of N, when the wall temperature varies very rapidly with x for large x, the predicted heat transfer is about 40 per cent small when $N = 10$. For cases between these two extremes the agreement is much better, as the following Table shows:

N	$\frac{1}{2}$	1	2	3	4	5
$\left(\frac{\partial T}{\partial y}\right)_w$	40% high	21% high	1% low	13% low	21% low	26% low

It should be noted, however, that these figures are only asymptotic, in the sense described below. The true position is less pessimistic. Consider a case where the wall temperature is uniform for $x \leqslant c$,

$$\frac{T_w}{T_z} - 1 = \beta\,, \tag{4.9}$$

and increases for $x \geqslant c$ with a square-root singularity, so that

$$\frac{T_w}{T_z} - 1 = \beta + \delta\left(\frac{x}{c} - 1\right)^{1/2}. \tag{4.10}$$

Then the error in the predicted heat transfer is zero when $x = c$, has increased to 17 per cent where $\{(T_w/T_z) - 1\}$ has increased to $1 \cdot 5\beta$, then to 24 per cent where $\{(T_w/T_z) - 1\}$ equals 2β, and asymptotically to 40 per cent far downstream.

5. *Cooling by Radiation at High Mach Number.* Lighthill[3] (1950) points out that for a projectile with an attached front shock it is probably reasonable to assume a uniform main stream (3.1) over the short front portion of the surface where the boundary layer is laminar. He goes on to examine the temperature distribution in this region by equating heat transfer to the body with radiation from it, which is probably a good physical approximation when the temperatures are high. An approximation to his accurate results can very rapidly be obtained by the method of the present paper.

Since ξ is again zero, the heat transfer to the wall per unit area is given by (4.7) and is thus

$$Q(x) = k\left(\frac{\partial T}{\partial y}\right)_w = 0 \cdot 332k\left(\frac{u_1}{\nu_w x}\right)^{1/2}\{T_z + T_w(0) - 2T_w(x)\}\,. \tag{5.1}$$

This must be balanced by radiation from the wall, which is

$$Q(x) = \alpha\{T_w(x)\}^4\,, \tag{5.2}$$

where α is the Stefan-Boltzmann constant multiplied by the emissivity of the wall. From (5.1) and (5.2) we deduce that

$$\begin{aligned} T_w{}^4 &= 0 \cdot 332\frac{k}{\alpha}\left(\frac{u_1}{\nu_w x}\right)^{1/2}\{T_z + T_w(0) - 2T_w\} \\ &= 0 \cdot 490\left(\frac{l}{x}\right)^{1/2} T^{*3}\{T_z + T_w(0) - 2T_w\}\,. \end{aligned} \tag{5.3}$$

Here l is a representative length, and T^* is a temperature defined by Lighthill as

$$T^{*3} = 0\cdot678\sigma^{1/3}\frac{k}{\alpha}\left(\frac{u_1}{\nu_w l}\right)^{1/2}, \tag{5.4}$$

σ being unity in the present application. It follows from (5.3), by letting $x \to 0$, that

$$T_w(0) = T_z, \tag{5.5}$$

for it could have no other finite value. Physically this is so because the boundary layer near $x = O$ is extremely thin, so that heat transfer is extremely effective, and the wall therefore takes up its equilibrium temperature (Lighthill[3] (1950)). Using (5.5) it follows that (5.3) becomes

$$T_w^4 = 0\cdot980\left(\frac{l}{x}\right)^{1/2} T^{*3}(T_z - T_w). \tag{5.6}$$

This equation is made non-dimensional by writing

$$T_w/T_z = F(z), \tag{5.7}$$

where

$$z = \left(\frac{T_z}{T^*}\right)^3 \left(\frac{x}{l}\right)^{1/2}, \tag{5.8}$$

whence it becomes

$$zF^4 = 0\cdot980(1 - F). \tag{5.9}$$

This simple algebraic equation can easily be solved to yield $F(z)$. The results are tabulated in Table 1, along with Lighthill's solution. We note that the present method yields results agreeing with Lighthill's at $z = 0$ (since the initial condition, $F(0) = 1$, is satisfied by both solutions), and that the percentage error increases as z increases. In the limit as $z \to \infty$ the function $z^{1/4} F(z)$ tends to $0\cdot995$ as compared with Lighthill's value of $0\cdot841$. The error is thus always less than 18 per cent, a most satisfactory result when one considers the ease with which the results presented here are obtained.

As in Lighthill's predictions, the wall temperature will fall from a maximum at the nose, so that at suitable Mach numbers melting may occur near the nose. The extent of the region in which melting is possible will be overestimated by the present method.

6. *Non-Uniform Wall Temperature and Boundary-Layer Separation.* It is now well established that, relative to incompressible flow with the same 'shape' of mainstream velocity, flow in a compressible boundary layer separates more readily when there is zero heat transfer and less readily when the wall is sufficiently cooled. Separation is predicted by the present method to occur when $m' = 0\cdot090$, but it is suggested in Ref. 4 that improved results may be obtained by allowing m' at separation to be a function of Mach number, which equals $0\cdot061$ when the Mach number is 4. For a linearly retarded mainstream

$$u_1 = u_0(1 - x/c), \tag{6.1}$$

this yields the results that separation occurs

(i) at $x = 0\cdot123c$ for incompressible flow

(ii) at $x = 0\cdot041c$ when the wall temperature is $T_w = T_z$ at $M_0 = 4$

(iii) at $x = 0\cdot248c$ when the wall temperature is $T_w = T_0$ at $M_0 = 4$.

In a physically more realistic case the wall temperature might well be a function of x, particularly at high Mach number. For example, in Section 5 we considered Lighthill's problem, where the wall temperature is determined by the balance between heat transfer to the wall and radiation from it. The wall temperature in this problem is given, for small x, by an expansion

$$T_w(x) = T_z\{1 - a(x/c)^{1/2} \ldots\} \tag{6.2}$$

for some value of a. Near the leading edge, therefore, $T_w \sim T_z$, and there is a tendency towards earlier separation than in incompressible flow. As x increases, however, the wall becomes cooler, and, if separation has not already occurred, the cooled wall conditions will tend to delay it further. It is interesting to consider whether separation may be greatly influenced by the latter effect.

In order to provide a partial answer to this question we consider a flow in which the main-stream velocity is (6.1), the Mach number is 4, and the wall temperature is given by the first two terms of the series (6.2). The solution by the simpler procedure, using (2.8) and (2.9), can easily be obtained, since $G_1(x)$ is influenced by wall-temperature distribution only through the value $T_w(0)$ at the leading edge. Accordingly, $G_1(x)$ is the same function as in the case $a = 0$, and by (2.8) and (2.9) it follows that

$$\begin{aligned} m'(x) &= m_0'(x)\frac{T_w(x)}{T_w(0)} \\ &= m_0'(x)\{1 - a(x/c)^{1/2}\}, \end{aligned} \tag{6.3}$$

where $m_0'(x)$ is the value of $m'(x)$ appropriate to the case $T_w = T_w(0)$, i.e., $T_w = T_z$, which function was computed in Ref. 4. So the function $m'(x)$ is easily obtained for any prescribed value of a. The results have been tabulated in Table 2 for the cases $a = \frac{1}{2}$, 1 and $\frac{3}{2}$. When $a = 0$, that is for the case of zero heat transfer, separation occurs when $x = 0 \cdot 041_3 c$. For the case $a = \frac{1}{2}$, separation is predicted at $x = 0 \cdot 047_7 c$, an increase of 15 per cent. The wall temperature falls from T_z at the leading edge to $0 \cdot 891 T_z$ at separation. For the case $a = 1$, separation is predicted at $x = 0 \cdot 058_3 c$, an increase (over the zero heat-transfer case) of 41 per cent. The wall temperature varies between T_z and $0 \cdot 759 T_z$. For the case $a = \frac{3}{2}$, separation is predicted at $x = 0 \cdot 084_8 c$, an increase of 105 per cent, whilst the wall temperature is $0 \cdot 563 T_z$ at separation.

These results give some indication of the effects of this type of wall-temperature distribution. By comparing the results for these cases we note that when the rate at which the temperature falls is made greater, the increase in the distance to separation increases more and more rapidly. This is just as one would expect, for the greater the distance to separation, the easier it is to delay separation even further by a reduction in wall temperature.

Results have also been obtained by the slightly longer but theoretically more acceptable procedure, using (3.21) and (2.9), for the case $a = 1$, to test the adequacy of the simpler method. It is found that separation is predicted at $x = 0 \cdot 060_2$. We note that the distance to separation agrees with the earlier estimate to within 3 per cent. What is perhaps more important, and is certainly a more severe test, is a comparison of the amounts which the two methods predict separation will be delayed by the cooling of the wall. According to the simpler method, separation is delayed from $x = 0 \cdot 041_3 c$ to $x = 0 \cdot 058_3 c$, that is $0 \cdot 017_0 c$, whereas the lengthier method predicts $0 \cdot 018_9 c$. As these differ by only 10 per cent, one would conclude that for this flow, in which the wall temperature at separation is $0 \cdot 759 T_z$, the simpler method yields acceptable results.

Consider now a problem in which the external flow conditions are as given for the above example, namely the linearly retarded velocity (6.1) at a Mach number of 4. When there is zero heat transfer, separation is predicted at $x = 0 \cdot 041 c$. If, however, the wall is cooled beyond this point, separation can be avoided. What wall-temperature distribution would just avoid separation? Equally well we might ask what step in wall temperature would be sufficient to provoke immediate separation at a point upstream of $x = 0\cdot041c$? Now by (6.3) we see that, according to the simpler method, the flow will be on the point of separation where

$$m_0'(x)\frac{T_w(x)}{T_w(0)} = m'(x) = 0\cdot061\,. \tag{6.4}$$

In the case of separation provoked upstream of $0\cdot041c$, since the wall temperature upstream of separation is everywhere $T_w = T_z$, the lengthier method also yields (6.4). However, for the case when separation is avoided by cooling the wall downstream of $x = 0\cdot041c$, the two methods would yield slightly differing results.

From (6.4) the wall temperature $T_w(x)$ which will just avoid (or provoke) separation is

$$T_w(x) = \frac{0\cdot061}{m_0'(x)}\,T_z\,, \tag{6.5}$$

where $m_0'(x)$ is given in Table 2. The solution of this equation is given in Table 3. For $x \leqslant 0\cdot041c$ the temperature $T_w(x)$ is that which would just cause the flow to separate if the wall temperature were discontinuously increased from T_z to T_w at that point. For $x \geqslant 0\cdot041c$ the temperature distribution is that which, according to the simpler method, would just prevent the flow from separating. We notice that a considerable cooling of the wall is required if the position of separation is to be delayed.

We note from Table 3 that separation is predicted at $x = 0\cdot03_0c$ if the wall temperature is increased to $T_w = 1\cdot30_0T_z$ at that position. It is a criticism of the simpler method that, provided the wall temperature equalled T_z at the leading edge and $1\cdot3T_z$ at $x = 0\cdot03_0c$, separation would be predicted at this position whatever the distribution of $T_w(x)$. The solution by the lengthier method, however, would depend upon the detailed distribution of wall temperature. Suppose, for example, that

$$T_w(x) = T_z(1 + 10\,x/c)\,, \tag{6.6}$$

so that the wall temperature increases linearly from T_z at $x/c = 0$ to $1\cdot3T_z$ at $x = 0\cdot03c$. Calculations by the lengthier method predict separation at $x = 0\cdot0305c$, as compared with $x = 0\cdot03c$ by the shorter method. Thus the predicted distances to separation differ by less than 2 per cent, and the predicted upstream movements of separation due to the increasing wall temperature differ by less than 5 per cent.

Calculations have also been carried out for the case of the velocity distribution (6.1) at a Mach number of 4 and a severely cooled wall, with $T_w = T_0 = \frac{5}{21}T_z$. According to the simpler method separation occurs at $x = 0\cdot248c$, whereas the longer method yields $x = 0\cdot256c$. Thus, even for this case of severe cooling the predicted distances to separation differ by only 3 per cent, the differences in the boundary-layer characteristics being smaller than this amount at positions upstream.

One would conclude from the results of this Section that in many practical cases it will be sufficient to use the simpler method.

7. *Allowance for an Initial Favourable Pressure Gradient.* It must be noted that the approximate temperature profile (2.5) was derived by integrating downstream from a sharp leading edge or from the pressure minimum. The method cannot be used upstream of the pressure minimum, save for the case of zero heat transfer, when the temperature profile is exact. Thus, for example, the predicted heat transfer is severely in error near a forward stagnation point. Accordingly some

other method must be applied from the forward stagnation point to the pressure minimum, and the present method used to continue the solution downstream. The method of Cohen and Reshotko[10] (1956) is ideally suited to this purpose. Their method is based on similarity solutions, so that one would expect them to be accurate in regions of favourable pressure gradient, but somewhat uncertain as separation is approached, judging from experience in incompressible flow. Indeed, Luxton and Young[11] (1958) have given some computed results, which indicate that, even without the empirical improvement mentioned in Section 6, the author's method yields a result of comparable accuracy to that of Cohen and Reshotko for the case of a linearly increasing pressure at a Mach number of 2, and a cooled wall, and that the author's method yields considerably better results for the case of a linearly retarded velocity at a Mach number of 4, with a cooled wall. Accordingly the use of the present method in the region downstream of the pressure minimum is to be preferred.

Upstream of the pressure minimum, Cohen and Reshotko's method should be very accurate, and is exact near to a forward stagnation point. The main result of their paper is that

$$\delta_2^2 = \frac{A\nu_1}{u_1}\left(\frac{T_z}{T_1}\right)^{(3\gamma-1)/2(\gamma-1)} M_1^{1-B}\int_0^x \left(\frac{T_z}{T_1}\right)^{-(3\gamma-1)/2(\gamma-1)} M_1^{B-1}\,dx\,, \tag{7.1}$$

where B is a function of T_w/T_z, assumed constant, and $A = 0{\cdot}44$. If we change A to $0{\cdot}45$, then (7.1) is identical with (2.8) for the case of zero heat transfer provided B is then equal to 6. Values of B which satisfy this condition, and at the same time agree with Cohen and Reshotko's values for cooled or heated walls in a favourable pressure gradient, are given by

$$B = 6\exp\left\{0{\cdot}8664\left(\frac{T_w}{T_z} - 1\right)\right\}. \tag{7.2}$$

When the wall temperature is not uniform, however, (7.1) is not valid, since it is the integral of equation (28) of Cohen and Reshotko's paper only when B is a constant. It is quite easy, however, to integrate their equation (28) in the case when B is a function of x, and this yields

$$\delta_2^2 = \frac{A\nu_1}{G_3}\left(\frac{T_z}{T_1}\right)^{(2\gamma-1)/(\gamma-1)}\int_0^x \left(\frac{T_z}{T_1}\right)^{-(2\gamma-1)/(\gamma-1)} \frac{G_3}{u_1}\,dx\,, \tag{7.3}$$

where

$$G_3 = \exp\left\{\int_{x_0}^x \frac{dM_1}{dx}\frac{B}{M_1}\,dx\right\}, \tag{7.4}$$

with $x = x_0$ representing the position of the pressure minimum, and T_z/T_1 being given in terms of the local main-stream Mach number M_1 by (2.6).

It is suggested that from the pressure minimum onwards, the present method should be used, so that from (3.15) and (3.17) we have

$$\frac{d}{dx}\left\{\frac{G_2\delta_2^2}{\nu_1}\right\} = \frac{0{\cdot}45}{u_1}G_2\,. \tag{7.5}$$

Upon integrating (7.5) from $x = x_0$, we have

$$\frac{G_2}{\nu_1}\delta_2^2 = 0{\cdot}45\int_{x_0}^x \frac{G_2}{u_1}\,dx + \frac{G_2(x_0)}{\nu_0}\,\delta_2^2(x_0)\,, \tag{7.6}$$

where $\delta_2^2(x_0)$ is given by (7.3). The remaining boundary-layer characteristics then follow from (2.9) to (2.11), when $x \geqslant x_0$, and from Cohen and Reshotko's method when $x \leqslant x_0$.

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TABLE 1

The Function $F(z)$

$z^{1/4}$	$F(z)$	
	Lighthill	Curle
0	1·00	1·00
0·5	0·93	0·95
1	0·67	0·72
2	0·38	0·43
3	0·26	0·30
4	0·20	0·23
$\gg 1$	$0 \cdot 841\, z^{-1/4}$	$0 \cdot 995\, z^{-1/4}$

TABLE 2

The Function $m'(x)$ for the case

$u_1 = u_0(1 - x/c), \quad M_0 = 4, \quad T_w = T_z(1 - a(x/c)^{\frac{1}{2}})$

x/c	$m_0'(x)$	$m'(x)$			$a = 1$
		$a = \frac{1}{2}$	$a = 1$	$a = \frac{3}{2}$	Lengthier method
0·00	0	0	0	0	0
0·01	0·0176	0·0167	0·0158	0·0149	0·0158
0·02	0·0330	0·0307	0·0283	0·0260	0·0282
0·03	0·0468	0·0428	0·0387	0·0347	0·0384
0·04	0·0595	0·0535	0·0476	0·0416	0·0470
0·05	0·0712	0·0633	0·0553	0·0473	0·0544
0·06	0·0824	0·0723	0·0622	0·0521	0·0609
0·07	0·0930	0·0807	0·0684	0·0561	—
0·08	0·1035	0·0888	0·0742	0·0595	—
0·09	0·1137	0·0967	0·0796	0·0626	—
0·10	0·1239	0·1043	0·0847	0·0650	—

TABLE 3

Wall Temperature $T_w(x)$ appropriate to Zero Skin Friction $u_1 = u_0(1 - x/c)$, $M_0 = 4$

x/c	$T_w(x)/T_z$
0·00	∞
0·01	3·47
0·02	1·85
0·03	1·30
0·04	1·03
0·05	0·856
0·06	0·741
0·07	0·656
0·08	0·590
0·09	0·536
0·10	0·492

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