R. & M. No. 2525
(9482)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

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LONDON: HER MAJESTY'S STATIONERY OFFICE
1952
PRICE 3*s* 6*d* NET

The Yawing Vibrations of an Aircraft

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COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR),
MINISTRY OF SUPPLY

*Reports and Memoranda No. 2525**
March, 1946

Summary.—This report gives a theoretical method for calculating the natural frequencies and modes of yawing vibration of a complete aircraft. The basic feature of the treatment is the replacement of the continuous mass system by one consisting of a finite number of discrete masses elastically interconnected. In the course of the analysis, use is made of the deflection coefficient artifice in the formation of the equations of motion, and the escalator process in their marshalling and numerical solution.

The method has been applied to a single-engined fighter aircraft, for which the results of a resonance test were available.

These results appear to be some 40 per cent. in excess of their calculated counterparts and no satisfactory explanation occurs to the authors to account for this incompatability.

1. *Introduction.*—This report deals with the yawing vibrations of an aircraft. As in previous reports[1,2], the basic assumption is made that the complete aircraft with its distributed mass and elasticity, corresponding to an infinite number of degrees of freedom, may be replaced by an approximately equivalent dynamical system consisting of a finite number of localised masses elastically interconnected. Advantage is taken of the escalator technique[3,4], in the marshalling and numerical solution of the appropriate equations of motion.

The case investigated in the present report is a single-engined fighter aircraft, on which resonance tests had been carried out.

Each wing was divided into three discrete portions for each of which the position of the centre of gravity and the mass were estimated from drawings. The centres of gravity of the masses were found to be very approximately on the flexural axis, which by calculation appeared to be straight and practically normal to the main axis of the aircraft. So far as the fuselage is concerned, the mass of the front part was concentrated in the engine propeller system, account being taken of the moment of inertia in yaw. The less concentrated rear part was divided into three portions by planes normal to the longitudinal axis and the tail was regarded as rigid.

As in the earlier reports referred to, a basic feature of the method is the employment of the deflection coefficient artifice.[1] The values of these coefficients were calculated directly from drawings, on the assumption that the wings and fuselage are separately encastré at their junction. It should be made clear, however, that this assumption is one of expediency in the specification of the elastic properties of the system. No such artificial condition is imposed in the analysis of the motion. A calculation made from drawings revealed that there is only a relatively insignificant coupling between pitching and yawing elastic deflections of the wings and fuselage. Although in the particular case the wing and fuselage flexural axes are not coplanar it was decided to treat them as such on the assumption that the effects of non-fulfilment of this condition are small enough to be neglected. In these circumstances the yawing vibrations may be separated, and the whole motion regarded as taking place in a horizontal plane.

* R.A.E. Report No. S.M.E. 4027—received 31st July, 1946.

The propeller was treated as rigid and non-rotating. This, of course, is not true in fact; even when the propeller is not rotating, pitching and yawing vibrations are coupled in virtue of the flexibility of the blades, their pitch and twist. This effect was ascertained and found to be insignificant. When the propeller is rotating there is in consequence a coupling between pitching and yawing vibrations due to gyroscopic action; this coupling was likewise found to be relatively inconsiderable.

On comparison of the theoretical results shown in Fig. 4 with the experimental results obtained by resonance test and given in Fig. 5, it will be noticed that there is only a superficial agreement. Indeed the test results appear to be some 40 per cent. in excess of the calculated results. It may be that the assumptions made in order to separate the yawing vibrations are not well founded and that in fact the mass centres are not on a rectilineal flexural axis, so that rolling vibrations are perforce present. However, these assumptions were made purposely in order to simplify the analytical treatment, and to elucidate the trend of the phenomena involved.

On the other hand, unqualified reliance cannot be placed on the experimental results, owing to the errors involved in their practical measurement, and the difficulty of exciting pure vibrations of any particular character as may be seen from Fig. 5 (*see* Ref. 5). Against this there is the case that much better agreement was obtained on similar assumptions in which pitching and rolling vibrations were treated as separating out.

To sum up, it appears from the foregoing that further consideration should be given to the yawing problem to ascertain the reasons for the apparent discrepancies in this particular case.

2. *General Theory*.—2.1. *Wings*.—Referring to Fig. 1, OX, OY are a system of fixed rectangular axes formed by the undeflected positions of the fuselage and wing flexural axes. Let the system, in vibration about its mean position, have small displacements relative to these axes as indicated in the figure. Each wing is regarded as consisting of three masses, viz., m_1, m_2, m_3.

Now the effective elastic displacements of the wing masses are as follows:

$$m_1:\ X_1 - l_1 \Phi_0\,.; \quad m_2:\ X_2 - l_2 \Phi_0\,.; \quad m_3:\ X_3 - l_3 \Phi_0\,.$$

Thus, making use of inertia forces, the equations of motion for small vibrations of a wing may be written

$$X_1 - l_1 \Phi_0 = m_1 \omega^2 X_1 y_{11} + m_2 \omega^2 X_2 y_{12} + m_3 \omega^2 X_3 y_{13}\,, \quad \ldots \quad (1)$$

$$X_2 - l_2 \Phi_0 = m_1 \omega^2 X_1 y_{12} + m_2 \omega^2 X_2 y_{22} + m_3 \omega^2 X_3 y_{23}\,, \quad \ldots \quad (2)$$

$$X_3 - l_3 \Phi_0 = m_1 \omega^2 X_1 y_{13} + m_2 \omega^2 X_2 y_{23} + m_3 \omega^2 X_3 y_{33}\,, \quad \ldots \quad (3)$$

where $\omega/2\pi$ is frequency of vibration; and y_{rs} is the deflection parallel to OX at point s due to unit load parallel to OX at point r, when the wing is encastré at its junction with the fuselage.

If we write

$$\begin{gathered} \sqrt{m_1}\, X_1 = x, \quad \sqrt{m_2}\, X_2 = y, \quad \sqrt{m_3}\, X_3 = z; \\ m_1 y_{11} = a_{11}, \quad m_2 y_{22} = a_{22}, \quad m_3 y_{33} = a_{33}\,, \\ \sqrt{m_1 m_2}\, y_{12} = a_{12}\,, \quad \sqrt{m_1 m_3}\, y_{13} = a_{13}, \quad \sqrt{m_2 m_3}\, y_{23} = a_{23}\,; \\ \text{and } \lambda = 1/\omega^2\,, \end{gathered}$$

the equations (1), (2), (3) may be written

$$(a_{11} - \lambda)x + a_{12} y + a_{13} z = -\, l_1 \sqrt{m_1}\, \lambda \Phi_0\,, \quad \ldots \quad (4)$$

$$a_{12} x + (a_{22} - \lambda) y + a_{23} z = -\, l_2 \sqrt{m_2}\, \lambda \Phi_0\,, \quad \ldots \quad (5)$$

$$a_{13} x + a_{23} y + (a_{33} - \lambda) z = -\, l_3 \sqrt{m_3}\, \lambda \Phi_0\,. \quad \ldots \quad (6)$$

Suppose now λ_r ($r = 1, 2, 3$) is a root of equations (4), (5), (6), with $\Phi_0 = 0$, and x_r, y_r, z_r, are its associated rectified modes, *i.e.* in the particular case $x_r^2 + y_r^2 + z_r^2 = 1$. These quantities are obtained conveniently by the escalator technique[3]. Multiply equations (4), (5), (6) by x_r, y_r, and z_r, respectively and add.

We obtain

$$(\lambda_r - \lambda)(xx_r + yy_r + zz_r) = -Q_r \lambda \Phi_0 , \quad \ldots \quad (7)$$

where

$$Q_r = l_1 \sqrt{m_1}\, x_r + l_2 \sqrt{m_2}\, y_r + l_3 \sqrt{m_3}\, z_r , \quad \ldots \quad (8)$$

and

$$r = 1, 2, 3 .$$

Now multiply equation (7) by x_r and take the summation for $r = 1$ to 3.

We obtain

$$\sqrt{m_1}\, X_1 = x = - \sum_1^3 \frac{Q_r x_r}{(\lambda_r - \lambda)} \lambda \Phi_0 \quad \ldots \quad (9)$$

$$\left[\text{N.B. } \sum_1^3 x_r^2 = 1, \quad \sum_1^3 x_r y_r = 0 \text{ etc.}^3\right]$$

Similarly, by multiplying equation (7) in turn by y_r, z_r, and taking their summations,

$$\sqrt{m_2}\, X_2 = y = - \sum_1^3 \frac{Q_r y_r}{(\lambda_r - \lambda)} \lambda \Phi_0 , \quad \ldots \quad (10)$$

$$\sqrt{m_3}\, X_3 = z = - \sum_1^3 \frac{Q_r z_r}{(\lambda_r - \lambda)} \lambda \Phi_0 , \quad \ldots \quad (11)$$

Further, multiply (9) (10), and (11), by $l_1 \sqrt{m_1}$, $l_2 \sqrt{m_2}$, $l_3 \sqrt{m_3}$, respectively, and add.

We obtain

$$m_1 l_1 X_1 + m_2 l_2 X_2 + m_3 l_3 X_3 = - \sum_1^3 \frac{Q_r^2}{(\lambda_r - \lambda)} \lambda \Phi_0 . \quad \ldots \quad (12)$$

2.2. *Fuselage (front part)*.—Similarly, for the front part of the fuselage we have:—

$$(Y_1)_f - Y_0 - L_1 \Phi_0 = (m_1)_f \omega^2 (Y_1)_f (y_{11})_f + (p_1)_f \omega^2 (\Phi_1)_f (z_{11})_f , \quad \ldots \quad (13)$$

$$(\Phi_1)_f - \Phi_0 = (m_1)_f \omega^2 (Y_1)_f (z_{11})_f + (p_1)_f \omega^2 (\Phi_1)_f (\phi_{11})_f , \quad \ldots \quad (14)$$

where the suffix f denotes that the fuselage is here under consideration.

$(m_1)_f$, $(p_1)_f$, are mass and moment of inertia respectively.

$(y_{11})_f$, $(z_{11})_f$, $(\phi_{11})_f$, are deflection coefficients with the front part of the fuselage regarded as encastré at its junction with the wings. They are defined as

$(y_{11})_f$ = linear deflection parallel to OY due to unit load parallel to OY,

$(z_{11})_f$ = angular deflection relative to OX due to unit load parallel to OY, or linear deflection parallel to OY due to unit couple acting in the sense OX to OY,

$(\phi_{11})_f$ = angular deflection relative to OX due to unit couple acting in the sense OX to OY.

Equations (13) and (14) may be written:—

$$(b_{11} - \lambda) x' + b_{12} y' = - \sqrt{(m_1)_f}\, \lambda (Y_0 + L_1 \Phi_0) , \quad \ldots \quad (15)$$

$$b_{12} x' + (b_{22} - \lambda) y' = - \sqrt{(p_1)_f}\, \lambda \Phi_0 , \quad \ldots \quad (16)$$

where

$$(m_1)_f (y_{11})_f = b_{11} ; \quad \sqrt{(m_1)_f (p_1)_f}\, (z_{11})_f = b_{12} ; \quad (p_1)_f (\phi_{11})_f = b_{22} ;$$

and

$$\sqrt{(m_1)_f}\, (Y_1)_f = x', \quad \sqrt{(p_1)_f}\, (\Phi_1)_f = y' ; \quad \lambda = 1/\omega^2 .$$

If λ_r', $(r = 1, 2)$ is a root of equations (15), (16), with $Y_0 = \Phi_0 = 0$, and x'_r, y'_r, are its associated rectified modes[3], we have by analysis similar to that for the wings,

$$\sqrt{(m_1)_f}\,(Y_1)_f = x' = -\left[\sum_1^2 \frac{P_r'\,x_r'}{(\lambda_r'-\lambda)}\,\lambda Y_0 + \sum_1^2 \frac{Q_r'\,x_r'}{(\lambda_r'-\lambda)}\,\lambda \Phi_0\right], \quad \ldots \quad (17)$$

$$\sqrt{(p_1)_f}\,(\Phi_1)_f = y' = -\left[\sum_1^2 \frac{P_r'\,y_r'}{(\lambda'_r-\lambda)}\,\lambda Y_0 + \sum_1^2 \frac{Q_r'\,y_r'}{(\lambda_r'-\lambda)}\,\lambda \Phi_0\right], \quad \ldots \quad (18)$$

where

$$P_r' = \sqrt{(m_1)_f}\,x_r'\,;\quad Q_r' = \sqrt{(m_1)_f}\,L_1\,x_r' + \sqrt{(p_1)_f}\,y_r'\,.$$

And further

$$(m_1)_f\,(Y_1)_f = -\left[\sum_1^2 \frac{P_r'^2}{(\lambda_r'-\lambda)}\,\lambda Y_0 + \sum_1^2 \frac{P_r'\,Q_r'}{(\lambda_r'-\lambda)}\,\lambda \Phi_0\right], \quad \ldots \quad (19)$$

$$(m_1)_f\,L_1\,(Y_1)_f + (p_1)_f\,(\Phi_1)_f = -\left[\sum_1^2 \frac{P_r'\,Q'_r}{(\lambda_r'-\lambda)}\,\lambda Y_0 + \sum_1^2 \frac{Q_r'^2}{(\lambda_r'-\lambda)}\,\lambda\Phi_0\right]. \quad \ldots \quad (20)$$

2.3. *Fuselage (rear part).*—For the rear part of the fuselage we have equations of motion as follows:

$$(Y_2)_f - Y_0 + L_2\Phi_0 = (m_2)_f\,\omega^2\,(Y_2)_f\,(y_{22})_f + (m_3)_f\,\omega^2\,(Y_3)_f\,(y_{23})_f + (m_4)_f\,\omega^2\,(Y_4)_f\,(y_{24})_f\,, \quad \ldots \quad (21)$$

$$(Y_3)_f - Y_0 + L_3\Phi_0 = (m_2)_f\,\omega^2\,(Y_2)_f\,(y_{23})_f + (m_3)_f\,\omega^2\,(Y_3)_f\,(y_{33})_f + (m_4)_f\,\omega^2\,(Y_4)_f\,(y_{34})_f\,, \quad \ldots \quad (22)$$

$$(Y_4)_f - Y_0 + L_4\Phi_0 = (m_2)_f\,\omega^2\,(Y_2)_f\,(y_{24})_f + (m_3)_f\,\omega^2\,(Y_3)_f\,(y_{34})_f + (m_4)_f\,\omega^2\,(Y_4)_f\,(y_{44})_f\,. \quad \ldots \quad (23)$$

The notation should now be self-explanatory.

Equations (21), (22), (23), may be written

$$(b_{33} - \lambda)z' + b_{34}u' + b_{35}\,v' = -\sqrt{(m_2)_f}\,\lambda\,(Y_0 - L_2\,\Phi_0)\,, \quad \ldots \quad (24)$$

$$b_{34}\,z' + (b_{44} - \lambda)\,u' + b_{45}\,v' = -\sqrt{(m_3)_f}\,\lambda\,(Y_0 - L_3\,\Phi_0)\,, \quad \ldots \quad (25)$$

$$b_{35}\,z' + b_{45}\,u' + (b_{55} - \lambda)\,v' = -\sqrt{(m_4)_f}\,\lambda\,(Y_0 - L_4\,\Phi_0)\,, \quad \ldots \quad (26)$$

Where

$$(m_2)_f\,(y_{22})_f = b_{33},\text{ etc.};\quad \sqrt{(m_2)_f\,(m_3)_f}\,(y_{23})_f = b_{34},\text{ etc.};$$

and

$$\sqrt{(m_2)_f}\,(Y_2)_f = z',\quad \sqrt{(m_3)_f}\,(Y_3)_f = u',\quad \sqrt{(m_4)_f}\,(Y_4)_f = v';\quad \lambda = 1/\omega^2\,.$$

If λ_r' ($r = 3, 4, 5$) is a root of equations (24), (25), (26), with

$Y_0 = \Phi_0 = 0$, and z'_r, u'_r, v'_r, are its associated rectified modes[3], we have

$$\sqrt{(m_2)_f}\,(Y_2)_f = z' = -\left[\sum_3^5 \frac{P_r'\,z_r'}{(\lambda_r'-\lambda)}\,\lambda Y_0 + \sum_3^5 \frac{Q_r'\,z_r'}{(\lambda_r'-\lambda)}\,\lambda \Phi_0\right], \quad \ldots \quad (27)$$

$$\sqrt{(m_3)_f}\,(Y_3)_f = u' = -\left[\sum_3^5 \frac{P_r'\,u_r'}{(\lambda_r'-\lambda)}\,\lambda Y_0 + \sum_3^5 \frac{Q_r'\,u_r'}{(\lambda_r'-\lambda)}\,\lambda \Phi_0\right], \quad \ldots \quad (28)$$

$$\sqrt{(m_4)_f}\,(Y_4)_f = v' = -\left[\sum_3^5 \frac{P_r'\,v_r'}{(\lambda_r'-\lambda)}\,\lambda Y_0 + \sum_3^5 \frac{Q_r'\,v_r'}{(\lambda_r'-\lambda)}\,\lambda \Phi_0\right], \quad \ldots \quad (29)$$

where

$$P_r' = \sqrt{(m_2)_f}\,z_r' + \sqrt{(m_3)_f}\,u_r' + \sqrt{(m_4)_f}\,v_r'\,,$$

$$Q_r' = -\,[\sqrt{(m_2)_f}\,L_2 z_r' + \sqrt{(m_3)_f}\,L_3 u_r' + \sqrt{(m_4)_f}\,L_4 v_r']\,.$$

And further

$$(m_2)_f\,(Y_2)_f + (m_3)_f\,(Y_3)_f + (m_4)_f\,(Y_4)_f = -\left[\sum_3^5 \frac{P_r'^2}{(\lambda_r'-\lambda)}\,\lambda Y_0 + \sum_3^5 \frac{P_r'\,Q_r'}{(\lambda_r'-d)}\,\lambda \Phi_0\right], \quad \ldots \quad (30)$$

$$(m_2)_f\,L_2\,(Y_2)_f + (m_3)_f\,L_3\,(Y_3)_f + (m_4)_f\,L_4\,(Y_4)_f = +\left[\sum_3^5 \frac{P_r'\,Q_r'}{(\lambda_r'-\lambda)}\,\lambda Y_0 + \sum_3^5 \frac{Q_r'^2}{(\lambda_r'-\lambda)}\,\lambda \Phi_0\right]. \quad (31)$$

2.4. *Derivation of Frequency Equation.*—From the equilibrium of the inertia forces parallel to OY, we have

$$2(m_1 + m_2 + m_3)\, Y_0 + (m_1)_f (Y_1)_f + (m_2)_f (Y_2)_f + (m_3)_f (Y_3)_f + (m_4)_f (Y_4)_f = 0\,; \quad \ldots \quad (32)$$

and by moments about O, in the plane XOY

$$(m_1)_f L_1 (Y_1)_f + (p_1)_f (\Phi_1)_f + m_1 l_1 X_1 + m_2 l_2 X_2 + m_3 l_3 X_3$$
$$- [(m_2)_f L_2 (Y_2)_f + (m_3)_f L_3 (Y_3)_f + (m_4)_f L_4 (Y_4)_f] = 0\,. \quad \ldots \quad (33)$$

Hence, on substituting the values given by (12), (19), (20), (30), (31), we obtain the equations

$$F_{11}(\lambda)\, \lambda\, Y_0 + F_{12}(\lambda)\, \lambda\, \Phi_0 = 0\,, \quad \ldots \quad (34)$$

$$F_{12}(\lambda)\, \lambda\, Y_0 + F_{22}(\lambda)\, \lambda\, \Phi_0 = 0\,, \quad \ldots \quad (35)$$

where

$$F_{11}(\lambda) = \sum_1^5 \frac{P_r'^2}{(\lambda_r' - \lambda)} - \frac{2(m_1 + m_2 + m_3)}{\lambda}\,, \quad \ldots \quad (36)$$

$$F_{12}(\lambda) = \sum_1^5 \frac{P_r' Q_r'}{(\lambda_r' - \lambda)}\,, \quad \ldots \quad (37)$$

$$F_{22}(\lambda) = \sum_1^3 \frac{Q_r^2}{(\lambda_r - \lambda)} + \sum_1^5 \frac{Q_r'^2}{(\lambda_r' - \lambda)}\,. \quad \ldots \quad (38)$$

The frequency equation is

$$F_{11}(\lambda) \times F_{22}(\lambda) - [F_{12}(\lambda)]^2 = 0\,, \quad \ldots \quad (39)$$

and when a root of this equation has been found the corresponding modes are given by (9), (10), (11), (17), (18), (27), (28), (29).

3. *Numerical Example.*—3.1. *Details of Calculation Leading to Formation of Frequency Equation.*—The units adopted are as follows:—

For force, pounds-weight; for length, inches; for time, seconds; in which circumstances the unit of mass will be

g (in/sec/sec) lb, or approximately 386 lb.

In the aircraft investigated, the calculated values of the various quantities were as follows:—

$m_1 = 2 \cdot 585$, $m_2 = 0 \cdot 610$, $m_3 = 0 \cdot 259$ in Lb sec units.
$l_1 = 53 \cdot 5$, $l_2 = 132 \cdot 0$, $l_3 = 190 \cdot 5$ in;
$y_{11} = 0 \cdot 2088 \times 10^{-6}$ in/Lb, $y_{12} = 0 \cdot 8536 \times 10^{-6}$ in/Lb, $y_{13} = 1 \cdot 3341 \times 10^{-6}$ in/Lb ,
$y_{22} = 8 \cdot 5047 \times 10^{-6}$ in/Lb, $y_{23} = 15 \cdot 928 \times 10^{-6}$ in/Lb, $y_{33} = 39 \cdot 129 \times 10^{-6}$ in/Lb .

$(m_1)_f = 5 \cdot 5285$, $(p_1)_f = 3150 \cdot 5$,
$(y_{11})_f = 121 \cdot 39 \times 10^{-6}$ in/Lb, $(z_{11})_f = 2 \cdot 756 \times 10^{-6}$ rad/Lb ,
$(\phi_{11})_f = 0 \cdot 25 \times 10^{-6}$ rad/Lb in, $L_1 = 63 \cdot 33$ in .

$(m_2)_f = 4 \cdot 780$, $(m_3)_f = 0 \cdot 4637$, $(m_4)_f = 0 \cdot 6140$ in Lb sec units ,
$(y_{22})_f = 0$, $(y_{23})_f = 0$, $(y_{24})_f = 0$,
$(y_{33})_f = 140 \times 10^{-6}$ in/Lb, $(y_{34})_f = 250 \times 10^{-6}$ in/Lb, $(y_{44})_f = 850 \times 10^{-6}$ in/Lb ,
$L_2 = 26 \cdot 0$, $L_3 = 116 \cdot 9$, $L_4 = 223 \cdot 2$ in .

Taking $\bar{\lambda} = 10^6\lambda$, and $\bar{\lambda}' = 10^6\,\lambda'$, it was found that:

For the wings

$\bar{\lambda}_1 = 14\cdot613$; $x_1 = 0\cdot10635$, $y_1 = 0\cdot56201$, $z_1 = 0\cdot82026$; $Q_1 = 146\cdot563$.
$\bar{\lambda}_2 = 0\cdot98186$; $x_2 = -0\cdot40316$, $y_2 = -0\cdot72972$, $z_2 = 0\cdot55225$; $Q_2 = -56\cdot274$.
$\bar{\lambda}_3 = 0\cdot25955$; $x_3 = 0\cdot90828$, $y_3 = -0\cdot38943$, $z_3 = 0\cdot14897$; $Q_3 = 52\cdot432$.

For the front part of the fuselage

$\bar{\lambda}_1' = 1097\cdot79$; $x_1' = 0\cdot64880$, $y_1' = 0\cdot76095$; $P_1' = 1\cdot5255$, $Q_1' = 139\cdot316$.
$\bar{\lambda}_2' = 360\cdot95$; $x_2' = 0\cdot76095$, $x'_2 = -0\cdot64880$; $P_2' = 1\cdot7892$, $Q_2' = 76\cdot886$.

For the rear part of the fuselage

$\bar{\lambda}'_3 = 557\cdot98$; $z'_3 = 0$, $u_3' = 0\cdot26116$, $v_3' = 0\cdot96529$; $P_3' = 0\cdot93423$; $Q_3' = 189\cdot614$.
$\bar{\lambda}_4' = 28\cdot831$; $z'_4 = 0$, $u_4' = 0\cdot96529$, $v_4' = -0\cdot26116$; $P_4' = 0\cdot45270$; $Q_4' = 31\cdot168$.
$\bar{\lambda}_5' = 0$; $z_5' = 1$, $u_5' = 0.$ $v_5' = 0$; $P_5' = 2\cdot1863$; $Q_5' = 56\cdot843$.

Hence,

$$F_{11}(\lambda) = 10^6\left[\frac{2\cdot3272}{(1097\cdot79-\bar{\lambda})} + \frac{0\cdot87278}{(557\cdot98-\bar{\lambda})} + \frac{3\cdot2012}{(360\cdot95-\bar{\lambda})} + \frac{0\cdot20494}{(28\cdot831-\bar{\lambda})} + \frac{11\cdot681}{-\bar{\lambda}}\right],$$

$$F_{12}(\lambda) = 10^6\left[\frac{212\cdot53}{(1097\cdot79-\bar{\lambda})} - \frac{177\cdot14}{(557\cdot98-\bar{\lambda})} + \frac{137\cdot56}{(360\cdot95-\bar{\lambda})} - \frac{14\cdot110}{(28\cdot831-\bar{\lambda})} - \frac{124\cdot274}{-\bar{\lambda}}\right],$$

$$F_{22}(\lambda) = 10^6\left[\frac{19{,}408\cdot9}{(1097\cdot79-\bar{\lambda})} + \frac{35{,}953\cdot5}{(557\cdot98-\bar{\lambda})} + \frac{5{,}911\cdot5}{(360\cdot95-\bar{\lambda})}\right.$$
$$+ \frac{971\cdot44}{(28\cdot831-\bar{\lambda})} + \frac{42{,}961\cdot3}{(14\cdot613-\bar{\lambda})}$$
$$\left.+ \frac{6{,}333\cdot57}{(0\cdot98186-\bar{\lambda})} + \frac{5{,}498\cdot13}{(0\cdot25955-\bar{\lambda})} + \frac{3{,}231\cdot1}{-\bar{\lambda}}\right].$$

3.2. *Numerical Solution of Frequency Equation.*—The solution of equation (39) was effected by rewriting it in the form

$$\frac{F_{11}(\lambda)}{F_{12}(\lambda)} = \frac{F_{12}(\lambda)}{F_{22}(\lambda)} = -\frac{\Phi_0}{Y_0},$$

and plotting curves for the functions

$$\frac{F_{11}(\lambda)}{F_{12}(\lambda)} \text{ and } \frac{F_{12}(\lambda)}{F_{22}(\lambda)}.$$

The points of intersection of these curves give the values of λ satisfying the frequency equation, and also the corresponding value of Φ_0/Y_0.

It should be noted that the curves intersect at $\bar{\lambda} = 1097\cdot79$, $557\cdot98$, $360\cdot95$ and $28\cdot831$, but that these values do not correspond to frequencies of the system. They are roots introduced artificially in virtue of the form in which the frequency equation is written. The graphs of these curves to give the four lowest frequencies are shown in Figs. 2 and 3.

3.3. *Summary of Results*

The calculated natural frequencies of the aircraft in yawing vibration were as follows:—

Fundamental, 5·46 c.p.s.; 1st Overtone, 8·44 c.p.s.; 2nd Overtone, 10·45 c.p.s.; 3rd Overtone, 30·0 c.p.s.

Conversion factors:

1 inch in		$= 25 \cdot 400$ millimetres mm
1 pound mass (Avoir)	lb	$= 0 \cdot 453\ 59$ kilogramme kg
1 pound weight	Lb	$= 0 \cdot 453\ 59$ kilogramme weight Kg
		$= 4 \cdot 448\ 2 \times 10^5$ dynes

To convert British to metric units multiply by the figure given.

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5	—	Resonance Test of Hurricane. R.A.E. Test Note No. S.M.E.2099. August, 1944. (Unpublished.)

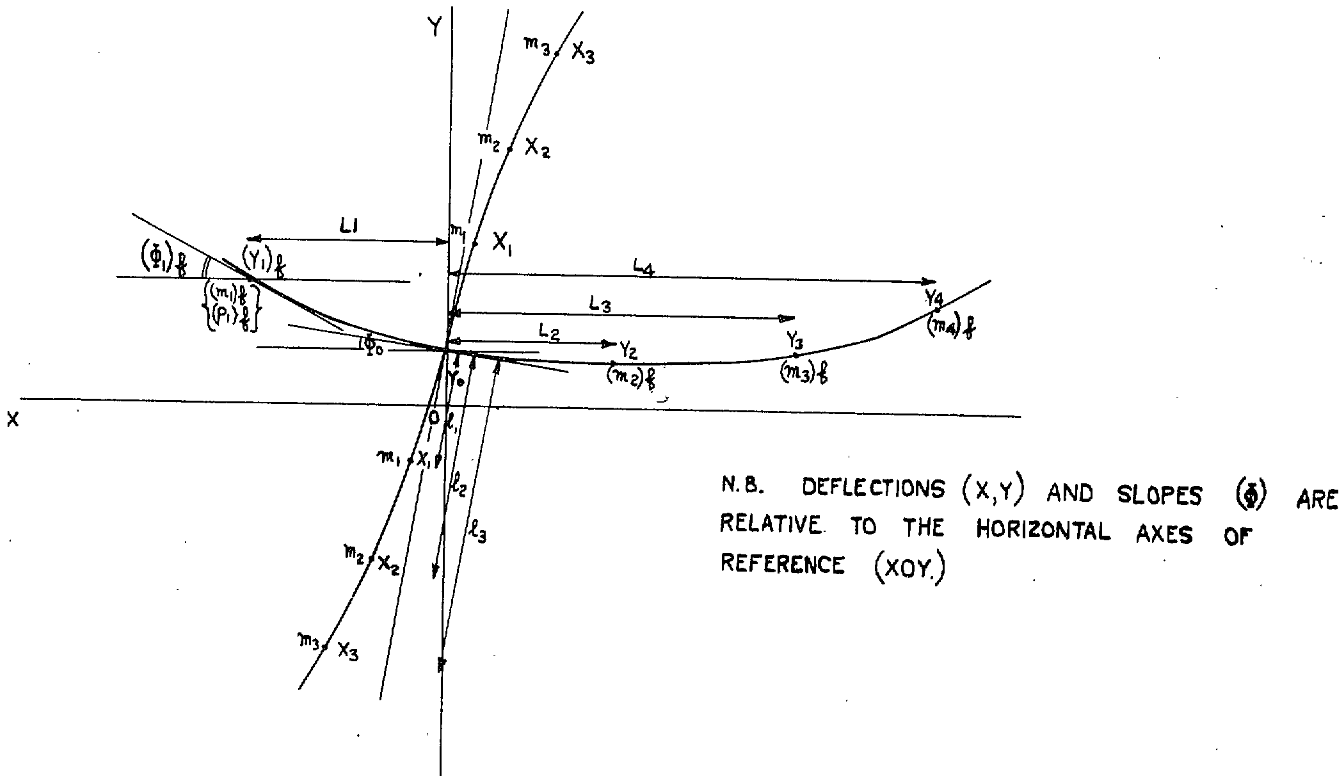

FIG. 1. Diagrammatic Outline of a Skeleton Aircraft Mass System in Yawing Vibration.

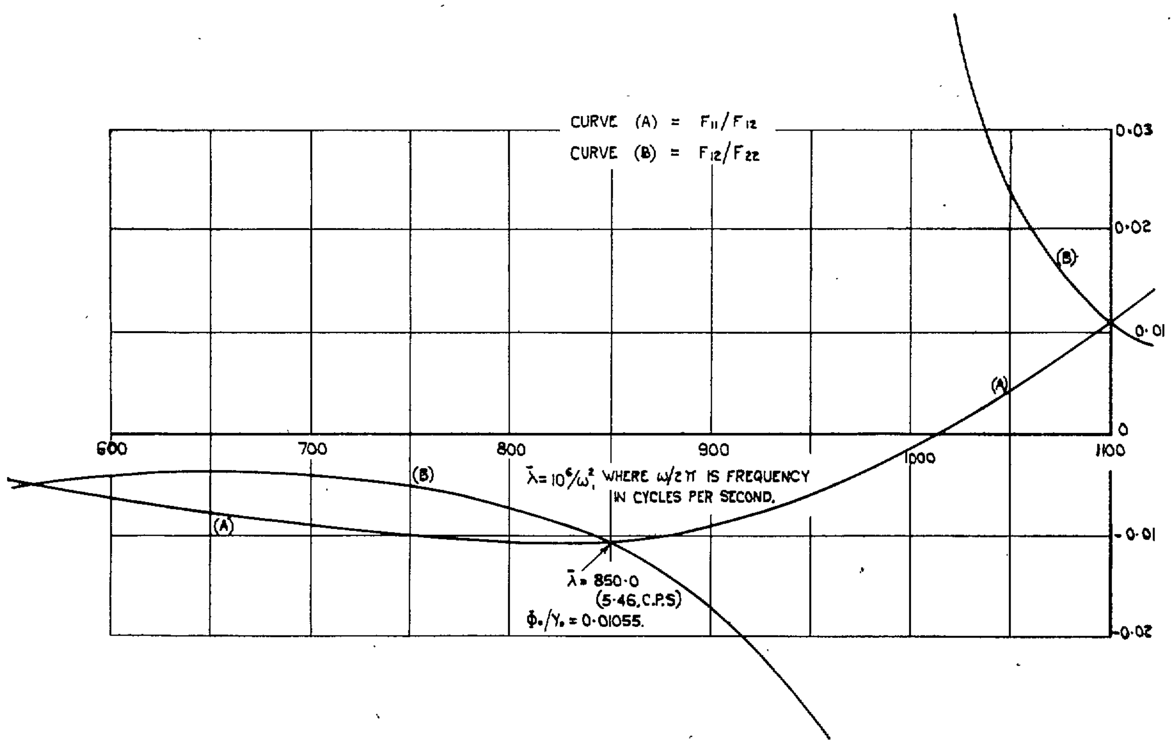

FIG. 2. Curves for Solution of Frequency Equation.

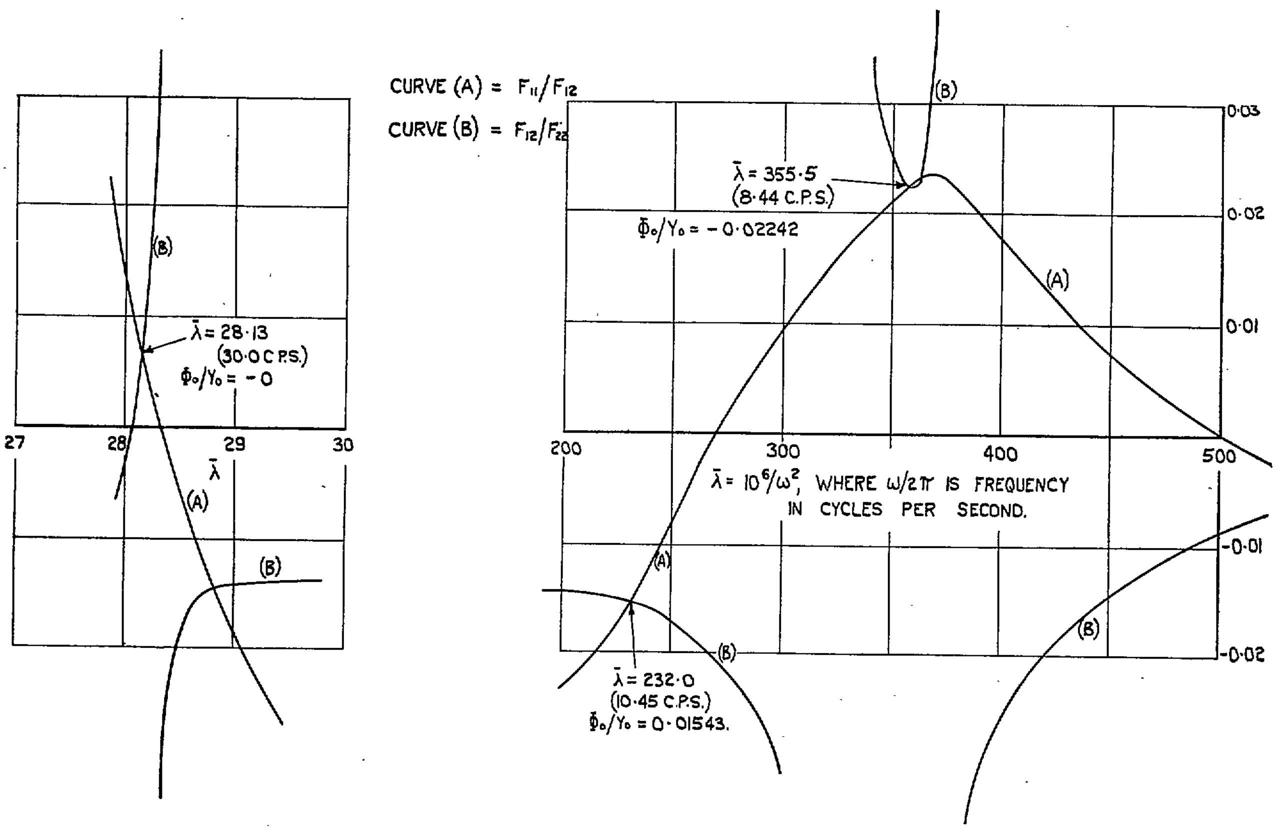

FIG. 3. Curves for Solution of Frequency Equation.

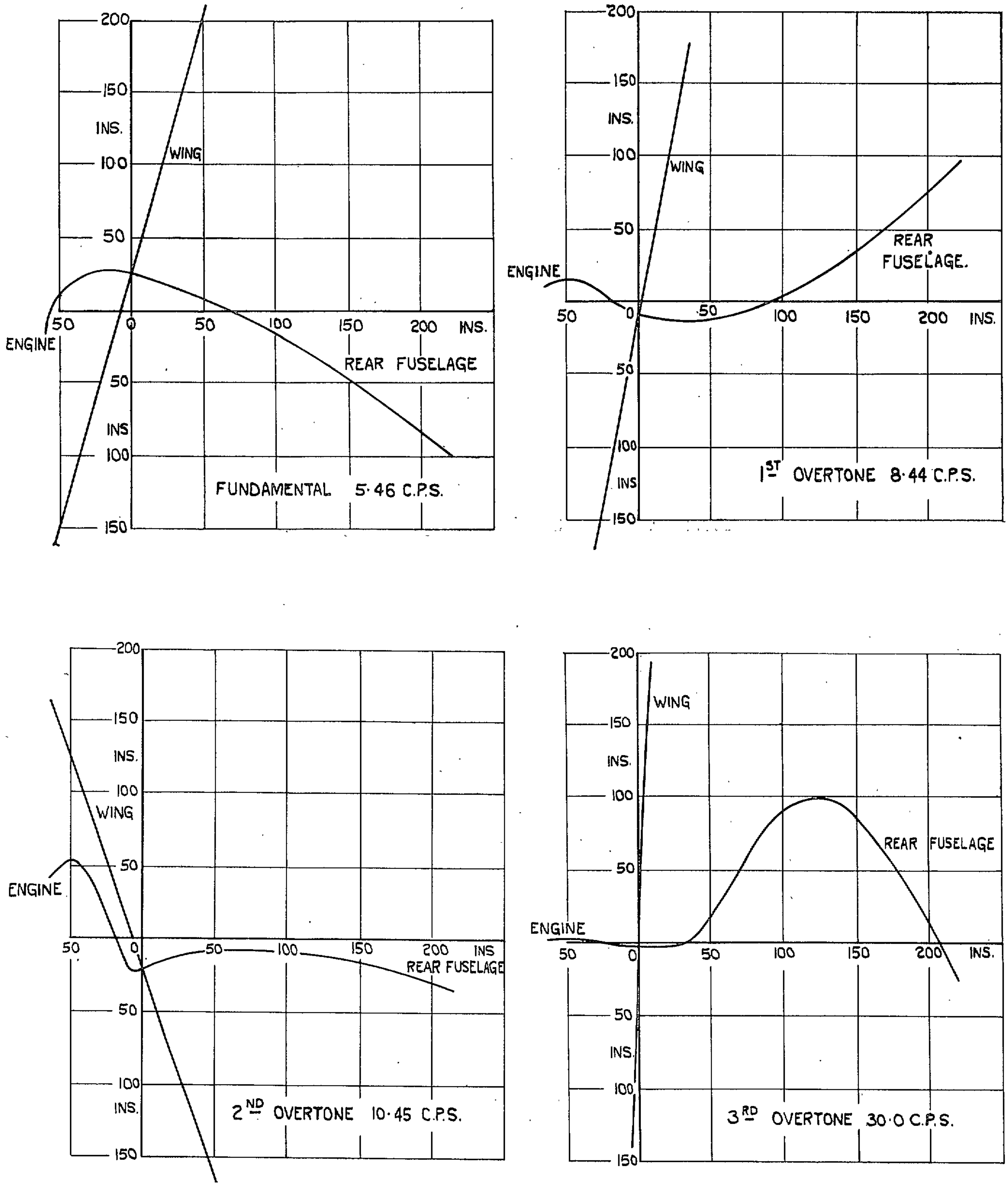

FIG. 4. Calculated Modes. All displacements in horizontal plane.

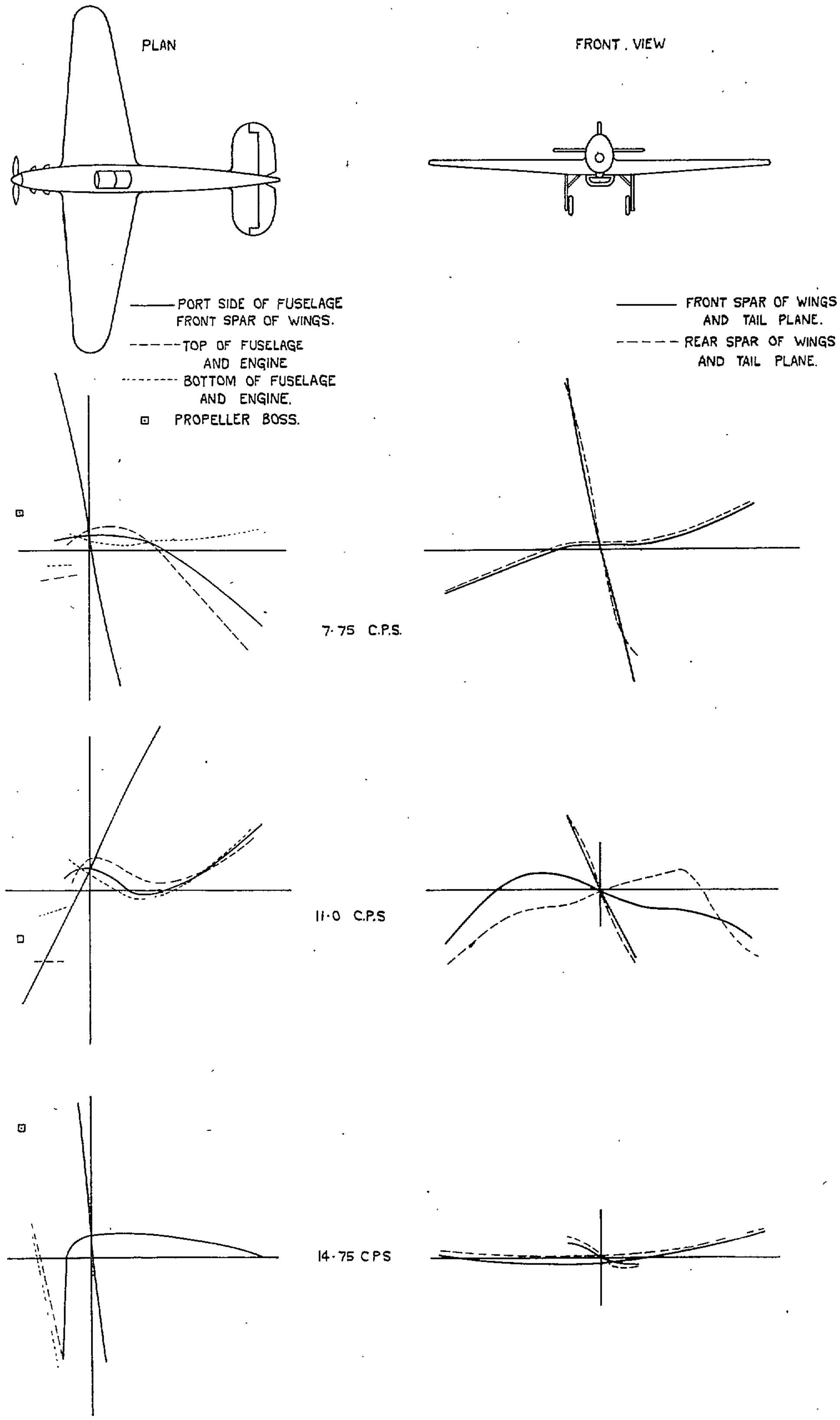

FIG. 5. Modes and Frequencies by Resonance Test.

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