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(New Zealand Scientific Defence Corps,
at present seconded to the Aerodynamics Division of the N.P.L.)

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Summary.—Spoilers, split flaps and sometimes (with thin aerofoils) incidence alone cause the flow to separate from the aerofoil surface. This flow often reattaches to form a closed bubble of virtually stationary air at a fairly constant pressure. The paper sets out a mathematical theory of the subsonic inviscid flow external to the bubble whose position is assumed. The influence of the bubble on the lift and moment coefficients is calculated and some comments are made about the stalling of thin aerofoils.

1. *Introduction.*—Recently interest has arisen in flows about aerofoils and wings in which separation of the flow followed by reattachment has occurred. Ref. 2 describes tunnel tests at low speeds of a delta wing fitted with split flaps. It was found in these tests that for small flap angles a type of flow exists in which the flow reattaches to the wing surface behind the flap, a type of flow also observed by Pearcey and Pankhurst^{4, 5} for aerofoils fitted with spoilers. Küchemann¹ has observed the same type of flow, although in his experiments the flow separation was not produced by spoilers or flaps, but by simply placing thin wings and aerofoils at high incidences. In this case flow separation occurs very close to the leading edge and for a certain range of incidence reattaches to the upper surface forming a closed bubble. In general as the incidence increases so does the bubble length, until eventually the flow does not reattach but forms an infinite wake. This phenomena in the case of aerofoils is known as the 'thin aerofoil stall', since the stall occurs at some stage between the first appearance of the bubble and the incidence at which reattachment fails to occur. A detailed experimental investigation of the thin aerofoil stall is presented in Ref. 3.

With sharp leading edges, spoilers and split flaps, the point of flow separation is clearly fixed, but the point of flow reattachment is, in general, initially unknown. It may be possible to use viscous flow theory to determine the reattachment point of the separated boundary layer⁶ but this difficult problem has not yet been solved. With rounded-nose aerofoils the point of flow separation for the leading-edge stall will be quite close to the leading edge, but its precise position will be unknown until a boundary-layer calculation has been made. In this paper the positions of both flow separation and reattachment will be assumed known.

The bubbles observed on aerofoils are not entirely at constant pressure. For example the leading-edge bubble is at constant pressure for about half of its length, and then some pressure recovery occurs over the rear portion up to the reattachment point. This has been explained thus. The boundary layer is laminar at separation and remains essentially laminar over the constant-pressure portion of the bubble. Transition then occurs resulting in some turbulent

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mixing over the rear part of the bubble. This turbulent mixing is responsible for the observed pressure recovery. The theory given in this paper could be made to allow for this pressure gradient, but in the absence of theoretical or empirical estimates of its magnitude, and of the position of transition, the author has developed the theory for constant-pressure bubbles only.

Fig. 1 gives an idea of the fairly general problem solved in the paper. AB is a spoiler or split flap† of length h and deflection angle ξ_1 , while EF is a normal type of flap. The flow separates from A and reattaches on the flap at G , so that AG is a constant pressure streamline. If the flap is deflected downwards, or if the point A is well above the aerofoil surface, the reattachment is unlikely to occur, and the bubble will be open to infinity. The theory of the flow external to such bubbles, usually termed 'wakes', has been given in Ref. 7. In the special case when the flap is undeflected and the spoiler is at the leading edge and of zero length the theory becomes appropriate to the case of the thin-aerofoil stall.

The theory developed below applies to subsonic compressible flow only.

2. Solution of Laplace's Equation within the Unit Circle with Mixed Boundary Conditions.—As shown in section 3 the determination of the compressible subsonic flow about an aerofoil can be approximately reduced to the solution of Laplace's equation within the unit circle. Although this solution (Poisson's integral) is well known when the boundary conditions are simple, it is not in the case when the boundary conditions are mixed. However, the required solution is easily derived from a result on mixed boundary conditions recently given by the author⁸.

The result referred to is that within the strip $-\infty \leq \delta \leq \infty$, $0 \leq \varepsilon \leq \pi/2$, a complex harmonic function, $f = r + i\theta$, is related to the boundary values of its real and imaginary parts by:

$$f(\zeta) = \frac{1}{\pi} \int_{-\infty}^{\infty} \{ \theta_0 \operatorname{cosech}(\delta^* - \zeta) + r_\varepsilon \operatorname{sech}(\delta^* - \zeta) \} d\delta^*, \quad \dots \quad (1)$$

where $\zeta = \delta + i\varepsilon$, and θ_0 and r_ε are the values of θ and r at $\delta = \delta^*$, on $\varepsilon = 0$ and $\varepsilon = \pi/2$ respectively. The ζ -plane is shown in Fig. 2a.

Now it is easily verified that the infinite strip in the ζ -plane is mapped outside the unit circle in the t -plane, shown in Fig. 2b, by:

$$\operatorname{anh} \zeta = -i \tan k \frac{1 - t e^{i\gamma_0}}{1 + t e^{i\gamma_0}}, \quad \dots \quad (2)$$

where

$$k = \frac{1}{4}(\gamma_1 - \gamma_2), \quad \dots \quad (3)$$

$$\gamma_0 = \frac{1}{2}(\gamma_1 + \gamma_2), \quad \dots \quad (4)$$

and γ_1 and γ_2 are defined in Fig. 2. The elimination of ζ between equations (1) and (2) yields the required solution of Laplace's equation with mixed boundary conditions within the unit circle.

On the circle, $t = -e^{-i\gamma}$, where γ is measured in a clockwise direction from CO (Fig. 2b), and equation (2) yields:

$$\tanh \delta = \frac{-\tan k}{\tan \frac{1}{2}(\gamma - \gamma_0)}, \quad \dots \quad (5)$$

on the surface AB , and

$$\operatorname{coth} \delta = \frac{-\tan k}{\tan \frac{1}{2}(\gamma - \gamma_0)}, \quad \dots \quad (6)$$

on CDA .

† For the purposes of this paper a spoiler is defined as a projection having a small value of h and large value of ξ_1 , whereas a split flap has large h and small ξ_1 .

3. *Compressible Subsonic Flow About an Aerofoil.*—The following notation will be used in the remainder of the paper:

(x, y) The physical plane

$z = x + iy$

n, s Distances measured normal to and along a streamline respectively

(q, θ) Velocity vector in polar co-ordinates

ρ, ρ_0 Local and stagnation densities respectively

∞ As a suffix to denote values at infinity

$U = q_\infty$

M Local Mach number

$\beta = (1 - M^2)^{1/2}$

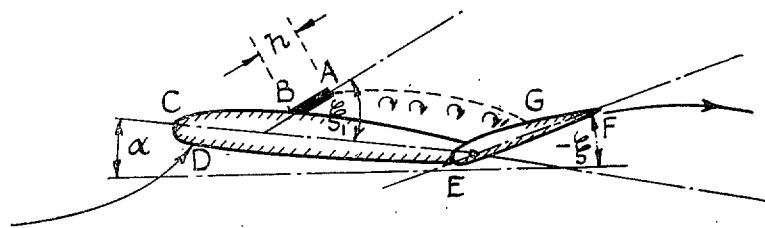
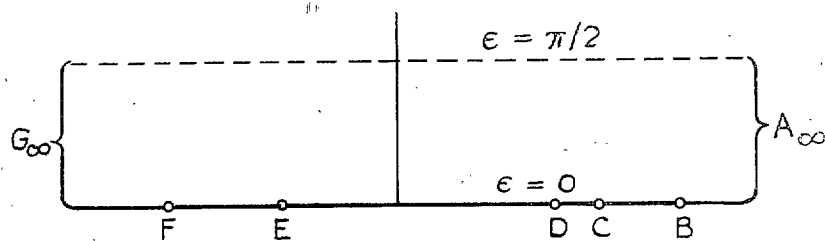
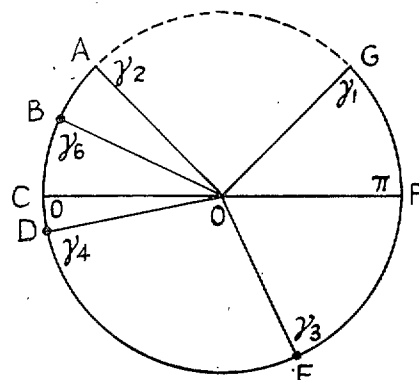


FIG. 1.



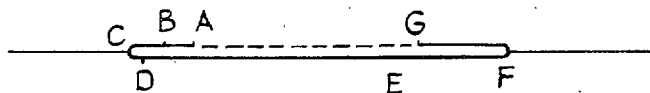
ζ - plane

FIG. 2a.



t - plane

FIG. 2b.



w - plane

FIG. 2c.

(ϕ, ψ) Plane of equipotentials ($\phi = \text{constant}$) and streamlines ($\psi = \text{constant}$), for zero circulation where:

$$d\phi = q ds, \quad d\psi = \frac{\rho}{\rho_0} q dn \quad \dots \quad (7)$$

r defined by:

$$r = \int_{a=U}^a \beta d(\log U/q), \quad \dots \quad (8)$$

f defined by

$$f = r + i\theta \quad \dots \quad (9)$$

m defined by

$$m = \beta \frac{\rho_0}{\rho} \quad \dots \quad (10)$$

w defined by

$$w = \phi + im_\infty \psi \quad \dots \quad (11)$$

Other symbols will be introduced as required in the text.

Now it has been shown¹⁰ that a good approximation to the differential equation of compressible subsonic flow is obtained by putting $m = m_\infty$ (von Kármán's approximation¹¹). Then we find that:

$$\frac{\partial^2 f}{\partial \phi^2} + \frac{1}{m_\infty^2} \frac{\partial^2 f}{\partial \psi^2} = 0,$$

so that from (11) we see that f is approximately an analytic function of w . This approximation is the basis of the following theory (the theory is exact in incompressible flow).

First consider the case of zero circulation. The w -plane (Fig. 2c) is transformed into the t -plane (Fig. 2b) by:

$$w = a(t + 1/t), \quad \dots \quad (12)$$

and the t -plane is in turn transformed into the ζ -plane by equation (2). From the analytic character of these transformations it follows that f is an analytic function of ζ (or approximately so) and is therefore given in terms of its boundary conditions by equation (1). On $\psi = 0$, $t = -e^{-i\psi}$, and (12) yields:

$$\phi = -2a \cos \gamma \quad \dots \quad (13)$$

We now make a basic approximation—the 'mapping' approximation, which is that the overall mapping from the z to the ζ -plane is only negligibly affected by small values of the incidence (α), flap deflection (ξ), and spoiler height (h) or split flap angle (ξ_1). In other words we calculate the first-order effects of small values of α , ξ , h (or ξ_1) in the ζ and t -planes for zero circulation and ignore the second-order effects of these quantities on the actual mapping itself. The same type of approximation was successfully employed in Refs. 7 and 9, and it is also commonly used in unsteady aerofoil theory.

Integrating equation (1) by parts we obtain:

$$f(\zeta) = r_A + i\theta_A + \frac{2}{\pi} \int_{\delta^* = -\infty}^{\infty} \tanh^{-1} \{ \exp(\delta^* - \zeta) \} d\theta_0(\delta^*) - \frac{2}{\pi} \int_{\delta^* = -\infty}^{\infty} \tan^{-1} \{ \exp(\delta^* - \zeta) \} d\gamma_e(\delta^*), \quad \dots \quad (14)$$

where, by the mapping approximation, ζ and δ^* refer to the ζ -plane derived from the z -plane of zero circulation, and where r_A and θ_A are the values of r and θ at $\delta = \infty$, i.e., at point A in Fig. 2. For a constant-pressure bubble lying between A and G , $d\gamma_e = 0$, and the last term of (14) vanishes.

This case will be assumed in the remainder of the paper. However, if a theory is developed to give the position of the boundary-layer transition point on the bubble and the subsequent pressure gradient, the last term of (14) could be retained to allow for this. For the constant-pressure bubble case:

$$f(\zeta) = r_A + i\theta_A + \frac{2}{\pi} \int_{\delta^* = -\infty}^{\infty} \tanh^{-1} \{ \exp(\delta^* - \zeta) \} d\theta_0(\delta^*) \dots \dots (15)$$

A condition not yet mentioned is that changes at the aerofoil can have no effect on the flow direction at infinity, *i.e.*,

$$\lim_{z \rightarrow \infty} f(z) = 0,$$

assuming directions of flow to be measured from the direction at infinity. Now $z \rightarrow \infty$ implies $w \rightarrow \infty$, that is from (2) and (12), $\zeta \rightarrow ik$. Thus the boundary condition at infinity is, from (15):

$$r_A + i\theta_A = -\frac{2}{\pi} \int_{\delta^* = -\infty}^{\infty} \tanh^{-1} \{ \exp(\delta^* - ik) \} d\theta_0(\delta^*) \dots \dots (16)$$

Equations (15) and (16) apply to an aerofoil of any thickness, but in order to simplify the algebra, we shall now confine our attention to thin aerofoils. This will enable us to calculate the first-order effects of the bubbles on aerofoil performance in quite simple forms. The 'skeleton' of our thin aerofoil will thus appear as in Fig. 3a. The values of θ_0 due to the incidence flap and spoiler (or split flap) are clearly those shown in Fig. 3b. In general the front stagnation point will be displaced from the leading edge C, resulting in the reversal in the flow direction in a small interval $\delta_5 - \delta_4$, as shown. The rear stagnation point will remain fixed in accordance with the Joukowski condition.

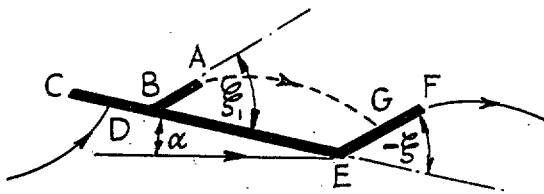


FIG. 3a.

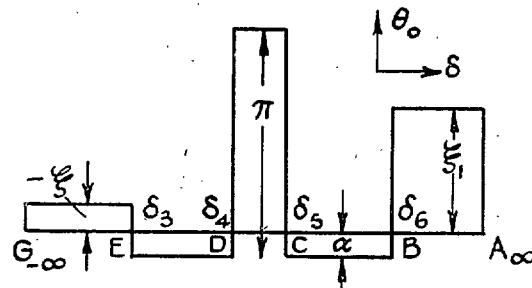


FIG. 3b.

Putting the values of θ_0 shown in Fig. 3 into (15) and (16), we obtain:

$$f(\zeta) = r_A - i(\alpha - \xi_1) + \frac{2\xi}{\pi} \tanh^{-1} \{ \exp(\delta_3 - \zeta) \} + 2 \tanh^{-1} \{ \exp(\delta_4 - \zeta) \} \\ - 2 \tanh^{-1} \{ \exp(\delta_5 - \zeta) \} + \frac{2\xi_1}{\pi} \tanh^{-1} \{ \exp(\delta_6 - \zeta) \}, \dots \dots (17)$$

and

$$r_A - i(\alpha - \xi_1) = -\frac{2\xi}{\pi} \tanh^{-1} \{ \exp(\delta_3 - ik) \} - 2 \tanh^{-1} \{ \exp(\delta_4 - ik) \} \\ + 2 \tanh^{-1} \{ \exp(\delta_5 - ik) \} - \frac{2\xi_1}{\pi} \tanh^{-1} \{ \exp(\delta_6 - ik) \}, \dots \dots (18)$$

since clearly

$$\theta_A = \xi_1 - \alpha.$$

The values of δ_3 and δ_6 depend on the positions of the flap and spoiler hinges. If these positions are defined by γ_3 and γ_6 in the t -plane (the angular co-ordinates (see Fig. 2b)), then from (5):

$$\tanh \delta_3 = \frac{-t_1}{\tan \frac{1}{2}(\gamma_3 - \gamma_0)}, \quad \tanh \delta_6 = \frac{-t_1}{\tan \frac{1}{2}(\gamma_6 - \gamma_0)}, \quad \dots \quad (19)$$

where $t_1 = \tan k$. (20)

In the following analysis it will be assumed that (i) $\gamma_1 \leq \gamma_3 \leq \gamma_1 + \pi$, and hence from (3) and (4), $k \leq \frac{1}{2}(\gamma_3 - \gamma_0) \leq \pi/2 + k$, so that from (19), $\delta_3 < 0$, and (ii) that $\gamma_2 - \pi \leq \gamma_6 \leq \gamma_2$, when from (3), (4) and (19), $\delta_6 > 0$. Under these conditions we deduce from (19) that:

$$\sinh \delta_3 = \frac{-t_1}{\sqrt{\{\tan^2 \frac{1}{2}(\gamma_3 - \gamma_0) - t^2\}}}, \quad \dots \quad (21)$$

and $\sinh \delta_6 = \frac{t_1}{\sqrt{\{\tan^2 \frac{1}{2}(\gamma_6 - \gamma_0) - t^2\}}}, \quad \dots \quad (22)$

results which are required later. From Figs. 2 and 3 it is apparent that $\gamma = 0$ when $\delta = \delta_5$; thus from equation (5):

$$\tanh \delta_5 = t_1/t_0, \quad \dots \quad (23)$$

where $t_0 = \tan \frac{1}{2}\gamma_0$. (24)

Hence $\sinh \delta_5 = \frac{t_1}{\sqrt{(t_0^2 - t_1^2)}}, \quad \cosh \delta_5 = \frac{t_0}{\sqrt{(t_0^2 - t_1^2)}}. \quad \dots \quad (25)$

It remains only to fix the value of δ_4 .

As

$$2 \tanh^{-1} e^{\delta - ik} = \frac{1}{2} \log \left(\frac{\cosh \delta + \cos k}{\cosh \delta - \cos k} \right) + i \tan^{-1} \left(\frac{\sin k}{\sinh \delta} \right) - i\pi \quad (\delta > 0)$$

$$+ 0 \quad (\delta < 0),$$

equation (18) yields the two equations

$$\gamma_A = -\frac{\xi}{2\pi} \log \left(\frac{\cosh \delta_3 + \cos k}{\cosh \delta_3 - \cos k} \right) - \frac{1}{2} \log \left(\frac{\cosh \delta_4 + \cos k}{\cosh \delta_4 - \cos k} \right) + \frac{1}{2} \log \left(\frac{\cosh \delta_5 + \cos k}{\cosh \delta_5 - \cos k} \right)$$

$$- \frac{\xi_1}{2\pi} \log \left(\frac{\cosh \delta_6 + \cos k}{\cosh \delta_6 + \cos k} \right), \quad \dots \quad (26)$$

and

$$\alpha = \frac{\xi}{\pi} \tan^{-1} \left(\frac{\sin k}{\sinh \delta_3} \right) + \tan^{-1} \left(\frac{\sin k}{\sinh \delta_4} \right) - \tan^{-1} \left(\frac{\sin k}{\sinh \delta_5} \right) + \frac{\xi_1}{\pi} \tan^{-1} \left(\frac{\sin k}{\sinh \delta_6} \right), \quad \dots \quad (27)$$

since we have assumed $\delta_3 < 0$ and $\delta_6 > 0$. Equation (27) can be written

$$\sinh \delta_5 = \frac{\sin k (\sinh \delta_4 + \sin k \tan \chi)}{\sin k - \tan \chi \sinh \delta_4}, \quad \dots \quad (28)$$

where

$$\chi = a - \frac{\xi}{\pi} \tan^{-1} \left(\frac{\sin k}{\sinh \delta_3} \right) - \frac{\xi_1}{\pi} \tan^{-1} \left(\frac{\sin k}{\sinh \delta_6} \right), \quad \dots \quad (29)$$

Elimination of $\sinh \delta_5$ from (25) and (28) gives:

$$\sinh \delta_4 = \sin k \cot (\chi + \mu), \quad \dots \quad \dots \quad \dots \quad \dots \quad (30)$$

where
$$\mu = \tan^{-1} \{ \sqrt{(t_0^2 - t_1^2)} \cos k \}. \quad \dots \quad \dots \quad \dots \quad \dots \quad (31)$$

Equations (29) to (31) fix the value of δ_4 .

4. The Pressure Distribution

From equations (21), (22) and (29):

$$\chi = \alpha + \frac{\xi}{\pi} \tan^{-1} [\sqrt{\{\tan^2 \frac{1}{2}(\gamma_3 - \gamma_0) - t_1^2\}} \cos k] - \frac{\xi_1}{\pi} \tan^{-1} [\sqrt{\{\tan^2 \frac{1}{2}(\gamma_6 - \gamma_0) - t_1^2\}} \cos k]. \quad \dots \quad \dots \quad (32)$$

In the case of a split flap ξ_1/π will be small (*see* footnote to section 1, para. 4) and since it has been assumed that α and ξ are small, it follows from (32) that χ is a small number, of the first order, say. On the other hand, suppose that instead of a split flap we have a spoiler. In this case ξ_1/π will not be small, but the spoiler height h will be. In section 5 it is shown that h is proportional to λ , where $\lambda = \gamma_2 - \gamma_6$. If we now assume that λ is of the first order in smallness we can write:

$$\sqrt{\{\tan^2 \frac{1}{2}(\gamma_6 - \gamma_0) - t_1^2\}} = \sec k \sqrt{(\lambda \tan k)}, \quad \dots \quad \dots \quad \dots \quad (33)$$

to first order. Hence (32) can be written:

$$\chi = \alpha - \frac{\xi}{\pi} \tan^{-1} [\cos k \sqrt{\{\tan^2 \frac{1}{2}(\gamma_3 - \gamma_0) - t_1^2\}}] - \frac{\xi_1}{\pi} \sqrt{(\lambda \tan k)} \dots \quad \dots \quad (34)$$

Thus in either case χ is a small first order quantity. In the remainder of this paper second order terms in χ will be ignored.

The approximation $\beta = \beta_\infty$ in equation (8) yields $r = \beta_\infty \log (U/q)$. Thus if C_p is the pressure coefficient, $C_p = 2r/\beta_\infty$. In particular if C_p is the pressure coefficient in the bubble, it follows from (26) that:

$$\begin{aligned} \bar{C}_p = \frac{2}{\beta_\infty} \left\{ -\frac{\xi}{2\pi} \log \left(\frac{\cosh \delta_3 + \cos k}{\cosh \delta_3 - \cos k} \right) - \frac{1}{2} \log \left(\frac{\cosh \delta_4 + \cos k}{\cosh \delta_4 - \cos k} \right) + \frac{1}{2} \log \left(\frac{\cosh \delta_5 + \cos k}{\cosh \delta_5 - \cos k} \right) \right. \\ \left. - \frac{\xi_1}{2\pi} \log \left(\frac{\cosh \delta_6 + \cos k}{\cosh \delta_6 - \cos k} \right) \right\}. \end{aligned}$$

With the aid of equations (21) to (25) and (29) to (31) this can be reduced to:

$$\begin{aligned} \bar{C}_p = \frac{2}{\beta_\infty} \left\{ -\chi \cot \frac{1}{2} \gamma_0 - \frac{\xi}{2\pi} \log \left(\frac{\tan \frac{1}{2}(\gamma_3 - \gamma_0) + \cos k \sqrt{\{\tan^2 \frac{1}{2}(\gamma_3 - \gamma_0) - t_1^2\}}}{\tan \frac{1}{2}(\gamma_3 - \gamma_0) - \cos k \sqrt{\{\tan^2 \frac{1}{2}(\gamma_3 - \gamma_0) - t_1^2\}}} \right) \right. \\ \left. - \frac{\xi_1}{2\pi} \log \left(\frac{\tan \frac{1}{2}(\gamma_0 - \gamma_6) + \cos k \sqrt{\{\tan^2 \frac{1}{2}(\gamma_0 - \gamma_6) - t_1^2\}}}{\tan \frac{1}{2}(\gamma_0 - \gamma_6) - \cos k \sqrt{\{\tan^2 \frac{1}{2}(\gamma_0 - \gamma_6) - t_1^2\}}} \right) \right\}. \quad (35) \end{aligned}$$

In the particular case of a spoiler and no trailing-edge flap, from (34)

$$\chi = \alpha - \frac{\xi_1}{\pi} \sqrt{(\lambda \tan k)}, \quad \dots \quad \dots \quad \dots \quad \dots \quad (36)$$

and $\gamma_0 - \gamma_6 = 2k + \lambda$, so that to first order in $\sqrt{\lambda}$, (35) yields:

$$\bar{C}_p = -\frac{2}{\beta_\infty} \left\{ \alpha \cot \frac{1}{2}\gamma_0 + \frac{\sin(\frac{1}{2}\gamma_0 - k)}{\sin \frac{1}{2}\gamma_0 \sin k} \frac{\xi_1}{\pi} \sqrt{\lambda \tan k} \right\} \chi. \quad \dots \dots (37)$$

On an aerofoil without a bubble the pressure at $\gamma = \gamma_0$ due to incidence alone is well known to be:

$$\bar{C}_p = -\frac{2}{\beta_\infty} \alpha \cot \frac{1}{2}\gamma_0,$$

so that it is interesting to note from the first term on the right-hand side of (37) that the pressure in a bubble produced by incidence alone is equal to the value it would have at the mid-point of the bubble (in angular measure) in its absence.

On the 'wetted' surface of the aerofoil $\zeta = \delta + i\pi/2$; hence from the real part of (17) the pressure coefficient is:

$$C_p - \bar{C}_p = \frac{2}{\beta_\infty} \left\{ \log \left| \frac{\tanh \frac{1}{2}(\delta_5 - \delta)}{\tanh \frac{1}{2}(\delta_4 - \delta)} \right| - \frac{\xi}{\pi} \log |\tanh \frac{1}{2}(\delta_3 - \delta)| \right. \\ \left. - \frac{\xi_1}{\pi} \log |\tanh \frac{1}{2}(\delta_6 - \delta)| \right\}. \quad \dots \dots (38)$$

We shall not express this in terms of the angular co-ordinates in the t -plane, as it is more convenient to calculate the lift and moment using δ as the independent variable. However, before calculating C_L and C_m it is convenient to derive the relationship between h and λ for a spoiler.

5. *The Spoiler Height.*—The spoiler height is given by:

$$h = \int_{\delta=\delta_6}^{\infty} \frac{1}{q} d\phi(\delta),$$

$$\text{Hence, from (13)} \quad \frac{hU}{4a} = \frac{1}{2} \int_{\delta=\delta_6}^{\infty} \frac{U}{q} \sin \gamma d\gamma(\delta). \quad \dots \dots \dots (39)$$

From (17) it follows that on the spoiler, *i.e.*, in $\delta_6 \leq \delta \leq \infty$,

$$\gamma = \int_{q=U}^q \beta d(\log U/q) \simeq \frac{\xi_1}{\pi} \log |\coth \frac{1}{2}(\delta_6 - \delta)| + r_A, \dots \dots \dots (40)$$

selecting only the dominant terms on the spoiler.

In the range of integration of (39) q varies from 0 at $\delta = \delta_6$ to approximately U at $\delta = \infty$, so that an average value of β in the range is approximately $\frac{1}{2}(1 + \beta_\infty)$. To make algebraic progress at this point it is necessary to replace β in equation (40) by this average value. This enables us to write:

$$\frac{U}{q} = \frac{U}{\tilde{q}} \coth^{\varepsilon} \left\{ \frac{1}{2}(\delta_6 - \delta) \right\},$$

in $\delta_6 \leq \delta \leq \infty$, where \tilde{q} is the (constant) velocity on the free streamline of the bubble, and:

$$\varepsilon \equiv \frac{2\xi_1}{\pi(1 + \beta_\infty)}. \quad \dots \dots \dots (41)$$

If c is the aerofoil chord, it is apparent from (13) that:

$$4a \simeq Uc, \quad \dots \dots \dots (42)$$

Hence (39) can be written:

$$\frac{h}{c} = \frac{1}{2} \frac{U}{q} \int_{\delta=\delta_0}^{\infty} \left(\frac{1 + e^{-\delta}/e^{-\delta_0}}{1 - e^{-\delta}/e^{-\delta_0}} \right)^{\epsilon} \sin \gamma \, d\gamma(\delta) \quad \dots \quad (43)$$

After some algebraic manipulation of equation (5) we find that:

$$\sin \gamma = \frac{2 \cos^2 \frac{1}{2} \gamma_0 (t_0 \tanh \delta - t_1) (\tanh \delta + t_1 t_0)}{(\tanh^2 \delta + t_1^2)}, \quad \dots \quad (44)$$

and
$$d\gamma = \frac{2t_1 \operatorname{sech}^2 \delta \, d\delta}{(\tanh^2 \delta + t_1^2)} \quad \dots \quad (45)$$

In the neighbourhood of $\delta = \infty$, (44) is approximately:

$$\sin \gamma = \frac{2 \cos^2 \frac{1}{2} \gamma_0 (t_0 - t_1) (1 + t_1 t_0)}{(1 + t_1^2)}$$

Hence equation (43) yields:

$$\frac{h}{c} = 8 \frac{\cos^2 \frac{1}{2} \gamma_0 t_1 (t_0 - t_1) (1 + t_1 t_0)}{(1 + t_1^2)^2 e^{2\delta_0}} \frac{2}{F^2},$$

where
$$F(\epsilon) = \left\{ \frac{1}{2} \int_0^1 y \left(\frac{1+y}{1-y} \right)^{\epsilon} dy \right\}^{-1/2}, \quad \dots \quad (46)$$

and the second order terms arising from U/\tilde{q} have been neglected. Now $\gamma_0 - \gamma_0 = 2k + \lambda$, so that from (5):

$$\tanh \delta_0 = t_1 \cot (k + \frac{1}{2}\lambda),$$

i.e.,
$$1 - 2e^{-2\delta_0} \simeq 1 - \frac{\lambda}{\sin 2k},$$

or
$$e^{-2\delta_0} \simeq \frac{\lambda}{2 \sin 2k}.$$

This result enables us to reduce the equation for h/c to:

$$h/c = \frac{2 \sin (\gamma_0 - 2k)}{F^2} \lambda \quad \dots \quad (47)$$

The function $F(\epsilon)$ has been tabulated in Ref. 7, where it arose in a similar calculation. It is reproduced below.

ϵ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
F	2.000	1.807	1.612	1.423	1.238	1.058	0.883	0.709	0.534	0.347	0

6. *The Lift Coefficient.*—The lift coefficient is given by:

$$\begin{aligned} C_L &= -\frac{1}{c} \oint C_p \cos \theta_0 \, ds, \\ &= -\frac{1}{c} \oint (C_p - \bar{C}_p) \cos \theta_0 \frac{2a \sin \gamma \, d\gamma}{q}, \text{ by (13)} \end{aligned}$$

$$\approx -\frac{1}{2} \left(\frac{4a}{Uc} \right) \int_{\gamma_1}^{\gamma_2} (C_p - \bar{C}_p) \sin \gamma \, d\gamma, \quad (48)$$

with neglect of second-order terms, and making use of the fact that $C_p - \bar{C}_p$ vanishes over the bubble surface. From (38), (42), (44), (45) and (48) we find that:

$$C_L = -\frac{4 \tan k}{\beta_\infty} \left\{ I_5 - I_4 - \frac{\xi}{\pi} I_3 - \frac{\xi_1}{\pi} I_6 \right\}, \quad (49)$$

where

$$I_i = \int_{-\infty}^{\infty} F(\delta) \log \left| \tanh \frac{1}{2}(\delta_i - \delta) \right| d\delta,$$

and

$$F(\delta) = \frac{\cos^2 \frac{1}{2} \gamma_0 \operatorname{sech}^2 \delta (t_0 \tanh \delta - t_1)(\tanh \delta + t_1 t_0)}{(\tanh^2 \delta + t_1^2)^2}$$

The integral I_i can be evaluated by integrating $F(z) \log \tanh \frac{1}{2}(\delta_i - z)$ around the infinite rectangle $-\infty \leq x \leq \infty, y = 0$; $-\infty \leq x \leq \infty, y = i\pi$, suitably indented to eliminate the logarithmic singularities. It is found that:

$$I_i = \frac{\pi \cos^2 k}{2} \left\{ \frac{\cos \gamma_0 \cos k \sinh \delta_i + \sin k \sin \gamma_0 \cosh \delta_i}{\sinh^2 \delta_i + \sin^2 k} \right\}. \quad (50)$$

Making use of (19) to (25), (30), (31), (32) and (50) we find, after some reduction, that (49) can be written:

$$\begin{aligned} C_L = \frac{2\pi \cos k}{\beta_\infty} & \left\{ \alpha \cos k + \frac{\xi}{2\pi} \left(2 \cos k \tan^{-1} [\cos k \sqrt{\{\tan^2 \frac{1}{2}(\gamma_3 - \gamma_0) - t_1^2\}}] \right. \right. \\ & - (\cos \gamma_3 + \cos \gamma_0) \sqrt{\{\tan^2 \frac{1}{2}(\gamma_3 - \gamma_0) - t_1^2\}} \\ & - \frac{\xi_1}{2\pi} \left(2 \cos k \tan^{-1} [\cos k \sqrt{\{\tan^2 \frac{1}{2}(\gamma_6 - \gamma_0) - t_1^2\}}] \right. \\ & \left. \left. - (\cos \gamma_6 + \cos \gamma_0) \sqrt{\{\tan^2 \frac{1}{2}(\gamma_6 - \gamma_0) - t_1^2\}} \right) \right\}, \quad (51) \end{aligned}$$

In the case when the split flap reduces to a spoiler and $\xi = 0$, we find from (33) and the definition of λ that (51) reduces to:

$$C_L = \frac{2\pi \cos k}{\beta_\infty} \left[\alpha \cos k - \frac{\xi_1}{\pi} \{ \cos k - \cos(\gamma_0 - k) \} \sqrt{(\lambda \tan k)} \right]. \quad (52)$$

Using (47) to eliminate λ we arrive at:

$$C_L = \frac{2\pi \cos k}{\beta_\infty} \left[\alpha \cos k - \frac{\xi_1}{\pi} \{ \cos k - \cos(\gamma_0 - k) \} F \sqrt{\left(\frac{(h/c) \tan k}{2 \sin(\gamma_0 - 2k)} \right)} \right]. \quad (53)$$

Special Cases

(a) *No Bubble at all.*—In this case $h = 0$, and from Fig. 3 it is clear that if the bubble vanishes we must have $\gamma_6 = \gamma_0$, and $\gamma_3 = 2\pi - \gamma_0$ (see paragraph following equation (20)). In this case (51) reduces to the well-known result:

$$C_L = \frac{2\pi}{\beta_\infty} \left\{ \alpha + \frac{\xi}{\pi} (\pi - \gamma_0 + \sin \gamma_0) \right\}.$$

(b) *Bubble Starting from Leading Edge.*—From equations (3) and (4) we find that in this case $k = \frac{1}{2}\gamma_0$. This case is quite important as it is appropriate to the 'thin-aerofoil stall' discussed in the introduction. Equation (51) reduces to:

$$C_L = \frac{2\pi}{\beta_\infty} \left\{ \alpha \cos^2 k + \frac{\xi}{\pi} \left(\cos^2 k \tan^{-1} \frac{\sqrt{\{\sin \frac{1}{2}\gamma_3 \sin (\frac{1}{2}\gamma_3 - 2k)\}}}{\cos (\frac{1}{2}\gamma_3 - k)} - \cos \frac{1}{2}(\gamma_3 + 2k) \sqrt{\{\sin \frac{1}{2}\gamma_3 \sin (\frac{1}{2}\gamma_3 - 2k)\}} \right) \right\}. \quad (54)$$

Some consequences of this result are discussed in section 8.

(c) *Bubble Closing at the Trailing Edge.*—Consider, for simplicity, the case of the spoiler only. Substituting $\gamma_0 = \pi - 2k$ in (53) we obtain:

$$C_L = \frac{2\pi \cos k}{\beta_\infty} \left\{ \alpha \cos k - \frac{\xi_1}{\pi} F \sqrt{\left(\frac{h}{2c} \cos 2k\right)} \right\} \quad (55)$$

It is interesting to compare this result with one obtained in Ref. 7 for a spoiler behind which extends a constant-pressure bubble of infinite extent. In this case the pressure in the bubble or wake will equal the value at infinity outside the wake. The result referred to, in the notation of this paper reads:

$$C_L = \frac{2\pi}{\beta_\infty} \cos k \left\{ \alpha \cos^3 k - \frac{\xi_1}{\pi} F \sqrt{\left(\frac{h}{c} \cos 2k\right)} \right\} \quad (56)$$

On comparison of (55) and (56) it is apparent that although having the bubble open to infinity improves the spoiler effectiveness it reduces the lift slope due to incidence.

7. *Moment Coefficient About Mid-chord Point.*—The moment coefficient about the mid-chord point is:

$$C_m = \frac{1}{c} \oint \left(\frac{x}{c} \cos \theta_0 + \frac{y}{c} \sin \theta_0 \right) C_p \frac{ds}{d\phi} \frac{d\phi}{dy} dy,$$

when the origin of the (x, y) -plane is at the mid-chord point: For thin aerofoils $Ux \doteq -2a \cos \gamma$, $y \doteq 0$.

Therefore
$$C_m = -\frac{1}{4} \left(\frac{4a}{Uc} \right)^2 \oint \frac{U}{q} C_p \sin \gamma \cos \gamma d\gamma$$

or
$$C_m = -\frac{1}{4} \int_{\gamma_1}^{\gamma_2} (C_p - \bar{C}_p) \sin \gamma \cos \gamma d\gamma, \quad (57)$$

with neglect of second-order terms. From equation (5):

$$\cos \gamma = \frac{\cos \gamma_0 (\tanh^2 \delta - t_1^2) + 2 \sin \gamma_0 t_1 \tanh \delta}{\tanh^2 \delta + t_1^2};$$

thus using (38), (44) and (45) we can write (57) in the form:

$$C_m = -\frac{2 \tan k}{\beta_\infty} \left\{ J_5 - J_4 - \frac{\xi}{\pi} J_3 - \frac{\xi_1}{\pi} J_6 \right\},$$

where

$$J_i = \int_{-\infty}^{\infty} \frac{\cos^2 \frac{1}{2}\gamma_0 (t_0 \tanh \delta - t_1) (\tanh \delta + t_1 t_0) \{ \cos \gamma_0 (\tanh^2 \delta - t_1^2) + 2 \sin \gamma_0 t_1 \tanh \delta \}}{\cosh^2 \delta (\tanh^2 \delta + t_1^2)^3} \times \log |\tanh \frac{1}{2}(\delta_i - \delta)| d\delta.$$

Evaluating this integral by a method similar to that for the corresponding integral (I_i) of the previous section we find:

$$J_i = \frac{\pi \cos^4 k \{ \sin k \cosh \delta_i \sin 2\gamma_0 (2 \tanh^2 \delta_i + t_1^2 \tanh^2 \delta_i - t_1^4) - \cos k \sinh \delta_i \cos 2\gamma_0 (2t_1^4 + t_1^2 - \tanh^2 \delta_i) \}}{4(\tanh^2 \delta_i + t_1^2) (\sinh^2 \delta_i + \sin^2 k)} \quad \dots \quad (58)$$

From (23) to (25), (30), (31) and (58) it is found that, to first order in χ :

$$J_5 - J_4 = \frac{\pi}{2} \sin k \cos^2 k (\cos \gamma_0 - \frac{1}{2} \cot^2 k) \chi, \quad \dots \quad (59)$$

while from (19) to (22) and (58):

$$J_3 = \frac{\pi}{4 \tan k} \sqrt{\{\sin^2 \frac{1}{2}(\gamma_3 - \gamma_0) - \sin^2 k\} [\cos \frac{1}{2}(\gamma_3 - \gamma_0) \cos 2\gamma_0 - \cos \frac{1}{2}(\gamma_3 + 3\gamma_0) \{\cos^2 k + \cos(\gamma_3 - \gamma_0)\}]},$$

$$J_6 = \frac{-\pi}{4 \tan k} \sqrt{\{\sin^2 \frac{1}{2}(\gamma_6 - \gamma_0) - \sin^2 k\} [\cos \frac{1}{2}(\gamma_6 - \gamma_0) \cos 2\gamma_0 - \cos \frac{1}{2}(\gamma_6 + 3\gamma_0) \{\cos^2 k + \cos(\gamma_6 - \gamma_0)\}]}$$

Substituting these values in the equation for C_m we obtain finally:

$$C_m = \frac{\pi}{2\beta_\infty} \left(\cos^4 k (1 - 2 \cos \gamma_0 \tan^2 k) \chi \right. \\ \left. + \frac{\xi}{\pi} \sqrt{\{\sin^2 \frac{1}{2}(\gamma_3 - \gamma_0) - \sin^2 k\} [\cos \frac{1}{2}(\gamma_3 - \gamma_0) \cos 2\gamma_0 - \cos \frac{1}{2}(\gamma_3 + 3\gamma_0) \{\cos^2 k + \cos(\gamma_3 - \gamma_0)\}]} \right. \\ \left. - \frac{\xi_1}{\pi} \sqrt{\{\sin^2 \frac{1}{2}(\gamma_6 - \gamma_0) - \sin^2 k\} [\cos \frac{1}{2}(\gamma_6 - \gamma_0) \cos 2\gamma_0 - \cos \frac{1}{2}(\gamma_6 + 3\gamma_0) \{\cos^2 k + \cos(\gamma_6 - \gamma_0)\}]} \right) \quad \dots \quad (60)$$

where χ is given by equation (32).

In the case when the aerofoil is fitted with a spoiler only it is found from (34), (47) and (60) that:

$$C_m = \frac{\pi \cos k}{2\beta_\infty} \left[\cos^3 k (1 - 2 \cos \gamma_0 \tan^2 k) \alpha \right. \\ \left. + \frac{\xi_1 F}{\pi} \sqrt{\left\{ \frac{(h/c) \tan k}{2 \sin(\gamma_0 - 2k)} \right\} \left\{ \cos 2k \cos(2\gamma_0 - k) + \sin k \cos k \sin(2\gamma_0 - k) - \cos^3 k (1 - 2 \cos \gamma_0 \tan^2 k) \right\}} \right] \quad \dots \quad (61)$$

Special Cases

(a) *No Bubble at all.*—In the same way as in the corresponding part of the previous section we find from (60) that:

$$C_m = \frac{\pi}{2\beta_\infty} \left\{ \alpha + \frac{\xi}{\pi} (\pi - \gamma_0 + \sin \gamma_0 \cos \gamma_0) \right\},$$

which agrees with the standard result for an aerofoil with a hinged flap.

(b) *Bubble Starting from Leading Edge.*—This special case is found by putting $k = \frac{1}{2}\gamma_0$ in equations (34) and (60). Consider for simplicity the case when $\xi = \xi_1 = 0$. Equations (32) and (60) yield:

$$C_m = \frac{\pi}{2\beta_\infty} \cos^2 k (2 - 5 \cos^2 k + 4 \cos^4 k) \alpha. \quad \dots \dots \dots (62)$$

For this case (54) reads:

$$C_L = \frac{2\pi}{\beta_\infty} \cos^2 k \alpha, \quad \dots \dots \dots (63)$$

so that the moment coefficient about the leading edge, C_m' , is

$$C_m' = C_m - \frac{1}{2}C_L,$$

i.e.,
$$C_m' = -\frac{\pi}{2\beta_\infty} \cos^4 k (5 - 4 \cos^2 k) \alpha. \quad \dots \dots \dots (64)$$

From (63) and (64) the centre of pressure is at a distance \bar{x} from the leading edge given by:

$$\frac{\bar{x}}{c} = \frac{\cos^2 k}{4} (5 - 4 \cos^2 k). \quad \dots \dots \dots (65)$$

(c) *Bubble Closing at the Trailing Edge.*—Putting $\gamma_0 = \pi - 2k$ in (61) we find:

$$C_m = \frac{\pi \cos k}{2\beta_\infty} \left\{ \cos^3 k (1 + 2 \cos 2k \tan^2 k) \alpha + \frac{\xi_1}{\pi} F \sqrt{\left\{ \frac{1}{2}(h/c) \sec 2k \right\} \left(\frac{\cos k \sin^2 k \cos 4k - \cos 3k \cos^2 2k}{\cos k} \right)} \right\}. \quad \dots \dots (66)$$

8. *Thin Aerofoil Stall**.—In this case the bubble starts from the leading edge³, and assuming that the controls are undeflected, from (37), (55), (62) and (65), we find that the appropriate equations are:

$$\tilde{C}_p = -\frac{2\alpha}{\beta_\infty} \cot k, \quad \left(\frac{1}{2}\gamma_0 = k \text{ in this case} \right) \quad \dots \dots (67)$$

$$C_L = \frac{2\pi\alpha}{\beta_\infty} \cos^2 k, \quad \dots \dots \dots (68)$$

$$C_m = \frac{\pi\alpha}{2\beta_\infty} \cos^2 k (2 - 5 \cos^2 k + 4 \cos^4 k), \quad \dots \dots \dots (69)$$

and

$$\frac{\bar{x}}{c} = \frac{\cos^2 k}{4} (5 - 4 \cos^2 k). \quad \dots \dots \dots (70)$$

The length (lc) of the bubble is given by (see Fig. 4):

$$cl = c \sin^2 2k. \quad \dots \dots \dots (71)$$

* See Ref. 11 for a more recent account, which differs in some details from that given here. The principal change is that Ref. 12 includes a closure condition (of the bubble on to the aerofoil) not given here. This results in small modifications to the equations in this section but the general conclusions are unaffected.

From (67) and (71):

$$C_L = \frac{\pi}{\beta_\infty} \{1 + \sqrt{(1-l)}\} \alpha. \quad \dots \quad (72)$$

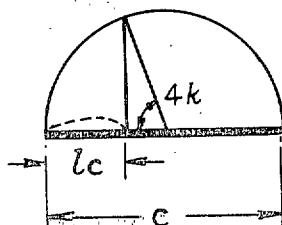


FIG. 4.

From these equations it is apparent that further progress requires a knowledge of the relationship between the length of the bubble and the incidence. At present there is insufficient experimental data on this relationship.

Fig. 5 shows the (l, α) relationship obtained experimentally³ for a 4.11 per cent thick double wedge and a 6 per cent thick symmetrical aerofoil. It will be noticed that the length of constant pressure in the bubble is much smaller than the total bubble length (over which reversed flow occurs). The theory of this paper is based on the assumption of a constant pressure over the whole bubble length. For purposes of a rough calculation we will take the average of the two lengths to be that appropriate to our theory. This average length is approximated to quite well by the straight line:

$$l = \lambda \begin{cases} 0 & \alpha < \alpha_1 \\ (\alpha - \alpha_1) & \alpha > \alpha_1 \end{cases}, \quad \dots \quad (73)$$

and α_1 being constants.

The stalling incidence α_s is the incidence at which $\delta C_L / \delta \alpha = 0$, i.e., the incidence corresponding to $C_{L \max}$. From (72) and (73) it is found that:

$$\alpha_s = \frac{2}{9\lambda} \{2 + 3\lambda\alpha_1 + \sqrt{4 + 3\lambda\alpha_1}\}, \quad \dots \quad (74)$$

and

$$l_s = \frac{1}{9} \{4 - 3\lambda\alpha_1 + 2\sqrt{4 + 3\lambda\alpha_1}\}, \quad \dots \quad (75)$$

where l_s is the bubble length when $\alpha = \alpha_s$. The relation between l_s and $\lambda\alpha_1$ is given in the following table:

$\lambda\alpha_1$	0	1	2	3	4
l_s	0.89	0.70	0.48	0.25	0

Clearly when $\lambda\alpha_1 \geq 4$ the stall will occur immediately the bubble starts to grow, and since λ is large these stalls will occur quite suddenly. They are known as 'leading-edge' stalls and are considered to be distinct from thin aerofoil stalls. It is the author's opinion that they are merely a limiting case of the thin aerofoil stall. This view would be strengthened if it could be demonstrated that $\lambda\alpha_1$ increases with aerofoil thickness from its value of about 0.75 (cf. Fig. 5) for very thin aerofoils, to a value of about 4 for aerofoils with thicknesses of the order of 10 per cent, which are known to stall suddenly. Above this thickness the stall is quite a different type, known as the trailing-edge stall, and the theory given in Ref. 7 is more appropriate.

If we put $\lambda\alpha_1$ equal to its experimental value of about 0.75 in (75), we find that $l_s \approx 0.75$, hence from Fig. 5, $\alpha_s \approx 7\frac{1}{2}$ deg for the double wedge and $\alpha_s = 10$ deg for NACA. 64A006. These values agree within $\frac{1}{2}$ deg with the experimental values given in Ref. 3.

From equations (70) and (71) it follows that the centre of pressure is related to the bubble length by:

$$\frac{\bar{x}}{c} = \frac{1}{8} \{1 + \sqrt{(1-l)}\} \{3 - 2\sqrt{(1-l)}\}. \quad \dots \dots \dots (76)$$

This relation is plotted in Fig. 6. The maximum rearward position of the centre of pressure is found to be at $\bar{x} = (25/64)c$, and this occurs when $l = 15/16$. For an infinitely long bubble it follows from the theory given in Ref. 7 that $\bar{x} = (5/16)c$. As the end of the bubble moves off the aerofoil surface with increasing incidence, the centre of pressure moves forward again. The same phenomena probably occurs when the bubble is due to a split flap or spoiler (if desired this point could be checked from the theory of this paper and Ref. 7) and if this is so it provides a possible explanation of the nose-up moments that have been experienced when the split-flap deflection increases beyond a certain critical value.

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REFERENCES

<i>No.</i>	<i>Author</i>	<i>Title, etc.</i>
1	D. Küchemann	Types of flow on swept wings with special reference to free boundaries and vortex sheets. R.A.E. Tech. Note Aero. 2234. A.R.C. 15,756. March, 1953.
2	J. F. Holford and J. W. Leathers	Low-Speed Tunnel tests of some split-flap arrangements on a 48 deg. delta wing. R. & M. 2996. September, 1952.
3	G. B. McCullough and D. E. Gault	Examples of three representative types of airfoil-section stall at low speed. N.A.C.A. Tech. Note 2502. September, 1951.
4	H. H. Pearcey and R. C. Pankhurst	Survey of progress in N.P.L. High-Speed Wind-Tunnel tests of spoilers on an aerofoil with 0.25c flaps. A.R.C. 15,291. October, 1952.
5	H. H. Pearcey, R. C. Pankhurst and G. F. Lee	Further results from N.P.L. High-Speed Wind-Tunnel tests of spoilers on an aerofoil with 0.25c flaps: Interim note on small spoilers on the trailing edge of the deflected flap. A.R.C. 15,415. November, 1952.
6	L. F. Crabtree	Comments on Mr. Falkner's 'Maximum Life' programme. R.A.E. Tech. Memo. Aero. 380. A.R.C. 16,159. September, 1953.
7	L. C. Woods	Theory of aerofoil spoilers, R. & M. 2969. May, 1953.
8	L. C. Woods	Compressible subsonic flow in two-dimensional channels with mixed boundary conditions. <i>Quart. J. Mech. App. Math.</i> Vol. III, Pt. 3, pp. 263 to 282. September, 1954.
9	L. C. Woods	The theory of aerofoils with hinged flaps in two-dimensional compressible flow. C.P. 138. August, 1952.
10	L. C. Woods	The two-dimensional subsonic flow of an inviscid fluid about an aerofoil of arbitrary shape. R. & M. 2811. November, 1950
11	L. C. Woods	Generalized aerofoil theory. <i>Proc. Roy. Soc. A.</i> Vol. 238. pp. 358 to 388. 1956.
12	T. von Kármán	Compressibility effects in aerodynamics. <i>J. Ae. Sci.</i> Vol. 8. p. 337. July, 1941.

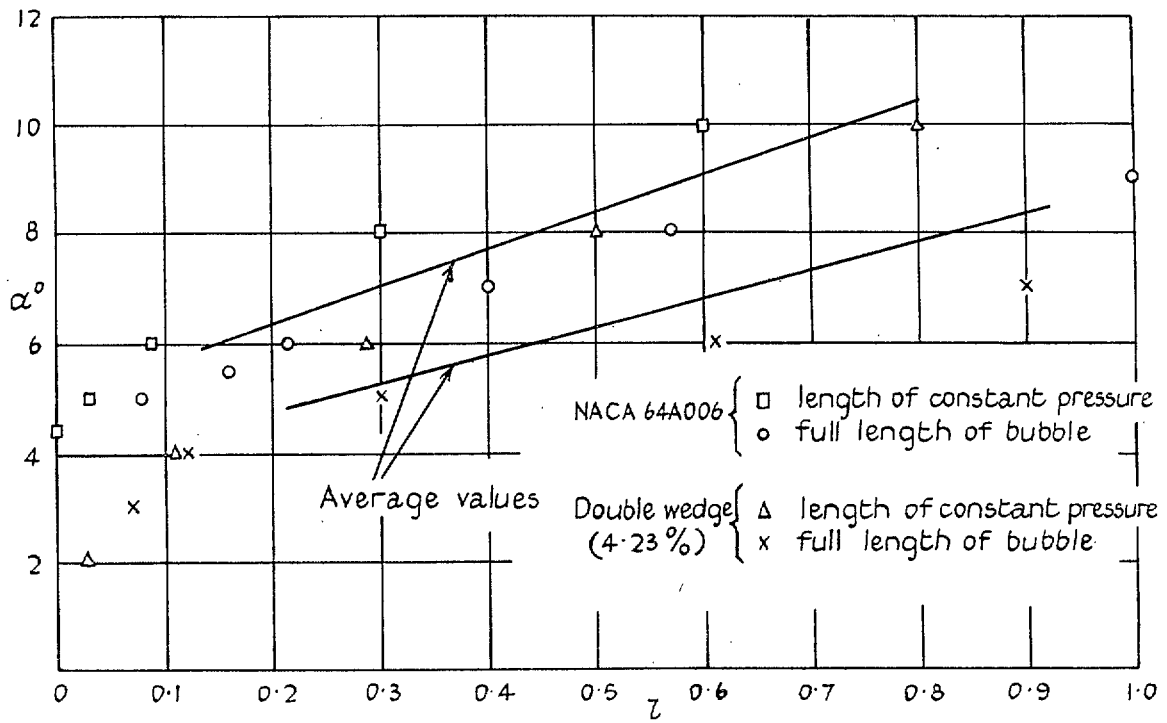


FIG 5. Relation between bubble length and incidence (From Ref. 3).

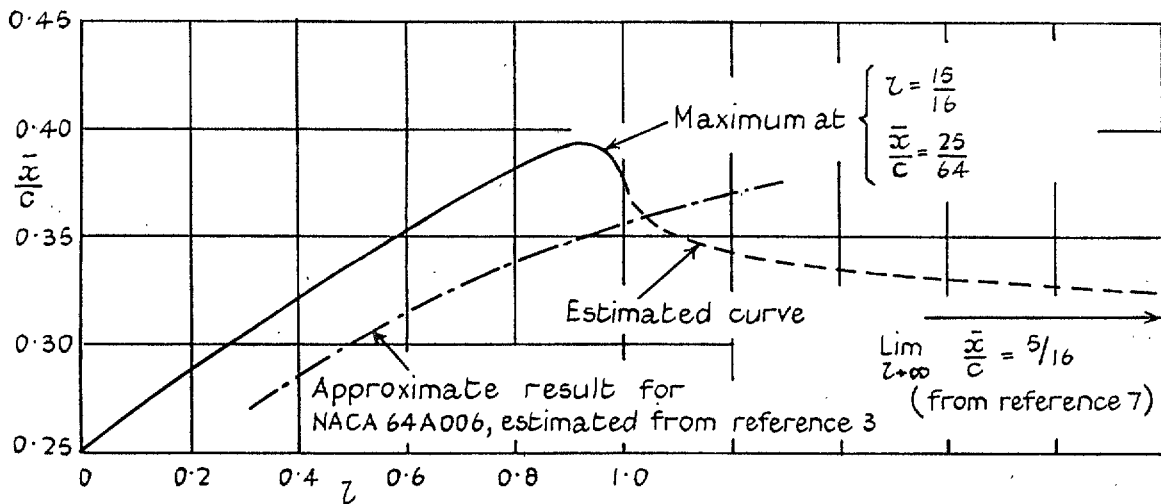


FIG 6. Relation between centre-of-pressure position and bubble length.

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