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Summary.—The equations governing the laminar compressible boundary layer on a yawed body of infinite span are transformed to give three non-dimensional equations defining two velocity components and the enthalpy.

Assuming that the Prandtl number is unity and that there is zero heat transfer, a relation is obtained between the stream Mach number and the angle of yaw for flows which give the same boundary-layer equations.

The further assumption of viscosity proportional to the absolute temperature is made and 'similar' solutions are found to be given by a family of surface Mach number distributions normal to the leading edge. 'Similar' solutions, obtained from a differential analyser, are presented for a range of two controlling parameters.

1. *Introduction.*—The practice of sweeping back the wings of aircraft designed for high-speed flight has emphasised the need for investigation of the compressible boundary-layer flow over a three-dimensional wing. In the following work, the problem is approached by considering a yawed wing of infinite span. This idealisation is expected to give results applicable to wings of moderate aspect ratio.

The analysis of the incompressible flow over a yawed infinite wing is simplified by the fact that the boundary-layer velocity in a direction normal to the leading edge is independent of the flow parallel to the leading edge and is, in fact, identical to the velocity distribution given by the unyawed wing case. The spanwise velocity distribution is then calculable from the chordwise distribution. A discussion of various solutions to this problem is given, with bibliography, by Rott and Crabtree[1].

Published work on the three-dimensional compressible boundary layer is scarce. Struminsky[2] treated the yawed wing of infinite span and solved the zero pressure gradient problem. He also found that the boundary-layer velocity profiles can be obtained separately (as in the incompressible case) provided the flow is isothermal.

Moore[3] derived the boundary-layer equations for compressible flow over a general three-dimensional body and gave a solution for the flat plate with arbitrary leading-edge contour and zero pressure gradient. He found that the flow was everywhere parallel to the free-stream flow, the velocity being given by the same equation as for the two-dimensional flat plate, except in a narrow region extending downstream of any point where the leading edge has discontinuous curvature.

Schuh[4] treated the yawed infinite-span wing considering several heat-transfer conditions at the surface. The drastic assumption was made that the heating did not affect the velocity distributions. The temperature distribution through the boundary layer was then calculated by using known incompressible-flow velocity solutions in the energy equation.

Crabtree[5] has approached the problem of the yawed infinite wing in compressible flow. Assuming viscosity proportional to the absolute temperature, Prandtl number of unity, and zero heat transfer, he applied the Illingworth-Stewartson transformation and noted that the chordwise-momentum equation differs from the incompressible-flow equation by an additional factor in the pressure gradient term while the spanwise-momentum equation is completely reduced to incompressible-flow form. The additional factor is governed by a parameter equivalent to K (equation (18)) in the present paper. He investigated the case where the chordwise compressible flow corresponds to the incompressible flow near a stagnation point, proposing a solution as a power series of K and evaluating the first two terms of the chordwise and spanwise velocity profiles. From this solution he concluded that a good approximation is given by neglecting the new term appearing in the chordwise-momentum equation—the transformation (involving the temperature) eliminating most of the interdependence of the two velocity components. He also derived the compressible-flow equations when the chordwise velocity distribution is transformed from the general Falkner-Skan type velocity distribution, stating that the solution to these equations is expected to be given to good approximation if the additional pressure gradient term is neglected.

The present paper represents an independent study of the problem treated by Crabtree. The two works are parallel, but the method here is somewhat different and some new features are revealed. The numerical solution given by Crabtree is found to be in error.*

In the following work, the possibility of solving the compressible boundary-layer equations for a yawed body of infinite span is investigated. The boundary-layer equations are transformed, using an extension of the method devised by Canetti[6], to define the two velocity components and the enthalpy. By assuming unit Prandtl number and zero heat transfer at the surface, the energy equation becomes soluble and the problem is reduced to solving the two momentum equations. These are further simplified by assuming the viscosity proportional to the absolute temperature. The two equations defining the chordwise and spanwise velocity distributions require simultaneous solution (unlike the incompressible-flow problem) due to the interdependence of the velocities on a common temperature.

Examination of the simplified momentum equations reveals a relation between the angle of yaw and the stream Mach number, for main stream flows which give the same boundary-layer equations. In addition, a family of chordwise Mach number distributions is found which gives ' similar ' velocity profiles. Some ' similar ' solutions are presented for several values of the two controlling parameters.

The theoretical work appearing in this paper formed part of the author's Ph.D. thesis (Queen Mary College, University of London, 1953). He wishes to acknowledge the use of the Massachusetts Institute of Technology Rockefeller Differential Analyzer for obtaining the numerical solutions.

2. *Boundary-Layer Equations and Transformation.*—The equations governing the motion of a compressible viscous fluid over a yawed wing of infinite span, simplified by the Prandtl boundary-layer approximations and assuming constant Prandtl number, are:

Equation of Continuity

$$(\rho u)_x + (\rho v)_y = 0 \qquad \text{(1a)}$$

Equations of Momentum

$$\rho u u_x + \rho v u_y = - p_x + (\mu u_y)_y \qquad \text{(1b)}$$

$$p_y = O[1] \qquad \text{(1c)}$$

$$\rho u w_x + \rho v w_y = (\mu w_y)_y \qquad \text{(1d)}$$

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* This computational error has been acknowledged in a private communication to the author.

Equation of Energy

$$\rho u i_x + \rho v i_y = u p_x + \frac{1}{\sigma}(\mu i_y)_y + \mu\{(u_y)^2 + (w_y)^2\} \quad \text{(1e)}$$

The boundary conditions are:

At $y = 0$, $u = v = w = 0$, and $i_y = 0$ for zero heat transfer

At $y \to \infty$, $u \to u_1$, $w \to w_1$, $i \to i_1$. (1f)

The physical properties of the gas are given by:

Equation of State

$$p = \frac{\gamma - 1}{\gamma} \rho i \quad \text{(1g)}$$

Viscosity-Temperature Relation

$$\mu \propto T^\omega \quad \text{(1h)}$$

The pressure and velocity of the main stream just outside the boundary layer are related by the Bernoulli equation

$$dp = -\rho_1(u_1\,du_1 + v_1\,dv_1 + w_1\,dw_1) \quad \text{(1i)}$$

In the present case this simplifies since $dv_1 = dw_1 = 0$.

By putting the equation of continuity in the form

$$\rho v = -\int_0^y (\rho u)_x\,dy \quad \text{(2)}$$

the above system can be reduced to three equations defining the three dependent variables u, w and i. The equations are now transformed by an extension of Canetti's method.

New variables are introduced as defined by:

$$x = \int_0^X F(X)\,dX \quad \text{(3a)}$$

$$y = \int_0^Y \lambda(X,Y)\,dY \quad \text{(3b)}$$

$$z = Z \quad \text{(3c)}$$

where $f(X)$ is an arbitrary function of X and $\lambda(X,Y)$ is an arbitrary function of X and Y.

After transformation, the equations are made non-dimensional according to

$$q = u/u_1; \quad s = w/w_1; \quad r = \rho/\rho_1; \quad m = \mu/\mu_1; \quad h = i/i_1 \quad \text{(4)}$$

and introducing

$$G = \mu_0\lambda/\mu \quad \text{(5)}$$

the boundary-layer equations can be written as:

$$mrqq_X = \frac{q_Y}{\mu_1\rho_1 u_1 G}\int_0^Y (\mu_1\rho_1 u_1 Gmrq)_X\, dY = mr\left(\frac{1}{\gamma} - q^2\right)\frac{(u_1)_X}{u_1} + \frac{F\mu_0^2}{\mu_1\rho_1 u_1 G}\left(\frac{q_Y}{G}\right)_Y \quad \ldots \quad \ldots \quad (6a)$$

$$mrqs_X - \frac{s_Y}{\mu_1\rho_1 u_1 G}\int_0^Y (\mu_1\rho_1 u_1 Gmrq)_X\, dY = \frac{F\mu_0^2}{\mu_1\rho_1 u_1 G}\left(\frac{s_Y}{G}\right)_Y \quad \ldots \quad \ldots \quad (6b)$$

$$mrqh_X - \frac{h_Y}{\mu_1\rho_1 u_1 G}\int_0^Y (\mu_1\rho_1 u_1 Gmrq)_X\, dY = -\frac{mq}{i_1}\{u_1(u_1)_X + rh(i_1)_X\}$$

$$+ \frac{1}{\sigma}\frac{F\mu_0^2}{\mu_1\rho_1 u_1 G}\left(\frac{h_Y}{G}\right)_Y + \frac{F\mu_0^2}{i_1\mu_1\rho_1 u_1 G}\{u_1^2(q_Y)^2 + w_1^2(s_Y)^2\}. \quad \ldots \quad \ldots \quad (6c)$$

By noting that

$$(i_1)_X = -u_1(u_1)_X \quad \ldots \quad \ldots \quad (7a)$$

and that

$$rh = 1 \quad \ldots \quad \ldots \quad (7b)$$

within the boundary layer, it is seen that the first expression on the right-hand side of the energy equation (6c) vanishes.

The boundary conditions for the problem are:

At $Y = 0$, $q = s = 0$, $h_Y = 0$ for zero heat transfer

At $Y \to \infty$, $q \to 1$, $s \to 1$, $h \to 1$. .. .. (8)

The arbitrary functions F and G are chosen to put the equations in the most suitable form for solution. For the present purposes they are specified by

$$G_Y = 0 \quad \ldots \quad \ldots \quad (9a)$$

$$\frac{(\mu_1\rho_1 u_1 G)_X}{\mu_1\rho_1 u_1 G} = \frac{F\mu_0^2}{\mu_1\rho_1 u_1 G^2} = \frac{1}{X} \quad \ldots \quad \ldots \quad (9b)$$

giving

$$G = \frac{AX}{\mu_1\rho_1 u_1} \qquad F = \frac{A^2 X}{\mu_0^2\mu_1\rho_1 u_1} \quad \ldots \quad \ldots \quad (9c)$$

where A is an arbitrary constant with dimensions of viscosity squared.

The relations between the two systems of co-ordinates are now completely determined.

$$x = \frac{A^2}{\mu_0^2}\int_0^X \frac{X\, dX}{\mu_1\rho_1 u_1} \quad \text{or} \quad X^2 = \frac{2\mu_0^2}{A^2}\int_0^x \mu_1\rho_1 u_1\, dx \quad \ldots \quad (10a)$$

$$y = \frac{1}{\rho_1 u_1}\{2\int_0^x \mu_1\rho_1 u_1\, dx\}^{1/2}\int_0^Y m\, dY \quad \text{or} \quad Y = \frac{\rho_1 u_1}{\{2\int_0^x \mu_1\rho_1 u_1\, dx\}^{1/2}}\int_0^y \frac{dy}{m} \quad (10b)$$

The three boundary-layer equations now take the forms :

$$Xmrqq_X - q_Y \int_0^Y mrq\, dY - Xq_Y \int_0^Y (mrq)_X\, dY = mr\left(\frac{1}{r} - q^2\right)\frac{X(u_1)_X}{u_1} + q_{YY} \quad \ldots \quad (11a)$$

$$Xmrqs_X - s_Y \int_0^Y mrq\, dY - Xs_Y \int_0^Y (mrq)_X\, dY = s_{YY} \quad \ldots \quad (11b)$$

$$Xmrqh_X - h_Y \int_0^Y mrq\, dY - Xh_Y \int_0^Y (mrq)_X\, dY = \frac{1}{\sigma} h_{YY} + \frac{1}{i_1}\{(u_1q_Y)^2 + (w_1s_Y)^2\}\,. \quad \ldots \quad (11c)$$

In general these equations require simultaneous solution but under certain circumstances independent solutions are obtainable.

(*a*) *Incompressible flow.*—In this case the boundary layer is isothermal giving $h = 1$ and hence $m = r = 1$. The first momentum equation (11a) then defines q with no dependence on s. This equation is identical with the equation for an unyawed wing provided $(u_1)_X/u_1$ is unchanged. Knowing q from (11a) allows (11b) to be solved for s.

(*b*) *Prandtl number unity and zero heat transfer.*—In this case the energy equation has the particular solution

$$i + \tfrac{1}{2}u^2 + \tfrac{1}{2}w^2 = \text{constant} \quad \ldots \quad (12a)$$

or in the present notation

$$h = 1 + \frac{\gamma - 1}{2}\left(\frac{u_1}{a_1}\right)^2(1 - q^2) + \frac{\gamma - 1}{2}\left(\frac{w_1}{a_1}\right)^2(1 - s^2)\,. \quad \ldots \quad (12b)$$

The two momentum equations remain to be solved simultaneously.

(*c*) *Zero pressure gradient.*—If s is put equal to q, the two momentum equations become identical and the energy equation is reduced to the extent that the resultant equations specify the two-dimensional compressible boundary-layer flow over a flat plate. The known solutions to the two-dimensional problem can therefore be applied directly to the yawed flat plate.

3. *Simplified Equations.*—The three equations (11a), (11b), (11c) are too complex for solution by ordinary methods. It is therefore necessary to seek simplifications. As the first simplifying measure it is natural to assume $\sigma = 1$ and zero heat transfer so that the direct solution (12b) to the energy equation can be exploited. Then guided by experience of the two-dimensional compressible boundary layer, the assumption of viscosity proportional to the absolute temperature ($\omega = 1$) is made. This reduces the product mr to unity.

The boundary-layer equations are now

$$Xqq_X - q_Y \int_0^Y q\, dY - Xq_Y \int_0^Y q_X\, dY = \left(\frac{1}{r} - q^2\right)\frac{X(u_1)_X}{u_1} + q_{YY} \quad \ldots \quad (13a)$$

$$Xqs_X - s_Y \int_0^Y q\, dY - Xs_Y \int_0^Y q_X\, dY = s_{YY} \quad \ldots \quad (13b)$$

and the enthalpy is given by (12b).

It is seen that the first of these equations is independent of s except for its implicit appearance in the function $1/r$ which is part of the pressure gradient term. Equation (13a) is identical with the corresponding equation in two-dimensional flow so that, under the present simplified conditions, the effect of yaw on the transformed boundary-layer equations is equivalent to a modification of the pressure gradient in the direction normal to the leading edge of the wing. It is useful to examine this pressure gradient term more fully.

By using the isentropic relations for the flow just outside the boundary layer and also noting that within the layer $1/r = h$.

$$\left\{\frac{1}{r} - q^2\right\}\frac{(u_1)_X}{u_1} = \left\{(1 - q^2) + \frac{\frac{\gamma - 1}{2}\left(\frac{w_1}{a_1}\right)^2}{1 + \frac{\gamma - 1}{2}\left(\frac{u_1}{a_1}\right)^2}(1 - s^2)\right\}\frac{\left(\frac{u_1}{a_1}\right)_X}{\frac{u_1}{a_1}}. \qquad (14)$$

The coefficient of $(1 - s^2)$ can be further reduced to the constant

$$\frac{w_1^2}{2i_s - w_1^2}. \qquad (15)$$

It is clear that this constant depends entirely on the outer flow and hence on the free-stream Mach number (M_∞) and the angle of yaw (Λ).

The free-stream Mach number can be written as

$$M_\infty^2 = \frac{Q_\infty^2 + w_1^2}{(\gamma - 1)i_\infty} \qquad (16)$$

where Q_∞ and w_1 are the velocity components normal and parallel to the leading edge respectively. Now, by using the isentropic relations in the main stream and noting that

$$\frac{w_1}{Q_\infty} = \tan \Lambda \qquad (17)$$

the required expression is obtained:

$$\frac{w_1^2}{2i_s - w_1^2} = \frac{\frac{\gamma - 1}{2}M_\infty^2}{1 + \left\{1 + \frac{\gamma - 1}{2}M_\infty^2\right\}\cot^2 \Lambda} = K. \qquad (18)$$

Equations (13a) and (13b) are now rewritten

$$q_{YY} + q_Y\int_0^Y q\,dY + Xq_Y\int_0^Y q_X\,dY - Xqq_X = -\{(1 - q^2) + K(1 - s^2)\}\frac{X\left(\frac{u_1}{a_1}\right)_X}{\frac{u_1}{a_1}} \qquad (19a)$$

$$s_{YY} + s_Y\int_0^Y q\,dY + Xq_Y\int_0^Y q_X\,dY - Xqs_X = 0. \qquad (19b)$$

Examination of these equations shows that the linkage between the two equations is confined to the single appearance of the parameter K in (19a). This is the only way in which yaw modifies the velocity profile in a plane at right-angles to the leading edge. It is clear that for $K = 0$ (corresponding either to zero yaw or zero Mach number) the two equations become independent.

Another feature of the equations is that for a given Mach number distribution normal to the leading edge (*i.e.*, a specified $(u_1/a_1)_X/(u_1/a_1)$) the boundary-layer equations and their solutions are the same for a range of combinations of angle of yaw and free stream Mach number as determined by $K =$ constant. Fig. 2, where lines of $K = 0{\cdot}001$, $0{\cdot}010$, $0{\cdot}050$, $0{\cdot}100$, $0{\cdot}500$ and $1{\cdot}000$ are plotted with M_∞ and Λ as variables, shows that an increase of Mach number requires a decrease in the angle of yaw for the same boundary-layer equations to hold.

It should be noted that the existence of the parameter K is independent of the assumption $\omega = 1$.

4. *Equations for 'Similar' Solutions.*—The known existence of 'similar' solutions for two-dimensional incompressible and compressible flows, and for incompressible flow over a yawed infinite wing prompts an investigation into the possible existence of 'similar' solutions for compressible flow over a yawed infinite wing.

For 'similar' velocity profiles, the boundary layer equations are required to become ordinary differential equations with Y as independent variable. On putting $q_X = s_X = 0$, the requirement for independence of X is fulfilled when

$$\frac{X\left(\dfrac{u_1}{a_1}\right)_X}{\left(\dfrac{u_1}{a_1}\right)} = n = \text{constant.}$$

Integration gives the velocity distribution as

$$\frac{u_1}{a_1} = A_n X^n \,. \qquad (20)$$

This is the same relation that gives 'similar' profiles in the unyawed compressible flow case[6], and is distinctly related to the original Falkner and Skan family,

$$u_1 = A_n x^n \,.$$

The equations defining the 'similar' velocity distributions are

$$q_{YY} + q_Y \int_0^Y q\, dY = -n[(1 - q^2) + K(1 - s^2)] \qquad (21a)$$

$$s_{YY} + s_Y \int_0^Y q\, dY = 0 \,. \qquad (21b)$$

The second of these can be integrated directly to give

$$s = \frac{\displaystyle\int_0^Y \exp\left[-\int_0^Y\!\!\int_0^Y q\, dY\, dY\right] dY}{\displaystyle\int_0^\infty \exp\left[-\int_0^Y\!\!\int_0^Y q\, dY\, dY\right] dY} \qquad (22)$$

when the boundary conditions are applied. Whether this solution will be helpful when solving equations (21a) and (21b) depends on the method of solution to be used. In general the two equations (21a) and (21b) require simultaneous solution for given values of the parameters n and K.

5. *'Similar' Solutions.*—For $K = 0$, equation (21a) defines the 'similar' velocity profiles in the chordwise direction in incompressible flow for either yawed or unyawed wings. Solutions to this equation for a range of values of n have been given by Hartree[7]. Cooke[8] used Hartree's solutions to calculate the corresponding spanwise velocity profiles as defined by equation (21b).

Compressible flow 'similar' solutions have been obtained for $K = 0{\cdot}01$, $0{\cdot}05$ and $0{\cdot}10$ and $n = 0{\cdot}5$ and $1{\cdot}0$ using the Rockefeller Differential Analyzer at the Massachusetts Institute of Technology. These results are presented in Table 1 and Fig. 3. For completeness, the results of Hartree and Cooke for $K = 0$ are included for the same values of n where these were available.

6. *Discussion of Results.*—It will be noticed from the tabulated values of q and s that the Differential Analyzer does not give the asymptotic values of unity. This appears to be due to cumulative errors in the integrators. As a check on the accuracy, the equations were solved for $K = 0$ and $n = 1{\cdot}0$ and the results compared with the solutions given by Hartree and Cooke. The equations were solved assuming the true values of

$$q_Y(0) = 1{\cdot}2326$$
$$s_Y(0) = 0{\cdot}5704$$

and the asymptotic values

$$q \rightarrow 0{\cdot}9981$$
$$s \rightarrow 0{\cdot}9994$$

for large values of Y were obtained. On assuming the slightly different initial values of

$$q_Y(0) = 1{\cdot}2327$$
$$s_Y(0) = 0{\cdot}5704$$

the asymptotic values of

$$q \rightarrow 1{\cdot}0049$$
$$s \rightarrow 0{\cdot}9990$$

were given.

These results indicate the sensitivity of the asymptotic values of q and s to small changes in the value of the initial slopes. It will be noted that the better asymptotic values occur with the first solution and are associated with the accurately known values of the initial slopes; therefore it can be assumed that the Differential Analyzer gives the initial slopes of the velocity profiles with good accuracy whilst the accuracy of the velocity profile itself is not so good.

The velocity profiles plotted in Fig. 3 show that the profiles become more convex as either n or K is increased. The chordwise profile is more affected than the spanwise profile by these changes. The increased convexity with increase of n and K is accompanied by an increase in the initial slopes of the velocity profiles. Curves of $q_Y(0)$ and $s_Y(0)$ are plotted against K for different values of n in Fig. 4. These curves illustrate the comparatively small dependence on K.

NOTATION

a	Velocity of sound
c_p	Specific heat at constant pressure
F	Function of X in transformation $x = \int_0^X F(X)\, dX$
G	Function defined by $G = \mu_0 \lambda / \mu$
h	Enthalpy ratio, $= i/i_1$
i	Enthalpy, $= c_p T$
k	Coefficient of thermal conductivity
K	Parameter relating M_∞ and Λ (equation (18))
m	Viscosity ratio, $= \mu/\mu_1$
M	Mach number
n	Parameter defining the main stream velocity variation (equation (20))
p	Pressure
q	Velocity ratio, $= u/u_1$
Q_∞	Component of stream velocity normal to leading edge of wing
r	Density ratio, $= \rho/\rho_1$
s	Velocity ratio, $= w/w_1$
T	Absolute temperature
u, v, w	Velocity components in directions x, y, z
x, y, z	Curvilinear co-ordinates (*see* Fig. 1)
X, Y, Z	Transformed co-ordinates
γ	Ratio of specific heats at constant pressure and volume
λ	Function of X and Y in transformation $y = \int_0^Y \lambda(X, Y)\, dY$
μ	Coefficient of viscosity
ρ	Density
σ	Prandtl number, $= c_p \mu / k$
ω	Index in viscosity-temperature relation, $\mu \alpha T^\omega$
Λ	Angle of Yaw
Suffixes	
$_1$	Value at edge of boundary layer
$_s$	Value at isentropic stagnation point
$_0$	Value at reference point
$_\infty$	Free-stream value

Suffix notation is used to denote partial differentiation where convenient.

Addition notation is defined as it is used.

REFERENCES

No.	Author	Title, etc.
1	N. Rott and L. F. Crabtree ..	Simplified laminar boundary-layer calculations for bodies of revolution and for yawed wings. *J.Ae. Sci.*, Vol. 19. No. 8, pp. 553 to 565. August, 1952.
2	V. V. Struminsky	Sideslip in a viscous compressible gas. Doklady Akademii Nauk, S.S.S.R., T. Liv. No. 9, pp. 769 to 772, 1946. N.A.C.A. Tech. Memo. 1276. April, 1951.
3	F. K. Moore	Three-dimensional compressible laminar boundary-layer flow. N.A.C.A. Tech. Note 2279. March, 1951.
4	H. Schuh	Aerodynamic heating on yawed infinite wings and on bodies of arbitrary shape. K.T.H. Aero. Tech. Note 35. July, 1953.
5	L. F. Crabtree	The laminar boundary layer on a yawed infinite cylinder. Ph.D. Thesis, Cornell University. September, 1952.
6	G. S. Canetti	A generalization of the problem of the compressible laminar boundary layer with solutions for an idealized fluid. Ph.D. Thesis, University of London. June, 1950.
7	D. R. Hartree	On an equation occurring in Falkner and Skan's approximate treatment of the equations of the boundary layer. *Proc. Camb. Phil. Soc.*, Vol. 33, pp. 223 to 239. 1937.
8	J. C. Cooke	The boundary layer of a class of infinite yawed cylinders. *Proc. Camb. Phil. Soc.*, Vol. 46, pp. 645 to 648. 1950.

TABLE 1

'Similar' Solutions

1	2	3	4	5	6	7	8	9	10	11
	$n = 0$		$n = 0·5$							
Y	K = any value		$K = 0$		$K = 0·01$		$K = 0·05$		$K = 0·10$	
	q	s	q	s	q	s	q	s	q	s
0	0·0000	Same as column 2	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000
0·1	0·0469		0·0903	0·0539	0·0906	0·0539	0·0922	0·0541	0·0941	0·0545
0·2	0·0939		0·1756	0·1078	0·1763	0·1078	0·1792	0·1083	0·1827	0·1089
0·3	0·1408		0·2558	0·1615	0·2569	0·1616	0·2609	0·1624	0·2660	0·1634
0·4	0·1876		0·3311	0·2151	0·3325	0·2153	0·3374	0·2162	0·3436	0·2174
0·5	0·2342		0·4015	0·2683	0·4030	0·2685	0·4087	0·2696	0·4159	0·2712
0·6	0·2806		0·4670	0·3209	0·4686	0·3211	0·4749	0·3226	0·4828	0·3244
0·7	0·3266		0·5276	0·3727	0·5293	0·3730	0·5361	0·3746	0·5444	0·3767
0·8	0·3720		0·5834	0·4235	0·5852	0·4239	0·5922	0·4256	0·6009	0·4279
0·9	0·4167		0·6344	0·4731	0·6365	0·4734	0·6436	0·4754	0·6524	0·4779
1·0	0·4606		0·6811	0·5211	0·6830	0·5215	0·6901	0·5236	0·6989	0·5262
1·2	0·5453		0·7615	0·6115	0·7633	0·6119	0·7701	0·6141	0·7783	0·6170
1·4	0·6244		0·8258	0·6928	0·8275	0·6932	0·8336	0·6955	0·8409	0·6985
1·6	0·6967		0·8760	0·7638	0·8775	0·7640	0·8827	0·7663	0·8889	0 7693
1·8	0·7610		0·9141	0·8236	0·9154	0·8238	0·9196	0·8258	0·9246	0·8286
2·0	0·8167		0·9421	0·8723	0·9431	0·8724	0·9465	0·8742	0·9503	0·8768
2·2	0·8633		0·9621	0·9104	0·9629	0·9105	0·9655	0·9120	0·9683	0·9143
2·4	0·9011		0·9760	0·9392	0·9765	0·9392	0·9785	0·9405	0·9804	0·9424
2·6	0·9306		0·9852	0·9601	0·9855	0·9600	0·9870	0·9610	0·9883	0·9627
2·8	0·9529		0·9913	0·9748	0·9914	0·9745	0·9924	0·9754	0·9932	0·9768
3·0	0·9691		0·9952	0·9846	0·9950	0·9843	0·9958	0·9850	0·9962	0·9863
3·2	0·9804		0·9974	0·9909	0·9972	0·9906	0·9978	0·9911	0·9979	0·9923
3·4	0·9880		0·9986	0·9948	0·9985	0·9945	0·9989	0·9949	0·9989	0·9960
3·6	0·9929		0·9993	0·9972	0·9990	0·9968	0·9996	0·9972	0·9994	0·9983
3·8	0·9959		0·9997	0·9985	0·9994	0·9982	0·9999	0·9986	0·9998	0·9996
4·0	0·9978		0·9999	0·9992	0·9995	0·9989	1·0000	0·9993	0·9998	1·0003
4·2	0·9988		0·9999	0·9996	0·9995	0·9993	1·0001	0·9996	0·9999	1·0006
4·4	0·9994		1·0000	0·9998	0·9995	0·9994	1·0001	0·9997	0·9999	1·0007
4·6	0·9997			0·9999	0·9995	0·9994	1·0002	0·9998	0·9999	1·0008
4·8	0·9999			1·0000	0·9995	0·9995	1·0002	0·9998		
5·0	0·9999				0·9995	0·9995	1·0002	0·9998		
5·2	1·0000									
$q_Y(0)$	0·4696		0·9277		0·9320		0·9489		0·9690	
$s_Y(0)$		0·4696		0·5390		0·5393		0·5423		0·5451

Columns 2, 4, 12 from Ref. 7.

Columns 3, 13 from Ref. 8.

TABLE 1—*continued*

1	12	13	14	15	16	17	18	19
	$n = 1·0$							
Y	$K = 0$		$K = 0·01$		$K = 0·05$		$K = 0·10$	
	q	s	q	s	q	s	q	s
0	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000
0·1	0·1183	0·0570	0·1188	0·0572	0·1213	0·0574	0·1243	0·0577
0·2	0·2266	0·1140	0·2278	0·1143	0·2324	0·1148	0·2380	0·1155
0·3	0·3252	0·1709	0·3269	0·1713	0·3332	0·1720	0·3408	0·1732
0·4	0·4144	0·2275	0·4164	0·2280	0·4241	0·2290	0·4333	0·2306
0·5	0·4946	0·2835	0·4969	0·2843	0·5055	0·2955	0·5160	0·2873
0·6	0·5662	0·3388	0·5687	0·3397	0·5779	0·3412	0·5892	0·3433
0·7	0·6298	0·3931	0·6324	0·3942	0·6420	0·3958	0·6537	0·3983
0·8	0·6859	0·4461	0·6884	0·4473	0·6982	0·4490	0·7101	0·4518
0·9	0·7350	0·4974	0·7376	0·4987	0·7472	0·5006	0·7591	0·5036
1·0	0·7778	0·5468	0·7803	0·5482	0·7897	0·5502	0·8012	0·5535
1·2	0·8467	0·6387	0·8488	0·6402	0·8573	0·6423	0·8677	0·6457
1·4	0·8968	0·7199	0·8987	0·7214	0·9058	0·7235	0·9147	0·7269
1·6	0·9324	0·7891	0·9338	0·7907	0·9396	0·7925	0·9468	0·7958
1·8	0·9569	0·8461	0·9579	0·8475	0·9625	0·8491	0·9679	0·8521
2·0	0·9732	0·8912	0·9740	0·8926	0·9773	0·8938	0·9814	0·8964
2·2	0·9841	0·9256	0·9843	0·9269	0·9866	0·9278	0·9895	0·9300
2·4	0·9905	0·9508	0·9908	0·9520	0·9924	0·9525	0·9943	0·9544
2·6	0·9946	0·9685	0·9947	0·9697	0·9957	0·9700	0·9969	0·9715
2·8	0·9971	0·9806	0·9971	0·9817	0·9976	0·9817	0·9984	0·9829
3·0	0·9985	0·9884	0·9983	0·9894	0·9986	0·9893	0·9990	0·9903
3·2	0·9992	0·9933	0·9991	0·9943	0·9991	0·9940	0·9992	0·9949
3·4	0·9996	0·9963	0·9993	0·9972	0·9993	0·9969	0·9992	0·9977
3·6	0·9998	0·9981	0·9996	0·9989	0·9993	0·9986	0·9992	0·9994
3·8	0·9999	0·9990	0·9996	0·9999	0·9993	0·9995	0·9992	1·0002
4·0	1·0000	0·9995	0·9997	1·0004	0·9993	1·0000	0·9991	1·0006
4·2		0·9998	0·9997	1·0005	0·9993	1·0002	0·9990	1·0009
4·4		0·9999	0·9997	1·0006	0·9992	1·0003	0·9989	1·0010
4·6		1·0000		1·0006				
4·8								
5·0								
5·2								
$q_Y(0)$	1·2326		1·2405		1·2670		1·3002	
$s_Y(0)$		0·5704		0·5719		0·5742		0·5783

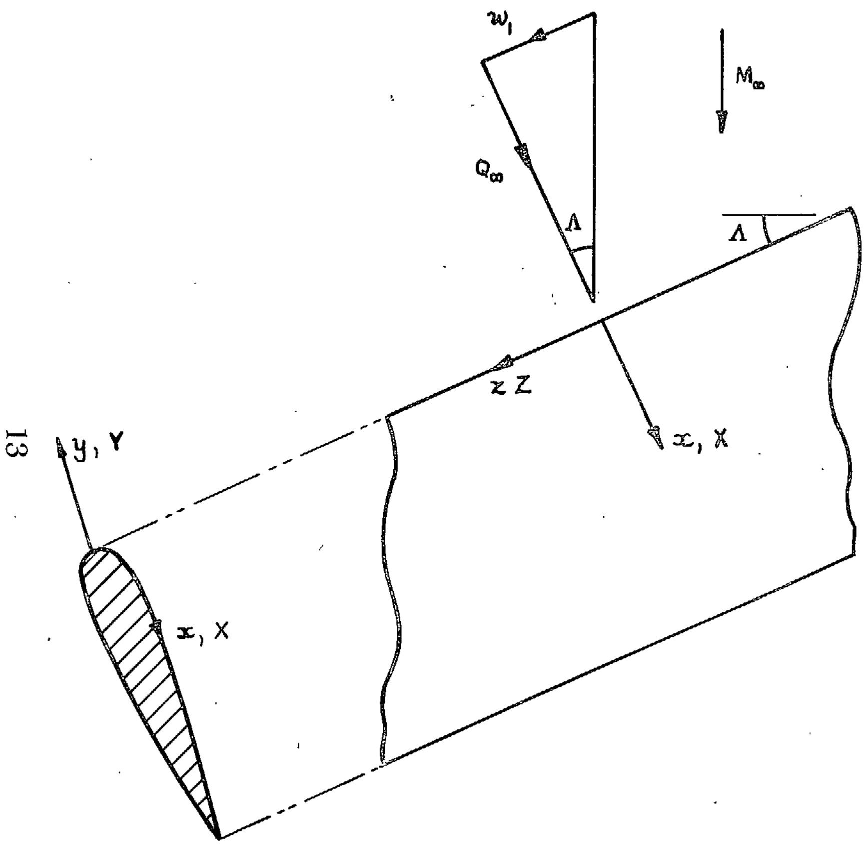

FIG. 1. Co-ordinate system.

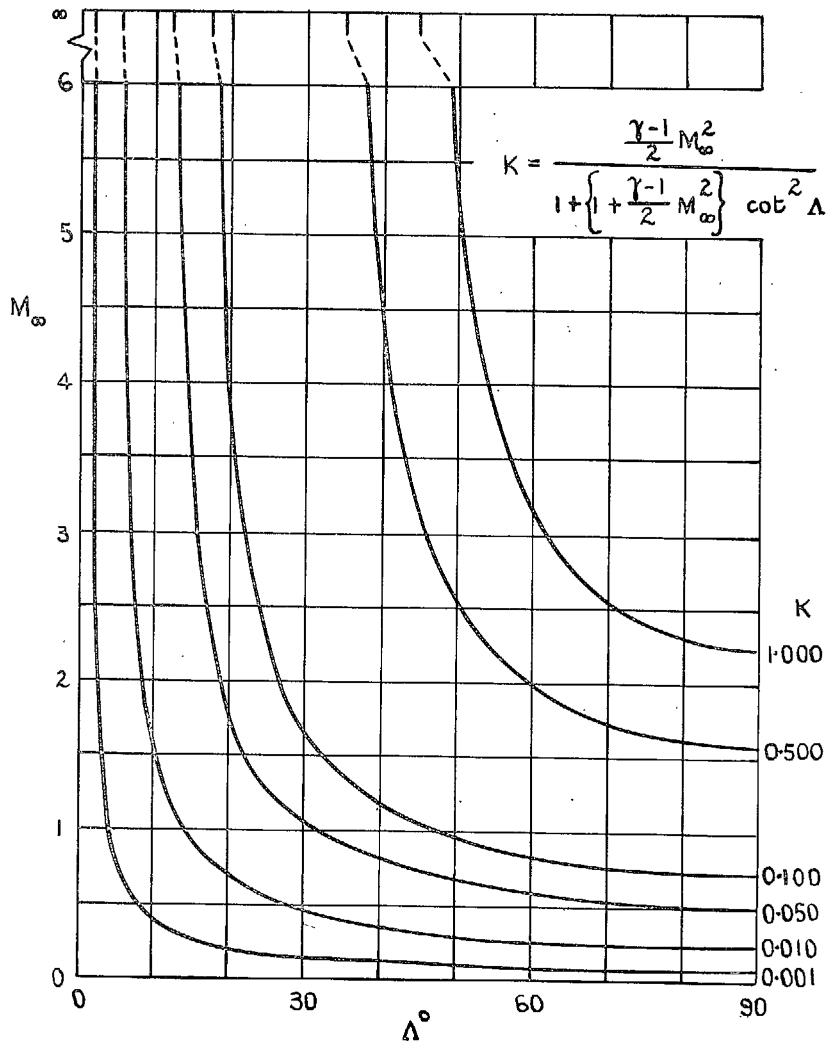

FIG. 2. Relation between Mach number and angle of yaw for identical boundary-layer equations.

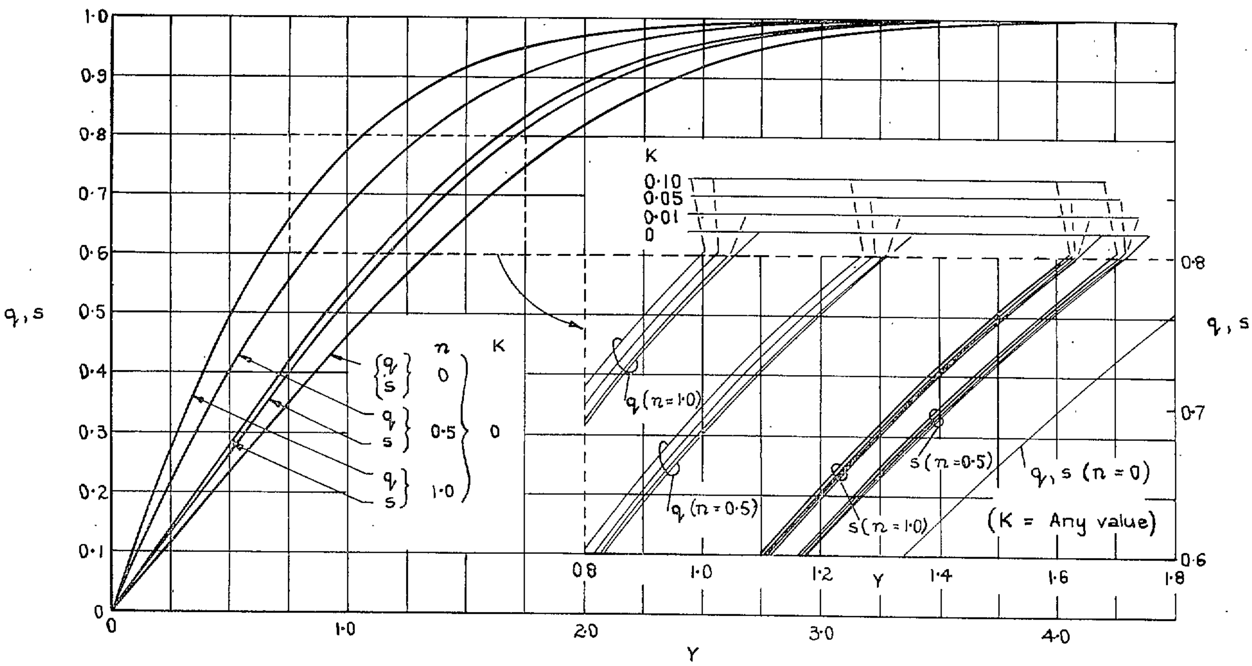

FIG. 3. 'Similar' velocity profiles.

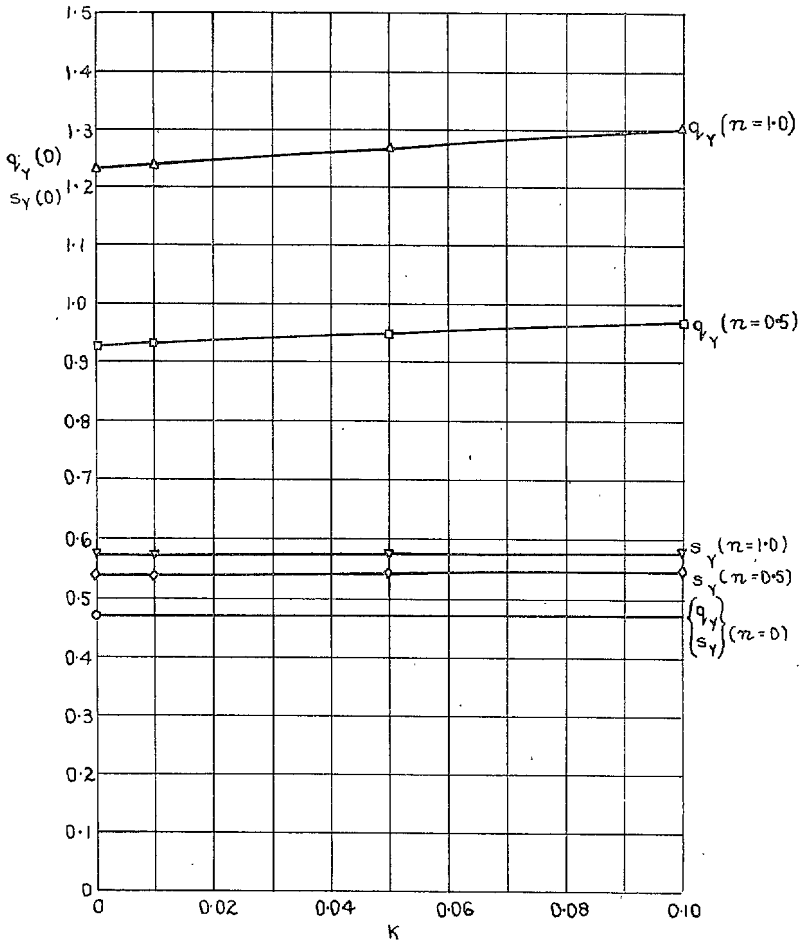

FIG. 4. Effect of K and n on the initial slope of the velocity profiles.

(4994) Wt. 20/9036 K 7 3/57 Hw.

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