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By

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INSTABILITY IN A SLOTTED WALL TUNNEL

by J. L. King, P. Boyle and J. B. Ogle

ABSTRACT

Large pressure fluctuations have been observed in the working section of A.R.L's new 30 in. water tunnel. The present report describes investigations carried out in a 7 in. wind tunnel to examine such fluctuations in detail and also theoretical investigations which explain and describe the phenomena.

It is shown theoretically that a jet of fluid circular in cross-section moving through the same fluid is unstable. The slotted wall is unable to stabilise all frequencies of disturbance. The length of the working section determines preferred wavelengths of disturbance which feed energy back to the upstream end of the jet by compressing the fluid in the reservoir surrounding the working section. In the absence of a reservoir, the instability disappears.

The present report is not concerned with the performance of the 30 in. tunnel or with measures now being taken to improve it.

INTRODUCTION

This work originated in the discovery of large pressure fluctuations in the working section of A.R.L's new 30 in. water tunnel. It is now known that these fluctuations were only excessive due to the presence of a mechanical resonance of the tunnel as a whole. However, it had already been decided that, in view of difficulties in instrumenting the tunnel, tests should be carried out in the 12 in. water tunnel, and in a specially constructed 7 in. wind tunnel. It was found that similar disturbances existed, although not so prominently. The difficulties in the 30 in. tunnel are being overcome but the results for the 7 in. wind tunnel were sufficiently interesting to be investigated in their own right. The present report therefore gives a complete description of the phenomena observed together with theoretical work explaining them.

2. The wind tunnel is shown in Figure 1. The slotted wall was made of perspex, the outer reservoir wall and the diffuser of acetate sheet and the nozzle was a dural extension to the wooden contraction. The ring was shaped in wax, with metal and wood backing. This is not a scale model of the water tunnel as it incorporates modifications introduced during the original tests. However, the underlying principles are valid for any slotted wall tunnel.

3. The theoretical work falls naturally into several small and distinct investigations. More detailed investigations would require considerable effort without adding anything to the basic principles outlined here. For the same reason, no attempt has been made to fill in the gaps in the experimental results, most of which were originally obtained for the rather different purpose of curing the 30 in. tunnel.

OUTLINE OF RESULTS

4. The first important result was obtained from the experiments by the use of smoke. It was discovered that the disturbances were axi-symmetric. Subsequent hot wire measurements showed that the oscillations were nearly sinusoidal. This suggested an instability in the flow. The self-sustained oscillations were determined theoretically by consideration of a closed loop comprising the jet of fluid in the working section, transfer of energy to the stationary fluid in the reservoir by the large surface disturbance at the downstream end and the transfer back of energy to the upstream end of the jet by the reaction of the pressure fluctuations in the reservoir on the jet or jet nozzle.

5. Self-sustained oscillations in a closed circuit demand that the total gain round the loop be unity and that the phase change round the loop be an integral number of cycles. In practice, the gain exceeds unity for small disturbances and the amplitude of the final disturbance is determined by non-linearities in the system. With this in mind, the various stages are detailed:-

(a) Jet instability. It is well known that, if one fluid moves over another with uniform velocity, the common surface may be unstable to certain disturbances (see, for example, Reference 1, p. 373). Particular examples are the generation of waves by wind and the flapping of a flag. If the fluids are of different densities, only disturbances of small wavelength (i.e. high frequency) are unstable but, if the fluids have the same density, all disturbances are unstable. Surface tension stabilises small wavelengths so that, with a suitable choice of parameters, including the densities of the fluids, both large and small wavelengths may be stable, leaving only a narrow band of frequencies unstable. One frequency in this band will eventually predominate so that the two-dimensional case leads to disturbances of the type found in the tunnel.

It is shown below that similar results apply to a circular jet. If it moves through a fluid of the same density, disturbances of all wavelengths are unstable as is shown by the breaking up of an open jet about one diameter downstream of the nozzle. A slotted wall may be regarded as a stabilising influence, restraining the radial motion of the jet boundary. The exact mechanism by which the slotted wall restrains the motion is not clear but it is found that large and small wavelengths may be stable. The measured amplification is considerably less than that calculated for an open jet, thus indicating that the slotted wall has a considerable stabilising influence even although complete stability is not achieved.

(b) Energy transfer to the reservoir. The experimental discovery that removing the reservoir wall (i.e. making the reservoir of infinite volume) removed the fluctuations fixed attention on the reservoir as the self-sustaining element. Measurements of the pressure fluctuations showed that their amplitude and phase were independent of position in the reservoir and working section. It was then realised that this was entirely a compressibility effect due to the fluctuations in volume of the constant mass of air enclosed between the oscillating jet boundary and the reservoir walls. Since the maximum amplitude of jet boundary occurs at the downstream end, the reservoir behaves as a tube open at one end and closed at the other and it is shown that, for the frequencies observed, the length of the reservoir is less than a quarter of the wavelength of a sound wave so that the pressure should indeed be fixed in phase with little variation in amplitude throughout the reservoir.

(c) Feed back to the jet. Precisely how energy is fed back into the jet at the upstream end is not clear; it is possible that the pressure fluctuations cause the nozzle to vibrate radially. On this assumption the wavelengths of the preferred disturbances can be calculated. To ensure a phase shift of an integral number of cycles round the loop, it is necessary that the jet fluctuations should produce the maximum pressure in the reservoir when the jet diameter is a minimum at the nozzle. This gives the result that the length of the working section must be an integral number of wavelengths less one quarter, a result borne out by experiment. It is found in experiment that the disturbance of two and three quarters wavelengths is preferred for low wind speeds with a change to the disturbance of one and three quarters wavelengths at higher speeds. Frequency increases linearly with speed so that the effect is of frequency increase to a certain amount and then a fall back to lower frequencies. The reason for this appears to be a resonance effect somewhere in the tunnel. Since the gain round the loop must be unity and since the energy feed back from the downstream to the upstream end of the jet cannot be very efficient, it follows that only disturbances which are highly amplified along the jet can be self-sustaining. The numerous ways in which the gain round the loop can be reduced indicate how the phenomenon can be eliminated in tunnel design.

6. In the presentation of results which follows, theory and experiment have been intermingled. The results are presented in three parts, as in the three stages above.

PART I. JET INSTABILITY

DERIVATION OF THE BASIC EQUATIONS

7. The cross-section of the tunnel working section is circular. The problem considered is the instability of a circular jet of incompressible fluid, of radius a, moving with velocity U through an infinite extent of the same fluid at rest. The disturbances whose stability is considered are axi-symmetric.

8. In this investigation it is adequate to assume that the reservoir is of infinite diameter. A recalculation of the theory for a finite reservoir demonstrated that the numerical results are practically unaltered. The assumption of axial symmetry is justified since only axi-symmetric disturbances change the volume in the reservoir and so provide feed back. It is assumed that the effect of the sixteen slots and gaps of the slotted wall can be averaged round the circumference.

9. The two co-ordinates of a point are taken as r, the radial distance from the axis, and z, the axial distance from the (arbitrary) origin. At any instant, the common surface of the jet and surrounding fluid is at

$$r = a + \eta,$$

where η is a function of z and t, the time. It is assumed that η is of the first order of smallness.

10. Inside the jet, the velocity potential takes the form

$$\phi = -Uz + \psi$$

and, outside, $\phi = \psi'$

where ψ and ψ' are the small disturbance potentials associated with η. The potentials satisfy Laplace's equation which, for axi-symmetric flow, takes the form

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = 0.$$

11. Strictly, boundary conditions have to be satisfied at $r = a + \eta$ but, since only first order effects will be considered, it is adequate to satisfy them at $r = a$.

The normal velocity of the surface is $\frac{\partial \eta}{\partial t}$ so that, outside the jet,

$$\frac{\partial \eta}{\partial t} = -\frac{\partial \psi'}{\partial r} \quad \text{at } r = a.$$

Inside the jet, the particles also have a translational velocity U so that the equivalent condition becomes

$$\frac{\partial \eta}{\partial t} + U\frac{\partial \eta}{\partial z} = -\frac{\partial \psi}{\partial r} \quad \text{at } r = a.$$

It is assumed that the slotted wall section imposes a pressure difference across the common surface. The pressure difference, P, is taken to be positive when the pressure is greater inside the jet. The relevant boundary condition becomes

$$\frac{\partial \psi}{\partial t} + U\frac{\partial \psi}{\partial z} = \frac{\partial \psi'}{\partial t} + \frac{P}{\rho} \quad \text{at } r = a,$$

where ρ is the density of the fluid.

12. A periodic surface disturbance takes the form

$$\eta = \eta_0 \exp i(\sigma t - kz).$$

The corresponding potential and pressure difference disturbances may be expressed as

$$\psi = \psi_0(r) \exp i(\sigma t - kz),$$

$$\psi' = \psi_0'(r) \exp i(\sigma t - kz),$$

$$P = P_0 \exp i(\sigma t - kz).$$

Then ψ_0 and ψ_0' satisfy the equation

$$\frac{d^2\psi_0}{dr^2} + \frac{1}{r}\frac{d\psi_0}{dr} - k^2\psi_0 = 0.$$

This is the modified Bessel equation of order zero, i.e. the Bessel equation with r replaced by ir. The two fundamental solutions are $I_0(kr)$ and $K_0(kr)$ in the notation of Reference 2, p.77. The conditions that ψ_0 should be finite at $r = 0$ and that $\psi_0' \to 0$ as $r \to \infty$ lead to the final forms

$$\psi_0 = A\, I_0(kr), \quad \psi_0' = A'\, K_0(kr).$$

The boundary conditions give

$$i\,\sigma\,\eta_0 - ik\,U\,\eta_0 = -A\,k\,I_0'(ka) = -A\,k\,I_1(ka),$$

$$i\,\sigma\,\eta_0 = -A'\,k\,K_0'(ka) = A'\,k\,K_1(ka),$$

$$i\,\sigma\,A\,I_0(ka) - i\,\sigma\,k\,UA\,I_0(ka) = i\,\sigma\,A'\,K_0(ka) + \frac{P_0}{\rho}.$$

The first two give

$$\frac{A}{\eta_0} = -\frac{i\,(\sigma - k\,U)}{k\,I_1(ka)},$$

$$\frac{A'}{\eta_0} = \frac{i\,\sigma}{k\,K_1(ka)}.$$

Elimination of A and A' from the third then gives

$$(\sigma - kU)^2\,\frac{I_0(ka)}{kI_1(ka)} + \sigma^2\,\frac{K_0(ka)}{kK_1(ka)} = \frac{P_0}{\rho\,\eta_0}.$$

13. A suitable non-dimensional form of this equation is obtained by the substitutions

$$X = \frac{\sigma}{k\,U},$$

$$\alpha = \frac{I_1(ka)\,K_0(ka)}{I_0(ka)\,K_1(ka)},$$

$$\beta = \frac{I_1(ka)}{ka\,I_0(ka)}\cdot\frac{a}{U^2}\cdot\frac{P_0}{\rho\,\eta_0},$$

when it reduces to

$$(X - 1)^2 + \alpha\,X^2 = \beta. \qquad \ldots\ldots (1)$$

The quantity α is a function of ka only and is tabulated in Table I and plotted in Figure 2.

TABLE I

α AS A FUNCTION OF ka

ka	α		ka	α
0	0			
0.1	0.012		1.2	0.377
0.2	0.036		1.4	0.432
0.3	0.066		1.6	0.484
0.4	0.100		1.8	0.530
0.5	0.136		2.0	0.567
0.6	0.171		2.4	0.634
0.7	0.208		2.8	0.679
0.8	0.244		3.2	0.717
0.9	0.279		3.6	0.749
1.0	0.312		4.0	0.771

14. There are two ways of investigating stability. In the problem considered, σ is real and, if equation (1) gives complex values of X, then k is complex and stability depends on the sign of the imaginary part. Although this is the practical case, it is the less convenient to work with as α and β are complicated functions of the complex variable k.

The usual way of investigating stability is to consider an initial sinusoidal surface disturbance and investigate its behaviour at subsequent times. Then k is taken to be real and equation (1) is used to determine X as a function of k as k takes all values from zero to infinity. The two cases are related by the assumption that the amplification per cycle in the latter case equals the amplification per wavelength in the former. Although this is not exactly true, it is approximately true provided that the amplification is not too large.

Let a typical root of equation (1) be $X = X_1 + i\,X_2$. Then $\sigma = \sigma_1 + i\,\sigma_2 = k\,U\,(X_1 + i\,X_2)$. All the unknowns η, ψ, ψ' and P contain a real exponential factor $\exp\,(-\sigma_2 t)$ and the motion is unstable when $\sigma_2 < 0$ or $X_2 < 0$.

15. The measurements taken at a given speed are the frequency, f, and wavelength, λ, of the disturbance and also ϵ, the amplification in a given length, l. These can be related to the theoretical quantities above.

It follows immediately that

$$k = \frac{2\pi}{\lambda}, \quad \sigma_1 = 2\pi f, \qquad \ldots\ldots (2)$$

and, since the amplification per wavelength equals the amplification per cycle,

$$\epsilon = \exp\,(-kl\,\sigma_2/\sigma_1). \qquad \ldots\ldots (3)$$

The ratio of wave velocity to main jet velocity is

$$\frac{f\lambda}{U} = \frac{\sigma_1}{k\,U} = X_1 .$$

Since α is a function of ka or $2\pi a/\lambda$, α is known from the wavelength. By means of equations (2) and (3), k, σ_1 and σ_2 are known and so X. Thus, in any practical case, the entire left hand side of equation (1) is known and β can be deduced.

THE OPEN JET

16. For an open jet, there is no stabilising mechanism and $\beta = 0$. Equation (1) becomes

$$(X - 1)^2 + \alpha X^2 = 0.$$

Since $\alpha > 0$, this is the sum of two squares and admits only the complex solutions

$$X = \frac{1 \pm i\,\alpha^{\frac{1}{2}}}{1 + \alpha},$$

one of which is unstable.

The ratio of wave velocity to main jet velocity is

$$X_1 = \frac{1}{1 + \alpha}, \qquad \ldots\ldots (4)$$

so that this ratio always lies between 0.5 and 1.

The amplification in length l is

$$\epsilon = \exp\,(k\,l\,\alpha^{\frac{1}{2}}). \qquad \ldots\ldots (5)$$

PARTICULAR FORMS OF STABILISATION

17. The effect of the bars is presumed to be to provide a pressure difference across the boundary of the jet. This pressure difference is a function of η and its derivatives which is probably non-linear. Linearising for small disturbances, the most general form for this pressure difference is

$$P = K_1 \eta + K_2 \frac{\partial \eta}{\partial t} - K_3 \frac{\partial^2 \eta}{\partial z^2} + K_4 \frac{\partial^4 \eta}{\partial z^4} .$$

The first term corresponds to a restoring force due to the stiffness of the bars on their supports, the second to a resistance damping as the fluid is forced through the bars, the third is a term analogous to surface tension and the fourth is a restoring force due to bending of the bars. The four constants K_1, K_2, K_3, K_4 are positive.

Then

$$\beta = \frac{I_1(ka)}{ka\, I_0(ka)} \cdot \frac{a}{\rho U^2} (K_1 + ik\, U K_2 X + k^2 K_3 + k^4 K_4)$$

$$= 2\, i\, \gamma X + \delta, \text{ say,} \qquad \ldots\ldots (6)$$

where γ and δ are positive functions of k.

18. Equation (1) now becomes

$$X^2 (1 + \alpha) - 2 X (1 + i \gamma) + (1 - \delta) = 0.$$

Some manipulation gives the roots of this in the form

$$X = \frac{(Z \pm 1)(i\, Y \pm 1)}{1 + \alpha}, \qquad \ldots\ldots (7)$$

where

$$\left. \begin{aligned} Y^2 - Z^2 &= \gamma^2 + \zeta , \\ YZ &= \gamma , \\ \zeta &= (1 + \alpha)(1 - \delta) - 1. \end{aligned} \right\} \qquad \ldots\ldots (8)$$

Y and Z are always taken as positive so that an unstable solution only exists if the solution of equation (8) gives $Z < 1$.

19. For an unstable solution, the ratio of wave velocity to main jet velocity is

$$X_1 = \frac{1 - Z}{1 + \alpha} \text{ so that } 0 < X_1 < \frac{1}{1 + \alpha} \qquad \ldots\ldots (9)$$

and the amplification in length l is

$$\epsilon = \exp (k\, l\, Y). \qquad \ldots\ldots (10)$$

20. When $\gamma = 0$, i.e. no damping takes place, stability depends on the sign of ζ. If $\zeta < 0$, $Y = 0$, $Z = (-\zeta)^{\frac{1}{2}}$ and the flow is stable while, if $\zeta > 0$, $Y = \zeta^{\frac{1}{2}}$, $Z = 0$ and the flow is unstable.

The condition $\zeta \lessgtr 0$ gives $\delta \gtrless \frac{\alpha}{1 + \alpha}$ so that increasing the restoring force of the bars past a critical amount stabilises the system.

21. When $\gamma \neq 0$, the criterion for instability is that $Z < 1$ so that, from equation (8), $Y > \gamma$ and so $Z^2 > -\zeta$. Thus instability demands $\zeta > -1$. The disturbance is therefore stable or unstable according as $\delta \gtrless 1$. It follows that in the presence of damping a greater restoring force is necessary to stabilise the flow. It also follows from equations (8) that, if ζ is constant, Y increases with γ. Thus damping destabilises the flow - a paradoxical result.

22. From the form of β given in equation (6), the following conclusions can be drawn:-

(a) For large wavelengths, i.e. k small, it is found from Ref. 2 that

$$I_0(ka) \sim 1, \; I_1(ka) \sim \tfrac{1}{2}\,ka; \; K_0(ka) \sim \log\frac{2}{ka}, \; K_1(ka) \sim \frac{1}{ka}.$$

Then

$$\beta \sim \frac{a}{2\,\rho\,U^2}\,(K_1 + i\,k\,U\,K_2\,X)$$

so that

$$\gamma \sim \frac{k\,K_2\,a}{4\,\rho\,U}, \quad \delta \sim \frac{K_1 a}{2\,\rho\,U^2}$$

and

$$\alpha \sim \tfrac{1}{2}\,k^2\,a^2 \log\frac{2}{ka}.$$

It follows that $\zeta \sim \dfrac{-K_1 a}{2\,\rho\,U^2}$ and disturbances are stable if

$$K_1 a > 2\,\rho\,U^2, \quad K_2 \neq 0 \text{ or } K_1 > 0, \; K_2 = 0.$$

(b) For small wavelengths, i.e. k large, it is found from Ref. 2 that

$$I_0(ka), \; I_1(ka) \sim \frac{e^{ka}}{(2\,\pi\,ka)^{\frac{1}{2}}}; \; K_0(ka), \; K_1(ka) \sim \left[\frac{\pi}{2\,ka}\right]^{\frac{1}{2}} e^{-ka}.$$

Then

$$\beta \sim \frac{1}{\rho\,U^2}\,(i\,U\,K_2\,X + k^3\,K_4), \; K_4 \neq 0,$$

$$\sim \frac{1}{\rho\,U^2}\,(i\,U\,K_2\,X + k\,K_3), \quad K_4 = 0, \; K_3 \neq 0.$$

The results obtainable from this are equally valid in the two cases; the case $K_4 \neq 0$ will be assumed. Then

$$\gamma \sim \frac{K_2}{2\rho U}, \quad \delta \sim \frac{k^3\,K_4}{\rho\,U^2}, \quad \alpha \sim 1.$$

It follows that $\zeta \sim 1 - \dfrac{2\,k^3\,K_4}{\rho\,U^2}$. Given K_4, $\zeta < 0$ or -1 for large enough k so that disturbances are stable if $k^3\,K_4 > \rho\,U^2$, $K_2 \neq 0$ or $k^3 K_4 > \frac{1}{2}\,\rho\,U^2$, $K_2 = 0$.

(c) Although large and small wavelengths may be stable, it is possible that an intermediate band of wavelengths will be unstable. In particular, increasing U, the speed of the jet, decreases the ranges of stability at high and low wavelengths, so that, even if all disturbances are stable at low speeds, at higher speeds instabilities will arise and the range of unstable wavelengths will increase as the speed is increased.

EXPERIMENTAL RESULTS

23. Figure 3 shows two sets of frequency analyses of the turbulence. They were taken on the axis of the jet with a hot wire anemometer and therefore relate to the longitudinal component of turbulence. The bandwidth of the frequency analyser was about 1% of the frequency. In neither case was the reservoir on so that no preferred frequencies were present. However, they differed in that the diffuser was on in one case and not in the other. On the assumption that the exponential law of amplification applies, these results have been extrapolated to give the amplification along the entire working section (taken as 40 in. from nozzle to stagnation point on the ring). Figure 4 gives the extrapolated results. Results agree at the lower frequencies but a systematic difference appears at higher frequencies. This effect of the diffuser agrees with that noted below in paragraph 28. Ignoring this, a mean curve is drawn which indicates that, at 75 feet per second, frequencies between 20 and 60 c.p.s. are unstable with a maximum amplification along the length of working section of 6.6 at 40 c.p.s.

That the amplification does obey the exponential law is shown by Figure 5. In this Figure, a straight line indicates an exponential relationship and it is seen how the amplification increases at the end of the bars when the diffuser is missing so that the jet becomes an open jet. The effect of the diffuser in damping out disturbances by creating a closed jet is also shown in Figure 5.

24. The amplification to be expected for an open jet is given by equations (4) and (5). This is a rapidly increasing function of frequency with the values of 13.5 at 20 c.p.s. and 360 at 30 c.p.s. at 75 feet per second. Thus the slotted wall has a considerable stabilising effect.

25. It was not possible to measure wavelengths on this occasion so that the method of computing β outlined in paragraph 15 cannot be carried out. On the assumption that $\gamma = 0$ so that $Z = 0$, $Y = \zeta^2$, equations (9) and (10) were used to compute ζ and equation (8) to compute δ as a function of frequency in the unstable band. Then

$$\frac{a P_0}{\rho U^2 \eta_0}$$

is computed; this should take the form

$$K_1 + k^2 K_3 + k^4 K_4$$

according to equation (6). In fact, it appears from Figure 6 that, over the small range of k considered, this quantity is almost exactly proportional to k. The experimental results thus shed no light on the mechanism of stabilisation other than confirmation of the general result that stabilisation of high and low frequencies is quite possible. A further observation very relevant to the mechanism of stabilisation is that it was at no time possible to observe any oscillatory motion of the bars of the slotted wall.

PART II. ENERGY TRANSFER TO THE RESERVOIR

EXPERIMENTAL RESULTS

26. The experimental discovery that sinusoidal disturbances only occurred when the reservoir was present led to the consideration of the reservoir as an integral part of the oscillating system. For this reason, the experimental results are given first.

27. Figure 7 gives comparable spectra for five different cases at 75 ft/sec. which are typical of the results obtained. Spectra, measured by hot wire anemometer, are given for:-

I. complete working section,

II. ring and diffuser removed,

III. diffuser only removed,

IV. outer reservoir wall only removed,

V. entire working section, including slotted wall, removed.

In the first four cases, the spectra were measured on the centre line of the jet in line with the end of the slotted wall; in the fifth case, the spectrum is measured at the exit to the nozzle as the jet breaks up very rapidly in this case.

28. The following facts are observed:-

(a) Removing the reservoir removes the large sinusoidal components (cf. case IV with cases I, II and III).

(b) Removing the ring reduces the amplitude of the large disturbances (cf. case II with cases I and III).

(c) Removing the diffuser affects the frequency of the major preferred component (cf. case III with case I). At 45 c.p.s. the analyser was overloaded in case III. The response falls again for higher frequencies but estimates of resonant frequency and amplitude on resonance cannot be made.

(d) As far as can be seen, removing the ring changes the preferred frequencies more than removing the diffuser does.

The difference between cases IV and V is basically that given in Figure 4 and demonstrates the amplification along the bars when no frequency preference mechanism is operating.

29. In order to investigate the effect of the reservoir, a small pressure microphone was used in addition to the hot wire anemometer, whose use in the reservoir is precluded since there is no main stream velocity there. The microphone was not calibrated and so no quantitative results are given. Figure 8 gives records of pressure measured at various positions in the jet and reservoir compared with the hot wire output at a reference position on the axis of the jet at the downstream end of the slotted wall section. It is seen that the pressure fluctuations at all points are in phase and that the variation in amplitude is not great. The phase relationship between hot wire and pressure fluctuation cannot be determined since the phase changes in the two amplifying and measuring circuits are not known.

30. Tests were carried out with holes cut in the outer reservoir to act as a pressure release mechanism. Hot wire measurements were taken on the axis of the jet near the downstream end of the working section. An example is given in Figure 9 with a row of holes of diameter $\frac{5}{16}$ in. set $\frac{1}{2}$ in. from the forward end of the reservoir. It is seen that the major component is reduced tenfold although the minor component is about doubled. Rows of holes at the downstream end of the reservoir were less effective.

THEORETICAL CONSIDERATIONS

31. Bernoulli's equation is usually expressed in the form

$$p + \tfrac{1}{2}\rho U^2 = \text{const.}$$

This is derived from the more general form, valid in the present case,

$$p + \tfrac{1}{2}\rho U^2 = F(t),$$

where $F(t)$ is a function of time to be determined.

Usually, the medium extends to infinity in some direction in which $U \to$ const., $p \to \pi$, the pressure at infinity. Thus $F(t)$ takes a constant value.

However, in the present case, the fluid in the reservoir does not extend to infinity and no reference point is available to determine $F(t)$.

32. The sinusoidal fluctuations observed in the pressure occur because, while the mass of fluid trapped in the reservoir remains constant, its volume fluctuates over a cycle due to the disturbance travelling downstream along the jet boundary. Thus compressibility determines the magnitude of the fluctuations. However, provided that the pressure fluctuations are independent of position in both amplitude and phase, as is observed to be the case, these disturbances do not affect the incompressible flow which depends only on pressure gradients and so the incompressible theory of the jet can still be used.

33. In the potential flow solution, pressure fluctuations producing gradients arise. The experimental measurements indicate that these must be small compared with the compressible effects. From Part I, the potential flow pressure fluctuations can be determined from the amplitude of the jet surface. Similarly the compressible fluctuations in the reservoir can be calculated from a knowledge of reservoir volume. These calculations confirm that the potential flow fluctuations are undetectable.

34. In view of the fact that the velocity of sound is finite, it is necessary to determine theoretically the distribution of phase and amplitude in the reservoir due to compressibility. An exact calculation is laborious but a simple approximate calculation can be made. It is assumed that the major contribution comes from the downstream end where the amplitude of the jet surface is greatest. Then the reservoir acts as an organ pipe with a vibrating diaphragm at $z = l$ and a fixed end at $z = 0$. The pressure is known to be given by

$$p \;\alpha\; \eta_0 \cos \frac{\sigma z}{c}\, e^{i\sigma t}$$

where c is the velocity of sound.

The pressure is a maximum at $z = 0$. Since $\sigma l < \frac{1}{2} \pi c$ in all cases of interest, the pressure is in phase everywhere and the minimum amplitude occurs at the downstream end and is $\cos \sigma l/c$ of the maximum. At 40 c.p.s., this quantity is about 0.7. In practice, with a continuously oscillating jet boundary, the discrepancy between minimum and maximum will be less than this so that the experimental results may be regarded as confirmed.

PART III. FEED BACK TO THE JET

THEORY

35. The requirement for self-sustained oscillations is that the phase change round the loop should be an integral number of cycles and the gain greater than unity in the linearised theory. It is not possible to make any useful deductions about the gain but considerations of phase lead to an estimate of the preferred frequencies.

36. The displacement of the surface of the jet may be expressed as

$$\eta = \eta_0 \exp i\,(\sigma t - kz),$$

where k is complex.

The change in volume of the reservoir is

$$V = V_0\, e^{i\sigma t}$$

where

$$\begin{aligned} V_0 &= 2\pi a \eta_0 \int_0^l e^{-ikz}\,dz \\ &= \frac{2\pi a \eta_0}{ik}\,(1 - e^{-ikl}) \\ &= \frac{2\pi a \eta_0}{k_1 + ik_2}\,\{-i + ie^{k_2 l}\,(\cos k_1 l - i \sin k_1 l)\} \\ &= \frac{2\pi a \eta_0}{k_1 + ik_2}\,\{e^{k_2 l} \sin k_1 l + i\,(e^{k_2 l} \cos k_1 l - 1)\} \end{aligned}$$

where $k = k_1 + i\,k_2$, $k_1, k_2 > 0$.

37. When V takes its maximum value, the reservoir volume is least and so the pressure is greatest. This pressure acts on either the jet or nozzle at the upstream end and it is assumed that at this point the jet diameter is a minimum. Thus V and η must be an amount π out of phase so that V_0/η_0 is real and negative. This condition is only satisfied if

$$\frac{e^{k_2 l}\sin k_1 l}{k_1} = \frac{e^{k_2 l}\cos k_1 l - 1}{k_2} < 0.$$

Experimentally it is known that $e^{k_2 l} \gg 1$ and $k_1 \gg k_2$. Thus $\sin k_1 l \doteq -1$ and

$$k_1 l = 2n\pi + \frac{3\pi}{2} + \xi, \qquad \ldots\ldots (11)$$

where n is an integer.

Since ξ is small,

$$-\frac{e^{k_2 l}}{k_1} = \frac{\xi e^{k_2 l} - 1}{k_2}$$

so that

$$\xi = e^{-k_2 l} - \frac{k_2}{k_1}.$$

Experimental results indicate that $e^{-k_2 l}$ and k_2/k_1 are comparable in magnitude so that

$$\xi \doteq 0.$$

Thus, from equation (11),

$$\frac{l}{\lambda} = n + \tfrac{3}{4}. \qquad \ldots\ldots (12)$$

Taking l as 40 inches, values for λ, ka and f/U are obtained and given in Table II.

TABLE II

n	λ (inches)	ka	f/U (feet^{-1})
0	53	0.41	0.20
1	23	0.96	0.40
2	15	1.51	0.56
3	11	2.06	0.71

f/U is obtained from equation (9) on the assumption that $Z = 0$.

38. When the diffuser is removed, this theory predicts that the selected wavelengths will be unaltered. When the ring is removed, however, the phase relationship between the jet boundary oscillation and the volume oscillation is changed and it is to be expected that the selected wavelengths will change. Also a smaller pressure oscillation should result in this case. These results are in accord with the experimental results noted in paragraph 28.

EXPERIMENTAL RESULTS

39. A typical set of hot wire records at the end of the working section is given in Figure 10. It is seen that, at low speeds, the record is more or less periodic although by no means sinusoidal. At speeds from 50 to 70 feet per second a marked sinusoidal component occurs, at 75 feet per second this is modulated by a beat frequency, from 80 to 95 feet per second a marked sinusoidal component occurs again and at 100 and 105 feet per second the record is again more random.

40. Figure 11 gives a plot of turbulence level, frequency and wavelength against speed. The first is obtained directly and the second was obtained by means of the frequency analyser. The third was obtained by using two hot wires, one fixed and the other moving, both on the axis of the jet, the sum of their outputs being measured. When the wires are separated by a wavelength, the signals reinforce and when they are separated by half a wavelength they tend to cancel. By this means the wavelength is estimated. The results for frequency and wavelength given in Table II are also shown in the Figure and it is seen that remarkably good agreement is obtained. Since the theoretical frequency curves assume that $Z = 0$, this agreement implies that $Z \ll 1$.

41. Figure 12 plots turbulence level against frequency. This indicates that frequencies between about 33 and 40 c.p.s. are accepted, presumably by some mechanical resonance in the tunnel system. It al , indicates - as suggested in Part I - that the amount of instability increases with speed. Thus the modes with $n = 3$ and above do not occur at low speeds since the gain round the loop is not great enough and the mode with $n = 0$ does not occur since it would only be accepted by the resonance at higher speeds than those at which the tunnel could be operated.

CONCLUSION

42. An explanation has been given above for the phenomena observed in the 7 in. wind tunnel. It has been shown how the observed frequencies and wavelengths of the oscillations agree with those calculated by theory.

43. Only two points are left unexplained.

(a) It is not clear how the slotted wall section stabilises the flow. As stated in paragraph 40, it is concluded from experiment that $Z \ll 1$. It seems most probable that the restoring forces produced by the slotted wall are more important than the damping in stabilising the flow, damping determining the amplification of unstable flows.

(b) The mechanism of frequency selection from the several preferred wavelengths is not known. It appears that it is frequencies which are preferred rather than wavelengths so that it may be assumed that a mechanical resonance is operating. The resonant frequency appears to depend slightly on speed and possibly more seriously on reservoir leakage as well as on whether or not the diffuser is in position so that the resonance presumably occurs at the downstream end of the reservoir.

44. It is considered that a more detailed investigation would enable these points to be cleared up. It is concluded that the description given in this report of the modus operandi of the phenomena is correct and that quantitative results can be predicted in all save a few details.

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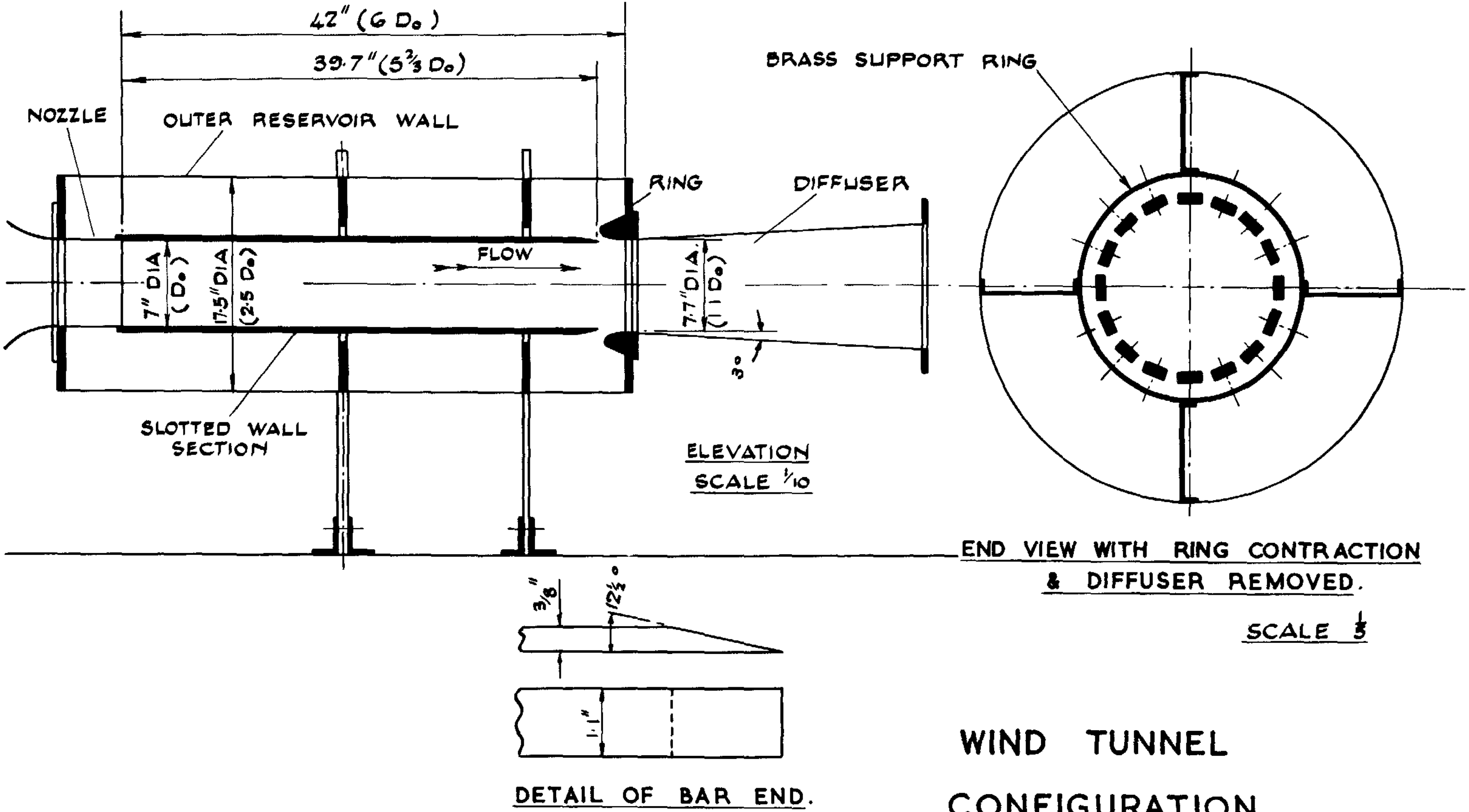
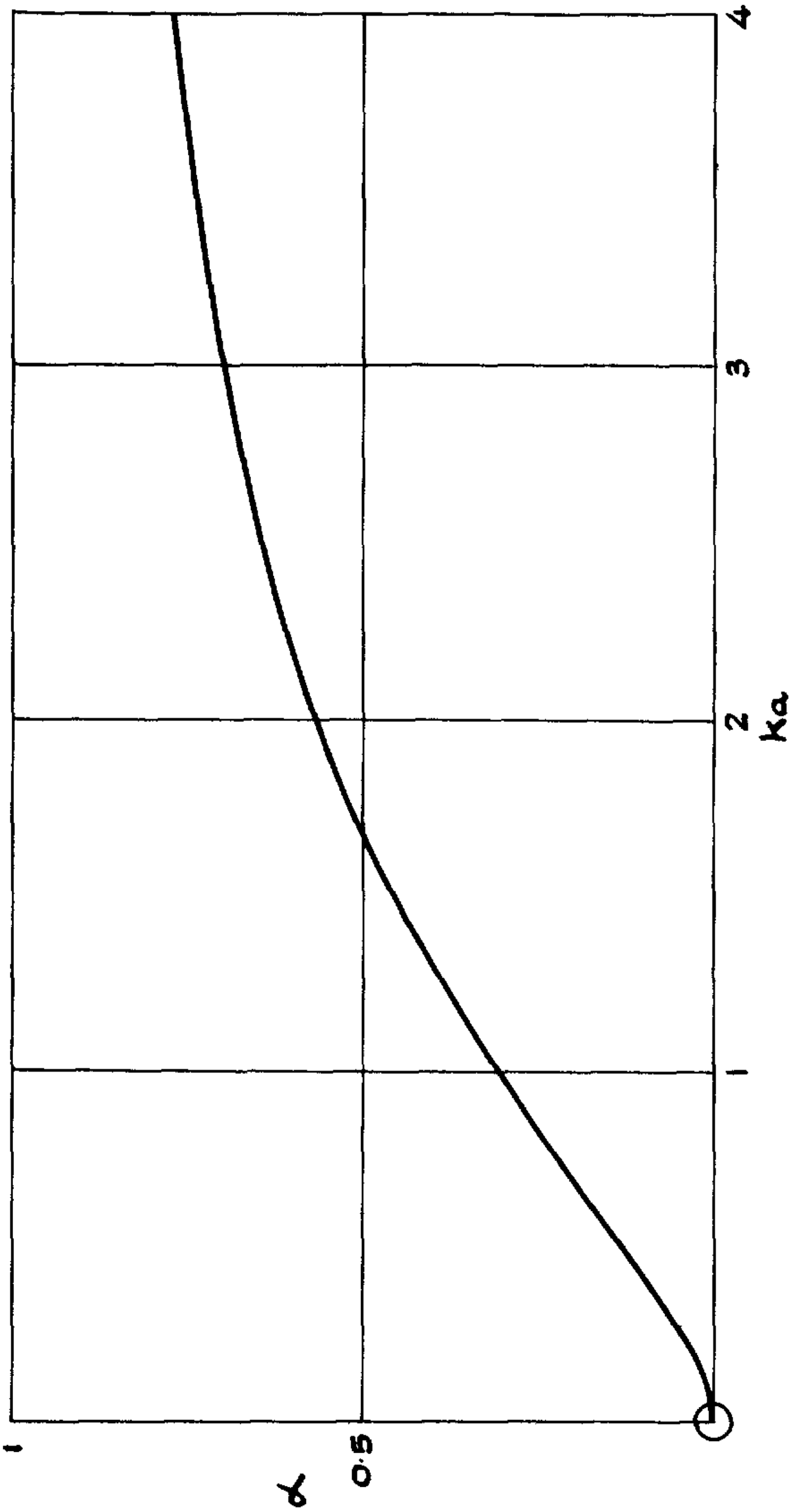
1. H. LAMB. Hydrodynamics, 6th Edition (C.U.P., 1932).

2. G.N. WATSON. A Treatise on the Theory of Bessel Functions, 2nd Edition (C.U.P., 1952).



WIND TUNNEL CONFIGURATION

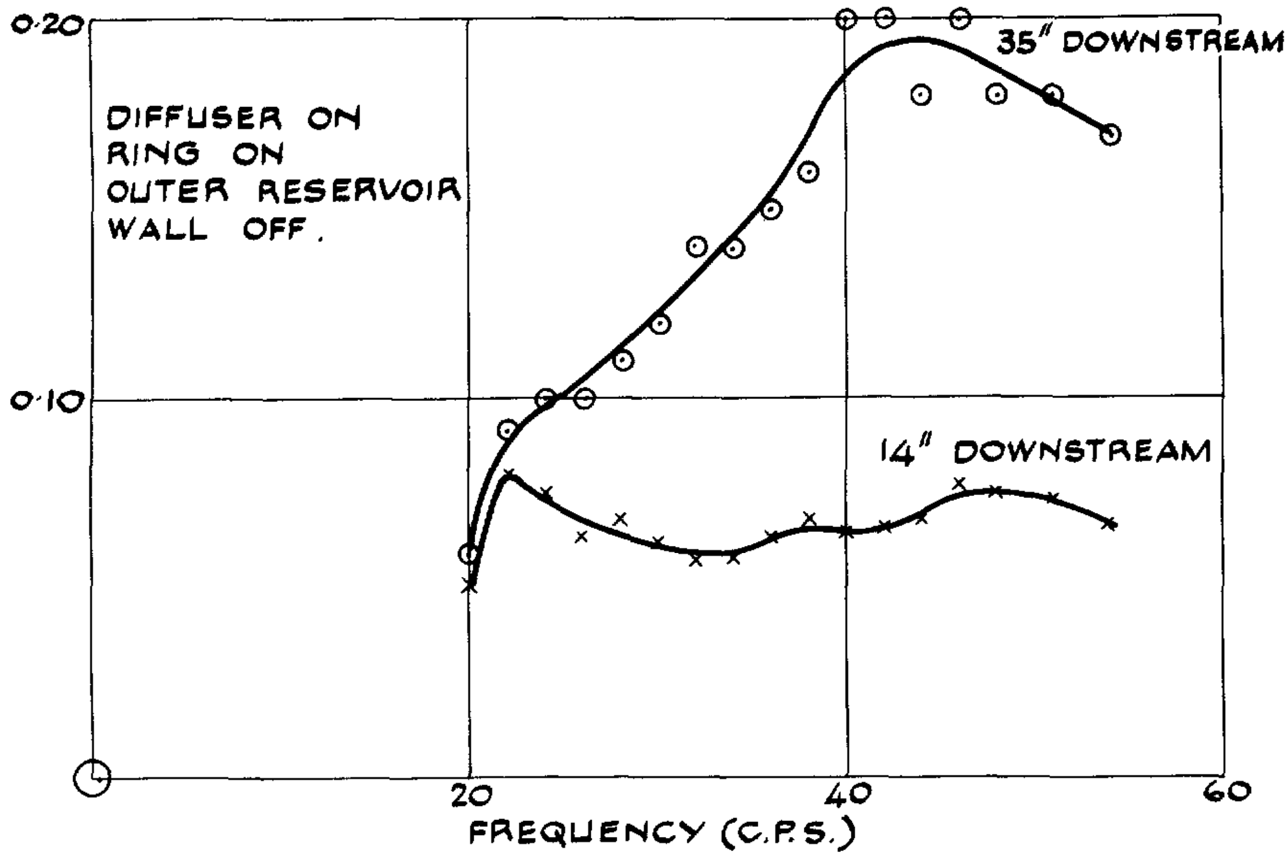
FIG. 1

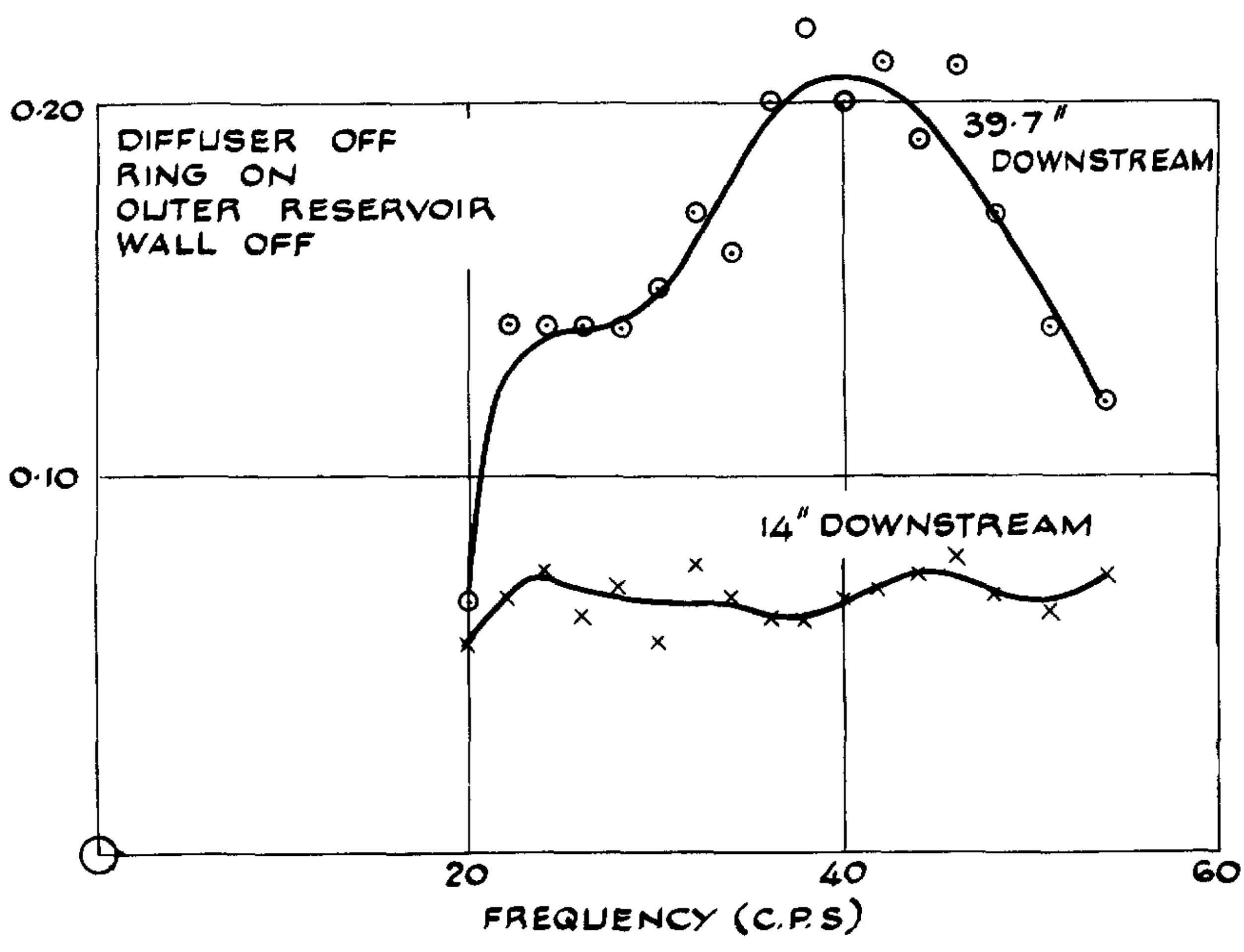


α PLOTTED AS A FUNCTION OF ka.

FIG. 2

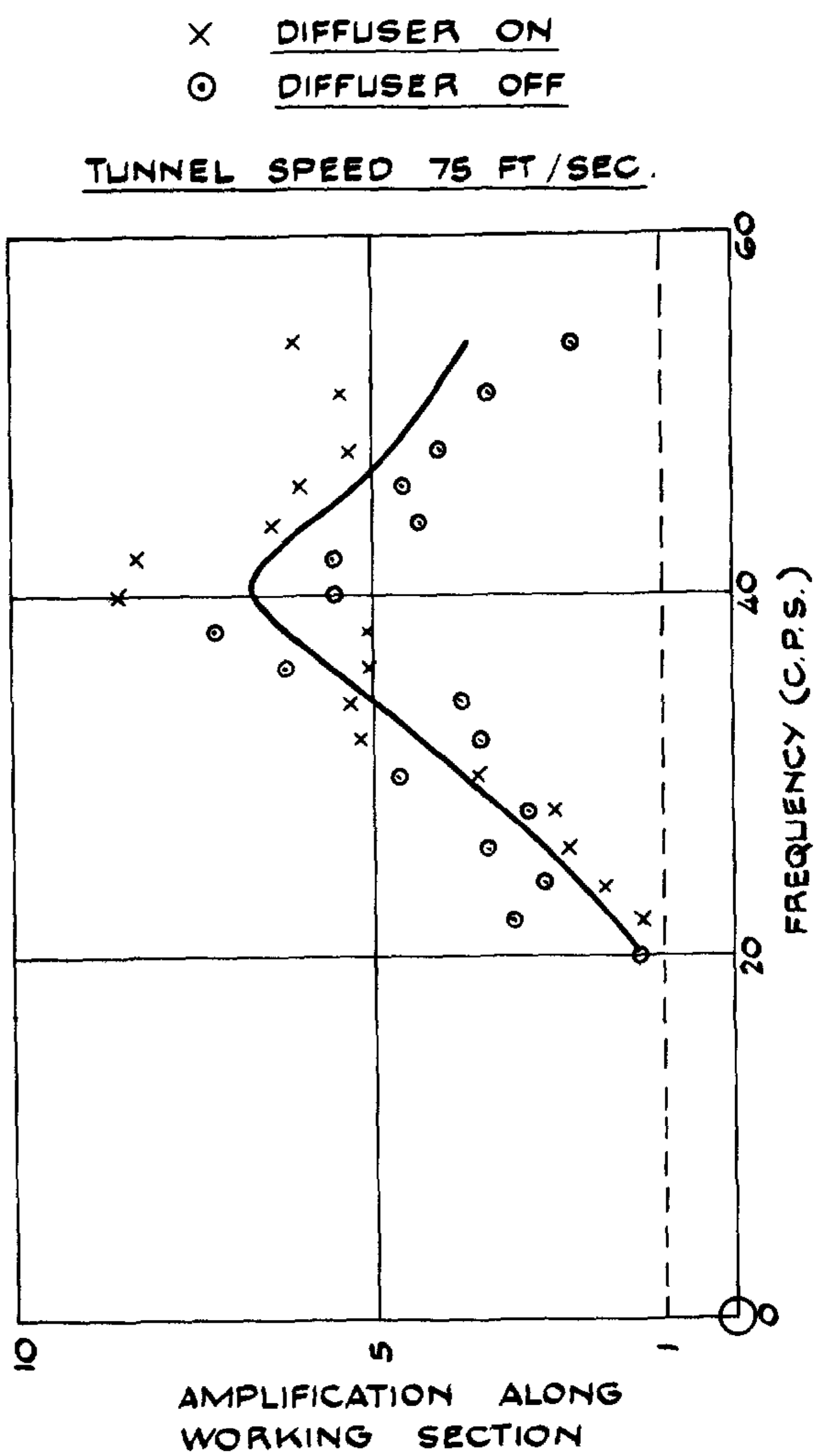
ORDINATES IN ARBITRARY UNITS
DISTANCES MEASURED FROM START OF WORKING SECTION.
TUNNEL SPEED 75 FEET/SEC.





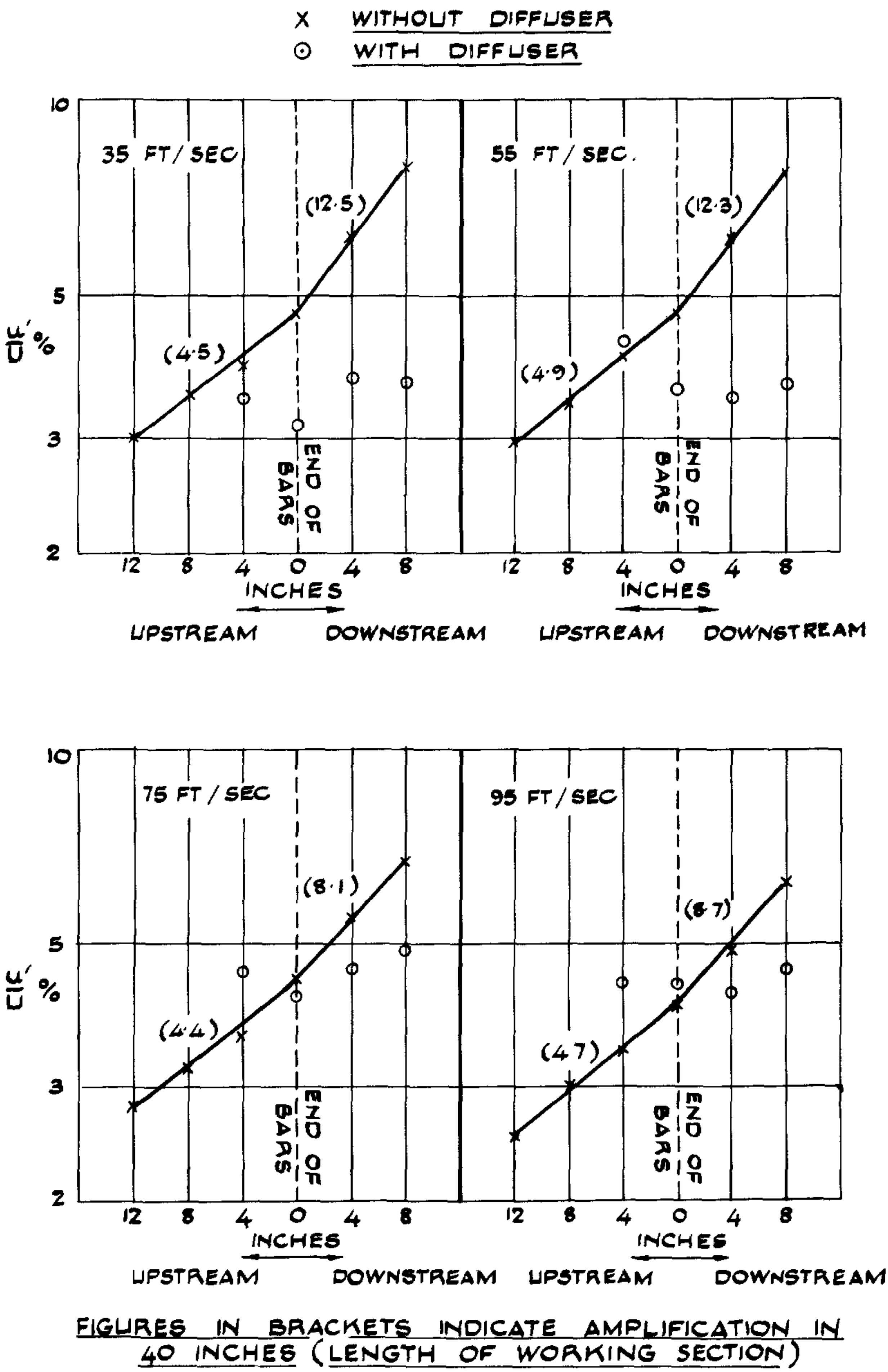
FREQUENCY ANALYSES OF HOT WIRE ANEMOMETER SIGNALS ON AXIS OF JET.

FIG. 3



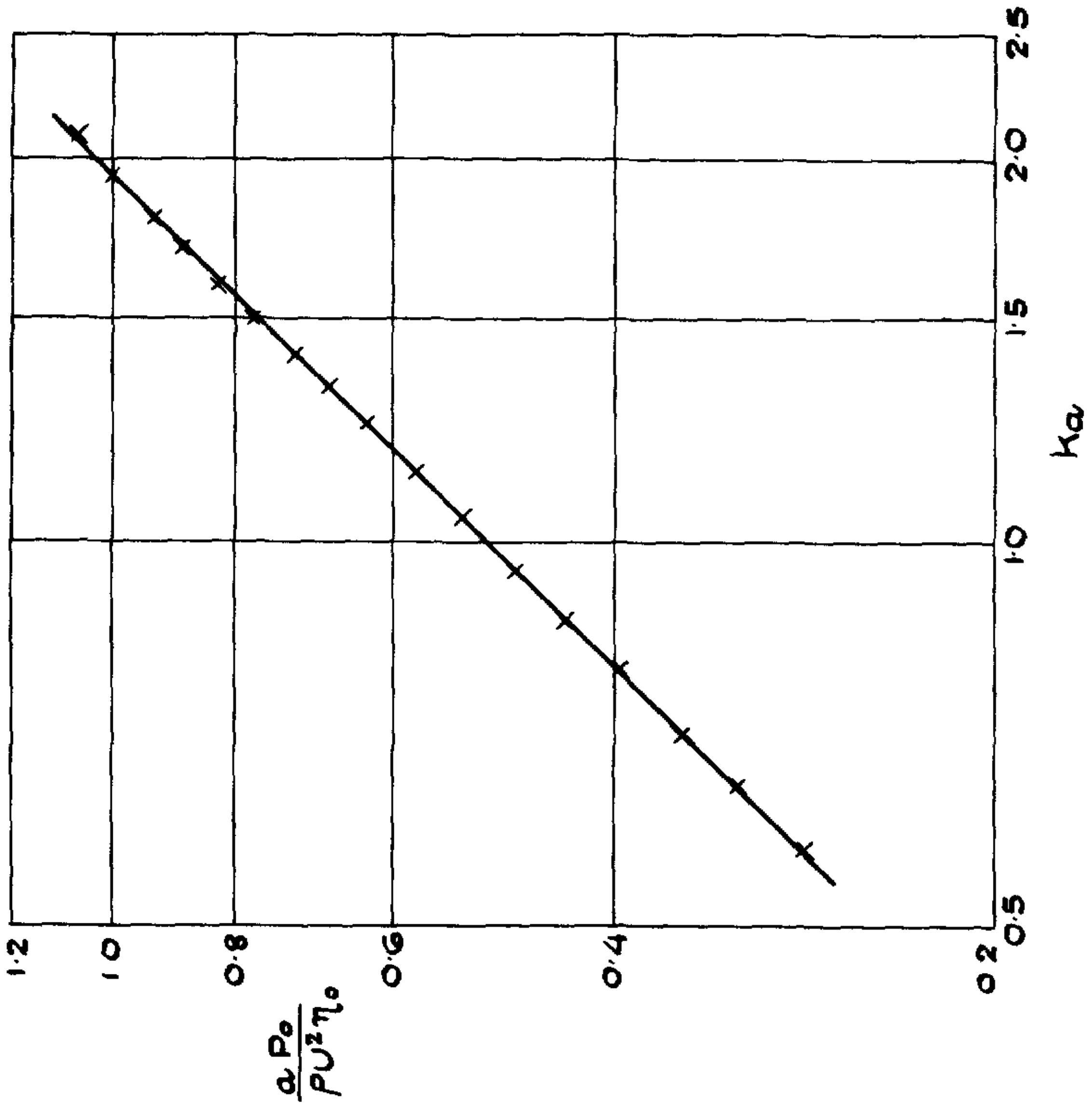
AMPLIFICATION AS A FUNCTION OF FREQUENCY.

FIG. 4



VARIATION OF TURBULENCE LEVEL ALONG AXIS OF JET.

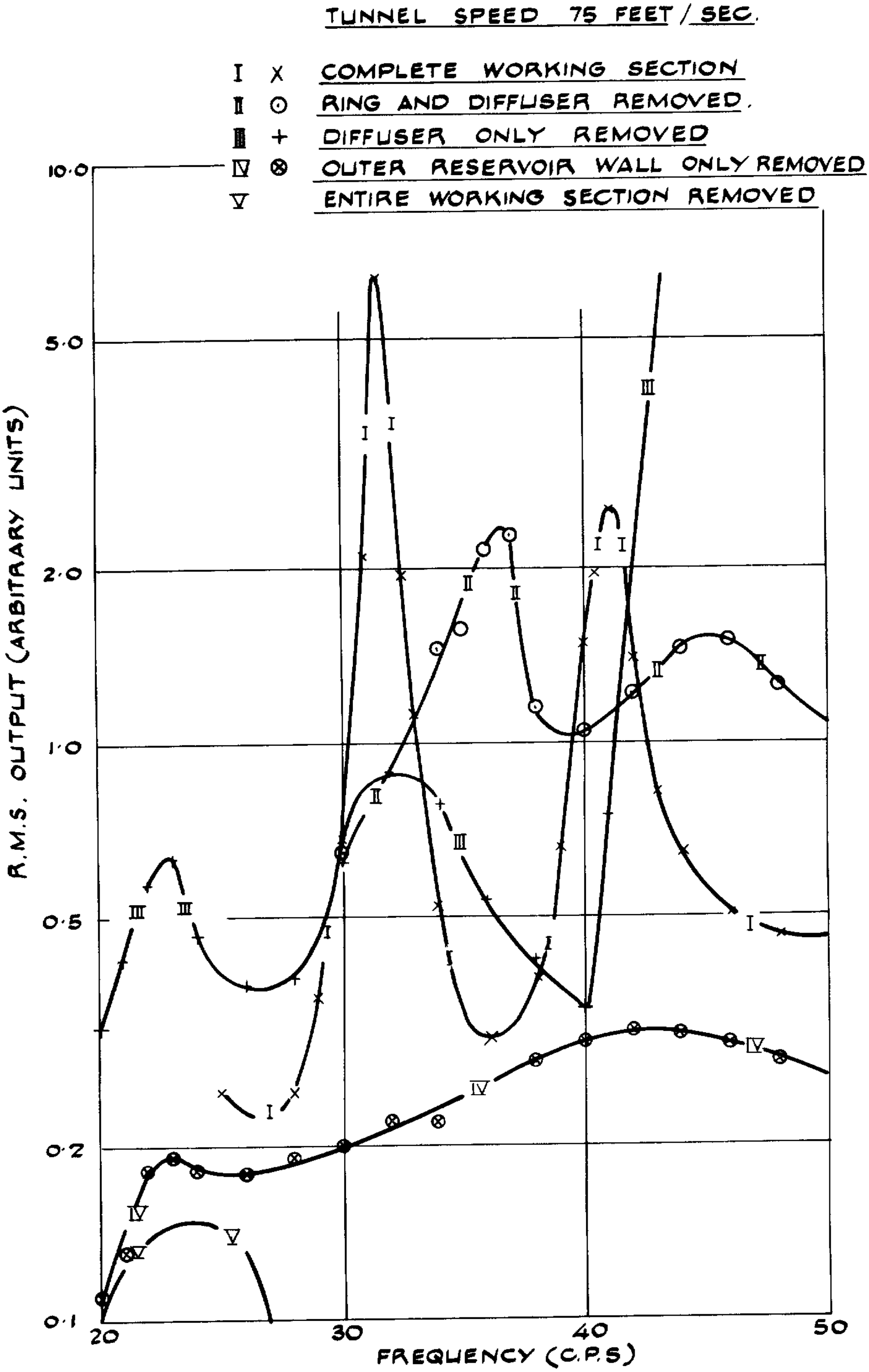
FIG. 5



TUNNEL SPEED 75 FEET/SEC.

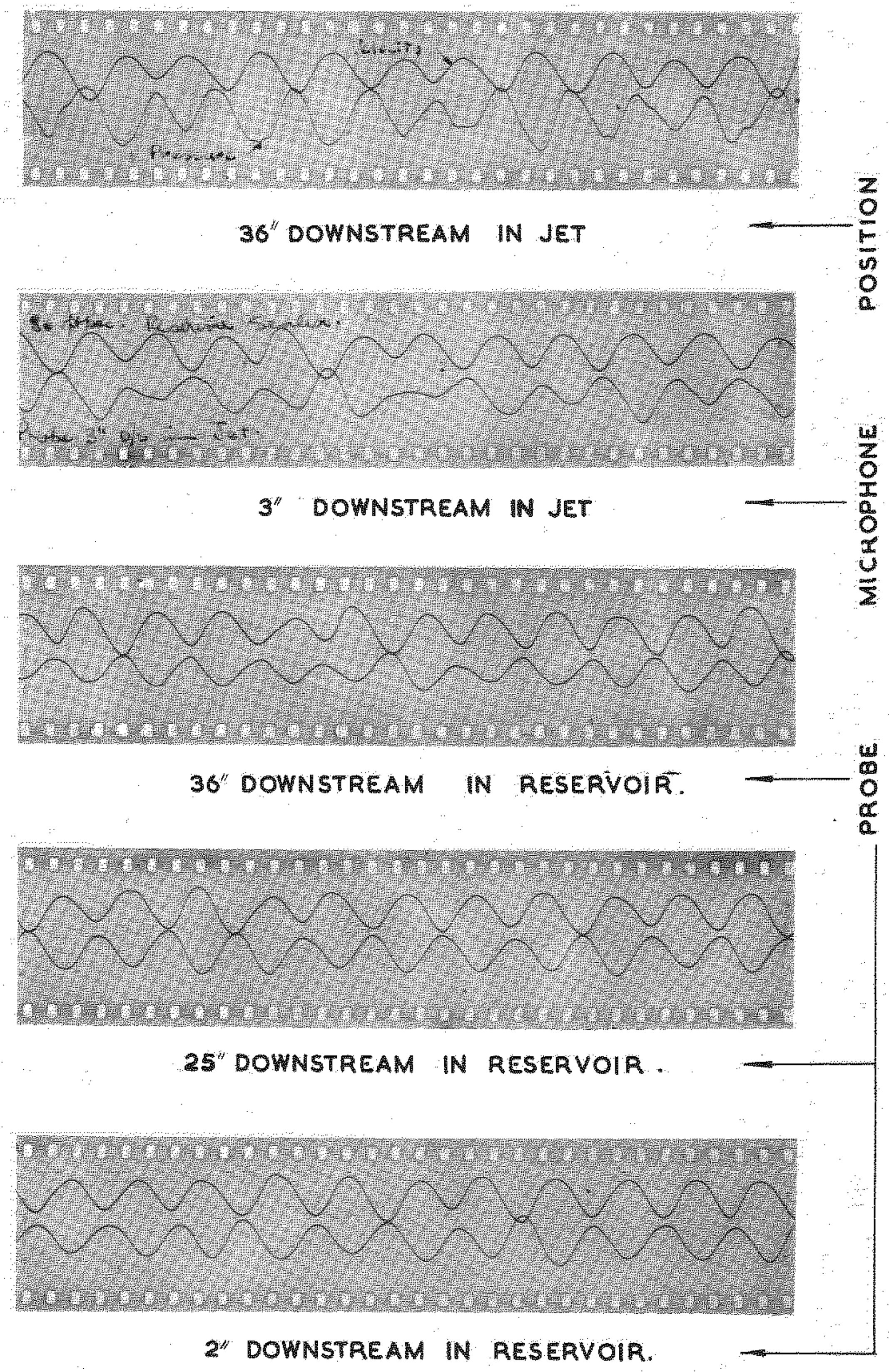
$\frac{aP_0}{\rho U^2 \eta_0}$ AS A FUNCTION OF Ka

FIG. 6



SPECTRA ON THE JET AXIS UNDER VARIOUS CONDITIONS.

FIG. 7

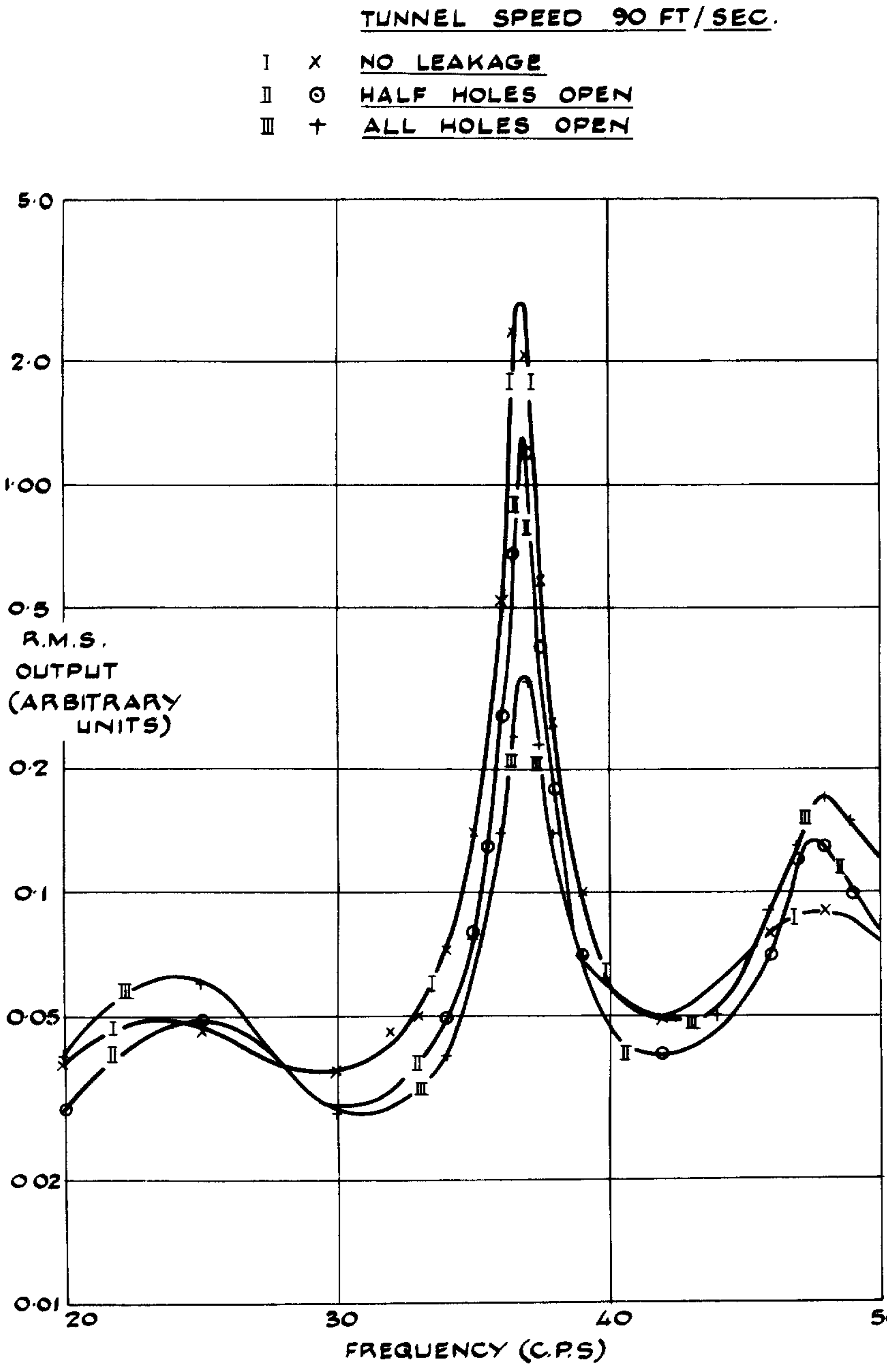


TUNNEL SPEED 80 FT/SEC.
ALL POSITIONS MEASURED FROM NOZZLE EXIT.

UPPER TRACES — REFERENCE HOT WIRE
LOWER TRACES — PROBE MICROPHONE

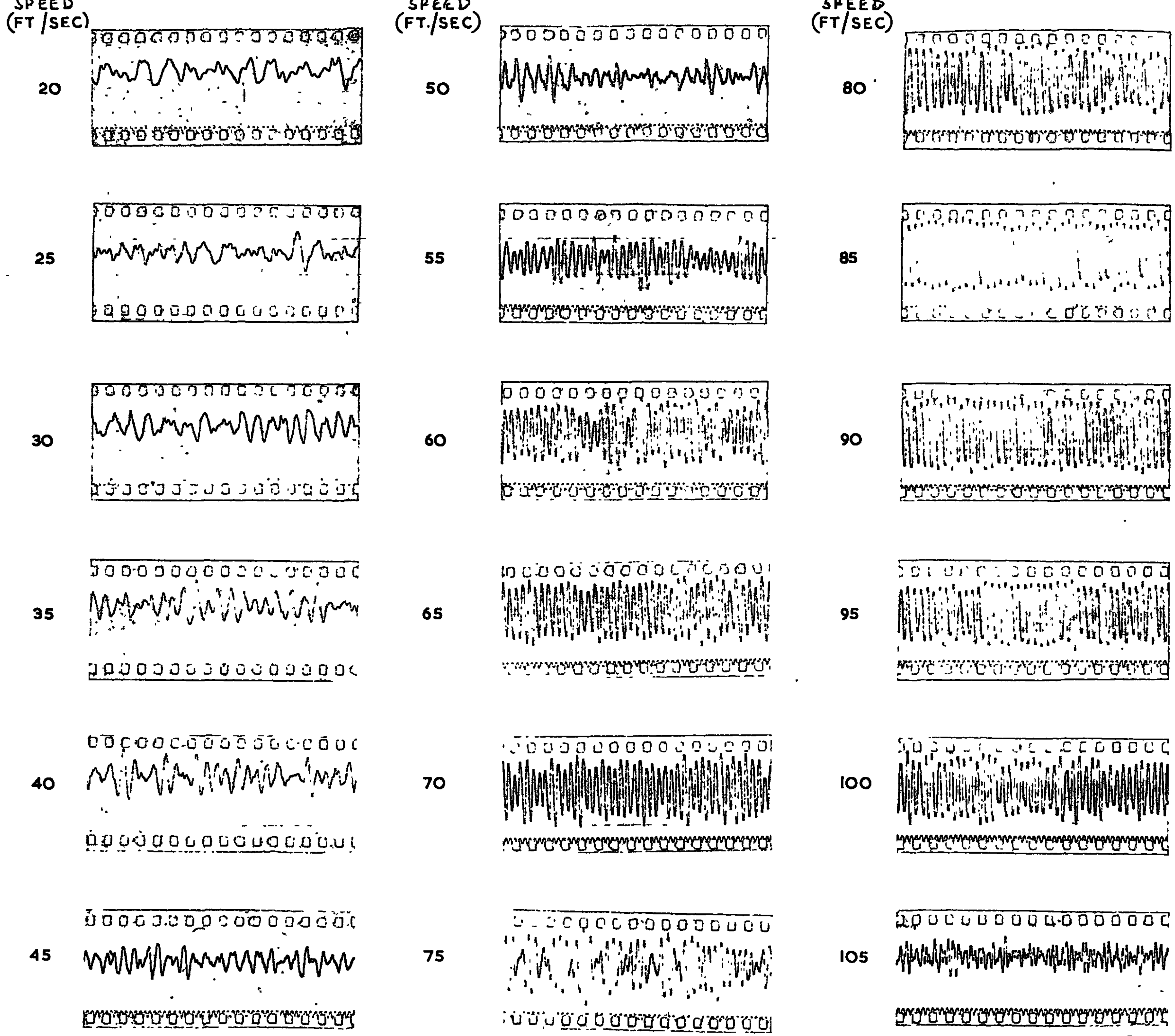
PHASE AND AMPLITUDE RELATIONSHIPS OF PRESSURE FLUCTUATIONS.

FIG. 8



EFFECT OF LEAKAGE HOLES. ON SPECTRUM.

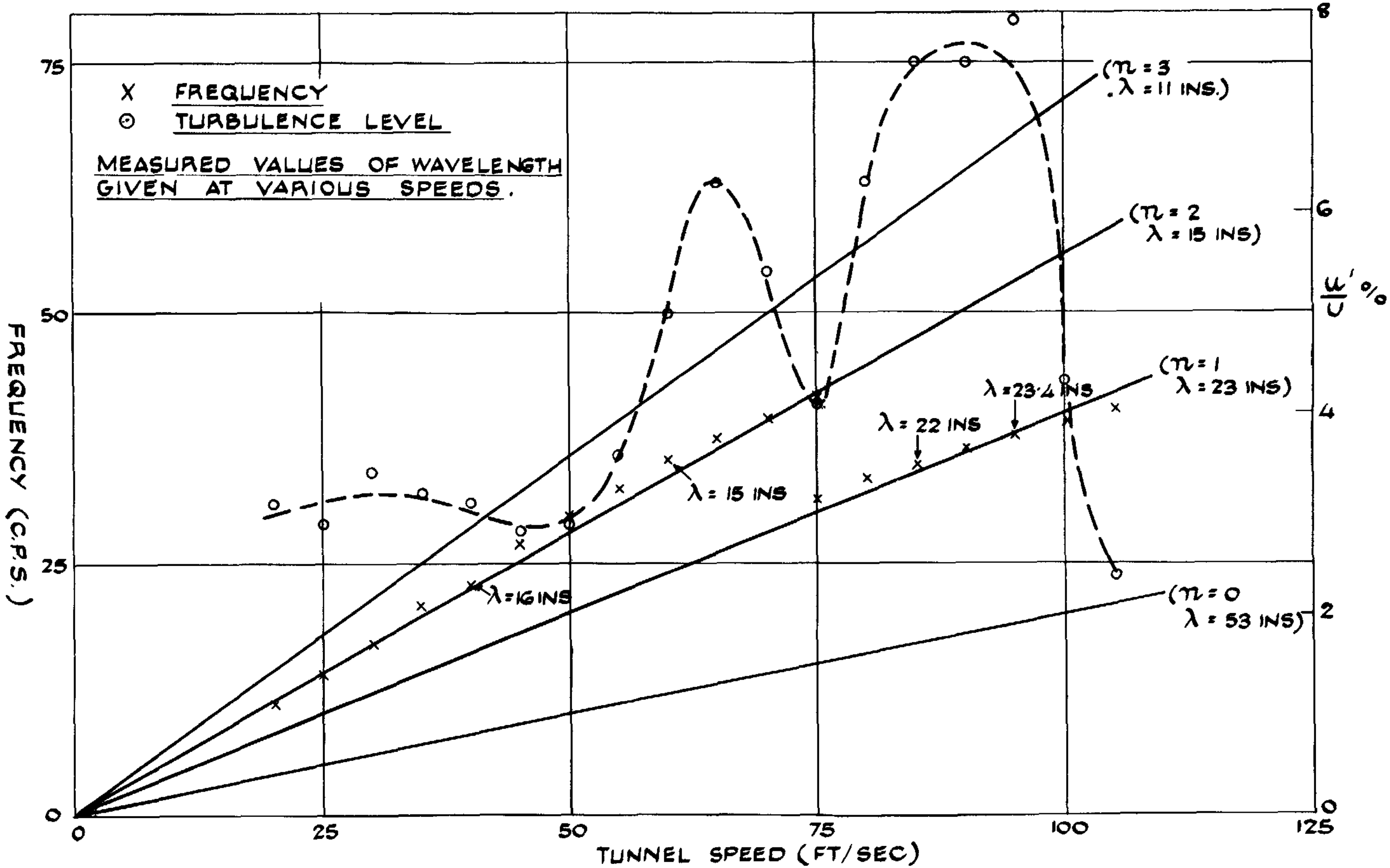
FIG. 9



OSCILLATIONS IN JET AT DOWNSTREAM END OF WORKING SECTION AS A FUNCTION OF SPEED.

FIG 10

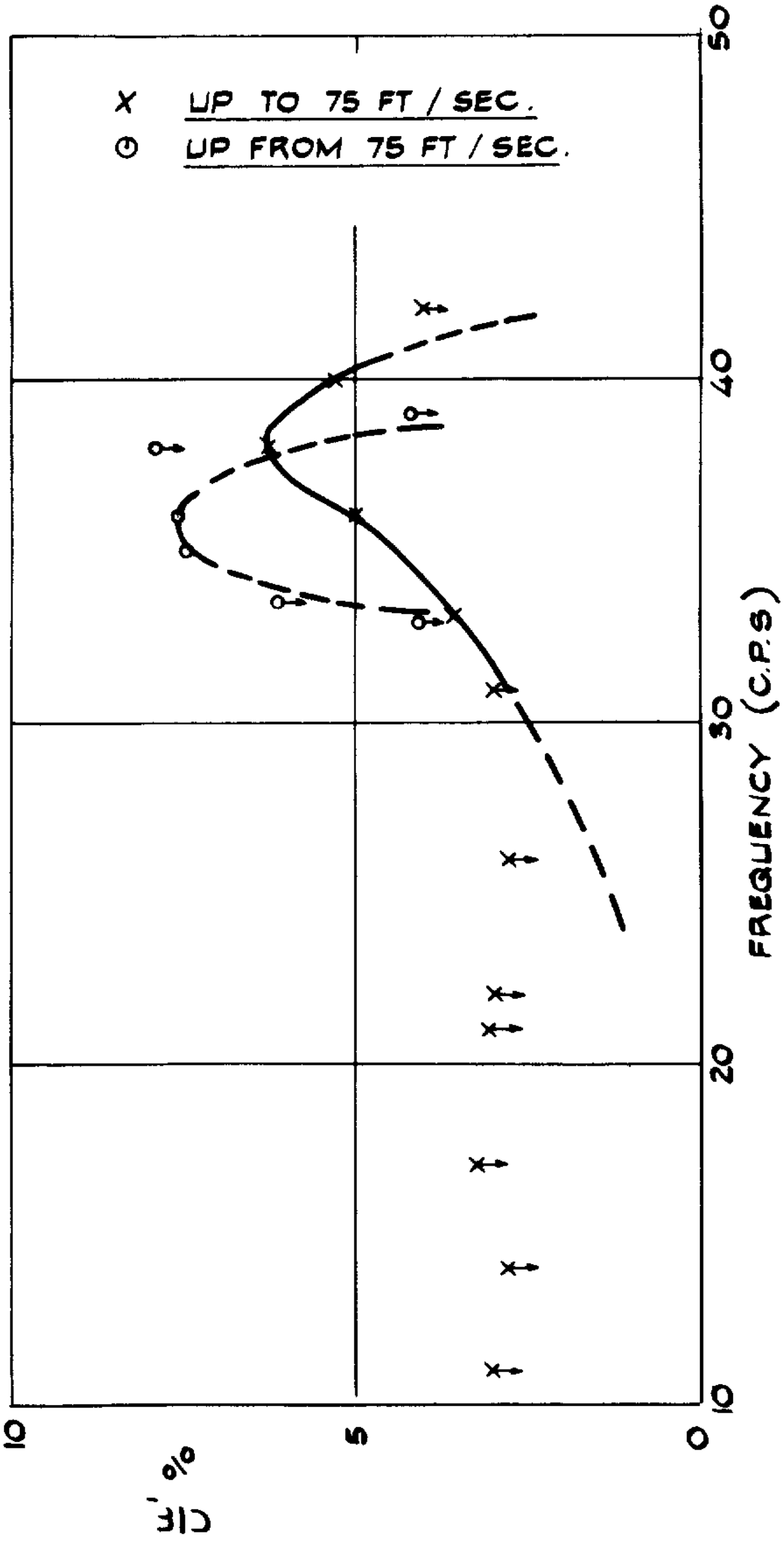
FIG 10



TURBULENCE LEVEL, FREQUENCY AND WAVELENGTH AS FUNCTIONS OF SPEED

FIG. II

ARROWS AGAINST A POINT INDICATE THAT SINCE THE SIGNAL IS NOT SINUSOIDAL THE TOTAL TURBULENCE LEVEL EXCEEDS THE PURE NOTE COMPONENT LEVEL.



TURBULENCE LEVEL AS A FUNCTION OF FREQUENCY.

FIG. 12

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