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The Flow at the Mouth of a Stanton Pitot

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Summary.—An arithmetical calculation is made of the flow at the mouth of a Stanton Pitot as the Reynolds number tends to zero. A stationary eddy is found under the lip of the Pitot. A figure is found for the height of the effective centre of the Pitot rather greater than the experimental value determined by Sir Geoffrey Taylor.

Introduction.—Sir Geoffrey Taylor calibrated a form of Stanton Pitot experimentally[4] and suggested that a theoretical investigation would be interesting. The present paper gives an arithmetical solution for a particular shape as the Reynolds number tends to zero. The quantity required is the position of the effective centre, *i.e.*, the height above the surface in the undisturbed stream where the velocity is that given by $p - p_0 = \frac{1}{2}\rho q_1^2$ and $p - p_0$ is the observed pressure rise in the Pitot. The streamlines are shown and the interesting fact emerges that there is a stationary eddy inside the mouth of the Pitot. The zero or surface streamline does not enter the tube at all but crosses to the lip outside the mouth.

Problem.—Figs. 1 and 3 show the assumed boundaries. The mouth of the opening is supposed to project above the lower or stationary wall of a two-dimensional passage. The upper wall is assumed to move with a constant velocity of 128 units/sec. The space between the walls is 64 units, *i.e.*, 8 times the mouth width, and is filled with a viscous liquid. Thus in the undisturbed portion of the passage we have

$$\psi = \lambda y^2$$

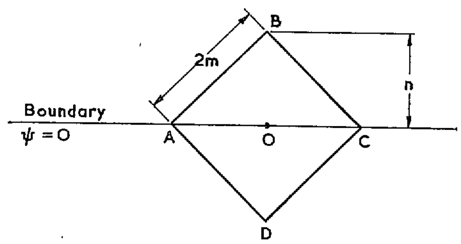
where λ is taken to be unity and y is the distance from the fixed wall. Hence $u = 2y$ and so is 128 on the upper wall. Thus one of the conditions of the solution is that, when $y = 64$, $\partial\psi/\partial y$ must be 128 all along. Evidently the gradient of ψ normal to the surface must be zero on all other boundaries. To simplify matters symmetry has been assumed so that the Pitot has a double mouth, as shown.

Method.—The method of solution is that already described and used by the author in Refs. 1, 2 and 3. Thus we operate on two fields simultaneously—the stream function ψ and the vorticity ζ.

For uniformity with earlier work write:

$$2\zeta \doteq \nabla^2\psi\,.$$

It was considered necessary to subdivide the field closely in the neighbourhood of the opening (*see* Fig. 5) but obviously this interval could not be maintained. In all, four sheets were used with boundaries as shown in Fig. 1, the size of square being halved on each successive sheet and the usual one square overlap being made. The boundary values for the ψ field are obvious—zero everywhere along the lower wall and along all surfaces of the 'tube'. A method of obtaining the boundary values for ζ has already been given in Ref. 1. To make matters clearer suppose for purposes of explanation that we have symmetry in ψ across a non-moving boundary. This of course gives the required condition of $\partial\psi/\partial y = 0$ at the boundary. Now consider a diamond lying across the boundary as shown.



Then $\psi_0 = \frac{1}{4}(\psi_A + \psi_B + \psi_C + \psi_D) - m^2\zeta_0$ .. (1)

but $\psi_A = \psi_0 = \psi_C = 0$.

Also $\psi_B = + \psi_D$.

Hence

$$\zeta_0 = \frac{\psi_B}{2m^2} = \frac{\psi_B}{n^2} . \qquad \text{.. .. .. ..} \qquad (2)$$

Thus when we have obtained an approximately settled ψ field we can calculate the value of the vorticity on the boundary. The ζ field can then be settled when (1) or whatever formula is in use can be used to resettle the ψ field.

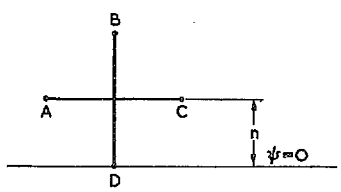
The above method of calculating the boundary values of ζ is good enough in the early stages, but it is obviously only an approximation assuming as it does that ζ is constant throughout the diamond shown. Ref. 1 gives a more accurate method but recently Dr. L. C. Woods has given an alternative method (Ref. 7) leading to

$$\zeta_0 = \frac{3\psi_B}{2n^2} - \frac{\zeta_B}{2} \ldots \qquad \text{.. .. .. ..} \qquad (3)$$

When he first showed me this formula I had practically settled all the fields. On application it eventually produced no change sufficient to affect the final values, but a word of warning should be given. On application it produces a fairly violent change in a faulty field but only a fraction of this change should be applied. The movement in ζ produces a movement in ψ of opposite sign which tends ultimately to return ζ_0 nearly to its original value.

The formula I had been using was

$$\zeta_D = \frac{\psi_A + \psi_C + \frac{1}{2}\psi_B}{2\frac{1}{2}n^2} - \frac{1}{5}(\zeta_A + \zeta_B + \zeta_C) \, . \qquad \text{..} \qquad (4)$$



This is easily deduced from the formula given in Ref. 1 and in some cases does not seem to produce such violent oscillations as Woods' much simpler form. The matter needs further investigation.

The body of the work was done by the 'large square' formulae (2), (3) and (4), Ref. 3. The difficulty of the sharp edge at the Pitot lip was overcome by the empirical method described in connection with Figs. 6 and 7 of Ref. 3. It must be clearly understood that the values of ζ_0 shown at this point in Fig. 5 of the present paper are artificial. A further subdivision of the field will raise these without seriously affecting the surrounding values. Presumably in the limit ζ_0 will be infinitely large but $m^2\zeta_0$ vanishingly small since m is the side of the square (*see, e.g.*, formula (1)). It will also be observed that as the squares or diamonds are made smaller and smaller the effect of the tip value tends to be swamped by that of the increasing number of neighbouring values in the boundary.

It will be appreciated that the solution of a problem of this nature is *very* slow. Each field has to be settled time after time as the boundary values continue to move. No claim is made that finality has been reached in Fig. 5, which shows the inner computation sheet. All that can be said is that I consider that the values given are likely to be correct to within a few figures in the last place. It might be mentioned that the solution was commenced by assuming no stationary eddy in the Pitot. This eddy gradually grew as the work proceeded until it settled at the size shown. For this reason I consider it certain that such an eddy does exist at zero Reynolds number.

Results.—The streamlines are shown in Figs. 3 and 4. The pressures along the boundary AB (Figs. 1 and 2) were obtained as in Ref. 1, and are shown in Fig. 2 in the form

$$p - p_0 = \frac{K}{R_1} \tfrac{1}{2}\rho U^2 \qquad \ldots \qquad (5)$$

where U is the velocity in the undisturbed passage at a height from the stationary wall equal to the height d of the Pitot (*see* Fig. 1) and

$$R_1 = \frac{U\rho d}{\mu} .$$

Put $p - p_0 = \frac{1}{2}\rho q_1^2$ and put $h =$ height of the effective centre of the Pitot, *i.e.*, the height at which q_1 obtains in the undisturbed stream. Then h is given by

$$\frac{h^2}{d^2} = \frac{K}{R_1} = \frac{K\mu}{U\rho d} . \qquad \ldots \qquad (6)$$

For actual use it is more convenient to write this as

$$\frac{h}{d} = \frac{K}{R_2} \qquad \ldots \qquad (7)$$

where

$$R_2 = \frac{q_1 \rho d}{\mu} .$$

This form permits of its application to an actual measurement without successive approximation as q_1 is calculated directly from the measured pressure. It may be of interest to compare the form of (5) with the value found in Ref. 5 for the pressure rise on the front generator of a cylinder, namely

$$(1 + c/R)\tfrac{1}{2}\rho V^2$$

where c has a value rising to about 7.

Comparison with Taylor's Experiments.—Sir Geoffrey Taylor (Ref. 4) carried out experiments with a Stanton Pitot and found a value of $K = 2{\cdot}4$ against $4{\cdot}0$ for the present computation. The arrangement used by Taylor differed in several respects from that assumed here:

(*a*) The Pitot shape was dissimilar

(*b*) The ratio of passage width to Pitot height was smaller, and

(*c*) The passage was the annular space between a rotating shaft and a cylinder.

The discrepancy may be partially due to these differences.

Comparison with Dean's Results (*Ref.* 6).—W. R. Dean in 1951 gave an approximate solution to the problem, the main difference being that he assumed the 'tube' to be long and not short and symmetrical as in the present paper. His results give, in the present notation, $K = 3{\cdot}34$ against 4 above.

The pressure distribution he obtains along the wall is shown (dotted) in Fig. 2 for comparison with mine. The leading edge of his tube has been taken to be at (− 8, 8).

Conclusion.—There seems to be a stationary eddy inside the mouth of a two-dimensional Stanton Pitot even as the Reynolds number tends to zero. The effective height of the centre of the Pitot is about $4\mu/\rho q_1$ above the wall for very low speeds where q_1 is calculated from the pressure rise by the usual Pitot formula. It will be noticed that the depth of the tube, d, does not appear in this expression.

NOTATION

y		Distance from lower wall
d		Depth of mouth of Pitot
h		Height of effective centre of Pitot
q_1		Velocity at effective centre $= \sqrt{\{2(p - p_0)/\rho\}}$
U		Undisturbed velocity when $y = d$
$2n$		Diagonal of square
ψ		Stream function
ζ		Vorticity
R_1	$=$	$U\rho d/\mu$
R_2	$=$	$q_1\rho d/\mu$
K		Given by $p - p_0 = K\rho U^2/2R_1$

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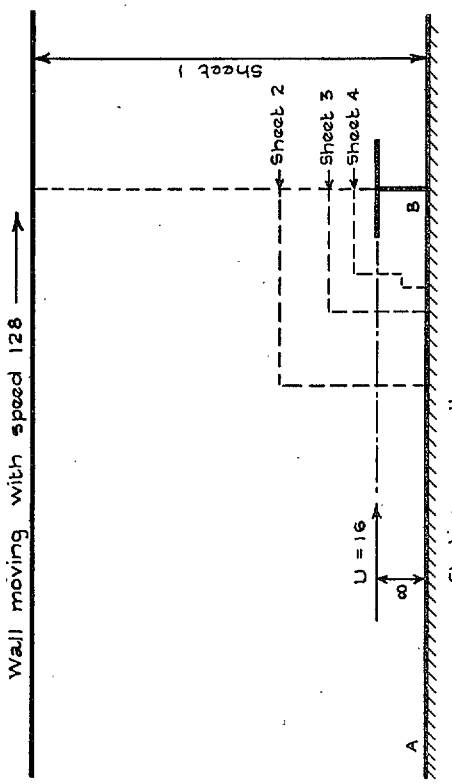

FIG. 1.

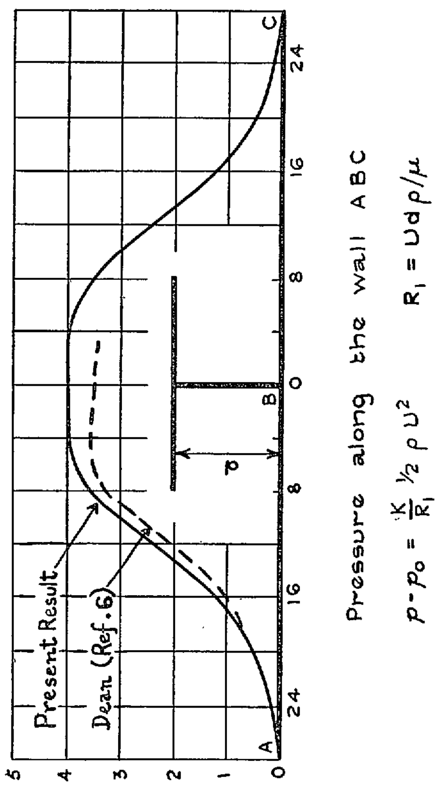

FIG. 2.

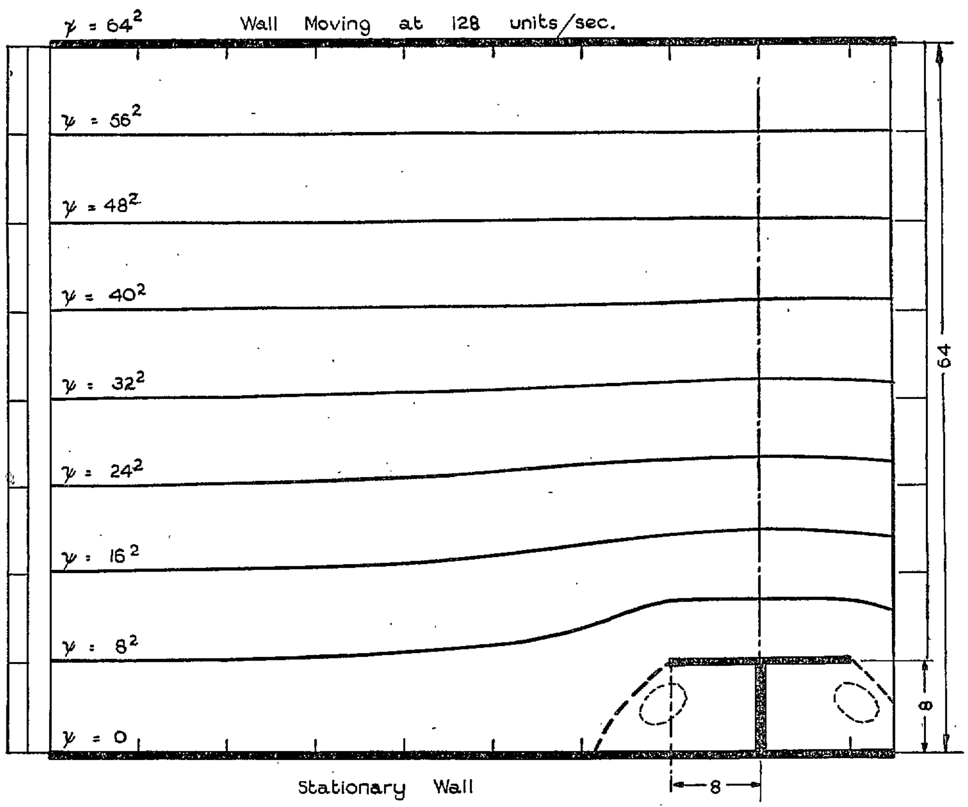

FIG. 3.

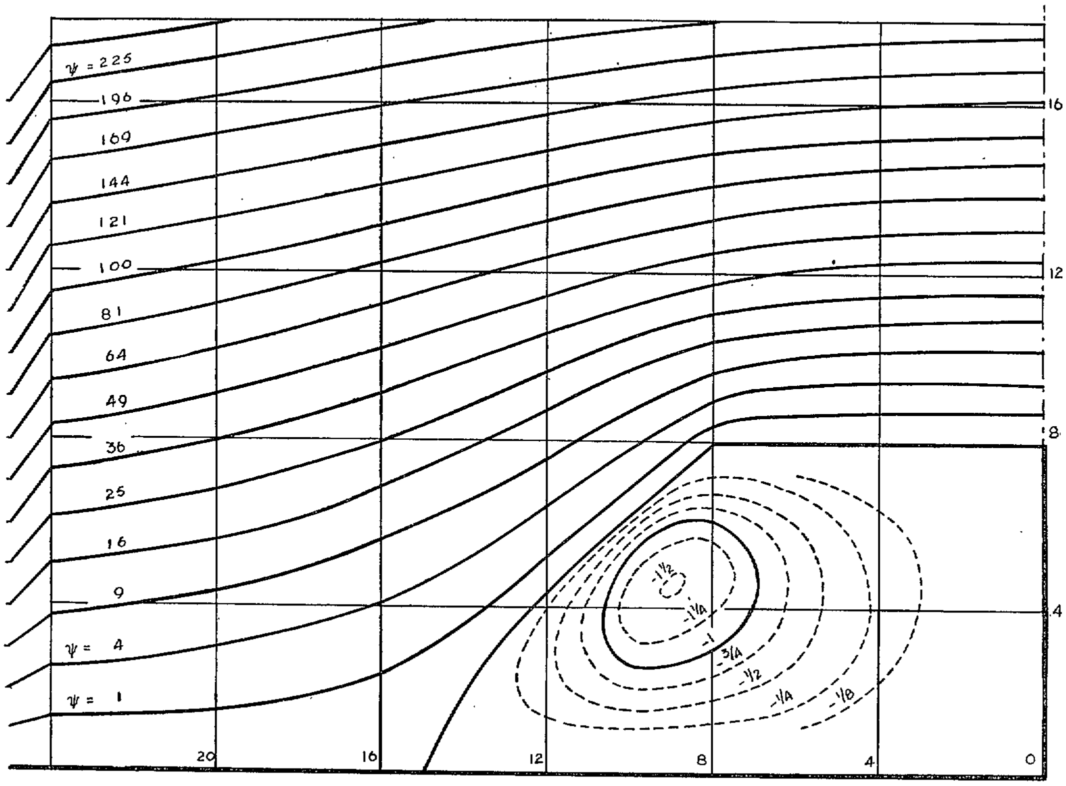

FIG. 4.

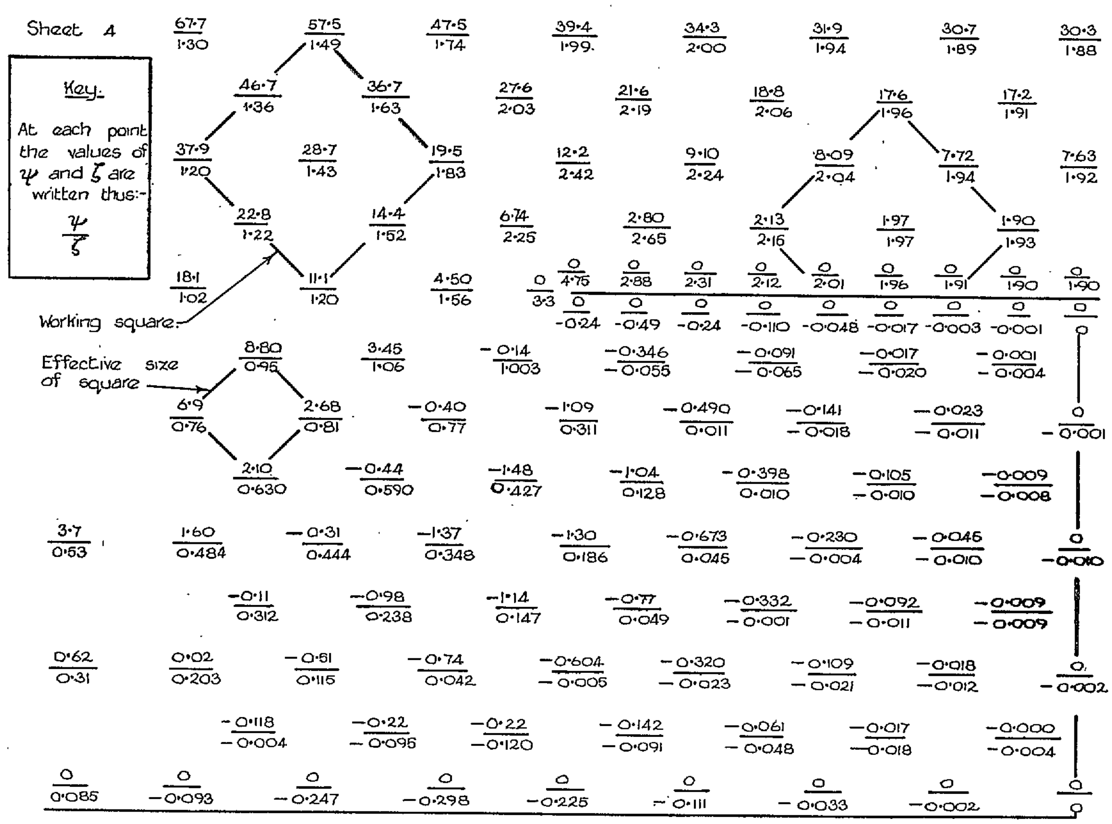

FIG. 5.

(4232) Wt. 19/8411 K9 12/56 Hw.

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