R. & M. No. 2961
(16,445)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

Calculation of Flutter Derivatives for Wings of General Plan-form

By

DORIS E. LEHRIAN, B.Sc.,
of the Aerodynamics Division, N.P.L.

LONDON: HER MAJESTY'S STATIONERY OFFICE
1958
PRICE 10*s* 6*d* NET

Calculation of Flutter Derivatives for Wings of General Plan-form

By

Doris E. Lehrian, B.Sc.,
of the Aerodynamics Division, N.P.L.

Reports and Memoranda No. 2961
January, 1954

Summary.—The vortex-lattice method of calculating flutter derivatives presented in this note is an extension to higher frequencies of the work on stability derivatives reported in R. & M. 2922. The method is a modified form of the scheme outlined in R. & M. 2470 and is suggested as an alternative to the latter method since it gives a simpler routine calculation for wings of general plan-form. Derivatives are calculated for the following wings describing plunging and pitching oscillations :

(*a*) Delta wings of aspect ratio $A = 1 \cdot 2$ and 3 and with a taper ratio 1/7

(*b*) Arrowhead wing of aspect ratio $1 \cdot 32$ with a taper ratio 7/18 and angle of sweep of $63 \cdot 4$ deg at quarter-chord.

The results for the delta wing $A = 1 \cdot 2$ and the arrowhead wing are compared with values of the pitching derivatives obtained in low-speed tests ; those for the delta wing $A = 3$ with the values calculated in R. & M. 2841. The comparison indicates that the present method gives reasonable accuracy for low–aspect-ratio wings in incompressible flow ; the method may be sufficiently reliable for use with the equivalent wing theory suggested in R. & M. 2855 for the calculation of flutter derivatives in compressible subsonic flow.

1. *Introduction.*—The vortex lattice method of Ref. 2 (R. & M. 2470) has been used to calculate derivatives for a delta wing $A = 3$ and gives satisfactory results for frequencies in the flutter range[3] (R. & M. 2841). However, there are some uncertainties in applying that method to a highly tapered wing, since it involves a lift function $C(\omega')$ and chordwise factors $L_0'(k)$, which are functions of the local frequency parameter $\omega' = pc/2V$. If the spanwise variation in $L_0'(k)$ is taken into account, the downwash calculation becomes laborious, and furthermore, because of the limiting form of $C(\omega')$ which involves $\log_e \omega'$, introduces erroneous effects if ω' is very small at the wing tips. It is not known if it is sufficient to use constant values of $L_0'(k)_m$ and $C(\omega_m')$ over the span corresponding to the mean frequency parameter ω_m. The present method attempts to avoid some of these difficulties by extending to higher frequencies the modified form of R. & M. 2470, which is used to calculate stability derivatives[1] (R. & M. 2922). For all plan-forms the method leads to a simpler routine than the method of Ref. 2 since the downwash computation, except for the correction for the oscillatory wake, is based on values of the downwash due to a rectangular vortex in steady motion. The present method does, however, involve chordwise factors $L_0'(k)$, which are dependent on the local parameter ω' (*see* Appendix). It is suggested that these factors may be expressed approximately over a range of ω' values, as polynomials in terms of the mean frequency parameter ω_m. In this case the corresponding downwash is obtained generally in terms of ω_m and it is only necessary to compute the wake correction for each particular value of ω_m by using the oscillatory downwash tables[7]. Except for the wake correction, the calculation reduces to a simple routine which appears suitable for use on a high-speed computing machine.

The method is applied in this note to low–aspect-ratio wings; derivatives are estimated for comparison with measured values and to obtain information on the accuracy of vortex lattice solutions. A reliable method for low–aspect-ratio wings in incompressible flow is required in order to apply the 'equivalent wing' theory of calculating flutter derivatives in compressible subsonic flow; it is shown in Ref. 4 that derivatives for a wing of aspect ratio A oscillating at a frequency f in compressible flow of Mach number M may be calculated by considering a wing of smaller aspect ratio $A\sqrt{(1 - M^2)}$, in incompressible flow, which is oscillating in a different mode at a higher frequency $f/(1 - M^2)$.

LIST OF SYMBOLS AND DEFINITIONS

V		Velocity of flow
x	$=$	$R(y) - \frac{1}{2}c \cos \theta$ } Definitions of chordwise parameters θ and ξ where $R(y)$ is the mid-chord line
	$=$	$R(y) + \frac{1}{2}c\, \xi$ }
x_l		Leading-edge co-ordinate, $\theta = 0$
x_t		Trailing-edge co-ordinate, $\theta = \pi$
y	$=$	$s\eta$ Definition of spanwise co-ordinate η
$c(y)$		Local chord
c_0		Root chord
c_m		Mean chord
s		Semi-span
S		Area of wing
A		Aspect ratio
$p/2\pi$		Frequency
ω	$=$	$2\omega' = pc/V$ Local frequency parameter
ω_m	$=$	pc_m/V Mean frequency parameter
$K\, e^{ipt}$		Doublet distribution (discontinuity in velocity potential)
$\Gamma\, e^{ipt}$		Bound velocity
$E\, e^{ipt}$		Free vorticity
$W\, e^{ipt}$		Induced downward velocity
$z'\, e^{ipt}$		Normal downward displacement of point (x, y) on the wing
δL	$=$	$\int_{x_l}^{x_t} \rho V \Gamma\, dx$. Local lift
δM	$=$	$\int_{x_l}^{x_t} \rho V \Gamma\, x\, dx$. Local moment
$L\, e^{ipt}$	$=$	$\int_{-s}^{s} \int_{x_l}^{x_t} \rho V \Gamma\, e^{ipt}\, dx\, dy$. Lift
$M\, e^{ipt}$		Pitching moment about axis $x = 0$ through wing apex
	$=$	$-\int_{-s}^{s} \int_{x_l}^{x_t} \rho V \Gamma\, e^{ipt}\, x\, dx\, dy$

$\Gamma_0 \quad = \quad 2 \cot \tfrac{1}{2}\theta$

$\Gamma_1 \quad = \quad -2 \sin \theta + \cot \tfrac{1}{2}\theta + i\omega'[\sin \theta + \tfrac{1}{2} \sin 2\theta]$

$\Gamma_n \quad = \quad -2 \sin n\theta + i\omega' \left[\frac{\sin (n+1)\theta}{n+1} - \frac{\sin (n-1)\theta}{n-1}\right]$ when $n \geqslant 2$

K_0 is defined by Equations (5) and (6)

$K_1 \quad = \quad \tfrac{1}{2}c[\sin \theta + \tfrac{1}{2} \sin 2\theta]$

$K_n \quad = \quad \tfrac{1}{2}c\left[\frac{\sin (n+1)\theta}{n+1} - \frac{\sin (n-1)\,\theta}{n-1}\right]$ when $n \geqslant 2$

$A_m \quad = \quad (s/c)T_m = s\eta^{m-1}\sqrt{(1-\eta^2)}$

Definition of Derivatives for Plunging and Pitching Oscillations

(*a*) Local derivative coefficients † at a spanwise position η, referred to an axis $x = 0$.

$$\frac{\delta L}{\rho V^2 c_m} = [l_z(\eta) + i\omega_m l_{\dot{z}}(\eta)]z + [l_\alpha(\eta) + i\omega_m l_{\dot{\alpha}}(\eta)]\alpha$$

$$\frac{\delta M}{\rho V^2 c_m{}^2} = [m_z(\eta) + i\omega_m m_{\dot{z}}(\eta)]z + [m_\alpha(\eta) + i\omega_m m_{\dot{\alpha}}(\eta)]\alpha$$

where $c_m z$ and α are the amplitudes of the vertical translational and angular displacements of the oscillating wing.

(*b*) Derivatives† referred to axis $x = 0$:

$$L/(\rho V^2 S) = (l_z + i\omega_m l_{\dot{z}})z + (l_\alpha + i\omega_m l_{\dot{\alpha}})\alpha$$

$$M/(\rho V^2 S c_m) = (m_z + i\omega_m m_{\dot{z}})z + (m_\alpha + i\omega_m m_{\dot{\alpha}})\alpha$$

where $c_m z$ and α are the amplitudes of the vertical translational and angular displacements of the oscillating wing.

(*c*) Derivatives referred to axis of oscillation $x = h'c_m = hc_0$ are obtained from definitions (*a*) and (*b*) by the usual transformation formulae :

$$l_z' = l_z$$

$$l_\alpha' = l_\alpha - h'l_z$$

$$-m_z' = -m_z - h'l_z$$

$$-m_\alpha' = -m_\alpha - h'(l_\alpha - m_z) + h'^2 l_z$$

and similar expressions for the damping derivatives.

† The derivatives $l_{\dot{z}}$, $l_{\dot{\alpha}}$, $-m_{\dot{z}}$, $-m_{\dot{\alpha}}$ include the aerodynamic inertia terms.

2. *Theory.*—The present method is a modified form of the vortex lattice scheme outlined in Ref. 2, which is based on linearized theory for a thin wing oscillating with small amplitude in incompressible inviscid flow. The approach suggested in Ref. 1, and used in that note to calculate stability derivatives by limiting the theory to first-order terms in frequency, is extended to include frequencies of interest in flutter research.

The basic equations of the method are not given here in detail since they are the same as in section 2, Ref. 1. The bound vorticity distribution Γe^{ipt} over the wing is assumed to be

$$\Gamma = V \sum_n \sum_m \Gamma_n C_{nm} A_m, \qquad (1)$$

where the chordwise distributions Γ_n (defined in the list of symbols) and the spanwise distributions $cA_m = sT_m = s\eta^{m-1}\sqrt{(1-\eta^2)}$ are the same as in Ref. 1, and C_{nm} are arbitrary coefficients. $K e^{ipt}$ over the wing and wake is

$$K = V \sum_n \sum_m K_n C_{nm} A_m \qquad (2)$$

and the corresponding downwash $W e^{ipt}$ induced at a point (x_1, y_1) on the wing is then given by

$$W = V \sum_n \sum_m W_{nm} C_{nm}. \qquad (3)$$

When the downwash values W_{nm} are known the arbitrary coefficients C_{nm} are determined, for a wing motion $z = z' e^{ipt}$, by satisfying the tangential flow condition

$$W = ipz' + V(\partial z'/\partial x_1). \qquad (4)$$

It can be seen from section 2, Ref. 1, that only the part of the downwash calculation dependent on the bound vorticity distribution $\Gamma_0 = 2 \cot \frac{1}{2}\theta$ requires extension in order to obtain solutions for values of the frequency parameter $\omega_m = pc_m/V$ in the flutter range. Therefore in the present note it is only necessary to consider the calculation of the downwash W_{0m} due to the bound vorticity $\Gamma_0 A_m$ and the corresponding doublet distribution $K_0 A_m$. The cordwise distribution K_0 is defined by the equations

$$K_0 \text{ (wing)} = K_0(\theta) = \tfrac{1}{2}c\, e^{-i\omega'\xi} \int_{-1}^{\xi} \Gamma_0 e^{i\omega'\xi}\, d\xi \ldots -1 \leqslant \xi \leqslant 1 \qquad (5)$$

$$\left.\begin{aligned} K_0 \text{ (wake)} &= K_0(\pi)\, e^{-i\omega'(\xi-1)} && \ldots \xi \geqslant 1 \\ &= K_0(\pi)\, e^{-ip(x-x_t)/V} && \ldots x \geqslant x_t \end{aligned}\right\} \qquad (6)$$

It follows from (5) that

$$K_0(\pi) = c\pi\, e^{-i\omega'} [J_0(\omega') - iJ_1(\omega')], \qquad (7)$$

where J_0 and J_1 are Bessel functions of the first kind. Then, as in Ref. 1, it is convenient to regard the doublet distribution K_0 over the wing and wake as the sum of two distributions K_0' and K_0'' such that

$$\left.\begin{aligned} K_0' &= K_0(\theta) \text{ over the wing} && 0 \leqslant \theta \leqslant \pi \\ &= K_0(\pi) \text{ over the wake} && x \geqslant x_t \\ K_0'' &= 0 \text{ over the wing} && \\ &= K_0(\pi)[e^{-ip(x-x_t)/V} - 1] \text{ over the wake } x \geqslant x_t && \end{aligned}\right\} \qquad (8)$$

The downwash W_{0m} can then be written as

$$W_{0m} = W_{0m}' + W_{0m}'', \qquad (9)$$

where W_{0m}' and W_{0m}'' are the downwash values induced at a point (x_1, y_1) by the doublet distributions $K_0' A_m$ and $K_0'' A_m$ as defined by equations (5) to (8).

2.1. *Calculation of the Downwash W'_{0m}*.—The doublet distribution $K'_0 A_m$ is of constant strength over the wake in the chordwise direction and may be replaced by a lattice of rectangular vortices as in steady motion. If the continuous chordwise distribution K'_0 is represented by k' vortices of strength $cL'_0(k)$, $k = 1, 2 \ldots . k'$, then a typical rectangular vortex at a chordwise position $(2k - 1)c/2k'$ and spanwise position $\eta_j s$ is of strength

$$cL'_0(k)\, A_m(\eta_j) = sL'_0(k)\, T_m(\eta_j), \qquad (10)$$

where $T_m = \eta^{m-1}\sqrt{(1 - \eta^2)}$. The factors $L'_0(k)$ are chosen on a two-dimensional basis, as described in the Appendix and give the correct value K'_0 at the trailing edge. Values of $L'_0(k)/\pi$ corresponding to $k' = 6$ and $k' = 2$ are tabulated in Table 1, with modified second differences, for a range of values of the local frequency parameter $\omega = 2\omega' = pc/V$. The downwash W'_{0m} at a collocation point (x_1, y_1) is then evaluated by summation ; the critical downwash tables[6] for steady motion give the downwash F at a point (X^*, Y^*)† due to a rectangular vortex of width $2s_1$ and strength $4\pi s_1$, so that W'_{0m} is obtained as

$$W'_{0m} = \frac{s}{4s_1} \sum_{\eta_j} \sum_{k} \left(\frac{L'_0(k)}{\pi} T_m(\eta_j) F \right). \qquad (11)$$

For a wing of general plan-form there is a spanwise variation in the parameter ω' and therefore in the factors $L'_0(k)$ for each position η_j of the lattice. It is suggested that the spanwise variation of these factors may be allowed for in the calculation (11) by expressing $L'_0(k)$ as polynomials in ω'. For example, if the factors are known for a range of values of ω', it is possible to fit polynomials

$$L'_0(k) = L'_a(k) + i\omega' L'_b(k) + \omega'^2 L'_c(k) + i\omega'^3 L'_d(k), \qquad (12)$$

where L'_a, $L'_b \ldots$ are real numbers, to give a good approximation over a range $\omega'_1 \leqslant \omega' \leqslant \omega'_2$ say. Then since

$$\omega' = \omega_m \left(\frac{c}{2c_m} \right) = \omega_m f(|\eta|), \qquad (13)$$

it follows that the downwash W'_{0m} can be expressed generally in terms of ω_m as

$$W'_{0m} = W'_{am} + i\omega_m W'_{bm} + \omega_m{}^2 W'_{cm} + i\omega_m{}^3 W'_{dm}, \qquad (14)$$

where W'_{am}, W'_{bm}, W'_{cm}, W'_{dm} are the downwash values due to a lattice of rectangular vortices of strength

$sL'_a(k)\, T_m(\eta_j)$,

$sL'_b(k)\, f(|\eta_j|) T_m(\eta_j)$,

$sL'_c(k)\, f^2(|\eta_j|) T_m(\eta_j)$,

$sL'_d(k)\, f^3(|\eta_j|) T_m(\eta_j)$ respectively.

It should be noted that the factors L'_a, $L'_b \ldots$ are applicable, provided

$$\omega'_1 \leqslant \omega_m f(|\eta_j|) \leqslant \omega'_2 \qquad (15)$$

for all positions η_j across the wing span and that the downwash W'_{0m} can be calculated from (14) for any value ω_m satisfying this condition. The range (ω'_1, ω'_2) of the polynomial representation (12) could be extended by considering a polynomial of higher order.

†The co-ordinates (X^*, Y^*) give the position of collocation point relative to the rectangular vortex in terms of semi-width s_1, where the positive X axis extends upstream.

2.2. *Calculation of the Downwash* W''_{0m}.—The downwash W''_{0m} at a collocation point (x_1, y_1) is calculated by using the lattice scheme of Ref. 2. The doublet distribution $K''_0 A_m$, as defined by (8), is replaced by narrow doublet strips of width $2s_1$ in the spanwise direction, which extend downstream from $x = x_t$ to $x = \infty$ and are of strength

$$K_0(\pi)[e^{-ip(x-x_t)/V} - 1]A_m(\eta_j).$$

The downwash W''_{0m} is then evaluated for a particular value of the mean frequency parameter ω_m by using the steady tables of Ref. 6, and the tables of Ref. 7† which give the downwash W_c at a point (t, n) due to an oscillating doublet strip of width $2s_1$ which extends from $X = 0$ to $X = \infty$ and is of strength $s_1 \exp(-ipX/V) = s_1 \exp(-i\bar{\omega}t)$. Hence

$$W''_{0m} = \frac{s}{4s_1} \sum_{\eta_j} \frac{K_0(\pi)}{\pi c} T_m(\eta_j)\,[4\pi W_c - F], \quad \ldots \ldots \ldots \ldots \ldots \quad (16)$$

where $K_0(\pi)$ is defined by equation (7) and is a function of the local frequency parameter ω'.

For wings of general plan-form, there is a spanwise variation of $K_0(\pi)$ with ω'. If the polynomial representation suggested in section 2.1 is used to calculate W'_{0m}, then the downwash W''_{0m} may be calculated by using in equation (16)

$$K_0(\pi) = c \sum_{k=1}^{k} L'_0(k), \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (17)$$

where $L'_0(k)$ and ω' are defined by (12) and (13) with the condition (15).

3. *Present Application of the Method : Results and Comparisons.*—In the present note, aerodynamic derivatives are calculated for three wings describing plunging and pitching oscillations for various values of the mean frequency parameter ω_m. The results are obtained from solutions in which :

(*a*) Distributions (1) to (3) are limited to two chordwise terms $n = 0$ and 1, and three spanwise terms corresponding to symmetrical motion $m = 1$, 3 and 5

(*b*) A 6-chordwise × 21-spanwise lattice with the corrector vortices‡ is used for the downwash calculation as in Ref. 8 ; the 6-chordwise vortices are reduced to 2 when any strip η_j is at a distance $\geqslant 10s_1$ from the collocation point (x_1, y_1)

(c) The six collocation points are placed on the 1/2 and 5/6 chord-lines at spanwise positions $\eta = 0{\cdot}2,\ 0{\cdot}6,\ 0{\cdot}8$.

The polynomial representation suggested in section 2.1 is used for the calculation of the downwash W'_{0m}. The factors $L_0(k)$ are representated by the polynomials of equation (12) for a frequency parameter range $0 \leqslant \omega' \leqslant 0{\cdot}4$ and, for convenience, the factors $L'_a(k)$ and $L'_b(k)$ are given the same values as those used in the $\omega_m \to 0$ method of Ref. 1. The factors $L'_c(k)$ and $L'_d(k)$ are then determined by a least-squares method so that the polynomial representation of $L'_0(k)$ gives an accuracy to within 1 or 2 per cent of the true $L'_0(k)$ values for the range $\theta \leqslant \omega' \leqslant 0{\cdot}4$. The factors L'_a, L'_b, L'_c, L'_d are tabulated in Table 2 for the 6 chordwise lattice $k = 1, 2 \ldots 6$ and the reduced 2 chordwise lattice $k = 1, 2$. The values of the downwash W_{nm} computed for the three wings are given in Tables 3(*a*) to 3(*c*).

† The tables are available for values of the parameter $\bar{\omega} = (s_1/2s)\omega_m A$ equal to 0·01 (0·01) 0·04 (0·02) 0·08, 0·09 0·12, 0·16, 0·18, 0·24 ; the co-ordinates (t, n) give the position of the collocation point relative to the doublet strip, in terms of the semi-width s_1, where positive t axis extends downstream.

‡ The corrector vortices are neglected in the calculation of W''_{0m}.

3.1. *Local Derivative Coefficients*.—For the delta wings $A = 1 \cdot 2$ and $A = 3$ and the arrowhead wing $A = 1 \cdot 32$, values of the local derivative coefficients are obtained at spanwise positions $\eta = 0\ (0 \cdot 2)\ 1 \cdot 0$ by use of the definitions given in the list of symbols. The local lift and moment for a wing of aspect ratio A are given by

$$\frac{\delta L}{\rho V^2 c_m} = \frac{\pi A}{2}\left[D_0 + \frac{i\omega'}{4} D_1\right]$$

$$\frac{\delta M}{\rho V^2 {c_m}^2} = -\frac{\bar{m}c}{c_m}\left(\frac{\delta L}{\rho V^2 c_m}\right) + \frac{\pi A c}{16 c_m}\left[2D_0 + \left(1 + \frac{i\omega'}{4}\right) D_1\right]$$

where $\quad x = R(y) = \bar{m}c$ is the mid-chord line

$$\omega' = \omega_m c/2c_m = \omega_m f(|\eta|)$$

$$D_n = \sum_m \eta^{m-1} C_{nm} \sqrt{(1 - \eta^2)}, \qquad m = 1, 3, 5$$

and the coefficients C_{nm} are for a particular value of the frequency parameter $\omega_{\bar{m}}$. In the solution for $\omega_m \rightarrow 0$ only first-order terms in frequency are retained as in Ref. 1.

For each wing, values of the local derivative coefficients are tabulated for one axis position $x = hc_0$. Values for the delta wing $A = 3$ are given in Table 4 for the axis $hc_0 = 0 \cdot 556c_0$ and mean frequency parameter values $\omega_m = 0,\ 0 \cdot 26,\ 0 \cdot 40$ and $0 \cdot 53$. The values for the delta wing $A = 1 \cdot 2$ with axis $0 \cdot 556c_0$ and $\omega_m = 0,\ 0 \cdot 33,\ 0 \cdot 67$ and those for the arrowhead wing $A = 1 \cdot 32$ with axis $0 \cdot 738c_0$ and $\omega_m = 0,\ 0 \cdot 30,\ 0 \cdot 61$ are given in Tables 5 and 6 respectively.

3.2. *Delta Wing of Aspect Ratio* 3, *Taper Ratio* 1/7.—Derivatives are calculated for two axis positions, $hc_0 = 0$ and $0 \cdot 556c_0$, for values of the mean frequency parameter $\omega_m = 0 \cdot 26,\ 0 \cdot 40$ and $0 \cdot 53$. The results are tabulated in Table 7 and the pitching derivatives for the axis $0 \cdot 556c_0$ are plotted in Figs. 1a and 1b. Values of stability and flutter derivatives previously obtained by vortex lattice methods[1,3] are quoted in Table 4 and used in drawing the graphs in Fig. 1. The correlation between the results is satisfactory and indicates the relative accuracy of the present method and the methods of Refs. 1 and 2. From Fig. 1, which covers the frequency parameter range $0 \leqslant \omega_m \leqslant 0 \cdot 8$, it can be seen that the first-order theory with wake correction of Ref. 1 gives a good idea of the rate of change of the derivatives with ω_m but becomes less accurate as ω_m increases to the higher values. The discrepancies between present results and those of Ref. 3 are less than 2 per cent and are probably due to differences between the present method and that of Ref. 2 and the assumptions made in the actual application of the method. Values of the pitching derivatives calculated by the Multhopp-Garner method[9] for $\omega_m \rightarrow 0$ and the experimental values of $-m_{\dot{\alpha}}$ obtained at the National Physical Laboratory by Bratt for low frequencies are also shown in Fig. 1.

3.3. *Delta Wing of Aspect Ratio* $1 \cdot 2$, *Taper Ratio* 1/7.—Derivatives are calculated for three axis positions $hc_0 = 0,\ 0 \cdot 431c_0$ and $0 \cdot 556c_0$ for values of $\omega_m = 0 \cdot 33$ and $0 \cdot 67$, and the results are tabulated in Table 8. Pitching derivatives for the two axis positions $0 \cdot 431c_0$ and $0 \cdot 556c_0$ are plotted in Figs. 2a to 2d. Values of the stability derivatives for $\omega_m \rightarrow 0$, calculated by the vortex lattice method[1] and the Multhopp-Garner method[9], are also shown in Fig. 2. Experimental values of the pitching derivatives have been obtained in low-speed tests at the N.P.L.[10] for the axis positions $0 \cdot 431c_0$ and $0 \cdot 556c_0$ and these results are plotted in Figs. 2a to 2d. The values shown in Fig. 2 are for zero mean incidence and the tests show no amplitude effects. The derivatives were measured for frequency parameter values $\omega_m = 0 \cdot 06$ to $0 \cdot 60$ and were approximately constant over this range.

3.4. *Arrowhead-Wing Aspect Ratio* $1 \cdot 32$, *Taper Ratio* 7/18, $\frac{1}{4}$-*Chord Angle* $= 63 \cdot 4$ *deg*.—Derivatives are calculated for three axis positions, $hc_0 = 0,\ 0 \cdot 613c_0$ and $0 \cdot 738c_0$ for values $\omega_m = 0 \cdot 30$ and $0 \cdot 61$, and the results are given in Table 9. The pitching derivatives for axis positions $0 \cdot 613c_0$ and $0 \cdot 738c_0$ are graphed in Figs. 3a to 3d; the values of the stability derivatives from Refs. 1 and 9 are also plotted in Fig. 3. Low-speed tests made on this wing at the N.P.L.[10] give values of the pitching derivatives for these axis positions; the values for zero mean incidence are plotted in Figs. 3a to 3d.

Concluding Remarks.—Comparison of the vortex lattice results and measured values of the derivatives indicates that the present method as applied in this note using a 21×6 lattice, gives reasonable accuracy for the calculation of flutter derivatives for low–aspect-ratio wings describing plunging and pitching oscillations in incompressible flow. It is noted that the results obtained in tests made at the Royal Aircraft Establishment for a delta wing $A = 3$ with body[11] at low values of the frequency parameter ω_m, are also in quite good agreement with the calculated values of the pitching derivatives. The method can be applied to wings oscillating in elastic modes, although it may then be necessary to use a finer lattice and more collocation points in the calculation.

Flutter derivatives for a wing in compressible flow may be calculated by applying the theory of Ref. 4 and using the present method to calculate the downwash values on the 'equivalent wing' in incompressible flow. If the original wing oscillates at a high frequency then it is apparent, from the relations given in the Introduction, that the frequency of oscillation of the equivalent wing could be very high. With this in view, the chordwise factors $L'_0(k)$ are tabulated in Table 1 of this note for very large values of the frequency parameter $\omega = 2\omega'$. It can be seen that these factors vary considerably when $\omega > 2$ and the use of polynomials in ω' to represent $L'_0(k)$, as suggested in section 2.1, then becomes impractical. Hence, if the equivalent wing is tapered and the equivalent frequency is high, the only practical way to allow for the spanwise variation of these factors is to use the set of values $L'_0(k)$, at each strip η_j of the lattice, which corresponds to the local frequency parameter value ω_j.

The downwash distribution on the equivalent wing has to satisfy a complicated tangential flow condition and it is suggested in Ref. 4 that if this condition is represented to first-order accuracy in frequency, then the simplified calculation should give results of reasonable accuracy for all practical purposes. The values of flutter derivatives for a finite wing would probably be sufficiently accurate for a Mach number up to about $0 \cdot 75$, but it is doubtful if the approximation would be good enough when high Mach number and high frequency are considered simultaneously. It is hoped that some information on this point will be obtained from calculations which are now in progress for rectangular wings.

Acknowledgment.—The numerical results given in this note were calculated by Mrs. S. D. Burney of the Aerodynamics Division.

REFERENCES

No.	*Author*	*Title, etc.*
1	D. E. Lehrian	Calculation of stability derivatives for oscillating wings. R. & M. 2922. February, 1953.
2	W. P. Jones	The calculation of aerodynamic derivative coefficients for wings of any plan form in non-uniform motion. R. & M. 2470. December, 1946.
3	Doris E. Lehrian	Aerodynamic coefficients for an oscillating delta wing. R. & M. 2841. July, 1951.
4	W. P. Jones	Oscillating wings in compressible subsonic flow. R. & M. 2855. October, 1951.
5	W. P. Jones	Aerodynamic forces on wings in non-uniform motion. R. & M. 2117. August, 1945.

REFERENCES—*continued*

No.	*Author*	*Title, etc.*
6	Staff of Maths. Div., N.P.L., with Preface by V. M. Falkner	Tables of complete downwash due to a rectangular vortex. R. & M. 2461. July, 1947.
7	Staff of Maths Div., N.P.L.	Downwash tables for the calculation of aerodynamic forces on oscillating wings. R. & M. 2956. July, 1952.
8	V. M. Falkner	The solution of lifting plane problems by vortex lattice theory. R. & M. 2591. September, 1947.
9	H. C. Garner	Multhopp's subsonic lifting surface theory of wings in slow pitching oscillations. R. & M. 2885. July, 1952.
10	C. Scruton, L. Woodgate and A. J. Alexander	Measurements of the aerodynamic derivatives for swept wings of low aspect ratio describing pitching and plunging oscillations in incompressible flow. R. & M. 2925. October, 1953.
11	G. F. Moss	Low speed wind tunnel measurements of longitudinal oscillatory derivatives on three wing planforms. R. & M. 3009. November, 1952.

APPENDIX

Calculation of the Chordwise Factors $L_0'(k)$

The chordwise doublet distribution K_0' is replaced by k' vortices of strength $cL_0'(k)$, $k = 1, 2 \ldots k'$, placed at positions $(2k - 1)c/2k'$, $k = 1, 2 \ldots . k'$, from leading edge of the chord. The factors $L_0'(k)$ are chosen so that the downwash value at each position kc/k', $k = 1, 2 \ldots (k' - 1)$, as calculated by two-dimensional theory to correspond to the k' vortices, is equal to the downwash W_0' corresponding to the continuous 2-dimensional distribution K_0'. These conditions give $(k' - 1)$ equations and the condition

$$\sum_{k=1}^{k=k'} c\, L_0'(k) = K_0'(\pi)$$

provides the kth equation needed to determine the factors.

The downwash W_0' is evaluated, for any position $x_1 = c(1 + \xi_1)/2$ from leading edge of the chord, as follows. It is known that

$$2\pi W_0' = -\int_0^\infty \frac{1}{x - x_1} \frac{\partial K_0'}{\partial x}\, dx ,$$

therefore

$$2\pi W_0' = -\int_0^\infty \frac{1}{x - x_1} \frac{\partial K_0}{\partial x}\, dx + \int_c^\infty \frac{1}{x - x_1} \frac{\partial K_0''}{\partial x}\, dx \qquad \text{by (8)}$$

$$\equiv 2\pi(W_0 - W_0'') \text{ say} . \qquad (18)$$

The downwash W_0 corresponding to a bound vorticity distribution $\Gamma_0 = 2 \cot \frac{1}{2}\theta$, and therefore to the doublet distribution K_0 is given in Ref. 5 (R. & M. 2117, Appendix I), as

$$W_0 = -\tfrac{1}{2}\pi(i\omega' e^{-i\omega'\xi_1})[H_1^{(2)}(\omega') + iH_0^{(2)}(\omega')], \qquad (19)$$

where $\omega' = pc/2V$ and $H_0^{(2)}$, $H_1^{(2)}$ are Hankel functions. In terms of Bessel functions of first and second kind

$$H_n^{(2)}(\omega') = J_n(\omega') - iY_n(\omega') . \qquad (20)$$

The downwash value W''_0, corresponding to the doublet distribution K''_0 over the wake, is deduced from equation (18) by use of equations (7) and (8).

$$W''_0 = -\int_1^{\infty} \frac{K_0(\pi)}{\pi c(\xi - \xi_1)} \frac{\partial}{\partial \xi} [e^{-i\omega'(\xi-1)} - 1]\, d\xi$$

$$= \frac{K_0(\pi)}{c\pi} \left\{ i\omega' \, e^{i\omega'} \int_1^{\infty} \frac{e^{-i\omega'\xi}}{\xi - \xi_1} d\xi \right\}$$

$$= i\omega' e^{-i\omega'\xi_1} [J_0(\omega') - iJ_1(\omega')] \int_{\alpha}^{\infty} \frac{e^{-it}}{t} dt \,, \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (21)$$

where $\qquad \alpha = \omega'(1 - \xi_1)$.

Since $-1 < \xi_1 < 1$, α is greater than 0 and the integral in (21) can be written as

$$\int_{\alpha}^{\infty} \frac{e^{-it}}{t} dt = -\,\mathrm{Ci}(\alpha) - i\,\{\tfrac{1}{2}\pi - \mathrm{Si}(\alpha)\}\,, \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (22)$$

where $\qquad \mathrm{Ci}(\alpha)$ is the cosine integral $-\int_{\alpha}^{\infty} \frac{\cos t}{t} dt$,

$\mathrm{Si}(\alpha)$ is the sine integral $\int_0^{\alpha} \frac{\sin t}{t} dt$.

It follows from equations (18) to (22) that the downwash W'_0 at a point ξ_1 is

$$W'_0 = -\,i\omega' \, e^{-i\omega\xi'_1} [\{Y_0(\omega') - iY_1(\omega')\} + \{J_0(\omega') - iJ_1(\omega')\}\{-\,\mathrm{Ci}\,(\alpha) + i\,\mathrm{Si}(\alpha)\}] \qquad \ldots \qquad (23)$$

where $\qquad \alpha = \omega'(1 - \xi_1)$.

The value $W'_0(\xi_1)$ is computed for values of the frequency parameter ω' by using tables† of the Bessel functions J_0, J_1, Y_0, Y_1 and of the sine and cosine integrals $Si\,(\alpha)$ and $Ci\,(\alpha)$.

Values of $L'_0(k)/\pi$, $k = 1, 2 \ldots 6$, for six vortices positioned at 1/12, 3/12 .... 11/12 chord are tabulated in Table 1(*a*) for values of the frequency parameter $\omega = 2\omega' = 0\,(0{\cdot}2)\,2{\cdot}0$ and in Table 1(*b*) for values $\omega = 2{\cdot}0\,(0{\cdot}4)\,6{\cdot}0$. Values of the modified second differences δ^2 are tabulated for use with the Everett interpolation formula $f\,(n) = (1 - n)\,f_0 + nf_1 + E''_0(n)\delta_0^{\,2} + E''_1(n)\delta_1^{\,2}$ where n is the fraction of the interval of tabulation ; $\delta_0^{\,2}$ and $\delta_1^{\,2}$ are modified second differences. The Everett coefficients :

$$E''_0 = -\frac{n(n-1)(n-2)}{3!}$$

$$E''_1 = \frac{(n+1)n(n-1)}{3!}$$

are tabulated in Tables XXI and XXVI of ' Interpolation and Allied Tables ' published by H.M.S.O.

Values of $L'_0(k)/\pi$, $k = 1, 2$ for two vortices positioned at $\frac{1}{4}$ and $\frac{3}{4}$ chord, are also tabulated with modified second differences in Table 1(*c*). These factors are used for the reduced lattice as indicated in section 3(*b*).

† British Association Mathematical Tables, Vol. VI, Tables of Bessel Functions. Bureau of Standards : Tables of Sine, Cosine and Exponential Integrals.

TABLE 1(*a*)

Factors $L_0'(k) = 1, 2 \ldots . 6$, *for Six Chordwise Vortices Positioned at* 1/12, 3/12 . . . . 11/12 *chord*

Values of $L_0'(k)/\pi$

$\omega = \frac{pc}{V}$	$k = 1$				$k = 2$				$k = 3$			
	Rl	δ^2	*Im*	δ^2	*Rl*	δ^2	*Im*	δ^2	*Rl*	δ^2	*Im*	δ^2
0	0·45117	−30	0	0	0·20508	−140	0	0	0·13672	−295	0	0
0·2	0·45102	−30	−0·01143	2	0·20438	−140	−0·02023	7	0·13525	−291	−0·02532	31
0·4	0·45057	−29	−0·02284	4	0·20228	−138	−0·04038	20	0·13087	−289	−0·05033	60
0·6	0·44983	−29	−0·03421	3	0·19880	−138	−0·06034	25	0·12361	−279	−0·07474	87
0·8	0·44880	−28	−0·04555	6	0·19394	−134	−0·08004	37	0·11356	−271	−0·09827	120
1·0	0·44749	−27	−0·05683	7	0·18774	−132	−0·09938	42	0·10081	−256	−0·12061	144
1·2	0·44591	−26	−0·06804	8	0·18022	−129	−0·11829	54	0·08550	−242	−0·14154	170
1·4	0·44407	−25	−0·07917	7	0·17141	−125	−0·13667	57	0·06778	−224	−0·16071	194
1·6	0·44198	−23	−0·09023	8	0·16135	−121	−0·15447	67	0·04783	−202	−0·17797	219
1·8	0·43966	−22	−0·10121	8	0·15008	−117	−0·17160	74	0·02586	−180	−0·19305	235
2·0	0·43712	−22	−0·11211	8	0·13764	−110	−0·18799	81	0·00209	−156	−0·20578	255

δ^2 are modified second differences (*see* Appendix)

TABLE 1(*a*)—*continued*

Values of $L_0'(k)/\pi$

$\omega = \frac{pc}{V}$	$k = 4$				$k = 5$				$k = 6$			
	Rl	δ^2	*Im*	δ^2	*Rl*	δ^2	*Im*	δ^2	*Rl*	δ^2	*Im*	δ^2
0	+0·09766	−478	0	0	+0·06836	−679	0	0	+0·04102	−889	0	0
0·2	0·09528	−471	−0·02884	68	0·06498	−672	−0·03128	126	0·03662	−863	−0·03217	200
0·4	0·08820	−458	−0·05700	135	0·05493	−637	−0·06131	247	0·02366	−800	−0·06235	394
0·6	0·07656	−434	−0·08381	201	0·03855	−582	−0·08888	363	+0·00277	−696	−0·08862	570
0·8	0·06059	−404	−0·10862	259	+0·01638	−511	−0·11284	468	−0·02502	−558	−0·10924	719
1·0	0·04059	−363	−0·13084	319	−0·01087	−421	−0·13215	556	−0·05833	−386	−0·12273	837
1·2	+0·01697	−317	−0·14989	367	−0·04230	−313	−0·14593	631	−0·09547	−198	−0·12792	914
1·4	−0·00981	−264	−0·16528	413	−0·07685	−200	−0·15344	685	−0·13456	+11	−0·12404	956
1·6	−0·03922	−205	−0·17656	446	−0·11338	−74	−0·15414	718	−0·17354	219	−0·11069	947
1·8	−0·07067	−139	−0·18339	476	−0·15064	+57	−0·14770	732	−0·21033	429	−0·08795	898
2·0	−0·10351	−76	−0·18548	496	−0·18733	+188	−0·13399	719	−0·24286	+622	−0·05631	805

δ^2 are modified second differences (*see* Appendix)

TABLE 1(*b*)

Factors $L'_0(k) = 1, 2 \ldots . 6$, *for Six Chordwise Vortices Positioned at* 1/12, 3/12 . . . . 11/12 *chord*

Values of $L'_0(k)/\pi$

$\omega = \frac{pc}{V}$	$k = 1$				$k = 2$				$k = 3$			
	Rl	δ^2	*Im*	δ^2	*Rl*	δ^2	*Im*	δ^2	*Rl*	δ^2	*Im*	δ^2
2·0	0·43712	−87	−0·11211	32	+0·13764	−444	−0·18799	324	+0·00209	−628	−0·20578	1019
2·4	0·43139	−82	−0·13368	25	0·10949	−406	−0·21829	368	−0·04988	−416	−0·22346	1134
2·8	0·42483	−81	−0·15500	20	0·07729	−359	−0·24491	413	−0·10598	−189	−0·22988	1204
3·2	0·41745	−85	−0·17612	16	0·04150	−312	−0·26741	449	−0·16395	+50	−0·22434	1232
3·6	0·40921	−98	−0·19708	13	+0·00259	−264	−0·28542	486	−0·22142	287	−0·20656	1213
4·0	0·40000	−104	−0·21790	16	−0·03895	−212	−0·29858	515	−0·27603	519	−0·17672	1157
4·4	0·38974	−117	−0·23855	28	−0·08260	−157	−0·30659	545	−0·32547	739	−0·13539	1054
4·8	0·37832	−124	−0·25892	39	−0·12781	−94	−0·30916	566	−0·36756	937	−0·08358	919
5·2	0·36567	−126	−0·27889	59	−0·17396	−31	−0·30607	589	−0·40032	1115	−0·02265	744
5·6	0·35177	−120	−0·29827	77	−0·22041	+39	−0·29711	599	−0·42199	1258	+0·04566	539
6·0	0·33668	−112	−0·31688	96	−0·26646	+117	−0·28217	603	−0·43114	+1369	+0·11931	309

δ^2 are modified second differences (*see* Appendix)

TABLE 1(*b*)—*continued*

Values of $L_0'(k)/\pi$

$\omega = \frac{pc}{V}$	$k = 4$				$k = 5$				$k = 6$			
	Rl	δ^2	*Im*	δ^2	*Rl*	δ^2	*Im*	δ^2	*Rl*	δ^2	*Im*	δ^2
2·0	−0·10351	−300	−0·18548	+1992	−0·18733	+753	−0·13399	+2904	−0·24286	+2513	−0·05631	+3262
2·4	−0·17073	+262	−0·17475	2036	−0·25383	1759	−0·08544	2554	−0·28766	3801	+0·02954	2037
2·8	−0·23534	818	−0·14393	1930	−0·30302	2611	−0·01195	1867	−0·29549	4507	0·13495	+362
3·2	−0·29184	1335	−0·09407	1682	−0·32658	3195	+0·07975	+928	−0·25954	4493	0·24362	−1520
3·6	−0·33513	1775	−0·02763	1302	−0·31879	3448	0·18043	−181	−0·18002	3716	0·33729	−3292
4·0	−0·36087	2102	+0·05164	819	−0·27720	3221	0·27922	−1334	−0·06454	2271	0·39876	−4666
4·4	−0·36582	2305	0·13896	+256	−0·20308	2817	0·36481	−2410	+0·07279	+343	0·41472	−5398
4·8	−0·34799	2356	0·22877	−343	−0·10139	1980	0·42666	−3287	0·21321	−1775	0·37812	−5337
5·2	−0·30688	2255	0·31514	−950	+0·01963	881	0·45618	−3869	0·33610	−3773	0·28963	−4448
5·6	−0·24350	2001	0·39208	−1519	0·14918	+375	0·44769	−4075	0·42205	−5330	+0·15797	−2829
6·0	−0·16036	+1611	+0·45396	−2018	0·27493	−1660	+0·39918	−3880	+0·45595	−6190	−0·00104	−684

δ^2 are modified second differences (*see* Appendix)

TABLE 1(*c*)

Factors $L_0'(k)$, $k = 1, 2$, for Two Chordwise Vortices Positioned at $\frac{1}{4}$ and $\frac{3}{4}$ Chord

Values of $L_0'(k)/\pi$

$\omega = \frac{pc}{V}$	$k = 1$				$k = 2$			
	Rl	δ^2	*Im*	δ^2	*Rl*	δ^2	*Im*	δ^2
0	+0·75000	—454	0	0	+0·25000	—2055	0	0
0·2	0·74774	—447	—0·05760	42	0·23979	—2019	—0·09167	+393
0·4	0·74101	—442	—0·11478	85	0·20952	—1913	—0·17943	775
0·6	0·72988	—425	—0·17112	124	0·16025	—1735	—0·25949	1130
0·8	0·71451	—405	—0·22623	156	0·09374	—1501	—0·32832	1448
1·0	0·69510	—382	—0·27978	191	+0·01233	—1206	—0·38276	1716
1·2	0·67188	—353	—0·33144	214	—0·08106	—870	—0·42014	1929
1·4	0·64513	—328	—0·38097	235	—0·18309	—501	—0·43835	2078
1·6	0·61511	—295	—0·42816	249	—0·29009	—109	—0·43591	2159
1·8	0·58213	—269	—0·47287	257	—0·39816	+293	—0·41202	2162
2·0	0·54646	—244	—0·51502	262	—0·50331	689	—0·36663	2102
2·0	0·54646	—969	—0·51502	1049	—0·50331	2774	—0·36663	8478
2·4	0·46803	—801	—0·59151	1031	—0·68927	5724	—0·21458	7128
2·8	0·38144	—721	—0·65775	977	—0·81915	8026	+0·00703	4817
3·2	0·28750	—719	—0·71419	940	—0·87045	9392	0·27562	+1848
3·6	0·18624	—796	—0·76112	964	—0·82980	9663	0·56215	—1422
4·0	+0·07696	—909	—0·79823	1090	—0·69455	8806	0·83465	—4583
4·4	—0·04138	—1007	—0·82422	1339	—0·47306	6938	1·06218	—7264
4·8	—0·16965	—1029	—0·83660	1712	—0·18357	4309	1·21849	—9153
5·2	—0·30796	—911	—0·83169	2180	+0·14820	+1232	1·28504	—10058
5·6	—0·45505	—610	—0·80490	2695	0·49215	—1917	1·25292	—9901
6·0	—0·60784	—83	—0·75122	3176	+0·81744	—4782	+1·12358	—8751

δ^2 are modified second differences (*see* Appendix)

TABLE 2

Approximate Representation of the Chordwise Factors $L'_0(k)$ for the Frequency Parameter Range $0 \leqslant \omega' \leqslant 0 \cdot 4$;

$$L'_0(k) \equiv L'_a + i\omega' L'_b + \omega'^2 L'_c + i\omega'^3 L'_d$$

Factors for 6 Vortices at 1/12, 3/12 . . . 11/12 Chord

k	L'_a	L'_b	L'_c	L'_d
1	$0 \cdot 4512\pi$	$-0 \cdot 1143\pi$	$-0 \cdot 0148\pi$	$0 \cdot 0026\pi$
2	$0 \cdot 2051\pi$	$-0 \cdot 2025\pi$	$-0 \cdot 0696\pi$	$0 \cdot 0149\pi$
3	$0 \cdot 1367\pi$	$-0 \cdot 2537\pi$	$-0 \cdot 1448\pi$	$0 \cdot 0499\pi$
4	$0 \cdot 0976\pi$	$-0 \cdot 2895\pi$	$-0 \cdot 2318\pi$	$0 \cdot 1123\pi$
5	$0 \cdot 0684\pi$	$-0 \cdot 3149\pi$	$-0 \cdot 3252\pi$	$0 \cdot 2051\pi$
6	$0 \cdot 0410\pi$	$-0 \cdot 3251\pi$	$-0 \cdot 4133\pi$	$0 \cdot 3244\pi$

Factors for 2 Vortices at $\frac{1}{4}$ and $\frac{3}{4}$ Chord

k	L'_a	L'_b	L'	L'_d
1	$0 \cdot 7500\pi$	$-0 \cdot 5767\pi$	$-0 \cdot 2219\pi$	$0 \cdot 0696\pi$
2	$0 \cdot 2500\pi$	$-0 \cdot 9233\pi$	$-0 \cdot 9776\pi$	$0 \cdot 6396\pi$

TABLE 3

Values of the Downwash W_{nm} at Collocation Points (ξ, η) for Frequency Parameter Values ω_m*

TABLE 3(*a*)

Delta Wing $A = 3$ (taper ratio 1/7)

$\omega_m = \frac{pc_m}{V}$	(ξ, η)	W_{01}	W_{03}	W_{05}	W_{11}	W_{13}	W_{15}
0·26	(0, 0·2)	2·545252−*i*0·082822	−0·181656+*i*0·031994	−0·109239+*i*0·020431	+0·645743	−0·025716	−0·013964
	(0, 0·6)	2·542664+*i*0·295211	+1·393250−*i*0·040106	+0·372680−*i*0·015113	0·869258	+0·324655	+0·082044
	(0, 0·8)	2·453213+*i*0·407252	+2·636882+*i*0·023664	+1·893298−*i*0·029642	+1·003016	+0·712047	+0·456151
	($0\cdot\dot{6}$, 0·2)	2·388822−*i*0·385899	−0·366899+*i*0·071600	−0·175960+*i*0·040352	−0·126059	−0·033161	−0·009490
	($0\cdot\dot{6}$, 0·6)	2·625207+*i*0·095922	+1·291855−*i*0·142992	+0·210472−*i*0·036674	−0·180251	−0·066988	−0·044739
	($0\cdot\dot{6}$, 0·8)	2·695214+*i*0·280461	+2·767484−*i*0·111123	+1·896697−*i*0·123987	−0·205678	−0·099858	−0·068765
0·40	(0, 0·2)	2·621509−*i*0·163755	−0·171779+*i*0·041998	−0·104626+*i*0·027997	+0·645743	−0·025716	−0·013964
	(0, 0·6)	2·669031+*i*0·391140	+1·409604−*i*0·069545	+0·378071−*i*0·026546	0·869258	+0·324655	+0·082044
	(0, 0·8)	2·573240+*i*0·563594	+2·663459+*i*0·026227	+1·903663−*i*0·048366	+1·003016	+0·712047	+0·456151
	($0\cdot\dot{6}$, 0·2)	2·419571−*i*0·634190	−0·350550+*i*0·098306	−0·167116+*i*0·056366	−0·126059	−0·033161	−0·009490
	($0\cdot\dot{6}$, 0·6)	2·763677+*i*0·077810	+1·299078−*i*0·225010	+0·213229−*i*0·059278	−0·180251	−0·066988	−0·044739
	($0\cdot\dot{6}$, 0·8)	2·831878+*i*0·362419	+2·790724−*i*0·178361	+1·902114−*i*0·190797	−0·205678	−0·099858	−0·068765

*Values computed using a 21 × 6 lattice (*see* section 3).

TABLE 3(*a*)—*continued*

Delta Wing $A = 3$ (*taper ratio* 1/7)

$\omega_m = \frac{pc_m}{V}$	(ξ, η)	W_{01}	W_{03}	W_{05}	W_{11}	W_{13}	W_{15}
$0 \cdot 5\dot{3}$	(0, 0·2)	2·709425−*i*0·276618	−0·161046+*i*0·048922	−0·099448+*i*0·034464	+0·645743	−0·025716	−0·013964
	(0, 0·6)	2·823964+*i*0·455474	+1·428525−*i*0·103370	+0·384006−*i*0·039381	0·869258	+0·324655	+0·082044
	(0, 0·8)	2·717751+*i*0·689617	+2·696466+*i*0·022756	+1·916463−*i*0·069598	+1·003016	+0·712047	+0·456151
	(0·6, 0·2)	2·433302−*i*0·919008	−0·332643+*i*0·120171	−0·156829+*i*0·070250	−0·126059	−0·033161	−0·009490
	(0·6, 0·6)	2·928592+*i*0·014488	+1·303425−*i*0·312299	+0·214633−*i*0·083504	−0·180251	−0·066988	−0·044739
	(0·6, 0·8)	2·996106+*i*0·407105	+2·817651−*i*0·252979	+1·907173−*i*0·260532	−0·205678	−0·099858	−0·068765

*Values computed using a 21 × 6 lattice (*see* section)

TABLE 3(b)

Delta Wing $A = 1\cdot2$ (taper ratio 1/7)

$\omega_m = \frac{pc_m}{V}$	(ξ, η)	W_{01}	W_{03}	W_{05}	W_{11}	W_{13}	W_{15}
$0\cdot\dot{3}$	(0, 0·2)	2·112126−i0·314994	−0·190966+i0·009856	−0·095695+i0·012854	+0·376591	−0·049566	−0·018831
	(0, 0·6)	1·949935+i0·174801	+1·230396−i0·115602	+0·322158−i0·050133	0·527714	+0·188470	+0·020035
	(0, 0·8)	1·552378+i0·372245	+2·167702−i0·035688	+1·603480−i0·077761	+0·647579	+0·475148	+0·290190
	($0\cdot\dot{6}$, 0·2)	1·798222−i0·592607	−0·443860+i0·066115	−0·198334+i0·039002	−0·035192	−0·035352	−0·013248
	($0\cdot\dot{6}$, 0·6)	1·916705+i0·003309	+1·030227−i0·215753	+0·085418−i0·063831	−0·034863	−0·023507	−0·032319
	($0\cdot\dot{6}$, 0·8)	1·900147+i0·243745	+2·336401−i0·185945	+1·616846−i0·182231	−0·024056	+0·004907	−0·005391
$0\cdot\dot{6}$	(0, 0·2)	2·101135−i0·663741	−0·189025+i0·016308	−0·091486+i0·023826	+0·376591	−0·049566	−0·018831
	(0, 0·6)	2·174305+i0·266866	+1·231967−i0·239939	+0·317546−i0·102952	0·527714	+0·188470	+0·020035
	(0, 0·8)	1·797448+i0·663994	+2·204600−i0·084402	+1·610081−i0·160178	+0·647579	+0·475148	+0·290190
	($0\cdot\dot{6}$, 0·2)	1·579886−i1·193350	−0·429191+i0·126825	−0·183584+i0·073309	−0·035192	−0·035352	−0·013248
	($0\cdot\dot{6}$, 0·6)	2·152709−i0·117903	+0·983320−i0·435760	+0·064299−i0·127723	−0·034863	−0·023507	−0·032319
	($0\cdot\dot{6}$, 0·8)	2·181827+i0·378005	+2·348403−i0·386502	+1·597716−i0·367748	−0·024056	+0·004907	−0·005391

TABLE 3(c)

Arrowhead Wing A = 1·32

$\omega_m = \frac{pc_m}{V}$	(ξ, η)	W_{01}	W_{03}	W_{05}	W_{11}	W_{13}	W_{15}
0·303	(0, 0·2)	2·482843−*i*0·202426	−0·093039−*i*0·003335	−0·070750+*i*0·008337	+0·508700	−0·046908	−0·016548
	(0, 0·6)	1·795205+*i*0·231324	+1·249513−*i*0·102610	+0·349294−*i*0·046877	0·681309	+0·207564	+0·019803
	(0, 0·8)	1·251431+*i*0·364138	+2·004976−*i*0·060076	+1·494930−*i*0·095898	+0·697997	+0·470449	+0·274126
	(0·$\dot{6}$, 0·2)	2·155778−*i*0·492366	−0·343441+*i*0·034328	−0·156266+*i*0·025548	−0·142266	−0·059275	−0·017160
	(0·$\dot{6}$, 0·6)	1·972589+*i*0·031702	+1·119696−*i*0·215699	+0·146579−*i*0·068215	−0·071080	−0·085286	−0·074402
	(0·$\dot{6}$, 0·8)	1·777549+*i*0·236414	+2·306562−*i*0·229443	+1·617099−*i*0·216950	+0·042384	−0·006498	−0·023228
0·606	(0, 0·2)	2·539712−*i*0·448094	−0·092402−*i*0·010293	−0·067778+*i*0·014866	0·508700	−0·046908	−0·016548
	(0, 0·6)	2·024731+*i*0·386421	+1·254423−*i*0·213611	+0·345821−*i*0·096258	0·681309	+0·207564	+0·019803
	(0, 0·8)	1·497862+*i*0·642022	+2·037023−*i*0·133855	+1·496510−*i*0·196248	+0·697997	+0·470449	+0·274126
	(0·$\dot{6}$, 0·2)	2·059323−*i*1·019561	−0·338042+*i*0·064539	−0·147713+*i*0·048060	−0·142266	−0·059275	−0·017160
	(0·$\dot{6}$, 0·6)	2·210724−*i*0·055103	+1·077825−*i*0·436377	+0·126270−*i*0·136590	−0·071080	−0·085286	−0·074402
	(0·$\dot{6}$, 0·8)	2·073506+*i*0·348347	+2·304622−*i*0·474958	+1·583621−*i*0·436332	+0·042384	+0·006498	−0·023228

TABLE 4

Local Derivative Coefficients referred to an Axis Position $0\cdot556c_0$ for the Delta Wing $A = 3$ describing Plunging and Pitching Oscillations

ω_m	η	$l_z(\eta)$	$l_{\dot{z}}(\eta)$	$l_\alpha(\eta)$	$l_{\dot{\alpha}}(\eta)$	$-m_z(\eta)$	$-m_{\dot{z}}(\eta)$	$-m_\alpha(\eta)$	$-m_{\dot{\alpha}}(\eta)$
→0	0	0	1·9972	1·9972	1·5864	0	−1·0021	−1·0021	0·3997
	0·2	0	1·9489	1·9489	1·4744	0	−0·5602	−0·5602	0·4820
	0·4	0	1·8028	1·8028	1·1529	0	−0·1409	−0·1409	0·4474
	0·6	0	1·5498	1·5498	0·7394	0	+0·2067	+0·2067	0·3304
	0·8	0	1·1460	1·1460	0·3891	0	+0·4045	+0·4045	0·2068
	1·0	0	0	0	0	0	0	0	0
$0\cdot2\dot{6}$	0	−0·0520	1·9466	1·9695	1·7133	−0·0109	−0·9763	−0·9971	0·3353
	0·2	−0·0450	1·8929	1·9068	1·5969	−0·0155	−0·5443	−0·5568	0·4461
	0·4	−0·0255	1·7457	1·7534	1·2719	−0·0155	−0·1364	−0·1432	0·4383
	0·6	−0·0023	1·4972	1·5035	0·8509	−0·0083	+0·2000	+0·1973	0·3460
	0·8	+0·0112	1·1034	1·1108	0·4769	+0·0014	+0·3894	+0·3907	0·2377
	1·0	0	0	0	0	0	0	0	0
0·4	0	−0·1282	1·9132	1·9587	1·7650	−0·0207	−0·9579	−1·0028	0·3219
	0·2	−0·1123	1·8516	1·8771	1·6439	−0·0329	−0·5322	−0·5593	0·4405
	0·4	−0·0684	1·7011	1·7124	1·3179	−0·0346	−0·1330	−0·1482	0·4376
	0·6	−0·0160	1·4554	1·4639	0·8961	−0·0202	+0·1944	+0·1877	0·3523
	0·8	+0·0166	1·0696	1·0815	0·5131	+0·0001	+0·3774	+0·3787	0·2507
	1·0	0	0	0	0	0	0	0	0
$0\cdot5\dot{3}$	0	−0·2432	1·8813	1·9540	1·8020	−0·0307	−0·9392	−1·0157	0·3103
	0·2	−0·2147	1·8087	1·8461	1·6747	−0·0551	−0·5192	−0·5656	0·4355
	0·2	−0·1371	1·6537	1·6655	1·3481	−0·0605	−0·1294	−0·1559	0·4364
	0·6	−0·0441	1·4114	1·4184	0·9281	−0·0380	+0·1885	+0·1754	0·3564
	0·8	+0·0167	1·0341	1·0486	0·5400	−0·0044	+0·3648	+0·3647	0·2600
	1·0	0	0	0	0	0	0	0	0

TABLE 5

Local Derivative Coefficients referred to an Axis Position $0 \cdot 556c_0$ for the Delta Wing $A = 1 \cdot 2$ describing Plunging and Pitching Oscillations

ω_m	η	$l_z(\eta)$	$l_{\dot{z}}(\eta)$	$l_\alpha(\eta)$	$l_{\dot{\alpha}}(\eta)$	$-m_z(\eta)$	$-m_{\dot{z}}(\eta)$	$-m_\alpha(\eta)$	$-m_{\dot{\alpha}}(\eta)$
$\to 0$	0	0	1·0336	1·0336	1·1638	0	−0·4249	−0·4249	0·2748
	0·2	0	1·0139	1·0139	1·1183	0	−0·2190	−0·2190	0·3356
	0·4	0	0·9511	0·9511	0·9511	0	−0·0309	−0·0309	0·3258
	0·6	0	0·8323	0·8323	0·7078	0	0·1229	0·1229	0·2741
	0·8	0	0·6237	0·6237	0·4425	0	0·2098	0·2098	0·2082
	1·0	0	0	0	0	0	0	0	0
0·33	0	−0·0662	1·0321	1·0472	1·1792	−0·0044	−0·4254	−0·4444	0·2705
	0·2	−0·0620	1·0057	1·0104	1·1292	−0·0113	−0·2188	−0·2302	0·3342
	0·4	−0·0466	0·9396	0·9364	0·9610	−0·0146	−0·0322	−0·0392	0·3257
	0·6	−0·0261	0·8211	0·8159	0·7185	−0·0123	0·1203	0·1155	0·2756
	0·8	−0·0089	0·6139	0·6112	0·4510	−0·0065	0·2063	0·2036	0·2108
	1·0	0	0	0	0	0	0	0	0
0·67	0	−0·2689	1·0403	1·0986	1·1927	−0·0197	−0·4288	−0·5061	0·2739
	0·2	−0·2509	0·9934	1·0102	1·1287	−0·0470	−0·2177	−0·2647	0·3393
	0·4	−0·1902	0·9158	0·9011	0·9562	−0·0590	−0·0334	−0·0618	0·3269
	0·6	−0·1098	0·7969	0·7728	0·7171	−0·0501	0·1155	0·0959	0·2746
	0·8	−0·0407	0·5923	0·5785	0·4507	−0·0276	0·1987	0·1868	0·2102
	1·0	0	0	0	0	0	0	0	0

TABLE 6

Local Derivative Coefficients referred to an Axis Position $0 \cdot 738c_0$ for the Arrowhead Wing $A = 1 \cdot 32$ describing Plunging and Pitching Oscillations

ω_m	η	$l_z(\eta)$	$l_{\dot{z}}(\eta)$	$l_\alpha(\eta)$	$l_{\dot{\alpha}}(\eta)$	$-m_z(\eta)$	$-m_{\dot{z}}(\eta)$	$-m_\alpha(\eta)$	$-m_{\dot{\alpha}}(\eta)$
→0	0	0	1·0064	1·0064	0·7600	0	−0·6059	−0·6059	0·0089
	0·2	0	0·9977	0·9977	0·7556	0	−0·3631	−0·3631	0·1109
	0·4	0	0·9628	0·9628	0·7112	0	−0·1385	−0·1385	0·1751
	0·6	0	0·8743	0·8743	0·6189	0	+0·0626	+0·0626	0·2187
	0·8	0	0·6771	0·6771	0·4615	0	+0·1980	+0·1980	0·2335
	1·0	0	0	0	0	0	0	0	0
0·30	0	−0·0415	0·9998	1·0130	0·7740	0·0080	−0·6012	−0·6124	0·0024
	0·2	−0·0394	0·9885	0·9963	0·7689	−0·0002	−0·3593	−0·3653	0·1071
	0·4	−0·0312	0·9519	0·9526	0·7248	−0·0068	−0·1366	−0·1400	0·1730
	0·6	−0·0195	0·8630	0·8586	0·6324	−0·0092	+0·0620	+0·0583	0·2190
	0·8	−0·0082	0·6666	0·6620	0·4726	−0·0072	+0·1950	+0·1911	0·2366
	1·0	0	0	0	0	0	0	0	0
0·61	0	−0·1700	0·9901	1·0413	0·7804	0·0334	−0·5938	−0·6379	0·0027
	0·2	−0·1615	0·9705	1·0000	0·7734	0	−0·3519	−0·3756	0·1085
	0·4	−0·1301	0·9280	0·9289	0·7310	−0·0268	−0·1330	−0·1464	0·1736
	0·6	−0·0847	0·8375	0·8173	0·6416	−0·0375	+0·0601	+0·0453	0·2200
	0·8	−0·0398	0·6432	0·6220	0·4827	−0·0308	+0·1881	+0·1718	0·2391
	1·0	0	0	0	0	0	0	0	0

TABLE 7

Derivatives of the Delta Wing $A = 3$
for Plunging and Pitching Oscillations

hc_0	ω_m	l_z	$l_{\dot{z}}$	l_α	$l_{\dot{\alpha}}$	$-m_z$	$-m_{\dot{z}}$	$-m_\alpha$	$-m_{\dot{\alpha}}$	Remarks
0	$\rightarrow 0$	0	1·539	1·539	2·423	0	+1·414	+1·414	2·623	Ref. 1
	$0\cdot1\dot{3}$	−0·003	1·533	1·534	2·462	−0·005	1·407	1·404	2·654	Ref. 1
	$0\cdot2\dot{6}$	−0·015	1·521	1·516	2·501	−0·023	1·389	1·371	2·685	Ref. 1
	$0\cdot2\dot{6}$	−0·017	1·490	1·483	2·480	−0·025	1·367	1·346	2·675	Present method
	0·40	−0·048	1·452	1·422	2·485	−0·066	1·331	1·269	2·682	Present method
	$0\cdot5\dot{3}$	−0·099	1·413	1·339	2·475	−0·131	1·293	1·163	2·674	Present method
	0·80	−0·316	1·346	1·039	2·520	−0·375	+1·234	+0·821	2·712	Ref. 3*
$0\cdot556c_0$	$\rightarrow 0$	0	1·539	1·539	0·926	0	−0·083	−0·083	0·346	Ref. 1
	$0\cdot1\dot{3}$	−0·003	1·533	1·537	0·970	−0·002	−0·085	−0·087	0·341	Ref. 1
	$0\cdot2\dot{6}$	−0·015	1·521	1·531	1·021	−0·009	−0·090	−0·096	0·340	Ref. 1
	$0\cdot2\dot{6}$	−0·017	1·490	1·499	1·030	−0·008	−0·082	−0·088	0·342	Present method
	0·40	−0·048	1·452	1·469	1·072	−0·019	−0·082	−0·096	0·345	Present method
	$0\cdot5\dot{3}$	−0·099	1·413	1·435	1·100	−0·035	−0·082	−0·106	0·346	Present method
	0·80	−0·316	1·346	1·347	1·211	−0·068	−0·076	−0·125	0·333	Ref. 3*

*Solutions with factors L_0' and $C(\omega')$ variable with ω across wing span.

TABLE 8

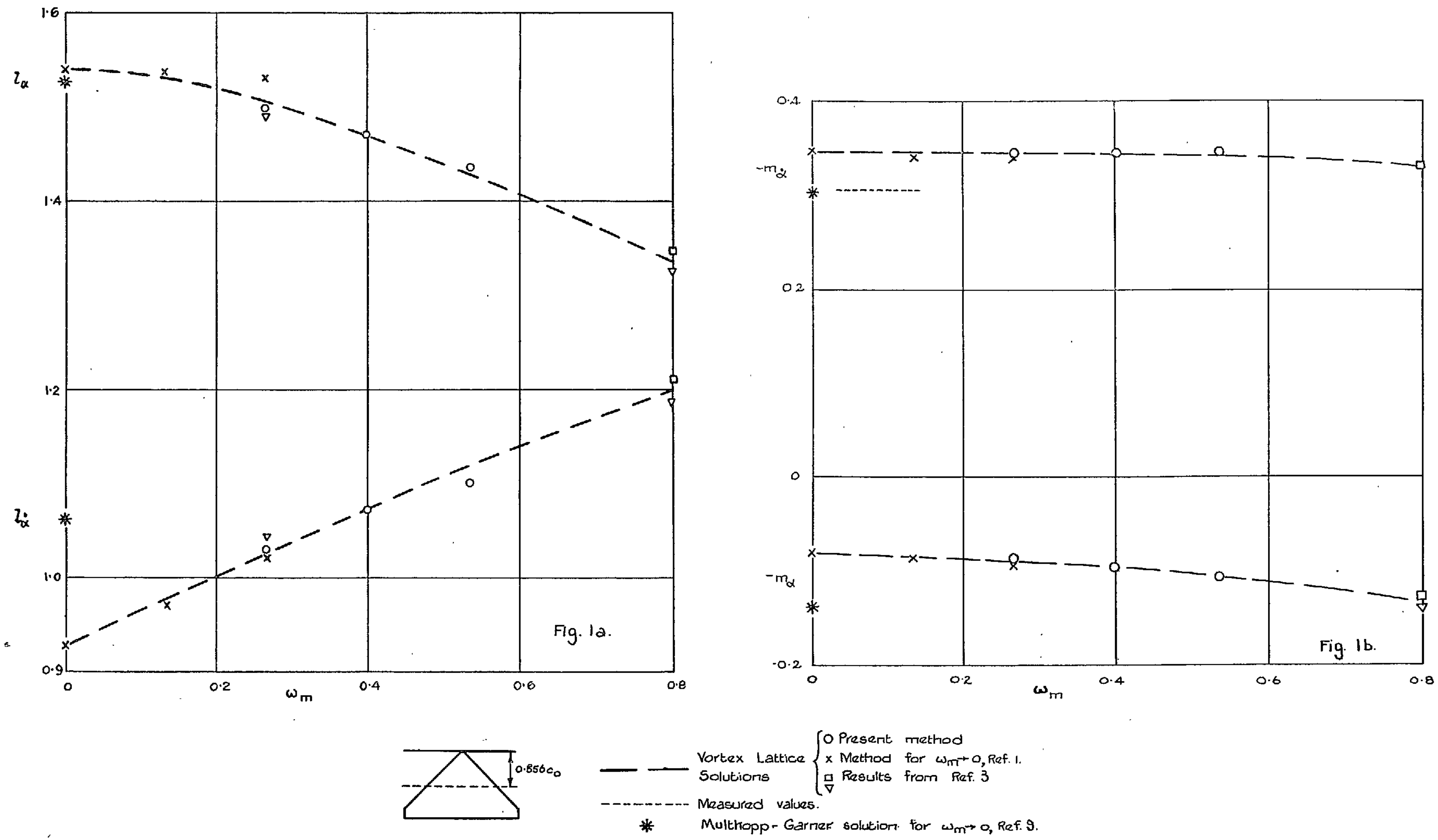
Derivatives of the Delta Wing $A = 1 \cdot 2$ for Plunging and Pitching Oscillations

hc_0	ω_m	l_z	$l_{\dot{z}}$	l_α	$l_{\dot{\alpha}}$	$-m_z$	$-m_{\dot{z}}$	$-m_\alpha$	$-m_{\dot{\alpha}}$	Remarks
0	$\rightarrow 0$	0	0·815	0·815	1·571	0	0·784	0·784	1·789	Ref. 1
	0·33	−0·036	0·805	0·771	1·571	−0·044	0·774	0·724	1·788	Present method
	0·67	−0·146	0·788	0·644	1·554	−0·182	0·753	0·545	1·768	Present method
$0 \cdot 431c_0$	$\rightarrow 0$	0	0·815	0·815	0·956	0	0·170	0·170	0·476	Ref. 1
	0·33	−0·036	0·805	0·798	0·964	−0·017	0·166	0·155	0·477	Present method
	0·67	−0·146	0·788	0·754	0·959	−0·072	0·159	0·114	0·477	Present method
$0 \cdot 556c_0$	$\rightarrow 0$	0	0·815	0·815	0·778	0	−0·008	−0·008	0·268	Ref. 1
	0·33	−0·036	0·805	0·805	0·788	−0·010	−0·010	−0·017	0·269	Present method
	0·67	−0·146	0·788	0·786	0·787	−0·040	−0·014	−0·043	0·270	Present method

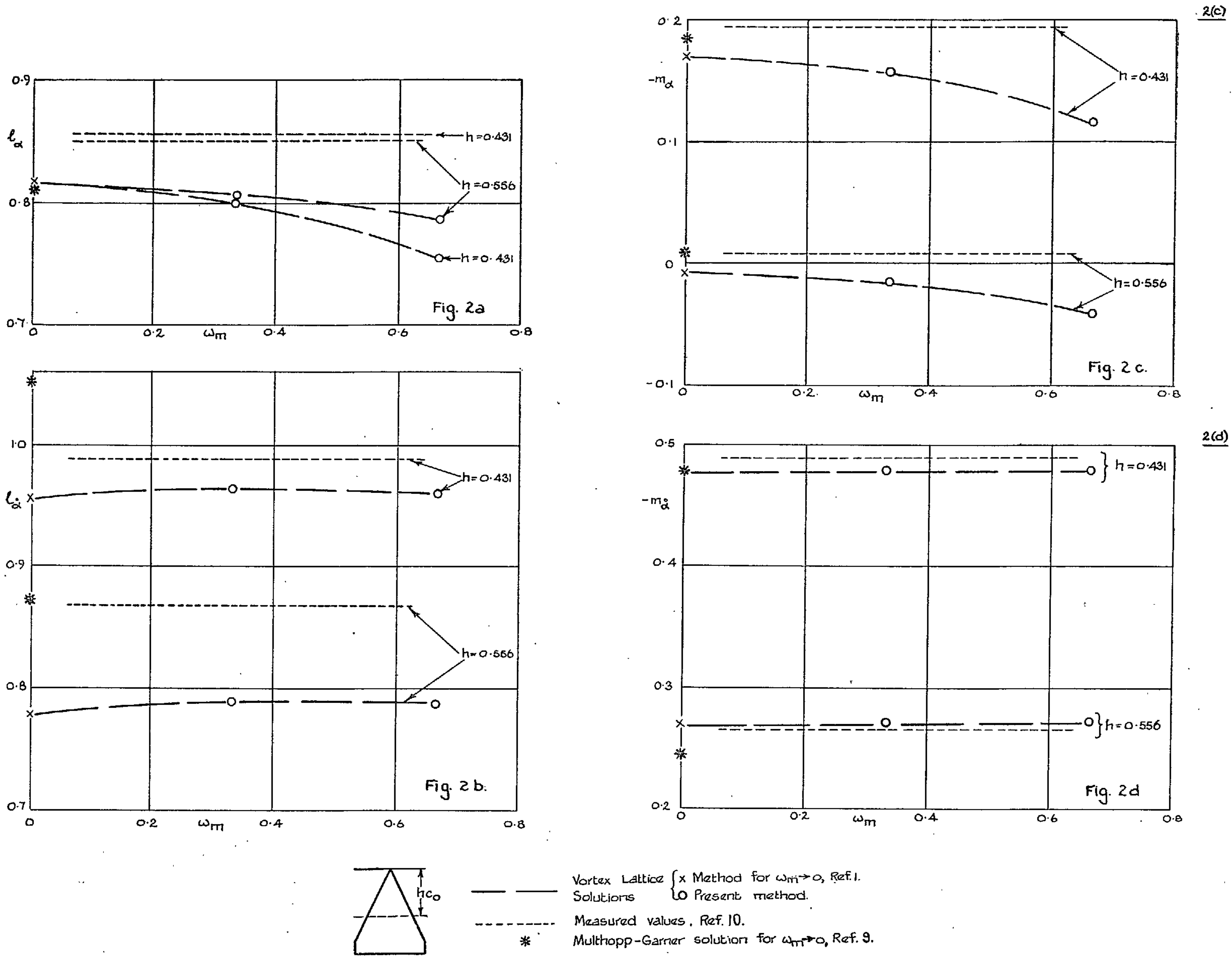
TABLE 9

Derivatives of the Arrowhead Wing $A = 1 \cdot 32$ for Plunging and Pitching Oscillations

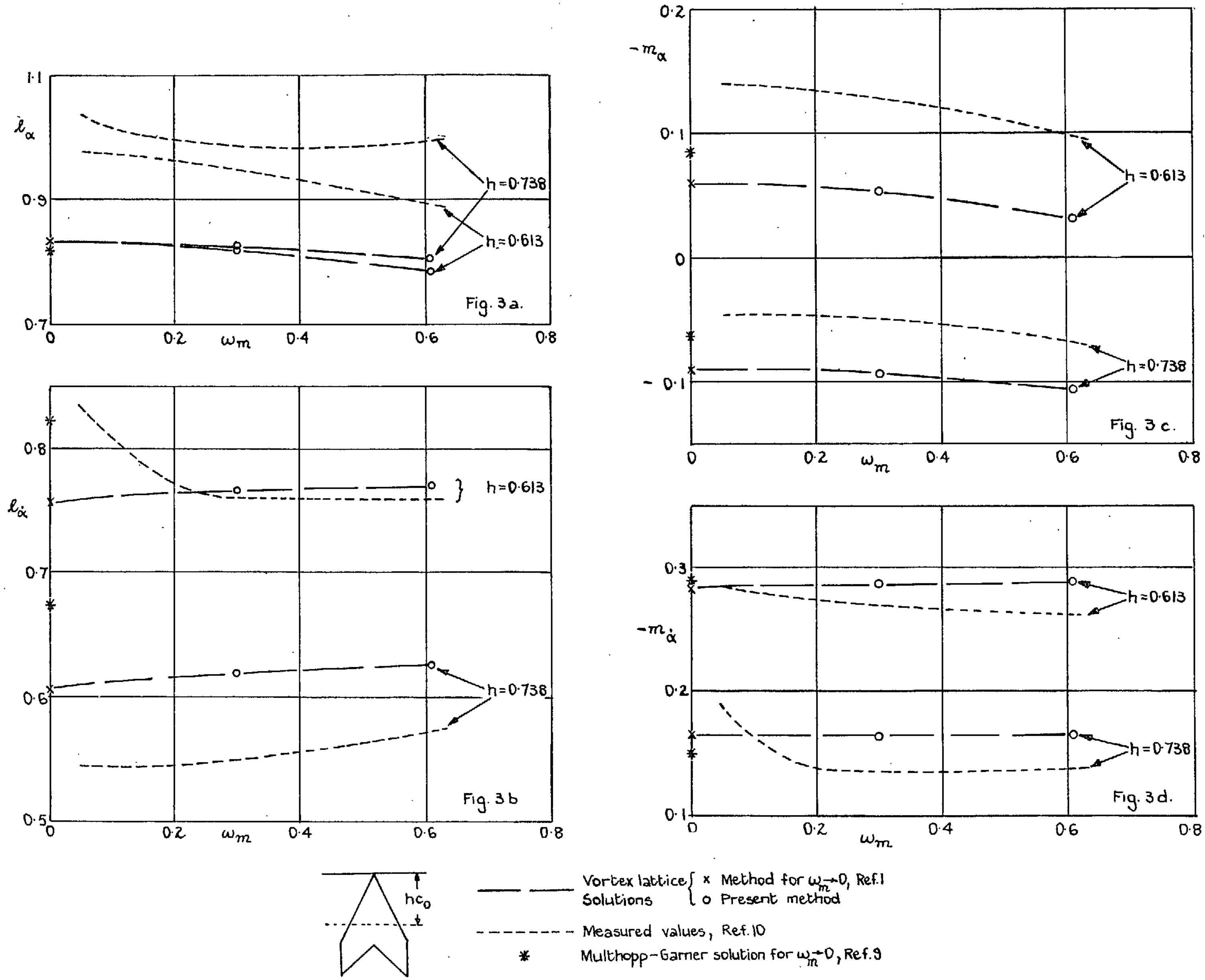
hc_0	ω_m	l_z	$l_{\dot{z}}$	l_α	$l_{\dot{\alpha}}$	$-m_z$	$-m_{\dot{z}}$	$-m_\alpha$	$-m_{\dot{\alpha}}$	Remarks
0	$\to 0$	0	0·833	0·833	1·491	0	+0·795	+0·795	1·653	Ref. 1
	0·30	−0·024	0·823	0·799	1·493	−0·030	0·785	0·750	1·655	Present method
	0·61	−0·101	0·802	0·697	1·478	−0·125	0·764	0·615	1·641	Present method
$0 \cdot 613c_0$	$\to 0$	0	0·833	0·833	0·756	0	0·060	0·060	0·284	Ref. 1
	0·30	−0·024	0·823	0·820	0·766	−0·009	0·059	0·053	0·286	Present method
	0·61	−0·101	0·802	0·786	0·769	−0·036	+0·056	+0·031	0·288	Present method
$0 \cdot 738c_0$	$\to 0$	0	0·833	0·833	0·606	0	−0·090	−0·090	0·165	Ref. 1
	0·30	−0·024	0·823	0·824	0·618	−0·004	−0·089	−0·094	0·164	Presnet method
	0·61	−0·101	0·802	0·805	0·625	−0·018	−0·089	−0·107	0·165	Present method



FIGS. 1a and 1b. Derivative coefficients for pitching oscillations of delta wing, $A = 3$ referred to the axis position $0 \cdot 556c_0$.



FIGS. 2a to 2d. Derivative coefficients for pitching oscillations of delta wing $A = 1 \cdot 2$, referred to axis positions hc_0.



FIGS. 3a to 3d. Derivative coefficients for pitching oscillations of arrowhead wing, $A = 1 \cdot 32$, referred to axis positions hc_0.

J4383 Wt.52/8943 K7 7/57 D&Co. 34/263

PRINTED IN GREAT BRITAIN

Publications of the Aeronautical Research Council

ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)

1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50s. (52s.)
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63s. (65s.)

1940 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Icing, Stability and Control, Structures, and a miscellaneous section. 50s. (52s.)

1941 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Stability and Control, Structures. 63s. (65s.)

1942 Vol. I. Aero and Hydrodynamics, Aerofoils, Airscrews, Engines. 75s. (77s.)
Vol. II. Noise, Parachutes, Stability and Control, Structures, Vibration, Wind Tunnels. 47s. 6d. (49s. 6d.)

1943 Vol. I. Aerodynamics, Aerofoils, Airscrews. 80s. (82s.)
Vol. II. Engines, Flutter, Materials, Parachutes, Performance, Stability and Control, Structures. 90s. (92s. 9d.)

1944 Vol. I. Aero and Hydrodynamics, Aerofoils, Aircraft, Airscrews, Controls. 84s. (86s. 6d.)
Vol. II. Flutter and Vibration, Materials, Miscellaneous, Navigation, Parachutes, Performance, Plates and Panels, Stability, Structures, Test Equipment, Wind Tunnels. 84s. (86s. 6d.)

1945 Vol. I. Aero and Hydrodynamics, Aerofoils. 130s. (132s. 9d.)
Vol. II. Aircraft, Airscrews, Controls. 130s. (132s. 9d.)
Vol. III. Flutter and Vibration, Instruments, Miscellaneous, Parachutes, Plates and Panels, Propulsion. 130s (132s. 6d.)
Vol. IV. Stability, Structures, Wind tunnels, Wind Tunnel Technique. 130s. (132s. 6d.)

ANNUAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

1937 2s. (2s. 2d.) 1938 1s. 6d. (1s. 8d.) 1939–48 3s. (3s. 5d.)

INDEX TO ALL REPORTS AND MEMORANDA PUBLISHED IN THE ANNUAL TECHNICAL REPORTS, AND SEPARATELY—

April, 1950 - - - - - R. & M. No. 2600. 2s. 6d. (2s. 10d.)

AUTHOR INDEX TO ALL REPORTS AND MEMORANDA OF THE AERONAUTICAL RESEARCH COUNCIL—

1909–January, 1954 - - - R. & M. No. 2570. 15s. (15s. 8d.)

INDEXES TO THE TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

December 1, 1936 — June 30, 1939.	R. & M. No. 1850.	1s. 3d. (1s. 5d.)
July 1, 1939 — June 30, 1945. -	R. & M. No. 1950.	1s. (1s. 2d.)
July 1, 1945 — June 30, 1946. -	R. & M. No. 2050.	1s. (1s. 2d.)
July 1, 1946 — December 31, 1946.	R. & M. No. 2150.	1s. 3d. (1s. 5d.)
January 1, 1947 — June 30, 1947. -	R. & M. No. 2250.	1s. 3d. (1s. 5d.)

PUBLISHED REPORTS AND MEMORANDA OF THE AERONAUTICAL RESEARCH COUNCIL—

Between Nos. 2251–2349. - -	R. & M. No. 2350.	1s. 9d. (1s. 11d.)
Between Nos. 2351–2449. - -	R. & M. No. 2450.	2s. (2s. 2d.)
Between Nos. 2451–2549. - -	R. & M. No. 2550.	2s. 6d. (2s. 10d.)
Between Nos. 2551–2649. - -	R. & M. No. 2650.	2s. 6d. (2s. 10d.)
Between Nos. 2651–2749. - -	R. & M. No. 5750.	2s. 6d. (2s. 10d.)

Prices in brackets include postage

HER MAJESTY'S STATIONERY OFFICE

York House, Kingsway, London W.C.2; 423 Oxford Street, London W.1; 13a Castle Street, Edinburgh 2; 39 King Street, Manchester 2; 2 Edmund Street, Birmingham 3; 109 St. Mary Street, Cardiff; Tower Lane, Bristol 1; 80 Chichester Street, Belfast, *or through any bookseller*

S.O. Code No. 23-2961

R. & M. No. 2961