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A Simple Method of Computing C_D from Wake Traverses at High- subsonic Speeds

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A Simple Method of Computing C_D from Wake Traverses at High-subsonic Speeds

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Summary.—This note gives a convenient method of obtaining C_D from a pitot-static traverse in an aerofoil wake, using Jones' equation as modified by Lock, Hilton and Goldstein² for compressible flow. Charts are provided from which the integrand C_D' can easily be obtained for any point in the traverse, but it is shown that in nearly all cases an accuracy of 1 per cent in C_D can be obtained by applying an integrating factor to the area under the total-head loss curve.

Three Appendices give (a) a summary of the standard theory and equations, (b) details of the construction of the charts and (c) an empirical equation giving C_D' in a simple analytical form.

1. *Introduction.*—To calculate the profile drag of an aerofoil from a pitot-static traverse across a wake at high subsonic speeds, the Jones formula modified for compressibility by Lock, Hilton and Goldstein², is generally used (*see* Appendix I). This formula gives C_D' , the drag contribution from any point in the wake traverse, in terms of the total and static pressures at that point and in the free stream, and C_D is obtained by integrating C_D' across the wake. The computation of C_D' is somewhat tedious, and though it may be simplified by the use of existing tables, these have the disadvantage of requiring interpolation (as in Ref. 2) or being limited in range (as in Ref. 1). This note gives simple methods of obtaining C_D' for the whole range likely to be encountered in flight or tunnel tests.

Drag calculations may be much simplified if, instead of using the 'point-by-point' method of evaluating C_D' for each point of the traverse, a factor F is used to convert the integrated total head loss to C_D . Methods of this type have been described before by Young⁷, Silverstein and Katzoff⁸, and others, but this note gives an assessment of the accuracy and a simple rule for determining F .

2. *Method of Obtaining C_D .*—2.1. *Point-by-Point Method.*—To obtain C_D from traverse readings by the point-by-point method, C_D' must first be evaluated for each point in the wake, and a curve plotted of C_D' against position y/c along the traverse. The area under this curve, between the limits where $C_D' = 0$, is then equal to C_D .

C_D' can be obtained from Fig. 1 as follows. Let h be the deficit of total head and p the excess of static pressure relative to the free stream, both expressed as fractions of the free-stream dynamic head $H_0 - P_0$, and let M be the free-stream Mach number. Then in Fig. 1, a straight line joining a point on the p scale to a point on the (M, h) grid intersects the C_D'/h scale at a point giving the value of C_D'/h corresponding to the given values of M , h and p . (In practice, instead

of ruling a line on the chart, it is preferable to use a strip of Perspex with a straight line engraved on it.) C_D'/h can thus be obtained for each point of the traverse, and multiplied by the appropriate value of h to give C_D' . The chart is arranged to give C_D'/h instead of C_D' in order to maintain good accuracy at low values of h .

Fig. 1 covers the range $h = 0$ to $h = 0.6$. Higher values are occasionally met with, and Fig. 2 is an auxiliary chart covering the range $h = 0.6$ to $h = 1.0$. Fig. 2 gives C_D' instead of C_D'/h , and is not so accurate.

2.2. *Integrating Factor F.*—When the wake is of normal shape, and η , the peak value of h in the wake, does not exceed about 0.6, a simpler method may be used. Instead of obtaining C_D'/h separately for each point in the wake, it is permissible to assume that the average value of C_D'/h is equal to its local value at $h = 0.75\eta$. This average value of C_D'/h can be used as an integrating factor F , and C_D is then given by the equation

$$C_D = F \int h d(y/c).$$

F can be obtained from Fig. 1, using the values of 0.75η and ϕ for the wake and the free-stream M (ϕ is assumed constant across the wake). By using this method the computation is much simplified, because it is not necessary to convert each pitot reading to dimensionless h units: the curve can be plotted as inches of manometer liquid, for instance, and its area multiplied by a factor to convert it to $\int h d(y/c)$.

It is, of course, possible to obtain F by any other method which gives C_D' , such as the Tables of Refs. 1 or 2, but the F factor should not be used when η is larger than 0.6.

The F method can be used only for a wake approximating to the normal shape typical of low-speed tests. In practice this includes any wake that looks roughly like either the sine curve or the 'error' curve in Fig. 3a. When the shape differs considerably from this, as in Fig. 3b, it may be split up into parts A and B, each of nearly normal shape, of which the areas are determined separately. For area A, η is taken as 0.065 and for B it is 0.570; a separate F factor is obtained for each, and

$$C_D = F_A \int_A h d(y/c) + F_B \int_B h d(y/c).$$

When η is small, practically any shape may be regarded as normal. Thus in Fig. 3b, if η had been 0.3 instead of 0.57 the area need not have been subdivided, but one F factor could have been used for the whole curve. In Fig. 3a, with $\eta = 0.3$, any of the four shapes could be taken as normal with an accuracy within 1 per cent.

3. *Corrections.*—Tunnel blockage involves corrections to M , h , and ϕ , but instead of applying these separately before calculating C_D' , it is generally more convenient to use uncorrected values throughout, and then to correct C_D by subtracting the correction appropriate to $\frac{1}{2}\rho V^2$. As long as the blockage correction is small, this will give the same result as applying the corrections separately.

A correction must be applied for the finite diameter of the pitot-tubes, and in the absence of high-speed data a low-speed value obtained by Young and Maas is used⁶. For a normal pitot-tube of external diameter d , the correction to be added to C_D is

$$\Delta C_D = 0.36d/c \times \text{maximum value of } C_D' \text{ in the wake.}$$

When the F method is used, this correction can conveniently be applied by adding $0.36\eta d/c$ to the area $\int h d(y/c)$.

4. *Accuracy.*—4.1. *Accuracy of Charts.*—Between the limits $p = 0$ to $p = 0.15$, Fig. 1 gives values of C_D'/h as accurately as they can be read from the chart, say to within ± 0.002 . For p between 0.15 and 0.2 the accuracy is less: the greatest errors (which occur at $p = 0.2$) being $+ 0.01$ at $M = 0.9$, $h = 0.6$ and $- 0.01$ at $M = 0$, $h = 0$.

Fig. 2 is as accurate as the scales permit for the range $p = 0$ to $p = 0.1$, but for higher values of p it is accurate only for $M = 0$. When the greatest possible accuracy is needed, a revised graduation on the p scale may be obtained for any given high values of M and p by using the fact that $C_D' = 0$ when $h = 1 - p$. For instance, when $M = 0.9$, $p = 0.15$, the values $C_D' = 0$ and $M = 0.9$, $h = 0.85$ give a point on the p scale at about 0.155 which can be used for determining C_D' for any other value of h . By this method, more accurate values of C_D' can be obtained from the chart, but it is doubtful whether the extra trouble is justified, in view of the uncertainty of the basic theory for high values of p .

Appendix II gives details of the construction of the charts, and explains how the p scale in Fig. 1 can be extended for negative values of p .

4.2. *Accuracy of Integrating Factor F .*—The integrating factor F is the mean value of C_D'/h across the wake, and the statement that it is equal to the value of C_D'/h at $h = 0.75\eta$ is strictly true only in certain cases, such as a sine-shaped wake at small values of η . The general application of the rule can only be justified empirically, and depends on the observed fact that most experimental wakes approximate to the 'error curve'

$$h = \eta \exp(-ky^2).$$

The effect of differences in the wake shape is illustrated in Fig. 4, which shows the errors produced by using the general rule for the four shapes shown in Fig. 3a. (These values were computed by means of the empirical equation (10) in Appendix III, which is convenient for such calculations.) These errors are generally small, and the rule can be used with confidence when the wake shape appears to be normal. For low values of η the errors become negligible for practically any shape, because of the small variation of C_D'/h between $h = 0$ and $h = \eta$. Thus when $\eta = 0.2$, $M = 0.8$, $p = 0$, the maximum possible errors are $- 2$ per cent or $+ 1$ per cent in C_D .

The wake shape shown in Fig. 3b was obtained from an NACA 0015 aerofoil with the comb 0.1c behind the trailing edge, and is a typical form when shock-waves are present on both surfaces. The error in using the standard F factor on the whole curve is $- 5$ per cent, but by treating the areas A and B separately, as described above (*see* section 2.2) the error is reduced to $- 0.5$ per cent.

Values of F obtained by this method agree reasonably well with low-speed values given in Ref. 7, where a somewhat artificial wake shape is chosen, and with the low-speed values in Ref. 8, where a sine shape is assumed. The high-speed values in Ref. 8 show large differences, however, and it is believed that there are errors in Table 1 of that report.

To obtain a high standard of accuracy in C_D , very great care is required both in the experimental work (an error of 1 per cent of $\frac{1}{2}\rho V^2$ in free-stream total head may mean 5 per cent or 10 per cent error in drag) and in the plotting and integration of the curves. For this reason a computing accuracy better than 1 per cent is rarely justified, and in nearly all cases the F method may be used.

LIST OF SYMBOLS

| | |
|-----------------|--|
| H_0 | Total head in free stream |
| H_1 | Total head in plane of traverse and also downstream where $P = P_0$ |
| P_0 | Static pressure in free stream |
| P_1 | Static pressure in plane of traverse |
| M or M_0 | Mach number in free stream |
| M_1 | Mach number in plane of traverse |
| M_2 | Mach number downstream where $P = P_0$ |
| a_0 | Velocity of sound in free stream |
| a_1 | Velocity of sound in plane of traverse |
| a_2 | Velocity of sound downstream where $P = P_0$ |
| h | $= (H_0 - H_1)/(H_0 - P_0)$ |
| p | $= (P_1 - P_0)/(H_0 - P_0)$ |
| η | Maximum value of h across wake |
| c | Aerofoil chord |
| y | Distance along traverse perpendicular to chord |
| C_D | Drag coefficient |
| C_D' | Integrand where $C_D = \int C_D' d(y/c)$ |
| F | Integrating factor where $C_D = F \int h d(y/c)$ |
| d | External diameter of pitot tube |
| γ | Ratio of specific heats of air, taken as 1.40 throughout |
| A_0, A_1, A_2 | } See equation (10), Appendix III |
| B_0 | |
| ξ, ζ | |

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APPENDIX I

Summary of Theory

(a) *Basic Assumptions.*—In order to obtain the drag of an aerofoil from a pitot-static exploration of its wake, the traverse should, in theory, be extended to cover the whole region in which the static pressure differs from that in the free stream. Since this is rarely practicable, the traverse is generally limited to the region where there is measurable loss of total head, and some arbitrary assumption must be made about how the static pressure comes back to its free-stream value. The usual assumption, due to Jones³, is that the total head remains constant along any streamline taken downstream from the plane of the traverse, and this is the only one considered in the present method of computation. Other possible assumptions are discussed in Ref. 4.

The static pressure term in any drag formula thus cannot be rigorously justified, but in practice its effect is not large, and several different assumptions give the same value for the first order term in the static pressure coefficient ϕ . Taylor⁵ has pointed out that in tunnel tests a further difficulty arises in defining the free-stream static pressure, but a discussion of this and the associated problem of tunnel blockage effects is outside the scope of this note.

(b) *Equations.*—The application of Jones' method to the case of compressible flow has been fully discussed by Lock, Hilton and Goldstein², who established the following formula:

$$C_D' = 2(H_1/H_0)^{2/7} (P_1/P_0)^{5/7} \left[\frac{1 - (P_1/H_1)^{2/7}}{1 - (P_0/H_0)^{2/7}} \right]^{1/2} \left[1 - \left\{ \frac{1 - (P_0/H_1)^{2/7}}{1 - (P_0/H_0)^{2/7}} \right\}^{1/2} \right]. \quad \dots \quad (1)$$

In their report² this equation is rearranged for computing purposes and numerical tables are provided. More detailed tables for low values of h have been given by Beavan and Manwell¹.

These tables have been used for calculating the C_D' values used for preparing Figs. 1 and 2, except for the following limiting cases.

When $M = 0$ (i.e., $P_0/H_0 = 1$), the expression reduces to the familiar Jones' equation

$$C_D' = 2\sqrt{(1 - h - \phi)[1 - \sqrt{(1 - h)}]}. \quad \dots \quad (2)$$

When $h = 0$ (i.e., $H_1 = H_0$) equation (1) becomes

$$\frac{C_D'}{h} = \frac{2}{7} (P_1/P_0)^{5/7} \left[\frac{1 - (P_1/H_0)^{2/7}}{1 - (P_0/H_0)^{2/7}} \right]^{1/2} \left[\frac{1 - P_0/H_0}{(H_0/P_0)^{2/7} - 1} \right]. \quad \dots \quad (3)$$

For values of h greater than 0.9 the following alternative form of (1) has been found more convenient:—

$$C_D' = 2 \left[\frac{P_1 a_2 M_1}{P_0 a_1 M_2} \right] \left[\frac{M_2}{M_0} \left(\frac{a_0}{a_2} - \frac{M_2}{M_0} \right) \right] \dots \dots \dots (4)$$

where

$$\left(\frac{H_1}{H_0} \right)^{2/7} = \frac{1 + \frac{1}{5}M_2^2}{1 + \frac{1}{5}M_0^2} = \left(\frac{a_0}{a_2} \right)^2 \dots \dots \dots (5)$$

and

$$\left(\frac{P_1}{P_0} \right)^{2/7} = \frac{1 + \frac{1}{5}M_2^2}{1 + \frac{1}{5}M_1^2} = \left(\frac{a_1}{a_2} \right)^2 \dots \dots \dots (6)$$

Equations (4), (5) and (6) do not give C_D' explicitly in terms of h and ϕ , but for any assumed values of M_1 and M_2 the values of C_D' , h , and ϕ may be computed without difficulty, using Table 1 of Ref. 2.

When $p = 0$, $P_1/P_0 = 1$, and the first term in square brackets in equation (4) is unity. When $p = 1 - h$, $P_1/H_1 = 1$, and equation (1) shows that $C_D' = 0$.

In equations (1) to (6) the following numerical coefficients and indices are functions of γ , which has been taken as 1.40:

$$\frac{2}{7} = \frac{\gamma - 1}{\gamma} \qquad \frac{5}{7} = \frac{1}{\gamma} \qquad \frac{1}{5} = \frac{\gamma - 1}{2} .$$

APPENDIX II

Construction of Charts

Fig. 1 is based on the empirical fact that the effect of the static pressure p may be represented very closely by the equation

$$C_D'/h = f_1(M, h) + p(1 + p)f_2(M, h) \dots \dots \dots (7)$$

It may be shown by elementary analysis that Fig. 1 represents an equation of this type, if the C_D'/h scale is uniformly graduated and distances on the p scale are proportional to $p(1 + p)$. Co-ordinates of points on the M, h grid can be calculated from f_1 and f_2 .

Since f_1 and f_2 are empirical functions, they were obtained for each pair of values of M and h by calculating C_D'/h for $p = 0$ and $p = 0.1$, and substituting in equation (7) to give two simultaneous equations for determining f_1 and f_2 . A set of key points on the grid was calculated in this way, using the values of C_D'/h in Table 1, and intermediate points were obtained by graphical interpolation. The fact that the lines of constant h are straight and concurrent appears to be purely fortuitous.

This method of construction ensures that the chart is exact (within the limits of graphical accuracy) at $p = 0$ and $p = 0.1$, and the empirical formula (7) is, in effect, used only to interpolate for other values of p . The error due to this approximation is in fact negligible between $p = -0.05$ and $+0.15$. The p scale as drawn includes only positive values of p , but may be extended to $p = -0.05$. (The graduation for $-p$ is $(1 - p)/(1 + p)$ times as far below zero as the graduation for p is above zero.)

For Fig. 2 a slightly different method of construction was used. When $M = 0$ the equation for C_D' is

$$C_D' = 2\sqrt{(1 - h - p)[1 - \sqrt{(1 - h)}]} \dots \dots \dots (8)$$

which may be written

$$\frac{1}{4}(C_D')^2 = (1 - h)[1 - \sqrt{(1 - h)}]^2 - p[1 - \sqrt{(1 - h)}]^2 \dots \dots \dots (9)$$

which is an equation of the same type as (7). In Fig. 2 the p scale is uniformly graduated, and distances on the C_D' scale are proportional to $(C_D')^2$; co-ordinates of points on the $M = 0$ line can be calculated without difficulty. This line represents equation (8) exactly over the whole range. For other values of M the same procedure was adopted as for Fig. 1, that is to say the grid is exact for $p = 0$ and $p = 0.1$ (or in the case of values of h greater than 0.9, for $p = 0$ and $p = 1 - h$).

The values of C_D' used for this chart are given in Table 2.

APPENDIX III

Approximate Equation for C_D'

1. *Description.*—A simple analytical expression for C_D' can be used for investigating the effect of the different variables, for special problems such as the F factor discussed in section 4.2, or for preparing tables or curves of C_D' in a form convenient for any particular application.

The following approximate equation gives C_D' to an accuracy of ± 0.2 per cent over the range $M = 0$ to 0.9 , $h = 0$ to 0.6 , $p = 0$ to 0.1 , and with an accuracy of 1 per cent up to $p = 0.2$.

$$C_D' = A_0 h - A_1 h^2 - A_2 \xi - p(1 + p)[B_0 h + \zeta] \quad \dots \quad \dots \quad \dots \quad (10)$$

where A_0, A_1, A_2, B_0 , are functions of M only

ξ and ζ are functions of h only.

Values of these functions are given in Tables 3 and 4; intermediate values may be obtained by plotting curves.

2. *Derivation.*— A_0 and A_1 are the first two coefficients in the expansion of C_D' in terms of h at $p = 0$, and are given by

$$A_0 = 2(1 - P_0/H_0)/\gamma M^2 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (11)$$

$$A_1 = \frac{1}{4}A_0^2(1 - M^2) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (12)$$

ξ is the sum of the higher powers of h at $M = 0$, taking $A_2 = 1$, and is given by

$$\xi = h - \frac{1}{4}h^2 - 2\sqrt{(1 - h)}[1 - \sqrt{(1 - h)}] = \frac{1}{8}h^3 + \frac{5}{64}h^4 + \dots \quad (13)$$

A_2 is a factor to give the sum of these higher powers when M is not zero, and is found empirically to be nearly independent of h .

In the pressure term the factor $p(1 + p)$ is used to give a better approximation than could be obtained with the p term only, but without the complication of additional terms in p^2 . Since the pressure term is a very small fraction of C_D' when p is small, the greatest accuracy in its coefficient is not required for the smallest values of p . Accordingly B_0 and ζ are chosen to give the best approximation to the pressure correction term when $p = 0.1$ for the range $h = 0$ to 0.6 . These functions are both empirical.

TABLE 1

| M | ϕ | Values of C_D'/h | | | |
|------|--------|--------------------|-----------|-----------|-----------|
| | | $h = 0$ | $h = 0.2$ | $h = 0.4$ | $h = 0.6$ |
| 0 | 0 | 1.000 | 0.944 | 0.873 | 0.775 |
| | 0.1 | 0.949 | 0.883 | 0.797 | 0.671 |
| 0.35 | 0 | 0.947 | 0.902 | 0.842 | 0.755 |
| | 0.1 | 0.903 | 0.849 | 0.772 | 0.657 |
| 0.5 | 0 | 0.897 | 0.861 | 0.812 | 0.734 |
| | 0.1 | 0.859 | 0.814 | 0.748 | 0.642 |
| 0.65 | 0 | 0.836 | 0.811 | 0.772 | 0.708 |
| | 0.1 | 0.807 | 0.773 | 0.717 | 0.623 |
| 0.8 | 0 | 0.768 | 0.752 | 0.726 | 0.675 |
| | 0.1 | 0.749 | 0.723 | 0.681 | 0.601 |
| 0.9 | 0 | 0.721 | 0.712 | 0.693 | 0.651 |
| | 0.1 | 0.709 | 0.691 | 0.656 | 0.585 |

TABLE 2

| M | ϕ | Values of C_D' | | | |
|------|--------|------------------|-----------|-----------|-----------|
| | | $h = 0.6$ | $h = 0.7$ | $h = 0.8$ | $h = 0.9$ |
| 0 | 0 | 0.465 | 0.495 | 0.494 | 0.432 |
| | 0.1 | 0.403 | 0.405 | 0.350 | 0 |
| 0.35 | 0 | 0.453 | 0.484 | 0.487 | 0.428 |
| | 0.1 | 0.394 | 0.397 | 0.346 | 0 |
| 0.5 | 0 | 0.440 | 0.474 | 0.479 | 0.425 |
| | 0.1 | 0.385 | 0.391 | 0.342 | 0 |
| 0.65 | 0 | 0.425 | 0.461 | 0.469 | 0.420 |
| | 0.1 | 0.374 | 0.383 | 0.337 | 0 |
| 0.8 | 0 | 0.405 | 0.444 | 0.456 | 0.412 |
| | 0.1 | 0.361 | 0.372 | 0.331 | 0 |
| 0.9 | 0 | 0.391 | 0.431 | 0.446 | 0.408 |
| | 0.1 | 0.351 | 0.365 | 0.327 | 0 |

TABLE 3

Coefficients for Equation (10)

| M | A_0 | A_1 | A_2 | B_0 |
|-----|--------|--------|-------|-------|
| 0 | 1.0000 | 0.2500 | 1.000 | 0.455 |
| 0.1 | 0.9955 | 0.2453 | 0.999 | 0.450 |
| 0.2 | 0.9822 | 0.2315 | 0.995 | 0.435 |
| 0.3 | 0.9608 | 0.2100 | 0.987 | 0.409 |
| 0.4 | 0.9320 | 0.1824 | 0.971 | 0.375 |
| 0.5 | 0.8970 | 0.1509 | 0.950 | 0.334 |
| 0.6 | 0.8571 | 0.1175 | 0.915 | 0.285 |
| 0.7 | 0.8136 | 0.0845 | 0.863 | 0.231 |
| 0.8 | 0.7678 | 0.0530 | 0.795 | 0.172 |
| 0.9 | 0.7209 | 0.0247 | 0.705 | 0.107 |

TABLE 4

Functions for Equation (10)

| h | ξ | ζ |
|------|---------|---------|
| 0 | 0 | 0 |
| 0.05 | 0.00001 | 0.0010 |
| 0.10 | 0.00013 | 0.0044 |
| 0.15 | 0.00046 | 0.0103 |
| 0.20 | 0.00113 | 0.0193 |
| 0.25 | 0.00226 | 0.0318 |
| 0.30 | 0.00433 | 0.0484 |
| 0.4 | 0.0108 | 0.097 |
| 0.5 | 0.0233 | 0.176 |
| 0.6 | 0.0451 | 0.300 |



THIS CHART GIVES EITHER
 $\frac{C'_D}{h}$ IN TERMS OF M, h, p
 OR F IN TERMS OF $M, 0.75\eta, p$
 (UP TO $\eta = 0.6$ ONLY.)

WHERE H = TOTAL HEAD IN WAKE
 P = STATIC PRESSURE IN WAKE
 H_0, P_0 ARE FREE STREAM VALUES
 η = PEAK VALUE OF h

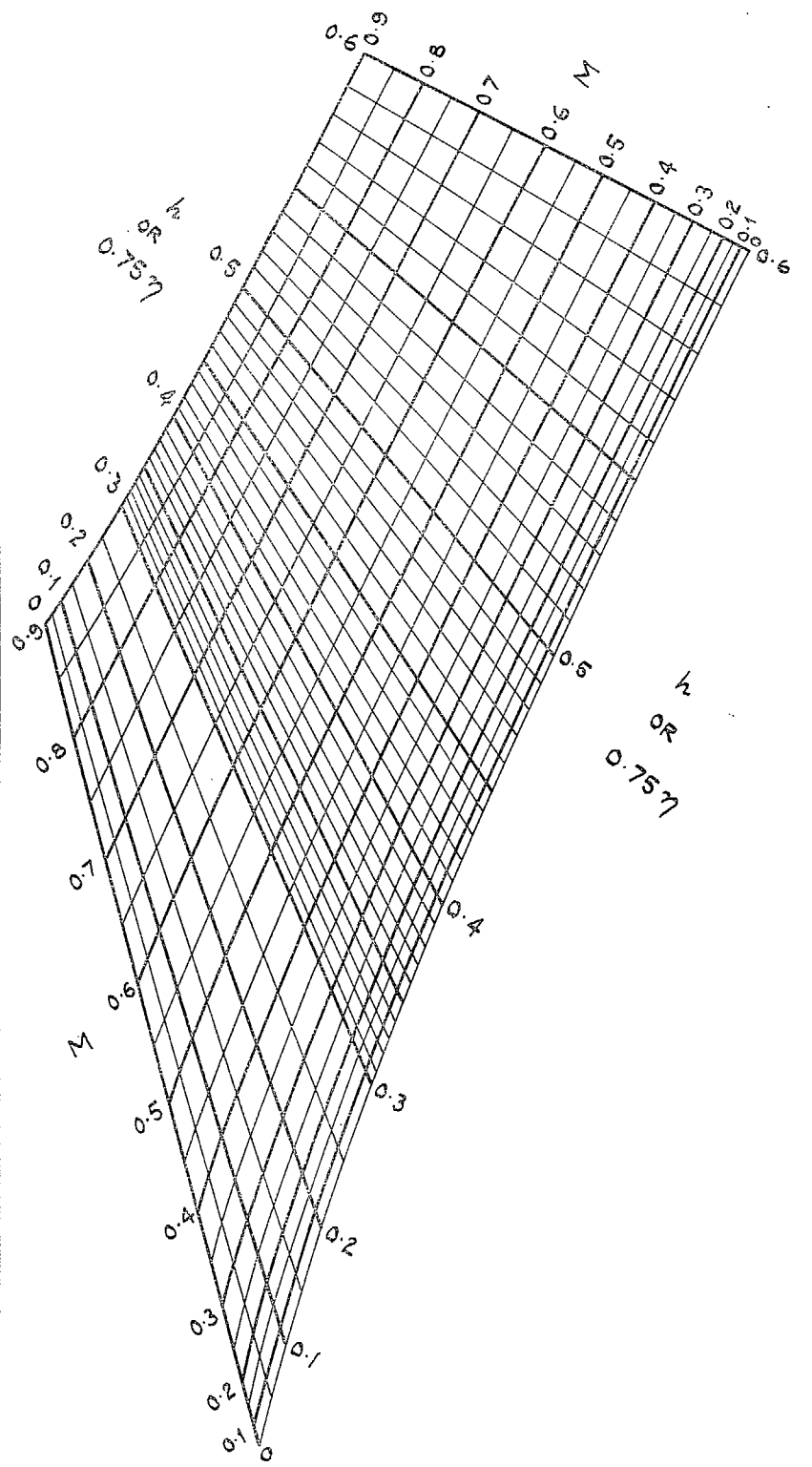
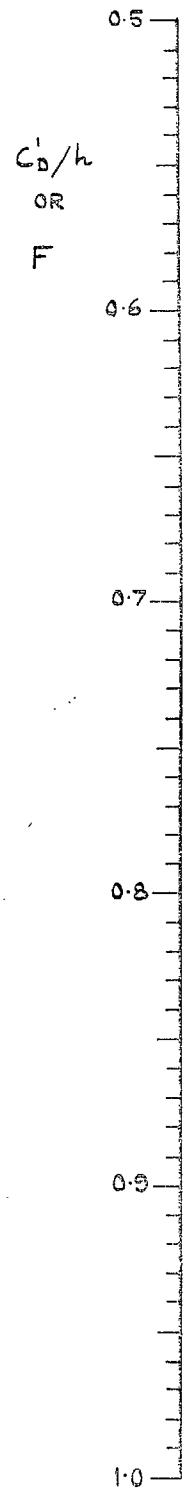
$$h = \frac{H_0 - H}{H_0 - P_0}$$

$$p = \frac{P - P_0}{H_0 - P_0}$$

$$C_D = \int C'_D d \frac{r}{r_0}$$

OR $C_D = F \int h d \frac{r}{r_0}$ (APPROX)

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THIS CHART GIVES C_D' IN TERMS OF M, h, p .

WHERE H = TOTAL HEAD.

P = STATIC PRESSURE.

H_0, P_0 ARE FREE STREAM VALUES.

$$h = \frac{H_0 - H}{H_0 - P_0}$$

$$p = \frac{P - P_0}{H_0 - P_0}$$

$$C_D = \int C_D' d\frac{y}{c}$$

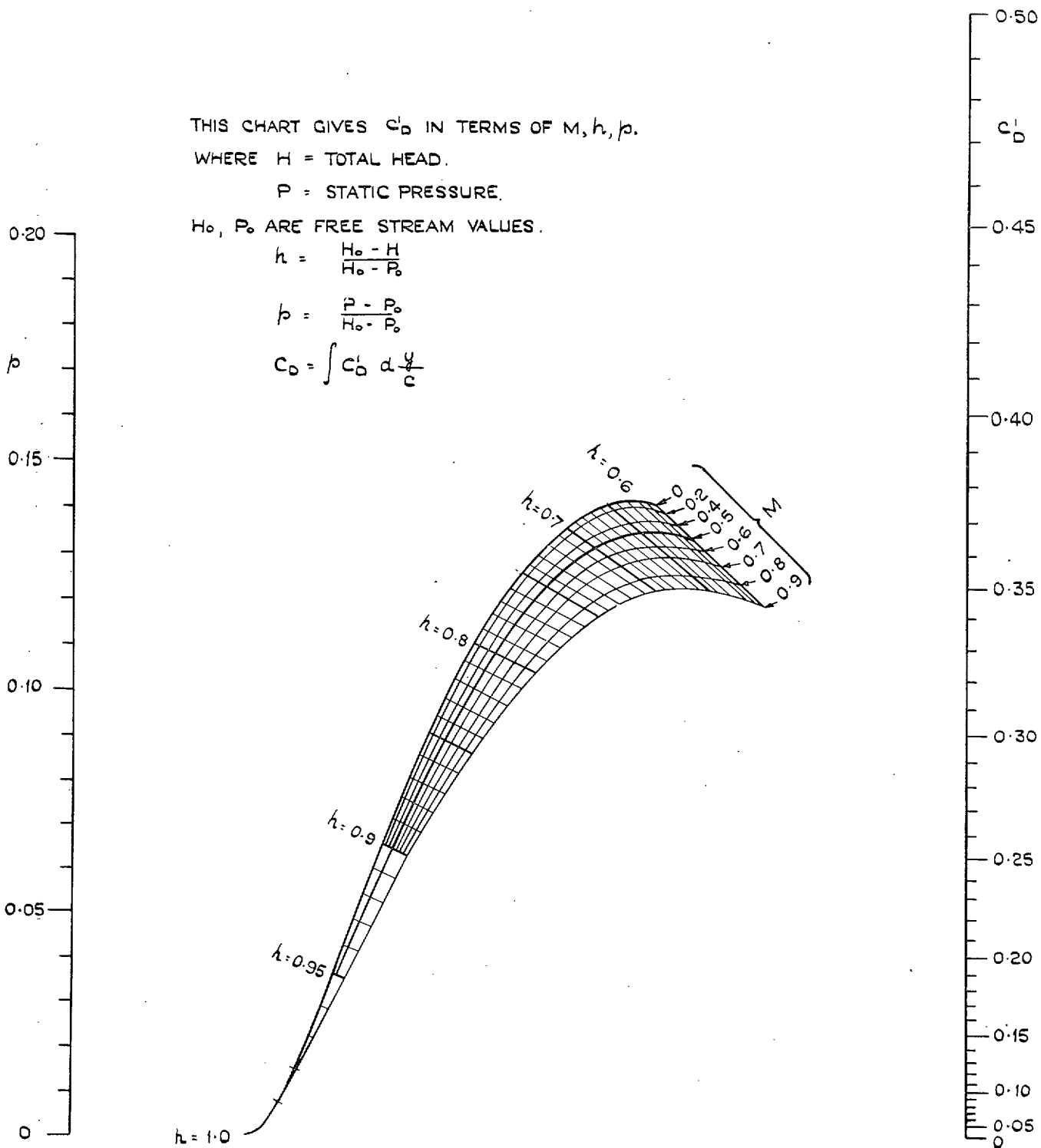


FIG. 2. Chart for C_D' .

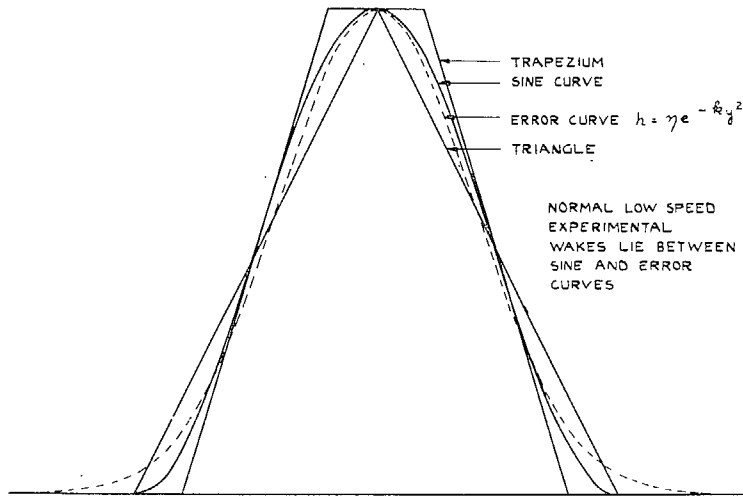


FIG. 3a.

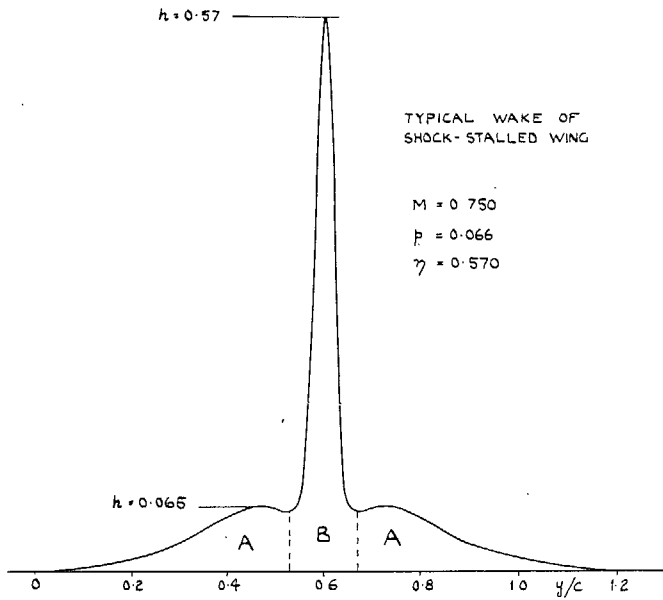
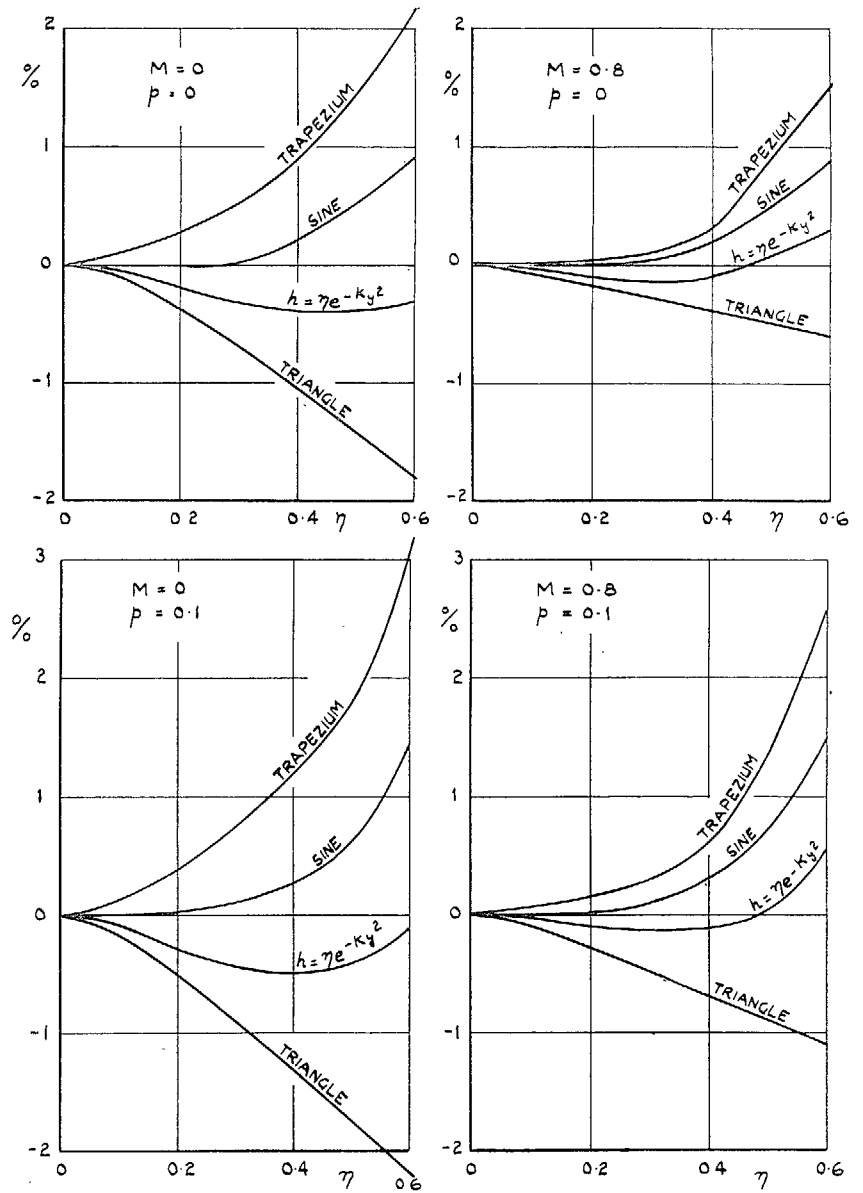


FIG. 3b.

FIGS. 3a and 3b. Wake shapes.



THESE CURVES GIVE THE ERROR IN C_D PRODUCED BY TAKING $F = C_D/h$ AT $h = 0.75\eta$

FIG. 4. Accuracy of integrating factor F .

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