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The Reduction to Standard Conditions of Take-off Measurements on Turbo-jet Aircraft

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Summary.—A reduction method intended for routine use is derived whereby the take-off distance required for a turbo-jet aircraft to clear a 50-ft screen under a specified set of standard conditions of air temperature and pressure, wind speed, aircraft weight and engine speed can be deduced from the distance measured in an arbitrary set of conditions. The method is basically similar to that used for piston-engined aircraft and the only information required in addition to that which can be observed is a numerical constant for the engine type. The method is shown to be not inconsistent with available experimental data.

1. *Introduction.*—To compare the take-off performance of different aircraft it is necessary to obtain a measure of the standard take-off distance, which is the distance from rest to reach a height of 50 ft above the runway under specified standard conditions. Since test conditions will very rarely be standard this distance must be deduced from one or more measurements made when not all the conditions are standard.

The standard distance can be obtained in two different ways. One may either make a number of measurements covering a range of values of each of the variables and deduce the standard distance by interpolation, or make only one measurement under an arbitrary set of conditions and make allowance for the departure from the standard.

An example of the first method applied to the take-off of a turbo-jet aircraft is the 'non-dimensional' method[1]. The disadvantages of this method are that many tests have to be made and the desired range of variables is not always easy to obtain. The standard distance can however be obtained from the test results alone without recourse to any other information.

A method of the second kind is preferable for routine tests on account of its greater economy of time and effort and is usually used for piston-engined aircraft. Such a method has also been applied to turbo-jet aircraft[2], but the calculations involved require estimates of effective thrust and drag during take-off. Fairly accurate thrust estimates can be obtained but it is undesirable to introduce the estimation of effective drag into the analysis of routine tests.

The method given in this Report has therefore been developed so that the standard distance can be deduced from the test results and the engine characteristics only. The approach is similar in principle to that which has been used for piston-engined aircraft[3].

* A.A.E.E. Report Res. 253, received 6th July, 1951.

2. *Specification of Standard Conditions.*—2.1. *General.*—The standard distance as defined in this report corresponds to standard values of the following parameters:—

(*a*) Runway gradient
(*b*) Wind velocity
(*c*) Atmospheric temperature and pressure
(*d*) Aircraft weight
(*e*) Engine speed.

The standard values are usually chosen to represent either the average or extreme operating conditions of the aircraft. In general:—

(*a*) The standard runway gradient is zero.
(*b*) The effect of a cross-wind component* is ignored and the take-off distance corrected either to zero head-wind or (for carrier take-offs) to a head-wind of 30 knots.
(*c*) Temperature and pressure correspond to a chosen height in a standard atmosphere (*e.g.*, I.C.A.N. atmosphere, sea level).

(*d*) and (*e*) depend on the aircraft and its engine.

The corrected distance is appropriate to the runway surface used for the tests.

The effect of humidity on the take-off distance of turbo-jet aircraft has not yet been investigated.

2.2. *Take-off Technique.*—Up to the present no correction method has been found which satisfactorily allows for the variations in piloting technique that appears to cause the large scatter in measured distances. In this report no attempt is made to correct the distance to a standard technique nor to a standard unstick speed and the corrected distance must be described as being appropriate to the particular technique used and a particular unstick speed. The only discrimination made is between a take-off in which the aircraft is accelerated to a safety speed before climbing and one in which an attempt is made to achieve the shortest practicable take-off distance.

3. *Outline of Method.*—3.1. *Assumptions.*—The accuracy of measured take-off distances is rarely better than ± 5 per cent and over-refinement of the correction method has therefore been avoided by introducing the following assumptions.

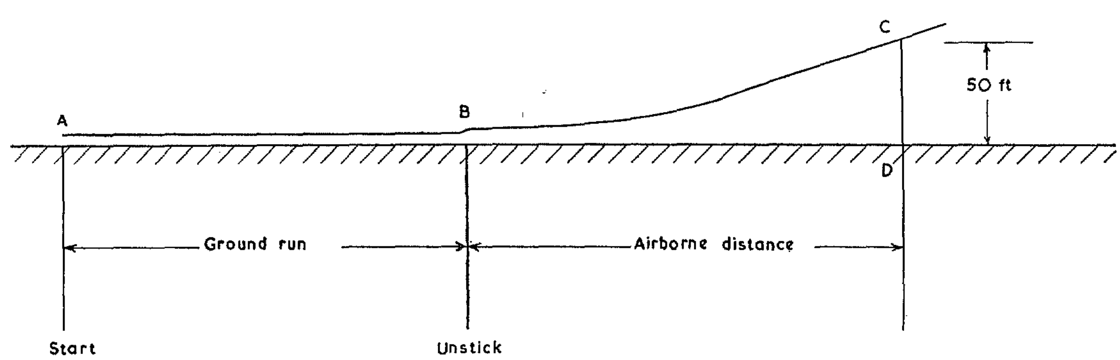

FIG. 1

* Take-off tests are not usually made in cross-winds greater than 10 knots.

(*a*) The take-off path ABC lies in a vertical plane

(*b*) The take-off is divided into two stages—the ground run AB and the airborne distance BD. Lift, drag and friction coefficients are constant during the ground run

(*c*) Engine thrust is either constant or varies linearly with the square of the equivalent air speed

(*d*) The head-wind is constant throughout the take-off

(*e*) Differences between observed and standard values of engine thrust, air density and aircraft weight are small

(*f*) The mean angle of climb during BC is small and proportional changes in distance measured along BC and along BD are equal

(*g*) The climbing speed is near the minimum drag speed.

3.2. *Correction to Zero Headwind and Zero Runway Slope.*—These corrections are made first, using methods established for piston-engined aircraft.

3.3. *Corrections to Standard Air Density, Engine Thrust and Aircraft Weight.*—These corrections are made by considering the energy equations for a take-off from level ground in still air.

For the ground run, the energy equation can be written as

$$S_g = \frac{W}{2g\sigma}\int_0^{V_{gi}} \frac{d(V_i^2)}{F - D - \mu(W - L)}, \qquad \text{(1)}$$

and for the airborne path

$$\int_{S_g}^{S_g + S_A} (F - D)\, dS = W\left[\frac{V_{Ai}^2 - V_{gi}^2}{2g\sigma} + 50\right], \qquad \text{(2)}$$

where W is the aircraft weight

σ	air density
V_i	equivalent air speed
V_{gi}	equivalent air speed at unstick
V_{Ai}	equivalent air speed at 50 ft
μ	coefficient of friction between aircraft and runway
F	total net engine thrust
D	total aerodynamic drag
L	total lift
S	distance measured from the start of the run along the take-off path ABC
S_g	ground run } *see* Fig. 1 above
S_A	airborne distance } *see* Fig. 1 above

From equation (1) we can deduce by differentiation a relation of the form

$$dS_g = \frac{\partial S_g}{\partial F} dF + \frac{\partial S_g}{\partial \sigma} d\sigma + \frac{\partial S_g}{\partial W} dW .$$

If then the ground run S_g is measured when the thrust is F, the correction ΔS_g to be added to S_g so as to give the ground run when the thrust has a standard value F_s is taken to be

$$\Delta S_g = -\frac{\partial S_g}{\partial F} . \Delta F \quad \text{where } \Delta F = F - F_s .$$

Similar expressions apply to the variables σ and W. Equation (2) can be treated in a like manner.

The details of the correction procedure which follows from this analysis are given in section 4 for the ground run and section 5 for the airborne distance.

4. *Correction of the Ground Run.*—4.1. *Correction to Zero Head-wind and Zero Runway Gradient.*—The assumptions under (*b*) and (*c*) of section 3.1 imply that the acceleration will fall off from the starting value linearly with the square of the true air speed. Hence it follows[3] that

$$\frac{S_{gw}}{S_{g0}} = 1 - \frac{\log\left[1 - r\left(\frac{w}{V_g}\right)^2\right] + \frac{w}{V_g}\sqrt{r}\log\left[\frac{1-\sqrt{r}}{1+\sqrt{r}} . \frac{1+(w/V_g)\sqrt{r}}{1-(w/V_g)\sqrt{r}}\right]}{\log(1-r)} \quad .. \quad (3)$$

where S_{g0} is the ground run in zero headwind

S_{gw}	ground run in headwind w
w	component of wind speed against take-off direction
V_g	true air speed at unstick (= ground speed + w)
r	$1 - \frac{\text{final acceleration}}{\text{initial acceleration}}$

Expanding the right-hand side of equation (3) in powers of r and w/V_g shows that, as r tends to zero, S_{gw}/S_{g0} tends to $(1 - w/V_g)^2$ and deviates from this value as r increases; the deviation increases as w/V_g increases. The magnitude of r for present-day jet aircraft is of the order of 0·2 and w/V_g is not likely to exceed 0·4.

If $r = 0{\cdot}2$ and $w/V_g = 0{\cdot}4$ then

$$\frac{S_{gw}}{S_{g0}} \Big/ \left(1 - \frac{w}{V_g}\right)^2 = 1{\cdot}013$$

and, if r is less than, say, 0·2, it is sufficiently accurate to use instead of equation (3) the simple expression

$$\frac{S_{gw}}{S_{g0}} = \left(1 - \frac{w}{V_g}\right)^2 . \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (4)$$

The value of r can be obtained from a speed-time curve for the ground run and if necessary S_{gw}/S_{g0} either calculated from (3) or taken from Fig. 1 of Ref. 3.

An uphill runway slope ϕ decreases the mean acceleration and increases the ground run beyond that required on level ground by the factor[3]

$$1 + \frac{\sin\phi}{(V_g - w)^2/2gS_{g1}} . \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (5)$$

Combining the factors for wind speed and runway gradient from (4) and (5) gives to the same order of accuracy

$$S_g = S_{g1}\left[\frac{1}{\left(1 - \frac{w}{V_g}\right)^2 + \frac{2gS_{g1}\sin\phi}{V_g^2(1 - w/V_g)^2}}\right]$$

where S_{g1} is the ground run measured in head-wind w, with uphill slope $\sin\phi$ of take-off path,

and S_g ground run corrected to zero head-wind and zero gradient of take-off path.

4.2. *Correction to Standard Thrust.*—It is shown in Appendix II that with the relevant assumptions of section 3 we have, to a very close approximation over a wide range of r

$$F_0\frac{\partial S_g}{\partial F_0} = -2g\sigma\frac{F_0}{W}\left(\frac{S_g}{V_{gi}}\right)^2\left\{1 - \left(\frac{1}{2} + \frac{r}{6}\right)\frac{\partial}{\partial F_0}(F_0 - F_g)\right\}$$

$$= -2g\frac{F_0}{W}\left(\frac{S_g}{V_g}\right)^2\left\{1 - \left(\frac{1}{2} + \frac{r}{6}\right)\frac{\partial}{\partial F_0}(F_0 - F_g)\right\} \qquad .. \qquad .. \qquad (6)$$

where F_0 is the static thrust

F_g net thrust at end of ground run.

For most jet aircraft r will not exceed about 0·2 and $\partial(F_0 - F_g)/\partial F_0$ will be less than 0·1. In such cases we may, subject to an error not exceeding about 6 per cent of the correction, omit the second term in the bracket and write

$$F_0\frac{\partial S_g}{\partial F_0} = -2g\frac{F_0}{W}\left(\frac{S_g}{V_g}\right)^2. \qquad .. \qquad .. \qquad .. \qquad .. \qquad (7)$$

This simple form is very convenient because in using it one need not know the mean acceleration, the coefficient of rolling friction μ, or the value of r. Writing F_{0s} for the static thrust under standard conditions, *i.e.*, at standard* air temperature and pressure and engine speed, the increment ΔS_g to be added to S_g is

$$\Delta S_g = A\frac{\Delta F_0}{F_{0s}}$$

where

$$A = 2g\frac{F_{0s}}{W}\left(\frac{S_g}{V_g}\right)^2.$$

It may be noted that F_0 is, strictly, the intercept at zero speed of the best rectilinear approximation to the curve of thrust against V^2 and may not be therefore equal to static thrust. This distinction will, however, in general be ignored when convenient.

The standard static thrust F_{0s} can be obtained from one of the following sources, in order of preference:—

(*a*) thrust measurements during take-offs, *e.g.*, by pitot-pressure observations in the jet-pipe (such measurements may give data for a curve of F against V^2 and so permit a more correct estimate of F_0 to be made)

* The error would be of second order if instead of F_{0s} the measured thrust were used.

(*b*) thrust measurements on the aircraft at rest

(*c*) manufacturer's estimates of the thrust of the engine installed in the aircraft

(*d*) manufacturer's estimates of the thrust of the bare engine.

It remains to express the quantity $\Delta F_0/F_{0s}$, which cannot easily be measured directly, in terms of the measurable values of engine speed, air temperature and air pressure which cause the change in thrust.

It is commonly accepted[1] that F/p is a function of the 'non-dimensional' parameters $N/\sqrt{\theta}$ and $V_i/\sqrt{p}$

i.e.,

$$\frac{F}{p} = f_1\left(\frac{N}{\sqrt{\theta}}, \frac{V_i}{\sqrt{p}}\right)$$

where p is atmospheric pressure

θ absolute atmospheric temperature

N engine speed

For the static thrust case the second term is zero, so that

$$\frac{F_0}{p} = f_2\left(\frac{N}{\sqrt{\theta}}\right).$$

Differentiating this equation,

$$\frac{dF_0}{F_0} = \left[\frac{N/\sqrt{\theta}}{F/p}\frac{\partial(F/p)}{\partial(N/\sqrt{\theta})}\right]\frac{dN}{N} - \frac{1}{2}\left[\frac{N/\sqrt{\theta}}{F/p}\frac{\partial(F/p)}{\partial(N/\sqrt{\theta})}\right]\frac{d\theta}{\theta} + \frac{dp}{p}$$

$$= k\left[\frac{dN}{N} - \frac{1}{2}\frac{d\theta}{\theta}\right] + \frac{dp}{p}, \quad \text{say} . \qquad (8)$$

k can be found as the slope, at the standard values of N and θ, of the curve connecting log (F/p) and log $(N/\sqrt{\theta})$. Some values of k have been calculated for current engine types and are given in Table 1 at the end of this report. The rate of change of k with $N/\sqrt{\theta}$ has been found to be negligible over the working range.

Hence

$$\Delta S_g = A\left[k\left(\frac{\Delta N}{N_s} - \frac{1}{2}\frac{\Delta\theta}{\theta_s}\right) + \frac{\Delta p}{p_s}\right]$$

where the suffix s indicates the standard value of the appropriate variable and ΔN, $\Delta\theta$ and Δp are the amounts by which the engine speed, air temperature and air pressure on the test take-off exceed their standard values.

4.3. *Correction to Standard Density.*—From equation (1),

$$\frac{\partial S_g}{\partial\sigma} = -\frac{S_g}{\sigma}$$

and, since

$$\frac{d\sigma}{\sigma} = \frac{dp}{p} - \frac{d\theta}{\theta} \quad \text{(from the Gas Law)},$$

then $$\Delta S_g = S_g\left(\frac{\Delta p}{p_s} - \frac{\Delta \theta}{\theta_s}\right).$$

ΔS_g is to be added to S_g .

4.4. *Correction to Standard Weight.*—The correction to be added to S_g for an excess weight ΔW is

$$\Delta S_g = -[A + S_g]\frac{\Delta W}{W_s}.$$

This result may be obtained by differentiation from equation (1): alternatively, it may be deduced from the relation[1]

$$\frac{S_g}{\theta} = f_3\left(\frac{N}{\sqrt{\theta}}, \frac{W}{p}\right)$$

which is true with the assumption of a constant lift coefficient at unstick which is made here.

After S_g has been corrected to standard values of N and θ ,

$$S_g = f_4\left(\frac{W}{p}\right).$$

Differentiating, $$\frac{dS_g}{S_g} = \left[\frac{W/p}{S}\cdot\frac{\partial S}{\partial(W/p)}\right]\frac{dW}{W} - \left[\frac{W/p}{S}\frac{\partial S}{\partial(W/p)}\right]\frac{dp}{p}.$$

Hence percentage corrections to S_g of equal magnitude and opposite sign should be applied for equal percentage increases in W and p, and the correction for weight variation can be found from the total correction for pressure variation in sections 4.2 and 4.3 above.

4.5. *Unstick Speed for Corrected Ground Run.*—It should be noted that after a weight correction has been applied, the ground run is then appropriate to an equivalent air speed at unstick of

$$V_{gi}' = V_{gi}\sqrt{\left(\frac{W_s}{W_s + \Delta W}\right)} \simeq V_{gi}\left(1 - \frac{1}{2}\frac{\Delta W}{W_s}\right)$$

if the lift coefficient at unstick is assumed to be constant.

5. *Correction of Airborne Distance.*—5.1. *Correction to Zero Head-wind.*—A simple form of corrected distance S_A , which neglects wind gradient, may be written[3]

$$S_A = S_{A1}\left[\frac{1}{1 - \dfrac{2w}{V_g + V_A}}\right]$$

where S_{A1} is the airborne distance in a headwind w.

5.2. *Effect of Different Take-off Techniques.*—The energy equation (2) for the airborne distance can be simplified by introducing mean values F_A and D_A of the thrust and drag such that

$$\int_{S_g}^{S_g + S_A}(F - D)\,dS = (F_A - D_A)S_A.$$

F_A, D_A and W together define a mean angle of steady climb γ_A, and if γ_A is small its value is given by

$$\gamma_A = \frac{F_A - D_A}{W} . \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (9)$$

Equation (2) then becomes

$$S_A = \frac{1}{\gamma_A}\left[\frac{V_{Ai}^2 - V_{gi}^2}{2g\sigma} + 50\right] . \quad .. \quad .. \quad .. \quad .. \quad (10)$$

There are two take-off techniques to be considered at this stage. The take-off may be continued after unstick by accelerating to a previously chosen indicated airspeed V_{Ai} (the take-off safety speed) before climbing, and making the initial climb at this chosen speed. This will be referred to as the safety-speed take-off; it is characterised by V_{Ai} remaining constant during the correction process. Alternatively the aircraft may be put into a climb as soon as possible after unstick in an attempt to reach a height of 50 ft in the shortest practicable (not necessarily the shortest possible) distance. For convenience, this will be referred to as the shortest distance take-off. It has been found that the speed at 50 ft depends on the angle of climb and V_{Ai} will therefore change during the correction process.

For the safety-speed take-off, corrections deduced as for the ground run by considering separately the partial derivatives of equation (10) are given in section 5.3.

For the shortest distance take-off, a semi-empirical relation is introduced to give the way in which V_{Ai} varies under varying conditions,

viz.,
$$\frac{V_{Ai}}{V_{gi}} = 1 + c\gamma_A \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (11)$$

where c is a constant.

An equation of similar form has previously been used[4] in the estimation of take-off distances.

Using the relation (11), equation (10) can be written

$$S_A = \frac{1}{\gamma_A}\left[c\gamma_A(2 + c\gamma_A)\frac{V_{gi}^2}{2g\sigma} + 50\right]$$

or approximately, since γ_A is small and c is of order unity[4],

$$S_A = \frac{cV_{gi}^2}{g\sigma} + \frac{50}{\gamma_A} . \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (12)$$

Corrections for the shortest distance take-off, deduced by considering separately the partial derivatives of equation (12), are given in section 5.4.

5.3. *Safety-speed Take-off. Correction to Standard Thrust, Air Density and Weight.*— 5.3.1. *Correction to standard thrust.*—From (10),

$$\frac{\partial S_A}{\partial F_A} = -\frac{S_A}{\gamma_A}\frac{\partial \gamma_A}{\partial F_A}$$

and from (9),
$$\frac{\partial \gamma_A}{\partial F_A} = \frac{1}{W} .$$

Hence, using also equation (8), the correction to be added to S_A is

$$\Delta S_A = \frac{S_A}{\gamma_A}\frac{F_A}{W_s}\left[k\left(\frac{\Delta N}{N_s} - \frac{1}{2}\frac{\Delta\theta}{\theta_s}\right) + \frac{\Delta p}{p_s}\right]$$

γ_A is to be calculated from equation (10),

i.e.,
$$\gamma_A = \frac{1}{S_A}\left[\frac{V_{Ai}^2 - V_{gi}^2}{2g\sigma} + 50\right].$$

The value of F_A can be taken at the average speed $\frac{1}{2}(V_g + V_A)$.

5.3.2. *Correction to standard density.*—From (10),

$$\frac{\partial S_A}{\partial\sigma} = -\frac{1}{\gamma_A}\frac{V_{Ai}^2 - V_{gi}^2}{2g\sigma}\cdot\frac{1}{\sigma}$$

and the correction to be added is

$$\Delta S_A = \left(S_A - \frac{50}{A}\right)\left(\frac{\Delta p}{p_s} - \frac{\Delta\theta}{\theta_s}\right).$$

5.3.3. *Correction to standard weight.*—From (10), the correction to be added is

$$\Delta S_A = -\frac{1}{\gamma_A}\left[\frac{F_A S_A}{W_s} - \frac{V_{gi}^2}{2g\sigma}\right]\frac{\Delta W}{W_s}.$$

This correction includes the effect of the change in unstick speed (section 4.5).

5.4. *Shortest Distance Take-off. Correction to Standard Thrust, Air Density and Weight.*—The process of deducing the corrections from equation (12) is similar to that used in section 5.3 and leads to the expressions given below.

5.4.1. *Correction to standard thrust.*

Add
$$\Delta S_A = \frac{50}{\gamma_A^2}\frac{F_A}{W_s}\left[k\left(\frac{\Delta N}{N_s} - \frac{1}{2}\frac{\Delta\theta}{\theta_s}\right) + \frac{\Delta p}{p_s}\right].$$

5.4.2. *Correction to standard density.*

Add
$$\Delta S_A = \left(S_A - \frac{50}{\gamma_A}\right)\left(\frac{\Delta p}{p_s} - \frac{\Delta\theta}{\theta_s}\right).$$

5.4.3. *Correction to standard weight.*

Add
$$\Delta S_A = -\left[S_A - \frac{50}{\gamma_A}\left(1 - \frac{1}{\gamma_A}\frac{F_A}{W_s}\right)\right]\frac{\Delta W}{W_s}.$$

5.4.4. *Equivalent air-speed at 50 ft for corrected shortest airborne distance.*—The equivalent air speed to which the corrected shortest distance take-off corresponds can be deduced from the equation

$$V_{Ai} = V_{gi}(1 + c\gamma_A).$$

By differentiation, $$dV_{Ai} = dV_{gi} + V_{Ai}\left(1 - \frac{V_{gi}}{V_{Ai}}\right)\frac{d\gamma_A}{\gamma_A}.$$

Also $$\gamma_A = \frac{F_A - D_A}{W}.$$

Therefore $$\frac{d\gamma_A}{\gamma_A} = \frac{F_A}{W_s\gamma_A}\left[\frac{dF_A}{F_A} - \frac{dW}{W_s}\right]$$

$$= \frac{F_A}{W_s\gamma_A}\Delta_2, \quad \text{say}.$$

Finally, from 4.5 $$dV_{gi} = \frac{V_{gi}}{2}\frac{\Delta W}{W_s}.$$

Hence the E.A.S. for the corrected run may be written

$$V_{Ais} = V_{Ai}\left[1 - \frac{c}{1 + c\gamma_A}\frac{F_A\Delta_2}{W_s}\right] - \frac{V_{gi}}{2}\frac{\Delta W}{W_s}$$

$$= V_{Ai}\left[1 - \frac{\Delta_2}{1 + \gamma_A}\frac{F_A}{W_s}\right] - \frac{V_{gi}}{2}\frac{\Delta W}{W_s}$$

if c is taken to be unity.

6. *Comparison with Experiment.*—The validity of the correction method can be checked by measuring the take-off distance of a particular aircraft a sufficient number of times and over a sufficiently large range of all the variables to enable the partial derivatives to be deduced. These derivatives can then be compared with the values estimated in the correction process. The twelve test results on a *Meteor* 1 aircraft reported in Ref. 1 have been used to draw up a comparison of this type in the following table. The 95 per cent limits of accuracy are quoted for the experimental results.

		$\frac{\theta}{S}\frac{\partial S}{\partial \theta}$	$\frac{N}{S}\frac{\partial S}{\partial N}$	$\frac{p}{S}\frac{\partial S}{\partial p}$ or $-\frac{W}{S}\frac{\partial S}{\partial W}$
Ground run	Experimental	$4 \cdot 1 \pm 0 \cdot 9$	$-6 \cdot 3 \pm 1 \cdot 9$	$-1 \cdot 8 \pm 1 \cdot 5$
	Estimated	$3 \cdot 8$	$-5 \cdot 6$	$-2 \cdot 5$
Airborne path (shortest distance)	Experimental	$1 \cdot 0 \pm 6 \cdot 9$	$0 \pm 13 \cdot 8$	$-2 \cdot 4 \pm 9 \cdot 7$
	Estimated	$2 \cdot 4$	$-3 \cdot 8$	$-1 \cdot 5$

Unpublished results of a series of twelve observations of the ground run of a *Meteor* 3 have also been analysed in the same way to give the following comparison.

		$\frac{\theta}{S}\frac{\partial S}{\partial \theta}$	$\frac{N}{S}\frac{\partial S}{\partial N}$	$\frac{p}{S}\frac{\partial S}{\partial p}$ or $-\frac{W}{S}\frac{\partial S}{\partial W}$
Ground run	Experimental	$3 \cdot 7 \pm 0 \cdot 9$	$-5 \cdot 3 \pm 1 \cdot 8$	$-2 \cdot 8 \pm 0 \cdot 9$
	Estimated	$4 \cdot 2$	$-6 \cdot 4$	$-2 \cdot 7$

Both sets of tests were limited in scope and comparatively few in number. A considerably greater number than twelve take-offs and a wider range of atmospheric conditions would be required to yield a complete and significant check on all aspects of the correction method, but it may be seen that the method is not inconsistent with available data.

7. *Routine Application of Method.*—In Appendix III the correction method is summarised and presented simply and briefly in a form convenient for routine use. The Appendix includes a simple extension to cover the case of an engine cut during the ground run for multi-engined aircraft.

8. *Conclusions.*—A method of reducing single measured take-offs of jet-propelled aircraft to standard conditions has been derived. This method is convenient for routine use and not inconsistent with available experimental data.

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APPENDIX I

LIST OF SYMBOLS

The following symbols are used in the text of the Report. Additional symbols used in Appendix III are defined therein.

D		Total aerodynamic drag
D_A		Mean drag over airborne path
F		Total net engine thrust
F_A		Mean value of F over airborne path
F_0		Static thrust
F_{0s}		Standard value of F_0
F_g		Thrust at unstick
ΔF_0	=	$F_0 - F_{0s}$
L		Total lift
N		Engine speed
N_s		Standard engine speed
ΔN	=	$N - N_s$
S		Distance from start along take-off path
S_A		Airborne distance in zero head-wind
S_{A1}		Measured airborne distance
ΔS_A		Correction to be added to S_A
S_g		Ground run in zero wind and zero runway gradient
S_{g0}		Ground run in zero head-wind
S_{gi}		Measured ground run
S_{gw}		Ground run into head-wind w
ΔS_g		Correction to be added to S_g
V_A		True air speed at 50 ft
V_{Ai}		Equivalent air speed at 50 ft
V_{Ai}'		Value of V_{Ai} corresponding to corrected shortest airborne distance
V_g		True air speed at unstick
V_i		Equivalent air speed
V_{gi}		Equivalent air speed at unstick
V_{gi}'		Value of V_{gi} corresponding to corrected shortest airborne distance
W		Weight of aircraft
W_s		Standard weight of aircraft
ΔW	=	$W - W_s$

LIST OF SYMBOLS—*continued*

A	$=$	$2g\frac{F_{0s}}{W}\left(\frac{S_g}{V_g}\right)^2$
c		Constant in equation relating V_{Ai} and V_{gi}
k		Thrust variation parameter for engine type
p		Atmospheric pressure
p_s		Standard atmospheric pressure
Δp	$=$	$p - p_s$
r	$=$	[1 — (final acceleration)/(initial acceleration)] for ground run
w		Component of wind speed opposing the take-off direction
α_0		Initial acceleration during ground run
α_g		Final acceleration during ground run
$\bar{a}$	$=$	$V_g^2/2S_g$
γ_A		Mean climb angle for airborne path
θ		Absolute air temperature
θ_s		Standard absolute air temperature
$\Delta\theta$	$=$	$\theta - \theta_s$
μ		Coefficient of friction
σ		Relative density of atmosphere
ϕ		Uphill slope of ground run

APPENDIX II

by K. J. Lush, B.Sc., D.I.C.

Variation of Ground Run with Thrust

It is proposed to derive below the relations quoted in equations (6) and (7) of the main text for the rate of variation of ground run with static thrust and to give an indication of the precision of the approximation involved.

With the relevant assumptions of section 3 of the main text the excess thrust term, which when inverted forms the integrand of equation (1), can be written

$$F_0 - \mu W - \{D_g - \mu L_g + F_0 - F_g\}\left(\frac{V_i}{V_{gi}}\right)^2 \quad \ldots \quad \ldots \quad \ldots \quad \text{(A1)}$$

where suffixes '0' and '*g*' denote respectively static conditions and conditions just before the aircraft is pulled off.

Let us now write

$$\alpha_0 = \text{initial acceleration}$$

$$\alpha_g = \text{final acceleration}$$

$$\bar{a} = V_{gi}^2/(2\sigma S_g)$$

and $$r = 1 - \frac{\alpha_g}{\alpha_0}.$$

We may now re-write the excess thrust in the form

$$\frac{W}{g}\alpha_0\left\{1 - r\left(\frac{V_i}{V_{gi}}\right)^2\right\}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(A2)}$$

Substituting in equation (1) of the main text and integrating we then have

$$S_g = \frac{V_{gi}^2}{2\sigma\alpha_0}\frac{1}{r}\log_e\left(\frac{1}{1-r}\right). \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(A3)}$$

Changes in F_0 will affect, in general, both α_0 and r, so if we differentiate (A3) with respect to F_0 we have

$$\frac{F_0}{S_g}\frac{\partial S_g}{\partial F_0} = \frac{\alpha_0}{S_g}\frac{\partial S_g}{\partial \alpha_0}\frac{F_0}{\alpha_0}\frac{\partial \alpha_0}{\partial F_0} + \frac{1}{S_g}\frac{\partial S_g}{\partial r}F_0\frac{\partial r}{\partial F_0}$$

$$= -1 \times \frac{F_0}{F_0 - \mu W} + \frac{1}{S_g}\frac{\partial S_g}{\partial r}F_0\frac{\partial r}{\partial F_0}. \quad \ldots \quad \ldots \quad \text{(A4)}$$

But differentiating (A3) with respect to r we have

$$\frac{\partial S_g}{\partial r} = \frac{V_{gi}^2}{2\sigma\alpha_0}\left[-\frac{1}{r}2\log_e\left(\frac{1}{1-r}\right) + \frac{1}{r}\frac{1}{1-r}\right]$$

i.e.,

$$\frac{1}{S_g}\frac{\partial S_g}{\partial r} = -\frac{1}{r} + \frac{1}{r}\frac{\bar{a}}{\alpha_0}\frac{1}{1-r}$$

$$= +\frac{1}{r}\left\{-1 + \frac{\bar{a}}{\alpha_0}\right\}. \qquad \text{(A5)}$$

But

$$r = \frac{1}{F_0 - \mu W}\{D_g - \mu L_g + F_0 - F_g\}$$

$$\frac{\partial r}{\partial F_0} = \frac{-1}{(F_0 - \mu W)^2}\{D_g - \mu L_g + F_0 - F_g\} + \frac{1}{F_0 - \mu W}\frac{\partial(F_0 - F_g)}{\partial F_0}$$

i.e.,

$$F_0\frac{\partial r}{\partial F_0} = \frac{rF_0}{F_0 - \mu W} + \frac{F_0}{F_0 - \mu W}\frac{\partial(F_0 - F_g)}{\partial F_0}. \qquad \text{(A6)}$$

Substituting in (A4) we have

$$\frac{F_0}{S_g}\frac{\partial S_g}{\partial F_0} = \frac{F_0}{F_0 - \mu W}\left[-1 + \frac{1}{r}\left(-1 + \frac{\bar{a}}{\alpha_g}\right)\left(-r + \frac{\partial(F_0 - F_g)}{\partial F_0}\right)\right]$$

$$= \frac{F_0}{F_0 - \mu W}\left[-\frac{\bar{a}}{\alpha_g} + \frac{\bar{a} - \alpha_g}{r\alpha_g}\frac{\partial(F_0 - F_g)}{\partial F_0}\right]$$

i.e.,

$$\frac{\partial S_g}{\partial F_0} = -\frac{gV_{gi}^2}{2\sigma W}\frac{1}{\alpha_0\alpha_g}\left\{1 - \frac{\bar{a} - \alpha_g}{r\bar{a}}\frac{\partial(F_0 - F_g)}{\partial F_0}\right\}. \qquad \text{(A7)}$$

Approximations.—It would be convenient if we could avoid using α_0 and α_g and write the equation in terms of $\bar{a}$. Let us substitute for α_0 and α_g in terms of $\bar{a}$ and r.

$$\frac{1}{\bar{a}} = \frac{2\sigma S_g}{V_{gi}^2}$$

$$= \frac{1}{r\alpha_0}\log_e\left(\frac{1}{1-r}\right) \quad \text{from equation (A3)}$$

$$= \frac{1}{\alpha_0}\left\{1 + \frac{r}{2} + \frac{r^2}{3} + \ldots\right\}.$$

Hence

$$\frac{\alpha_0\alpha_g}{(\bar{a})^2} = (1-r)\left\{1 + \frac{r}{2} + \frac{r^2}{3} + \ldots\right\}^2$$

$$= (1-r)\left\{1 + r + \frac{11r^2}{12} + \ldots\right\}$$

$$= 1 - \frac{r^2}{12} \quad \text{approximately,}$$

i.e.,

$$< 1 \text{ and } > 0{\cdot}99 \text{ for } |r| < 0{\cdot}35.$$

For most jet aircraft r is about $0 \cdot 2$. A quite negligible error is, therefore, introduced by substituting $(\bar{a})^2$ for $\alpha_0\alpha_g$ in equation (A7) and so obtaining

$$F_0 \frac{\partial S_g}{\partial F_0} = -\frac{g}{2}\frac{V_{gi}^2}{\sigma}\frac{F_0}{W}\left(\frac{2\sigma S_g}{V_{gi}^2}\right)^2\left\{1 - \frac{\bar{a} - \alpha_g}{r\bar{a}}\frac{\partial(F_0 - F_g)}{\partial F_0}\right\}. \quad \ldots \quad \ldots \quad (A8)$$

But

$$\frac{\bar{a} - \alpha_g}{r\bar{a}} = \frac{1}{r} - \frac{1-r}{r}\left\{1 + \frac{r}{2} + \frac{r^2}{3} + \ldots\right\}$$

$$= \frac{1}{2} + \frac{r}{6} \text{ approximately.}$$

It is shown below that $|\partial(F_0 - F_g)/\partial F_0|$ is less than about $0 \cdot 1$. We may therefore write, with sufficient accuracy

$$F_0 \frac{\partial S_g}{\partial F_0} = -2g\sigma\frac{F_0}{W}\left(\frac{S_g}{V_{gi}}\right)^2\left\{1 - \left(\frac{1}{2} + \frac{r}{6}\right)\frac{\partial(F_0 - F_g)}{\partial F_0}\right\}. \quad \ldots \quad \ldots \quad \ldots \quad (A9)$$

The right-hand side of this equation is very insensitive to r and a rough estimate would always suffice. Hence no precise estimation of α_0 (which involves μ) or α_g is required. The equation may be applied with confidence to all but very extreme cases (*e.g.*, for all $|r| < 0 \cdot 5$). The expression $\partial(F_0 - F_g)/\partial F_0$ is, however, an inconvenience and we will therefore consider when it may be omitted.

With a rocket $\partial(F_0 - F_g)/\partial F_0$ would be zero. For the conventional turbine-jet engine let us consider the usual 'non-dimensional' thrust curves, in which F/p is a function of $N/\sqrt{\theta}$ and $V_i/\sqrt{p}$ only. Changes in F_0 will result either from

(*a*) changes in air pressure p

or (*b*) changes in engine speed N or air temperature θ .

With regard to the former, increases in F_0 resulting from changes in p at constant N , θ and, of course, V_i will leave F_0/p unaltered but will decrease $(F_0 - F_g)/p$ slightly because of the associated decrease of $V_i/\sqrt{p}$. This effect is slight and we may, for such changes, expect $\partial(F_0 - F_g)/\partial F_0$ to be sensibly equal to $(F_0 - F_g)/F_0$, which is less than $0 \cdot 1$.

With regard to the latter, increases in F_0 resulting from changes in $N/\sqrt{\theta}$ at constant V_i and p will have a very small effect on $F_0 - F_g$ of either sign.

The above remarks apply to typical current engines. For such engines it may be assumed for all practical purposes that

$$\left|\frac{\partial}{\partial F_0}(F_0 - F_g)\right| < 0 \cdot 1 .$$

In case of doubt a check is easily made and reversion made, if necessary to the more precise form of equation (A9), writing it as

$$F_0 \frac{\Delta S_g}{\Delta F_0} = -2g\frac{F_0}{W}\left(\frac{S_g}{V_g}\right)^2\left\{1 - \left(\frac{1}{2} + \frac{r}{6}\right)\frac{\Delta(F_0 - F_g)}{\Delta F_0}\right\}. \quad \ldots \quad \ldots \quad \ldots \quad (A10)$$

TABLE 1

Values of Thrust Variation Parameter k for some Current Engines

Engine type and Mark number	k
Derwent 5	3·9
Nene 2	3·9
Avon 2	3·7
Avon 3	4·4
Tay	3·7
Goblin 4	3·4
Ghost 5	3·1

k is the slope of the line giving log (F/p) as a function of log $(N/\sqrt{\theta})$.
The values of F/p have been taken from the appropriate engine manufacturers' brochures.

APPENDIX III

Scheme for Routine Reduction of Take-off Tests on Jet-propelled Aircraft

1. *Statement of Standard Conditions.*—The following quantities specify the standard conditions to which each measured take-off is reduced.

(*a*) Standard atmospheric conditions:—

Standard temperature	θ_s deg K.
Standard pressure	p_s Atm

(*b*) Standard conditions for the aircraft and engine:—

Standard take-off weight	W_s lb
Standard engine speed	N_s r.p.m.

2. *Engine Performance Data.*—Obtain the net engine thrust at (θ_s, p_s, N_s), at zero speed and at the forward speed quoted in Table 2 below.

Calculate the parameter k, given by

$$k = \frac{N/\sqrt{\theta}}{F/p} \frac{\partial(F/p)}{\partial(N/\sqrt{\theta})},$$

which is the slope of the curve of log (F/p) against log $(N/\sqrt{\theta})$ when $N = N_s$ and $\theta = \theta_s$. Values of k for some current engine types are listed in Table 1.

3. *Observations.*—The following quantities are measured for each take-off.

(*a*) Atmospheric conditions at runway level:—

Air temperature	θ deg K.
Air pressure	p Atm
Head-wind component	w ft/sec

(*b*) Aircraft and engine conditions:—

Take-off weight	W lb
Engine speed	N r.p.m.

(*c*) Distances and speeds:—

Ground run from start to unstick	S_g ft
Airborne distance from unstick to 50 ft above the unstick point	S_A ft
Ground speed at unstick	v_g ft/sec
Ground speed at 50 ft	v_A ft/sec

(*d*) Uphill gradient $\sin\phi$ of the ground run.

4. *Calculation of Corrected Distances in Zero Head-wind.*—From the observations obtain the following quantities:—

$$\Delta_1 = \frac{p - p_s}{p_s} - \frac{\theta - \theta_s}{\theta_s} - \frac{W - W_s}{W_s}$$

$$\Delta_2 = k\left(\frac{N - N_s}{N_s} - \frac{1}{2}\frac{\theta - \theta_s}{\theta_s}\right) + \left(\frac{p - p_s}{p_s} - \frac{W - W_s}{W_s}\right)$$

$$V_g = v_g + w$$

$$V_A = v_A + w$$

$$V_A^2 - V_g^2 = (V_A - V_g)(V_A + V_g)\ .$$

The corrected distances for each part of the run is given by the expression

$$(1 + \Delta_1)S + B\Delta_2 - C\Delta_1$$

where S , B and C take the appropriate values given in Table 2 below.

The corrected distance corresponds to the following equivalent air-speeds.

At unstick, $V_{gi}\left(1 - \frac{1}{2}\frac{\Delta W}{W_s}\right)$

At 50 ft, $\begin{cases} V_{Ai} & \text{for a safety-speed take-off} \\ V_{Ai}\left[1 - \frac{\Delta_2}{1 + \gamma_A}\frac{F}{W_s}\right] - \frac{V_{gi}}{2}\frac{\Delta W}{W_s} & \text{for a shortest distance take-off.} \end{cases}$

F in the last formula is the 'climb-away' thrust.

TABLE 2

	Ground run	Airborne distance: Safety-speed take-off	Airborne distance: Shortest distance take-off
A	$\left(1 - \frac{w}{V_g}\right)^2 + \frac{\sin\phi}{V_g^2(1 - w/V_g)^2/2gS_g}$	$1 - \frac{w}{\frac{1}{2}(V_g + V_A)}$	$1 - \frac{w}{\frac{1}{2}(V_g + V_A)}$
S	$\frac{S_g}{A}$	$\frac{S_A}{A}$	$\frac{S_A}{A}$
γ_A	—	$\frac{1}{S}\left[\frac{V_A^2 - V_g^2}{2g} + 50\right]$	$\frac{1}{S}\left[\frac{V_A^2 - V_g^2}{2g} + 50\right]$
Speed for mean thrust F	Zero	$\frac{1}{2}(V_g + V_A)$	$\frac{1}{2}(V_g + V_A)$
B	$2g\frac{F}{W}\left(\frac{S}{V_g}\right)^2$	$\frac{S}{\gamma_A}\frac{F}{W}$	$\frac{50}{\gamma_A^2}\frac{F}{W}$
C	0	$\frac{50}{\gamma_A}$	$\frac{50}{\gamma_A}$

Note that F is the total net thrust for all the engines of the aircraft.

5. *Ground Run in Arbitrary Headwind.*—To find the ground run in an arbitrary head-wind w_s, multiply the corrected ground run by

$$\left(1 - \frac{w_s}{V_g}\right)^2$$

where V_g must be interpreted as being the true unstick speed for the corrected run.

6. *Engine Cut Take-off Tests.*—If one engine of a multi-engined aircraft is cut during the ground run, modify the procedure as follows.

6.1. *Observations.*—(*a*) Distances and speeds

Ground run from start to engine cut	S_1 ft
Ground run from engine cut to unstick	S_2 ft
Ground speed at engine cut	v_1 ft/sec
Ground speed at unstick	v_g ft/sec

(*b*) Uphill gradient $\sin\phi_1$ for S_1

$\sin\phi_2$ for S_2.

6.2. *Calculation of Corrected Distance.*—From the observations obtain

$$V_1 = v_1 + w$$

$$V_g = v_g + w$$

$$V_g^2 - V_1^2 = (V_g - V_1)(V_g + V_1).$$

The corrected distance for each part of the run is given by the expression

$$(1 + \Delta_1)S + B\Delta_2$$

where S and B take the appropriate values from the following table. Add the two corrected distances to give the corrected ground run.

	Ground run	
	Before engine cut	After engine cut
A	$\left(1 - \frac{w}{V_1}\right)^2 + \frac{\sin \phi_1}{V_1^2(1 - w/V_1)^2/2gS_1}$	$\left(1 - \frac{w}{\frac{1}{2}(V_g + V_1)}\right) + \frac{\sin \phi_2}{(V_g - V_1)\left(\frac{V_g + V_1}{2} - w\right)/gS_2}$
S	$\frac{S_1}{A}$	$\frac{S_2}{A}$
Speed for mean thrust	$0{\cdot}7V_1 + 0{\cdot}3w$	$0{\cdot}5(V_1 + V_g)$
B	$2g\frac{F_1}{W}\left(\frac{S_1}{V_1}\right)^2$	$2g\frac{F_2}{W}\frac{S_2^2}{V_g^2 - V_1^2}$

F_1 is the total net engine thrust before engine cut

F_2 is the total net engine thrust after engine cut.

The corrected distance corresponds to an equivalent air speed at unstick of $V_{gi}[1 - \frac{1}{2}(\Delta W/W_s)]$.

7. *Presentation of Results.*—The reduced results of a series of measured ground runs are conveniently presented in the form of a plot of ground run against the square of the corresponding unstick speed. These two quantities would be proportional one to the other if the acceleration during the ground runs were constant. Similarly, airborne distance can be plotted against the quantity

$$\frac{V_{Ai}^2 - V_{gi}^2}{2g} + 50\,,$$

where the speeds correspond to the reduced distances.

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