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Some Notes on Shock-Wave Boundary-Layer Interactions, and on the Effect of Suction on the Separation of Laminar Boundary Layers

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SUMMARY

A brief review is given of a theory, due to Crocco and Lees (Ref. 1), which can be used for a wide range of laminar and turbulent boundary layer problems. The results of applying this theory to the problem of shock wave boundary layer interactions are described from Ref. 3.

The same method is then applied to the problem of estimating the extent to which critical Mach number effects on a wing can be delayed by boundary layer suction. In this report, a number of crude approximations have been made to find the order of magnitude of this effect. It is found that a sucked boundary layer may require almost three times as big a disturbance as an unsucked layer, in order to cause separation. As this preliminary result is quite encouraging, some suggestions are made for more accurate calculations.

1. Outline of Theory

The mixing theory of Crocco and Lees (Refs. 1 and 2) is very suitable for calculating the interaction between shock waves and laminar or turbulent boundary layers, because it is able to take into account boundary layer separation, and compressibility effects.

Basic assumptions of the theory are:

(i) constant pressure across the boundary layer.

(ii) zero heat transfer to a solid surface.

(iii) that the boundary layer velocity profiles form a one-parameter family. A quantity $\mathcal{K}$, defined as the ratio of mean velocity in the boundary layer to free stream velocity, is chosen as the profile parameter. It is a measure of the "fullness" of the velocity profile. The properties of any boundary layer are then expressed in terms of three functions of $\mathcal{K}$, which can be shown to be independent of Mach number and Reynolds number.

Using/

Using these functions, the von Kármán momentum integral may be evaluated, and the growth of any boundary layer may be calculated. The effect of the boundary layer on the free stream is also given. In Ref. 4 the theory has been extended to allow for heat transfer between the fluid and a solid surface.

2. Definitions

If δ is the boundary layer thickness

m = mass flux in boundary layer = $\int_0^\delta \rho u \, dy$

I = momentum flux in boundary layer = $\int_0^\delta \rho u^2 \, dy$

P = pressure = constant across layer

M = Mach number

u_e = free stream velocity

then mean quantities in the layer may be defined as

$$\bar{u} = \frac{I}{m}$$

$$\bar{\rho} = \frac{m}{\bar{u}\delta}$$

$$\bar{T} = \frac{P}{\bar{\rho}R}$$

Dimensionless ratios are written as follows:

$\mathcal{H} = \dfrac{\bar{u}}{u_e}$ = velocity profile parameter

$w = \dfrac{u_e}{a^o}$ = a reduced free stream velocity, where a^o = stagnation speed of sound = constant

$\sigma = \dfrac{\tau_w}{(u_e-\bar{u})\frac{dm}{dx}}$ = ratio of momentum destroyed by skin friction to momentum by mixing from free stream

$t = 1 - \dfrac{\gamma-1}{2} w^2 = \dfrac{T_e}{T^o}$ = free stream temperature ratio

$\psi = \dfrac{\bar{T}}{T^o} \dfrac{1}{\mathcal{H}} - t\,\mathcal{H}$ = temperature function

We also use the following coefficients:-

$$C_F \;=\; \text{skin friction coefficient} \;=\; \frac{\tau_w}{\tfrac{1}{2}\rho_e u_e^2}$$

$$C_M \;=\; \text{mixing coefficient} \;=\; \frac{d\delta}{dx} - \phi - C_Q$$

(where ϕ = free stream deflection angle).

$$C_Q \;=\; \text{suction coefficient} \;=\; \frac{\text{mass sucked/unit area}}{\rho_e u_e}$$

$C \;=\; C_m \dfrac{1}{q}\dfrac{m}{\mu_e}$ where q – a factor allowing for variation of friction coefficient with M.

The properties of any boundary layer may be described by the quantities σ (momentum ratio), ψ (temperature function) and C (mixing rate function), which are found empirically to be functions only of the velocity profile parameter $\mathcal{H}$. Stewartson's transformation shows them to be independent of compressibility effects, so they may be obtained from incompressible boundary layer solutions, or from experiments. Mean curves may be drawn, see Figs. 1, 2 and 3.

Each value of $\mathcal{H}$ corresponds to a particular velocity profile: for instance

separation	:	$\mathcal{H} = 0.665$
Blasius	:	$\mathcal{H} = 0.700$
asymptotic suction	:	$\mathcal{H} = 0.750$

3. Equations

The equations given below include terms to allow for an arbitrary amount of boundary layer suction, but if the suction coefficient C_Q is set equal to zero, the equations reduce to those of Ref. 3.

Continuity:

$$\frac{dm}{dx} = \rho_e u_e C_M \qquad \ldots (1)$$

Momentum:

$$\frac{d\mathcal{H}}{dx} = \frac{1}{m}\frac{dm}{dx}(1-\mathcal{H})\left(1-\sigma+\frac{C_Q}{C_M}\right) + \frac{\psi}{wt}\frac{dw}{dx} \qquad \ldots (2)$$

Interaction:/

Interaction:

$$\left.\begin{aligned} \frac{h(\mathcal{X}, w)}{wt}\frac{dw}{dx}\left[k(\mathcal{X}, w) - \frac{t\phi}{C_m} + \frac{C_Q}{C_m}\left\{\psi' - \mathcal{X}(\psi'+t)\right\}\right] - \frac{1}{m}\frac{dm}{dx} &= 0 \\ h(\mathcal{X}, w) &= \psi(\psi'+\gamma w^2) + \mathcal{X}t(w^2-t) \\ k(\mathcal{X}, w) &= \psi + \psi'(1-\sigma)(1-\mathcal{X}) - \sigma t(1-\mathcal{X}) \end{aligned}\right\} \quad \ldots (3)$$

Outer Flow:

$$\phi = \frac{\sqrt{M_0^2-1}}{w_0}(w_0-w) + \text{const.}(w_0-w)^2 + \ldots \qquad \ldots (4)$$

We have found four equations, for the variables m, $\mathcal{X}$, w and ϕ as functions of x. If the functions $\sigma(\mathcal{X})$, $C(\mathcal{X})$ and $\psi(\mathcal{X})$ are given, the equations may be solved numerically. A number of simple analytic solutions have also been found, for special cases.

4. Shock-boundary Layer Interactions without Suction

The complete equations (1) to (4) were integrated numerically, in Ref. 3 for the case with a free stream Mach number of 2.0, and an initial Reynolds number of 10^5. Five different shock strengths were considered. The results were presented as pressure distribution on the wall, plotted against distance downstream, and a typical result is shown in Fig. 4. Some unpublished calculations for the case with a free stream Mach number of 2.0 and an initial Reynolds number of 5×10^5, which were carried out at Princeton University, showed that the pressure ratio at separation lay about half way between the values for $R_e = 10^5$ and $R_e = \infty$.

These results generally appear to agree quite well with available experimental pressure distributions, although the calculated pressure gradient at reattachment is certainly too small. There is a sharp "knee" in the pressure curve, at the point where the boundary layer separates, and the ratio between this separation pressure (P_s) and the initial pressure (P_0) is found to be a measure of the size of the disturbance required to cause separation. The boundary layer separates more easily, as Reynolds number is increased.

By cross plotting the results at different shock strengths, the graph of Fig. 5 is obtained for $M = 2.0$.

Note that no allowance is made for transition to turbulent flow, so that these results only make physical sense if the separated region is quite small. There is experimental evidence which suggests that laminar reattachment can take place under these conditions, if the Reynolds number is small.

5. Boundary Layers with Suction

At the suggestion of Dr. G. V. Lachmann an attempt was made to estimate whether weak shock waves are likely to cause separation of, and hence, early transition in, a laminar boundary layer in the presence of suction. A number of crude approximations are made, to find the order of magnitude of the stabilising effect of suction on a laminar boundary layer.

We/

We assume flow along a flat plate, with suction being applied at a sufficient number of strips for the boundary layer velocity profile ahead of the interaction to be an asymptotic profile. However, the mass of air sucked is assumed to be small enough to be neglected in the equation of momentum (2). That is $C_Q = 0$ in the region of the interaction. Any suction which takes place in the region of the interaction will have an additional stabilising effect, and delay separation.

As a first approximation, we will neglect skin friction at the wall, and the mixing of air between the boundary layer and the free stream ($C_F = 0$, $C_M = 0$). This amounts to assuming that the Reynolds number approaches infinity, which is not a very good approximation for a laminar layer. However, by making the same assumptions for the cases with and without suction, an estimate of the effect of suction can be obtained.

With these assumptions, the momentum equation (2) may be written

$$\frac{d\mathcal{H}}{dx} = \frac{\psi(\mathcal{H})}{M}\frac{dM}{dx} \qquad \dots (5)$$

where x = distance along plate

M = Mach number

$\mathcal{H}$ = velocity profile parameter

$\psi(\mathcal{H})$ = a boundary layer function evaluated in Ref. 3.

The solution of equation (5) is

$$M = M_o\, e^{\int_{\mathcal{H}_o}^{\mathcal{H}} \frac{d\mathcal{H}}{\psi(\mathcal{H})}} \qquad \dots (6)$$

Where M_o is the Mach number, just ahead of the disturbance.

Equation (6) can easily be evaluated, if $\psi(\mathcal{H})$ is known, and if the value $\mathcal{H}_o$ of the profile parameter when $M = M_o$ is given. By integrating this expression from $\mathcal{H} = \mathcal{H}$(Blasius) to $\mathcal{H}$ (separation) and from $\mathcal{H} = \mathcal{H}$(asymptotic) to $\mathcal{H}$ (separation) we obtain the Mach number ratio to cause separation, starting from a Blasius profile and an asymptotic profile respectively.

6. Results of Suction Calculations, with $R_\theta \to \infty$

The ratio of initial free stream Mach number (M_o) to Mach number at the separation point, is found to be 1.023 starting with a Blasius profile, and 1.075 starting with an asymptotic profile. Or, if $M \simeq 1$, a pressure increase of about 3.2% is needed to separate a flat plate boundary layer; whereas an 8.6% is needed to separate an asymptotic layer. The asymptotic layer will stand almost three times as large a pressure change as the flat plate layer, before separating. This result is very encouraging.

7. Suggestions for Further Calculations

The figures above apply strictly only to the case where the Reynolds number approaches infinity, which is not physically attainable because of transition. Also, the thickness of a sucked boundary layer is likely to be quite small, corresponding to a low Reynolds number, and it is therefore desirable to know the effect of finite Reynolds number on separation of an asymptotic boundary layer. From the results of Ref. 3 it seems likely that, as Reynolds number is reduced, shock induced separation will be further delayed.

More accurate calculations at a given Reynolds number would settle this point. Such calculations would present no special difficulties, as the method of Ref. 3 could be used without alteration, except to the boundary conditions; but the computation is rather lengthy, and almost certainly requires an automatic digital computer.

It would also be interesting to compute the effect of suction in the interaction region itself, instead of just starting with an asymptotic profile and then ignoring suction. This would be easy enough in the case where C_Q = constant, but it would add a little to the cost of integration, because of the extra terms in the equations. However, the case where C_Q is a function of x in the interaction region would be very expensive indeed to integrate.

Another use of the equations of Section 5 would be to calculate the growth of a laminar boundary layer with suction, with free stream velocity distribution (either subsonic or supersonic) and suction coefficient specified. Only equations (1) and (2) would be required. The general case, with arbitrary free stream velocity and arbitrary suction could be solved numerically without too much difficulty, and it would be interesting to make a comparison with other methods. A particularly simple solution can be found for flow over a flat plate, with C_Q and C_M constant. This may be written:

$$R_{e_\theta} = \frac{\nu_e x_o}{u_e} \frac{1}{C_M(1-\mathcal{H})} e^{\int_{\mathcal{H}_o}^{\mathcal{H}} \frac{d\mathcal{H}}{(1-\mathcal{H})(1-\sigma + C_Q/C_M)}} \quad \ldots (7)$$

where $\mathcal{H} = \mathcal{H}_o$ at the point $x = x_o$.

This expression will give R_{e_θ} very quickly, if C_Q is known. However, C_M = constant is not a very good approximation for a laminar layer. A better answer would undoubtedly be obtained by using the exact relation for C_M in which case a solution to the differential equation

$$\frac{d\mathcal{H}}{dm} = \frac{(1-\mathcal{H})[1-\sigma(\mathcal{H})]}{m} + \frac{C_Q}{\mu_e q} \frac{(1-\mathcal{H})}{C(\mathcal{H})} \quad \ldots (8)$$

is required. This could easily be obtained by numerical means.

The well known solution for a boundary layer with asymptotic suction is a singular point in equation (8) from which the propertie of the asymptotic profile can be derived quite simply. For example, it is easy to show that

$$C_F = \frac{C_Q}{2}$$

8. Conclusions

Simple calculations suggest that an asymptotic boundary layer on a flat plate requires nearly three times as large a pressure change as a comparable unsucked boundary layer, to cause separation. This result is encouraging, because it seems reasonable to suppose that a shock wave, which is not strong enough to cause separation, will not cause transition either; and therefore that no appreciable drag rise should be felt, before the direct effect which the local supersonic flow has on the pressure drag becomes important.

The method suggested here may also be applied to a large number of problems concerning boundary layers with suction, and boundary layers in interactions with shock waves, with or without heat transfer.

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4	K. N. C. Bray	H.P. Res. Rep. (Unpublished).

FIG 1.

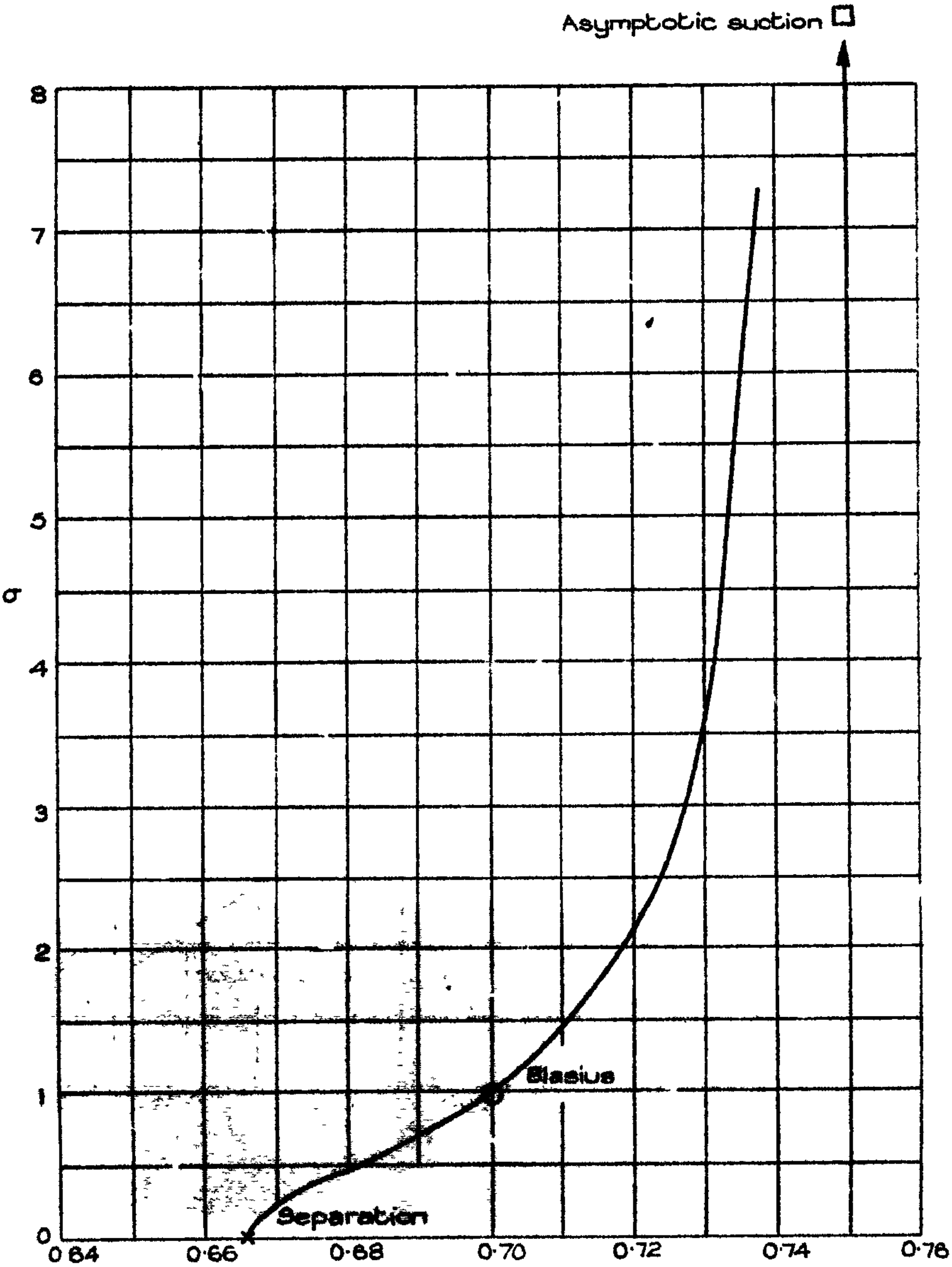

Asymptotic suction
8
7
6
5
4
3
2
1
0
σ
Blasius
Separation
0·64
0·66
0·68
0·70
0·72
0·74
0·76
H

FIG. 2

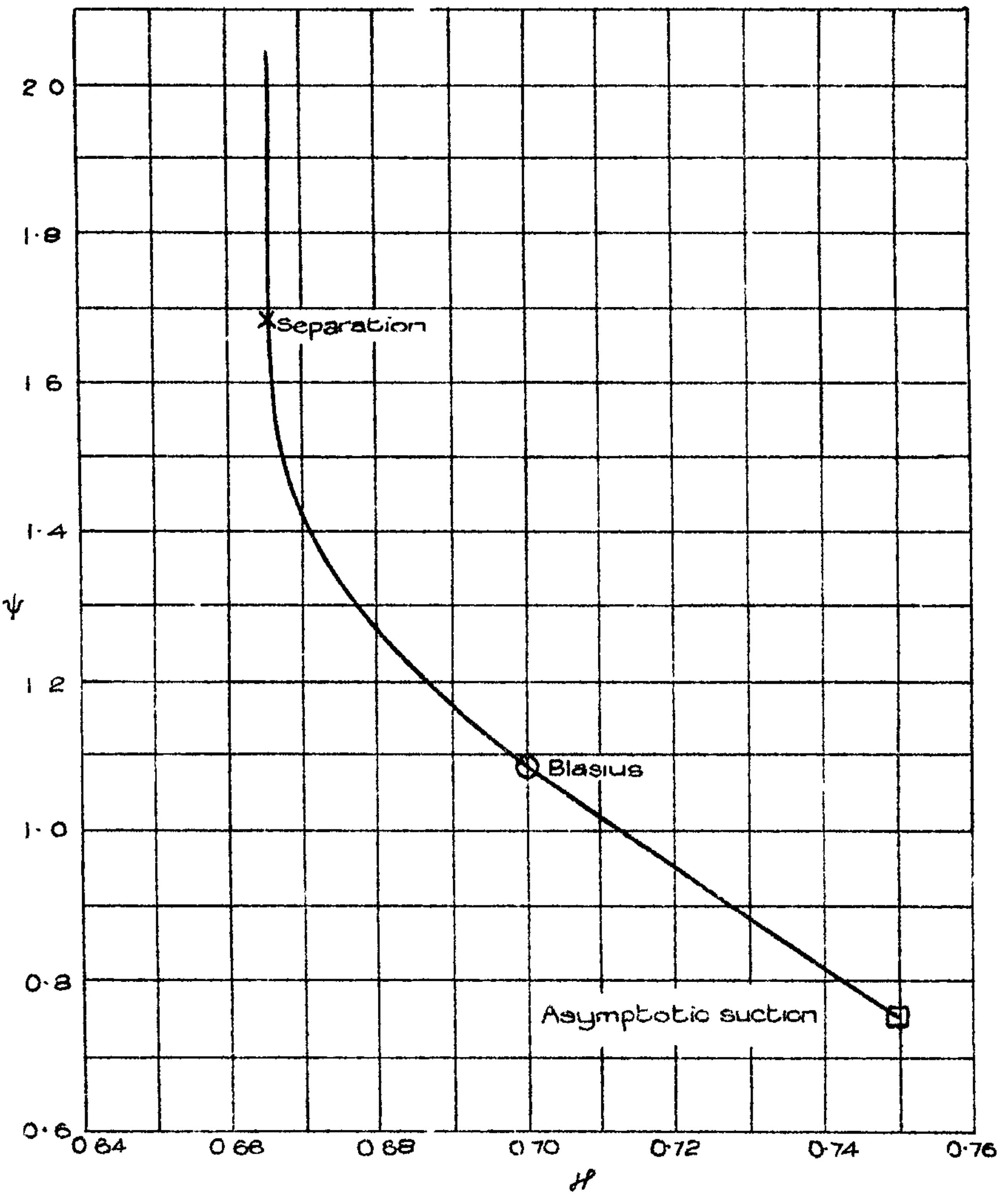

2 0
1·8
1 6
1·4
ψ
1 2
1·0
0·8
0·6
0 64
0 66
0 68
0 70
0·72
0·74
0·76
H
Separation
Blasius
Asymptotic suction

FIG. 3

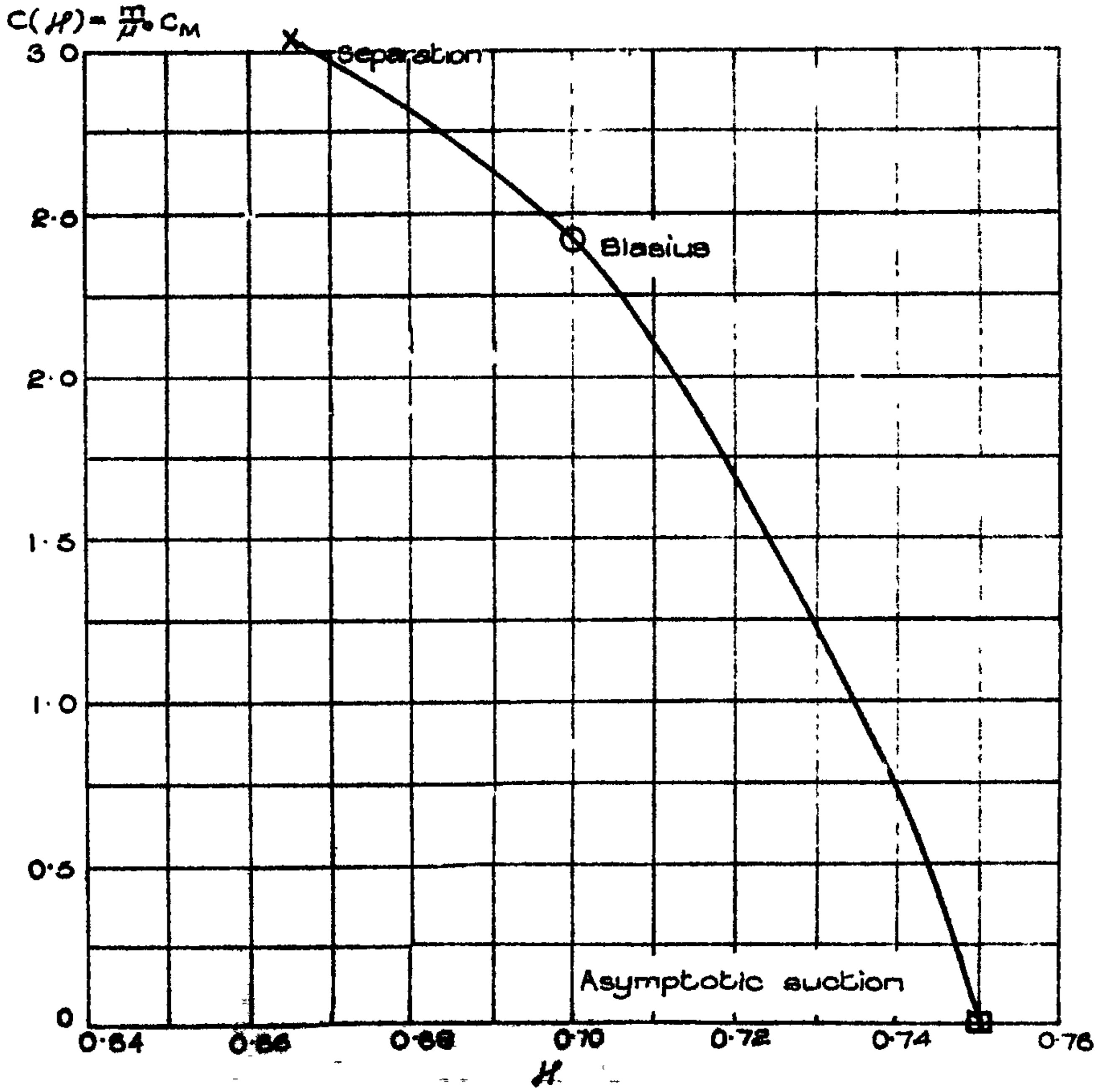
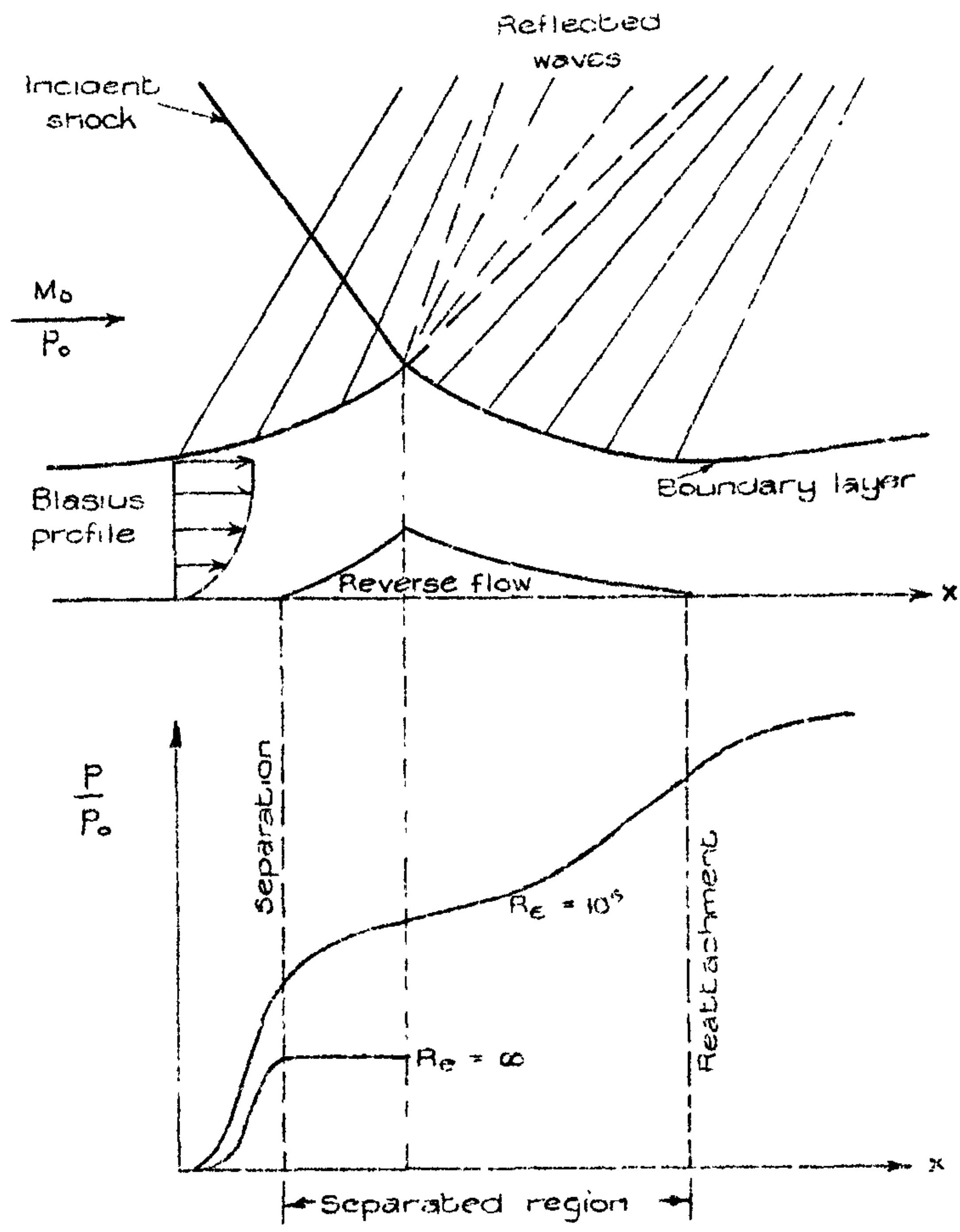

C(H) = m/μ₀ c_M
3.0
2.5
2.0
1.5
1.0
0.5
0
0.64
0.66
0.68
0.70
0.72
0.74
0.76
H
Separation
Blasius
Asymptotic suction

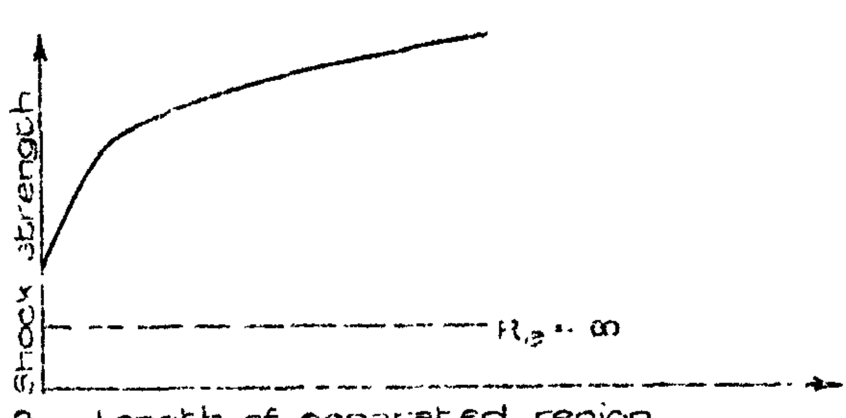
FIG 4 & 5

FIG 4



Typical pressure distribution.

FIG 5



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