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Summary.—In Part I of this paper the method of two-dimensional aerofoil design in incompressible flow due to Lighthill[1] (1945) is extended to compressible subsonic flow. Lighthill's equations are derived as special cases of more general equations due to the author, and some advances are made in the application of these equations to aerofoil design. It is shown, for example, how the designer can control the nose radius of curvature. The method of Ref. 1 requires that the velocity distribution be prescribed analytically, whereas this paper deals with distributions defined numerically, a development especially important for compressible flow design. The compressible flow theory is based on an approximation to the equation of flow not unlike, and with at least the same accuracy, as the Kármán – Tsien approximation for calculating the flow about a given aerofoil. A method of estimating the effects of a modification to the designed aerofoil shape, on the velocity distribution is also given.

In Part 2 five examples have been calculated. Aerofoil I is symmetrical, with a ' roof-top ' distribution at a given angle of incidence, showing how a given nose radius can be achieved ; Aerofoil II is symmetrical, designed for $M_\infty = 0$, and $\alpha = 0$ (M_∞ being the Mach number at infinity, and α the absolute angle of incidence), while Aerofoil III has been designed for the same distribution but at $M_\infty = 0{\cdot}7$. A comparison is made between Aerofoil III and that obtained from Aerofoil II by linear pertubation theory. It is shown, as would be expected, that this theory underestimates the reduction in thickness necessary to produce a compressible flow aerofoil from one designed for incompressible flow and the same velocity distribution. Aerofoils IV and V are asymmetric aerofoils designed for $M_\infty = 0{\cdot}7$, the former being designed to have a given distribution over each surface at incidence, while the latter is designed so that the upper surface has a given distribution at incidence and the lower surface has a given distribution at zero incidence. The design of an asymmetric aerofoil by the author's method is about two days' work for one computer.

Introduction.

NOMENCLATURE

(x, y)		$z = x + iy$, $(i = \sqrt{-1})$, the physical plane
(ϕ, ψ)		$w = \phi + i\psi$, the plane of equipotentials (ϕ = constant) and streamlines (ψ = constant) for *zero circulation*
(q, θ)		Velocity vector in polar co-ordinates for zero circulation
α		Angle of incidence measured from zero lift angle
(q', θ')		Velocity vector for angle of incidence
U		Velocity at infinity
L	$\equiv$	$\log (U/q)$
s		Distance measured along the aerofoil surface

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(η, γ)	Elliptic co-ordinates defined by $w = \phi + i\psi = -2a \cosh(\eta + i\gamma)$. The aerofoil surface is $\eta = 0$, when $\phi = -2a \cos\gamma$.
$2\tau_a$, $2\tau_b$	Leading- and trailing-edge angles respectively
θ_s, θ_a	Symmetric and antisymmetric parts of θ on the aerofoil surface
M	Mach number
β	$\equiv (1 - M^2)^{1/2}$
r	$\equiv \int_{q=U}^{q} \beta d \left\{ \log(U/q) \right\}$
∞	As a suffix to denote values at infinity.

In incompressible flow L $(\log(U/q))$ and θ are conjugate harmonic functions in the (x, y) and (ϕ, ψ)-planes, and so if either is specified on the aerofoil boundary the other can be calculated. In the (ϕ, ψ)-plane, the aerofoil is simply a slit on $\psi = 0$, and so in this plane the boundary conditions are very simple ; we are thus able to find an equation for $L(\phi, \psi)$ given $\theta(\phi, 0)$ on the aerofoil—or conversely find an equation for $\theta(\phi, \psi)$ given $L(\phi, 0)$ on the aerofoil. The calculation of the flow about a given aerofoil has been set out in detail in a previous report[2] and in this report we shall consider the converse problem—the calculation of the aerofoil shape for a given velocity distribution. This problem has already been solved by Lighthill[1] (1945) for the case of incompressible flow. The main purpose of this paper is to extend the method of Ref. 1 to a fairly accurate treatment of subsonic compressible flow.

However, it was found necessary to develop the application of the design equations along somewhat different lines from those of Ref. 1. For example, Lighthill's method requires that the velocity distribution be specified analytically as a function of γ (*see* Fig. 1) ; in fact L must be specified as an analytical function of γ simple enough to enable the conjugate to be evaluated. This limitation means that a profile with a given distribution defined by a complex formula or numerically cannot be designed by the method of Ref. 1. In the compressible flow theory of this paper L is replaced by $r \equiv \int \beta \, dL$, which yields a complicated functional relation between r and q (*see* equation (61)). If we are prepared to define r as an analytical function of γ, possessing an explicit conjugate we can, following Lighthill, readily find the corresponding aerofoil shape, but then we are scarcely designing an aerofoil for a given distribution of q. These considerations lead the author to develop the numerical method set out in section 4.

It is shown below how the nose radius of an aerofoil, an important parameter in the theory of aerofoils, can be controlled in the design problem. Lighthill's aerofoils in most cases had a zero nose radius of curvature, R_n ; he did give a method of obtaining a finite R_n but not of controlling it, which however fails for aerofoils of zero incidence (*see* Ref. 1, equation (IX. 14)). If the velocity distribution is specified over the whole aerofoil chord, the resulting 'aerofoil' will, in general, not be a closed one, and further the nose radius will be automatically determined. In this paper we therefore specify the velocity distribution over only about 80 per cent of the aerofoil chord, leaving the distribution near the nose and trailing edge to be determined by the 'closure' conditions, the specified trailing-edge angle, and the specified value of R_n. In addition adverse pressure gradients near the nose are avoided by requiring that, in the region of the unspecified distribution near the nose, $dq/ds \geqslant 0$, where s is the distance measured along the aerofoil surface from the front stagnation point.

The extension to compressible flow is relatively simple. We merely replace L by r in almost all the design equations and proceed in exactly the same way as for incompressible flow. It is a simple matter to tabulate r as a function of q/U for a given value of M_∞. Aerofoils II and

III give an interesting comparison between the author's theory and linear pertubation at $M_\infty = 0 \cdot 7$ (*see* Fig. 3). The author believes his theory to be reasonably accurate for aerofoils not having supersonic patches. By defining β ($\equiv (1 - M^2)^{1/2}$) to be $(M^2 - 1)^{1/2}$ in supersonic patches, the theory can be extended to this case, but the accuracy of this has yet to be studied.

It has been mentioned by Glauert[5] (1947) that the method of Ref. 1 can not be used to estimate the effects of a modification of design shape on the velocity distribution. It is important to be able to do this for the designed aerofoil may possess minor undesirable features, such as concavities towards the trailing edge, which cannot always be predicted beforehand. A convenient method of modifying designed aerofoils is given in section 4 and illustrated in section 13.

The method of obtaining the design equations given below is quite different from Lighthill's derivation. The equations are obtained as special cases of formulae for L and θ at any point in the field. Of course for design problems it is usually only necessary to have equations giving values on the aerofoil alone. However, it is possible to calculate compressible flow aerofoils accurately by using a relaxation technique on the non-linear differential equations for L (Woods[6], 1949), but before this can be done, at least approximate values of θ on an outer boundary surrounding the aerofoil need to be estimated, for otherwise the relaxation field is unbounded. These outer boundary values of θ can be obtained with reasonable accuracy by the author's more general equations (*see* equation (1)). The field is then filled in by relaxation on the exact differential equation, and values of θ on the aerofoil determined, completing the design. This relaxation method is a comparatively long process, but, provided the lattice is fine enough, provides accurate answers even in the transonic region of flow.

PART I

Mathematical Theory

1. *Two Methods of Design in Incompressible Flow.*—We shall first consider the case of zero incidence. Suppose that $w = \phi + i\psi$, where ϕ and ψ are the equipotential and stream functions respectively for zero circulation, then in the w-plane the aerofoil will be represented by a slit extending from the trailing edge (T.E.) to the leading edge (L.E.), *i.e.*, in Fig. 1, from A to B, a distance of 4a. If $L \equiv \log (U/q)$, q and U being the local and undisturbed stream velocities respectively, and if θ is the flow direction, assumed to be zero at infinity, then it is easily shown that, for incompressible flow, the complex function defined by $f \equiv L + i\theta$ satisfies Laplace's equation. If L on the aerofoil surface, for zero circulation, is given as a function γ^*, where $\gamma^* \equiv \cos^{-1}(-\phi^*/2a)$, then it has been shown that (Woods[2], 1950)

$$f(\eta, \gamma) = \frac{1}{2\pi}\int_{-\pi}^{\pi} L(\gamma^*) \coth \tfrac{1}{2}(i\gamma^* - i\gamma - \eta)\, d\gamma^*, \qquad (1)$$

where $$w = \phi + i\psi = -2a \cosh (\eta + i\gamma), \qquad (2)$$

and the star distinguishes the integrated variable. In terms of these elliptic co-ordinates, (η, γ), it follows from (2) that the aerofoil surface is defined by

$$\eta = 0, \quad \text{when} \quad \phi = -2a \cos \gamma. \qquad (3)$$

We shall take the L.E. and T.E. to be at $\gamma = 0$ and $\gamma = \pi$ respectively.

Integrating (1) by parts we have

$$f(\eta, \gamma) = -\frac{i}{\pi}\int_{\gamma^*=-\pi}^{\pi} L(\gamma^*)\, d[\log \sinh \tfrac{1}{2}(i\gamma^* - i\gamma - \eta)]$$

$$= -\frac{i}{\pi}\int_{\gamma^*=-\pi}^{\pi} L(\gamma^*)\, d\Big[\tfrac{1}{2}\log\Big(\sinh^2\frac{\eta}{2}\cos^2\tfrac{1}{2}(\gamma^* - \gamma) + \sin^2\tfrac{1}{2}(\gamma^* - \gamma)\cosh^2\frac{\eta}{2}\Big)$$

$$- i\tan^{-1}\coth\frac{\eta}{2}\tan\tfrac{1}{2}(\gamma^* - \gamma)\Big] .$$

Taking the limit as $\eta \rightarrow -0$, we have

$$L(\gamma) + i\theta(\gamma) = -\frac{i}{\pi}\int_{\gamma^*=-\pi}^{\pi} L(\gamma^*)\, d[\log \sin \tfrac{1}{2}(\gamma^* - \gamma)]$$

$$-\frac{1}{\pi}\lim_{\eta\rightarrow -0}\int_{\gamma^*=-\pi}^{\pi} L(\gamma^*)\, d\Big[\tan^{-1}\coth\frac{\eta}{2}\tan\tfrac{1}{2}(\gamma^* - \gamma)\Big] .$$

Now the function $\lim\limits_{\eta\rightarrow -0} \tan^{-1}\coth\frac{\eta}{2}\tan\frac{1}{2}(\gamma^* - \gamma)$ is a step function with a discontinuity of $-\pi$ at $\gamma^* = \gamma$. Evaluating the Stieltjes integral, we therefore have

$$L(\gamma) + i\theta(\gamma) = -\frac{i}{\pi}\int_{\gamma^*=-\pi}^{\pi} L(\gamma^*)\, d[\log \sin \tfrac{1}{2}(\gamma^* - \gamma)] + L(\gamma) .$$

Thus on the aerofoil surface equation (1) reduces to

$$\theta(\gamma) = -\frac{1}{2\pi}\int_{-\pi}^{\pi} L(\gamma^*)\cot\tfrac{1}{2}(\gamma^* - \gamma)\, d\gamma^* . \qquad (4)$$

This special case of equation (1) was derived by another method by Lighthill[1] (1945), and is the basis of his method of aerofoil design.

It has been shown that (Woods[2], 1950), on the Joukowski Hypothesis, at an absolute angle of incidence α, the velocity (q', θ') at (η, γ) is related to the zero circulation velocity, (q, θ), by

$$\frac{q'}{U}e^{-i\theta'} = \Big\{\cos\alpha + i\sin\alpha\coth\tfrac{1}{2}(\eta + i\gamma)\Big\}\frac{q}{U}e^{-i\theta}$$

$$= \frac{\sinh\frac{1}{2}(\eta + i\gamma + 2i\alpha)}{\sinh\frac{1}{2}(\eta + i\gamma)}\frac{q}{U}e^{-i\theta} . \qquad (5)$$

Taking logarithms we find

$$f' = f - \log\frac{\sinh\frac{1}{2}(\eta + i\gamma + 2i\alpha)}{\sinh\frac{1}{2}(\eta + i\gamma)} , \qquad (6)$$

which on the aerofoil surface, $\eta = 0$, becomes

$$L' = L - \log \frac{\sin(\frac{1}{2}\gamma + \alpha)}{\sin \frac{1}{2}\gamma} \quad \ldots \quad (7)$$

If this result is used in equation (4) we find the otherwise obvious result

$$\theta = -\frac{1}{2\pi}\int_{-\pi}^{\pi} L' \cot \tfrac{1}{2}(\gamma^* - \gamma)\, d\gamma^* + \alpha \, .$$

Of course in design problems q', and hence L', is usually defined as a function of s, the distance along the aerofoil surface, but it is shown in Appendix I that, given $q'(s)$, we can deduce $q'(\gamma)$ without much difficulty.

When θ has been calculated from (4) the profile co-ordinates follow from

$$\left.\begin{aligned} x &= \int_{-2a}^{\phi} \frac{\cos\theta}{q}\, d\phi = 2a\int_{0}^{\gamma} \frac{\cos\theta \sin\gamma\, d\gamma}{q} \\ y &= \int_{-2a}^{\phi} \frac{\sin\theta}{q}\, d\phi = 2a\int_{0}^{\gamma} \frac{\sin\theta \sin\gamma\, d\gamma}{q} \end{aligned}\right\} \quad \ldots \quad (8)$$

An alternative, but inferior method of design, is as follows. The author has shown that (Woods[2], 1950), if $z = x + iy$, then $x - \phi/U$, and $y - \psi/U$ are conjugate harmonic functions and corresponding to equation (1) there is

$$z(\eta, \gamma) - \frac{w}{U} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left(x - \frac{\phi}{U}\right) \coth \tfrac{1}{2}(\eta + i\gamma - i\gamma^*)\, d\gamma^* \, .$$

On the aerofoil surface this becomes

$$y(\gamma) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left(x - \frac{\phi}{U}\right) \cot \tfrac{1}{2}(\gamma - \gamma^*)\, d\gamma^* \, ,$$

but from equation (8), $x - \dfrac{\phi}{U} = 2a\displaystyle\int_{0}^{\gamma} \left(\frac{\cos\theta}{q} - \frac{1}{U}\right) \sin\gamma^*\, d\gamma^*$,

hence

$$y(\delta) = \frac{a}{\pi}\int_{-\pi}^{\pi} \left\{ \int_{0}^{\gamma} \left(\frac{\cos\theta}{q} - \frac{1}{U}\right) \sin\gamma^*\, d\gamma^* \right\} \cot\frac{\delta - \gamma}{2}\, d\gamma \quad \ldots \quad (9)$$

This equation has the disadvantage of being an integral equation requiring an iterative method of solution.

In the following sections we shall develop a method of applying equation (4) to aerofoil design which differs in detail from, and has some advantages over the method of Ref. 1. These details will also shorten the explanation of the compressible flow design technique.

2. *Conditions to be Satisfied by the Specified Velocity Distribution.*—Integrating equation (4) by parts we have

$$\theta(\gamma) = \frac{1}{\pi} \int_{\gamma^*=-\pi}^{\pi} \log \sin \tfrac{1}{2}(\gamma^* - \gamma)\, dL(\gamma^*) \ ,$$

and then dividing $L(\gamma)$ into symmetrical and antisymmetrical parts defined by

$$L_s = \tfrac{1}{2}\{L(\gamma) + L(-\gamma)\} \quad , \quad L_a = \tfrac{1}{2}\{L(\gamma) - L(-\gamma)\} \ , \qquad \ldots \quad \ldots \quad (10)$$

we find that

$$\theta(\pm\gamma) = \pm \frac{1}{\pi} \int_{\gamma^*=0}^{\pi} \log \frac{\sin \frac{1}{2}(\gamma^* - \gamma)}{\sin \frac{1}{2}(\gamma^* + \gamma)}\, dL_s(\gamma^*)$$

$$+ \frac{1}{\pi} \int_{\gamma^*=0}^{\pi} \log \sin \tfrac{1}{2}(\gamma^* - \gamma) \sin \tfrac{1}{2}(\gamma^* + \gamma)\, dL_a(\gamma^*) \ . \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

Note that dL_s is the derivative of the symmetric part—not the symmetric part of the derivative, and similarly for dL_a. From equation (7) it follows that

$$\left.\begin{aligned} L_s &= L'_s + \tfrac{1}{2} \log \sin (\tfrac{1}{2}\gamma + \alpha) \sin (\tfrac{1}{2}\gamma - \alpha) - \log \sin \tfrac{1}{2}\gamma \ , \\ \text{and} \quad & \\ L_a &= L_a' + \tfrac{1}{2} \log \frac{\sin (\frac{1}{2}\gamma + \alpha)}{\sin (\frac{1}{2}\gamma - \alpha)} \end{aligned}\right\} \ . \quad \ldots \quad (12)$$

One advantage of dividing L into symmetrical and antisymmetrical parts lies in the fact that even when L is infinite at the leading and trailing edges, *i.e.*, the aerofoil is not cusped at $\gamma = 0$ and $\gamma = \pi$, L_a will be bounded—in fact zero—at these points, and it is only necessary to consider the infinities in L_s.

Putting $\eta = -\infty$ (i.e., $z = w = \infty$) in (1) we find

$$f_\infty = -\frac{1}{2\pi} \int_{-\pi}^{\pi} L(\gamma^*)\, d\gamma^* = \log \frac{U}{U} + i\theta = 0 \ ,$$

but since $\theta_\infty = 0$, $q_\infty = U$, then $f_\infty = 0$,

i.e., L must satisfy $\displaystyle\int_{-\pi}^{\pi} L(\gamma^*)\, d\gamma^* = 0$.

From equations (10) it follows that this is equivalent to

$$\int_0^{\pi} L_s(\gamma)\, d\gamma = 0 \ . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

Equation (13) is the condition that the velocity at infinity is equal to U.

Now $dz/dw = e^{i\theta}/q$, i.e., the z- and w-planes are related by

$$z = \int \frac{e^{i\theta}}{q}\, dw = U \int e^f\, dw \ . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

Closure of the profile in the z-plane requires that

$$\oint e^f\, dw = 0 \ ,$$

where the integral is taken around any contour enclosing the profile.

Thus $$\oint (1 + f + \tfrac{1}{2}f^2 + \ldots)\, dw = 0\,,$$
but $f = 0$ at $w = \infty$, and so from Cauchy's theorem of residues f must be of the form
$$f = \frac{A}{w^2} + \frac{B}{w^3} + \ldots\,,$$
i.e., no '$1/w$' term in the expansion. The closure condition is therefore equivalent to
$$\oint f\, dw = 0\,. \qquad (15)$$
$$\oint f \sinh(\eta + i\gamma)\, d(\eta + i\gamma) = 0\,,$$
which on the aerofoil surface yields
$$\int_{-\pi}^{\pi} L(\gamma) \sin\gamma\, d\gamma = \int_{-\pi}^{\pi} \theta(\gamma) \sin\gamma\, d\gamma = 0\,. \qquad (16)$$
With the aid of equation (4)
$$\int_{-\pi}^{\pi} \theta(\gamma) \sin\gamma\, d\gamma = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} L(\gamma^*) \cot\tfrac{1}{2}(\gamma - \gamma^*)\, d\gamma^* \sin\gamma\, d\gamma\,,$$
i.e.,
$$0 = \int_{-\pi}^{\pi} L(\gamma^*) \cos\gamma^*\, d\gamma^*\,. \qquad (17)$$
Equations (16) and (17) can be generalized, with the aid of (7), to
$$\int_{-\pi}^{\pi} L'(\gamma) \sin\gamma\, d\gamma + \pi \sin 2\alpha = 0$$
and
$$\int_{-\pi}^{\pi} L'(\gamma) \cos\gamma\, d\gamma + 2\pi \sin^2\alpha = 0\,.$$
We notice from (12) that since $L_a = 0$ at $\gamma = 0, \pi$, then $L_a' = 0$, at $\gamma = 0, \pi$. Thus using equation (10) and integrating by parts we can write the first of these equations as
$$\sin 2\alpha = -\frac{2}{\pi}\int_0^{\pi} L_a'(\gamma) \sin\gamma\, d\gamma\,,$$
i.e.,
$$\sin 2\alpha = -\frac{2}{\pi}\int_{\gamma=0}^{\pi} \cos\gamma\, dL_a'(\gamma)\,. \qquad (18)$$
Similarly equation (17) yields
$$\int_{\gamma=0}^{\pi} \sin\gamma\, dL_s(\gamma) = 0\,. \qquad (19)$$
Equations (13) and (19) must be satisfied by L_s, while equation (18) can be regarded as defining α, which clearly depends very largely on the antisymmetric part of the distribution. $L_s(\gamma)$ (or $L_s'(\gamma)$) must be specified in at least a two-parameter form so that (13) and (19) can be satisfied, and so if we desire to control, say, n other features, such as the leading-edge radius, the trailing-edge angle, etc., $L_s(\gamma)$ must be specified in a $n + 2$ parameter form. From (13) however, we see that one of these parameters need be merely an additive constant, which will have no effect on θ (equation (11)), the nose radius (equation (34)), or equation (19).

3. *The Effect on Velocity Distribution of a Modification to Aerofoil Shape.*—It is convenient at this stage to outline the author's method[2] of calculating the flow about a *given* aerofoil. The conjugate equation to (1) is (merely interchange L and $i\theta$ in the integral)

$$f(\eta, \gamma) = \frac{i}{2\pi}\int_{-\pi}^{\pi} \theta(\gamma^*) \coth \tfrac{1}{2}(i\gamma^* - i\gamma - \eta)\, d\gamma^* ,$$

or integrating by parts,

$$f(\eta, \gamma) = -\frac{1}{\pi}\int_{\gamma^*=-\pi}^{\pi} \log \sinh \tfrac{1}{2}(i\gamma^* - i\gamma - \eta)\, d\theta(\gamma^*) .$$

On the aerofoil surface $\eta = 0$, and so

$$L(\gamma) = -\frac{1}{\pi}\int_{\gamma^*=-\pi}^{\pi} \log \sin \tfrac{1}{2}(\gamma^* - \gamma)\, d\theta(\gamma^*) . \qquad \ldots \quad (20)$$

If θ is divided into symmetrical and antisymmetrical parts defined by

$$\theta_s(\gamma) = \tfrac{1}{2}\{\theta(\gamma) + \theta(-\gamma)\} , \qquad \theta a(\gamma) = \tfrac{1}{2}\{\theta(\gamma) - \theta(-\gamma)\} ,$$

then (20) can be written

$$L(\pm \gamma) = \pm\frac{1}{\pi}\int_{\gamma^*=0}^{\pi} \log\frac{\sin \frac{1}{2}(\gamma^* - \gamma)}{\sin \frac{1}{2}(\gamma^* + \gamma)}\, d\theta_s(\gamma^*) .$$

$$-\frac{1}{\pi}\int_{\gamma^*=0}^{\pi} \log \sin \tfrac{1}{2}(\gamma^* - \gamma) \sin \tfrac{1}{2}(\gamma^* + \gamma)\, d\theta_a(\gamma^*). \qquad \ldots \quad (21)$$

If there is a simple discontinuity in θ of τ, at $\gamma^* = \pi$, viz., a T.E. angle of 2τ, then from (21) this jump in θ contributes $-(2\tau/\pi) \log \cos \frac{1}{2}\gamma$ to $L(\pm \gamma)$. Similarly a discontinuity of $\frac{1}{2}\pi$ at the L.E. (a rounded nose) contributes $-\log \sin \frac{1}{2}\gamma$ to $L(\pm \gamma)$. These results will be used in the next section.

Suppose now that we have designed an aerofoil to give a certain velocity distribution, and have found that some undesirable feature has appeared in the shape of the designed aerofoil, *e.g.*, concavities towards the T.E., then it is useful to be able to calculate the effect on the velocity distribution of modifying the shape to eliminate the undesirable feature. Equation (21) enables this to be done. In the actual computation we proceed as follows.

When θ is continuous, $d\theta = (d\theta/d\gamma)\, d\gamma$, otherwise at sharp corners there are simple discontinuities in θ, say $\Delta\theta$. We subdivide the range $(0, \tau)$ into $(\gamma_2, \gamma_3, \ldots \gamma_i \ldots \gamma_{n-1}, \gamma_n)$, where $\gamma_2 = 0$, $\gamma_n = \pi$, such that we can assume, with negligible error that $d\theta/d\gamma$ is constant in each interval, say equal to $(d\theta/d\gamma)_i$ in the interval (γ_i, γ_{i+1}). Equation (21) then becomes

$$L(\pm \gamma_k) = -\frac{1}{\pi}\sum_{i=2}^{n-1}\left[\pm\left(\frac{d\theta_s}{d\gamma}\right)_i \int_{\gamma_i}^{\gamma_{i+1}} \log\frac{\sin \frac{1}{2}(\gamma^* - \gamma_k)}{\sin \frac{1}{2}(\gamma^* + \gamma_k)}\, d\gamma^* \right.$$
$$\left. + \left(\frac{d\theta_a}{d\gamma}\right)_i \int_{\gamma_i}^{\gamma_{i+1}} \log \sin \frac{\gamma^* - \gamma_k}{2} \sin \frac{\gamma^* + \gamma_k}{2}\, d\gamma^* \right]$$
$$-\frac{1}{\pi}\sum_j\left[\pm \Delta\theta_{sj} \log\frac{\sin \frac{1}{2}(\gamma_j - \gamma_k)}{\sin \frac{1}{2}(\gamma_j + \gamma_k)} + \Delta\theta_{aj} \log \sin \tfrac{1}{2}(\gamma_j - \gamma_k) \sin \tfrac{1}{2}(\gamma_j + \gamma_k)\right] , \qquad (22)$$

where the discontinuities in θ occur at γ_j. In the case of a symmetrical aerofoil $\theta_s = 0$, and $\theta_a = \theta$.

If $I(x) \equiv -\frac{1}{\pi}\int_0^{\pi} \log \sin \tfrac{1}{2}t \, dt$, which has been tabulated[5] then

$$-\frac{1}{\pi}\int_{\gamma_i}^{\gamma_{i+1}} \log \frac{\sin \frac{1}{2}(\gamma^* - \gamma_k)}{\sin \frac{1}{2}(\gamma^* + \gamma_k)} \, d\gamma^* = \Big[I(\gamma^* - \gamma_k) - I(\gamma^* + \gamma_k) \Big]_{\gamma_i}^{\gamma_{i+1}},$$

and

$$-\frac{1}{\pi}\int_{\gamma_i}^{\gamma_{i+1}} \log \sin \tfrac{1}{2}(\gamma^* - \gamma_k) \sin \tfrac{1}{2}(\gamma^* + \gamma_k) \, d\gamma^* = \Big[I(\gamma^* - \gamma_k) + I(\gamma^* + \gamma_k) \Big]_{\gamma_i}^{\gamma_{i+1}}.$$

We shall write the discontinuities in θ at the T.E. ($\gamma = \pi$) and L.E. ($\gamma = 0$) as τ_b and τ_a respectively. If matrices A, B, C and D are defined by

$$A_{ik} = -\frac{1}{\pi}\int_{\gamma_i}^{\gamma_{i+1}} \log \frac{\sin \frac{1}{2}(\gamma^* - \gamma_k)}{\sin \frac{1}{2}(\gamma^* + \gamma_k)} \, d\gamma^*, \qquad i, k = 2, 3, \ldots n-1$$

$$B_{ik} \begin{cases} = -\frac{1}{\pi}\int_{\gamma_i}^{\gamma_{i+1}} \log \sin \frac{1}{2}(\gamma^* - \gamma_k) \sin \frac{1}{2}(\gamma^* + \gamma_k) \, d\gamma^*, & i, k = 2, 3, \ldots n-1 \\ = -\frac{2}{\pi} \log \sin \frac{1}{2}\gamma_k, & i = 1, k = 2, 3, \ldots n-1 \\ = -\frac{2}{\pi} \log \cos \frac{1}{2}\gamma_k, & i = n, k = 2, 3, \ldots n-1 \\ = -\frac{2}{\pi}\int_{\gamma_i}^{\gamma_{i+1}} \log \sin \frac{1}{2}\gamma^* \, d\gamma^*, & i = 2, 3 \ldots n-1, k = 1 \\ = -\frac{2}{\pi}\int_{\gamma_i}^{\gamma_{i+1}} \log \cos \frac{1}{2}\gamma^* \, d\gamma^*, & i = 2, 3, \ldots n-1, k = n \end{cases}$$

$$C_i \begin{cases} = \left(\frac{d\theta_a}{d\gamma}\right)_i, & i = 2, 3, \ldots n-1 \\ = \tau_a, & i = 1 \\ = \tau_b, & i = n \end{cases}$$

$$D_i = \left(\frac{d\theta_s}{d\gamma}\right)_i, \qquad i = 2, 3, \ldots n-1,$$

then (22) can be written concisely as

$$L(\pm \gamma_k) = \sum_{i=1}^{n} C_i B_{ik} \pm \sum_{i=2}^{n-i} D_i A_{ik}, \qquad k = 2, 3, \ldots n-1, \qquad (23)$$

in which, to save space, it has been assumed that the only discontinuities in θ are at the T.E. and L.E. If any other discontinuities exist their effects can easily be calculated from (22).

One small modification needs to be made to the above scheme, a modification which is really more important in the case of aerofoil design. Clearly $d\theta_s/d\gamma = 0$ at $\gamma = 0$, and so instead of assuming $(d\theta_s/d\gamma)_2$ to be a constant, it is better to write $(d\theta_s/d\gamma)$ in the form $A \sin\gamma$ in the interval $(0, \lambda)$ where λ is the value of γ at the end of the first interval. (We choose $A \sin\gamma$ as $d\theta_s/d\gamma$ is an antisymmetrical function vanishing at $\gamma = 0$.) If then we redefine $D_2 = A \sin \frac{1}{2}\lambda$

and
$$A_{2k} = -\frac{1}{\pi \sin\frac{1}{2}\lambda}\int_0^{\lambda} \sin\gamma^* \log\frac{\sin\frac{1}{2}(\gamma^* - \gamma_k)}{\sin\frac{1}{2}(\gamma^* + \gamma_k)}\, d\gamma^* .$$
$$= \frac{1}{\pi \sin\frac{1}{2}\lambda}\left\{2\sin\tfrac{1}{2}(\gamma_k + \lambda)\sin\tfrac{1}{2}(\gamma_k - \lambda)\log\frac{\sin\frac{1}{2}(\gamma_k - \lambda)}{\sin\frac{1}{2}(\gamma_k + \lambda)} + \lambda\sin\gamma_k\right\},$$

(23) remains unchanged in form.

Suitable matrices A and B are given in Tables 2 and 3. In these tables $(0, \pi)$ has been subdivided into (0°, 6°, 12°, 18°, 24°, 30° 40°, . . . 160°, 170°, 180°), and γ_k has the values 3°, 9°, 15°, 21°, 27°, 35°, 45°, . . . 165° and 175°. These tables, which are also used for aerofoil design (*see* equation (31)), should be quite sufficient for all but the most unusual aerofoil shapes. The interval is reduced near the nose to allow for the greater rate of change of θ in this neighbourhood.

There are equations which must be satisfied by $(d\theta_a/d\gamma)$, $(d\theta_s/d\gamma)$, τ_a and τ_b which are equivalent to the closure conditions (18) and (19) of the previous section. From (16) and the conjugate of (17) they are
$$\int_{-\pi}^{\pi} \theta(\gamma)\sin\gamma\, d\gamma = 0 \text{ and } \int_{-\pi}^{\pi} \theta(\gamma)\cos\gamma\, d\gamma = 0 .$$

In addition
$$\int_{\gamma=-\pi}^{\pi} d\theta(\gamma) = 0 ,$$

expresses the obvious requirement that θ be cyclic. Integration by parts, and a separation of θ into symmetric and antisymmetric parts results in
$$\int_{\gamma=0}^{\pi} \cos\gamma\, d\theta_a(\gamma) = \int_{\gamma=0}^{\pi} \sin\gamma\, d\theta_s(\gamma) = \int_{\gamma=0}^{\pi} d\theta_a(\gamma) = 0 ,$$

and ignoring all except the T.E. and L.E. discontinuities in θ, it follows that
$$\left.\begin{aligned} &\int_0^{\pi}\cos\gamma\left(\frac{d\theta_a}{d\gamma}\right)d\gamma + \tau_a - \tau_b = 0 \\ &\int_0^{\pi}\left(\frac{d\theta_a}{d\gamma}\right)d\gamma \qquad + \tau_a + \tau_b = 0 \\ \text{and} \quad &\int_0^{\pi}\sin\gamma\left(\frac{d\theta_s}{d\gamma}\right)d\gamma \qquad\qquad = 0 \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (24)$$

Now a given closed aerofoil *does* satisfy these exactly, but if the given aerofoil is replaced by an approximating one, for which $(d\theta/d\gamma)_i$ is constant in the i^{th} interval, then instead of (24) the equations

$$\left.\begin{aligned} &\sum_{i=2}^{n-1}\left(\frac{d\theta_a}{d\gamma}\right)_i [\sin\gamma]_i + \tau_a - \tau_b = 0 \\ &\sum_{i=2}^{n-1}\left(\frac{d\theta_a}{d\gamma}\right)_i [\gamma]_i \qquad + \tau_a + \tau_b = 0 \\ &\sum_{i=2}^{n-1}\left(\frac{d\theta_s}{d\gamma}\right)_i [\cos\gamma]_i \qquad\qquad = 0\,, \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

where $[X]_i$ is the jump in X in the i^{th} interval, must be satisfied exactly. This is ensured by using the first two equations to *define* τ_a and τ_b, completely ignoring the actual values of these angles. The last equation can be satisfied simply by adjusting the value of $(d\theta_s/d\gamma)_i$.

Of course θ is not given as a function of γ when calculating the flow about a given aerofoil, but this difficulty is easily overcome by making use of the Cauchy-Riemann equations relating L and θ. In the design problem the result of the calculation will be θ as a function of γ, and if over any interval (γ_s, γ_r) say, the shape is undesirable, we can modify it by changing θ to $\theta + \bar{\theta}$. We can then calculate the modification to $\log U/q$ by using $\bar{\theta}$ (which we have as a function of γ) in equation (23), ensuring beforehand that equations (25) are satisfied. An example of this calculation appears in section 13.

4. *Calculation of the Aerofoil Profile.*—Returning now to the design problem, we shall suppose that we have a given velocity distribution which *does* satisfy (13), (18) and (19), and that we have to calculate the corresponding aerofoil co-ordinates. In section 6 methods of modifying a given distribution to satisfy these equations are given, but it is convenient to postpone this for the moment.

First we find $q'(\gamma)$ (*see* Appendix I), then calculate L_s' and L_a'. The angle of incidence follows from (18), and then equations (12) yield L_s and L_a. The infinities in L_s can be dealt with as follows. In the previous section it was shown that $L_s \sim -(2\tau/\pi)\log\cos\frac{1}{2}\gamma$ at the T.E. and $L_s \sim -\log\sin\frac{1}{2}\gamma$ at the L.E. Consequently the auxiliary function defined by

$$L_m = L_s + \begin{cases} \log(\sin\frac{1}{2}\gamma/\sin\frac{1}{2}\lambda),\ 0 \leqslant \gamma \leqslant \lambda \\ 0 \qquad\qquad\qquad\qquad ,\ \lambda \leqslant \gamma \leqslant \pi - \varepsilon \\ \dfrac{2\tau}{\pi}\log(\cos\frac{1}{2}\gamma/\sin\frac{1}{2}\varepsilon),\ \pi - \varepsilon \leqslant \gamma \leqslant \pi, \end{cases} \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

remains bounded at $\gamma = 0$, and $\gamma = \pi$. If other discontinuities in θ are required on the aerofoil surface further appropriate terms can be added to the right-hand side of (26), but for simplicity τ shall be assumed that this is not the case, so that L_m is bounded in $0 \leqslant \gamma \leqslant \pi$.

From (12)

$$L_m = L'_s + \tfrac{1}{2}\log\frac{\sin(\frac{1}{2}\gamma - \alpha)\sin(\frac{1}{2}\gamma + \alpha)}{\sin^2(\frac{1}{2}\gamma)} + \begin{cases} \log\dfrac{\sin\frac{1}{2}\gamma}{\sin\frac{1}{2}\lambda}\,,\ 0 \leqslant \gamma \leqslant \lambda \\ 0 \qquad\qquad\quad \lambda \leqslant \gamma \leqslant \pi - \varepsilon \\ \dfrac{2\tau}{\pi}\log\dfrac{\cos\frac{1}{2}\gamma}{\sin\frac{1}{2}\varepsilon}\,,\ \pi - \varepsilon \leqslant \gamma \leqslant \pi, \end{cases} \quad \ldots \quad \ldots \quad (27)$$

and so

$$\frac{dL_m}{d\gamma} = \frac{dL'_s}{d\gamma} + \tfrac{1}{4}\left\{\cot(\tfrac{1}{2}\gamma + \alpha) + \cot(\tfrac{1}{2}\gamma - \alpha) - 2\cot\tfrac{1}{2}\gamma\right\} + \begin{cases} \frac{1}{2}\cot\frac{1}{2}\gamma,\ 0 \leqslant \gamma \leqslant \lambda \\ 0 \qquad\qquad ,\ \lambda \leqslant \gamma \leqslant \pi - \varepsilon \\ -\dfrac{\tau}{\pi}\tan\frac{1}{2}\gamma,\ \pi - \varepsilon \leqslant \gamma \leqslant \pi\,. \end{cases} \quad (28)$$

In the special case of a symmetrical aerofoil $L_a = 0$, *i.e.*, from (12)

$$L'_a = -\tfrac{1}{2}\log\frac{\sin(\frac{1}{2}\gamma + \alpha)}{\sin(\frac{1}{2}\gamma - \alpha)}, \quad \text{and so}$$

$$L'_s = L' - L'_a = L' + \tfrac{1}{2}\log\frac{\sin(\frac{1}{2}\gamma + \alpha)}{\sin(\frac{1}{2}\gamma - \alpha)}, \quad \text{when (27) and (28) become}$$

$$L_m = L' + \log\frac{\sin(\frac{1}{2}\gamma + \alpha)}{\sin\frac{1}{2}\gamma} + \begin{cases} \log\dfrac{\sin\frac{1}{2}\gamma}{\sin\frac{1}{2}\lambda}, & 0 \leqslant \gamma \leqslant \lambda \\ 0, & \lambda \leqslant \gamma \leqslant \pi - \varepsilon \\ \dfrac{2\tau}{\pi}\log\dfrac{\cos\frac{1}{2}\gamma}{\sin\frac{1}{2}\varepsilon}, & \pi - \varepsilon \leqslant \gamma \leqslant \pi, \end{cases} \quad \ldots \quad (29)$$

and

$$\frac{dL_m}{d\gamma} = \frac{dL'}{d\gamma} + \tfrac{1}{2}\left\{\cot(\tfrac{1}{2}\gamma + \alpha) - \cot\tfrac{1}{2}\gamma\right\} + \begin{cases} \frac{1}{2}\cot\frac{1}{2}\gamma, & 0 \leqslant \gamma \leqslant \lambda \\ 0, & \lambda \leqslant \gamma \leqslant \pi - \varepsilon \\ -\dfrac{\tau}{\pi}\tan\frac{1}{2}\gamma, & \pi - \varepsilon \leqslant \gamma \leqslant \pi\,. \end{cases} \quad (30)$$

Differentiating (26) and substituting the result in (11) we find

$$\theta(\pm\gamma_k) = \pm\left\{J(\lambda, \gamma_k) - \frac{2\tau}{\pi}J(\varepsilon, \pi - \gamma_k)\right\} \pm \frac{1}{\pi}\int_{\gamma^*=0}^{\pi} \log\frac{\sin\frac{1}{2}(\gamma^* - \gamma_k)}{\sin\frac{1}{2}(\gamma^* + \gamma_k)}\, dL_m(\gamma^*)$$

$$+ \frac{1}{\pi}\int_{\gamma^*=0}^{\pi} \log\sin\tfrac{1}{2}(\gamma^* - \gamma_k)\sin\tfrac{1}{2}(\gamma^* + \gamma_k)\, dL_a(\gamma^*)\,,$$

where

$$J(x, y) \equiv -\frac{1}{2\pi}\int_0^x \log\frac{\sin\frac{1}{2}(\gamma^* - y)}{\sin\frac{1}{2}(\gamma^* + y)}\cot\tfrac{1}{2}\gamma^*\, d\gamma^*\,.$$

This integral is readily evaluated and is discussed in Appendix II. Subdividing the range of integration just as for the conjugate equation we find corresponding to (23) the equation

$$\theta(\pm\gamma_k) = \pm\left\{G_k(\tau) - \sum_{i=2}^{n-1} E_i A_{ik}\right\} - \sum_{i=2}^{n-1} F_i B_{ik} \pm \frac{1}{\pi}\sum_j \Delta L_{mj}\log\frac{\sin\frac{1}{2}(\gamma_j - \gamma_k)}{\sin\frac{1}{2}(\gamma_j + \gamma_k)}$$

$$+ \frac{1}{\pi}\sum_j \Delta L_{aj}\log\sin\tfrac{1}{2}(\gamma_j - \gamma_k)\sin\tfrac{1}{2}(\gamma_j + \gamma_k)\,, \quad k = 1, 2, \ldots n\,,$$

or $\theta(\pm\gamma_k) = \pm\,\theta_J \pm \theta_m + \theta_a$, say , ... (31)

where $G_k(\tau) = J(\lambda, \gamma_k) - \dfrac{2\tau}{\pi}J(\varepsilon, \pi - \gamma_k) = \theta_J, \quad k = 1, 2, \ldots n$

$$E_i = \left(\frac{dL_m}{d\gamma}\right)_i \qquad i = 3, \ldots\ldots n - 1$$

$$E_2 = A\sin\tfrac{1}{2}\lambda\,,$$

$$F_i = \left(\frac{dL_a}{d\gamma}\right)_i \qquad i = 2, 3, \ldots n - 1,$$

and ΔL_{mj} and ΔL_{aj} are jumps in L_m and L_a at $\gamma = \gamma_j$. Of course if a jump occurs in the i^{th} range, then $(dL/d\gamma)_i$ is the slope calculated *after* the jump has been removed. It will be noticed from the form of E_2 that we have assumed $dL_m/d\gamma$ to be equal to $A \sin \gamma$ in the first interval $(0, \lambda)$. The significance of this will be made clear in the next section.

5. *The Nose Radius of Curvature.*—If R is the radius of curvature at any point, and p is the semi-perimeter (very nearly equal to the chord for conventional aerofoils), then

$$\frac{p}{R} = p\frac{d\theta}{ds} = p\frac{d\theta}{d\gamma}\frac{d\gamma}{d\phi}\frac{d\phi}{ds} = \frac{pq}{2a\sin\gamma}\frac{d\theta}{d\gamma},$$

and so from the first equation of section 2

$$\frac{p}{R} = \frac{1}{\pi}\left(\frac{pU}{4a}\right)\left(\frac{q}{U\sin\gamma}\right)\int_{\gamma^*=-\pi}^{\pi} \cot\tfrac{1}{2}(\gamma^* - \gamma)\, dL(\gamma^*) . \qquad (32)$$

If we denote L_m at $\gamma = 0$ by h, then from (26)

$$\lim_{\gamma\to 0}\frac{q}{U\sin\gamma} = \lim_{\gamma\to 0}\frac{e^{-L_m}}{\sin\frac{1}{2}\lambda}\,\frac{\sin\frac{1}{2}\gamma}{\sin\gamma} = \frac{1}{2e^h\sin\frac{1}{2}\gamma} . \qquad (33)$$

Thus the nose radius of curvature is given by

$$\frac{p}{R_n} = \frac{1}{\pi}\left(\frac{pU}{4a}\right)\frac{e^{-h}}{2\sin\frac{1}{2}\lambda}\int_{\gamma^*=-\pi}^{\pi}\cot\tfrac{1}{2}\gamma^*\, dL(\gamma^*)$$

$$= \frac{1}{\pi}\left(\frac{pU}{4a}\right)\frac{e^{-h}}{\sin\frac{1}{2}\lambda}\int_{\gamma^*=0}^{\pi}\cot\tfrac{1}{2}\gamma^*\, dL_s(\gamma^*), \text{ i.e., using (26)}$$

$$\frac{p}{R_n} = \frac{1}{\pi}\left(\frac{pU}{4a}\right)\frac{e^{-h}}{\sin\frac{1}{2}\lambda}\left\{\cot\tfrac{1}{2}\lambda + \tfrac{1}{2}\lambda + \frac{\tau\varepsilon}{\pi} + \int_{\gamma^*=0}^{\pi}\cot\tfrac{1}{2}\gamma^*\, dL_m(\gamma^*)\right\} . \qquad (34)$$

In the previous section we wrote $dL_m/d\gamma$ in the form $A \sin\gamma$ in the neighbourhood of $\gamma = 0$. We shall now find an approximate relation between R_n and the constant A. If

$$\frac{dL_s}{d\gamma} = A\sin\gamma - \tfrac{1}{2}\cot\tfrac{1}{2}\gamma , \quad 0 \leqslant \gamma \leqslant \delta ,$$
$$= 0 \qquad\qquad , \quad \delta \leqslant \gamma \leqslant \pi ,$$

where δ is a small angle greater than or equal to λ, then from (26)

$$\frac{dL_m}{d\gamma} = \left\{\begin{array}{ll} A\sin\gamma & , \quad 0 \leqslant \gamma \leqslant \lambda \\ A\sin\gamma - \frac{1}{2}\cot\frac{1}{2}\gamma , & \quad \lambda < \gamma < \delta \\ 0 & , \quad \delta \leqslant \gamma \leqslant \pi \end{array}\right\} . \qquad (35)$$

Substitution of these values in (34) yields

$$\frac{p}{R_n} = \frac{1}{\pi}\left(\frac{pU}{4a}\right)\frac{e^{-h}}{\sin\frac{1}{2}\lambda}\left\{\cot\tfrac{1}{2}\delta + \tfrac{1}{2}\delta + A(\delta + \sin\delta) + \frac{\tau\varepsilon}{\pi}\right\} .$$

Therefore $$\frac{p}{R_n} \simeq \frac{4}{\pi}\left(\frac{pU}{4a}\right)\frac{e^{-h}}{\lambda\delta}(1 + A\delta^2)\,,$$

since δ, λ and $\tau\varepsilon/\pi$ are small, but A may be a relatively large number. Integrating the equation for $dL_s/d\gamma$ we find $L_s = -A\cos\gamma - \log\sin\frac{1}{2}\gamma + \text{constant}$, $0 \leqslant \gamma \leqslant \delta$. The constant follows from (26), and the result, $L_m = h$ at $\gamma = 0$. It is found that

$$L_s = A(1 - \cos\gamma) + h + \log\frac{\sin\frac{1}{2}\lambda}{\sin\frac{1}{2}\gamma}\,, 0 \leqslant \gamma \leqslant \delta$$

$$= A(1 - \cos\delta) + h + \log\frac{\sin\frac{1}{2}\lambda}{\sin\frac{1}{2}\delta}\,, \delta \leqslant \gamma \leqslant \pi\,.$$

Substituting this in (13) and ignoring a term $O(\delta^2)$ we find

$$e^{-h} = \frac{\lambda}{\delta}\exp\left(\tfrac{1}{2}A\delta^2 + \frac{\delta}{\pi}\right) \simeq \frac{\lambda}{\delta}\left\{\exp\left(\tfrac{1}{2}A\delta^2\right)\right\}\left\{1 + \frac{\delta}{\lambda}\right\} \simeq \frac{\lambda}{\delta}\exp\left(\tfrac{1}{2}A\delta^2\right)$$

and hence

$$\frac{p}{R_n} \simeq \frac{4}{\pi}\left(\frac{pU}{4a}\right)\frac{\exp\left(\frac{1}{2}A\delta^2\right)}{\delta^2}(1 + A\delta^2)$$

i.e., $$\frac{p}{R} \simeq \frac{4}{\pi\delta^2}e^{\chi}\left(\frac{pU}{4a}\right)(1 + 2\chi)\,, \qquad \text{(36)}$$

where $\chi = \frac{1}{2}A\delta^2$.

For given values of δ and p/R_n this approximate equation yields a value of A to use in (35). $pU/4a$ can be taken to have some value slightly less than unity at first (*see* equation (38)). An example of the application (36) is given below in section 11.

Since $$ds = \frac{d\phi}{q} = \frac{2a\sin\gamma\, d\gamma}{q}\,,$$

then $$2p = 2a\left\{\int_0^{\pi}\frac{\sin\gamma\, d\gamma}{q(\gamma)} + \int_0^{-\pi}\frac{\sin\gamma\, d\gamma}{q(-\gamma)}\right\},$$

i.e., $$\frac{pU}{4a} = \frac{1}{4}\int_0^{\pi}\left\{\frac{U}{q(\gamma)} + \frac{U}{q(-\gamma)}\right\}\sin\gamma\, d\gamma\,. \qquad \text{(37)}$$

For thin aerofoils

$$\frac{U}{q(\gamma)} + \frac{U}{q(-\gamma)} = e^{L(\gamma)} + e^{L(-\gamma)} \simeq 2\{1 + L_s(\gamma)\}\,,$$

and so $$\frac{pU}{4a} \simeq 1 + \frac{1}{2}\int_0^{\pi}L_s(\gamma)\sin\gamma\, d\gamma\,. \qquad \text{(38)}$$

6. *A Method of Modifying a Given Distribution to Yield a Closed Aerofoil.*—Substituting (26) in (19) and (13) we find that L_m must satisfy

$$\int_{\gamma=0}^{\pi} \sin\gamma \, dL_m(\gamma) = \tfrac{1}{2}(\lambda + \sin\lambda) - \frac{\tau}{\pi}(\varepsilon + \sin\varepsilon) = P, \text{ say,} \qquad (39)$$

and

$$\int_0^{\pi} L_m \, d\gamma = -\left(I(\lambda) + \frac{2\tau}{\pi} I(\varepsilon) \right) - \lambda \log\sin\tfrac{1}{2}\lambda - \frac{2\tau\varepsilon}{\pi}\log\sin\tfrac{1}{2}\varepsilon = Q, \text{ say,} \qquad (40)$$

where $$I(x) \equiv -\frac{1}{\pi}\int_0^x \log\sin\tfrac{1}{2}t \, dt .$$

Suppose $\tilde{L}_m$ denotes values not satisfying (39) and (40), then if we take

$$L_m = \tilde{L}_m + \begin{cases} -C(\cos\sigma + \cos\gamma) , & \pi - \sigma \leqslant \gamma \leqslant \pi \\ 0 , & \delta \leqslant \gamma \leqslant \pi - \sigma \\ B(\cos\delta - \cos\gamma) , & 0 \leqslant \gamma \leqslant \delta \end{cases} ,$$

such that L_m satisfies (39) and (40), then $\tilde{L}_m$ is unaltered in $\delta \leqslant \gamma \leqslant \pi - \varepsilon$, and further, if $\tilde{L}_m$ is continuous in $0 \leqslant \gamma \leqslant \pi$, so is L_m. Equations (39) and (40) become

$$\int_0^{\pi} \tilde{L}_m \, d\gamma + B(\delta\cos\delta - \sin\delta) - C(\sigma\cos\sigma - \sin\sigma) = Q,$$

i.e.,

$$B(\delta\cos\delta - \sin\delta) - C(\sigma\cos\sigma - \sin\sigma) = Q + \int_{\gamma=0}^{\pi} \gamma \, d\tilde{L}_m(\gamma) - \pi\tilde{L}_m(\gamma) , \qquad (41)$$

and

$$\tfrac{1}{2}B(\delta - \tfrac{1}{2}\sin 2\delta) + \tfrac{1}{2}C(\sigma - \tfrac{1}{2}\sin 2\sigma) = P - \int_{\gamma=0}^{\pi} \sin\gamma \, d\tilde{L}_m(\gamma), \qquad (42)$$

respectively. These simultaneous equations are solved for B and C, and then $dL_m/d\gamma$ is replaced by

$$\frac{dL_m}{d\gamma} = \frac{d\tilde{L}_m}{d\gamma} + \begin{Bmatrix} C\sin\gamma, & \pi - \sigma \leqslant \gamma \leqslant \pi \\ 0 , & \delta \leqslant \gamma \leqslant \pi - \sigma \\ B\sin\gamma, & 0 \leqslant \gamma \leqslant \delta \end{Bmatrix} . \qquad (43)$$

The smaller σ and δ are, the larger will B and C become in magnitude, but the need to avoid adverse pressure gradients near the nose imposes an upper limit to the value of B, i.e., a lower limit to the values of σ and δ. In equation (41) $L_m(\pi)$ is obtained from

$$\tilde{L}_m(\pi) = \tilde{L}_m(\gamma^*) + \int_{\gamma=\gamma^*}^{\pi} d\tilde{L}_m(\gamma) , \qquad (44)$$

where $\delta \leqslant \gamma^* \leqslant \pi - \sigma$.

It is also necessary that the antisymmetric part of L' satisfies

$$\frac{\pi}{2} \sin 2\alpha + \int_{\gamma=0}^{\pi} \cos \gamma \frac{dL'_a}{d\gamma} d\gamma = 0 . \qquad \text{(18 bis.)}$$

Suppose that $d'_a/d\tilde{L}\gamma$ does not satisfy this equation, but that

$$\frac{dL'_a}{d\gamma} = \frac{d\tilde{L}'_a}{d\gamma} + \begin{Bmatrix} R, 0 \leqslant \gamma \leqslant \lambda \\ S, \lambda \leqslant \gamma \leqslant \delta \\ 0, \delta \leqslant \gamma \leqslant \pi \end{Bmatrix}, \qquad (45)$$

does, then integrating from $\gamma = 0$, at which $L'_a = \tilde{L}'_a = 0$, to $\gamma = \delta$, we have

$$L'_a(\delta) = \tilde{L}'_a(\delta) + R\lambda + S(\delta - \lambda) ;$$

hence $\tilde{L}'_a$, $\delta \leqslant \gamma \leqslant \pi$, will be unaltered if $R = -S(\delta - \lambda)/\lambda$. Using (45) we find that equation (18) will be satisfied if S is calculated from

$$S\left(\frac{\delta \sin \lambda}{\lambda} - \sin \delta\right) = \int_{0}^{\pi} \cos \gamma \left(\frac{dL'_a}{d\gamma}\right) d\gamma + \tfrac{1}{2}\pi \sin 2\alpha . \qquad (46)$$

If we assume, as in section 4, that $dL_m/d\gamma$ and $dL_a/d\gamma$ are constant over small intervals, and that $dL_m/d\gamma = A \sin \gamma$ in the first interval $(0, \lambda)$, where λ is small, then (41), (42) and (46) can be written as

$$B(\delta \cos \delta - \sin \delta) - C(\sigma \cos \sigma - \sin \sigma) = Q + \frac{A\lambda^3}{3} + \sum_{i=3}^{n-1} \left(\frac{dL_m}{d\gamma}\right)_i [\tfrac{1}{2}\gamma^2]_i - \pi \tilde{L}_m(\pi) , \qquad (47)$$

$$\tfrac{1}{2}B(\delta - \tfrac{1}{2} \sin 2\delta) + \tfrac{1}{2}C(\sigma - \tfrac{1}{2} \sin 2\sigma) = P - \frac{A\lambda^3}{3} - \sum_{i=3}^{n-1} \left(\frac{dL_m}{d\gamma}\right)_i [-\cos \gamma]_i , \qquad (48)$$

and

$$S\left(\frac{\delta \sin \lambda}{\lambda} - \sin \delta\right) = \frac{\pi}{2} \sin 2\alpha + \sum_{i=2}^{n-1} \left(\frac{dL'_a}{d\gamma}\right)_i [\sin \gamma]_i , \qquad (49)$$

respectively. Similarly equation (34) can be written.

$$\frac{p}{R} = \frac{1}{\pi}\left(\frac{pU}{4a}\right)\frac{e^{-h}}{\sin \frac{1}{2}\lambda}\left\{\cot \tfrac{1}{2}\lambda + \tfrac{1}{2}\lambda + \frac{\tau\varepsilon}{\pi} + A(\lambda + \sin \lambda) + \sum_{i=3}^{n-1} \left(\frac{dL_m}{d\gamma}\right)_i \left[2 \log \sin \frac{\gamma}{2}\right]_i\right\}. \qquad (50)$$

For the particular subdivision of $(0, \pi)$ used in Tables 2, and 3, Table 1 sets out the values of $[\cos \gamma]_i$, $[\frac{1}{2}\gamma^2]_i$, $[2 \log \sin \frac{1}{2}\gamma]_i$ and $[\sin \gamma]_i$. In this case $\lambda = 6°$, $\varepsilon = 10°$, and so P and Q are given by

$$P = 0 \cdot 1046 - 0 \cdot 1741 \left(\frac{2\tau}{\pi}\right) , \qquad (51)$$

and

$$Q = -\left\{0 \cdot 1047 + 0 \cdot 1744 \left(\frac{2\tau}{\pi}\right)\right\} . \qquad (52)$$

7. *Equations for Design in Compressible Flow.*—The exact equations[7]

$$\frac{\partial \theta}{\partial n} - (1 - M^2)\frac{\partial L}{\partial s} = 0, \quad \frac{\partial \theta}{\partial s} + \frac{\partial L}{\partial n} = 0 ,$$

where n is distance measured normal to a streamline
s is distance measured along a streamline
and M is the local Mach number,
when transformed to the (ϕ, ψ)-plane defined by

$$d\phi = q\,ds\,, \quad d\psi = \frac{\rho}{\rho_0} q\,dn\,, \qquad \text{(53)}$$

in which ρ and ρ_0 are the local and stagnation densities respectively, become

$$\frac{\partial \theta}{\partial \psi} - \frac{\rho_0}{\rho}(1 - M^2)\frac{\partial L}{\partial \phi} = 0\,, \quad \frac{\partial \theta}{\partial \phi} + \frac{\rho}{\rho_0}\frac{\partial L}{\partial \psi} = 0\,. \qquad \text{(54)}$$

The (ϕ, ψ)-plane introduced here is the *compressible* flow plane for zero circulation. The variables ϕ, ψ, of the previous sections will be distinguished by a suffix i in the sequence.

If, following von Kármán[3] (1941), we write

$$dr \equiv (1 - M^2)^{1/2}\,dL = -\frac{U}{q}\,\beta\,d\left(\frac{q}{U}\right), \qquad \text{(55)}$$

and

$$m \equiv \frac{\rho_0}{\rho}(1 - M^2)^{1/2} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{1/(\gamma-1)}(1 - M^2)^{1/2}\,, \qquad \text{(56)}$$

where γ here is the ratio of the specific heats, equations (54) becomes

$$\frac{\partial \theta}{\partial \psi} - m\frac{\partial r}{\partial \phi} = 0\,, \quad \frac{\partial \theta}{\partial \phi} + \frac{1}{m}\frac{\partial r}{\partial \psi} = 0\,. \qquad \text{(57)}$$

Since from equation (56)

$$m = 1 - \frac{\gamma + 1}{8}M^4 + O(M^6)\,,$$

then $$m - m_\infty = \frac{\gamma + 1}{8}(M_\infty^4 - M^4) + O(M_\infty^6 - M^6)\,,$$

and so for subsonic flow about aerofoils of moderate thickness/chord ratio it is sufficiently accurate to write

$$m \simeq m_\infty \equiv \left(1 + \frac{\gamma - 1}{2}M_\infty^2\right)^{1/(\gamma-1)}(1 - M_\infty^2)^{1/2}\,, \qquad \text{(58)}$$

an approximation first introduced by von Kármán[3].

Using this approximation in (57) we have

$$\frac{\partial \theta}{\partial (m_\infty \psi)} = \frac{\partial r}{\partial \phi} \; ; \quad \frac{\partial \theta}{\partial \phi} = - \frac{\partial r}{\partial (m_\infty \psi)} \quad \ldots \quad (59)$$

From these Cauchy-Riemann equations we conclude that $g \equiv r + i\theta$, is an analytic function of $w \equiv \phi + im_\infty \psi$. Using dashes to denote values applying to a non-zero circulation, and undashed symbols for zero-circulation values, we have thus proved

$$g' = g'(w') \; , \quad \ldots \quad (60)$$

a particular case of which is $g = g(w)$.

We note here that if equation (58) is regarded as being exact for some imaginary gas, this gas proves to be a 'tangent' gas, that is a gas whose $(p, 1/\rho)$-curve is a straight line tangential to the $(p, 1/\rho)$-curve of an ideal gas. The point of tangency is $(p_\infty, 1/\rho_\infty)$. At this stage we have a choice between (a) taking (58) as an approximation and using the *exact* relations for an ideal gas between p, ρ, M and q, and (b) developing an exact theory for the tangent gas defined by (58). In other words we have to choose between an approximate theory for an 'exact' gas, and an exact theory for an 'approximating' gas. In this paper we choose the former; in Ref. 4, which deals with aerofoil design with mixed boundary conditions, the latter point of view was adopted. There is probably little to choose between them.

Using the exact adiabatic relation between q and β, and taking $\gamma = 1 \cdot 4$, we can integrate equation (55) to find

$$r = \frac{\sqrt{6}}{2} \log \left| \frac{\sqrt{6} - \beta}{\sqrt{6} - \beta_\infty} \, \frac{\sqrt{6} + \beta_\infty}{\sqrt{6} + \beta} \right| + \tfrac{1}{2} \log \left| \frac{1 - \beta_\infty}{1 - \beta} \, \frac{1 + \beta}{1 + \beta_\infty} \right| , \quad \ldots \quad (61)$$

in which the constant of integration has been selected so that $r = 0$ when $q = U$. Thus we can deduce a relation

$$r = r(q/U) \; , \quad \ldots \quad (62)$$

although it is much easier to obtain this by numerical integration of equation (55) (*see* Table 6).

(*a*). *The Case of Zero Circulation.*—Corresponding to equation (2) we now define elliptic co-ordinates by

$$w = \phi + im_\infty \psi = - 2a \cosh (\eta + i\gamma) \, . \quad \ldots \quad (63)$$

In this $(\eta + i\gamma)$-plane the differential equation and boundary conditions for g are identical in form with those of the function f defined in section 1—in fact g can be regarded as a generalisation of f to compressible flow—and hence corresponding to equation (1) we have

$$g(\eta, \gamma) = \frac{1}{2\pi} \int_{-\pi}^{\pi} r(\gamma^*) \coth \tfrac{1}{2}(i\gamma^* - \gamma i - \eta) \, d\gamma^* \, . \quad \ldots \quad (64)$$

On the aerofoil surface, $\eta = 0$, and (64) reduces to

$$\theta(\gamma) = - \frac{1}{2\pi} \int_{-\pi}^{\pi} r(\gamma^*) \cot \tfrac{1}{2}(\gamma^* - \gamma) \, d\gamma^* \, . \quad \ldots \quad (65)$$

When θ has been calculated from (65), equations (8), which in virtue of (53) are still applicable, yield the profile co-ordinates.

(*b*) *Non-zero Circulation.*—From $dz = dx + i\,dy = e^{i\theta}\,ds$, $dz = i\,e^{i\theta}\,dn$, and equations (53) it is found that

$$dz = \frac{e^{i\theta}}{q}\left(d\phi + i\frac{\rho_0}{\rho}\,d\psi\right). \qquad (66)$$

Unlike the case of incompressible flow (*cf.* equation (14)), the right-hand side of (66) is not an analytic function of w, i.e., z is not an analytic function of w. In incompressible flow $w' = w'(z)$, and $z = z(w) = z(\eta + i\gamma)$, from (63). Hence $w' = w'(\eta + i\gamma)$. We can thus write $f' = f(\eta + i\gamma) + \delta(\alpha, \eta + i\gamma)$, where δ is the increment due to incidence. For compressible flow, however, this argument cannot be applied, and so we cannot separate out the effects of incidence so conveniently.

However for small values of α the w- and w'-planes are almost coincident. Approximating in the position of the solution g', we can write $g' \simeq g'(w) = g'(\eta + i\gamma)$, and hence for small values of α

$$g' = g(\eta + i\gamma) + \delta(\alpha, \eta + i\gamma) \qquad (67)$$

Equation (67) is an approximation, exactly true in either of the limits $\alpha \to 0$, or $M_\infty \to 0$. It neglects the second-order effects of α on the mapping from w' to $\eta + i\gamma$. It is an adequate approximation over the range of α for which it is legitimate to linearly superimpose the effects of thickness and incidence. This is sufficient for practical purposes, as high-speed aerofoils operate over fairly small incidence ranges.

The increment δ in (67) satisfies the same boundary conditions as δ for incompressible flow (flow direction reversed over $-2\alpha \leqslant \gamma \leqslant 0$), and therefore from (6) we conclude that

$$g' = g - \log\frac{\sinh\frac{1}{2}(\eta + i\gamma + 2i\alpha)}{\sinh\frac{1}{2}(\eta + i\gamma)}.$$

Thus corresponding to equations (7) and 12) we have

$$r' = r - \log\frac{\sin\frac{1}{2}(\gamma + 2\alpha)}{\sin\frac{1}{2}\gamma}; \qquad (68)$$

and

$$\begin{cases} r_s = r'_s + \frac{1}{2}\log\sin(\frac{1}{2}\gamma + \alpha)\sin(\frac{1}{2}\gamma - \alpha) - \log\sin\frac{1}{2}\gamma, & (69) \\ r_a = r'_a + \frac{1}{2}\log\dfrac{\sin(\frac{1}{2}\gamma + \alpha)}{\sin(\frac{1}{2}\gamma - \alpha)}, & (70) \end{cases}$$

respectively.

The equation conjugate to (65) is

$$r(\gamma) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \theta(\gamma^*)\cot\tfrac{1}{2}(\gamma^* - \gamma)\,d\gamma^*.$$

If we redefine r to be $\int(M^2 - 1)^{1/2}\,dL$ in supersonic patches, then since $M_\infty < 1$, i.e., m_∞ is real, equation (65) is still valid on the assumption that (58) holds. This extension of the theory is not meant to provide an accurate method of dealing with supersonic patches, but it permits a calculation to be carried out even if supersonic patches are present; the aerofoil shape outside the supersonic patch may still be reasonably accurate.

8. *Conditions to be Satisfied by the Compressible Velocity Distribution.*—Since we have taken r to vanish when $q = U$, it follows that $r_\infty = 0$; also if θ is measured from the direction of flow at infinity, $\theta_\infty = 0$. Hence $g_\infty = 0$. Now z, $w \to \infty$ as $\eta \to -\infty$, and so from (64) it follows that

$$g_\infty = \frac{1}{2\pi}\int_{-\pi}^{\pi} r(\gamma^*)\, d\gamma^* .$$

Hence $\displaystyle\int_0^{\pi} r_s(\gamma)\, d\gamma = 0\,,$ .. .. (71)

is the equation corresponding to (13).

From (63) for large w

$$e^{\eta + i\gamma} = -\frac{a}{w} + O\left(\frac{a}{w}\right)^2 ,$$

hence if equation (64) is expressed as a power series in $\exp(\eta + i\gamma)$, and then transformed into a power series in (a/w), it will be found that in the neighbourhood of the point at infinity

$$g = -\frac{a}{\pi w}\int_{-\pi}^{\pi} r e^{-i\gamma}\, d\gamma + O\left(\frac{a}{w}\right)^2 . \qquad (72)$$

Now from (55) it follows that well away from the aerofoil

$$q/U = e^{-r/\beta_\infty} ,$$

and hence we see that from the form of g, q/U will be of the form $1 + A/|w|$. Since there are no sources or sinks at infinity this result is contrary to the assumption of zero circulation on which (64) is based. Hence from (72),

$$\int_{-\pi}^{\pi} r \cos\gamma\, d\gamma = 0, \qquad \int_{-\pi}^{\pi} r \sin\gamma\, d\gamma = 0 .$$

By just the same algebra as in section 3 these equations can be written in the form

$$\sin 2\alpha = -\frac{2}{\pi}\int_0^{\pi} r_a' \sin\gamma\, d\gamma , \qquad (73)$$

and $\displaystyle\int_{\gamma=0}^{\pi} \sin\gamma\, dr_s(\gamma) = 0\,.$ .. .. (74)

It should be clear now that most of the equations given in sections 1 to 6 for incompressible flow can be made applicable to compressible flow merely by changing f, L, ϕ_i and ψ_i into g, r, ϕ and $m_\infty\psi$ respectively. Amongst the numbered equations the only exceptions to this rule are (9), (14), (34), (36) and (38). The author has not found an equation corresponding to (9) for compressible flow, the integrated form of equation (66) corresponds to equation (14), while equation (38) is approximate in any case; its exact form is equation (37) which also applies to compressible flow. It is shown in Appendix III that

$$\lim_{\gamma\to 0}\frac{q}{U\sin\gamma} = \frac{\exp\left(-h - \frac{1}{4}M_\infty^2\right)}{2\sin\frac{1}{2}\lambda} ,$$

where h is now defined as the value of r_m at $\gamma = 0$. Thus corresponding to (33) we have

$$\frac{p}{R_n} = \frac{1}{\pi}\left(\frac{pU}{4a}\right)\frac{\exp\left(-h - \frac{1}{4}M_\infty^2\right)}{2\sin\frac{1}{2}\lambda}\left\{\cot\tfrac{1}{2}\lambda + \tfrac{1}{2}\lambda + \frac{\tau\varepsilon}{\pi} + \int_{\gamma^*=0}^{\pi}\cot\tfrac{1}{2}\gamma^*\, dr_m(\gamma^*)\right\}; \qquad (75)$$

equation (50) is similarly modified. Likewise, if r_m replaces L_m in (35) then (36) becomes

$$\frac{p}{R} = \frac{4\exp(-\frac{1}{4}M_\infty^2)}{\pi\delta^2}\frac{pU}{4a}e^\chi(1+2\chi) \; . \qquad (76)$$

9. *A Simple Approximation for Compressible Flow.*—The incompressible flow plane is defined by

$$d\phi_i = q_i\,ds,\; d\psi_i = q_i\,dn \; .$$

At infinity $q_i = q = U$, and so from (53) and (56) it follows that at infinity the (ϕ_i, ψ_i) and (ϕ, ψ)-planes are related by

$$d\phi_i = d\phi,\; d\psi_i = \frac{\rho_0}{\rho_\infty}d\psi = \frac{m_\infty}{\beta_\infty}d\psi \; . \qquad (77)$$

If we now assume that $r = \beta_\infty L$ and (77) hold throughout the field, and not merely at infinity, then (65) can be written

$$L(\eta, \gamma) + \frac{i\theta(\eta, \gamma)}{\beta_\infty} = \frac{1}{2\pi}\int_{-\pi}^{\pi} L(\gamma^*) \coth \tfrac{1}{2}(i\gamma^* - i\gamma - \eta)\, d\gamma^* \; , \qquad (78)$$

where $\quad -2a\cosh(\eta + i\gamma) = \phi_i + i\beta_\infty\psi_i \; .$

On the aerofoil surface (78) becomes

$$\theta(\gamma) = \frac{1}{2\pi}\int_{-\pi}^{\pi} L(\gamma^*) \cot \tfrac{1}{2}(\gamma^* - \gamma)\, d\gamma^* \; . \qquad (79)$$

It is unnecessary to pursue this theory further, as it is obvious that all the incompressible flow equations can be modified to apply to compressible flow merely by changing L into $\beta_\infty L$. For a *given* velocity distribution over a symmetrical aerofoil at zero incidence, compressibility (on this theory) reduces θ by the factor β_∞, which for thin aerofoils (and this approximate theory is only applicable to thin aerofoils) is equivalent to reducing the ordinate y by the factor β_∞, a simple result which could also be deduced from the linear pertubation theory of compressible flow. This, and the more accurate theory of the preceeding section is compared in Fig. 3.

PART II

Examples of Aerofoil Design

10. *Summary of the Aerofoils Designed.*—Five aerofoils have been designed, each one illustrating some feature in the method. Aerofoil I is symmetrical with a 'roof-top' distribution at a given angle of incidence, showing how a given nose radius can be achieved by the theory of section 5; Aerofoil II is symmetrical, designed for $M_\infty = 0$, and $\alpha = 0°$, and illustrates the use of equations (47) and (48), while Aerofoil III has been designed for the same distribution but at $M_\infty = 0{\cdot}7$. The theory of section 3 has been illustrated on Aerofoil III in the modification of the rear part into a simple wedge. Aerofoils IV and V are asymmetric aerofoils designed for $M_\infty = 0{\cdot}7$, the former being designed to have a given distribution over each surface at incidence, while the latter is designed so that the upper surface has a given distribution at incidence and the lower surface has a given distribution at zero incidence. The design of an asymmetric aerofoil by the author's method is about two days' work for one computor.

11. *Aerofoil I: An Aerofoil with a Given Nose Radius.—Specification:* A symmetrical roof-top aerofoil, T.E. angle 18°, $M_\infty = 0$, $C_L \simeq 0\cdot25$, $c/R_n = 100 \pm 2$ per cent, and the 'roof-top' extending to about $0\cdot75c$.

Now $C_L = (8a\pi \sin \alpha)/cU$ (*see* Ref. 8, section 7.13), and assuming $(cU/4a) = 0\cdot9$, we find from this equation that if $C_L = 0\cdot25$, $\alpha \simeq 2°6'$. Using this value of α we can fill in columns 2 and 3 of Table 4. In column 4 we have taken the value of $dL'/d\gamma$ as zero in the range (12°, 120°)—the '12°' being arbitrary, while 120° has been selected in the anticipation that it will be the interval end nearest to the given value $x = 0\cdot75c$ for the termination of the roof-top. The exact position of the end of the flat portion of the velocity distribution will be known only when the design is complete. Since $dL'/d\gamma = -(1/q')(dq'/d\gamma)$, a positive value of $dL'/d\gamma$ implies an adverse pressure gradient which we want to avoid in (0°, 120°); apart from this restriction we shall select $dL'/d\gamma$ in (0°, 12°) and (120°, 180°) so that the specified nose radius is achieved and the aerofoil is closed. Column 5 follows from columns 2, 3 and 4 (*see* equation (30)), in which $\lambda = 6°$, $\varepsilon = 10°$, and the entries in (120°, 180°) are based on the temporary assumption that $dL'/d\gamma$ vanishes in this range also.

If in (36) we take $p/R_n = 100$, $(pU/4a) = 0\cdot9$, $\delta = 12°$, then $\frac{1}{4}\pi(p/R_n)\delta^2(4a/pU) \simeq 3\cdot84 = e^\chi(1 + 2\chi)$. Therefore $\chi = 0\cdot58$, and so $A \simeq 26$. Then from (35) we have that $26 \sin 3°$ and $-\frac{1}{2}\cot 4\frac{1}{2}° + 26 \sin 9° = -2\cdot2858$, are the appropriate entries for the first two places of column 5. In this example we shall not control the velocity *magnitude*, but will allow it to be determined by the condition that the velocity at infinity is U. This is done by putting $B = 0$ in equations (47) and (48), and then calculating $L_m(\pi)$ from

$$\pi L_m(\pi) = Q + \frac{A\lambda^3}{3} + \sum_{i=3}^{n-1}\left(\frac{dL_m}{d\gamma}\right)_i [\tfrac{1}{2}\gamma^2]_i \; . \qquad \qquad (80)$$

From (51), with $\tau = 9°$, we find $P = 0\cdot0872$, and so with $\sigma = 60°$ in (48) we have

$$0\cdot3071C = 0\cdot0872 - \frac{A\lambda^3}{3} - \sum_{i=3}^{n-1}\left(\frac{d\bar{L}_m}{d\gamma}\right)_i [-\cos\gamma]_i \; .$$

With $\lambda = 6°$, $A = 26$ and with the aid of column 1, Table 1 we have

$$0\cdot3071C = 0\cdot0872 - 0\cdot0099 + 0\cdot1842 ,$$

i.e., $C = 0\cdot851$.

Thus if we add (*see* (43)) $0\cdot851 \sin\gamma$ to $d\bar{L}_m/d\gamma$ in (120°, 180°) we obtain column 6 which *does* satisfy the closure condition.

Equation (52) yields $Q = -0\cdot1221$, and from (80), in which the sum is evaluated by multiplying column 2, Table 1 into column 6, Table 4, we find

$$L_m(\pi) = 0\cdot2165.$$

Remembering that we have assumed $dL_m/d\gamma$ to be constant in each interval we can integrate from $\gamma = \pi$ back to $\gamma = 0$ to find $L_m(\gamma)$ (column 7), and then use linear interpolation to find $L_s(\gamma)$ at the mid-range points (column 8). Over the last interval

$$\int_0^\lambda \frac{dL_m}{d\gamma}\,d\gamma = A\int_0^\lambda \sin\gamma\,d\gamma \simeq \tfrac{1}{2}A\lambda^3 ,$$

i.e., $L_m(0°) \equiv h = L_m(6°) - \frac{1}{2}A\lambda^2$. $L_s(3°)$ follows from (26), *i.e.*,

$$L_s(3°) = L_m(0°) + \tfrac{1}{8}A\lambda^2 + \log\frac{\sin\frac{1}{2}\lambda}{\sin\frac{1}{4}\lambda} = 0\cdot1816 + 0\cdot0356 + 0\cdot6932 = 0\cdot9104 ;$$

while for $L_s(175°)$ we have

$$L_s(175°) = \tfrac{1}{2}\{L_m(\pi) + L_m(170°)\} + \frac{2\tau}{\pi}\log\frac{\sin\frac{1}{2}\varepsilon}{\sin\frac{1}{4}\varepsilon}$$

$$= \tfrac{1}{2}(0\cdot2068 + 0\cdot2165) + \frac{1}{10} \times 0\cdot6932 = 0\cdot2810 .$$

Column 10 follows from columns 1 and 9, the first entry being, from equation (33), $2\,e^h \sin\frac{1}{2}\lambda = 2\,e^{0\cdot1816}\sin 3° = 0\cdot1255$.

Equation (37) for a symmetrical aerofoil becomes

$$\left(\frac{pU}{4a}\right) = \tfrac{1}{2}\int_0^\pi \frac{U}{q}\sin\gamma\,d\gamma .$$

and so using column 10, and the integration formulae of Appendix IV we find that $(pU/4a) = 0\cdot9115$. Equation (50) becomes

$$\frac{p}{R_n} = \frac{0\cdot9115 \times 2}{\pi \times 0\cdot1255}\left\{19\cdot115 + 0\cdot2093A + \sum_{i=3}^{n-1}\left(\frac{dL_m}{d\gamma}\right)_i\left[2\log\sin\frac{\gamma}{2}\right]_i\right\} ,$$

which, with the aid of column 3, Table 1, gives $p/R_n = 92\cdot137$. If in equation (50) we replace A by $A + \Delta A$, when h will be replaced by $h - \frac{1}{2}\Delta A\delta^2 = h - 0\cdot0217\Delta A$ ($\delta = 12°$), the first two entries in column 6 will be altered, and then

$$\frac{p}{R_n} = 4\cdot6235\,e^{0\cdot0217\Delta A}\,(19\cdot928 + 0\cdot435\Delta A) = 100 .$$

Therefore $\quad \Delta A \simeq 2$.

Thus to obtain the required nose radius we should take $28\sin 3°$ and $-\frac{1}{2}\cot 4\frac{1}{2}° + 28\sin 9° = -1\cdot9730$ as the first two entries in column 6.

However, if $dL_m/d\gamma > \frac{1}{2}\cot(4\frac{1}{2}° + \alpha) - \frac{1}{2}\cot 4\frac{1}{2}° = -2\cdot0317$, $dL'/d\gamma > 0$, and an adverse gradient pressure exists. We shall therefore take $A = 29$ in (0°, 6°), and fixthe value of $dL_m/d\gamma$ at $\gamma = 9°$ to be $-2\cdot0371$; this increase in A will probably compensate for the lower value of $dL/_m d\gamma$ at $\gamma = 9°$. With these values in column 6 and the rest of the entries unchanged, we find that now $0\cdot3071C = -0\cdot0038$; Therefore $C = -0\cdot0124$. Applying this correction over the range (120°, 180°) we obtain column 11, then, just as before, we find $L_m = 0\cdot2133$, and are able to complete columns 12 to 16. Column 18 (*see* equation (31)) follows from columns 5 and 6 of Table 1, while column 17 follows from column 11 and Table 2. The first entry in column 11 is $29\sin 3° = 1\cdot5179$ (*see* paragraph following equation (23)). Columns 19, 20 and 22 are obvious, and then equation (8) enables us to fill in columns 21 and 23. The integration formulae of Appendix IV are used at this stage. Columns 24 and 25 follow from 21 and 23 on division by $(cU/2a)$—the last entry of column 23. It will be noticed from column 25 that the aerofoil fails to 'close' by a small amount ($0\cdot0006$), but this is an inevitable consequence of our numerical methods. It is eliminated simply by introducing a thin wedge into the profile—and this in turn introduces some slight uncertainty into the velocity distribution. If we have a symmetrical wedge such that θ has discontinuities of τ at $\gamma = 0$ and $-\tau$ at $\gamma = \pi$, then from the compressible flow equation corresponding to (22) we find

$$r(\pm\gamma) = -\frac{2\tau}{\pi}\log\tan\tfrac{1}{2}\gamma .$$

For a wedge of $\tau = 0\cdot0006$, $r(\pm\gamma) \simeq 0\cdot0004 \log \tan \frac{1}{2}\gamma$, which exceeds unity in the third figure after the decimal place only outside the range $12' < \gamma < 89°48'$. Column 26 sets out the adjusted values of (y/c).

Using equation (50) in which $(pU/4a)$ is replaced by $(cU/4a)$ we find that $c/R_n = 99\cdot78$, which is within the specified range for this parameter. Also $C_L = (2\pi \sin 2°6')/(cU/4a) = 0\cdot259$. Finally, L', and hence q'/U, follows from column 13, and equation (7). The shape and velocity distributions for this aerofoil are shown in Fig. 2.

12. *Aerofoil II.—Specification:* Symmetrical, T.E. angle 12°, $c/R_n \simeq 50$, q/U varying linearly from $1\cdot1794$ at $x/c \simeq 0\cdot25$ to $1\cdot0766$ at $x/c \simeq 0\cdot75$, $M_\infty = 0$.

Thus between 60° and 120° (these values corresponding roughly to $x/c = 0\cdot25$ and $x/c = 0\cdot75$) we have

$$\frac{dL}{d\gamma} = \frac{2a \sin\gamma}{pq}\frac{dL}{d\gamma} = \frac{1}{2}\left(\frac{4a}{pq}\right) \sin\gamma \times \frac{0\cdot1650 - 0\cdot0740}{0\cdot5} \simeq 0\cdot0910 \sin\gamma \, ,$$

the last step being on the assumption that

$$\frac{4a}{pq} \simeq 1 \, . \quad \text{With } \frac{p}{R_n} = 50 \, , \left(\frac{pU}{4a}\right) = 0\cdot9 \text{ and } \delta = 12°$$

in equation (36), we find $A \simeq 11$, and so we put $11 \sin 3°$ and $11 \sin 9° - \frac{1}{2} \cot 4\frac{1}{2}°$ in the first two places of column 2, Table 5. The values of $d\tilde{L}_m/d\gamma$ in (60°, 120°) are $0\cdot0910 \sin\gamma$; the values in (12°, 60°) have been written down arbitrarily, while the values in (120°, 170°) are $\frac{1}{15} \tan \frac{1}{2}\gamma$. It follows from equation (36) that near a discontinuity in θ of τ, $dL_s/d\gamma \sim \tau/\pi \tan \frac{1}{2}\gamma$, and $dL_m/d\gamma = 0$ at $= 175°$. For this aerofoil $\tau/\pi = 1/30$, and so by taking $d\tilde{L}_m/d\gamma = \frac{1}{15} \tan \frac{1}{2}\gamma$ instead of the seemingly more appropriate value $\frac{1}{30} \tan \frac{1}{2}\gamma$, we cause concavities to appear in the surface near the trailing edge. We have introduced these concavities so that we can eliminate them in the next section thereby illustrating the theory of section 3.

At $\gamma = 120°$ we wish to fix the value of q/U as $1\cdot0766$, i.e., $L(120°) = -0\cdot0740$. Using an obvious notation, we can write equation (44) as

$$\begin{aligned} \tilde{L}_m(\pi) &= L_m(120°) + \sum_{120°}^{180°} \left(\frac{d\tilde{L}_m}{d\gamma}\right)_i [\gamma]_i \\ &= -0\cdot0740 + \frac{\pi}{3} \sum_{120°}^{180°} \left(\frac{d\tilde{L}_m}{d\gamma}\right)_i \\ &= 0\cdot1543. \end{aligned}$$

From (51) and (52), in which $\tau = 6°$, we find $P = 0\cdot0930$ and $Q = -0\cdot1163$. Taking $\sigma = \delta = 60°$ and using these values of P, Q, and $\tilde{L}_m(\pi)$ in equations (47) and (48) we have

$$-0\cdot3424B + 0\cdot3424C = 0\cdot000383A - 0\cdot1163 - 0\cdot1543 + \sum_{i=3}^{n-1} \left(\frac{dL_m}{d\gamma}\right)_i [\tfrac{1}{2}\gamma^2]_i$$

$$0\cdot3071B + 0\cdot3071C = -0\cdot000383A + 0\cdot0930 - \sum_{i=3}^{n-1} \left(\frac{dL_m}{d\gamma}\right)_i [-\cos\gamma]_i \, .$$

Evaluating the sums in the usual way, putting $A = 11$, and solving these simultaneous equations, we find $B = 0\cdot0496$, $C = 0\cdot0706$. Column 3 is now obtained from column 2 and equation (43). Columns 4 to 17 now follow just as for the corresponding columns in Table 4. An application of equation (50) yields $c/R_n = 52\cdot8$. The aerofoil profile and velocity distribution are shown in Fig. 3.

13. *A Modification to Aerofoil II.*—Column 10, Table 5 shows that Aerofoil II has concavities near the trailing edge. We shall calculate the effect on the velocity distribution of changing the rear 28 per cent of the aerofoil into a simple wedge and thus eliminating these concavities. Table 8 sets out the calculation : column 1 is taken from column 10, Table 5 ; column 2 follows from column 1 by interpolation, and column 3 is calculated from the first differences of column 2. Interpolating in column 16, Table 5, we find $x/c = 0\cdot7225$ at $\gamma = 120°$. We shall start the wedge at this point. We therefore fill in column 4 in (125°, 175°) so that $d\theta/d\gamma + d\bar{\theta}/d\gamma = 0$. We shall also enter non-zero values of $d\bar{\theta}/d\gamma$ at $\gamma = 95°$, 105° and 115° so that the modified rear section joins the front section smoothly. $d\bar{\theta}/d\gamma$ will be zero in (0°, 85°). If, for convenience, we assume that $d\bar{\theta}/d\gamma$ has a constant value, say $(d\theta/d\gamma)^*$ at $\gamma = 95°$, 105° and 115°, then, with an obvious notation, the first two of equations (25) can be written

$$\sum_{120°}^{180°}\left(\frac{d\bar{\theta}}{d\gamma}\right)_i\left[\sin\gamma\right]_i - \tau_b + \left(\frac{d\theta}{d\gamma}\right)^*\left[\sin\gamma\right]_{90°}^{120°} = 0$$

and

$$\sum_{120°}^{180°}\left(\frac{d\bar{\theta}}{d\gamma}\right)_i\left[\gamma\right]_i + \tau_b + \left(\frac{d\theta}{d\gamma}\right)^*\left[\gamma\right]_{90°}^{120°} = 0\ .$$

Solving these equations with the aid of Table 1 we find $(d\theta/d\gamma)^* = -0\cdot0415$ and $\tau_b = 0\cdot0284$, which values are entered in column 4. This procedure ensures that the modified profile is closed. Column 6 is obtained by integrating column 5, while column 7 follows from column 4 and Table 3 (*see* equation (23)). Column 8 is the sum of columns 5, Tables 5 and 7, Table 8 ; column 9 is the required modified velocity distribution. The modification alters all of column 6, and half of column 10, Table 5. All of column 13 and 16 will therefore be changed, but only by very small amounts over (0°, 90°). The modified profile and distribution are shown in Fig. 3.

The same procedure applies to compressible flow with the exception that Table 6 replaces logarithm tables.

14. *Compressible Flow Version of Aerofoil II.—Specification :* As for Aerofoil II, but with $M_\infty = 0\cdot7$.

It is first necessary to construct Table 6 setting out r and β as differenced functions of q/U. Column 2 of this table was calculated from the equation preceding (63), and column 4 was obtained by numerical integration of column 3 (*see* equation (55)).

For Aerofoil II $(cU/4a) = 0\cdot9026$ (foot of column 15, Table 5), and $c/R_n = 52\cdot8$; we shall suppose that compressibility changes c/R_n to $c/R_n\beta_\infty$ and that $cU/4a$ is unchanged, then an application of equation (76), with $\delta = 12°$, yields $A \simeq 22$. This value of A enables us to make the first two entries in column 3, Table 7. Column 1 of this table was obtained by taking antilogarithms of column 4, Table 5, while column 2 was obtained from column 1 and Table 6. Column 3 follows from column 2, and as we are assuming that $d\tilde{r}_m/d\gamma$ is constant in each interval, these derivatives are obtained directly from the first differences. An application of equations (47) and (48) to column 3 yields

$$-0\cdot3424B + 0\cdot3424C = -0\cdot0401 + 0\cdot00294\Delta A$$

$$0\cdot3071B + 0\cdot3071C = 0\cdot0183 - 0\cdot00294\Delta A,$$

where the term ΔA gives the effect of changing A to $A + \Delta A$ in the range (0°, 12°) on these two equations. We want to keep the velocity distribution of Aerofoil III as close as possible to that of Aerofoil II, and so if we take $\Delta A = 9\cdot730$ in the above equations we find that $B = 0$, and $C = -0\cdot0355$, when the distribution for this aerofoil will be exactly the same as for Aerofoil II in the range (12°, 120°). It is interesting to note that, from this example, it appears that a distribution giving a closed profile at one Mach number does not generally produce a closed profile at another Mach number. Applying (43), in which $\tilde{L}_m$ is replaced by $\tilde{r}_m$, and

$A = 31\cdot73$, we obtain a new column $dr_m/d\gamma$. Checking this new column (always a necessary step) in equations (47) and (48) we find that, $B = C = 0$, if $A = -0\cdot072$. This adjustment yields column 4. The remaining columns of this Table follow as for the previous aerofoils, with only two exceptions—column 7 must be obtained from column 6 with the aid of Table 6, and the value of $U \sin \gamma/q$ at $\gamma = 0$ must be calculated from equation (89), Appendix III. The aerofoil and corresponding velocity distribution is shown in Fig. 3. Also shown in this figure is the profile obtained by multiplying column 17 of Table 5 by β_∞, *i.e.*, the profile predicted by linear pertubation theory (*see* column 18, Table 5). We see that the point of maximum thickness, linear pertubation theory, yields only 80 per cent of the reduction in profile thickness obtained by the author's theory.

15. *Aerofoil IV: An Asymmetric Aerofoil with Both Surfaces Designed at the Same Incidence.—Specification:* q'/U to have the following values:—

γ (deg)	40 to 90	100	110	120	130	140	-40 to -140
q'/U	$1\cdot2675$	$1\cdot2241$	$1\cdot1845$	$1\cdot1492$	$1\cdot1177$	$1\cdot0910$	$1\cdot0366$

T.E. angle $= 12°$, $M_\infty = 0\cdot7$, and

(*a*) no adverse pressure gradients near the nose at incidence,

(*b*) velocity peak on lower surface at zero incidence to be (at worst) of about the same order of magnitude as the upper surface magnitude of $1\cdot2675$, at incidence.

Using Table 6 and the specified values of q'/U we can set out columns 1 to 6 in Table 9, in the range (40°, 140°). The other figures appearing in these columns are filled in at later stages of the calculation. In (140°, 180°) we can assume any reasonable distribution (vanishing at 180°) for r_a' (column 5). We have taken $r_a = \{(180° - \gamma)/180°\}\{0\cdot0169\}$. In (0°, 40°) however we have to take some care to satisfy conditions (*a*) and (*b*) of the specifications. We now have to estimate α. To do this we fill in column 7 in (40°, 140°), and integrate it with respect to γ. We now put $\delta = 40°$ in equation (91), Appendix V, and assume that the last two terms in the bracket cancel. We then find $\sin 2\alpha = \dfrac{2}{\pi(1 - \frac{2}{9})} \times 0\cdot07595 \simeq 0\cdot0622$, *i.e.*, $\alpha = 1°47'$.

Condition (*a*) of the specification will be satisfied if, in (6°, 40°) $r(-\gamma) = r'(\gamma)$

$$= r(\gamma) - \log \frac{\sin(\frac{1}{2}\gamma + \alpha)}{\sin \frac{1}{2}\gamma}, \text{ from (7)},$$

i.e.,

$$r_a = \tfrac{1}{2} \log \frac{\sin(\frac{1}{2}\gamma + \alpha)}{\sin \frac{1}{2}\gamma},$$

in which we put $\alpha = 1°47'$. r_a at $\gamma = 0$, must, of course, vanish. We can thus fill in columns 8 and 9 in (0°, 40°). Another application of equation (91), but this time using column 9 and $\alpha = 1°47'$, yields

$$\sin 2\alpha = \frac{18}{7\pi}(0\cdot07595 - 0\cdot01017 - 0\cdot00841 + 0\cdot01980) = 0\cdot06316,$$

i.e., $\alpha = 1°49'$, a value so close to 1°47′ that we shall leave column 8 (and hence 9) unaltered in (0°, 40°). With this new value of α we can now fill in columns 11 and 12, and then, from the second of equations (12) we can fill in column 8 in (40°, 180°) from column 6, and column 6 in (0°, 40°) from column 8. Column 10 follows from the first differences of column 8. From the first of equations (12) we can now fill in column 13, and hence column 14, in the range (40°, 140°). Condition (*a*) of the specification requires that in (0°, 40°)

$$\frac{q'}{U} \leqslant 1\cdot2675, \text{ i.e., } r' \geqslant -0\cdot1394\,; \text{ therefore } r_s' \geqslant -0\cdot1394 - r_a',$$

and so from (12),

$$r_m \geqslant -0\cdot1394 - r_a' + \tfrac{1}{2} \log \frac{\sin(\frac{1}{2}\gamma + \alpha)\sin(\frac{1}{2}\gamma - \alpha)}{\sin \frac{1}{2}\gamma} \quad \ldots \quad (81)$$

These limiting values of r_m are entered in column 13. Since $r_m(40°)$ must remain unaltered, the corresponding values of $d\tilde{r}_m/d\gamma$ in (0°, 40°) must not be exceeded or condition (81) will be violated. The values assumed for $d\tilde{r}_m/d\gamma$ in (140°, 180°) are $\frac{1}{15}\tan\frac{1}{2}\gamma$, (*see* column 2, Table 5).

With $\sigma = \delta = 40°$, equations (47) and (48) become $-0\cdot1089B + 0\cdot1080C = -0\cdot1163 + 0\cdot000383A + 0\cdot5308 - \pi \times 0\cdot1351$ and $0\cdot1029B + 0\cdot1029C = 0\cdot0930 - 0\cdot000383A - 0\cdot0616$, which are satisfied by $A = 54\cdot67$, $B = 0$, and $C = 0\cdot1019$. B is taken to be zero, as a positive value would violate (81) and a negative value would increase A, which must not exceed $74\cdot22$ (*see* Appendix VI, equation (93), in which $\lambda = 6°$, $\alpha_2 = 0°$ and $\alpha_1 = 1°49'$). A reapplication of equations (47) and (48) to the new column $dr_m/d\gamma$ given by (43), yields $A = 54\cdot89$, $C = 0\cdot0009$, $B = 0$; column 15 is the final result. Integrating column 15 we can now complete column 13, which enables us to complete columns 5, 3, 4, 1 and 2 in that order. Columns 16 and 17, and hence 18 and 19, follow from columns 13 and 8. Columns 1 and 2 of Table 10 are then obtained with the aid of Table 6. Columns 20 and 22 of Table 9 follow from Table 2 and column 15, and Table 3 and column 10, respectively (*see* equation (31)). Column 21 happens to be the same as column 10, Table 7. Columns 20, 21 and 22 yield column 4, Table 10. The aerofoil co-ordinates given in columns 11 and 12 of Table 10, are obtained just as for the other aerofoils. The velocity distributions and aerofoil shape are shown in Fig. 4.

16. *Aerofoil V: An Asymmetric Aerofoil with the Surfaces Designed for Difference Incidences.—Specification :*

γ (deg)	40 to 90	90 to 140
$\frac{q'}{U}(\gamma)$	$1\cdot28$	$1\cdot28 + 0\cdot2611\cos\gamma$
$\frac{q}{U}(-\gamma)$	$1\cdot16$	$1\cdot16 + 0\cdot1044\cos\gamma$,

$M_\infty = 0\cdot7$, T.E. angle = 24°, and

$$\left.\begin{array}{l}\dfrac{q'}{U} \leqslant 1\cdot28 \text{ in } (0°, 40°)\\[2ex] \dfrac{q}{U} \leqslant 1\cdot16 \text{ in } (0°, -40°)\end{array}\right\} \qquad \text{(82)}$$

From the specified velocity distribution, columns 1 to 5, Table 11, can be completed in (40°, 140°). Column 5 is completed in (0°, 40°) on the assumption that $\{r'(\gamma) - r(-\gamma)\}$ remains constant and equal to its value at $\gamma = 40°$, while the values in (140°, 180°) are assumed to vanish. These assumptions are only temporary, and are made merely to enable us to estimate an appropriate value of α from equation (92), Appendix V. In this way we find that $\alpha \simeq 1°21'$—we shall take a smaller value, say $\alpha = 1°12'$. Column 7 follows from columns 3 and 6, (*see* equation (7)), and then columns 8 and 9 can be calculated—but only in the range (40°, 140°). Column 9 is completed in (140°, 180°) arbitrarily, except that $r_a(180°) = 0$.

The inequalities (82) are equivalent to

$$r'(\gamma) \geqslant -0\cdot1433, \quad r(-\gamma) \geqslant -0\cdot0959.$$

From equation (7) it follows that

$$r_s(\gamma) \geqslant \tfrac{1}{2}(-0\cdot1433 - 0\cdot0959) + \tfrac{1}{2}\log\frac{\sin(\frac{1}{2}\gamma + \alpha)}{\sin\frac{1}{2}\gamma}, \qquad \text{(83)}$$

while assuming the equalities to hold

$$r_a(\gamma) = \tfrac{1}{2}(0\cdot0959 - 0\cdot1433) + \tfrac{1}{2}\log\frac{\sin(\frac{1}{2}\gamma + \alpha)}{\sin\frac{1}{2}\gamma}\,. \qquad (84)$$

Column 9 was completed in (6°, 40°) from equation (84), but these figures were entered in pencil only. Column 10 then follows from the first differences of column 9. Putting $\alpha = 0$ in equation (49) we have

$$S\left(\frac{\delta\sin\lambda}{\lambda} - \sin\delta\right) = \sum_{i=2}^{n-i}\left(\frac{dr_a}{d\gamma}\right)_i [\sin\gamma]_i\ .$$

With $\delta = 40°$, $\lambda = 6°$ in this equation, we find from column 10, Table 11, and column 4, Table 1, that $S = -0\cdot0684$, and so $R \equiv -S(\delta - \lambda)/\lambda = 0\cdot3876$. Equations (45) then yield column 11. Integrating column 11 we can now write in the final values of r_a in column 9, in (0°, 40°), over the original pencilled values. Column 12 follows from column 8 in (40°, 140°), from the equality in (83) in (6°, 40°), and from

$$\frac{d\tilde{r}_m}{d\gamma} = \frac{\tau}{\pi}\tan\tfrac{1}{2}\gamma = \tfrac{1}{15}\tan\tfrac{1}{2}\gamma \text{ in } (140°, 180°)$$

(*see* remarks in first paragraph of section 12). Column 13 is the result of applying equations (47) and (48) to column 12 in just the same way as for Aerofoil IV. Columns 14 to 22 can now be deduced in turn, and columns 1 to 8 can be completed outside the range (40°, 140°). Table 12 sets out the final stages of the calculation leading to the aerofoil co-ordinates. The velocity distributions and aerofoil shape appear in Fig. 5.

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APPENDIX I

Velocity Distribution Specified as a Function of Perimeter Distance

If we take the origin of s at the trailing edge where $\phi = 2a$, then, since $d\phi/ds = q$,

$$\phi = \int_0^s q\,ds + 2a\ ,$$

i.e.,

$$-2a(1 + \cos\gamma) = p\int_0^{s/p} q\,d\left(\frac{s}{p}\right)\,.$$

$$\text{Therefore } \cos^2 (\tfrac{1}{2}\gamma) = -\left(\frac{pU}{4a}\right)\int_0^{s/p} \frac{q}{U}\, d\left(\frac{s}{p}\right). \qquad (85)$$

This equation gives the relation between γ and s/p if q/U is specified as a function of s/p. When $\gamma = 0$, $s \simeq p$ (equivalent to assuming that the front stagnation point for zero circulation is at the semi-perimeter point), and (85) becomes

$$\frac{4a}{pU} = -\int_0^1 \frac{q}{U}\, d\left(\frac{s}{p}\right),$$

and so
$$\cos^2 (\tfrac{1}{2}\gamma) = \int_0^{s/p} \frac{q}{U}\, d\left(\frac{s}{p}\right) \Big/ \int_0^1 \frac{q}{U}\, d\left(\frac{s}{p}\right). \qquad (86)$$

With this relation between γ and s/p we can readily find $q = q(\gamma)$ from the given $q(s/p)$ distribution.

A rough approximation is given by $q = U$, when $\cos^2 (\tfrac{1}{2}\gamma) = s/p$. This approximation is useful in the case of incidence. Using it we can find $q'(\gamma)$ from the given $q'(s/p)$. $L'(\gamma)$ and $L_a'(\gamma)$ can then be calculated ; α follows from equation (21). From (5)

$$\frac{q'}{U} = \frac{\sin (\tfrac{1}{2}\gamma + \alpha)}{\sin \tfrac{1}{2}\gamma} \frac{q}{U},$$

and so we can find $q/U(\gamma)$, which can then be used in

$$\frac{s}{p}(\gamma) = \left(\frac{2a}{pU}\right)\int_0^{\gamma} \frac{U}{q} \sin \gamma \, d\gamma,$$

and
$$\frac{pU}{2a} = \int_0^{\pi} \frac{U}{q} \sin \gamma \, d\gamma ,$$

to find a new relation between γ and s. This new relation enables us to find $q'(\gamma)$ from the given $q'(s/p)$, and so on. For conventional aerofoils only one or two iterations are necessary.

APPENDIX II

Evaluation of $J(\lambda, \gamma) \equiv -\dfrac{1}{2\pi}\displaystyle\int_0^{\lambda} \cot \tfrac{1}{2}\gamma^* \log \frac{\sin \tfrac{1}{2}(\gamma^* - \gamma)}{\sin \tfrac{1}{2}(\gamma^* + \gamma)}\, d\gamma^*$

If $p = \tan \tfrac{1}{2}\gamma$, $t = \tan \tfrac{1}{2}\gamma^*$, $a = \tan \tfrac{1}{2}\lambda$, then

$$J = -\frac{1}{2\pi}\int_0^a \log\left(\frac{1 - t/p}{1 + t/p}\right) \frac{2\, dt}{t(1 + t^2)},$$

and so for $p > t$, $t < 1$,

$$J = \frac{2}{\pi p}\int_0^a \left(1 + \frac{1}{3}\left(\frac{t}{p}\right)^2 + \frac{1}{5}\left(\frac{t}{p}\right)^4 + \ldots\right)(1 - t^2 + t^4 - \ldots)\, dt$$

$$= \frac{2}{\pi p}\left\{a + \frac{1}{3}\left(\frac{1}{3p^2} - 1\right)a^3 + \frac{1}{5}\left(\frac{1}{5p^4} - \frac{1}{3p^2} + 1\right)a^5 + \ldots\right\},$$

which converges rapidly. This series enables all of column **5**, Table **1**, except the first entry to be calculated. For the first entry, $p < t$, and we proceed as follows :—

$$J(\lambda, \gamma) = -\frac{1}{2\pi}\int_0^\pi \log\frac{\sin\frac{1}{2}(\gamma^* - \gamma)}{\sin\frac{1}{2}(\gamma^* + \gamma)}\cot\tfrac{1}{2}\gamma^*\, d\gamma^* + \frac{1}{2\pi}\int_\lambda^\pi \log\frac{\sin\frac{1}{2}(\gamma^* - \gamma)}{\sin\frac{1}{2}(\gamma^* + \gamma)}\cot\tfrac{1}{2}\gamma^*\, d\gamma^*$$

$$= \frac{\pi}{2} + \frac{1}{2\pi}\int_0^{\pi-\lambda} \log\frac{\cos\frac{1}{2}(\gamma^* + \gamma)}{\cos\frac{1}{2}(\gamma^* - \gamma)}\tan\tfrac{1}{2}\gamma^*\, d\gamma^*$$

$$= \frac{\pi}{2} + \frac{1}{2\pi}\int_0^{\cot\frac{1}{2}\lambda} \log\left(\frac{1 - tp}{1 + tp}\right)\frac{2t\, dt}{(1 + t^2)}\, .$$

If $tp = x$ and $k = \tan\frac{1}{2}\gamma/\tan\frac{1}{2}\lambda$, then,

$$J(\lambda, \gamma) = \frac{\pi}{2} + \frac{1}{\pi}\int_0^k \log\left(\frac{1 - x}{1 + x}\right)\frac{x\, dx}{p^2 + x^2}\, ,$$

and so if $x < 1$, which is the case if $k < 1$,

$$J(\lambda, \gamma) = \frac{\pi}{2} - \frac{2}{\pi}\int_0^k \frac{x^2}{p^2 + x^2}\left(1 + \frac{x^2}{3} + \frac{x^4}{5} + \ldots\right) dx$$

$$= \frac{\pi}{2} - \frac{2}{\pi}\left[\left(k - p\tan^{-1}\frac{k}{p}\right)\left(1 - \frac{p^2}{3} + \frac{p^4}{5} - \ldots\right) + \frac{k^3}{3}\left(\frac{1}{3} - \frac{p^2}{5} + \frac{p^4}{7}\ldots\right)\right.$$

$$\left. + \frac{k^5}{5}\left(\frac{1}{5} - \frac{p^2}{7} + \ldots\right) + \ldots\right].$$

Now $k = (\tan\frac{1}{4}\lambda)/(\tan\frac{1}{2}\lambda) \simeq \frac{1}{2}$, for the first entry in column **5**, Table **1**, and so this expansion is valid in this case.

Column **6** of Table **1** was obtained similarly.

APPENDIX III

Relation Between 'r' and 'L' near a Stagnation Point

From equation (55) $r = \int (1 - M^2)^{1/2}\, dL$,

but
$$\left(\frac{q}{U}\right)^2 = \frac{M^2}{M_\infty^2}\frac{\left(1 + \frac{\gamma - 1}{2}M_\infty^2\right)}{\left(1 + \frac{\gamma - 1}{2}M^2\right)} = e^{-2L}\, , \qquad \qquad (87)$$

and so, with $\gamma = 1\cdot4$, it follows that

$$r = -\int \frac{(1-M^2)^{1/2}\,dM^2}{2M^2(1+0\cdot2M^2)} = -\tfrac{1}{2}\int \{1-0\cdot7M^2+O(M^4)\}\frac{dM^2}{M^2}$$

$\simeq -\frac{1}{2}\log M^2 + 0\cdot35\,M^2 +$ constant, if M is small. Now $r = 0$, when $q = U$, i.e., $M = M_\infty$, and we can evaluate the constant in this equation. We find, with the aid of equation (87)

$$r = -\tfrac{1}{2}\log\frac{M^2}{M_\infty^{\;2}} + 0\cdot35(M^2 - M_\infty^{\;2}) = -\tfrac{1}{2}\log\left(\frac{q}{U}\right)^2 - \tfrac{1}{2}\log\left|\frac{1+0\cdot2M^2}{1+0\cdot2M_\infty^{\;2}}\right| + 0\cdot35\,(M^2 - M_\infty^{\;2}).$$

Thus, using (87), and ignoring terms $O(M^4)$, we find

$$r = L + \tfrac{1}{4}M_\infty^{\;2}\left\{\left(\frac{q}{U}\right)^2 - 1\right\}. \qquad \text{(88)}$$

This result enables us to calculate $\lim_{\gamma\to 0}(U/q)\sin\gamma$ for compressible flow. Corresponding to equation (26) we have

$$r_m = r_s + \log\frac{\sin\frac{1}{2}\gamma}{\sin\frac{1}{2}\lambda}, \quad 0 \leqslant \gamma \leqslant \lambda$$

i.e.,

$$L_s = r_m - \tfrac{1}{4}M_\infty^{\;2}\left\{\left(\frac{q}{U}\right)^2 - 1\right\} - \log\frac{\sin\frac{1}{2}\gamma}{\sin\frac{1}{2}\lambda}.$$

Therefore $\lim_{\gamma\to 0}\frac{U}{q}\sin\gamma = 2\{\exp(h + \frac{1}{4}M_\infty^{\;2})\}\sin\frac{1}{2}\lambda$, ... (89)

where $h \equiv r_m$ at $\gamma = 0$.

APPENDIX IV

Integration Formulae

Consider $\int_0^{5a} y\,d\gamma$, where y is given at $= 0$, a, $3a$ and $5a$. Fitting a third-degree curve through the ordinates we can readily deduce that the area under the curve is given by

$$\int_0^{5a} y\,d\gamma = \frac{5a}{96}(8y_0 + 25y_1 + 50y_3 + 13y_5) + O(a^4),$$

a result we use in the ranges (0°, 15°) and (180°, 165°) (*see* Table 4). Similarly if we have y at $\gamma = 0$, $4a$ and $9a$,

$$\int_0^{9a} y\,d\gamma = \frac{9a}{40}(5y_0 + 27y_4 + 8y_9) + O(a^3),$$

which is required for the range (27°, 45°). Otherwise we use the well-known formulae,

$$\int_0^{2a} y\,d\gamma = \frac{2a}{6}(y_0 + 4y_1 + y_2) + O(a^4),$$

and

$$\int_0^{2a} y\,d\gamma = \frac{3a}{8}(y_0 + 3y_1 + 3y_2 + y_3) + O(a^4).$$

APPENDIX V

Equations for the Angle of Incidence

From equation (70),

$$r_a' = r_a - \tfrac{1}{2}\log\frac{\sin(\frac{1}{2}\gamma+\alpha)}{\sin(\frac{1}{2}\gamma-\alpha)},$$

and so

$$\int_0^\delta r_a'\sin\gamma\,d\gamma = \int_0^\delta r_a\sin\gamma\,d\gamma - \tfrac{1}{2}\delta\sin 2\alpha - \tfrac{1}{2}(\cos 2\alpha - \cos\delta)\log\frac{\sin(\frac{1}{2}\delta+\alpha)}{\sin(\frac{1}{2}\delta-\alpha)},$$

but

$$\int_0^\pi r_a'\sin\gamma\,d\gamma = -\tfrac{1}{2}\pi\sin 2\alpha, \qquad (90)$$

and hence by subtraction we find

$$\sin 2\alpha = \left(\frac{2}{\pi-\delta}\right)\left[-\int_0^\pi r_a'\sin\gamma\,d\gamma - \int_0^\gamma r_a\sin\gamma\,d\gamma + \sin(\tfrac{1}{2}\delta-\alpha)\sin(\tfrac{1}{2}\delta+\alpha)\log\frac{\sin(\frac{1}{2}\delta+\alpha)}{\sin(\frac{1}{2}\delta-\alpha)}\right]. \qquad (91)$$

This result is used in the design of Aerofoil IV.

With an obvious notation we have (*see* equation (68)),

$$r_{a1}(\gamma) = r(\gamma) - \log\frac{\sin(\frac{1}{2}\gamma+\alpha_1)}{\sin\frac{1}{2}\gamma},\quad r(\gamma) = r_{a2}(\gamma) + \log\frac{\sin(\frac{1}{2}\gamma+\alpha_2)}{\sin\frac{1}{2}\gamma}.$$

therefore $\frac{1}{2}\{r_{a1}(\gamma) - r_{a1}(-\gamma)\} = \frac{1}{2}\{r_{a1}(\gamma) - r_{a2}(-\gamma)\} + \frac{1}{2}\log\dfrac{\sin(\frac{1}{2}\gamma-\alpha_1)}{\sin(\frac{1}{2}\gamma-\alpha_2)}.$

From (90)

$$\sin 2\alpha_1 = -\frac{2}{\pi}\int_0^\pi \tfrac{1}{2}\{r_{a1}(\gamma) - r_{a1}(-\gamma)\}\sin\gamma\,d\gamma$$

$$= -\frac{1}{\pi}\int_0^\pi\{r_{a1}(\gamma) - r_{a2}(-\gamma)\}\sin\gamma\,d\gamma - \frac{1}{\pi}\int_0^\pi\sin\gamma\log\frac{\sin(\frac{1}{2}\gamma-\alpha_1)}{\sin(\frac{1}{2}\gamma-\alpha_2)}\,d\gamma,$$

which, after some algebra, reduces to

$$\sin 2\alpha_1 + \sin 2\alpha_2 = -\frac{2}{\pi}\int_0^\pi\{r_{a1}(\gamma) - r_{a2}(-\gamma)\}\sin\gamma\,d\gamma - \frac{4}{\pi}\left\{\log\frac{\cos\alpha_1}{\cos\alpha_2} + \sin^2\alpha_1\log\tan\alpha_1 - \sin^2\alpha_2\log\tan\alpha_2\right\}.$$

If $\alpha_1 = \alpha$, and $\alpha_2 = 0$, this equation becomes

$$\sin 2\alpha = -\frac{2}{\pi}\int_0^{\pi} \{r_a(\gamma) - r(-\gamma)\} \sin \gamma \, d\gamma - \frac{4}{\pi}\{\log \cos \alpha + \sin^2 \alpha \log \tan \alpha\}, \qquad (92)$$

an equation used in the design of Aerofoil V.

APPENDIX VI

The Maximum Value of 'A' in Equation (35)

We shall suppose, as is commonly the case, that we have to satisfy the following inequalities near the nose (in order to avoid adverse pressure gradients) :

$$\left(\frac{q}{U}\right)_{a1} \leqslant K, \quad \lambda \geqslant \gamma \geqslant 0\,; \quad \left(\frac{q}{U}\right)_{a2} \leqslant H, \quad -\lambda \leqslant \gamma \leqslant 0,$$

where H and K are constants. These inequalities are equivalent to

$$r_{a1}(\gamma) \geqslant \bar{K}, \quad r_{a2}(-\gamma) \geqslant \bar{H}, \quad 0 \leqslant \gamma \leqslant \lambda,$$

where $\bar{K}$ and $\bar{H}$ are constants derived from K and H.

From equation (7) we then have

$$r(\gamma) \geqslant \bar{K} + \log \frac{\sin(\frac{1}{2}\gamma + \alpha_1)}{\sin \frac{1}{2}\gamma}, \quad 0 \leqslant \gamma \leqslant \lambda,$$

$$r(-\gamma) \geqslant \bar{H} + \log \frac{\sin(\frac{1}{2} - \alpha_2)}{\sin \frac{1}{2}\gamma}, \quad 0 \leqslant \gamma \leqslant \lambda,$$

therefore

$$r_s(\gamma) \geqslant \tfrac{1}{2}(\bar{K} + \bar{H}) + \tfrac{1}{2}\log \frac{\sin(\frac{1}{2}\gamma + \alpha_1)\sin(\frac{1}{2}\gamma - \alpha_2)}{\sin^2 \frac{1}{2}\gamma}, \quad 0 \leqslant \gamma \leqslant \lambda.$$

Differentiating this equation, and using the compressible flow version of (26), we find, on the assumption that $r_s(\lambda)$ is fixed in value, that

$$\frac{dr_m}{d\gamma} \leqslant \tfrac{1}{4}\{\cot(\tfrac{1}{2}\gamma + \alpha_1) + \cot(\tfrac{1}{2}\gamma - \alpha_2)\}, \quad 0 \leqslant \gamma \leqslant \lambda.$$

From (35) it follows that

$$A \leqslant \frac{1}{4 \sin \gamma}\{\cot(\tfrac{1}{2}\gamma + \alpha_1) + \cot(\tfrac{1}{2}\gamma - \alpha_2)\}.$$

But λ, α_1 and α_2 are usually small, so

$$A \leqslant \frac{1}{\gamma^2}\frac{\left(1 + \frac{\alpha_1 - \alpha_2}{\gamma}\right)}{\left(1 + \frac{2\alpha_1}{\gamma}\right)\left(1 - \frac{2\alpha_2}{\gamma}\right)}, \quad 0 \leqslant \gamma \leqslant \lambda,$$

will be sufficiently accurate. This equation will be satisfied if

$$A \leqslant \frac{1}{\lambda_2}\frac{\left(1 + \frac{\alpha_1 - \alpha_2}{\lambda}\right)}{1 + \left(\frac{2\alpha_1}{\lambda}\right)\left(1 - \frac{2\alpha_2}{\lambda}\right)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (93)$$

TABLE 1

	1	2	3	4	5	6
γ	$[-\cos\gamma]_i$	$\frac{1}{2}[\gamma^2]_i$	$[2\log\sin\frac{1}{2}\gamma]_i$	$[\sin\gamma]_i$	$J(6\text{ deg}, \gamma)$	$J(10\text{ deg}, \pi-\gamma)$
0					1·5708	0
3	0·0055	0·0055	—	+0·1045	1·26837	0·00146
9	0·0164	0·0146	1·3835	0·1034	0·44880	0·00438
15	0·0271	0·0275	0·8064	0·1011	0·25794	0·00733
21	0·0376	0·0384	0·5690	0·0977	0·18150	0·01032
27	0·0475	0·0494	0·4380	0·0933	0·13959	0·01337
35	0·1000	0·1066	0·5546	0·1428	0·10607	0·01756
45	0·1232	0·1372	0·4232	0·1232	0·08062	0·02307
55	0·1428	0·1675	0·3363	0·1000	0·06411	0·02899
65	0·1580	0·1980	0·2746	0·0737	0·05238	0·03548
75	0·1684	0·2284	0·2279	0·0451	0·04350	0·04273
85	0·1736	0·2589	0·1907	+0·0152	0·03639	0·05103
95	0·1736	0·2894	0·1601	−0·0152	0·03056	0·06078
105	0·1684	0·3198	0·1341	−0·0451	0·02558	0·07258
115	0·1580	0·3503	0·1108	−0·0737	0·02124	0·08739
125	0·1428	0·3807	0·0909	−0·1000	0·01736	0·10706
135	0·1232	0·4112	0·0723	−0·1232	0·01381	0·13480
145	0·1000	0·4416	0·0550	−0·1428	0·01051	0·17775
155	0·0737	0·4721	0·0387	−0·1580	0·00739	0·25519
165	0·0451	0·5026	0·0230	−0·1684	0·00439	0·44698
175	0·0152	0·5330	0·0076	−0·1736	0·00146	1·28485
180					0	1·5708

TABLE 2

$A \times 10^4$

γ		3	9	15	21	27	35	45	55	65	75
3		361	284	160	128	98	74	57	45	37	31
9		242	928	474	305	228	171	128	102	83	69
15		136	474	1097	607	414	298	220	173	140	116
21		96	305	607	1205	698	452	322	249	200	164
27		73	228	414	698	1284	675	443	333	265	216
35		93	286	499	761	1152	1986	1135	786	605	487
45		71	215	368	539	742	1135	2103	1232	868	673
55		56	170	288	415	557	786	1232	2185	1301	924
65		46	138	234	334	442	605	868	1301	2241	1346
75		38	115	193	274	361	487	673	924	1346	2276
85		32	96	161	228	299	399	543	719	959	1372
95		27	80	135	191	249	331	445	578	745	976
105		22	67	113	159	207	275	366	471	595	753
115		19	56	94	132	171	227	301	384	479	595
125		15	46	76	108	140	184	244	309	384	471
135		12	36	61	86	111	146	193	244	301	366
145		9	28	46	65	84	111	146	184	227	275
155		7	19	33	46	59	78	102	129	158	191
165		4	12	19	27	35	46	61	76	94	113
175		1	4	6	9	12	15	20	25	31	37

γ		85	95	105	115	125	135	145	155	165	175
3		26	20	19	15	13	10	8	6	4	0
9		57	48	40	34	27	22	17	12	7	2
15		97	81	68	56	46	36	28	20	12	4
21		137	114	96	79	65	51	39	27	16	5
27		179	149	124	103	84	67	51	36	21	7
35		399	331	275	227	184	146	111	78	46	15
45		543	445	366	301	244	192	146	102	61	20
55		719	578	471	384	309	244	184	129	76	25
65		959	745	595	479	384	301	227	158	94	31
75		1372	976	753	595	471	366	275	191	113	37
85		2293	1381	976	745	578	445	331	229	135	45
95		1381	2293	1372	959	719	543	399	275	161	53
105		976	1372	2276	1346	924	673	487	331	193	63
115		745	959	1346	2241	1301	868	605	405	234	76
125		578	719	924	1301	2185	1232	786	508	288	94
135		445	543	673	868	1232	2103	1135	669	368	118
145		331	399	487	605	786	1135	1986	995	499	156
155		229	275	331	405	508	669	995	1816	783	225
165		135	161	193	234	288	368	499	783	1542	402
175		45	53	63	76	94	118	156	225	402	915

TABLE 3

$B \times 10^4$

γ	0	3	9	15	21	27	35	45	55	65	75
0	—	23191	16203	12963	10838	9262	7650	6115	4919	3955	3160
3	2633	2546	1755	1376	1144	976	805	643	517	415	332
9	1710	1755	2168	1523	1207	1012	826	655	525	422	337
15	1362	1376	1523	1999	1390	1099	872	682	543	435	347
21	1137	1144	1207	1390	1890	1299	956	727	572	455	362
27	971	976	1012	1099	1299	1811	1117	797	614	484	384
35	1339	1343	1379	1458	1603	1890	2605	1627	1172	902	709
45	1070	1072	1094	1139	1215	1334	1627	2489	1529	1090	834
55	860	862	877	907	955	1026	1172	1529	2407	1461	1034
65	691	693	704	725	760	809	902	1090	1461	2351	1416
75	553	554	562	579	605	641	709	834	1034	1416	2315
85	436	437	444	458	479	507	560	652	788	998	1390
95	339	340	346	357	374	398	441	514	617	762	981
105	258	258	263	273	288	308	344	405	488	600	754
115	190	190	195	203	216	234	265	318	388	480	600
125	134	134	138	146	158	174	202	248	310	388	488
135	88	89	92	100	111	125	151	193	249	318	405
145	53	54	57	64	74	87	111	151	202	265	344
155	27	27	31	37	47	60	82	120	168	228	301
165	10	10	13	20	29	42	64	100	146	203	273
175	1	2	5	11	20	33	54	90	135	191	260
180	—	2	19	55	107	178	302	504	763	1084	1474

γ	85	95	105	115	125	135	145	155	165	175	180
0	2497	1940	1474	1084	763	504	302	153	55	6	—
3	262	204	155	114	80	53	32	16	6	0	0
9	266	207	158	117	83	55	34	18	8	3	2
15	274	214	164	122	87	60	38	22	12	7	6
21	287	224	173	130	94	66	44	28	17	12	11
27	304	239	185	140	104	75	52	36	25	19	19
35	560	441	344	265	202	151	111	82	64	54	53
45	652	514	405	318	248	193	151	120	100	90	88
55	788	617	488	388	310	248	202	168	146	135	134
65	998	762	600	480	388	318	265	228	203	191	190
75	1390	981	754	600	488	405	344	301	273	260	258
85	2298	1381	981	762	617	514	441	390	357	341	339
95	1381	2298	1390	998	788	652	560	497	458	439	436
105	981	1390	2315	1416	1034	834	709	628	579	555	553
115	762	998	1416	2351	1461	1090	902	791	726	695	691
125	617	788	1034	1461	2407	1529	1172	1000	907	865	860
135	514	652	834	1090	1529	2489	1627	1288	1139	1077	1070
145	441	560	709	902	1172	1627	2605	1767	1458	1351	1339
155	390	497	628	791	1000	1288	1767	2775	1978	1732	1708
165	357	458	579	726	907	1139	1458	1978	3049	2359	2284
175	341	439	555	695	865	1077	1351	1732	2359	3676	3821
180	1940	2497	3160	3955	4919	6155	7650	9743	12963	19941	—

TABLE 4

Aerofoil I

1	2	3	4	5	6	7	8	9	10	11	12	13
γ	$\frac{1}{2}\cot\frac{1}{2}\gamma$	$\frac{1}{2}\cot(\frac{1}{2}\gamma+\alpha)$	$\frac{dL'}{d\gamma}$	$\frac{d\bar{L}_m}{d\gamma}$	$\frac{dL_m}{d\gamma}$	L_m	L_s	$\frac{q}{U}(\gamma)$	$\frac{U\sin\gamma}{q}$	$\frac{dL_m}{d\gamma}$	$L_m(\gamma)$	$L_s(\gamma)$
0									0·1255			
						0·1816					0·1415	
3	19·0940	7·9475		1·3624	1·3624		0·9104	0·4024	0·1301	1·5179		0·8744
						0·3241					0·3005	
9	6·3530	4·3213		—2·2858	—2·2858		0·2044	0·8151	0·1919	—2·0317		0·1942
						0·0847					0·0878	
15	3·7979	2·9562	0	—0·8417	—0·8417		0·0407	0·9602	0·2695	—0·8417		0·0438
						—0·0033					—0·0003	
21	2·6977	2·2368	0	—0·4609	—0·4609		—0·0275	1·0281	0·3486	—0·4609		—0·0245
						—0·0516					—0·0486	
27	2·0827	1·7908	0	—0·2919	—0·2919		—0·0669	1·0691	0·4246	—0·2919		—0·0639
						—0·0822					—0·0791	
35	1·5858	1·4042	0	—0·1816	—0·1816		—0·0980	1·1031	0·5200	—0·1816		—0·0950
						—0·1138					—0·1108	
45	1·2071	1·0921	0	—0·1150	—0·1150		—0·1238	1·1318	0·6248	—0·1150		—0·1208
						—0·1339					—0·1309	
55	0·9605	0·8801	0	—0·0804	—0·0804		—0·1409	1·1513	0·7115	—0·0804		—0·1379
						—0·1479					—0·1449	
65	0·7849	0·7248	0	—0·0601	—0·0601		—0·1531	1·1654	0·7777	—0·0601		—0·1501
						—0·1584					—0·1554	
75	0·6516	0·6044	0	—0·0472	—0·0472		—0·1625	1·1763	0·8212	—0·0472		—0·1596
						—0·1667					—0·1637	
85	0·5457	0·5070	0	—0·0387	—0·0387		—0·1701	1·1852	0·8405	—0·0387		—0·1670
						—0·1734					—0·1704	
95	0·4581	0·4256	0	—0·0325	—0·0325		—0·1762	1·1925	0·8354	—0·0325		—0·1733
						—0·1791					—0·1761	
105	0·3837	0·3554	0	—0·0283	—0·0283		—0·1816	1·1987	0·8058	—0·0283		—0·1786
						—0·1840					—0·1810	
115	0·3186	0·2933	0	—0·0253	—0·0253		—0·1862	1·2049	0·7522	—0·0253		—0·1832
						—0·1884					—0·1854	
125	0·2603	0·2374		—0·0229	0·6742		—0·1296	1·1385	0·7195	0·6640		—0·1275
						—0·0708					—0·0696	
135	0·2071	0·1859		—0·0212	0·5804		—0·0202	1·0204	0·7028	0·5717		—0·0197
						0·0305					0·0302	
145	0·1577	0·1377		—0·0200	0·4681		0·0714	0·9311	0·6160	0·4610		0·0704
						0·1122					0·1106	
155	0·1108	0·0918		—0·0190	0·3406		0·1419	0·8677	0·4871	0·3353		0·1399
						0·1716					0·1692	
165	0·0659	0·0472		—0·0187	0·2015		0·1892	0·8276	0·3127	0·1983		0·1865
						0·2068					0·2038	
175	0·0219	0·0035		—0·0184	0·0558		0·2810	0·7550	0·1154	0·0547		0·2779
						0·2165					0·2133	
180									0			

TABLE 4—*continued*

Aerofoil I

14	15	16	17	18	19	20	21	22	23	24	25	26
γ	$\frac{q}{U}(\gamma)$	$\frac{U \sin \gamma}{q}$	θ_m	θ_J	θ	$\frac{U \sin \gamma}{q} \sin \theta$	$\frac{Uy}{2a}$	$\frac{U \sin \gamma}{q} \cos \theta$	$\frac{Ux}{2a}$	$\frac{x}{c}$	$\frac{y}{c}$	$(y/c)_m$
0		0·1205	0	+1·5708	+1·5708	+0·1205	−0·0011	0	0	0	−0·0006	0
3	0·4171	0·1255	+0·0137	1·2682	1·2819	0·1203	0·0052	0·0358	0·0008	0·00047	0·0029	0·0035
9	0·8235	0·1900	0·2101	0·4484	0·6585	0·1163	0·0172	0·1503	0·0109	0·00612	0·0097	0·0102
15	0·9572	0·2704	0·2117	0·2572	0·4689	0·1254	0·0300	0·2396	0·0314	0·01765	0·0169	0·0174
21	1·0249	0·3497	0·1813	0·1805	0·3618	0·1238	0·0430	0·3270	0·0610	0·03426	0·0242	0·0248
27	1·0661	0·4258	0·1550	0·1383	0·2933	0·1231	0·0560	0·4076	0·0997	0·05598	0·0314	0·0320
35	1·0992	0·5082	0·1251	0·1043	0·2294	0·1156	0·0747	0·4949	0·1619	0·09091	0·0419	0·0424
45	1·1286	0·6265	0·0934	0·0783	0·1717	0·1068	0·0920	0·6173	0·2594	0·1457	0·0517	0·0522
55	1·1479	0·7136	0·0659	0·0612	0·1271	0·0905	0·1115	0·7077	0·3755	0·2109	0·0626	0·0631
65	1·1621	0·7799	0·0398	0·0488	0·0886	0·0688	0·1233	0·7768	0·5052	0·2837	0·0692	0·0696
75	1·1730	0·8235	+0·0142	0·0392	0·0534	0·0438	0·1353	0·8223	0·6453	0·3624	0·0760	0·0764
85	1·1817	0·8430	−0·0128	0·0313	+0·0185	+0·0154	0·1384	0·8428	0·7907	0·4440	0·0777	0·0780
95	1·1890	0·8378	−0·0428	0·0245	−0·0183	−0·0153	0·1406	0·8376	0·9380	0·5267	0·0789	0·0791
105	1·1953	0·8081	−0·0802	0·0183	−0·0619	−0·0500	0·1328	0·8066	1·0815	0·6073	0·0746	0·0748
115	1·2010	0·7546	−0·1347	0·0125	−0·1222	−0·0920	0·1227	0·7490	1·2180	0·6840	0·0689	0·0691
125	1·1368	0·7206	−0·2385	0·0067	−0·2318	−0·1656	0·0989	0·7013	1·3435	0·7544	0·0555	0·0557
135	1·0199	0·6933	−0·2564	+0·0003	−0·2561	−0·1755	0·0684	0·6707	1·4638	0·8220	0·0384	0·0385
145	0·9321	0·6154	−0·2316	−0·0073	−0·2389	−0·1456	0·0399	0·5979	1·5751	0·8845	0·0224	0·0224
155	0·8694	0·4861	−0·1808	−0·0181	−0·1989	−0·0961	0·0187	0·4765	1·6697	0·9376	0·0105	0·0105
165	0·8299	0·3119	−0·1136	−0·0403	−0·1539	−0·0478	0·0063	0·3082	1·7387	0·9764	0·0036	0·0036
175	0·7574	0·1151	−0·0378	−0·1270	−0·1648	−0·0189	0·0009	0·1135	1·7757	0·9971	0·0005	0·0005
180		0	0	−0·1571	−0·1571	0	0	0	1·7808	1·0000	0	0

TABLE 5

Aerofoil II

1	2	3	4	5	6	7	8	9	10
γ	$\frac{d\tilde{L}_m}{d\gamma}$	$\frac{dL_m}{d\gamma}$	L_m	L_s	$\frac{q}{U}(\gamma)$	$\frac{U \sin \gamma}{q}$	θ_m	θ_s	θ
0			+0·3971		0	0·1558	0	+1·5708	+1·5708
3	+0·5757	+0·5757	−0·4574	+1·1054	0·3318	0·1580	+0·1034	1·2683	1·3717
9	−4·6326	−4·6248	−0·0269	+0·2153	0·8064	0·1941	0·4561	0·4485	0·9046
15	−0·9511	−0·9383	−0·1251	−0·0760	1·0791	0·2398	0·3120	0·2575	0·5695
21	−0·1000	−0·0822	−0·1337	−0·1294	1·1387	0·3148	0·1933	0·1808	0·3741
27	−0·0825	−0·0600	−0·1400	−0·1369	1·1472	0·3957	0·1383	0·1387	0·2770
35	−0·0825	−0·0540	−0·1494	−0·1447	1·1560	0·4962	0·0955	0·1049	0·2004
45	−0·0825	−0·0474	−0·1577	−0·1536	1·1663	0·6063	0·0622	0·0791	0·1413
55	−0·0825	−0·0419	−0·1650	−0·1613	1·1752	0·6970	+0·0224	0·0622	0·0846
65	+0·0825	+0·0825	−0·1506	−0·1578	1·1711	0·7739	−0·0189	0·0500	+0·0311
75	0·0879	0·0879	−0·1353	−0·1430	1·1539	0·8370	−0·0463	0·0407	−0·0056
85	0·0907	0·0907	−0·1194	−0·1274	1·1363	0·8766	−0·0673	0·0330	−0·0343
95	0·0907	0·0907	−0·1036	−0·1115	1·1181	0·8910	−0·0837	0·0265	−0·0572
105	0·0879	0·0879	−0·0883	−0·0960	1·1009	0·8774	−0·0982	0·0207	−0·0775
115	0·0825	0·0825	−0·0739	−0·0811	1·0847	0·8355	−0·1146	0·0154	−0·0992
125	0·1281	0·1862	−0·0414	−0·0576	1·0595	0·7731	−0·1349	0·0102	−0·1247
135	0·1609	0·2110	−0·0046	−0·0230	1·0234	0·6902	−0·1487	+0·0048	−0·1439
145	0·2114	0·2521	+0·0394	+0·0174	0·9829	0·5835	−0·1553	−0·0013	−0·1566
155	0·3007	0·3307	0·0971	0·0383	0·9340	0·4525	−0·1548	−0·0096	−0·1644
165	+0·5064	0·5248	0·1886	0·1428	0·8671	0·2985	−0·1356	−0·0254	−0·1610
175	0	+0·0062		+0·2353	0·7904	0·1103	−0·0383	−0·0842	−0·1225
180			+0·1897		0	0	0	−0·1047	−0·1047

	11	12	13	14	15	16	17	18
γ	$\frac{U \sin \gamma}{q} \sin \theta$	$\frac{Uy}{2a}$	$\frac{y}{c}$	$\frac{U \sin \gamma}{q} \cos \theta$	$\frac{Ux}{2a}$	$\frac{x}{c}$	(y/c_m)	$\beta_\infty (y/c)_m$
0	+0·1558	−0·0008	−0·0004	0	0	0	0	0
3	0·1549	+0·0072	+0·0040	0·0311	0·0007	0·0004	0·0044	0·0031
9	0·1526	0·0237	0·0130	0·1199	0·0088	0·0049	0·0135	0·0096
15	0·1294	0·0384	0·0213	0·2021	0·0256	0·0142	0·0217	0·0155
21	0·1151	0·0511	0·0283	0·2931	0·0514	0·0285	0·0288	0·0206
27	0·1083	0·0628	0·0348	0·3806	0·0869	0·0481	0·0354	0·0252
35	0·0988	0·0771	0·0427	0·4862	0·1459	0·0808	0·0431	0·0307
45	0·0854	0·0933	0·0517	0·6003	0·2426	0·1344	0·0520	0·0371
55	0·0590	0·1061	0·0588	0·6945	0·3539	0·1962	0·0591	0·0422
65	+0·0241	0·1134	0·0628	0·7735	0·4841	0·2682	0·0631	0·0450
75	−0·0046	0·1149	0·0637	0·8370	0·6250	0·3462	0·0640	0·0457
85	−0·0296	0·1120	0·0620	0·8763	0·7749	0·4292	0·0622	0·0443
95	−0·0511	0·1048	0·0581	0·8895	0·9293	0·5148	0·0583	0·0415
105	−0·0679	0·0945	0·0523	0·8749	1·0837	0·6003	0·0525	0·0375
115	−0·0827	0·0812	0·0450	0·8314	1·2331	0·6830	0·0451	0·0322
125	−0·0969	0·0657	0·0364	0·7669	1·3728	0·7604	0·0365	0·0260
135	−0·0991	0·0481	0·0266	0·6833	1·4997	0·8307	0·0267	0·0190
145	−0·0911	0·0317	0·0176	0·5765	1·6100	0·8918	0·0176	0·0125
155	−0·0740	0·0168	0·0093	0·4464	1·6995	0·9414	0·0093	0·0066
165	−0·0479	0·0065	0·0036	0·2946	1·7646	0·9774	0·0036	0·0026
175	−0·0135	+0·0006	+0·0003	0·1095	1·8004	0·9973	0·0003	0·0002
180	0	0	0	0	1·8054	1·0000	0	0

TABLE 6

$r = r(q/U)$, *at* $M_\infty = 0{\cdot}7$

1	2			3	4		
$\frac{q}{U}$	β	differences of $\beta \times 10^4$		$\frac{U\beta}{q}$	r	differences of $r \times 10^4$	
0·72	0·8703	−169		1·2088	0·2642	−466	
0·76	0·8534	−185	−16	1·1229	0·2176	−433	33
0·80	0·8349	−201	−16	1·0437	0·1743	−402	31
0·84	0·8148	−219	−18	0·9700	0·1341	−374	28
0·88	0·7929	−239	−20	0·9011	0·0967	−348	26
0·92	0·7690	−261	−22	0·8359	0·0619	−321	27
0·96	0·7429	−288	−27	0·7738	0·0298	−298	23
1·00	0·7141			0·7141	0		
1·00	0·7141	−154		0·7141	0	−140	
1·02	0·6987	−162	−8	0·6850	−0·0140	−134	
1·04	0·6825	−172	−10	0·6562	−0·0274	−128	7
1·06	0·6653	−180	−8	0·6276	−0·0402	−123	5
1·08	0·6473	−191	−11	0·5994	−0·0525	−117	6
1·10	0·6282	−202	−11	0·5711	−0·0642	−112	5
1·12	0·6080	−215	−13	0·5428	−0·0754	−105	7
1·14	0·5865	−230	−15	0·5144	−0·0859	−100	5
1·16	0·5635	−246	−16	0·4858	−0·0959	−95	5
1·18	0·5389	−265	−19	0·4567	−0·1054	−88	7
1·20	0·5124	−286	−21	0·4270	−0·1142	−82	6
1·22	0·4838	−314	−28	0·3965	−0·1224	−77	5
1·24	0·4524	−346	−32	0·3649	−0·1301	−69	8
1·26	0·4178	−388	−42	0·3316	−0·1370	−63	7
1·28	0·3790	−446	−58	0·2961	−0·1433	−55	8
1·30	0·3344			0·2572	−0·1488		

TABLE 7

Aerofoil III

	1	2	3	4	5	6	7	8	9
γ	q/U	r_m	$\frac{d\tilde{r}_m}{d\gamma}$	$\frac{dr_m}{d\gamma}$	r_m	r_s	q/U	$\frac{U \sin\gamma}{q}$	θ_m
0	0				−0·0458		0	0·1000	0
3			1·1506	1·6570		0·6908	0·4549	0·1151	−0·0180
	0·6329	+0·3586			0·1277				
9			−2·9122	−1·4015		0·0544	0·9291	0·1684	+0·1106
	1·0274	−0·0190			−0·0190				
15			−0·6054	−0·6054		−0·0507	1·0771	0·2403	0·1053
	1·1333	−0·0824			−0·0824				
21			−0·0487	−0·0487		−0·0850	1·1383	0·3148	0·0591
	1·1431	−0·0875			−0·0875				
27			−0·0344	−0·0344		−0·0893	1·1467	0·3959	0·0397
	1·1503	−0·0911			−0·0911				
35			−0·0304	−0·0304		−0·0937	−1·1555	0·4964	0·0242
	1·1611	−0·0964			−0·0964				
45			−0·0264	−0·0264		−0·0987	1·1658	0·6065	+0·0084
	1·1707	−0·1010			−0·1010				
55			−0·0229	−0·0229		−0·1030	1·1748	0·6973	−0·0070
	1·1792	−0·1050			−0·1050				
65			+0·0453	+0·0453		−0·1010	1·1707	0·7742	−0·0277
	1·1624	−0·0971			−0·0971				
75			0·0504	0·0504		−0·0927	1·1535	0·8374	−0·0417
	1·1448	−0·0883			−0·0883				
85			0·0527	0·0527		−0·0837	1·1357	0·8772	−0·0526
	1·1269	−0·0791			−0·0791				
95			0·0550	0·0550		−0·0743	1·1181	0·8910	−0·0603
	1·1093	−0·0695			−0·0695				
105			0·0550	0·0550		−0·0647	1·1009	0·8774	−0·0685
	1·0926	−0·0599			−0·0599				
115			0·0544	0·0544		−0·0551	1·0844	0·8358	−0·0770
	1·0765	−0·0504			−0·0504				
125			0·1226	0·0951		−0·0421	1·0630	0·7706	−0·0889
	1·0424	−0·0290			−0·0338				
135			0·1461	0·1224		−0·0231	1·0335	0·6842	−0·0984
	1·0049	−0·0035			−0·0124				
145			0·1844	0·1652		+0·0020	0·9974	0·5751	−0·1061
	0·9614	+0·0287			0·0164				
155			0·2504	0·2362		0·0370	0·9508	0·4445	−0·1098
	0·9076	0·0724			0·0576				
165			+0·4183	+0·4096		0·0933	0·8835	0·2929	−0·1003
	0·8282	+0·1454			0·1291				
175			0	−0·0029		+0·1750	0·7993	0·1090	−0·0273
180	0	0·1454			0·1286		0	0	0

TABLE 7—*continued*

Aerofoil III

	10	11	12	13	14	15	16	17	18
γ	θ_J	θ	$\frac{U}{q}\sin\gamma\sin\theta$	$\frac{y}{2a}$	y/c	$\frac{U}{q}\sin\gamma\cos\theta$	$\frac{x}{2a}$	$\frac{x}{c}$	$(y/c)_m$
0	+1·5708	+1·5708	+0·1000	0	0	0	0	0	0
3	1·2683	1·2503	0·1092	+0·0054	+0·0030	0·0362	0·0009	0·0005	0·0030
9	0·4485	0·5591	0·0893	0·0156	0·0086	0·1428	0·0106	0·0059	0·0086
15	0·2575	0·3628	0·0853	0·0248	0·0137	0·2247	0·0299	0·0165	0·0137
21	0·1808	0·2399	0·0748	0·0332	0·0183	0·3058	0·0577	0·0318	0·0183
27	0·1387	0·1784	0·0703	0·0407	0·0224	0·3896	0·0941	0·0519	0·0224
35	0·1049	0·1291	0·0639	0·0501	0·0276	0·4923	0·1557	0·0859	0·0276
45	0·0791	0·0875	0·0530	0·0603	0·0333	0·6042	0·2517	0·1388	0·0333
55	0·0622	0·0552	0·0385	0·0684	0·0377	0·6936	0·3655	0·2015	0·0377
65	0·0500	+0·0223	+0·0173	0·0734	0·0405	0·7740	0·4939	0·2724	0·0405
75	0·0407	−0·0010	−0·0008	0·0746	0·0412	0·8374	0·6348	0·3501	0·0412
85	0·0330	−0·0196	−0·0172	0·0732	0·0404	0·8770	0·7849	0·4328	0·0404
95	0·0265	−0·0338	−0·0301	0·0688	0·0380	0·8905	0·9394	0·5180	0·0381
105	0·0207	−0·0478	−0·0419	0·0628	0·0346	0·8764	1·0941	0·6033	0·0347
115	0·0154	−0·0616	−0·0515	0·0543	0·0306	0·8342	1·2437	0·6858	0·0307
125	0·0102	−0·0787	−0·0606	0·0448	0·0247	0·7682	1·3839	0·7632	0·0248
135	+0·0048	−0·0936	−0·0640	0·0535	0·0185	0·6812	1·5106	0·8330	0·0186
145	−0·0013	−0·1074	−0·0617	0·0228	0·0126	0·5718	1·6204	0·8936	0·0127
155	−0·0096	−0·1194	−0·0529	0·0127	0·0070	0·4413	1·7090	0·9424	0·0071
165	−0·0254	−0·1257	−0·0367	0·0048	0·0026	0·2906	1·7733	0·9779	0·0027
175	−0·0842	−0·1115	−0·0121	+0·0004	+0·0002	0·1083	1·8086	0·9973	0·0003
180	−0·1047	−0·1047	0	−0·0001	−0·0001	0	1·8134	1·0000	0

TABLE 8

Modification to Aerofoil II

	1	2	3	4	5	6	7	8	9
γ	θ	θ	$\left(\frac{d\theta}{d\gamma}\right)$	$\left(\frac{d\bar{\theta}}{d\gamma}\right)$	$\frac{d\theta}{d\gamma}+\frac{d\bar{\theta}}{d\gamma}$	$\theta+\bar{\theta}$	$\bar{L}_s$	$L_s+\bar{L}_s$	q/U
0				0			—	—	0
3				0			−0·0004	+1·1050	0·3312
9				,,			−0·0004	+0·2149	0·8066
15				,,			−0·0004	−0·0764	1·0792
21				,,			−0·0004	−0·1298	1·1387
27				,,			−0·0005	−0·1374	1·1473
35				,,			−0·0005	−0·1452	1·1562
45				,,			−0·0006	−0·1542	1·1667
55				,,			−0·0008	−0·1621	1·1759
65				,,			−0·0011	−0·1589	1·1722
75				,,			−0·0016	−0·1446	1·1556
85	−0·0343	−0·0461		0		−0·0461	−0·0029	−0·1303	1·1393
95	−0·0572	−0·0676	−0·1232	−0·0415	−0·1647	−0·0748	−0·0067	−0·1182	1·1256
105	−0·0775	−0·0879	−0·1163	−0·0415	−0·1578	−0·1024	−0·0045	−0·1005	1·1060
115	−0·0992	−0·1114	−0·1346	−0·0415	−0·1761	−0·1331	+0·0039	−0·0772	1·0803
125	−0·1247	−0·1354	−0·1375	+0·1375	0	,,	0·0258	−0·0318	1·0323
135	−0·1439	−0·1511	−0·0900	0·0900	0	,,	0·0245	+0·0015	0·9985
145	−0·1566	−0·1611	−0·0573	0·0573	0	,,	+0·0145	0·0319	0·9686
155	−0·1644	−0·1644	−0·0189	+0·0189	0	,,	−0·0012	0·0671	0·9352
165	−0·1610	−0·1477	+0·0957	−0·0957	0	,,	−0·0230	0·1198	0·8872
175	−0·1225		+0·2464	−0·2464	0		−0·0309	+0·2044	0·8152
180	−0·1047	−0·1047		0·0284		−0·1331	—	—	0

TABLE 9

Aerofoil IV

	1	2	3	4	5	6	7	8	9	10	11
γ	$\frac{q'}{U}(\gamma)$	$\frac{q'}{U}(-\gamma)$	$r'(\gamma)$	$r'(-\gamma)$	r_s'	r_a'	$r_a' \sin \gamma$	r_a	$r_a \sin \gamma$	$\frac{dr_a}{d\gamma}$	$\frac{1}{2} \log \frac{\sin(\frac{1}{2}\gamma + \alpha)}{\sin(\frac{1}{2}\gamma - \alpha)}$
0								0	0		
3										+2·2231	
9	1·2675	0·4059	−0·1394	+0·7996	+0·3301	−0·4695		+0·2328	0·0244	−0·9864	0·7023
15	1·2675	0·7541		0·2242	+0·0422	−0·1820		0·1295	0·0269	−0·3820	0·3115
21	1·2675	0·8902	,,	0·0876	−0·0259	−0·1135		0·0895	0·0277	−0·1700	0·2030
27		0·9892		+0·0078	−0·0608	−0·0786		0·0717	0·0292	−0·1337	0·1503
35		1·0244	−0·1394	−0·0170	−0·0782	−0·0612		0·0577	0·0288	−0·1570	0·1189
45		1·0366	−0·1394	−0·0252	−0·0823	−0·0571	−0·0367	0·0303	0·0194	−0·1106	0·0874
55	,,	,,					−0·0437	+0·0110		−0·0751	0·0681
65			,,		,,	,,	−0·0494	−0·0021		−0·0556	0·0550
75							−0·0537	−0·0118		−0·0430	0·0453
85				,,			−0·0561	−0·0193		−0·0407	0·0378
95	1·2675		−0·1394		−0·0823	−0·0571	−0·0571	−0·0264		+0·0160	0·0307
105	1·2241	,,	−0·1240		−0·0746	−0·0494	−0·0486	−0·0236		0·0229	0·0258
115	1·1845		−0·1074		−0·0663	−0·0411	−0·0387	−0·0196		0·0263	0·0215
125	1·1492		−0·0906		−0·0579	−0·0327	−0·0283	−0·0150		0·0275	0·0177
135	1·1177		−0·0741		−0·0497	−0·0245	−0·0188	−0·0102		0·0258	0·0143
145	1·0910	1·0366	−0·0590	−0·0252	−0·0421	−0·0169	−0·0109	−0·0057		0·0069	0·0112
155	1·0096	0·9768	−0·0067	+0·0167	+0·0040	−0·0127	−0·0064	−0·0045		0·0080	0·0082
165	0·9266	0·9081	+0·0565	0·0735	0·0650	−0·0085	−0·0029	−0·0031		0·0098	0·0054
175	0·8200	0·8117	+0·1538	+0·1622	+0·1580	−0·0042	−0·0007	−0·0014		+0·0080	
180	0	0				0	0	0			0

TABLE 9—*continued*

Aerofoil IV

	12	13	14	15	16	17	18	19	20	21	22
γ	$\frac{1}{2}\log\frac{\sin(\frac{1}{2}\gamma+\alpha)\sin(\frac{1}{2}\gamma-\alpha)}{\sin^2\frac{1}{2}\gamma}$	r_m	$\frac{d\tilde{r}_m}{d\gamma}$	$\frac{dr_m}{d\gamma}$	r_s	r_a	$r(\gamma)$	$r(-\gamma)$	θ_m	θ_s	θ_a
0		−0·2003							0	1·5708	−0·2873
3				+2·8729	+0·5681	+0·1164	+0·6845	+0·4517	−0·0717	1·2683	−0·2627
	−0·2296	+0·1005									
9			−1·0170	−1·0170	+0·0473	0·1811	0·2284	−0·1338	−0·0380	0·4485	−0·0380
	−0·0482	−0·0060									
15			−0·3896	−0·3896	−0·0264	0·1095	0·0831	−0·1359	+0·0591	0·2575	+0·0072
	−0·0209	−0·0468									
21			−0·2454	−0·2454	−0·0597	0·0806	+0·0209	−0·1403	0·0478	0·1808	0·0192
	−0·0117	−0·0725									
27			−0·1270	−0·1270	−0·0792	0·0647	−0·0145	−0·1439	0·0337	0·1387	0·0296
	−0·0076	−0·0858									
35			−0·0046	−0·0046	−0·0862	0·0440	−0·0422	−0·1302	+0·0130	0·1049	0·0416
	−0·0043	−0·0866									
45			+0·0092	+0·0092	−0·0858	0·0207	−0·0651	−0·1065	−0·0016	0·0791	0·0434
	−0·0027	−0·0850									
55			0·0040	0·0040	−0·0847	+0·0044	−0·0803	−0·0891	−0·0094	0·0622	0·0395
	−0·0020	−0·0843									
65			0·0029	0·0029	−0·0840	−0·0070	−0·0910	−0·0770	−0·0188	0·0500	0·0343
	−0·0015	−0·0838									
75			0·0017	0·0017	−0·0837	−0·0156	−0·0993	−0·0681	−0·0271	0·0407	0·0274
	−0·0012	−0·0835									
85			0·0011	0·0011	−0·0834	−0·0228	−0·1062	−0·0606	−0·0370	0·0330	0·0195
	−0·0010	−0·0833									
95			0·0452	0·0452	−0·0794	−0·0250	−0·1044	−0·0544	−0·0503	0·0265	+0·0067
	−0·0008	−0·0754									
105			0·0482	0·0482	−0·0712	−0·0216	−0·0928	−0·0496	−0·0615	0·0207	−0·0013
	−0·0007	−0·0670									
115			0·0487	0·0487	−0·0628	−0·0173	−0·0801	−0·0455	−0·0705	0·0154	−0·0052
	−0·0006	−0·0585									
125			0·0470	0·0470	−0·0544	−0·0126	−0·0670	−0·0418	−0·0809	0·0102	−0·0095
	−0·0006	−0·0503									
135			0·0436	0·0436	−0·0465	−0·0080	−0·0545	−0·0385	−0·0959	0·0048	−0·0105
	−0·0006	−0·0427									
145			0·2114	0·2703	−0·0191	−0·0051	−0·0242	−0·0140	−0·1286	−0·0013	−0·0092
	−0·0005	+0·0045									
155			0·3007	0·3441	+0·0345	−0·0038	+0·0307	+0·0383	−0·1402	−0·0096	−0·0091
	−0·0005	0·0645									
165			+0·5064	0·5332	0·1110	−0·0022	0·1088	0·1132	−0·1281	−0·0254	−0·0094
	−0·0005	0·1575									
175			0	+0·0090	+0·2045	−0·0007	0·2038	0·2052	−0·0355	−0·0842	−0·0091
180		+0·1591							0	−0·1047	−0·0094

TABLE 10

Aerofoil IV

	1	2	3	4	5	6	7	8	9	10	11	12
γ	q'/U	q/U	$\frac{U}{q}\sin\gamma$	θ	$\frac{U}{q}\sin\gamma\sin\theta$	$\frac{y}{2a}$	$\frac{U}{q}\sin\gamma\cos\theta$	$\frac{x}{2a}$	y/c	x/c	$(y/c)_m$	$(x/c)_m$
−180	0	0	0	+0·0953	0	+0·0300	0	+1·8193	+0·0165	+1·0016	+0·0163	+1·0000
−175		0·7712	0·1130	0·1106	+0·0125	0·0299	0·1123	1·8144	0·0164	0·9989	0·0162	0·9973
−165	0·8117	0·8622	0·3002	0·1441	0·0431	0·0242	0·2971	1·7780	0·0133	0·9789	0·0131	0·9773
−155	0·9081	0·9492	0·4452	0·1407	0·0624	0·0151	0·4408	1·7130	0·0083	0·9431	0·0081	0·9416
−145	0·9768	1·0200	0·5623	0·1207	0·0677	+0·0032	0·5582	1·6257	+0·0018	0·8950	+0·0016	0·8936
−135	1·0366	1·0558	0·6697	0·0806	0·0539	−0·0071	0·6676	1·5187	−0·0039	0·8361	−0·0041	0·8347
−125	”	1·0626	0·7709	0·0612	0·0472	−0·0160	0·7694	1·3931	−0·0088	0·7670	−0·0090	0·7658
−115	”	1·0684	0·8483	0·0499	0·0424	−0·0236	0·8483	1·2514	−0·0130	0·6890	−0·0130	0·6879
−105	”	1·0752	0·8984	0·0395	0·0355	−0·0307	0·8977	1·0987	−0·0169	0·6049	−0·0170	0·6039
−95	”	1·0832	0·9197	0·0305	0·0281	−0·0360	0·9192	0·9397	−0·0198	0·5174	−0·0199	0·5166
−85	”	1·0938	0·9108	0·0235	0·0214	−0·0405	0·9105	0·7796	−0·0223	0·4292	−0·0224	0·4285
−75	”	1·1068	0·8727	0·0318	0·0120	−0·0433	0·8726	0·6236	−0·0239	0·3433	−0·0240	0·3427
−65	”	1·1230	0·8070	+0·0031	+0·0025	−0·0447	0·8070	0·4766	−0·0246	0·2624	−0·0247	0·2620
−55	”	1·1463	0·7146	−0·0131	−0·0095	−0·0441	0·7145	0·3434	−0·0243	0·1891	−0·0243	0·1888
−45	”	1·1824	0·5980	−0·0341	−0·0204	−0·0415	0·5976	0·2286	−0·0228	0·1259	−0·0228	0·1257
−35	1·0366	1·2403	0·4625	−0·0763	−0·0352	−0·0367	0·4612	0·1360	−0·0202	0·0749	−0·0202	0·0748
−27	1·0244	1·2821	0·3541	−0·1428	−0·0504	−0·0307	0·3505	0·0795	−0·0169	0·0438	−0·0169	0·0437
−21	0·9892	1·2702	0·2821	−0·2094	−0·0586	−0·0250	0·2760	0·0467	−0·0138	0·0257	−0·0138	0·0257
−15	0·8902	1·2558	0·2061	−0·3094	−0·0628	−0·0186	0·1963	0·0219	−0·0102	0·0120	−0·0102	0·0120
−9	0·7541	1·2504	0·1251	−0·5254	−0·0627	−0·0120	0·1083	+0·0058	−0·0066	+0·0032	−0·0066	+0·0032
−3	0·4059	0·5877	0·0891	−1·4593	−0·0885	−0·0044	0·0099	−0·0005	−0·0024	−0·0002	−0·0024	−0·0002
0		0	0·857	−1·8581 1·2835	±0·0822	0	±0·0243	0	0	0	0	0

TABLE 10—*continued*

Aerofoil IV

	1	2	3	4	5	6	7	8	9	10	11	12
γ	q'/U	q/U	$\frac{U}{q}\sin\gamma$	θ	$\frac{U}{q}\sin\gamma\sin\theta$	$\frac{y}{2a}$	$\frac{U}{q}\sin\gamma\cos\theta$	$\frac{x}{2a}$	y/c	x/c	$(y/c)_m$	$(x/c)_m$
3	1·2675	0·4579	0·1143	+0·9339	+0·0919	0·0046	0·0680	0·0025	0·0025	0·0014	0·0025	0·0014
9		0·7506	0·2084	0·4485	0·0904	0·0141	0·1878	0·0163	0·0077	0·0090	0·0077	0·0090
15	”	0·8953	0·2891	0·3238	0·0920	0·0237	0·2741	0·0405	0·0130	0·0223	0·0130	0·0223
21	”	0·9719	0·3687	0·2478	0·0904	0·0332	0·3574	0·0736	0·0183	0·0405	0·0183	0·0406
27	”	1·0207	0·4448	0·2020	0·0892	0·0426	0·4358	0·1152	0·0235	0·0634	0·0235	0·0635
35	”	1·0632	0·5395	0·1595	0·0857	0·0549	0·5326	0·1829	0·0302	0·1007	0·0302	0·1009
45	”	1·1016	0·6419	0·1209	0·0774	0·0691	0·6372	0·2853	0·0381	0·1571	0·0381	0·1574
55	”	1·1293	0·7254	0·0923	0·0669	0·0818	0·7223	0·4042	0·0450	0·2225	0·0451	0·2229
65	”	1·1501	0·7880	0·0655	0·0516	0·0922	0·7863	0·5362	0·0508	0·2952	0·0509	0·2957
75	”	1·1671	0·8276	0·0410	0·0339	0·0997	0·8269	0·6773	0·0549	0·3729	0·0550	0·3735
85	”	1·1818	0·8429	+0·0155	+0·0131	0·1039	0·8428	0·8234	0·0572	0·4533	0·0573	0·4540
95	1·2675	1·1778	0·8458	−0·0171	−0·0145	0·1038	0·8457	0·9707	0·0572	0·5345	0·0573	0·5354
105	1·2241	1·1554	0·8360	−0·0421	−0·0352	0·0992	0·8352	1·1178	0·0546	0·6154	0·0547	0·6164
115	1·1845	1·1289	0·8028	−0·0603	−0·0484	0·0920	0·8014	1·2609	0·0506	0·6942	0·0508	0·6953
125	1·1492	1·1049	0·7414	−0·0802	−0·0594	0·0824	0·7390	1·3959	0·0454	0·7685	0·0456	0·7697
135	1·1177	1·0834	0·6527	−0·1016	−0·0662	0·0715	0·6493	1·5173	0·0394	0·8354	0·0396	0·8367
145	1·0910	1·0351	0·5541	−0·1391	−0·0769	0·0591	0·5487	1·6219	0·0325	0·8930	0·0328	0·8946
155	1·0096	0·9588	0·4408	−0·1589	−0·0697	0·0459	0·4352	1·7080	0·0253	0·9403	0·0256	0·9418
165	0·9266	0·8669	0·2986	−0·1629	−0·0484	0·0356	0·2947	1·7723	0·0196	0·9757	0·0199	0·9773
175	0·8200	0·7725	0·1128	−0·1288	−0·0145	0·0297	0·1119	1·8084	0·0164	0·9956	0·0167	0·9972
180	0	0	0	−0·1141	0	0·0291	0	1·8134	0·0160	0·9984	0·0163	1·0000

TABLE 11

Aerofoil V

	1	2	3	4	5	6	7	8	9	10	11
γ	$\frac{q'}{U}(\gamma)$	$\frac{q}{U}(-\gamma)$	$r'(\gamma)$	$r(-\gamma)$	$\sin\gamma \times \{r'(\gamma) - r(-\gamma)\}$	$\log \frac{\sin(\frac{1}{2}\gamma + \alpha)}{\sin\frac{1}{2}\gamma}$	$r(\gamma)$	$r_m(\gamma)$	$r_a(\gamma)$	$\frac{d\tilde{r}_a}{d\gamma}$	$\frac{dr_a}{d\gamma}$
0					0			—0·0557	0		
3	1·0964	1·1600	—0·0621	—0·0959	—0·0050	0·3360	+0·2739	+0·0890	+0·1849	+1·3780	+1·7657
9	1·1222		—0·0766		—0·0099	0·1815	0·1049	+0·0045	0·1004	—0·7382	—0·8066
15	1·1495		—0·0907		—0·0146	0·1240	+0·0333	—0·0313	0·0646	—0·2741	—0·3425
21	1·1793		—0·1051		—0·0193	0·0938	—0·0113	—0·0536	0·0423	—0·1442	—0·2126
27	1·2128		—0·1195		—0·0237	0·0750	—0·0445	—0·0702	0·0257	—0·0898	—0·1582
35	1·2800	,,	—0·1433		—0·0305	0·0557	—0·0876	—0·0917	+0·0041	—0·0556	—0·1240
45				,,	—0·0363	0·0437	—0·0996	—0·0978	—0·0019	—0·0344	—0·0344
55			,,		—0·0410	0·0354	—0·1079	—0·1019	—0·0060	—0·0235	—0·0235
65	,,				—0·0445	0·0293	—0·1140	—0·1050	—0·0090	—0·0172	—0·0172
75					—0·0467	0·0244	—0·1189	—0·1074	—0·0115	—0·0143	—0·0142
85	1·2800	1·1600	—0·1433	—0·0959	—0·0474	0·0205	—0·1228	—0·1093	—0·0134	—0·0109	—0·0109
95	1·2347	1·1419	—0·1281	—0·0869	—0·0406	0·0172	—0·1109	—0·0989	—0·0120	+0·0080	+0·0080
105	1·1907	1·1243	—0·1102	—0·0777	—0·0305	0·0143	—0·0960	—0·0869	—0·0092	0·0160	0·0160
115	1·1495	1·1078	—0·0907	—0·0686	—0·0191	0·0118	—0·0789	—0·0738	—0·0051	0·0235	0·0235
125	1·1122	1·0929	—0·0711	—0·0601	—0·0084	0·0095	—0·0616	—0·0608	—0·0008	0·0246	0·0246
135	1·0800	1·0800	—0·0525	—0·0525	0	0·0074	—0·0451	—0·0488	+0·0037	0·0258	0·0258
145	0·9931	0·9979	+0·0051	+0·0015		0·0054	+0·0105	+0·0060	0·0045	+0·0046	+0·0046
155	0·9091	0·9133	0·0711	0·0676		0·0035	0·0746	0·0716	0·0030	—0·0086	—0·0086
165	0·7778	0·8074	+0·1980	+0·1666		0·0016	+0·1696	+0·1681	+0·0015	—0·0086	—0·0086
175										—0·0086	—0·0086
180	0	0						0·1708	0		

TABLE 11—*continued*

Aerofoil V

	12	13	14	15	16	17	18	19	20	21	22
γ	$\frac{d\bar{r}_m}{d\gamma}$	$\frac{dr_m}{d\gamma}$	$r_s(\gamma)$	$r_a(\gamma)$	$r(\gamma)$	$\frac{q}{U}(\gamma)$	$r(-\gamma)$	$\frac{q}{U}(-\gamma)$	θ_m	θ_J	θ_a
0						0		0	0	+1·5708	−0·2177
3		+1·3823	0·6737	+0·0925	+0·7662	0·4204	+0·5812	0·5107	−0·0222	1·2682	−0·1980
9	−0·8069	−0·8069	0·0468	0·1426	0·1894	0·7856	−0·0959	1·1600	+0·0629	0·4482	−0·0175
15	−0·3419	−0·3419	−0·0134	0·0825	0·0691	0·9120			0·0752	0·2570	+0·0202
21	−0·2129	−0·2129	−0·0424	0·0535	+0·0111	0·9853			0·0680	0·1801	0·0308
27	−0·1585	−0·1585	−0·0619	0·0340	−0·0279	1·0408			0·0611	0·1378	0·0345
35	−0·1232	−0·1232	−0·0810	0·0149	−0·0661	1·1033	”	”	0·0497	0·1037	0·0339
45	−0·0350	−0·0350	−0·0948	+0·0011	−0·0937	1·1556			0·0288	0·0775	0·0241
55	−0·0235	−0·0235	−0·0999	−0·0040	−0·1039	1·1768			+0·0136	0·0603	0·0185
65	−0·0178	−0·0178	−0·1035	−0·0075	−0·1110	1·1906			−0·0001	0·0477	0·0144
75	−0·0137	−0·0137	−0·1062	−0·0103	−0·1165	1·2055			−0·0139	0·0378	0·0105
85	−0·0109	−0·0109	−0·1083	−0·0125	−0·1208	1·2160	−0·0959	1·1600	−0·0297	0·0296	0·0061
95	+0·0596	+0·0596	−0·1041	−0·0127	−0·1168	1·2058	−0·0914	1·1510	−0·0514	0·0225	+0·0004
105	0·0688	0·0688	−0·0929	−0·0106	−0·1035	1·1705	−0·0823	1·1330	−0·0681	0·0159	−0·0041
115	0·0751	0·0751	−0·0804	−0·0072	−0·0876	1·1433	−0·0732	1·1160	−0·0816	0·0096	−0·0072
125	0·0745	0·0745	−0·0673	−0·0029	−0·0702	1·1105	−0·0644	1·1003	−0·0946	+0·0031	−0·0084
135	0·0688	0·0688	−0·0548	+0·0015	−0·0533	0·1820	−0·0563	1·0864	−0·1110	−0·0042	−0·0078
145	0·2114	0·3139	−0·0214	0·0041	−0·0173	1·0249	−0·0255	1·0370	−0·1452	−0·0132	−0·0041
155	0·3007	0·3762	+0·0388	0·0038	+0·0426	0·9437	+0·0350	0·9534	−0·1541	−0·0266	−0·0007
165	+0·5064	0·5526	0·1198	0·0022	0·1220	0·8526	0·1176	0·8555	−0·1372	−0·0552	+0·0009
175	0	+0·0155	+0·2618	+0·0008	+0·2626	0·7214	+0·2610	0·7227	−0·0388	−0·1699	0·0017
180						0		0	0	−0·2094	+0·0016

TABLE 12

Aerofoil V

	1	2	3	4	5	6	7	8	9	10	11	12
γ	$(q/U)'$	q/U	$\frac{U}{q}\sin\gamma$	θ	$\frac{U}{q}\sin\gamma\sin\theta$	$\frac{y}{2a}$	$\frac{U}{q}\sin\gamma\cos\theta$	$\frac{x}{2a}$	y/c	x/c	$(y/c)_m$	$(x/c)_m$
−180	0	0	0	+0·2110	0	+0·0171	0	+1·7779	+0·0096	+1·0007	+0·0088	+1·0000
−175	0·0855	0·7227	0·1206	0·2104	+0·0252	0·0159	0·1179	1·7729	0·0090	0·9979	0·0082	0·9972
−165	0·9085	0·8555	0·3025	0·1933	0·0581	+0·0083	0·2969	1·7359	+0·0047	0·9827	+0·0039	0·9820
−155	0·9898	0·9534	0·4433	0·1800	0·0794	−0·0037	0·4361	1·6712	−0·0021	0·9407	−0·0029	0·9400
−145	1·0671	1·0370	0·5531	0·1543	0·0850	−0·0185	0·5465	1·5853	−0·0104	0·8924	−0·0111	0·8918
−135	1·0761	1·0864	0·6509	0·1074	0·0698	−0·0319	0·6471	1·4813	−0·0180	0·8338	−0·0187	0·8332
−125	1·0862	1·1003	0·7445	0·0831	0·0618	−0·0433	0·7419	1·3598	−0·0244	0·7654	−0·0250	0·7649
−115	1·0976	1·1160	0·8121	0·0648	0·0526	−0·0535	0·8104	1·2239	−0·0301	0·5889	−0·0306	0·6884
−105	1·1084	1·1330	0·8525	0·0481	0·0410	−0·0615	0·8515	1·0785	−0·0346	0·6071	−0·0351	0·6067
−95	1·1184	1·1510	0·8655	0·0293	0·0254	−0·0675	0·8652	0·9283	−0·0380	0·5225	−0·0384	0·5221
−85	1·1109	1·1600	0·8588	+0·0062	+0·0053	−0·0702	0·8588	0·7776	−0·0395	0·4377	−0·0398	0·4374
−75	1·1019	,,	0·8377	−0·0134	−0·0112	−0·0696	0·8326	0·6296	−0·0392	0·3544	−0·0395	0·3541
−65	1·0905	,,	0·7813	−0·0332	−0·0259	−0·0663	0·7808	0·4885	−0·0373	0·2750	0·0375	0·2748
−55	1·0754	,,	0·7062	−0·0554	−0·0391	−0·0606	0·7051	0·3585	−0·0341	0·2018	−0·0343	0·2017
−45	1·0541	,,	0·6096	−0·0822	−0·0500	−0·0528	0·6075	0·2436	−0·0297	0·1371	−0·0298	0·1370
−35	1·0204	,,	0·4945	−0·1195	−0·0589	−0·0433	0·4910	0·1475	−0·0244	0·0830	−0·0245	0·0829
−27	0·9886	,,	0·3914	−0·1644	−0·0641	−0·0347	0·3861	0·0862	−0·0195	0·0485	−0·0195	0·0485
−21	0·9409	,,	0·3089	−0·2173	−0·0666	−0·0278	0·3016	0·0502	−0·0157	0·0282	−0·0157	0·0282
−15	0·8477	,,	0·2231	−0·3120	−0·0685	−0·0208	0·2123	0·0232	−0·0117	0·0131	−0·0117	0·0131
−9	0·6117	1·1600	0·1349	−0·5286	−0·0742	−0·0133	0·1126	+0·0061	−0·0075	+0·0034	−0·0075	+0·0034
−3		0·5812	0·0901	−1·4440	−0·0894	−0·0049	0·0114	−0·0004	−0·0027	−0·0002	−0·0027	−0·0002
0		0	0·0990	−1·7885 1·3531	∓0·0967	0	∓0·0241	0	0	0	0	0

TABLE 12—*continued*

Aerofoil V

	1	2	3	4	5	6	7	8	9	10	11	12
γ	$(q/U)'$	q/U	$\frac{U}{q}\sin\gamma$	θ	$\frac{U}{q}\sin\gamma\sin\theta$	$\frac{y}{2a}$	$\frac{U}{q}\sin\gamma\cos\theta$	$\frac{x}{2a}$	y/c	x/c	$(y/c)_m$	$(x/c)_m$
3	1·0964	0·4204	0·1245	+1·0480	+0·1079	0·0054	0·0622	0·0022	0·0030	0·0012	0·0030	0·0012
9	1·1222	0·7856	0·1991	0·4936	0·0943	0·0157	0·1753	0·0149	0·0089	0·0084	0·0089	0·0084
15	1·1495	0·9120	0·2838	0·3524	0·0979	0·0258	0·2663	0·0381	0·0145	0·0214	0·0145	0·0214
21	1·1793	0·9853	0·3637	0·2789	0·1001	0·0362	0·3497	0·0704	0·0204	0·0396	0·0204	0·0396
27	1·2128	1·0408	0·4362	0·2334	0·1009	0·0467	0·4244	0·1110	0·0263	0·0625	0·0264	0·0625
35	1·2800	1·1033	0·5199	0·1873	0·0968	0·0609	0·5108	0·1765	0·0343	0·0994	0·0344	0·0995
45	,,	1·1556	0·6119	0·1304	0·0795	0·0762	0·6067	0·2741	0·0429	0·1543	0·0430	0·1544
55	,,	1·1768	0·6961	0·0924	0·0643	0·0887	0·6931	0·3877	0·0500	0·2182	0·0502	0·2183
65	,,	1·1906	0·7612	0·0620	0·0472	0·0985	0·7598	0·5145	0·0555	0·2898	0·0557	0·2900
75	,,	1·2055	0·8013	0·0344	0·0276	0·1051	0·8008	0·6515	0·0591	0·3667	0·0594	0·3669
85	1·2800	1·2160	0·8192	+0·0060	+0·0049	0·1080	0·8192	0·7931	0·0608	0·4464	0·0612	0·4467
95	1·2347	1·2058	0·8262	−0·0285	−0·0235	0·1065	0·8259	0·9367	0·0599	0·5273	0·0603	0·5277
105	1·1907	1·1705	0·8252	−0·0563	−0·0465	0·1001	0·8239	1·0809	0·0563	0·6084	0·0568	0·6088
115	1·1495	1·1433	0·7927	−0·0792	−0·0627	0·0906	0·7902	1·2225	0·0510	0·6881	0·0515	0·6885
125	1·1122	1·1105	0·7376	−0·0999	−0·0735	0·0785	0·7339	1·3554	0·0442	0·7629	0·0448	0·7634
135	1·0800	1·0820	0·6535	−0·1230	−0·0802	0·0652	0·6485	1·4770	0·0367	0·8314	0·0374	0·8320
145	0·9931	1·0249	0·5596	−0·1625	−0·0905	0·0503	0·5522	1·5812	0·0283	0·8900	0·0290	0·8906
155	0·9091	0·9437	0·4478	−0·1814	−0·0808	0·0349	0·4405	1·6680	0·0196	0·9389	0·0204	0·9396
165	0·7778	0·8526	0·3036	−0·1915	−0·0578	0·0229	0·2980	1·7331	0·0129	0·9756	0·0137	0·9763
175		0·7214	0·1208	−0·2070	−0·0248	0·0152	0·1182	1·7707	0·0085	0·9967	0·0093	0·9974
180	0	0	0	−0·2078	0	0·0142	0	1·7752	0·0080	0·9993	0·0088	1·0000

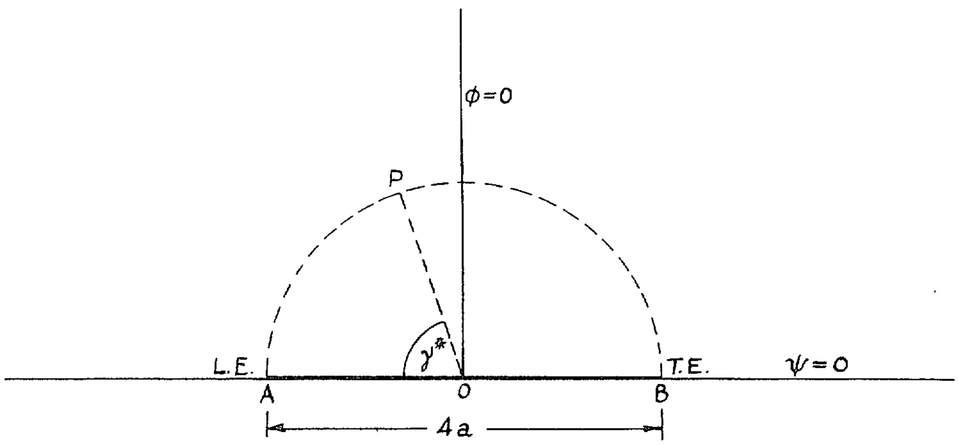

FIG. 1.

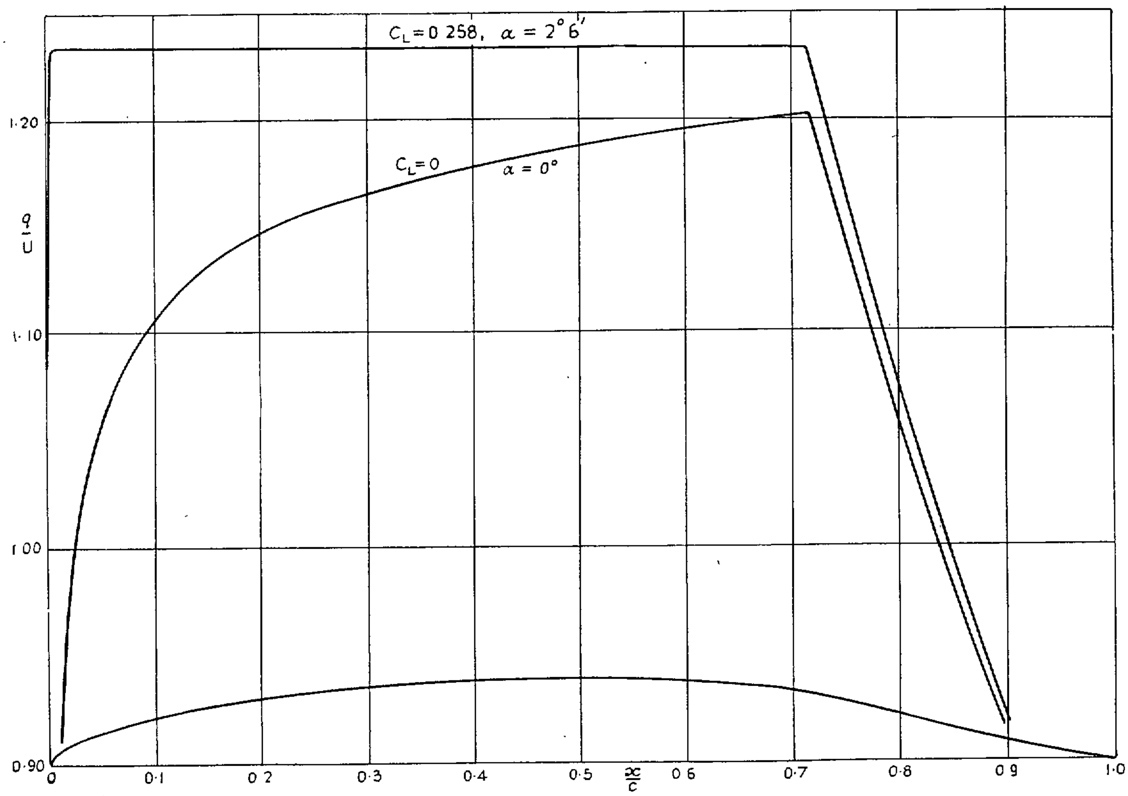

FIG. 2. Aerofoil I.

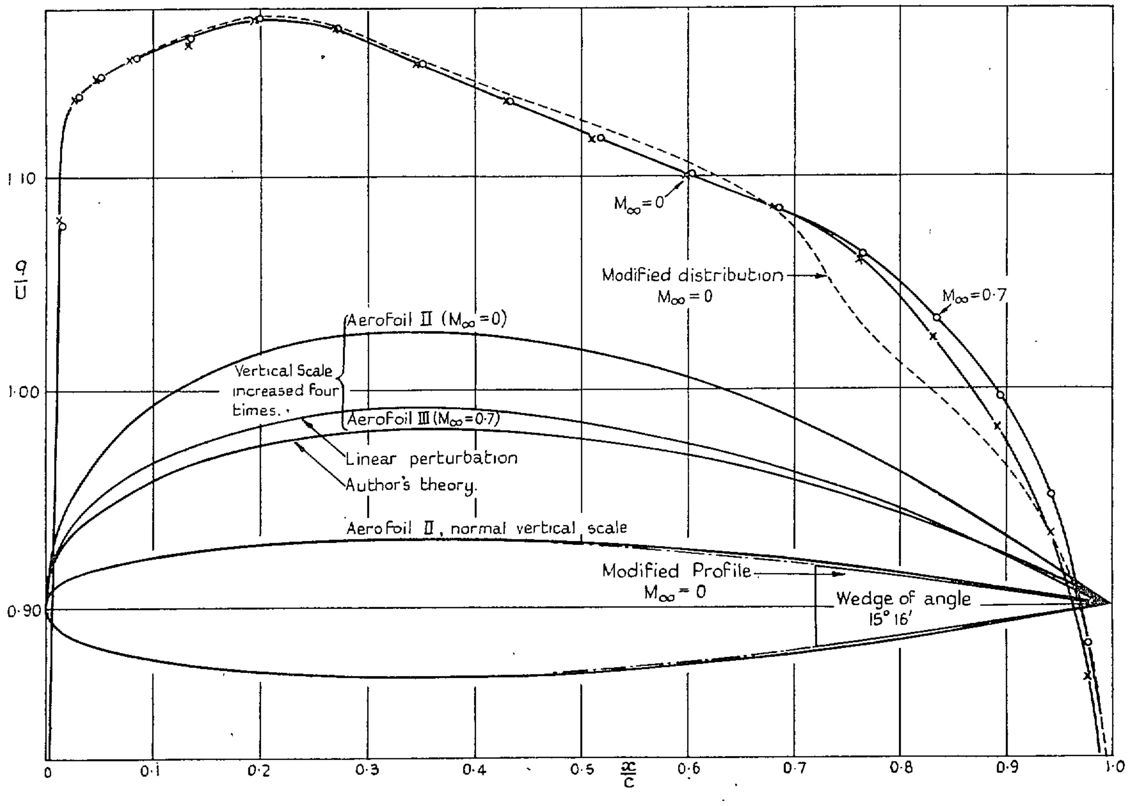

FIG. 3. Aerofoils II and III.

D

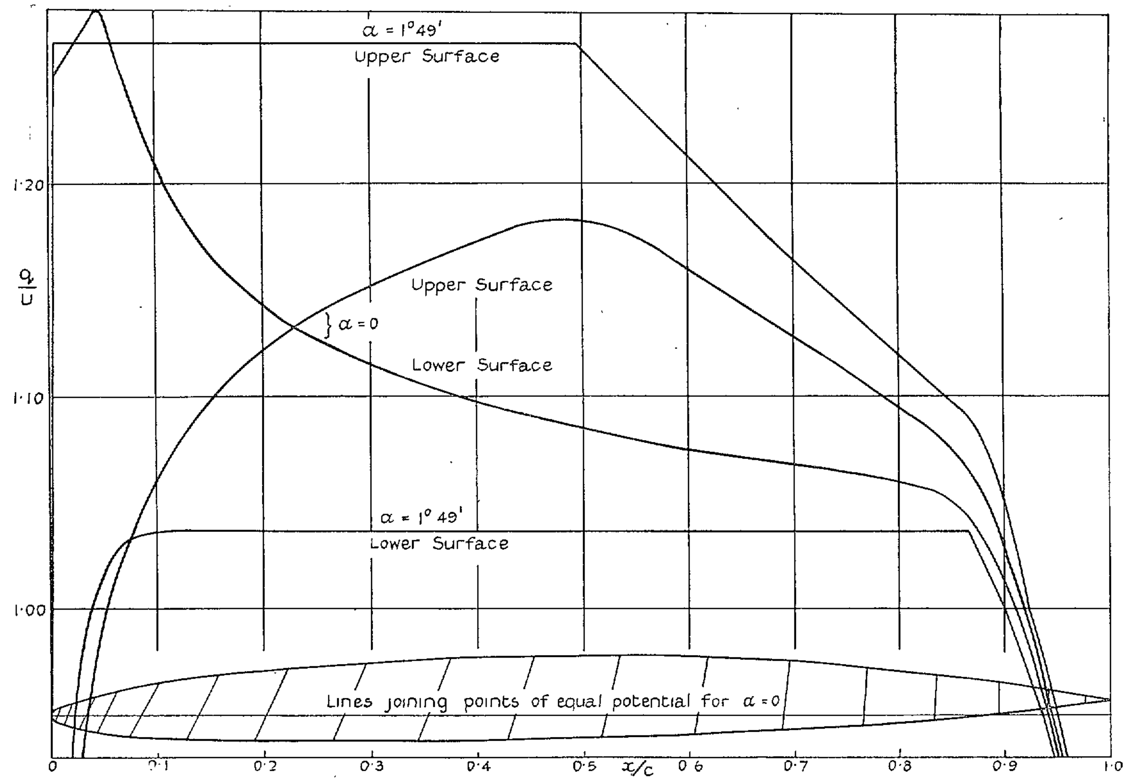

FIG. 4. Aerofoil IV.

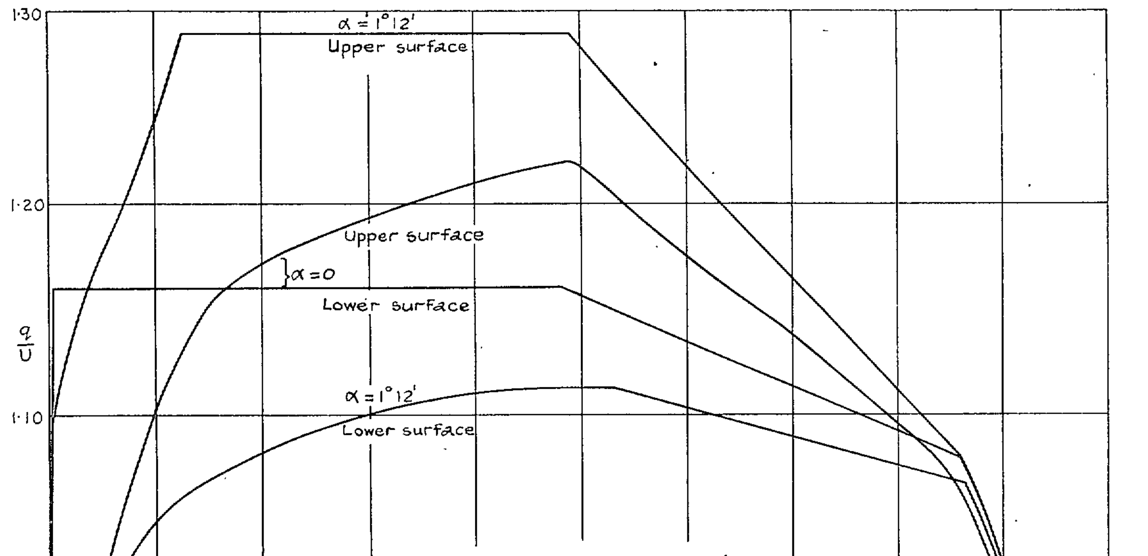

FIG. 5. Aerofoil V.

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