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Criteria for the Prevention of Flutter of Tab Systems

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Summary.—An investigation has been made into the flutter characteristics of an idealised tab system in which the three degrees of freedom normal translation of the lifting surface, rotation of the control surface, and rotation of the tab are represented. Specific cases of this idealised system represent similar idealised forms of the standard trimming, spring, servo, and geared tab systems. From a consideration of the relationships existing between the systems, criteria for flutter prevention have been developed from the criteria evolved earlier for trimming tabs[1]. As initially derived, the criteria are applicable to the stick-fixed condition (in the case of spring and servo-tabs), to the case with no aerodynamic balance on either control surface or tab, and to the case where the control surface is statically balanced about its hinge.

Comparison is made between the criteria for spring-tabs and the existing Collar-Sharpe criteria[2,3]. Design implications are deduced from the criteria for spring-tabs, and the general application of the criteria to actual systems is considered in some detail.

A comprehensive survey of the results is given in section 11. Points of major importance are as follows:—

(*a*) The criteria are liberally provided with generalised constants whose values can if necessary be adjusted in the light of practical experience.

(*b*) The backward limit set to the tab centre of gravity will normally be less severe than in the case of the Collar-Sharpe criterion.

(*c*) Satisfying the criteria of this report is likely to be most difficult with elevators carrying a tab on one side only, and from a flutter point of view such systems should be avoided if possible.

1. *Introduction.*—Specific types of tab dealt with in this report are the spring-tab, the servo-tab, the geared-tab, and the trimming-tab. These terms are generally accepted, but to avoid any misunderstanding they are defined precisely in section 3.

Because of their relatively greater susceptibility to flutter, spring and servo-tabs have received most attention. Early investigations into the flutter characteristics of spring-tab systems established certain features of the phenomenon and provided the background for the first systematic approach to the problem by Collar and Sharpe[2,3]. More recently it has been shown that the binary system (control-surface rotation β, tab rotation γ) on which the Collar-Sharpe criterion was originally based is inadequate and that it is necessary to include motion of the main lifting surface (wing, tailplane, or fin). Current recommendations[4] for the prevention of spring-tab flutter represent an extension of the Collar-Sharpe criterion, on a somewhat approximate basis, to the ternary (z, β, γ) case, z representing normal translation of the lifting surface.

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Trimming and geared-tabs have on the whole been considered much less seriously. Compared with spring-tabs, they have been regarded as having no separate degree of freedom and current design requirements stipulate mass-balancing only above 350 m.p.h. E.A.S. and of course the avoidance of backlash. In actual fact, however, the difference between trimming and geared-tabs on the one hand and spring-tabs on the other is one of degree only. The design requirements for trimming and geared-tabs rely in effect upon the stiffness of their connections being reasonably high ; and if this is not so flutter may in fact occur. A recent investigation by Wittmeyer[1] into the flutter characteristics of trimming-tab systems established a criterion for the ternary (z, β, γ) case. This criterion, even allowing for the assumptions involved, is the most comprehensive flutter criterion that has yet been evolved for any type of tab system ; and it has in fact demonstrated the need for similar criteria applicable to other tab systems, especially spring and servo-tabs.

The criterion for trimming-tabs (which is re-stated in section 2 below) was evolved from a systematic analysis of a series of theoretical calculations in which the relevant parameters were varied appropriately. Criteria for the other types of tab system could be evolved by the same direct method, but the labour involved would be considerable. In the present report an attempt is made to reduce the labour by a process of induction.

On the principle that the several tab systems considered may be regarded as specific cases of a single generalised tab system (hereafter referred to as the S-system), it is reasonable to suppose that the criterion already derived for trimming tabs should bear some relation to the criteria that would be derived for other types of tab following the same method and assumptions. It is therefore proposed to use the results already obtained for trimming-tabs and from these results alone, as far as they are applicable, to derive criteria appropriate to the other tab systems based on a comparison between the trimming-tab system and the generalised S-system. It cannot be expected, of course, that the criteria derived in this way should necessarily be as comprehensive as criteria derived by the direct method, and as will appear later the criteria can in fact be proved to ensure freedom from certain types of flutter only. Intuition suggests however that the criteria should in practice prevent other types of flutter as well, and it is reasonable to expect that the criteria should prove superior to any that have been established so far.

It is appropriate here to recapitulate the assumptions upon which the earlier work on trimming-tabs was based, since the criteria evolved in the present report will be automatically subject to the same assumptions. The system considered has a simple geometrical plan form (*see* later in section 3) and is allowed to have the three simple degrees of freedom specified (*see* also section 3). Application of the criteria to actual systems for which these geometric and modal assumptions may not be valid is discussed in section 10.

The flutter analysis is based on the theory of small oscillations, and the air forces have been calculated by strip-theory using the values of Dietze for the case of a thin aerofoil with two hinged flaps in tandem in an incompressible medium. No aerodynamic balance has been taken on either control surface or tab.

2. *The Trimming-Tab Criterion already Derived.*—It is useful to re-state here the criterion derived for trimming-tabs[1], since it forms the basis of the development in the following sections of this report. The criterion is designed to prevent ternary (z, β, γ) flutter of the system. The trimming-tab criterion I requires that the following conditions should be met:

$$f_\gamma / f_\beta \geqslant 2k_1 \qquad (1)$$

$$f_\gamma / f_z \geqslant 2k_2 \qquad (2)$$

$$f_\beta/f_z \leqslant k_3 \qquad (3)$$

$$F_2 + \delta_2 \leqslant p_t \leqslant F_1 - \delta_1 \qquad (4)$$

where

$$F_2 = a_3 - a_4(a_2 - \bar{q}) - k_5 \sqrt{\left(\frac{a_2 - \bar{q}}{a_1 \bar{q}}\right)} \qquad (5)$$

$$F_1 = \frac{3}{4}\left[\frac{E_1}{E_1 - E_2}\right]\left[\frac{1}{6}\left(1 - 4\frac{E_2}{E_1}\right) i_t + a_3 - a_4(a_2 - \bar{q}) + k_4 \sqrt{\left(\frac{a_2 - \bar{q}}{a_1 \bar{q}}\right)}\right] \qquad (6)$$

$$a_1 = 0\cdot222 + 0\cdot013 i_t + i_c(-0\cdot0145 + 0\cdot00149 i_t) \qquad (7)$$

$$a_2 = 1\cdot12 - 0\cdot0267 i_t + i_c(0\cdot0365 - 0\cdot001 i_t) \qquad (8)$$

$$a_3 = -0\cdot164 - 0\cdot0965 i_t + i_c(0\cdot0778 + 0\cdot00489 i_t) \qquad (9)$$

$$a_4 = \frac{i_t}{0\cdot448 + 0\cdot277 i_t} - 0\cdot238 i_c \qquad (10)$$

$$\bar{q} = jq \qquad (11)$$

$$j = \sqrt{\left(\frac{E_2}{E_1}\right)}\left[0\cdot93 + 1\cdot28(1\cdot97 - E_1)\left(0\cdot745 - \frac{E_2}{E_1}\right)\right]. \qquad (12)$$

The constants $k_i (i = 1 \text{ to } 5)$ are equal to unity and the safety margins are $\delta_1 = \delta_2 = 0\cdot5$. Symbols are defined in section 14. The range of validity of the criterion is $\mu = 0$ to 50 (probably), $i_c = 1$ to $7\cdot78$, $p_c = 0$, $i_t = 1\cdot31$ to $13\cdot1$, $E_1 = 0\cdot2$ to $0\cdot4$, $E_2/E_1 = 0\cdot13$ to $0\cdot25$. To provide scope for adjustment to wider ranges and to the results of experience, the constants k_i are retained in their general form.

By appropriate simplifications the trimming-tab criterion I was reduced successively to the trimming-tab criteria II and III, criterion III being recommended for general design use. It is not necessary to re-state criteria II and III here, but mention is made of them because they will be referred to again later.

3. *The Generalised S-system and its Specific Cases.*—A description of the generalised S-system considered is now given, together with an account of the specific cases in which the S-system can be reduced to the various standard types of tab system. For convenience the trimming-tab system is referred to as the T-system.

3.1. *The S-system.*—Fig. 1 shows the S-system, which consists of a rectangular lifting surface of span s, to which is hinged a rectangular control surface also of span s. The control surface is in turn fitted with a tab of span qs. The degrees of freedom given to the system are:

(*a*) translation of the lifting surface in a direction normal to the air stream (z)

(*b*) rotation of the control surface about hinge C relative to the lifting surface (β) and

(*c*) rotation of the tab about hinge D relative to the control surface (γ).

Lifting surface, control surface, and tab are taken to be structurally rigid within themselves. Motion of the system is thus completely defined by the three co-ordinates z, β, and γ, each of which is constant over the span.

Kinematically, the three surfaces are regulated by a system of links and springs. The links, numbered (0) to (5) in Fig. 1, are supposedly rigid. Each of the links (0) to (4) is freely pivoted at both ends independent of each other. Link (0) may be regarded as the pilot's control. Points A and C are fixed to the lifting surface, B and D to the control surface.

The springs of the system are as follows:

(i) a spring K_w constraining the translation of the lifting surface,

(ii) a spring K_c constraining the rotation of the control surface,

(iii) a spring K_o representing the elasticity of the control circuit,

(iv) the 'main spring' K_m constraining the rotation of lever (2) relative to the control surface,

(v) the 'subsidiary spring' K_s constraining the relative rotation between levers (2) and (3).

The symbols denoting the various springs are also used quantitatively to represent the actual spring rates, in lb per ft in the case of K_w and in lb ft per radian for the remaining springs.

3.2. *Specific Cases of the S-system.*—Under certain conditions the S-system of Fig. 1 reduces to a spring, servo, trimming, or geared-tab system. In each case the specific tab system concerned has the same simple geometrical plan form and the same basic degrees of freedom.

(*a*) *The Spring-Tab System.*—The S-system as it stands represents the spring-tab system in what may be regarded as its most general form. If the motion of the system is consistent with normal operation of the control concerned, K_c will however be zero. The spring K_c may be effectively present when the motion is not consistent with normal operation. More specifically, in the case of the elevator for instance, normal operation of the elevator control involves symmetric motion of the two halves of the elevator, and in symmetric flutter K_c will be zero. In the case of anti-symmetric flutter, however, any structural interconnection between the two halves of the elevator will exert an additional elastic restraint on each half, which restraint is represented by the spring K_c.

The follow-up ratio N is commonly defined as the ratio γ/β when the control surface is moved with the point E fixed relative to the lifting surface and spring K_s remaining undeflected. Although this action essentially involves deflection of the spring K_m, the follow-up ratio is a purely geometric parameter and is given (in Fig. 1) by $N = l_1 l_3 / l_2 l_4$. For spring-tabs N is usually positive.

Throughout the subsequent development of flutter criteria in sections 4 to 7 the point O (pilot's control) is regarded as fixed. Consideration is given later in section 10 to the possibilities of flutter with the stick (or pedal) free.

(*b*) *The Servo-Tab System.*—The S-system becomes a servo-tab system if $K_m = 0$. Zero or finite values of K_c apply as in the case of the spring-tab system. Follow-up ratio N is usually positive but may be zero, in which case the tab is termed a pure servo-tab.

(*c*) *The Trimming-Tab System* (*T-system*).—A trimming-tab system may be represented as an S-system with $K_o = 0$ and K_m very large (virtually infinite). Control operation to either control surface or tab is not actually represented in this case, but the kinematic representation, which is all that matters for the present purpose, is authentic. The system becomes effectively that of Fig. 2, where the spring K_c now represents the stiffness of the control circuit operating the control surface, and the S-system linkage is effectively replaced by the spring K_t ($K_t = K_s l_4^2 / l_3^2$). It is evident that for the system of Fig. 1, with K_m virtually infinite, it is impossible to move the control surface relative to link (2), and in the sense of the physical definition of N the follow-up ratio is therefore effectively zero.

(*d*) *The Geared-Tab System.*—A geared-tab system may be represented as an S-system with $K_m = 0$ and K_o normally very large. Spring K_c again represents the stiffness of the control circuit operating the control surface. For the tab to act as a balance tab the follow-up ratio N must be negative and is then (with positive sign) termed the gear ratio. The system is then effectively that of Fig. 3a. If N is positive, the tab behaves as an anti-balance (Fig. 3b).

Having established the above relationships between the various specific tab systems and the generalised S-system, the next stage is to consider the comparative flutter characteristics of the generalised S-system and the T-system.

4. *Flutter Equations for the T-system and the S-system.*—Throughout this report, when comparing T- and S-systems, the index $^+$ is used to denote quantities appropriate to the T-system. Quantities without the index $^+$ refer to the S-system.

The flutter equations are the equations of motion for the system in the critical flutter condition when the several parts of the system are executing simple harmonic motion with a common frequency. For the T-system in the ternary case, the flutter equations may be written in absolute form as

$$\sum_{j=1}^{3}\left[-(w_f^+)^2 A_{ij}^+ + D_{ij}^+ + E_{ij}^+\right] q_j^+ = 0 \quad (i = 1,2,3) \qquad \ldots \quad \ldots \quad \ldots \quad (13)$$

where w_f^+ is the circular frequency, the A_{ij}^+ are the inertia coefficients (including the virtual inertia of the air) and the E_{ij}^+ are the elastic stiffness coefficients. The D_{ij}^+ are the air-force coefficients excluding the virtual inertia of the air, and are complex functions involving the air speed, the frequency, and the dimensions of the plan-form. q_j^+ are the generalised Lagrangian co-ordinates appropriate to the degrees of freedom considered. For the case already investigated[1], $q_1^+ = z$, $q_2^+ = \beta$, $q_3^+ = \gamma$. The inertia coefficients are then

$$\left.\begin{aligned}
A_{11}^+ &= m_w^+ + a_{11}^+ \\
A_{12}^+ &= m_c^+ x_c^+ + a_{12}^+ \\
A_{13}^+ &= m_t^+ x_t^+ + a_{13}^+ \\
A_{22}^+ &= I_c^+ + a_{22}^+ \\
A_{23}^+ &= m_t^+ x_t^+ (E_1 - E_2) c_w + I_t^+ + a_{23}^+ \\
A_{33}^+ &= I_t^+ + a_{23}^+ \\
\text{with} \quad A_{ij}^+ &= A_{ji}^+ \,,\; a_{ij}^+ = a_{ji}^+
\end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad (14)$$

(the a_{ij} representing the virtual inertias of the air), and the elastic coefficients are

$$\left.\begin{aligned}
&E_{11}^+ = K_w^+ \,,\; E_{22}^+ = K_c^+ \,,\; E_{33}^+ = K_t^+ \\
&E_{ij}^+ = 0 \text{ for } i \neq j
\end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

For the S-system in the ternary case, the flutter equations may likewise be written

$$\sum_{j=1}^{3}\left[-w_f^2 A_{ij} + D_{ij} + E_{ij}\right] q_j = 0 \quad (i = 1,2,3) \,. \qquad \ldots \quad \ldots \quad \ldots \quad (16)$$

If the plan-form of the S-system is the same as that of the T-system, and if the same degrees of freedom are considered (i.e., $q_1 = z$, $q_2 = \beta$, $q_3 = \gamma$), then

$$a_{ij} = a_{ij}^{+} \qquad \ldots \quad (17)$$

and the coefficients A_{ij} are given by equations (14) with the index $^{+}$ omitted. .. .. (18)

Also, for the same speed and frequency,

$$D_{ij} = D_{ij}^{+} . \qquad \ldots \quad (19)$$

Because in the S-system K_m is finite and $K_o \neq 0$, a similar relationship does not exist between the elastic coefficients of the two systems. For the S-system the elastic coefficients (as derived in Appendix I) are

$$E_{11} = K_w ,\ E_{12} = 0 ,\ E_{13} = 0 \qquad \ldots \quad (20)$$

$$E_{22} = K_c + \frac{K_o(K_m + K_s)}{\dfrac{K_o}{N_1^2} + K_m + K_s} \qquad \ldots \quad (21)$$

$$E_{23} = -\frac{1}{N}\frac{K_o K_s}{\dfrac{K_o}{N_1^2} + K_m + K_s} \qquad \ldots \quad (22)$$

$$E_{33} = N_1^2 \frac{K_s\left(\dfrac{K_o}{N_1^2} + K_m\right)}{N^2\left(\dfrac{K_o}{N_1^2} + K_m + K_s\right)} \qquad \ldots \quad (23)$$

$$E_{ij} = E_{ji} \text{ for } i \neq j \qquad \ldots \quad (24)$$

where $N_1 = l_1/l_2$ (the eccentricity ratio) .. .. (25)

and $N = \dfrac{l_1}{l_2}\dfrac{l_3}{l_4}$ (the follow-up ratio) . .. .. (26)

It can easily be verified that, with $K_o = 0$ and $K_m = \infty$, the expressions for the coefficients (20) to (24) for the S-system reduce to those of (15) for the T-system.

5. *General Recommendation for all Cases of the S-system.*—In the development of a criterion for the prevention of flutter of T-systems[1], calculated values of the reduced critical speed v_c^{+} were plotted against the non-dimensional tab out-of-balance moment p_t. Special interest was attached to the two vertical asymptotes ($v_c \rightarrow \infty$) lying nearest to the v_c^{+}-axis, and approximate expressions for the abscissae (p_t values) of these asymptotes were deduced as follows:

For the right-hand asymptote

$$p_{t\ (1)}^{+} = F_1(\mu^{+}, i_c^{+}, i_t^{+}, p_c^{+}, \bar{q}^{+}) \qquad \ldots \quad (27)$$

and for the left-hand asymptote

$$p_{t\ (2)}^{+} = F_2(\mu^{+}, i_c^{+}, i_t^{+}, p_c^{+}, \bar{q}^{+}) . \qquad \ldots \quad (28)$$

The functions F_1 and F_2 are those defined by equations (5) and (6) of the present report, and it will be remembered that their derivation is strictly valid only for certain ranges of the parameters concerned (*see* section 2). It is convenient to mention here that the above asymptotes both move to the left if i_t is increased, *i.e.*,

$$\frac{\partial F_1}{\partial i_t^+} < 0 \qquad \text{(29)}$$

and

$$\frac{\partial F_2}{\partial i_t^+} < 0 . \qquad \text{(30)}$$

The exact values of $p_t^+{}_{(1)}$ and $p_t^+{}_{(2)}$ would be obtained as solutions of the flutter equations (13) as v_c^+ (and w_f^+) $\rightarrow \infty$. They are therefore the solutions of the complex determinantal equation

$$\lim_{w_f^+ \rightarrow \infty} \left| -A_{ij}^+ + \frac{D_{ij}^+}{(w_f^+)^2} + \frac{E_{ij}^+}{(w_f^+)^2} \right| = 0 \qquad \text{(31)}$$

and are therefore independent of the values of the elastic coefficients E_{ij}^+. This explains why the functions F_1 and F_2 of (27) and (28) are functions of the geometry and inertias only. The terms $D_{ij}^+/(w_f^+)^2$ in (31) tend to finite values as $w_f^+ \rightarrow \infty$, and the asymptotes (as one would expect) are therefore dependent upon the aerodynamic characteristics.

For the S-system, the vertical asymptotes of the curve $v_c = f(p_t)$ as obtained from the flutter equations (16) will likewise be given by the equation

$$\lim_{w_f \rightarrow \infty} \left| -A_{ij} + \frac{D_{ij}}{w_f^2} + \frac{E_{ij}}{w_f^2} \right| = 0 \qquad \text{(32)}$$

and will also be independent of the elastic coefficients E_{ij}. Since, as shown in section 4, the only intrinsic difference between the S-system and the T-system is in the values of the elastic coefficients, it is evident that if the S-system has the same geometry and the same distribution of masses as the T-system then the abscissae of the asymptotes will also be the same, *i.e.*, $p_{t(1)} = p_t^+{}_{(1)}$ and $p_{t(2)} = p_t^+{}_{(2)}$. It follows therefore that the asymptotes for an S-system are given by equations (27) and (28) with the index $^+$ omitted.

The significance of these asymptotes lies in their relation to the existence of a flutter-free region, that is, a range of p_t values for which there is no flutter at any speed. If such a flutter-free region can be defined, under certain conditions, then these conditions in combination with a restriction that the value of p_t must lie within the limits of the flutter-free region constitute a criterion for the prevention of flutter. In the development of a criterion for the T-system[1], conditions were formulated under which no branch of the curve $v_c^+ = f(p_t)$ lay between the above asymptotes, in which case the flutter-free region could be defined as that between the asymptotes, *i.e.*, $p_t^+{}_{(2)} \leqslant p_t \leqslant p_t^+{}_{(1)}$. The conditions under which a flutter-free region can be defined must inevitably be concerned with finite branches of the flutter curve, and, unlike the conditions for the vertical asymptotes, must therefore be related to the values of the elastic coefficients. The criterion already derived for the T-system cannot therefore be directly applied to the S-system.

In the following sections of this report consideration is given to the conditions under which a flutter-free region may be defined for the several specific cases of the S-system. For the moment however, it can be stated that for the generalised S-system, and therefore for any of its specific cases, the value of p_t should at least not lie outside the region bounded by the above asymptotes. This means that the value of p_t should at least satisfy the conditions

$$p_t \geqslant F_2(\mu, i_c, i_t, p_c, \bar{q}) \qquad \text{(33a)}$$

$$p_t \leqslant F_1(\mu, i_c, i_t, p_c, \bar{q}). \qquad \text{(33b)}$$

This of course constitutes only a recommendation, as distinct from a sufficient condition for the prevention of flutter, but it is stated here because of its applicability to all cases of the S-system. For practical use it is advisable to introduce safety margins, as was done in the case of the T-system[1], in which case the recommendation (which is stated in full in section 11.1), takes the form $F_2 + \delta_2 \leqslant p_t \leqslant F_1 - \delta_1$.

6. *Derivation of Criteria for the Normal Spring-Tab System* (*S-system with* $N > 0$).—As already explained (section 5), a criterion for the prevention of flutter is based upon the definition of a flutter-free region. It is therefore necessary to consider under what conditions flutter at a finite speed can be prevented. For the S-system with positive N, it is proposed to derive these conditions in the following way. For any given S-system fluttering at a given speed and frequency it is possible to derive an equivalent T-system which will flutter at the same speed and frequency. From the earlier work on trimming-tabs[1] the conditions for prevention of flutter of the equivalent T-system are already known, and from the relationships existing between the two systems similar conditions for the S-system can then be derived. The conditions derived in this way for the S-system are, as will be seen later, subject to certain restrictions. They are applicable only to the case $N > 0$, and they can only be proved to prevent flutter of the type in which the flutter frequency f_f is greater than the natural tab frequency f_γ. The derivation of these conditions will now be given in detail.

Comparing the flutter equations (13) and (16) of a T-system and an S-system respectively, it is seen that the two sets of equations become identical if

$$D_{ij}^{+} = D_{ij} \qquad \text{(34)}$$

and

$$-(w_f^{+})^2 A_{ij}^{+} + E_{ij}^{+} = -w_f^2 A_{ij} + E_{ij}. \qquad \text{(35)}$$

Equations (34) are satisfied if the two systems have the same geometrical plan-form and the same critical speeds and frequencies. In Appendix II a set of relationships between the inertial and elastic characteristics of the two systems is derived which satisfies equations (35), with $w_f^{+} = w_f$. These relationships thus represent the transformation of an S-system into an equivalent T-system of the same geometrical plan-form which flutters at the same speed and frequency and with the same mode (*i.e.*, amplitude ratios) as the S-system.

If the common flutter frequency is written as

$$w_f^2 = f(w_3^{+})^2 = g(w_3)^2 \qquad \text{(36)}$$

where w_3^{+} and w_3 are the natural circular frequencies of the tab for the T- and S-systems respectively, then it is shown in Appendix II that, for the two systems related as above, if $f \geqslant 1$ then $g \geqslant 1$, and *vice versa*. This may be expressed by the following commutative relationship

$$f \geqslant 1 \rightleftarrows g \geqslant 1\,. \qquad \text{(37)}$$

The equations governing the transformation of an S-system into an equivalent T-system as derived in Appendix II, are

$$m_w^+ = m_w \qquad \text{(38a)}$$

$$m_c^+ x_c^+ = m_c x_c \qquad \text{(38b)}$$

$$m_t^+ x_t^+ = m_t x_t \qquad \text{(38c)}$$

$$I_c^+ = I_c \qquad \text{(38d)}$$

$$I_t^+ = \left(1 + \frac{\bar{N}}{g}\right) I_t + \frac{\bar{N}}{g} a_{33} \qquad \text{(38e)}$$

$$w_1^+ = w_1 \qquad \text{(38f)}$$

$$w_2^+ = w_2 \qquad \text{(38g)}$$

$$(w_3^+)^2 = \left(1 - \bar{N}\frac{1-g}{g+\bar{N}}\right) w_3^2 \qquad \text{(38h)}$$

where

$$\bar{N} = N \frac{K_o}{K_o + N_1^2 K_m}. \qquad \text{(39)}$$

The factor g defines the flutter frequency in terms of the natural tab frequency f_γ, as in (36).

The quantity $\bar{N}$ is the 'modified' follow-up ratio and is an important parameter in the criteria shortly to be derived. Its physical definition is that it is the ratio γ/β (*see* Fig. 1) when the control surface is moved by a moment acting only on the control surface, the link (0) being fixed. As the tab is unloaded there is no deflection of the spring K_s, but the modified follow-up ratio does take account of the action of the springs K_m and K_o. Equation (39) for $\bar{N}$ can be easily derived on the above physical basis. It should be noted that $\bar{N} = N$ if either $K_o = \infty$ or $K_m = 0$, either of which conditions corresponds to the point E in Fig. 1 being fixed. The definition of $\bar{N}$ is thus consistent with the definition of the follow-up ratio N given in section 3.2.

6.1. *Derivation of Spring-Tab Criterion I.*—In the earlier report on trimming-tab flutter[1] it was found that under certain conditions a T-system could flutter with very low critical speeds. As an example, Fig. 3c of that report shows low critical speeds associated with Branch III of the flutter curve. The flutter frequencies appropriate to these low critical speeds—at least as far as Branch III is concerned—were invariably greater than the natural tab frequency, i.e., $f \geqslant 1$. It is therefore possible in the absence of any preventive restriction for a T-system equivalent to a given S-system to experience this type of flutter; in which case the S-system would also flutter at the same low critical speed and, by relationship (37), with $g \geqslant 1$. It is obviously desirable that this type of flutter, more than any other, should be prevented. Although it is impossible by the method of the present report to derive criteria as comprehensive as those already obtained for trimming-tabs, it is nevertheless considered worthwhile to establish restrictions for a spring-tab system that will prevent flutter of the type for which $g \geqslant 1$ and for which the critical speeds would be very low.

It is convenient to state the proposed criterion first and then to prove its validity, for the conditions stipulated, afterwards. It is therefore suggested that the flutter of a spring-tab system for which

$$N \text{ (and therefore } \bar{N}) \geqslant 0 \qquad \text{(40)}$$

and $$f_f \geqslant f_\gamma \quad \ldots \quad (41)$$

will be prevented by the following conditions

$$f_\gamma/f_\beta \geqslant 2k_1 \quad \ldots \quad (42)$$

$$f_\gamma/f_z \geqslant 2k_2 \quad \ldots \quad (43)$$

$$f_\beta/f_z \leqslant k_3 \quad \ldots \quad (44)$$

$$p_t \geqslant F_2(\mu, i_c, i_t, p_c, \bar{q}) \quad \ldots \quad (45)$$

$$p_t \leqslant F_1(\mu, i_c, (1 + \bar{N})i_t + \alpha_{33}\bar{N}, p_c, \bar{q}) \quad \ldots \quad (46)$$

where the k_i ($i = 1, 2, 3$) are to be taken equal to unity. The functions F_1 and F_2 depend in the same way on their arguments as is defined in (5) to (12). The above criterion is in fact identical with the trimming-tab Criterion I as given in section 2 except for the fact that the function F_1 in (46) is evaluated using $(1 + \bar{N})i_t + \alpha_{33}N$ in place of i_t. α_{33} is the non-dimensional value of the virtual inertia coefficient a_{33} and is given by

$$\alpha_{33} = \frac{a_{33}}{\dfrac{\pi}{16}\rho c_w^{\,4} q s E_2^{\,3}} = \frac{f_{12}(E_2)}{E_2^{\,3}} < 1\cdot45\, E_2 \quad \ldots \quad (47)$$

where $f_{12}(E_2)$ is the function introduced by Dietze[5].

The validity of the above criterion can be proved indirectly by supposing that an S-system satisfying the conditions of the criterion does in fact flutter with a frequency $f_f = g f_\gamma$ with

$$g \geqslant 1\,. \quad \ldots \quad (48)$$

The characteristics of the equivalent T-system, which will also flutter at the frequency f_f, can then be obtained from the relationships (38a) to (38h).

Because of (40) and (48), equation (38h) gives

$$w_3^{+} \geqslant w_3\,. \quad \ldots \quad (49)$$

Because of (38f), (38g) and (49), conditions (42), (43), and (44) then give

$$f_\gamma^{+}/f_\beta^{+} \geqslant 2k_1 \quad \ldots \quad (50)$$

$$f_\gamma^{+}/f_z^{+} \geqslant 2k_2 \quad \ldots \quad (51)$$

$$f_\beta^{+}/f_z^{+} \leqslant k_3\,. \quad \ldots \quad (52)$$

Also, (38e) gives, because of (40) and (48),

$$I_t \leqslant I_t^{+} \leqslant (1 + \bar{N})I_t + a_{33}\bar{N}$$

or, in non-dimensional form,

$$i_t \leqslant i_t^{+} \leqslant (1 + \bar{N})i_t + \alpha_{33}\bar{N}\,. \quad \ldots \quad (53)$$

Since the geometrical plan-forms of the two systems are identical, it follows that

$$\bar{q}^+ = \bar{q} \, . \qquad \ldots \qquad (54)$$

Relationships (38a) to (38d) also give

$$\mu^+ = \mu \, ; \; p_c^+ = p_c \, ; \; p_t^+ = p_t \, ; \; i_c^+ = i_c \, . \qquad \ldots \qquad (55)$$

Considering the left-hand side of the inequality (53) together with (54) and (55), and remembering (29), condition (45) then gives

$$p_t^+ \geqslant F_2(\mu^+, i_c^+, i_t^+, p_c^+, \bar{q}^+) \, . \qquad \ldots \qquad (56)$$

Considering the right-hand side of the inequality (53) together with (54) and (55), and remembering (30), condition (46) then gives

$$p_t^+ \leqslant F_1(\mu^+, i_c^+, i_t^+, p_c^+, \bar{q}^+) \, . \qquad \ldots \qquad (57)$$

The characteristics of the equivalent T-system, which must flutter if the original S-system flutters, are thus given by the conditions (50), (51), (52), (56) and (57). But these are exactly the conditions which, by the trimming-tab Criterion I of section 2, are sufficient to prevent flutter of the T-system. The hypothesis that the original S-system could flutter under the conditions (40) to (46) is therefore wrong, and the conditions (42) to (46) are therefore proved to constitute a valid criterion for the prevention of flutter of an S-system with $\bar{N} \geqslant 0$ at a frequency $f_f \geqslant f_\gamma$.

The constants k_i ($i = 1, 2, 3$) are retained in the criterion for convenience of adjustment in the light of further experience. For design use, safety margins δ_1 and δ_2 are introduced into the conditions (45) and (46), as was done in the case of the trimming-tab Criterion I of section 2. At the same time condition (46) may be simplified by neglecting the term $\alpha_{33}N$; the value of α_{33}, as shown by (47), is small (normally less than 3 per cent of i_t) and the error involved is well covered by the safety margin δ_1. In this form the criterion is given in full in section 11.2.

6.2. *Derivation of Spring-Tab Criteria II and III.*—In section 6.1 the spring-tab Criterion I has been developed from the original trimming-tab Criterion I derived previously[1] and quoted in section 2. As mentioned in section 2, simplified forms of the trimming-tab Criterion I were also evolved, known as the trimming-tab Criteria II and III. Spring-tab criteria corresponding to these simplified trimming-tab criteria can obviously be derived in a similar way and will bear the same relationship to them as exists between the spring-tab Criterion I and the trimming-tab Criterion I. The spring-tab criteria are in fact derived from the trimming-tab criteria by simply replacing i_t by $(1 + \bar{N})i_t$, or I_t by $(1 + \bar{N})I_t$, in the formula for the upper limit of p_t. Spring-tab Criteria II and III derived in this way are given in full in section 11.2.

In section 11.2 the frequency condition (O 1) of the spring-tab Criterion III is given in the alternative form (O 2) involving the relevant inertias and stiffnesses. This is derived simply by writing (O 1) in the form

$$4k_1^2 \leqslant \frac{f_\gamma^2}{f_\beta^2} = \frac{E_{33}\,I_c + a_{22}}{E_{22}\,I_t + a_{33}} \simeq \frac{E_{33}\,I_c}{E_{22}\,I_t} \qquad \ldots \qquad (58)$$

and substituting for E_{22} and E_{33} from (21) and (23), whence (O 2) follows.

7. *Other Specific Cases of the S-system.*—The kinematics of the various standard tab systems have been considered in section 3 on the basis of representing them as specific cases of the generalised S-system. Criteria for the normal spring-tab system (S-system with $N \geqslant 0$) have been derived in section 6, and it now remains to consider how the developments of sections 5 and 6 may be applied to the derivation of criteria for the remaining specific cases of the S-system.

7.1. *The Trimming-Tab*.—With $K_m \neq 0$ and $K_o = 0$, and with $N_1^2/N \neq 0$, (39) gives $\bar{N} = 0$. The spring-tab criteria of section 6 are therefore applicable, and putting $\bar{N} = 0$ it is seen that they become the trimming-tab criteria of the earlier report[1].

7.2. *The Servo-Tab*.—With $N \geqslant 0$ and $K_m = 0$ the spring-tab criteria of section 6 are directly applicable.

7.3. *The Geared-Tab*.—With $K_m = 0$ and $K_o \neq 0$, and $\bar{N} = N$ by equation (39), the spring-tab criteria of section 6 are applicable if $N > 0$, that is to the case of the anti-balance tab (*see* section 3). For the normal balance-type of tab ($N < 0$), the spring-tab criteria are not applicable since they have been proved valid only for $N \geqslant 0$.

It is therefore necessary to give special consideration to the case of the geared balance tab with $\bar{N} = N < 0$. The method used in section 6 to derive criteria for spring-tabs is however still applicable. For a given geared-tab system and its equivalent T-system, fluttering at the same speed and frequency, relationship (38e) gives, with $\bar{N} < 0$ and $g > 0$,

$$I_t^+ < I_t$$

and therefore

$$i_t^+ < i_t \,. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (59)$$

Following an argument similar to that used in section 6.1, it can then be deduced that for the geared balance-tab necessary conditions for the prevention of flutter are

$$p_t \geqslant F_2(\mu, i_c, i_t, p_c, \bar{q}) + \delta \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (60)$$

$$p_t \leqslant F_1(\mu, i_c, i_t, p_c, \bar{q}) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (61)$$

where δ is a positive quantity. This means that, of the two conditions (33a) and (33b) given in section 5 as a general recommendation, (33b) is for the geared balance-tab a sufficient condition whilst (33a) is not. On the other hand, (45) and (46) mean that for the spring-tab (33a) is a sufficient condition whilst (33b) is not. This difference between the two cases arises directly from the change of sign of $\bar{N}$.

In the case of the geared balance-tab there is unfortunately no ready-made substitute for (33a), such as (46) proved to be for (33b) in the case of the spring-tab with $g \geqslant 1$. It is therefore proposed to cover condition (60) by an arbitrary increase in the value of the constant k_6 of condition (P) of the spring-tab Criterion III (section 11.2), which corresponds to condition (45) of the spring-tab Criterion I and therefore to condition (60) with $\delta = 0$. In this form, with the constant k_9 replacing k_6, condition (60) becomes condition (R) of section 11.3. Condition (61), which is identical with the corresponding condition in the trimming-tab Criterion I of section 2, is at the same time put in the form of the trimming-tab Criterion III and as such is given by (S) of section 11.3.

Conditions (60) and (61) will not alone be sufficient to prevent flutter of the geared balance-tab without additional conditions for the frequencies corresponding to (1), (2), and (3) for the T-system. These conditions are not however immediately derivable as in the case of the spring-tab, but it is fairly certain that they will amount to a similar requirement that the tab frequency f_γ should be high compared with the natural frequencies f_β and f_z. For the geared-tab system this requirement will generally be automatically satisfied by the normally high stiffness of the tab linkage. It is therefore not considered worthwhile to go further into details regarding the frequency conditions. Backlash in the tab linkage should of course be avoided.

The criterion for geared-tabs is given in full in section 11.3.

8. *Comparison with the Collar-Sharpe Criteria for Spring-Tabs.*—Using the notation of the present report, the Collar-Sharpe criterion in its first form[2] may be written as

$$\frac{(1 + N)I_t + (E_1 - E_2)c_w m_t x_t}{I_c} < K \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (62)$$

and in its second form (due to Sharpe)[3] as

$$\frac{(1 + N)I_t + (E_1 - E_2)c_w m_t x_t}{I_c} < K' p^{3/2} . \qquad \ldots \quad \ldots \quad \ldots \quad (63)$$

Both (62) and (63) are based on binary (β,γ) considerations. It is instructive to compare these with the criteria for spring-tabs derived here in section 6. A similar comparison has already been made for the case of the trimming-tab[1].

For the purpose of this comparison the most suitable choice is the spring-tab Criterion III, involving the three conditions (O 1), (P) and (Q 1) of section 11.2. Condition (Q 1) is the equivalent of (62) or (63), whereas conditions (Q 1) and (P) have no counterpart in the Collar-Sharpe criterion and therefore represent additional restrictions.

Comparing condition (Q 1) of spring-tab Criterion III with the alternative criteria (62) and (63) of Collar and Sharpe, it is seen that the follow-up ratio N in the Collar-Sharpe criteria is replaced by the modified follow-up ratio $\bar{N}$. Since, with non-zero values of K_o and K_m, equation (39) gives $\bar{N} < N$, this represents a relaxation on the Collar-Sharpe criterion ; though in most cases the effect will be small because K_m is normally small compared with K_o. This was in fact realised by Collar and Sharpe, who assumed K_m zero for reasons of simplicity.

A more important difference is that the constants K' and K of the Collar-Sharpe criteria are replaced by C and $C_1 = Cp^{3/2}$ (with the present unit value of k_7), both of which are functions of i_c, $(1 + \bar{N})i_t$, p, q, and E_1. In Table 1 the values of C and C_1 as given by the formulae (Q 2) and (Q 3) of section 11.2 for a range of values of the relevant parameters are compared with the Collar-Sharpe values for K' and K. The table is in fact identical with that given in the similar comparison already made for the case of the trimming-tab[1], and the conclusions are the same: for the sake of completeness they are however re-iterated here. For all cases except Case 1 the Sharpe (K') criterion is more restrictive than that of (Q 1), (Q 2), and (Q 3). In the extreme Case 1 the small value of C comes from high values of $(1 + \bar{N})i_t$, i_c, and small values of p, q, and E_1. Following Cases 1 to 5 in Table 1 step-by-step it is apparent that C increases by decreasing i_c and i_t and by increasing p and E_1. Comparing Cases 1 and 1a and Cases 5 and 5a respectively it is seen that for small values of i_c, i_t the value of C decreases if q increases, but for large values of i_c and i_t the value of C increases if q increases. Inspection of the formulae (Q 2) and (Q 3) confirms these conclusions drawn from Table 1 and shows in addition that for small values of i_c and large values of q the value of C might increase if i_t increases. Further it should be noted that the influence of i_c upon C is much greater than that of i_t. This influence of i_c is such that a large value of I_c is not as beneficial as it would be if C in condition (Q 1) were a constant, as K' is in the Sharpe criterion (63). It should also be noted that through i_c and i_t the value of C depends upon the air density, in such a way that C will normally decrease with altitude. Comparing finally the values of C and C_1 in Table 1 it is seen that the former varies less than the latter, so that of the two Collar-Sharpe criteria (63) is at any rate an improvement on (62).

Of the two additional conditions (O 1) and (P) occurring in the spring-tab Criterion III of section 11.2, the second condition (P) simply sets a forward limit to the tab c.g. position which is slightly behind the tab hinge*. The first condition (O 1) is concerned with the spring stiffnesses, and it is important to consider the implications of this requirement.

* It should be noted however that under the conditions of the spring-tab Criterion I this forward limit may be in front of the hinge-line.

Taking the equivalent condition (O 2) and in the first place putting $K_m = K_c = 0$ and $k_1 = 1$, the requirement becomes

$$\frac{N^2 I_t}{I_c} \leqslant 0 \cdot 25 . \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (64)$$

In Table 2 the values of $N^2 I_t/I_c$ are given for the various specific cases considered by Sharpe[3] in his statistical survey. It is seen that the values are nowhere greater than $0 \cdot 182$ and therefore satisfy the condition (64). For the 'normal' case (*see* section 3.2) in which K_c is zero, and provided also that K_m is small compared with K_s, condition (O 2) is therefore unlikely to prove an embarrassment. The same conclusion may be deduced by considering (64) in its non-dimensional form,

$$\frac{N^2 I_t}{I_c} = \frac{N^2 i_t p^3 q}{i_c} \leqslant 0 \cdot 25 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (65)$$

and by taking not too favourable values for the non-dimensional parameters, *viz.*, $N = 4$, $i_t/i_c = 2$, $p = 0 \cdot 2$, $q = 0 \cdot 7$, the left-hand side of (65) becomes $N^2 I_t/I_c = 0 \cdot 179$.

Considering, however, cases in which K_m and K_c in condition (O 2) are not zero, it is evident that condition (O 2) may become restrictive if either K_s is comparable with K_m or K_c is comparable with K_o. Difficulty may therefore be experienced with condition (O 2) if either

(*a*) K_s is low, resulting primarily in a low tab frequency f_γ, or

(*b*) for the case in which the motion is not consistent with normal operation (*see* section 3.2) K_c is such as to make the control-surface frequency f_β comparable with the tab frequency f_γ.

9. *Design Implications Relating to Spring-Tabs.*—From the criteria produced in this report it is possible to deduce directly those factors which will be beneficial from a flutter point of view. In all cases, however, the tab is designed to perform a function concerned primarily with the control of the aeroplane, and considerations of flutter prevention represent in fact only a subsidiary though an important feature of the design. In the case of spring-tabs these two sets of considerations are very closely bound up and it is desirable that they should as far as possible be taken together. An attempt is therefore made to deduce the factors having a beneficial effect on the flutter characteristics, subject to the over-riding condition that the tab system is designed to perform a given control function.

9.1. *Theoretical Deductions.*—The control function specified for this purpose is simply that the tab system is so designed that the pilot performs a given amount of work in moving the control surface through a given angle β at a given speed. It is convenient in this connection to postulate a factor E representing the tab effectiveness, defined by the statement that $(1 - E)$ is the ratio of the actual work done to the work that would be required if the tab were inoperative (*i.e.*, with $K_m = K_s = \infty$).

The tab effectiveness E is a function of the geometric and aerodynamic characteristics of the tab and control surface and of the elastic stiffnesses and geometry of the system. In Appendix III this relationship is derived in the following form, convenient for the present purpose:

$$N\frac{b_3}{b_2}q = \frac{1 - \dfrac{\lambda_m N_1^2}{b_2}\left(1 - \dfrac{b_2}{\lambda_o}\right)E}{1 - 2E\left(1 - \dfrac{b_2}{\lambda_o}\right) - \dfrac{2b_2}{\lambda_o} - p^2\dfrac{Nc_3}{b_3}\left(1 + \dfrac{\lambda_m}{\lambda_s}\right)\left(1 - \dfrac{b_2}{\lambda_o} - \dfrac{b_2}{N_1^2\lambda_m}\right)} . \qquad \ldots \quad \ldots \qquad (66)$$

Here λ_o, λ_m, λ_s are the non-dimensional forms of the spring stiffnesses K_o, K_m, K_s (*see* section 14). Coefficients b_2, b_3, c_3, define the aerodynamic hinge moments about the control-surface and tab hinges (*see* Appendix III) as follows:

$$M_c = (b_2\beta + qb_3\gamma)sc_c^2Q \quad \ldots \quad (67)$$

$$M_t = c_3\gamma sqc_t^2Q \quad \ldots \quad (68)$$

It should be mentioned that equation (66), as derived in Appendix III, is an approximation which is not valid for small values of λ_m. It cannot therefore be applied to the extreme case of a servo-tab ($\lambda_m = 0$).

The question to be considered is how the spring-tab criteria of this report may best be satisfied, subject to the over-riding conditions that equation (66) must also be satisfied with a given value of E. Choosing for simplicity the spring-tab Criterion III of section 11.2, the three conditions involved are (O 1), (P) and (Q 1). In non-dimensional form, the condition (P) amounts simply to a requirement that $p_t \geqslant k_6$. The other two conditions however require more detailed consideration.

Condition (Q 1) is easily reducible to the non-dimensional form

$$N\bar{q}\left[\left(\frac{1+\bar{N}}{N}\right)pi_t + \frac{2}{N}(1-p)p_t\right] \leqslant k_7C\frac{j}{\sqrt{p}}i_c = C_2 . \quad \ldots \quad (69)$$

It will be assumed that i_c changes little as other changes are made. Also, since the proportional effect of a change in $(1 + \bar{N})i_t$ on the value of C is always numerically less than the effect on the left-hand side of (69), and the same is also true for changes in q (*see* for instance Table 1), then from a purely qualitative point of view C_2 in (69) may be regarded as constant.

From the definition of $\bar{q}$ given elsewhere (*see* also section 14 of this report) it is evident that

$$\bar{q} \simeq C_3\frac{b_3}{b_2}q \quad \ldots \quad (70)$$

where C_3 is a factor of proportionality and the sign ($\simeq$) of approximate equality is taken because b_3 and b_2 represent the actual values of the aerodynamic parameters whereas $\bar{q}$ has originally been defined by theoretical values.

Combining (69) and (70) gives, for the condition (Q 1) of the spring-tab Criteria III,

$$N\frac{b_3}{b_2}q\left[\left(\frac{1+\bar{N}}{N}\right)pi_t + \frac{2}{N}(1-p)p_t\right] \leqslant C_4 \quad \ldots \quad (71)$$

where again C_4 may for the present purpose be regarded as constant.

The factors beneficial from a flutter point of view, as far as conditions (P) and (Q 1) of the flutter criterion are concerned, may now be deduced, subject to the over-riding condition that equation (66) is at all times satisfied. Comparing (71) and (66), it is evident that anything which reduces either the term in (71) within the square brackets or the right-hand side of (66) will be beneficial while (66) remains operative. (It should be remembered incidentally that b_2, b_3, c_3 will normally be negative.) Small i_t, p_t ($\nless k_6$ by condition Q), and λ_m/λ_s are thus seen to be favourable; also a small p if p_t is small enough. The effect of $\bar{N}$ is rather mixed and will depend upon the values of the other parameters. It is interesting to note from (66) that the adoption of as small a value of E as possible will help the flutter problem.

From (66) it might also be deduced that small values of $-b_2$ and $-c_3$ would be beneficial, assuming $-b_2/\lambda_o$ normally small. Any appreciable reduction of these values can however only be effected by aerodynamic balance in the form of a setback hinge, and since the flutter criterion is not applicable to such cases (*see* section 1) this deduction cannot therefore be made.

There remains the frequency condition (O 1) of the flutter criterion. Using the alternative form (O 2) of section 11.2, this is quickly reducible to

$$\frac{N^2 p^3 q i_t}{i_c} = N\frac{b_3}{b_2}q\left[N\frac{b_2}{b_3}p^3\frac{i_t}{i_c}\right] \leqslant \frac{0\cdot 25}{k_1^{\,2}} \cdot \frac{1 + N_1^{\,2}\dfrac{\lambda_m}{\lambda_o}}{\left(1 + \dfrac{\lambda_m}{\lambda_s}\right)\left(1 + \dfrac{\lambda_c}{\lambda_o}\right) + \dfrac{\lambda_c}{N_1^{\,2}}} \quad \ldots \quad \ldots \quad (72)$$

The value of λ_m/λ_o in (72) will normally be small. As far as (72) is concerned therefore, and again remembering (66), it is seen that small values of N, p, i_t, λ_m/λ_s, and λ_c will be favourable.

For the flutter criterion as a whole, in combination with the specified control function as represented by equation (66), it can therefore be said that small values of i_t, p_t ($\not< k_6$), λ_m/λ_s, and λ_c—and of p if p_t is small enough—will be favourable. A small N, though clearly favourable to condition (O 1), may not necessarily be so in the case of condition (Q 1). Finally, the tab effectiveness E should be no greater than the minimum desirable from the control point of view.

9.2. *Practical Interpretation of the Deductions.*—The effect of the follow-up ratio N will in practice depend very much upon the particular conditions. As shown in the preceding section 9.1, a small N is clearly helpful only in the case of the frequency condition (O 1). This condition will not however normally present any difficulty (*see* section 8), except in the case of a low subsidiary spring stiffness K_s or a high stiffness K_c. In such cases a reduction in N might be resorted to, provided there is no difficulty with the inertia condition (Q 1). From the control point of view, too low a value of N might result in an undesirable 'sponginess' of feel[6].

The value of p_t will normally be small enough to make a small value of p (ratio of tab chord to control-surface chord) beneficial. From the control point of view a small value of p is also desirable, though it should not be less than about $0\cdot 10$. Values of p below this limit might in any case involve unduly low torsional stiffness of the tab.

From a flutter point of view it would of course be unwise to reduce p_t down to the lower limit (k_6 in the case of the spring-tab Criteria II and III). The greatest margin of safety is obtained with a value of p_t roughly half-way between the upper and lower limits, and as far as possible this condition should be achieved.

To provide as small a value of i_t as possible is largely a matter of basic design. The main structural member should be located near the hinge and the remaining structure made as light as possible consistent with other requirements. Here again, care must be taken that the torsional stiffness of the tab is not seriously reduced.

The recommendation that λ_m/λ_s, the ratio of the main spring stiffness to the subsidiary spring stiffness, should be small may seriously conflict with control requirements[6]. A satisfactory variation of stick force characteristic with speed can sometimes only be obtained with a fairly flexible subsidiary spring and not too flexible a main spring. In such cases a detailed examination of the possibilities in relation to the other parameters already discussed will be necessary, but a compromise between control and flutter requirements may eventually have to be made in the case of λ_m/λ_s.

As explained in section 3.2, the spring K_c can be present only in cases where the control-surface motion is not consistent with normal operation, and the requirements that λ_c (or K_c) should be small applies therefore only to such cases. For the elevator, where the problem is likely to be most acute, the appropriate motion is antisymmetric and the stiffness K_c is that provided by the structural interconnection between the two halves of the elevator. If it becomes necessary to reduce K_c this can only be done by reducing the stiffness of the interconnection. There is in fact no reason why the interconnection should not be deleted, thus reducing K_c to zero, provided that each half-elevator is adequately mass-balanced against fuselage torsion to prevent ordinary elevator anti-symmetric flutter. In this case it would of course be essential to have a spring-tab on each half-elevator. From a control point of view a symmetric system is preferable to an unsymmetric system, whether there is an interconnection or not.

An alternative solution to this problem would be to forsake the criteria and make K_c very large by increasing the stiffness of the interconnection. Although the frequency condition $f_\gamma \geqslant 2f_\beta$ would then be far from satisfied and flutter would therefore possibly occur at some finite speed, this speed would be very high in virtue of the high stiffness K_c. Unfortunately there is no quantitative evidence as to the minimum value of K_c for this condition, though in a specific case it could be determined by detailed flutter calculations.

It may be wondered how the tip-to-tip elevator torsional stiffness criterion of Ministry of Supply publication A.P.970 (Chap. 504) stands in relation to the above considerations. In point of fact the A.P.970 requirement is based on static control considerations and is appropriate to the case of a plain elevator operated directly at its centre. It therefore provides no criterion for the stiffness of the interconnection in the sense discussed above. Furthermore, the A.P.970 requirement does not in principle prohibit deletion of the interconnection though in such a case the requirement could not of course be applied in exactly its present form.

10. *Application to Actual Systems.*—The criteria developed in sections 6 and 7 are strictly appropriate to the idealised system of section 3 and Fig. 1. They are also subject to the assumptions outlined in section 1. It now remains to consider in what manner and to what extent these criteria may be applied to actual systems in practice. It will, of course, be possible to adjust the criteria to the results of experience by alterations in the values of the constants k_1 to k_9 occurring in the criteria, but any adjustment of this sort must primarily be in the nature of a refinement.

10.1. There is firstly the broad question of how an actual system is to be replaced by an equivalent idealised system appropriate to the criteria. A strict investigation has not been made into this question, and the following recommendations should therefore be regarded as provisional.

The spans of the idealised lifting surface and control surface will normally be taken equal to the span of the actual control surface. Some reduction of this span will be made only at that end where the control surface ends together with the lifting surface. The tab will be given the same span as it has in reality, possibly with a similar reduction as mentioned above. The chord c_w will be taken as the mean chord of that part of the lifting surface covered by the control surface. The chords c_c and c_t will be taken as the root-mean-squares of the actual control surface and tab chords.

The non-dimensional quantities q, E_1, E_2, p, μ, i_c, i_t, p_c, p_t, N can then be calculated. Knowing the stiffnesses K_o and K_m of the circuit and main spring (K_o being preferably obtained by test), $\bar{N}$ can then be calculated from N. In point of fact $\bar{N}$ will normally be very slightly less than N, and the use of N in the criteria in place of $\bar{N}$ will therefore represent only a slight (and conservative) error.

In the development of the criteria the mode assumed for the lifting surface is one of pure normal translation. The appropriate mode for the lifting surface of the actual system will be that fundamental mode of the structure which involves appreciable normal translation of the lifting surface (*e.g.*, wing fundamental bending in the case of the aileron). This mode will in general contain some pitching or will be such that the normal translation z is not constant over the span covered by the control surface, amounting effectively to a rolling constituent in the mode. Small amounts of pitching or rolling will not seriously affect the validity of the criteria: the safety margins occurring in the criteria provide some latitude, which can if necessary be enlarged by appropriate adjustments to the constants k_1 to k_9 as mentioned above. Moderate amounts of rolling may be allowed for by using a modified value for p_t in conditions (A), (J), (N 1), (P), (R) of section 11. In the expression for p_t, $m_t x_t$ should then be replaced by

$$\sum_T \frac{\delta m_t xy}{y_m}$$

where $\sum_T$ denotes summation over the tab

δm_t is a mass element of the tab

x is distance of the element aft of the tab hinge

y is the distance of the element from the effective rolling axis

y_m is the mean y-value of all elements of the part of the lifting surface (including control surface and tab) under consideration.

(For the manner of calculating mass couplings of parts of actual aeroplanes, reference should be made to Duncan, Ellis and Gadd[7].)

The above modified p_t value is applied only to the conditions (A), (J), (N 1), (P), (R) because only these conditions represent a restriction of the mass coupling between lifting-surface and tab motions.

The criteria should not be applied to cases where the mode of the lifting surface contains a considerable amount of pitching. It may be that modifications to the constants k_1 to k_9 might be found to cover such cases, but further investigation of this aspect is required.

It should be mentioned that in the case where the actual lifting surface has a span greater than that of the control surface (*e.g.*, wing and aileron) the part of the lifting surface without control surface will tend to damp the flutter motion in those modes in which the wing motion plays an essential part. (It is assumed here that the system is flutter free without the tab.) In such a case the constant k_5 in conditions (A) and (J) may be allowed to have higher values and the constants k_6 and k_9 in (N 1), (P) and (R) lower values.

10.2. The next question to be considered is that of the more specific aspects concerned with design clearance of the aircraft. In the first place there are three criteria to choose from in the case of spring, servo, and trimming-tabs. From the point of view of convenience in the design stage, Criterion III is in each case the obvious choice. Apart from being simpler in form it involves only the frequencies f_γ and f_β of the tab and control surface, which can be calculated (using for instance the formulae of this report) provided the relevant stiffnesses are known or can be estimated. Criterion III does, of course, apply a greater restriction on the forward limit of the tab c.g. than does Criterion I, but this will not normally prove an embarrassment. For practical use, Criterion III is therefore to be generally recommended.

With the relevant design data assembled, modified as necessary according to the general recommendations of section 10.1 above, the design can then be checked in the light of the appropriate criterion and suitable modifications made if required.

It is important, however, that each tab system should be checked for both the 'normal' case (in which the motion is consistent with normal operation) and the 'non-normal' case (in which the motion is not consistent with normal operation). This point cannot be too strongly emphasised. In the case of the elevator, for instance, the 'normal' case involves symmetric motion and the 'non-normal' anti-symmetric motion. The frequency f_z is in the 'normal' case that of the fuselage fundamental vertical bending mode, and in the 'non-normal' case that of the fuselage fundamental torsion mode: the frequency f_β is in the 'normal' case that of the elevator oscillating as a whole against its circuit, and in the 'non-normal' case that of the elevator oscillating anti-symmetrically, one half against the other. The tab frequency f_γ will also not necessarily be quite the same in the 'normal' and 'non-normal' cases. It is therefore evident that the natural frequencies will in general be different in the two cases ; and, as already pointed out in section 9.2, difficulty may be experienced in meeting the frequency condition $f_\gamma/f_\beta \geqslant 2k_1$ in the 'non-normal' case, particularly for an elevator whose two halves are directly connected. For aileron systems the 'non-normal' (symmetric) case rarely presents any difficulty, unless there is a separate balance circuit of exceptionally high stiffness. For a single integral rudder there is of course no 'non-normal' case.

A further important point arises in connection with elevator systems in which there is a tab on one side of the elevator only. Any flutter motion occurring on such a system is likely to be unsymmetric, that is a mixture of symmetric and anti-symmetric, and a type of flutter is therefore possible in which that side of the elevator carrying the tab oscillates while the other side remains more or less stationary. To cover this possibility, the inertia conditions of the criterion must be covered for the case in which only the half elevator is effective. This means that the elevator intertia I_c and span s are halved and the proportional tab span q is doubled, compared with the case in which the full elevator is effective. Non-dimensional quantities i_c, i_t, p_t are however unaffected. For a spring-tab system, for instance, the effect would therefore be to halve the value of I_c and double the value of q in condition (Q 1) of the spring-tab criterion III, or simply to double the value of q in conditions (J) and (K) of the spring-tab criterion I. In either case the criteria become more difficult to meet compared with the case in which the full elevator is effective.

It should be noted incidentally that, either directly or indirectly through the values of the non-dimensional parameters i_c, i_t, p_t, the criteria are a function of air density. In any particular case the criteria should therefore be satisfied over the complete height range concerned, though it will generally be found in practice that the maximum height involves the greatest restriction on the structural parameters.

A final check on the design values of the parameters used in the criteria will be obtained from test measurements. Resonance tests on the complete aeroplane give the natural frequencies f_γ, f_β and f_z of the tab, control surface, and lifting surface appropriate to both the 'normal' and the 'non-normal' cases for each particular system. Natural frequencies f_γ and f_β of the tab and control surface are best obtained by direct excitation of the surface concerned, under the conditions implied by the definitions of these frequencies in section 14. This means that for the tab frequency f_γ the tab is excited with the control surface locked to the lifting surface, and for the control-surface frequency f_β the control surface is excited with the tab locked to the control surface: in each case the appropriate cockpit control is held fixed. The frequency f_z of the lifting surface will be obtained from the tests in which excitation is applied to the main structure: at the same time the mode associated with f_z is determined and an assessment can be made of the possible effect of the modal characteristics on the applicability of the criteria. This last point is important: even if the criterion used does not directly involve f_z (as is the case for instance with the spring-tab criterion III) the applicability of the criterion is still dependent on the modal characteristics of the lifting surface in the manner already discussed. Finally, an experimental check should be made of the inertias and out-of-balance moments of control surface and tab, using for instance the standard weighing and swinging techniques.

A final design point worth mentioning is that if conditions (P) and (Q 1) say of the spring-tab criterion III (the same applies also to servo and trimming-tabs) result in there being no range of tab c.g. positions that will satisfy the criterion—which would be the case if the limits appropriate to the two conditions were coincident or overlapped—then a design might still be possible on the basis of the criterion I which allows a more forward limit to the tab c.g. position. The principle of weight economy will of course favour a backward tab c.g., and criterion III, which prohibits static or over-balance of the tab, will normally prove no embarrassment. In unfortunate cases it may however prove expedient to turn to criterion I, which may allow tab c.g. positions on or forward of the hinge; but in such cases it should be remembered that criterion I requires the frequency conditions (G) and (H) (in the case of the spring tab) to be satisfied.

10.3. The final question to be discussed is that of the conditions for which the criteria of this report are patently inapplicable. Briefly, these relate to the stick condition, aerodynamic balance of control surface and tab, state of mass-balance of the control surface, and the modal characteristics of the lifting surface.

Consideration of the stick condition arises in the case of spring-tab and servo-tab systems. The criteria which have been evolved for such systems in sections 6 and 7 of this report apply to the stick (or pedal) -fixed condition. For the 'non-normal' case the stick may be regarded as effectively fixed, but for the 'normal' case it is evident that in practice the stick-free condition must also be considered. Two broad types of stick-free flutter are possible in which the stick is either in or out of phase with the control-surface lever (link (2) of Fig. 1). The question is discussed in greater detail in Appendix IV, but what evidence there is suggests that the in-phase type of flutter will not normally occur. Possible exceptions to this are cases where the follow-up ratio is very low or where there is a bob-weight attached to the stick (*see* Appendix IV): such cases require special investigation. For the out-of-phase type of flutter, in which there is a node C somewhere in the control circuit between stick and control surface, the system becomes effectively that of Fig. 1 but with a higher control circuit stiffness (say $\bar{K}_o$) than in the stick-fixed case. For a given position of the node, the criteria of this report could be applied using the appropriate value for $\bar{K}_o$ and the corresponding effective values $\bar{f}_\gamma$ and $\bar{f}_\beta$ of the tab and control surface natural frequencies.

In Appendix IV it is shown that if either criterion II or criterion III for spring or servo-tabs can be satisfied using, in place of f_γ and f_β, the effective values $\bar{f}_\gamma$ and $\bar{f}_\beta$ for the case $\bar{K}_o = \infty$, then out-of-phase types of flutter will be prevented irrespective of the flutter frequency. The case $\bar{K}_o = \infty$ corresponds to the node being at the control-surface lever, and experimental values for the frequencies $\bar{f}_\gamma$ and $\bar{f}_\beta$ can therefore be obtained from resonance tests in which the control-surface lever is locked to the lifting surface. For a given system the criteria will be more difficult to satisfy using $\bar{f}_\gamma$ and $\bar{f}_\beta$ as above than when using f_γ and f_β appropriate to the stick fixed conditions: if, however, the actual circuit stiffness K_o is large compared with K_m, the difference will not be very great.

The effects of aerodynamic balance of the control surface or tab are not covered by the criteria of this report. A qualitative estimate of these effects can however be made based on the general statement by Voigt and Walter[8,9] that the 'effective' natural frequency of a control surface in an air stream will be reduced by aerodynamic balance. Considering the frequency conditions (F), (G) and (H) of section 11.2, aerodynamic balance of the control surface will in this respect have a good effect and aerodynamic balance of the tab will have a bad effect. From the control point of view, aerodynamic balance of the control surface will require a smaller tab effectiveness E (*see* section 9) and this in turn will have favourable repercussions on the flutter characteristics.

The criteria of this report are strictly valid only for the case $p_c = 0$, representing static balance of the control surface (including tab) about the control-surface hinge. Slight departures from this condition will not materially affect the validity of the criteria, but the criteria as they stand cannot be applied to cases in which the control surface is appreciably under- or over-mass-balanced. From the work of Buxton[10], it is known that decreasing p_c results mainly in a movement of the left-hand asymptote of the $v_c \sim p_t$ curve (*see* section 5) to the left, thus reducing the lower limit of permissible p_t values. It is possible therefore that variations in the state of mass-balance of the control surface might be covered in the existing criteria by changes in the values of k_5, k_6 and k_9 in conditions (A), (J), (N 1), (P) and (R) of section 11: these changes would be such as to make the criteria less restrictive with over-mass-balance and more restrictive with under-mass-balance.

As already mentioned in section 10.1, the criteria should not be applied to cases where the mode of the lifting surface contains a considerable amount of pitching.

In all cases where the criteria are patently inapplicable, design should preferably be based (at least finally) on detailed flutter calculations appropriate to the system concerned. Where the effects not covered by the criteria are uncertain or are known to be unfavourable, flutter calculations are essential to prove the safety of the system. Cases of this kind discussed above are (a) aerodynamic balance on the tab, (b) under-mass-balance of the control surface, (c) considerable pitching in the lifting-surface mode, and (d) investigation of the in-phase type of stick-free flutter where the follow-up ratio is very low or where a bob-weight is attached to the stick (*see* Appendix IV).

11. *General Summary*.—An investigation has been made into the flutter characteristics of an idealised tab system in which the three degrees of freedom normal translation of the lifting surface, rotation of the control surface, and rotation of the tab are represented. Specific cases of this idealised system which differ basically only in their elastic characteristics, represent similar idealised forms of the standard trimming, spring, servo, and geared-tab systems. From a consideration of the relationships existing between the systems, criteria for flutter prevention have been developed from the criteria evolved earlier for trimming-tabs[1]. As initially derived, the criteria are applicable only to the stick-fixed condition (in the case of spring and servo-tabs), to the case with no aerodynamic balance on either control surface or tab, and to the case where the control surface is statically balanced about its hinge.

Comparison is made between the criteria for spring-tabs and the existing Collar-Sharpe criteria[2,3]. Design implications are deduced from the criteria for spring-tabs, and the general application of the criteria to actual systems is considered in some detail.

11.1. *General Recommendation for all Tab Systems*.—The vertical asymptotes of the $v_c \sim p_t$ curve nearest to the vertical axis bound a region which under certain conditions can become a flutter-free region. Since these asymptotes are independent of the elastic characteristics of the system, they provide a necessary (though not sufficient) condition for the prevention of flutter of any type of tab system. The condition is (with provisionally $\delta_1 = \delta_2 = 0\cdot 5$; $k_4 = k_5 = 1$):

$$p_t \geqslant a_3 - a_4(a_2 - \bar{q}) - k_5\sqrt{\left(\frac{a_2 - \bar{q}}{a_1\bar{q}}\right)} + \delta_2 \qquad \text{(A)}$$

$$p_t \leqslant \frac{3}{4}\left[\left(\frac{1}{1-p}\right)\right]\left[\frac{1}{6}(1 - 4p)i_t + a_3 - a_4(a_2 - \bar{q}) + k_4\sqrt{\left(\frac{a_2 - \bar{q}}{a_1\bar{q}}\right)}\right] - \delta_1 \qquad \text{(B)}$$

where

$$a_1 = 0\cdot 222 + 0\cdot 013 i_t + i_c(-0\cdot 0145 + 0\cdot 00149 i_t) \qquad \text{(C 1)}$$

$$a_2 = 1\cdot 12 - 0\cdot 0267 i_t + i_c(0\cdot 0365 - 0\cdot 001 i_t) \quad \ldots \quad \text{(C 2)}$$

$$a_3 = -0\cdot 164 - 0\cdot 0965 i_t + i_c(0\cdot 0778 + 0\cdot 00489 i_t) \quad \ldots \quad \text{(C 3)}$$

$$a_4 = \frac{i_t}{0\cdot 448 + 0\cdot 277 i_t} - 0\cdot 238 i_c \quad \ldots \quad \text{(C 4)}$$

$$\bar{q} = jq \quad \ldots \quad \text{(D 1)}$$

with $$j = \sqrt{p}[0\cdot 93 + 1\cdot 28(1\cdot 97 - E_1)(0\cdot 745 - p)]. \quad \ldots \quad \text{(D 2)}$$

The range of validity of this recommendation is $\mu \simeq 6$; $i_c = 1$ to $7\cdot 78$: $p_c = 0$; $i_t = 1\cdot 31$ to $13\cdot 1$; $E_1 = 0\cdot 2$ to $0\cdot 4$; $p = 0\cdot 13$ to $0\cdot 25$ .. .. (E)

It should however be possible to use this recommendation for values of μ up to $\mu = 50$.

The above recommendation will in general be insufficient to ensure freedom from flutter without additional restrictions. Such additional restrictions have already been established for trimming-tab systems[1], and similar criteria have now been deduced for spring, servo and geared-tab systems.

11.2. *Criteria for Spring-Tabs with $N \geqslant 0$.*—Three criteria have been deduced, corresponding respectively to the three criteria already established for trimming-tabs[1]. In the order I, II, III the criteria become simpler though more approximate, but for most practical purposes Criterion III should be adequate.

For the case $N \geqslant 0$ the criteria are proved to prevent only certain types of flutter characterised by low critical speeds. This is not to say that they will not prevent other types of flutter as well, but only that such has not so far been proved.

Spring-Tab Criterion I

The conditions to be satisfied are, with provisionally k_i ($i = 1$ to 5) $= 1$ and $\delta_1 = \delta_2 = 0\cdot 5$:

$$\frac{f_\gamma}{f_\beta} \geqslant 2k_1 \quad \ldots \quad \text{(F)}$$

$$\frac{f_\gamma}{f_z} \geqslant 2k_2 \quad \ldots \quad \text{(G)}$$

$$\frac{f_\beta}{f_z} \leqslant k_3 \quad \ldots \quad \text{(H)}$$

$$p_t \geqslant a_3 - a_4(a_2 - \bar{q}) - k_5\sqrt{\left(\frac{a_2 - \bar{q}}{a_1\bar{q}}\right)} + \delta_2 \quad \ldots \quad \text{(J)}$$

$$p_t \leqslant \frac{3}{4}\left[\frac{1}{1-p}\right]\left[\frac{1}{6}(1 - 4p)\bar{i}_t + \bar{a}_3 - \bar{a}_4(\bar{a}_2 - \bar{q}) + k_4\sqrt{\left(\frac{\bar{a}_2 - \bar{q}}{\bar{a}_1\bar{q}}\right)}\right] - \delta_1 \quad \ldots \quad \text{(K)}$$

where a_1, a_2, a_3, a_4, $\bar{q}$ are to be calculated by the formulae (C 1), (C 2), (C 3), (C 4), (D 1), (D 2); and $\bar{a}_1$, $\bar{a}_2$, $\bar{a}_3$, $\bar{a}_4$ are to be calculated by replacing in a_1, a_2, a_3, a_4 the quantity i_t by $\bar{i}_t$, where

$$\bar{i}_t = (1 + \bar{N})i_t \quad \ldots \quad \text{(L)}$$

and

$$\bar{N} = N\frac{K_o}{K_o + N_1^2 K_m}. \quad \ldots \quad \text{(L 2)}$$

The range of validity of the criterion is the same as under (E) with the additional restriction

$$\bar{i}_t \simeq 1\cdot 31 \text{ to } 13\cdot 1\,. \qquad \text{(M)}$$

In practice some relaxation may be possible in the form of a reduction in the values of k_1, k_2 and an increase in the values of k_3, k_4, k_5. This is anticipated because the original calculations relating to the trimming-tab system[1] used mainly rather low values of μ and employed full theoretical aerodynamic derivatives, the effects of which are in each case likely to be conservative. With k_1, k_2, k_3 all unity, the condition (F) is of course superfluous.

Spring-Tab Criterion II

The spring-tab Criterion II results from the spring-tab Criterion I by omitting (G) and replacing (J) by

$$p_t \geqslant k_6 \qquad \text{(N 1)}$$

with provisionally

$$k_6 = 0\cdot 1\,. \qquad \text{(N 2)}$$

The range of validity of the criterion is the same as for the spring-tab Criterion I.

Spring-Tab Criterion III

The conditions to be satisfied are, with provisionally $k_1 = 1$, $k_6 = 0\cdot 1$, $k_7 = 1$:

$$\frac{f_\gamma}{f_\beta} \geqslant 2k_1 \qquad \text{(O 1)}$$

which may alternatively be written as

$$\frac{N^2 I_t}{I_c} \leqslant \frac{0\cdot 25}{k_1^{\,2}} \frac{N_1^{\,2} K_s\left(\dfrac{K_o}{N_1^{\,2}} + K_m\right)}{K_o(K_m + K_s) + K_c\left(\dfrac{K_o}{N_1^{\,2}} + K_m + K_s\right)} \qquad \text{(O 2)}$$

also

$$m_t x_t \geqslant 0\cdot 4\, k_6 \rho c_w c_t^{\,2} q s \qquad \text{(P)}$$

and

$$\frac{(1 + \bar{N})I_t + (E_1 - E_2)c_w m_t x_t}{I_c} \leqslant k_7 C p^{3/2} \qquad \text{(Q 1)}$$

where

$$C = \frac{\sqrt{p}}{j}\left\{-0\cdot 0435 + \frac{0\cdot 751}{i_c} + \frac{0\cdot 69}{(1 + \bar{N})i_t} + jq\left[0\cdot 25 - \frac{0\cdot 14}{i_c} - \frac{1}{(1 + \bar{N})i_t}\left(0\cdot 634 + \frac{1\cdot 27}{i_c}\right)\right]\right\} \qquad \text{(Q 2)}$$

$$j = \sqrt{p}[0\cdot 93 + 1\cdot 28(1\cdot 97 - E_1)(0\cdot 745 - p)] \qquad \text{(Q 3)}$$

and

$$\bar{N} = N\frac{K_o}{K_o + N_1^{\,2}K_m}\,. \qquad \text{(Q 4)}$$

The range of validity of the criterion is similar to that of the spring-tab Criterion II, except that it is strictly appropriate to values of μ higher than 6 (such values as are in fact now typical).

11.3. *Criteria for Trimming, Servo, and Geared-Tabs.*—A trimming-tab system may be regarded as effectively a spring-tab system with $\bar{N} = 0$. Putting $\bar{N} = 0$ in the spring-tab Criteria I, II and III of section 11.2 gives the corresponding trimming-tab criteria of the earlier report[1] (*see* also section 2).

A servo-tab system is a spring-tab system with the main spring K_m deleted. Criteria for servo-tabs are therefore obtained by putting $K_m = 0$ in the spring-tab criteria of section 11.2.

A geared-tab system may be regarded as effectively a spring-tab system with $K_m = 0$. The spring-tab criteria of section 11.2 are applicable if $N > 0$ (anti-balance tab) but not if $N < 0$ (normal balance tab). For $N < 0$ the general recommendation of section 11.1 is of course still applicable, but as already stated this is not sufficient to prevent flutter. A criterion for the prevention of flutter of geared balance tabs has been deduced, corresponding to the spring-tab Criterion III.

Geared-tab criterion ($N < 0$)

The conditions to be satisfied are, with provisionally $k_8 = 1$, $k_9 = 0 \cdot 5$;

$$m_t x_t \geqslant 0 \cdot 4\, k_9 \rho c_w c_t^2 q s \quad \ldots \quad (R)$$

$$\frac{I_t + (E_1 - E_2) c_w m_t x_t}{I_c} \leqslant k_8 C p^{3/2} \quad \ldots \quad (S)$$

where C and j are as given by formulae (Q 2) (Q 3), but with $\bar{N} = 0$.

The tab connection to the lifting surface should in addition be fairly stiff, and free from backlash.

The criterion is very approximate in that it contains no specific frequency conditions and in the fact that the value of k_9 has been only roughly estimated. It is to be expected therefore that the value of k_9 may be modified in the light of experience.

11.4. *Comparison with Collar-Sharpe Criteria for Spring-Tabs*[2,3].—The Collar-Sharpe criterion, which is based on binary (β,γ) considerations, occurs in two forms, both similar to condition (Q 1) of the spring-tab Criterion III. The right-hand side of (Q 1) is replaced in the first form by K ($= 0 \cdot 02$) and in the second form by $K' p^{3/2}$ ($K' = 0 \cdot 1$). Comparison between the spring-tab Criterion III and the Collar-Sharpe criteria shows the following differences:

(*a*) N in the Collar-Sharpe criteria is replaced by $\bar{N}$. This represents a (normally slight) relaxation.

(*b*) Neither $k_7 C$ nor $k_7 C p^{3/2}$, the counterparts of K' and K respectively, are constant in the spring-tab Criterion III, but are functions of i_c, $(1 + \bar{N}) i_t$, p, q, and E_1. Numerical analysis shows $k_7 C$ to be less variable than $k_7 C p^{3/2}$, thus confirming the second (K') form of the Collar-Sharpe criterion to be an improvement on the first (K) form.

(*c*) The same numerical analysis shows K' to be more restrictive than $k_7 C$ (with $k_7 = 1$) except in an extreme case involving high values of i_c, $(1 + \bar{N}) i_t$ and small values of p, q, E_1.

(*d*) Increasing the value of I_c is not as beneficial in the spring-tab Criterion III as it is in the Collar-Sharpe criteria.

(*e*) The spring-tab Criterion III involves two additional restrictions—conditions (O 1) and (P)—which have no counterpart in the Collar-Sharpe criterion. Condition (P) sets a forward limit to the tab c.g. Condition (O 1), which is concerned with the natural frequencies of the tab and control surface, may prove difficult to meet with a very flexible subsidiary spring or where K_c in the 'non-normal' case (motion inconsistent with normal operation) is fairly high.

(*f*) The spring-tab Criterion III contains air density as a parameter.

11.5. *Design Implications Relating to Spring-Tabs.*—From the spring-tab Criterion III it is deduced that, with certain assumptions and on the hypothesis that from control considerations the system is designed to provide a given tab effectiveness (*see* section 9.1), the following factors will be beneficial from the flutter point of view: small values of i_t, p_t ($\nless k_6$), K_m/K_s, K_c, p (if p_t is small enough)—and possibly of N in certain cases. At the same time the tab effectiveness should be no greater than the minimum required from control considerations. The practical interpretation of these deductions is as follows:

(*a*) The effect of reducing the follow-up ratio N is in general problematical. In practice a reduction of N is unlikely to be worthwhile, except possibly in cases where particular difficulty is experienced in meeting the frequency condition (O 1). From the control point of view, too low a value of N may in any case be undesirable.

(*b*) Small values of p will normally be beneficial, subject to a lower limit of about 0·10 set by other considerations.

(*c*) The value of p_t should preferably lie about mid-way between the upper and lower limits given by the criterion.

(*d*) To achieve as low a value of i_t as possible is largely a matter of basic design. Care should be taken however that the tab torsional stiffness is not seriously reduced.

(*e*) Reduction of the stiffness ratio K_m/K_s may conflict with the achievement of a satisfactory variation of stick force characteristic with speed. In such cases a compromise between flutter and control requirements may be necessary.

(*f*) The necessity for reducing K_c applies only to the 'non-normal' case and is most likely to arise acutely in the case of the elevator (anti-symmetric motion). Deletion of the interconnection between the two halves of the elevators is then the most effective expedient, provided that each half-elevator is adequately mass-balanced against fuselage torsion. This would also necessitate a spring-tab being fitted to each half-elevator, a system favoured incidentally from the control point of view. The alternative of making K_c very large, in which case flutter could occur though at a high speed, might be followed: the minimum value of K_c for such a design would then have to be established by flutter calculations.

11.6. *Application to Actual Systems.*—For design purposes it is important to know how and to what extent the criteria of this report may be applied to actual systems.

Provision is made in the criteria, in the form of possible changes in the values of the constants k_1 to k_9, for adjustment in the light of experience ; or possibly for a subsequent extension of the criteria to conditions for which they are not at the moment strictly valid.

Certain general principles can be laid down governing the transformation of an actual system into an equivalent simple system appropriate to the criteria. These are given in detail in section 10.1.

Points to be noted in relation to the use of the criteria in design are as follows:

(*a*) For spring, servo, and trimming-tabs Criterion III is recommended for general practical use. Criterion I may allow some relaxation on the forward limit of the tab c.g. and may therefore prove useful on occasions.

(*b*) Both 'normal' and 'non-normal' cases (motion respectively consistent with and not consistent with normal operation) should be considered for each system concerned.

(*c*) In the case of an elevator carrying a tab on one side only, the inertia conditions of the criterion should be met assuming only the half-elevator to be effective.

(*d*) Since the criteria contain air density as a parameter, they should be checked over the full height range. Maximum height will normally prove the most restrictive.

(*e*) Design values should be checked where possible by full-scale measurements. In particular, inertias and out-of-balance moments should be measured and natural frequencies and the modal characteristics of the lifting surface obtained from resonance tests on the complete aircraft.

Conditions for which the criteria are patently inapplicable are the stick-free condition (in the 'normal' case) and cases where there is aerodynamic balance on control surface or tab, where the control surface is other than statically balanced ($p_c \neq 0$), or where the mode of the lifting surface contains considerable pitching. The criteria can be adapted to cover certain types of stick-free flutter (*see* section 10.3). Qualitatively it is estimated that aerodynamic balance or over-mass-balance of the control surface will be beneficial*, aerodynamic balance of the tab or under-mass-balance of the control surface unbeneficial. Flutter calculations will be necessary to prove the safety of the system in the following cases:

(i) aerodynamic balance on the tab

(ii) under-mass-balance of the control surface

(iii) considerable pitching in the lifting surface mode

(iv) in-phase type of stick-free flutter where the follow-up ratio is very low or where a bob-weight is attached to the stick.

12. *Conclusions.*—For the avoidance of flutter of spring-tabs under the conditions considered certain qualitative conclusions can be drawn from the results obtained, *viz.* that the following effects will be favourable:

(*a*) High stiffness of the subsidiary spring.

(*b*) Low moment of inertia of the tab about its hinge.

(*c*) Tab chord reasonably small (assuming control-surface chord fixed by aerodynamic considerations).

(*d*) Tab c.g. normally aft of its hinge (unless control surface is over-mass-balanced).

(*e*) Tab effectiveness as low as possible compatible with control considerations.

* This statement may not be true for special types of aerodynamic balance, such as a sealed balance.

(f) Either giving each part of a divided control surface its own share of a spring-tab and not joining them directly, or connecting them together very stiffly.

(g) Avoidance of backlash in the tab connection.

Quantitative requirements for the avoidance of flutter are obtained in the form of alternative Criteria I, II and III, given in detail in section 11.2. For general design use Criterion III is recommended.

Servo-tabs and trimming-tabs can be treated as special cases of spring-tabs with respect to these recommendations and criteria (*see* section 11.3). For geared-tabs a separate criterion is given in section 11.3. A general recommendation (though not sufficient condition) for all tab systems is given in section 11.1.

13. *Further Developments.*—It is considered desirable that the investigation should be extended as follows:

(a) Research, possibly by wind-tunnel tests, to obtain more accurate values for the factors k_1 to k_9 in the criteria.

(b) Effect of under- and over-mass-balance of the control surface and extension of the present range of values of tab and control-surface inertias and of lifting-surface mass.

(c) Effect of aerodynamic balance of the tab or control surface.

(d) Effect of pitch in the lifting-surface mode.

(e) Investigation of stick-free flutter, particularly of the in-phase type.

(f) Statistical analysis of actual cases to check the applicability of the criteria in practice.

LIST OF SYMBOLS

Index $^+$ denotes a value appropriate to the trimming-tab system

Suffix c denotes a value appropriate to the control surface

Suffix f denotes a value appropriate to the critical flutter condition

Suffix t denotes a value appropriate to the tab

Suffix w denotes a value appropriate to the lifting surface (wing, tailplane, or fin).

a_1, a_2, a_3, a_4	Functions defined in section 11.1 formulae (C 1), (C 2), (C 3), (C 4)
$\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4$	Functions derived from a_1, a_2, a_3, a_4 by replacing i_t by $\bar{i}_t$
a_{ij}	Virtual inertia coefficients of the air for the tab system
A_{ij}	Total inertia coefficients, including virtual inertia, for the tab system
b_2, b_3	Control-surface hinge-moment derivatives for steady flow (*see* definition of C_H)

c_2, c_3	Tab hinge-moment derivatives for steady flow (*see* definition of C_T)	
c_c	Control-surface chord	
c_t	Tab chord	
c_w	Lifting-surface chord	
C_H	Control-surface hinge-moment coefficient in steady flow $= M_c/sc_c^2Q = b_2\beta + qb_3\gamma$	
C_T	Tab hinge-moment coefficient in steady flow $= M_t/qs\, c_t^2Q = c_2\beta + c_3\gamma$	
D_{ij}	Air force coefficients, excluding virtual inertia, for the tab system in the flutter condition	
E	Control effectiveness of the tab, as defined in Appendix III	
E_1	Ratio c_c/c_w	
E_2	Ratio c_t/c_w	
E_{ij}	Elastic stiffness coefficients for the tab system	
$f(p_t)$	Function of p_t representing dependence of critical speed on p_t	
f_8, f_{10}, f_{12}	Functions of E_1, E_2 defined by Dietze[5]	
f	Ratio w_f^2/w_3^{+2}	
f_f	Flutter frequency in cycles per second	
f_z	Natural frequency (c.p.s.) appropriate to freedom z*	for the tab system, including effect of virtual inertia
f_β	Natural frequency (c.p.s.) appropriate to freedom β*	
f_γ	Natural frequency (c.p.s.) appropriate to freedom γ*	
$\bar{f}_\beta$, $\bar{f}_\gamma$	Effective values of f_β, f_γ for stick-free case with a node in the circuit	
F_1, F_2	Functions defined in section 2 (5) and (6)	
g	Ratio w_f^2/w_3^2	
i_c	$= 16I_c/\pi\rho c_w s c_c^3$ non-dimensional inertia of the control surface	
i_t	$= 16I_t/\pi\rho c_w s c_t^3 q$ non-dimensional inertia of the tab	
$\bar{i}_t$	$= (1 + \bar{N})i_t$	
I_c	Moment of inertia of control surface about its hinge-line	
I_t	Moment of inertia of the tab about its hinge-line	
j	Factor defined in section 11.1, formula (D 2), by which tab span qs is multiplied to give span $\bar{q}s$ of equivalent tab (with $E_1 = 0\cdot3$, $E_2 = 0\cdot075$) having same value of qb_3/q_2	
k_1 to k_9	Constants occurring in the tab criteria (*see* section 11)	
K, K'	Constants occurring in the Collar-Sharpe criteria for spring-tabs	
K_o, K_m, K_s, K_c	Spring rates (moment per radian) of the tab system as defined in section 3 and Fig. 1	
K_t	Spring rate (moment per radian) of the spring between trimming-tab and control surface (*see* section 3.2 and Fig. 2)	
K_w	Spring rate (force per unit length) constraining the normal translation of the lifting surface	
$\bar{K}_o$	Effective value of K_o for stick-free case with a node in the circuit	
l_1, l_2, l_3, l_4	Lever arms of the tab system (*see* Fig. 1)	

* Natural frequency of the system when only the degree of freedom concerned is allowed.

m_w		Mass of the lifting surface
m_c		Mass of the control surface (including tab)
m_t		Mass of the tab
M_c, (M_t)		Moment about the control-surface (tab) hinge, positive when acting in a clockwise direction (*see* Fig. 1)
N		Follow-up ratio $(= l_1 l_3/l_2 l_4)$
N_1		Eccentricity ratio $(= l_1/l_2)$
$\bar{N}$	=	$\dfrac{NK_o}{K_o + N_1^2 K_m}$ modified follow-up ratio (*see* section 6)
$\bar{\bar{N}}$		Effective value of $\bar{N}$ for stick-free case with a node in the circuit
P		Ratio $\dfrac{E_2}{E_1} = \dfrac{c_t}{c_c}$
p_c	=	$8m_c x_c/\pi \rho c_w c_c^2 s$ non-dimensional mass-moment of control surface
p_t	=	$8m_t x_t/\pi \rho c_w c_t^2 qs$ non-dimensional mass-moment of the tab
q		Ratio of tab span to control-surface span
$\bar{q}$	=	jq (*see* definition of j)
q_1, q_2, q_3		Generalised co-ordinates for the tab system
$\bar{q}_1$, $\bar{q}_2$, $\bar{q}_3$		Amplitudes of q_1, q_2, q_3 in the flutter condition
Q		Dynamic pressure $(= \frac{1}{2}\rho V^2)$
s		Span of the control surface
S_c		Area of control surface (including tab)
S_t		Area of tab
V		Air speed
V_f		Flutter speed
W_p		Work done by the pilot to produce angular movement β of the control surface
W_{p0}		Value of W_p with tab inoperative
x		Distance of a tab element aft of the tab hinge
x_c		Distance of control-surface c.g. aft of its hinge-line
x_t		Distance of tab c.g. aft of its hinge-line
X_1		Force applied to link (0) of the tab system (*see* Fig. 1)—equivalent to pilot's force
X_2		Compressive force in link (4) of the tab system (*see* Fig. 1)
y		Distance of an element of the lifting surface from axis of rolling
y_m		Mean value of y
z		Normal translation of the lifting surface
α_{33}		Non-dimensional value of a_{33} (*see* section 6.1 formula (47))
β		Rotation of control surface about its hinge relative to the lifting surface, positive clockwise
β_1		Rotation of link (2) (*see* Fig. 1) relative to the control surface, positive clockwise
β_2		Rotation of link (3) (*see* Fig. 1) relative to the control surface, positive clockwise

γ		Rotation of the tab about its hinge relative to the control surface, positive clockwise	
δ_1, δ_2		Safety margins occurring in the criteria	
λ_c	=	K_c/sc_c^2Q	non-dimensional values of the spring rates K_c, K_o, K_m, K_s
λ_o	=	K_o/sc_c^2Q	
λ_m	=	K_m/sc_c^2Q	
λ_s	=	K_s/sc_c^2Q	
μ	=	$4m_w/\pi\rho c_w^2 s$	non-dimensional mass of the lifting surface
ρ		Air density	
w_1	=	$2\pi f_z$	circular frequencies of the natural frequencies f_z, f_β, f_γ
w_2	=	$2\pi f_\beta$	
w_3	=	$2\pi f_\gamma$	
w_f	=	$2\pi f_f$	circular flutter frequency

REFERENCES

No.	Author	Title, etc.
1	H. Wittmeyer	Trimming-Tab Flutter and its Prevention. R. & M. 2671. October, 1948.
2	A. R. Collar and G. D. Sharpe ..	Mass-balance for Spring-Tab Flutter Prevention. R. & M. 2637. June, 1946.
3	G. D. Sharpe	Consideration of the Effect of Tab Dimensions on Spring-Tab Flutter. R. & M. 2637. June, 1946.
4		The Prevention of Flutter of Spring Tabs and Aerodynamic Servo-Tabs. Airworthiness Tech. Note 171. 1949.
5	F. Dietze	The Air forces on a Harmonically Oscillating, Deformable Plate (two-dimensional problem) *e.g.*, for the Calculation of a Wing with Control Surface and Tab. (Die Luftkräfte der harmonisch schwingenden, in sich verformbaren Platte (Ebenes Problem). U.a. zur Berechnung des Flügels mit Ruder und Hilfsruder). *Luftf.-Forschung*, Bd. 16, p. 84/96. 1939.
6	F. B. Baker	Spring Tabs. *Aircraft Engineering*, Vol. XIX. 1947.
7	W. J. Duncan, D. L. Ellis and A. G. Gadd	Experiments on Servo-rudder Flutter. R. & M. 1652. 1934.
8	H. Voigt, F. Walter and W. Heger ..	Flutter Experiments on the Influence of the Aerodynamic Balance Given by a Geared Tab. (Flatterversuche über den Einfluss des aerodynamischen Ausgleichs durch Weggesteuertes Hilfsruder) Jahrbuch 1941 der Deutschen Luftfahrtforschung, Vol. I pp. 338/550.
9	F. Walter	Flutter Experiments on the Influence of a Control Surface Balanced by Setting Back the Hinge. (Flatterversuche über den Einfluss des aerodynamischen Ruderinnenausgleischs) Jahrbuch 1942 der Deutschen Luftfahrtforschung, Vol. I p. 549.
10	G. H. L. Buxton and G. D. Sharpe ..	Ternary Tailplane-Elevator-Tab Flutter. R. & M. 2418. November, 1946.
11	H. Wittmeyer	Theoretical Investigations on the Flutter Characteristics of Wings and Tail Units with Control Surfaces and Tabs. R.A.E. Library Translation No. 196. (Theoretische Untersuchungen der Flattereigenschaften von Flügeln und Leitwerken mit Rudern und Hilfsrudern. *ZWB* (FB) 1468. 1941. Report of the Focke-Wulf Aircraft Co., Bremen. A.R.C. 11,058. 1947.
12	H. Wittmeyer	On the Influence of a Tab on the Critical Speed of an Aeroplane I. (Über den Einfluss eines Hilfsruders auf die kritische Geschwindigkeit von Flugzeugen I.) *Technische Berichte* 3/1939, *and Jahrbuch der Deutschen Luftfahrtforschung*. 1940.

Substituting (I.9) and (I.10) in (I.5) and (I.6) and comparing with (I.3) and (I.4) gives finally

$$E_{22} = K_c + \frac{K_o(K_m + K_s)}{\frac{K_o}{N_1^2} + K_m + K_s} \quad \ldots \quad \text{(I.11)}$$

$$E_{23} = E_{32} = -\frac{1}{N}\frac{K_o K_s}{\frac{K_o}{N_1^2} + K_m + K_s} \quad \ldots \quad \text{(I.12)}$$

$$E_{33} = \frac{N_1^2}{N^2}\frac{K_s\left(\frac{K_o}{N_1^2} + K_m\right)}{\frac{K_o}{N_1^2} + K_m + K_s} \quad \ldots \quad \text{(I.13)}$$

where

$$N = \frac{l_1 l_3}{l_2 l_4} \text{ (follow-up ratio)} \quad \ldots \quad \text{(I.14)}$$

$$N_1 = \frac{l_1}{l_2} \text{ (eccentricity ratio)}\,. \quad \ldots \quad \text{(I.15)}$$

APPENDIX II

Derivation of the Relationship between Equivalent S- and T-systems.

Two systems, one an S-system and the other a T-system, are here said to be equivalent if the corresponding dimensions of the various surfaces are the same and if the two systems flutter at the same speed with the same frequency and mode.

Considering the flutter equations (13) of the T-system and (16) of the S-system, it is evident that on the above hypothesis the air-forces will be the same, *viz.*,

$$D_{ij} = D_{ij}^{+} \quad \ldots \quad \text{(II.1)}$$

and that the two sets of equations then become reconcileable if

$$-w_f^2 A_{ij}^{+} + E_{ij}^{+} = -w_f^2 A_{ij} + E_{ij} \quad (i, j = 1,2,3)\,. \quad \ldots \quad \text{(II.2)}$$

The A_{ij} are the inertia coefficients (including virtual inertia of the air) and the E_{ij} the stiffness coefficients of the S-system, quantities with the index $^{+}$ being the corresponding coefficients of the T-system. w_f is the circular flutter frequency common to both systems.

1. *Transformation of a T-system into an S-system.*—For given values of A_{ij}^+ and E_{ij}^+ a set of values can be deduced for A_{ij} and E_{ij} which will satisfy equations (II.2). An S-system having these values of A_{ij} and E_{ij} will then be equivalent to a T-system having the given values of A_{ij}^+ and E_{ij}^+.

We write first

$$w_1^2 = \frac{E_{11}}{A_{11}}\,; \quad w_2^2 = \frac{E_{22}}{A_{22}}\,; \quad w_3^2 = \frac{E_{33}}{A_{33}} \qquad \text{(II.3)}$$

$$w_1^{+2} = \frac{E_{11}^+}{A_{11}^+}\,; \quad w_2^{+2} = \frac{E_{22}^+}{A_{22}^+}\,; \quad w_3^{+2} = \frac{E_{33}^+}{A_{33}^+} \qquad \text{(II.4)}$$

$$w_f^2 = f w_3^{+2} \qquad \text{(II.5)}$$

where w_1, w_2, w_3, w_1^+, w_2^+, w_3^+ are the natural circular frequencies of the S- and T-systems respectively, allowing for the virtual inertia of the air.

From equation (II.2) with $i = j = 1$

$$A_{11}^+\left(1 - \frac{w_1^{+2}}{w_f^2}\right) = A_{11}\left(1 - \frac{w_1^2}{w_f^2}\right)$$

which can be satisfied by

$$A_{11} = A_{11}^+\,; \quad w_1 = w_1^+\,. \qquad \text{(II.6)}$$

Similarly equation (II.2) with $i = j = 2$ can be satisfied by

$$A_{22} = A_{22}^+\,; \quad w_2 = w_2^+\,. \qquad \text{(II.7)}$$

Because $E_{1j} = E_{1j}^+ = 0$ $(j \neq 1)$, equation (II.2) with $i = 1, j = 2$ and with $i = 1, j = 3$ is satisfied uniquely by

$$A_{12} = A_{12}^+ \qquad \text{(II.8)}$$

$$A_{13} = A_{13}^+\,. \qquad \text{(II.9)}$$

Equation (II.2) with $i = 2, j = 3$ together with (II.9) in combination with (14), (17), (18) gives

$$A_{33} = A_{33}^+ + \frac{E_{23}}{f w_3^{+2}}\,. \qquad \text{(II.10)}$$

Equation (II.2) with $i = j = 3$ yields directly

$$A_{33}\left(1 - \frac{w_3^2}{f w_3^{+2}}\right) = A_{33}^+\left(1 - \frac{1}{f}\right). \qquad \text{(II.11)}$$

Eliminating A_{33}^+ from (II.10) and (II.11) gives

$$\left(A_{33} - \frac{E_{23}}{f w_3^{+2}}\right)\left(1 - \frac{1}{f}\right) = A_{33}\left(1 - \frac{w_3^2}{f w_3^{+2}}\right). \qquad \text{(II.12)}$$

Dividing (II.12) by A_{33} and putting

$$-\frac{E_{23}}{E_{33}} = \bar{N} \quad \ldots \quad \text{(II.13)}$$

gives finally

$$w_3^2 = \frac{w_3^{+2}}{1 - \bar{N}\left(\frac{1-f}{f}\right)} \quad \ldots \quad \text{(II.14)}$$

In combination with (II.13) and (II.14), (II.10) then gives

$$A_{33} = \left[1 - \frac{1}{f}\left(\frac{\bar{N}}{1+\bar{N}}\right)\right] A_{33}^{+} \quad \ldots \quad \text{(II.15)}$$

and finally (II.13) together with (II.14) and (II.15) gives

$$E_{23} = - A_{33}^{+} w_3^{+2} \frac{\bar{N}}{1+\bar{N}} \quad \ldots \quad \text{(II.16)}$$

The value of A_{23} follows directly from (14), (17), (18) and is given below under (II.17).

Summarising, the equations (II.2) give finally (II.6), (II.7), (II.8), (II.9), (II.14), (II.15), (II.16), which constitute the equations of transformation of a T-system into an equivalent S-system, as follows:—

$$\left.\begin{aligned}
& A_{11} = A_{11}^{+}\,; \quad A_{12} = A_{12}^{+}\,; \quad A_{13} = A_{13}^{+} \\
& A_{22} = A_{22}^{+}\,; \quad A_{33} = \left[1 - \frac{1}{f}\left(\frac{\bar{N}}{1+\bar{N}}\right)\right] A_{33}^{+} \\
& A_{22} = A_{33} + (E_1 - E_2)c_w A_{13} - [a_{33}^{+} + (E_1 - E_2)c_w a_{13}^{+}] + a_{23}^{+} \\
\text{with} \quad & A_{ij} = A_{ji} \\
& w_1 = w_1^{+}\,; \quad w_2 = w_2^{+}\,; \quad w_3^2 = \frac{w_3^{+2}}{1 - \bar{N}\left(\frac{1-f}{f}\right)} \\
& E_{23} = - A_{33}^{+} w_3^{+2} \frac{\bar{N}}{1+\bar{N}} \\
\text{where} \quad & f = \frac{w_f^2}{w_3^{+2}}
\end{aligned}\right\} \quad \ldots \quad \text{(II.17)}$$

In order that the S-system given by (II.17) can be physically realised it is necessary and sufficient that the matrix of the stiffness coefficients and that of the inertia coefficients of the S-system should be positive definite. These conditions are certainly satisfied for the case $\bar{N} = 0$ because then the S-system is identical with the T-system. By reasons of continuity there will therefore be a range $0 \leqslant \bar{N} \leqslant \varepsilon$ for which the S-system given by (II.17) can be physically realised.

To the equations (II.17) there should therefore strictly be added the condition

$$0 \leqslant \bar{N} \leqslant \varepsilon \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \text{(II.18)}$$

for the derived equivalent system to be a physically real system.

2. *Transformation of an S-system into a T-system.*—The transformation of an S-system into an equivalent T-system will be given by the reciprocal of the relationships (II.17), but the flutter frequency must first be expressed in terms of w_3 in place of $w_3{}^+$. The factor g is therefore introduced, defined by

$$w_f{}^2 = g w_3{}^2. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \text{(II.19)}$$

From (II.5), (II.17) and (II.19) the relationship between g and f is obtained as

$$\frac{g}{f} = \frac{w_3{}^{+2}}{w_3{}^2} = 1 - \bar{N}\Big(\frac{1-f}{f}\Big)$$

or

$$f = \frac{g + \bar{N}}{1 + \bar{N}}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \text{(II.20)}$$

Incidentally, from (II.20) it is evident that

$$\text{if } g \geqslant 1 \text{ then } f \geqslant 1 \text{ and } \textit{vice versa} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \text{(II.21)}$$

$$\text{if } g < 1 \text{ then } f < 1 \text{ and } \textit{vice versa}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \text{(II.22)}$$

By substituting (II.20) into (II.17) the equations of transformation of an S-system into an equivalent T-system are obtained, as follows:—

$$\left.\begin{array}{ll}
 & A_{11}{}^+ = A_{11}\,; \quad A_{12}{}^+ = A_{12}\,; \quad A_{13}{}^+ = A_{13} \\
 & A_{22}{}^+ = A_{22}\,; \quad A_{33}{}^+ = 1 + \dfrac{\bar{N}}{g} A_{33} \\
 & A_{23}{}^+ = A_{33}{}^+ + (E_1 - E_2)c_w A_{13}{}^+ - [a_{33} + (E_1 - E_2)c_w a_{13}] + a_{23} \\
\text{with} & A_{ij}{}^+ = A_{ji}{}^+ \\
 & w_1{}^+ = w_1\,; \quad w_2{}^+ = w_2\,; \quad w_3{}^{+2} = \Big(1 - \bar{N}\dfrac{1-g}{g+\bar{N}}\Big) w_3{}^2 \\
\text{where} & g = \dfrac{w_f{}^2}{w_3{}^2}
\end{array}\right\} \quad \ldots \quad \ldots \text{(II.23)}$$

Here again a restriction on the value of $\bar{N}$ should strictly be added to (II.23) if the equivalent T-system is to be a physically real system. For the present investigation, however, this restriction is unimportant.

For application in section 6, the relationships (II.23) are transformed, by (14) and (18) of section 4, into

$$\left.\begin{aligned} &m_w^+ = m_w\,; \quad m_c^+x_c^+ = m_cx_c\,; \quad m_t^+x_t^+ = m_tx_t \\ &I_c^+ = I_c\,; \quad I_t^+ = \left(1 + \frac{\bar{N}}{g}\right)I_t + \frac{\bar{N}}{g}a_{33} \\ &w_1^+ = w_1\,; \quad w_2^+ = w_2\,; \quad w_3^{+\,2} = \left(1 - \bar{N}\frac{1-g}{g+\bar{N}}\right)w_3^2 \\ \text{where} \quad &g = w_f^2/w_3^2 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \text{(II.24)}$$

APPENDIX III

Control Effectiveness of a Spring-Tab

For the purpose of this report, the control effectiveness (E) of a spring-tab is defined as follows. If W_p is the work done by the pilot to deflect the control surface through a given angle β at a given speed V, and W_{p0} is the value of W_p with the tab inoperative ($K_m = K_s = \infty$), then

$$1 - E = W_p/W_{p0}\,. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \text{(III.1)}$$

If $E = 0$ then $W_p = W_{p0}$, and if $E = 1$ then $W_p = 0$. As the tab effectiveness increases, the work required decreases. On this basis, the effectiveness of the tab system as a lift-producer is compared with that of the simple system in which the control surface is operated directly by the pilot through the same control circuit. Tabs of the same effectiveness will produce roughly the same lift for the same expenditure of pilot's energy.

From a consideration of the equilibrium of the system of Fig. 1, the relation between the tab effectiveness and the relevant parameters of the system can be deduced.

The aerodynamic hinge moments about the control-surface and tab hinges, due to angular movements β and γ, are respectively

$$M_c = (b_2\beta + qb_3\gamma)sc_c^2Q \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \text{(III.2)}$$

$$M_t = (c_2\beta + c_3\gamma)qsc_t^2Q\,. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \text{(III.3)}$$

b_3 is here the value of $\partial C_H/\partial\gamma$ appropriate to a tab of the same chord as the actual tab but with span s : the contribution of the tab to M_c is thus assumed proportional to q.

If, for the sake of simplicity, it is assumed that $c_2 = 0$, (III.3) becomes

$$M_t = c_3\gamma qsc_t^2Q\,. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \text{(III.4)}$$

If the pilot's force is X_1, and the displacement of its point of application (*see* Fig. 1) is s_1, corresponding to the control surface deflection β, then

$$W_p = \tfrac{1}{2}X_1 s_1 . \qquad \text{(III.5)}$$

X_1 and s_1 are each proportional to β, and W_p is therefore proportional to β^2. The actual relationships are obtained from the following six equations, which represent the conditions of force equilibrium and of geometrical consistency appropriate to the system.

Equilibrium of forces about the hinge A:

$$X_1 l_1 = M_c . \qquad \text{(III.6)}$$

Consistency of displacements s_1, β, β_1:

$$s_1 = l_1^2 \frac{X_1}{K_o} + l_1\beta + l_2\beta_1 . \qquad \text{(III.7)}$$

Equilibrium of link (2) about hinge B:

$$K_m\beta_1 = l_2X_1 + K_s(\beta_2 - \beta_1) . \qquad \text{(III.8)}$$

Equilibrium of link (3) about hinge B:

$$l_3X_2 + K_s(\beta_2 - \beta_1) = 0 . \qquad \text{(III.9)}$$

Equilibrium of tab about hinge D:

$$l_4X_2 = M_t . \qquad \text{(III.10)}$$

Consistency of displacements β_2, γ:

$$l_3\beta_2 + l_4\gamma = 0 . \qquad \text{(III.11)}$$

For the case with $K_m = K_s = \infty$ (tab inoperative), $\beta_1 = \beta_2 = \gamma = 0$ and (III.5) then gives, together with (III.2), (III.4), (III.6), and (III.7).

$$W_{p0} = -\tfrac{1}{2}b_2\left(1 - sc_c^2Q\frac{b_2}{K_o}\right)sc_c^2Q\,\beta^2 . \qquad \text{(III.12)}$$

For the general case with the tab operative, X_1 and s_1 are obtained from equations (III.6) to (III.11) by eliminating X_2, β_1, β_2, γ, using also (III.2) and (III.4). Equations (III.1), (III.5), (III.12) then give, after considerable reduction, the approximate relationship

$$1 - E \simeq \frac{qNb_3 - \lambda_m N_1^2 + b_2\left(1 + \frac{\lambda_m}{\lambda_o}N_1^2\right) - qp^2N^2c_3\left(1 + \frac{\lambda_m}{\lambda_s}\right)\left(\frac{b_2}{N_1^2\lambda_m} + \frac{b_2}{\lambda_o} - 1\right)}{(2qNb_3 - \lambda_m N_1^2)\left(1 - \frac{b_2}{\lambda_o}\right)} \qquad \text{(III.13)}$$

where λ_o, λ_m, λ_s are non-dimensional forms of the stiffnesses K_o, K_m, K_s (*see* section 14).

The accuracy of the approximation rests on the assumption that q is small and λ_m not too small, this assumption being made in the algebraic reduction to the form (III.13).

For application in section 9, (III.13) is transformed, at the same time dispensing with the approximate sign of equality, into

$$N\frac{b_3}{b_2}q = \frac{1 - \dfrac{\lambda_m N_1^2}{b_2}\left(1 - \dfrac{b_2}{\lambda_o}\right)E}{1 - 2E\left(1 - \dfrac{b_2}{\lambda_o}\right) - \dfrac{2b_2}{\lambda_o} - p^2 N\dfrac{c_3}{b_3}\left(1 + \dfrac{\lambda_m}{\lambda_s}\right)\left(1 - \dfrac{b_2}{\lambda_o} - \dfrac{b_2}{N_1^2\lambda_m}\right)} \quad \text{(III.14)}$$

APPENDIX IV

A Note on Stick-free Flutter of Spring and Servo-Tabs

In the main development of the criteria in this report, the idealised system of Fig. 1 has been considered with the link (0) fixed, corresponding to the stick (or pedal) fixed condition. The stick is however in practice free to rotate about its axis. It is therefore necessary to consider the possibility of flutter in which the stick freedom is present.

Consideration is restricted to the 'normal' case, in which the flutter motion is consistent with normal operation (*e.g.*, symmetric in the case of the elevator). In such cases, $K_c = 0$. Friction or damping in the control circuit is neglected.

With these restrictions, all flutter cases of the stick-free system can be divided into two groups only. Either

(*a*) there is a node in the circuit

or (*b*) there is no node in the circuit.

Since the stick is subjected to elastic and inertia forces only, it is evident that for a given harmonic oscillation at the control-surface end of the circuit (point E of Fig. 1) the stick motion is determined. The stick in fact responds as it would to a forced oscillation applied at E. If f_c is the flutter frequency and f_{st} the natural frequency of the stick on the circuit with E fixed, it follows that group (*a*) above is characterised by $f_c > f_{st}$ (stick out of phase with E) and group (2) by $f_c < f_{st}$ (stick in phase with E).

1. *Out-of-phase flutter* ($f_c > f_{st}$).—With a node in the circuit the system is effectively still that of Fig. 1, but with a higher circuit stiffness (say $\bar{K}_o$) because of the relatively shorter length of circuit between the node and point E. The criteria of this report could then be applied with the actual stiffness K_o replaced by $\bar{K}_o$; which would have the effect of replacing actual values of f_γ, f_β, $\bar{N}$ by corresponding effective values $\bar{f}_\gamma$, $\bar{f}_\beta$, $\bar{\bar{N}}$.

As f_c decreases to f_{st} the node moves further from the stick and $\bar{K}_o$ increases. In the limit $f_c = f_{st}$ the node is at E and $\bar{K}_o = \infty$. From (I.11), (I.13) with $K_c = 0$, the frequencies f_β, f_γ are given by

$$f_\beta^2 = \frac{1}{4\pi_2} \frac{K_o(K_m + K_s)}{\dfrac{K_o}{N_1^2} + K_m + K_s} \frac{1}{A_{22}} \quad \ldots \quad (\mathrm{IV}.1)$$

$$f_\gamma^2 = \frac{1}{4\pi^2} \frac{N_1^2}{N^2} \frac{K_s\left(\dfrac{K_o}{N_1^2} + K_m\right)}{\dfrac{K_o}{N_1^2} + K_m + K_s} \frac{1}{A_{33}} \quad \ldots \quad (\mathrm{IV}.2)$$

and therefore

$$\frac{f_\gamma^2}{f_\beta^2} = \frac{N_1^2}{N^2} \frac{K_s\left(\dfrac{1}{N_1^2} + \dfrac{K_m}{K_o}\right)}{K_m + K_s} \frac{A_{22}}{A_{33}}. \quad \ldots \quad (\mathrm{IV}.3)$$

Also, from (39),

$$\bar{N} = \frac{N}{1 + N_1^2 \dfrac{K_m}{K_o}}. \quad \ldots \quad (\mathrm{IV}.4)$$

Expressions for $\bar{f}_\beta$, $\bar{f}_\gamma$, $\bar{\bar{N}}$ are given by replacing K_o by $\bar{K}_o$ in (IV.1), (IV.2), (IV.4). As $\bar{K}_o$ decreases it is thus seen that $\bar{f}_\beta$, $\bar{f}_\gamma$, $\bar{\bar{N}}$ decrease and $\bar{f}_\gamma/\bar{f}_\beta$ increases. In particular, if $\bar{f}_{\beta\infty}$, $\bar{f}_{\gamma\infty}$, $\bar{\bar{N}}_\infty$ $(= N)$ are the values corresponding to $\bar{K}_o = \infty$, then the values $\bar{f}_\beta$, $\bar{f}_\gamma$, $\bar{\bar{N}}$ corresponding to finite $\bar{K}_o$ will be such that

$$\bar{f}_\beta < \bar{f}_{\beta\infty}; \quad \frac{\bar{f}_\gamma}{\bar{f}_\beta} > \frac{\bar{f}_{\gamma\infty}}{\bar{f}_{\beta\infty}}; \quad \bar{\bar{N}} < \bar{\bar{N}}_\infty. \quad \ldots \quad (\mathrm{IV}.5)$$

Out-of-phase flutter of any frequency f_c $(> f_{st})$ will be prevented if either Criterion II or Criterion III of section 11.2 is satisfied using $\bar{f}_{\beta\infty}$, $\bar{f}_{\gamma\infty}$, $\bar{\bar{N}}_\infty$ in place of f_β, f_γ, $\bar{N}$. To prove this, suppose that flutter does occur with either of the criteria satisfied so. Since the flutter must involve a node somewhere in the circuit (because $f_c > f_{st}$) the values $\bar{f}_\beta$, $\bar{f}_\gamma$, $\bar{\bar{N}}$ appropriate to the particular nodal position must satisfy the relationships (IV.5). But these relationships are such as to ensure that if the criteria have been satisfied with $\bar{f}_{\beta\infty}$, $\bar{f}_{\gamma\infty}$, $\bar{\bar{N}}_\infty$, then they will also be satisfied by $\bar{f}_\beta$, $\bar{f}_\gamma$, $\bar{\bar{N}}$. The supposition that flutter could occur is therefore wrong, and the original statement is proved.

It should be noted however that the above procedure cannot be used in conjunction with the Criterion I of section 11.2. Condition (G) of the criterion, together with the fact that $\bar{f}_\gamma$ decreases as $\bar{K}_o$ decreases, invalidates the process in this case.*

* The procedure could however be used with the Criterion I if condition (G) were independently satisfied for the stick-fixed condition, in which case (G) would also be satisfied for any position of the node along the circuit (because $\bar{f}_\gamma > f_\gamma$).

A rider should strictly be added to the effect that, since for the stick-fixed condition the criteria have been proved valid only for $f_c \geqslant f_\gamma$, then the above application to the stick-free case is likewise valid only for $f_c \geqslant \bar{f}_\gamma$. The latter condition is satisfied if $f_c \geqslant f_{\gamma 1}$, where $f_{\gamma 1} (> f_{st})$ is the natural frequency of the stick-free system in an out-of-phase motion with control surface fixed to the lifting surface. There is however no justification in applying this restriction if the criteria for the stick-fixed condition are not similarly restricted.

To satisfy the criteria with K_o effectively infinite is of course more difficult than satisfying them with the actual value of K_o appropriate to the stick-fixed condition. Condition (O 1) of Criterion III (section 11.2) will in practice normally be the most important, and from (IV.3) it is evident that if K_m/K_o (K_o being the stick-fixed value) is fairly small there will be little extra difficulty in meeting the condition with K_o infinite. In the case of a servo-tab ($K_m = 0$) there will be no difference at all.

It should be noted that in the case of flutter with a node in the circuit the stick is effective only in respect of its moment of inertia. Any bob-weight attached to the stick on a horizontal arm (say for g-restriction purposes) has therefore no effect on this type of flutter, except in so far as it contributes to the stick moment of inertia.

2. *In-phase Flutter* ($f_c < f_{st}$).—For the in-phase type of motion the circuit stiffness K_o is effectively negative; the criteria of this report cannot therefore be used in this case. Something is however known, from separate investigations, about the characteristics of this in-phase type of flutter.

A report by Wittmeyer[11] describes some theoretical investigations on an idealised system with three degrees of freedom. Fig. 16 (a1) of that report gives results for the stick-free case with rigid control circuit and rigid subsidiary spring: they show that flutter is impossible provided that the moment of inertia of the stick (non-dimensional value ϑ_{2z}) is high compared with the moment of inertia of the tab (non-dimensional value ϑ_2), and provided also that the follow-up ratio N (denoted there by $-1/\alpha$) is not too low. Investigations in America by Curtiss-Wright on a binary system, but with flexible control circuit, showed similarly that flutter could be prevented by increasing the stick inertia.

Returning to Wittmeyer's results, Fig. 16 (a2) of his report[11] shows that for the system considered flutter was impossible if

$$\frac{I_{st}}{I_t} \geqslant \frac{0 \cdot 47N^2}{N - 0 \cdot 26}, \quad (N > 0 \cdot 26) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \text{(IV.6)}$$

where I_{st}, I_t are the moments of inertia of stick and tab respectively, the former with respect to angular rotation about the control-surface hinge. The system considered had the following parameters: $\mu = 5 \cdot 7$, $i_c = 7 \cdot 8$, $i_t = 13 \cdot 1$, $p_c = 0$, $p_t = 0$, $E_1 = 0 \cdot 3$, $p = 0 \cdot 25$, $q = 1$, $K_m = 0$.

The moment of inertia of the stick will in practice be considerably greater than the limiting value of (IV.6). For values of the other parameters similar to those considered, it is unlikely therefore that in-phase flutter would occur.

This example, combined with practical experience of spring-tab systems, invites the supposition that the moment of inertia I_{st} of the stick and the follow-up ratio N will normally be high enough to prevent the in-phase type of flutter. It must be admitted however that the supposition is based on rather slight evidence, and that a thorough investigation of this type of flutter would be desirable. In any such investigation account would have to be taken of any out-of-balance moment of the stick (as for instance exerted by a bob-weight) and of the fuselage mode.

TABLE 1

Comparison Between the Spring-Tab Criterion III (section 11.2) and the Collar-Sharpe Criteria

(*a*) Values of the constants C, C_1 in the requirement

$$\frac{(1 + \bar{N})I_t + m_t x_t(E_1 - E_2)c_w}{I_c} \leqslant k_7 C p^{3/2} = k_7 C_1 \quad \ldots \quad \ldots \quad \ldots \quad (Q.1)$$

with

$$C = \frac{\sqrt{p}}{j}\left\{-0\cdot0435 + \frac{0\cdot751}{i_c} + \frac{0\cdot69}{(1 + \bar{N})i_t} + jq\left[0\cdot25 - \frac{0\cdot14}{i_c} - \frac{1}{(1 + \bar{N})i_t}\left(0\cdot634 + \frac{1\cdot27}{i_c}\right)\right]\right\} \quad (Q.2)$$

$$j = \sqrt{p}[0\cdot93 + 1\cdot28(1\cdot97 - E_1)(0\cdot745 - p)] \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (Q.3)$$

for different sets of values of the relevant parameters.

No.	i_c	$(1 + \bar{N})i_t$	p	q	E_1	C	$C_1 = C_p{}^{3/2}$
1	7	10	0·15	0·25	0·2	0·0727	0·00422
2	3	10	0·15	0·25	0·2	0·140	0·0081
3	3	3	0·15	0·25	0·2	0·177	0·0103
4	3	3	0·3	0·25	0·2	0·205	0·0338
5	3	3	0·3	0·25	0·5	0·228	0·0375
1a	7	10	0·15	1	0·2	0·116	0·00673
5a	3	3	0·3	1	0·5	0·165	0·0273

(*b*) Values of the constants K and K' according to Collar and Sharpe (independent of the value of the structural paramaters).

Reference	K' (compare with C for $k_7 = 1$)	K (compare with C_1 for $k_7 = 1$)
Collar and Sharpe	—	0·02
Sharpe	0·1	0·015 if greater than $K'p^{3/2}$

TABLE 2

Values of N^2I_t/I_c for the Examples of Spring-Tab Systems given by Sharpe[3]

Spring-tab system No. A = Aileron E = Elevator R = Rudder	Flutter + yes — no	N	I_t slugs × ft²	I_c slugs × ft²	$\frac{N^2I_t}{I_c}$
1 (A)	+	2·75	0·00405	0·168	0·182
2 (R)	+	2·73	0·0370	6·00	0·0460
3 (A)	+	3·00	0·0130	1·22	0·0960
4 (A)	—	1·00	0·0230	0·904	0·0255
5 (E)	+	3·39	0·143	22·0	0·0745
6 (A)	+	4·00	0·000245	0·0856	0·0458
7 (A)	+	1·83	0·0124	1·49	0·0278
8 (A)	+	3·50	0·00487	1·05	0·0568
9 (R)	—	1·52	0·00842	1·27	0·0153
10 (A)	+	3·00	0·00710	1·40	0·0456
11 (A)	—	2·51	0·000540	0·231	0·0147
12 (A)	—	1·85	0·00149	0·152	0·0335
13 (A)	—	4·54	0·00120	0·390	0·0635
14 (R)	—	2·66	0·00141	0·991	0·0101
15 (R)	—	2·90	0·00965	2·36	0·0344
16 (A)	—	2·38	0·00866	2·72	0·0180
17 (E)	—	3·55	0·00398	1·96	0·0257
18 (R)	—	2·66	0·00225	0·991	0·0160
19 (A)	—	3·50	0·00175	1·23	0·0174
20 (A)	—	3·33	0·00367	2·49	0·0163
21 (E)	—	3·00	0·00633	7·15	0·00795
22 (R)	—	2·17	0·00370	4·06	0·00428
23 (E)	—	2·00	0·00475	5·25	0·00362
24 (R)	—	1·55	0·00345	3·07	0·00270
25 (E)	—	1·80	0·00311	5·10	0·00197
26 (R)	—	1·26	0·00636	1·46	0·0069

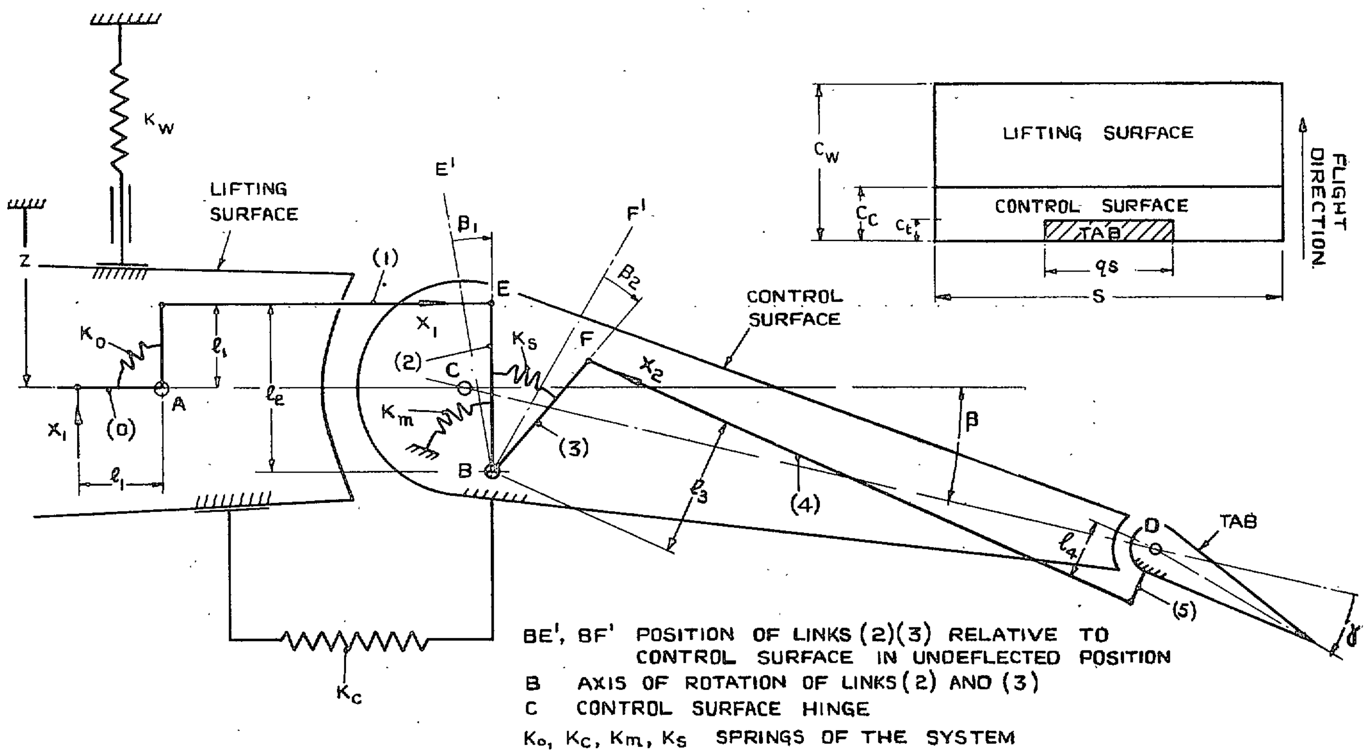

FIG. 1. The S-system.

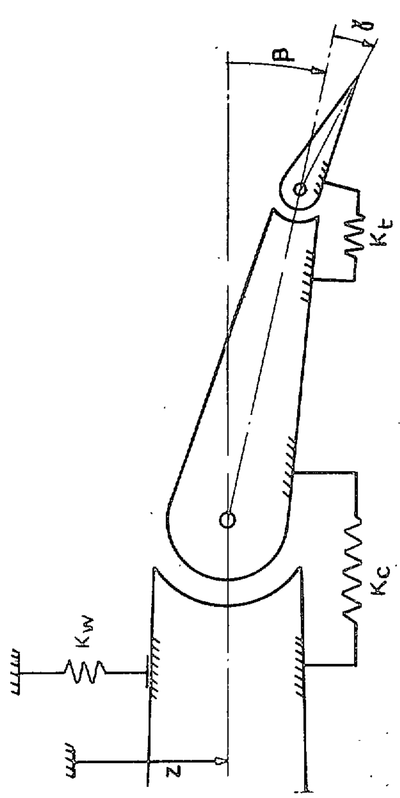

FIG. 2. The trimming-tab. T-system.

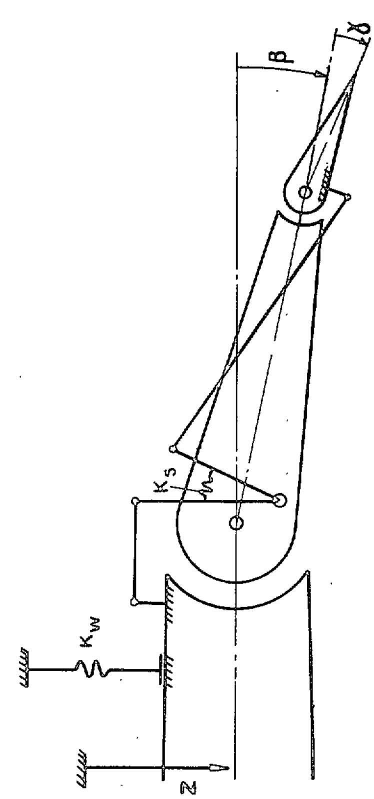

FIG. 3a. Geared (anti-balance) tab system ($N > 0$).

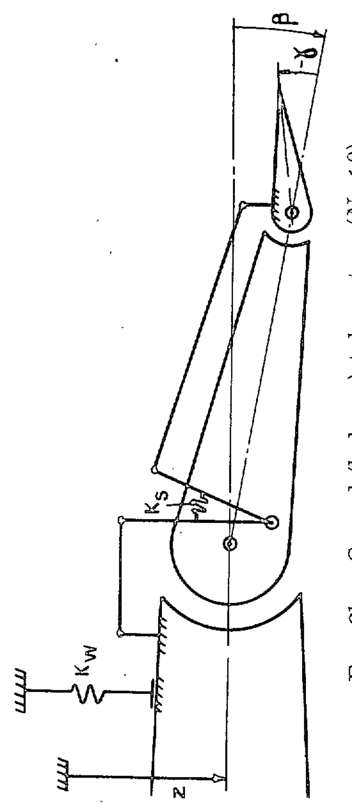

FIG. 3b. Geared (balance) tab system ($N < 0$).

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