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The Simple Harmonic Motion of a Helicopter Rotor with Hinged Blades

By

J. K. ZBROZEK, Dipl.Eng.

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COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR), MINISTRY OF SUPPLY

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Summary.—In simple harmonic oscillation of the helicopter with hinged blades, the tip-path plane is tilted with respect to the shaft in the plane of oscillation and in the plane perpendicular to it. The angles of tilt can be expressed as functions of angular velocity and acceleration. The influence of the acceleration term on the dynamic stability of the helicopter is small.

The expressions for angles of tilt due to angular velocity can be simplified to the expressions obtained in previous work under assumptions of quasi-static conditions.

1. *Introduction.*—It has been shown in Ref. 1 that when the rotor shaft of a helicopter tilts with constant pitching velocity, the tip-path plane lags behind the shaft and also tilts sideways. These angles of lag and sidetilt were found to be proportional to the pitching velocity of the shaft and the following expressions were derived:

$$\frac{\partial a_1}{\partial q} = -\frac{16}{\gamma \Omega}\,; \qquad \qquad (1)$$

$$\frac{\partial b_1}{\partial q} = \frac{1}{\Omega}\,; \qquad \qquad (2)$$

where a_1 is the angle of lag between the rotor shaft and the tip-path plane axis measured in the pitching plane and q is the pitching velocity ; b_1 is the angle of sidetilt, γ is Lock's inertia number and Ω is the rotational velocity of the rotor.

In R. & M. 2509[1] the value of $\partial a_1/\partial q$ as given by equation (1) was used in the analysis of dynamic stability. Strictly speaking equation (1) applies only to a motion with constant rate of pitch. Where the motion of the helicopter approximates to a simple harmonic motion the use of the derivative given by equation (1) may not be justified in the study of the dynamic stability.

The present note gives a more detailed analysis of rotor derivatives and shows that the expressions (1) and (2) for rotary derivatives are accurate enough for any practical application. However, the analysis indicates the existence of acceleration derivatives, but their influence on the dynamic stability of the helicopter is small.

* R.A.E. Report Aero. 2319, received 14th July, 1949.

2. *The Approach to the Problem.*—It is assumed that the helicopter is hovering with its shaft vertical and tip-path plane horizontal. Suddenly the helicopter and its shaft begin to oscillate in a pitching plane about the rotor centre according to the equation:

$$\theta = A \sin \nu t \; ; \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (3)$$

where A is some arbitrary amplitude and ν the circular frequency of rotor shaft oscillation.

The main difficulty in the present problem is estimating the magnitude and direction of the flow through the disc. Lacking any data it is assumed that the flow through the disc is in the direction opposite to the instantaneous direction of lift. In other words no allowance for any downwash lag is made.

It is further assumed that the blade hinge has zero offset, and the hinge bearing has no friction.

The diagram of axes and angles is shown in Fig. 1. The reference axes are chosen as horizontal and vertical, *i.e.*, the axis of the rotor in its initial conditions. All the values are measured from these axes, positive in an anti-clockwise direction for Fig. 1. The following quantities are defined:

θ the angle of the shaft tilt measured from the vertical and given by equation (3)

β the flapping angle measured between blade axis and horizon

Δa the angle of lag, between the shaft and the tip-path plane axis, positive in positive direction of θ.

The equation of motion is written, assuming that all the quantities are measured from OX-plane.

Due to the blade hinge being fixed relative to the shaft, the tilt of the shaft in the pitching plane by an angle θ produces, in the horizontal plane, feathering in the form:

$$-\theta \sin \psi \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (4)$$

where $\psi = \Omega t$ is the blade azimuth angle measured in the OX-plane from the downwind blade position. The angle of shaft tilt θ is a function of time given by equation (3) and hence the feathering is given by:

$$-A \sin \nu t \sin \Omega t \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (5)$$

and the blade pitch angle by:

$$\vartheta = \vartheta_0 - A \sin \nu t \sin \Omega t \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (6)$$

where ϑ_0 is the collective pitch angle.

The moments acting on the blade about the flapping hinge are as follows:

Moment due to centrifugal force:

$$-I_1 \Omega^2 \beta \, . \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (7)$$

Moment due to angular acceleration of blade in flapping motion:

$$-I_1 \ddot{\beta} \, . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (8)$$

The aerodynamic moment:

$$\int_0^R \tfrac{1}{2}\rho a\left(\vartheta_0 - A \sin \nu t \sin \Omega t - \frac{U}{\Omega r} - \frac{\dot\beta}{\Omega}\right)\Omega^2 r^3 c \, dr \qquad \ldots \qquad (9)$$

where U is velocity of flow through and perpendicular to the disc. It is assumed that the coning angle a_0 is small and that cos a_0 can be taken as unity. The flow velocity U is taken to be constant. Integrating the aerodynamic moment along the blade and introducing Lock's inertia number, the aerodynamic moment can be expressed as:

$$\tfrac{1}{8}I_1\gamma\Omega^2\left(\vartheta_0 - \tfrac{4}{3}\lambda - \frac{\dot\beta}{\Omega} - A \sin \nu t \cos \Omega t\right) \qquad \ldots \qquad (10)$$

where

$$\gamma = \frac{\rho a \bar{c} R^4}{I_1} \qquad \ldots \qquad (11)$$

is Lock's inertia number. $\bar{c}$ is a mean chord the exact value of which can be defined by comparison of equations (9) and (10), but with sufficient accuracy the mean chord can be defined as

$$\bar{c} = \frac{\int_0^R c r^3 \, dr}{\int_0^R r^3 \, dr} . \qquad \ldots \qquad (12)$$

λ is the coefficient of the flow through the disc defined as:

$$\lambda = U/\Omega R . \qquad \ldots \qquad (13)$$

In the evaluation of integral (9) no allowance has been made for tip and root losses.

Neglecting the gravity term, the equation of motion of the blade is obtained from the expressions (7), (8) and (10)

$$\ddot\beta + \tfrac{1}{8}\gamma\Omega\dot\beta + \Omega^2\beta = \tfrac{1}{8}\gamma\Omega^2(\vartheta_0 - \tfrac{4}{3}\lambda) - \tfrac{1}{8}\gamma\Omega^2 A \sin \nu t \sin \Omega t . \qquad \ldots \qquad (14)$$

Remembering that the coning angle a_0 is given by R. & M. 1127[2] as

$$a_0 = \tfrac{1}{8}\gamma(\vartheta_0 - \tfrac{4}{3}\lambda) ; \qquad \ldots \qquad (15)$$

and introducing trigonometrical substitutions equation (14) can be put into the following alternative form for ease of integration

$$\ddot\beta + \tfrac{1}{8}\gamma\Omega\dot\beta + \Omega^2\beta = a_0\Omega^2 + \tfrac{1}{16}A\gamma\Omega^2 \cos(\Omega + \nu)t - \tfrac{1}{16}A\gamma\Omega^2 \cos(\Omega - \nu)t . \qquad \ldots \qquad (16)$$

The complete solution of equation (16) takes the form:

$$\begin{aligned}\beta = a_0 &+ e^{\lambda_F t}\{C_1 \cos \nu_F t + C_2 \sin \nu_F t\} \\ &+ C_3 \cos(\Omega + \nu)t + C_4 \sin(\Omega + \nu)t \\ &+ C_5 \cos(\Omega - \nu)t + C_6 \sin(\Omega - \nu)t ; \qquad \ldots \qquad (17)\end{aligned}$$

where C_1, C_2, etc., are constants of integration and are functions of A, Ω, ν and γ only.

The first part of equation (17) shows that due to an initial disturbance the blade will oscillate with frequency given by

$$\nu_F = \Omega \sqrt{\left[1 - \left(\frac{\gamma}{16}\right)^2\right]} . \qquad \qquad (18)$$

This oscillation is very heavily damped and the damping factor is given by:

$$\lambda_F = -\frac{\gamma \Omega}{16} . \qquad \qquad (19)$$

After the blade motion due to the initial disturbance is damped out, which in practice takes about $\frac{1}{4}$ sec, the mean value of blade flapping angle approaches a_0, the coning angle. The second part of equation (17) shows that the blade will oscillate steadily about its flapping hinge with two frequencies, $\Omega + \nu$ and $\Omega - \nu$.

After some re-arrangement of equation (17) and omitting the rapidly damped part of the motion, the expression for steady blade flapping measured from horizontal will be in the form:

$$\beta' = a_0 + (C_7 \cos \nu t + C_8 \sin \nu t) \cos \psi + (C_9 \cos \nu t + C_{10} \sin \nu t) \sin \psi ; \qquad \qquad (20)$$

where $\psi = \Omega t$ and C_7, C_8, etc., are constants and functions of A, Ω, ν and γ.

The coefficient of $\cos \psi$ gives the longitudinal tilt of the tip-path plane and the coefficient of $\sin \psi$ the lateral tilt of the tip-path plane, both measured from horizontal.

The angle between the shaft and OZ-axis as given by equation (3) is

$$\theta = A \sin \nu t \qquad \qquad (3)$$

and the angle of the longitudinal lag, *i.e.*, the angle between the tip-path plane axis and shaft axis (positive in positive direction of θ) is:—

$$\Delta a = (C_7 \cos \nu t + C_8 \sin \nu t) - A \sin \nu t . \qquad \qquad (21)$$

Remembering that:

$$\left.\begin{aligned} \theta &= A \sin \nu t \\ \dot{\theta} &= A \nu \cos \nu t \\ \ddot{\theta} &= -A \nu^2 \sin \nu t \end{aligned}\right\} \qquad \qquad (22)$$

and putting

$$\nu/\Omega = \bar{\nu} \qquad \qquad (23)$$

the angle between the shaft and tip-path plane axis, measured in the plane of the shaft oscillation is given by the expression:

$$\begin{aligned} \Delta a = &-\frac{2}{\Omega}\frac{\gamma}{8}\left\{\left(\frac{\gamma}{8}\right)^2 + \bar{\nu}^2(4 - \bar{\nu}^2)\right\}\frac{1}{D}\dot{\theta} \\ &-\frac{1}{\Omega^2}\left\{\left(\frac{\gamma}{8}\right)^4(1 - \bar{\nu}^2) - \left(\frac{\gamma}{8}\right)^2(4 - 3\bar{\nu}^2 + 2\bar{\nu}^4) - \bar{\nu}^2(4 - \bar{\nu}^2)^2\right\}\frac{1}{D}\ddot{\theta} . \qquad \qquad (24) \end{aligned}$$

The corresponding lateral tilt of tip-path plane is given by:

$$\Delta b = \frac{1}{\Omega}\left(\frac{\gamma}{8}\right)^2\left\{\left(\frac{\gamma}{8}\right)^2(1 - \bar{\nu}^2) - \bar{\nu}^4\right\}\frac{1}{D}\dot{\theta}$$

$$- \frac{1}{\Omega^2}\left(\frac{\gamma}{8}\right)\left\{\left(\frac{\gamma}{8}\right)^2(3 - \bar{\nu}^2) + \bar{\nu}^2(4 - \bar{\nu}^2)\right\}\frac{1}{D}\ddot{\theta} \; ; \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

and the tilt of the rotor tip-path plane is *positive towards the advancing blade.* In equations (24) and (25), D is an abbreviation for the following expression:

$$D = \left(\frac{\gamma}{8}\right)^4(1 - \bar{\nu}^2)^2 + 2\bar{\nu}^2\left(\frac{\gamma}{8}\right)^2(4 - 3\bar{\nu}^2 + \bar{\nu}^4) + \bar{\nu}^4(4 - \bar{\nu}^2)^2 . \qquad \ldots \quad \ldots \quad (26)$$

3. *Discussion.*—3.1. *Longitudinal Disc Tilt.*—The equation (24) can be rewritten:

$$\Delta a = -\frac{16}{\gamma\Omega}\Lambda\,\dot{\theta} + \frac{1}{\Omega^2}\left\{\left(\frac{16}{\gamma}\right)^2 - 1\right\}\Gamma\ddot{\theta} \; ; \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

where Λ and Γ are functions of γ and $\bar{\nu}$ only, and for small values of $\bar{\nu} = \nu/\Omega$ can be taken as unity. The full expressions for Λ and Γ are as follows:—

$$\Lambda = \left(\frac{\gamma}{8}\right)^2\left\{\left(\frac{\gamma}{8}\right)^2 + \bar{\nu}^2(4 - \bar{\nu}^2)\right\}\frac{1}{D} \; ; \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

$$\Gamma = \frac{\left(\frac{\gamma}{8}\right)^2\left\{\left(\frac{\gamma}{8}\right)^4(1 - \bar{\nu}^2) - \left(\frac{\gamma}{8}\right)^2(4 - 3\bar{\nu}^2 + 2\bar{\nu}^4) - \bar{\nu}^2(4 - \bar{\nu}^2)^2\right\}}{\left\{\left(\frac{\gamma}{8}\right)^2 - 4\right\} \times D} \; ; \qquad \ldots \quad \ldots \quad (29)$$

where D is given by the equation (26).

The numerical values of these functions are given in Fig. 2, and were calculated for a range of values of $\bar{\nu}$ and γ. The values of $\bar{\nu}$ as found in practice are usually very small ; for example in the case of the Sikorsky R4-B helicopter, which has $\gamma = 13$, the values of $\bar{\nu}$ are about 0·02 for longitudinal and about 0·05 for lateral oscillations. The value of $\bar{\nu}$ is seldom expected to exceed 0·05 for full-scale rotors. Also, for most of the small helicopter models the value of $\bar{\nu} = \nu/\Omega$ remains of the same order due to the large values of Ω necessary if the corresponding tip speed is to be maintained on the model. It can be seen from Fig. 2 that within the practical range of values the ratio of oscillations, $\bar{\nu}$ has only a small effect upon the values of rotor derivatives. Only for very heavy blades, $\gamma < 6$, can the values of Λ and Γ differ appreciably from unity, and in those cases for the purpose of stability calculations the corrections of Fig. 2 can be used.

For normal values of blade inertia number the following approximation is justified:

$$\Lambda = \Gamma = 1 , \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

and thus simplify the equation (27) to the form:

$$\Delta a \simeq -\frac{16}{\gamma\Omega}\dot{\theta} + \frac{1}{\Omega^2}\left\{\left(\frac{16}{\gamma}\right)^2 - 1\right\}\ddot{\theta} . \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

The derivative of the tilt of the tip-path plane relative to the shaft with respect to angular velocity of pitch is:

$$\frac{\partial a_1}{\partial q} = -\frac{16}{\gamma\Omega}, \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

the tip-path plane lagging behind the shaft. The equation (32) is in agreement with equation (1), which was obtained from the simplified analysis of Ref. 1.

The derivative of the tilt of the tip-path plane relative to the shaft with respect to angular acceleration of pitch is:

$$\frac{\partial a_1}{\partial \dot{q}} = \frac{1}{\Omega^2}\left\{\left(\frac{16}{\gamma}\right)^2 - 1\right\} = \left(\frac{\partial a_1}{\partial q}\right)^2 - \frac{1}{\Omega^2}. \qquad \ldots \quad \ldots \quad \ldots \quad (33)$$

The approximate expression for the acceleration derivative (33) and the expression for the correction factor Γ (29) are valid only for $\gamma \neq 16$; for the values of γ equal or near 16 the full expression (24) for the acceleration derivative has to be used.

It is interesting to note that the angular velocity derivative, equation (32), is always negative and always produces a damping of the angular motion of the helicopter. The sign of the angular acceleration derivative however depends on the value of blade inertia number. For practical values of blade inertia number ($\gamma < 16$) the acceleration derivative is positive, as shown by equation (33), *i.e.*, the apparent displacement derivative is negative, and it can be shown that the positive acceleration term has a stabilising effect on the helicopter stability. However the numerical calculations based on the Sikorsky R4-B helicopter show a very small effect due to this acceleration derivative. It has to be borne in mind that the present analysis does not take into account any effect of the time lag of the aerodynamic forces acting on the blade. This effect could be of considerable importance in determining the blade motion.

For very light blades, $\gamma > 16$, the acceleration derivative changes sign and becomes negative*. It is worth noting that according to equation (18) for values of inertia number $\gamma > 16$ the disturbed flapping motion of blade ceases to be oscillatory and becomes a pure subsidence.

To give a better illustration of the disc and shaft behaviour in simple harmonic motion, the diagrams of Fig. 3 were prepared. The thick line represents motion of the rotor shaft according to the equation:

$$\theta = A \sin \nu t\,; \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

The dotted line shows motion of the tip-path plane with respect to the horizon. The angle α is the angle between tip-path plane axis and the vertical. The motion of the tip-path plane can be described by an equation:

$$\alpha = \Phi A \sin(\nu t + \phi)\,; \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (34)$$

The constants Φ and ϕ can be found from equation (20) and are functions of $\bar{\nu}$ and γ only.

For the practical range of values of γ and $\bar{\nu}$, the motion of the rotor disc is shown in Fig. 3, top diagram. The ratio of amplitudes, Φ is slightly less than unity, but for practical purposes can be taken as unity. The phase angle ϕ is small and negative. For infinitely heavy blades,

* The exact value of γ at which the acceleration derivative becomes negative is slightly less than 16 and depends on the value of $\bar{\nu}$.

$\gamma \to 0$, Fig. 3, bottom diagram, the amplitude ratio Φ approaches zero value, and at the same time the phase angle $\phi \to 90$ deg. For the limiting case when $\gamma = 0$ and $\Phi = 0$, the tip-path plane remains horizontal, irrespective of the shaft motion, which is quite obvious for physical reasons. For very light blades, when the inertia number γ is large, the amplitude ratio Φ is increasing and becomes larger than unity (Fig. 3, bottom diagram). Increasing the value of γ to infinity brings the value of Φ back to unity. The phase angle ϕ is decreasing steadily with increasing γ, and for $\gamma \to \infty$ the phase angle approaches zero. For infinitely light blades, $\Phi = 1$ and $\phi = 0$, the rotor disc follows exactly the motion of the shaft.

It might be of interest to point out that the vertical shift between the thick and dotted lines of Fig. 3 is proportional to the tilt of the disc with respect to the shaft, and at the point $\nu t = 0$ this shift is proportional to the velocity derivative and at the point $\nu t = \pi/2$ to the acceleration derivative. Bearing this in mind, it can be deduced immediately from Fig. 3, that the angular velocity derivative always remains negative, but the acceleration derivative may change sign depending on the value of the inertia number.

To give some illustrations of the values involved, Fig. 4 was prepared, where the value of the amplitudes ratio, Φ, is plotted against inertia number, γ, for different values of frequency ratio, $\bar{\nu}$. The scale to the right is too small to show the shape of the Φ-curve for very large values of γ, and the dotted line shows only the tendency of the Φ-curve.

Remembering that the values of $\bar{\nu}$ and γ likely to meet in practice are $\bar{\nu} < 0{\cdot}05$ and $\gamma > 5$, it can be seen that the value of Φ differs from unity by less than 1 per cent.

3.2. *Lateral Tilt.*—For the practical range of values of $\bar{\nu} = \nu/\Omega$ the equation (25) for the lateral tilt of the tip-path plane can be simplified to:

$$\Delta b = \frac{1}{\Omega}\dot{\theta} - \frac{1}{\Omega^2}\frac{24}{\gamma}\ddot{\theta} = \frac{1}{\Omega}\dot{\theta} + \frac{3}{2}\frac{1}{\Omega}\frac{\partial a_1}{\partial q}\ddot{\theta} \,. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (35)$$

During the longitudinal oscillations of the helicopter with frequency ν, the tip-path plane tilts not only in the plane of oscillations, but in the plane perpendicular to it as well. This angle of tilt, Δb, can be split up into two parts as in equation (35), first that due to angular velocity and second that due to the angular acceleration of the helicopter.

The corresponding derivatives of lateral tilt can be written:

$$\frac{\partial b_1}{\partial q} = \frac{1}{\Omega} \,. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (36)$$

The value of $\partial b_1/\partial q$ for constant rate of helicopter pitch was deduced in Ref. 1, Appendix II, from kinematic considerations, and is in agreement with present result.

Similarly the acceleration derivative of the sidetilt is:

$$\frac{\partial b_1}{\partial \dot{q}} = -\frac{1}{\Omega^2}\frac{24}{\gamma} = \frac{3}{2}\frac{1}{\Omega}\frac{\partial a_1}{\partial q} \,. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (37)$$

4. *Conclusions.*—The mathematical analysis of the dynamics of the flapping blade during simple harmonic motion of the helicopter shows the existence of angular velocity and angular acceleration derivatives. The effect of the accleration derivative on the dynamic stability of the helicopter is quite small, and for the usual values of blade inertia number it can be neglected. The effect of the frequency ratio $\bar{\nu} = \nu/\Omega$ on the values of the rotor derivatives is also small, mainly due to the fact that for all helicopters and for helicopter models *built to dynamic and aerodynamic scale*, the value of $\bar{\nu}$ is likely to be much less than $0{\cdot}1$.

It seems that only for the case of helicopters with very heavy blades, say $\gamma < 8$, need the effects of the acceleration term and of frequency ratio be taken into account.

It has to be borne in mind that the analysis given here does not take into account any changes in slipstream structure due to the oscillation of the helicopter ; and further, the analysis covers only simple harmonic motion in hovering.

It is proposed to analyse other modes of helicopter motion in hovering, say an exponential mode of motion, corresponding to motion due to stick displacement.

A further step will be the extension of the present analysis to forward flight.

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a	Lift slope of blade section
a_0	Coning angle
a_1	Coefficient of Fourier's series for flapping angle, and corresponding to backward tilt of the rotor disc
b_1	Coefficient of Fourier's series for flapping angle, and corresponding to lateral tilt of the rotor disc, positive towards the advancing blade.
B	Tip-loss coefficient ; usually defined: $B = 1 - c/2R$
c	Blade chord
$\bar{c}$	Mean blade chord
I_1	Blade moment of inertia about flapping hinge
$q = \dot{\theta}$	Pitching velocity of helicopter
t	Time
R	Rotor radius
U	Velocity of flow through the disc
α	Angle between rotor disc and horizon
β	Flapping angle, measured between longitudinal axis of the blade and horizon
$\gamma = \dfrac{\rho a \bar{c} R^4}{I_1}$	Blade inertia number
θ	Angle of shaft tilt in pitching plane and measured from vertical
ϑ	Pitch angle of rotor blade ; measured from blade airfoil chord
ϑ_0	Collective pitch
$\lambda = \dfrac{U}{\Omega R}$	Coefficient of flow through the disc
λ_F	Damping coefficient of disturbed flapping motion
ν_F	Frequency of disturbed flapping motion
ν	Circular frequency of shaft oscillation
$\bar{\nu} = \nu/\Omega$	Frequency ratio
ρ	Air density
$\psi = \Omega t$	Blade azimuth angle, measured from rearmost position
Ω	Rotational velocity of rotor
Λ	Correction to angular velocity derivative
Γ	Correction to angular acceleration derivative
Φ	Ratio of disc to shaft amplitudes
ϕ	Phase angle between shaft and disc motions

APPENDIX

The Comparison with Calculations for Constant Angular Velocity and Constant Angular Acceleration

Since the completion of the present report, the attention of the author was drawn to the work of Miehl, Ref. 4. Miehl extended Lock's analysis (R. & M. 1127[2]) to steady curvilinear flight, and his investigations lead to the following expressions for the rotor tilt derivatives:

$$\left.\begin{aligned} \frac{\partial a_1}{\partial q} &= -\frac{16}{\gamma\Omega}\frac{1}{B^2(B^2 - 1/2\,\mu^2)} \\ \frac{\partial b_1}{\partial q} &= \frac{1}{\Omega}\frac{1}{1 - \mu^2/2B^2} \end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (43)$$

where B is the usual tip-loss coefficient. The formulae (43) are in agreement with the present report if we assume no tip loss, $B = 1$, and very slow oscillation, $\bar{\nu} = 0$. It can be seen from equation (43) that the forward speed has a very small effect on the values of the angular velocity derivatives.

Miehl (Ref. 4) in his work is considering one particular case of blade motion during uniformly accelerated rotation of helicopter in hovering. The formulae for the rotor tilt derivatives due to constant acceleration are in agreement with the present report, assuming frequency ratio, $\bar{\nu} = 0$.

The comparison of the theoretical calculations with experimental values obtained from model tests, Ref. 4, have shown that in order to obtain satisfactory agreement it was necessary to modify the distribution of the induced velocities. It was assumed in Ref. 4 that the increments of induced velocity are proportional to the new mass forces present in curvilinear accelerated flight.

The calculations have shown that the re-distribution of induced velocities has a small effect on the values of angular derivatives, and is increasing slightly the value of $\partial a_1/\partial q$, the value of $\partial b_1/\partial q$ not being affected. However, re-distribution of induced velocities has a considerable effect on the acceleration derivative, and increases the value of $\partial a_1/\partial \dot{q}$; the value of $\partial b_1/\partial \dot{q}$ is not affected. The assumed distribution of induced velocities gave good agreement between theory and experiment.

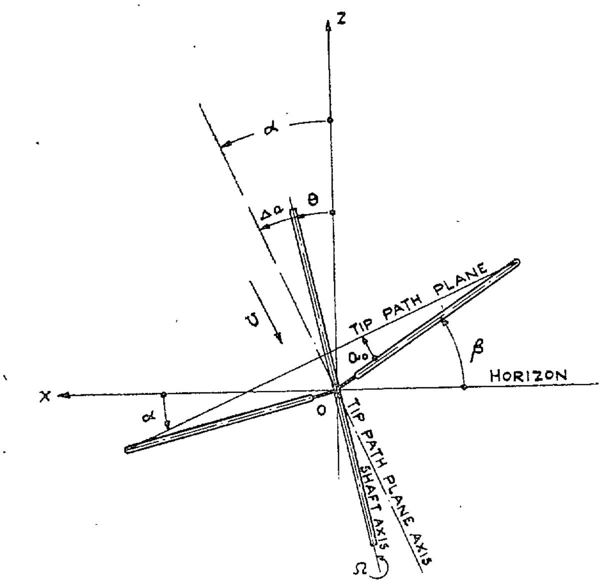

FIG. 1. Diagram of rotor in simple harmonic motion.

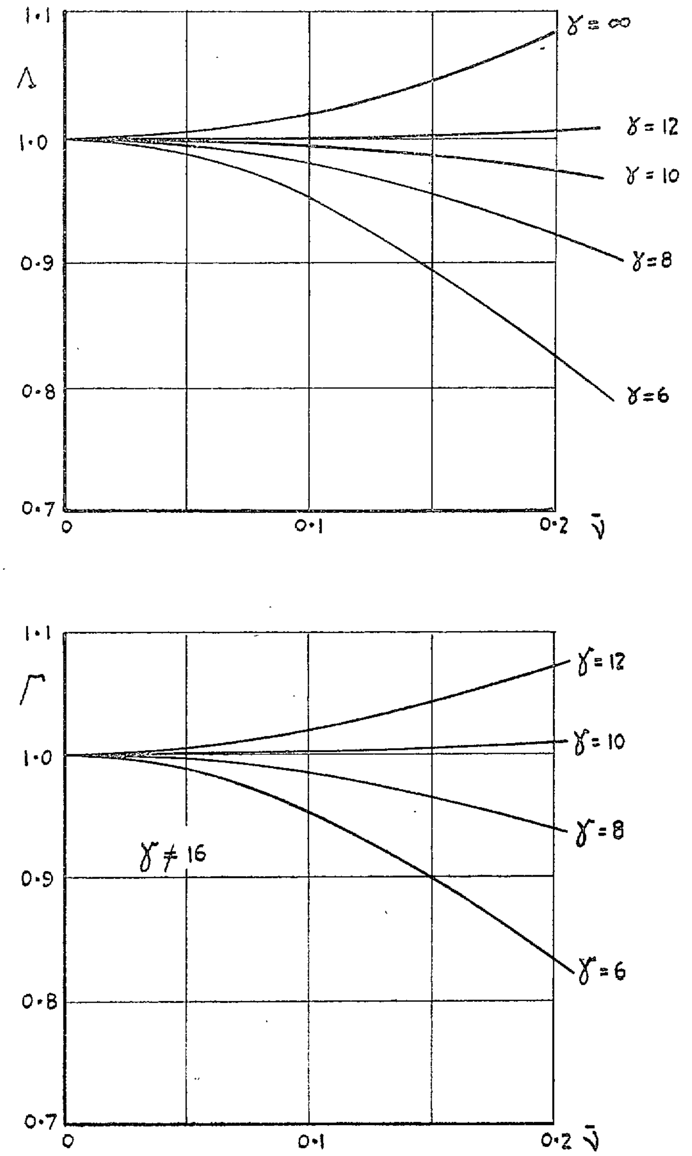

FIG. 2. Corrections to velocity and acceleration derivatives.

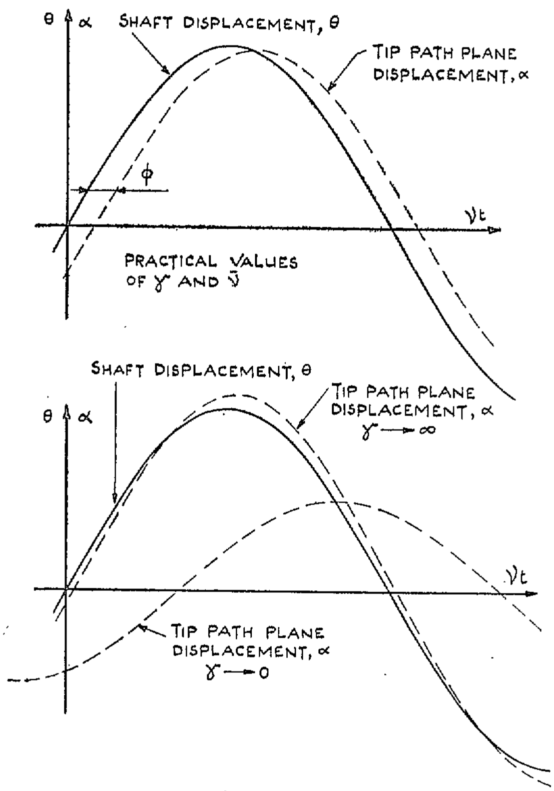

FIG. 3. Attitude *versus* time curves of rotor shaft and tip-path plane axis in steady harmonic motion.

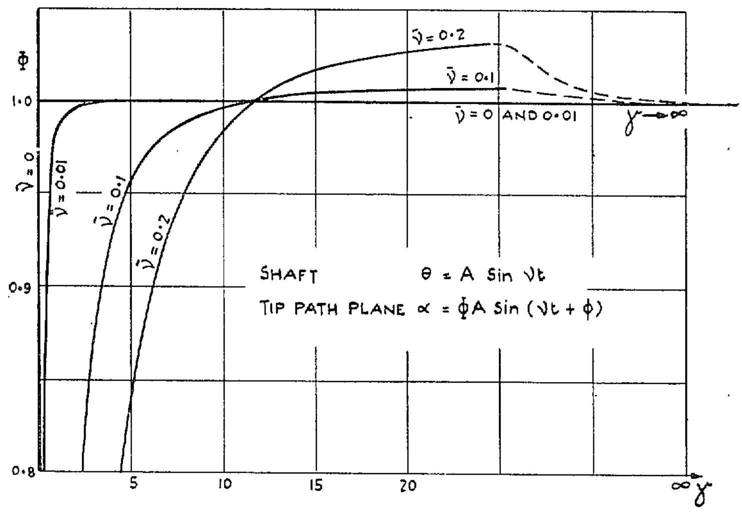

FIG. 4. Ratio of tip-path plane to shaft amplitudes in simple harmonic oscillation.

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