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Resonant Vibration of Helicopter Rotor Blades

By

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Summary.—The blades of the operative rotors of helicopters are usually hinged both in the lift and rotational planes and it is because of this articulation that the blades in the course of rotation are akin dynamically to 'pendulum vibration dampers'.

If the fundamental frequency of this species of pendulum vibration is numerically equal to nN where n is the number of blades and N is the frequency of rotation of the rotor then serious resonant forced vibration may ensue and it would appear that this is quite likely to occur in practical cases with the blades in vibration in the plane of rotation of the rotor.

Introduction.—For the effective control of helicopters it is common practice to arrange for the rotor blades to be articulated; that is, so hinged as to allow for a restricted range of amplitude in angular motion relative to their hub both in the lift and rotational planes; and it is this feature which gives rise to a variety of problems not the least important of which is severe vibration which may affect the comfort of the occupants and the reliablity of the craft. One of these problems which appears to yield to comparatively simple treatment, provided the rotor blades are regarded as rigid, is the torsional vibration of the engine crankshaft, transmission shafting, and rotor system. Now the layout of the power drive in ordinary practice consists of engine, fan, relatively flexible driving shaft, gearbox, rotor; so that as a first approximation there is the engine–fan, and the rotor–gearbox systems, each of which can be treated as separating out owing to the very flexible connecting driving shaft. The engine–fan system presents no new feature except perhaps that special care has to be exercised to ensure that the fan blades are free from resonant vibration at the engine operating speeds. The vibration of the rotor system however is far more difficult to evaluate more particularly if the flexibility of the blade is taken into account as it should be.

Owing to the articulation of the blades of the rotor they acquire, in the course of rotation, dynamical characteristics akin to 'pendulum vibration dampers'. And it is because of this fact that the effective polar moment of inertia of a helicopter rotor system can assume any value between plus and minus infinity according to the 'order' of the forcing vibrations which may act on the rotor. It seems that if the blades were rigid in the plane of rotation the rotor–gear system will in general, have two frequencies of vibration about the axis of the rotor. The lower of these frequencies may in actual helicopter rotors be in the region of nN where n is the number of blades of the rotor and N is its frequency of rotation. But nN is the 'order' of aerodynamic forced vibration due to the rotor blades rhythmically passing the hull and to their cyclic change in pitch. For instance in a particular helicopter with a three-blade rotor the aforementioned fundamental frequency of vibration was found to be $3{\cdot}05N$. The effect of the flexibility of the blades will result in the reduction of this value. In the event of this frequency being equal

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to $3N$ the blades might be subject to powerful forced aerodynamic vibrations, particularly when being driven with the full power of the engine.

1. *Rotor System.*—Fig. 1 is a schematic plan view of the rotor hub, articulated system.

If Ω is the mean angular velocity of rotation of the hub, we assume that at time t the hub will have turned through an angle $\Omega t + \theta$, relative to the reference line O_1Z, where θ is a small angle of variation due to vibration. All the symmetrically disposed blades are assumed to be displaced similarly at time t each being at a small angle ϕ to their associated hub radius R. G is the mass centre of a typical blade, the length O_2G being l_1; m is the mass of the blade, and mk^2 its second moment or moment of inertia about the vertical axis through G; and all the blades are assumed to be rigid.

O_1x, O_1y respectively parallel to and normal to the blade O_2G, are taken as moving axes of reference at O_1. α_x, α_y are the accelerations of m in the directions of these axes. The co-ordinates of G are, to the first order, $x = R + l_1$, $y = -R\phi$, and the angular velocity of the blade and thus that of the moving axes of reference, is $\Omega + \dot{\theta} + \dot{\phi}$. In these circumstances it may be shown that

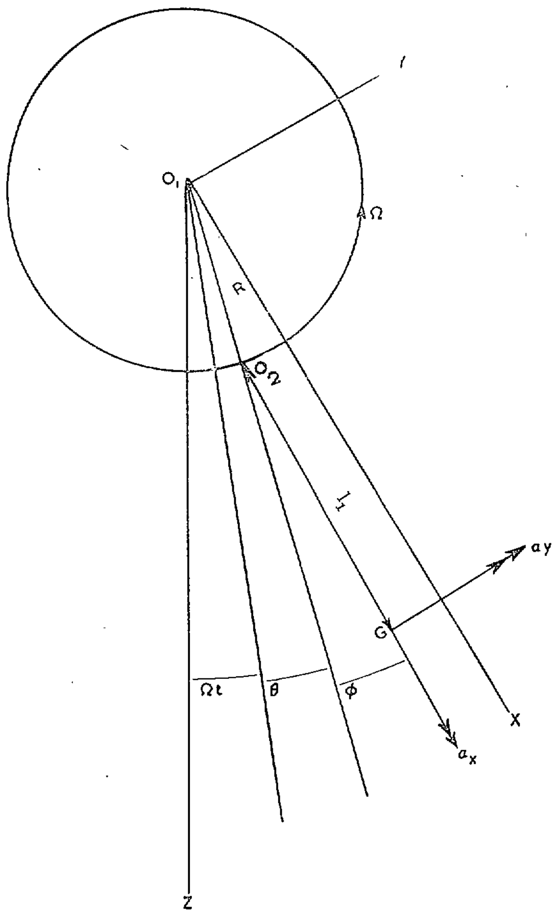

FIG. 1.

$$\alpha_x = -(R + l_1)\Omega^2 - 2\Omega[(R + l_1)\dot{\theta} + l_1\dot{\phi}] , \qquad (1)$$

$$\alpha_y = (R + l_1)\ddot{\theta} + l_1\ddot{\phi} + R\Omega^2\phi. \qquad (2)$$

For the rate of change of angular momentum of the blade about O_2, on the assumption that the hinge at O_2 is perfectly smooth, we have

$$m[\alpha_y l_1 + k^2(\ddot{\theta} + \ddot{\phi})] = 0; \qquad (3)$$

or, making use of (2)

$$(Rl_1 + l_1^2)\ddot{\theta} + l_1^2\ddot{\phi} + Rl_1\Omega^2\phi + k^2(\ddot{\theta} + \ddot{\phi}) = 0, \qquad (4)$$

which may be written

$$(Rl_1 + l_2{}^2)\ddot{\theta} + l_2{}^2\ddot{\phi} + Rl_1\Omega^2\phi = 0, \quad \ldots \quad (5)$$

where

$$ml_2{}^2 = m(k^2 + l_1{}^2) \quad \ldots \quad (6)$$

is the second moment of the blade about the vertical axis through the drag hinge.

The rate of change of angular momentum of the hub and its associated blades about O_1 will be

$$p_h\ddot{\theta} + nm[\alpha_y(R + l_1) + \alpha_x R\phi + k^2(\ddot{\theta} + \ddot{\phi})]\,, \quad \ldots \quad (7)$$

where p_h is the polar moment of inertia of the hub alone and n is the number of blades.

Making use of (3) and (5) we derive for (7) the expression

$$\left[p_h + nmR\left\{(R + l_1) + \frac{(Rl_1 + l_2{}^2)l_1(\Omega^2 - D^2)}{(Rl_1\Omega^2 + l_2{}^2D^2)}\right\}\right]\ddot{\theta}, \quad \ldots \quad (8)$$

where D is the operator d/dt.

Thus for a forcing vibration of frequency $\omega/2\pi$ acting on the hub rotor system, the effective equivalent polar moment of inertia of the system will be

$$p_h + nmR\left[\frac{(R^2 + 2Rl_1 + l_2{}^2)l_1\Omega^2 - R(l_2{}^2 - l_1{}^2)\omega^2}{(Rl_1\Omega^2 - l_2{}^2\omega^2)}\right], \quad \ldots \quad (9)$$

and this, for computational convenience, may be written

$$p_e = p_h + nmR^2\frac{l_1}{l_2}\left[\frac{\left(\frac{R}{l_2} + 2\frac{l_1}{l_2} + \frac{l_2}{R}\right)\Omega^2 - \left(\frac{l_2}{l_1} - \frac{l_1}{l_2}\right)\omega^2}{\left(\frac{R}{l_2}\cdot\frac{l_1}{l_2}\Omega^2 - \omega^2\right)}\right]. \quad \ldots \quad (10)$$

We notice from this expression that

$$\lim_{\Omega/\omega\to\infty} p_e = p_h + nmRl_2\left(\frac{R}{l_2} + 2\frac{l_1}{l_2} + \frac{l_2}{R}\right), \quad \ldots \quad (11)$$

and that

$$\lim_{\Omega/\omega\to 0} p_e = p_h + nmR^2\frac{l_1}{l_2}\left(\frac{l_2}{l_1} - \frac{l_1}{l_2}\right). \quad \ldots \quad (12)$$

2. *Torsional Vibration of the Rotor–Gear System.*—The rotor–gear system may be driven through the agency of a relatively highly flexible shaft and in such circumstances the rotor–gear system may be assumed, with a fair degree of approximation, to separate out from the remainder of the power system so far as the torsional vibration is concerned. Thus we shall have the frequency equation

$$\omega^2 = c_{eg}\left(\frac{1}{p_e} + \frac{1}{p_g}\right), \quad \ldots \quad (13)$$

where p_g is the effective polar moment of inertia of the gear system alone and c_{eg} is the torsional stiffness of the shaft scantling connecting p_e and p_g; also

$$\frac{1}{p_e} = \left(\frac{R}{l_2}\cdot\frac{l_1}{l_2}\Omega^2 - \omega^2\right)\frac{1}{p}, \quad \ldots \quad (14)$$

where

$$p = \left[p_h\frac{R}{l_2}\cdot\frac{l_1}{l_2} + nmR^2\frac{l_1}{l_2}\left(\frac{R}{l_2} + 2\frac{l_1}{l_2} + \frac{l_2}{R}\right)\right]\Omega^2 - \left[p_h + nmR^2\frac{l_1}{l_2}\left(\frac{l_2}{l_1} - \frac{l_1}{l_2}\right)\right]\omega^2. \quad (15)$$

For the case in which the connecting shaft between p_e and p_g is infinitely stiff the frequency equation is given by

$$p_e + p_g = 0, \quad \ldots \quad (16)$$

or

$$p + \left(\frac{R}{l_2}\cdot\frac{l_1}{l_2}\Omega^2 - \omega^2\right)p_g = 0, \quad \ldots \quad (17)$$

where p is given by (15) above.

Thus in such case there will be only one frequency, viz., $\omega/2\pi$, given by the equation

$$\frac{\omega^2}{\Omega^2} = \frac{\left(p_h + p_g\right)\dfrac{R}{l_2}\cdot\dfrac{l_1}{l_2} + nmR^2\dfrac{l_1}{l_2}\left(\dfrac{R}{l_2} + 2\dfrac{l_1}{l_2} + \dfrac{l_2}{R}\right)}{\left(p_h + p_g\right) + nmR^2\dfrac{l_1}{l_2}\left(\dfrac{l_2}{l_1} - \dfrac{l_1}{l_2}\right)} \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

and in these circumstances unless ω/Ω corresponds to the order of a powerful forced vibration acting on the system no serious resonant vibration can occur so far as the rotor system is concerned.

3. *Numerical Examples.*—Let us consider a case taken from practice for which

$$R = 9\cdot125, \quad l_1 = 99\cdot275, \quad l_2 = 132\cdot41, \quad \text{all in in.}; \ n = 3;$$

$$m = 0\cdot367, \quad p_h + p_g = 79\cdot7, \quad \text{both in lb wt in. sec units.}$$

Inserting these values in the expression (18) of section 2 we find that

$$\omega = 3\cdot05\,\Omega \text{ approx.}$$

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