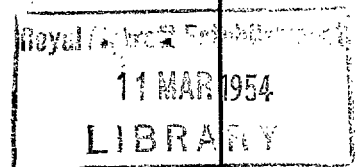




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The Elastic Stability of Sandwich Plates

By

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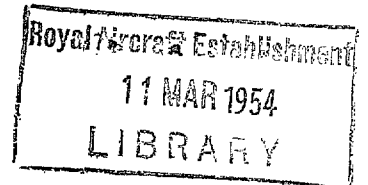
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Summary.—This paper treats the elastic stability of supported rectangular plates of sandwich construction with isotropic and aeolotropic fillings under compression and shear loading. Formulae are developed for critical stresses for flat and curved panels in compression and flat panels in shear for the buckling of the whole panel, also for the wrinkling or local failure of the faces of flat panels in compression.

It is established that for a wide range of conditions the critical stress for panels buckling in compression is independent of the form of the filling providing it is symmetrical about the normal; of the elastic constants of the filling only the transverse shear is of concern. As a result a simple extension of the equivalent plate theory of greatly improved accuracy is developed enabling the use of equations treating the plate as a whole.

NOTE: This paper was presented as a thesis for the Diploma of the College of Aeronautics, June 1948.

1. *Summary of Results.*—An isotropic and two forms of aeolotropic filling are considered. Of the latter both are symmetrical about the normal to the panel[†], but one has stiffnesses only in normal planes and the other has comparable stiffnesses in all planes: the first of these will be referred to as a honeycomb filling, as a fabricated paper honeycomb filling is a good example of the material in mind, and the other simply as aeolotropic.

Failure may be either a buckling of the panel as a whole or a short-wave buckling or wrinkling of the faces. The latter has only been investigated for simply supported flat panels. The critical stress for this type of failure is not dependent on the dimensions of the panel other than the ratio of the skin thickness to the overall thickness; from this and from dimensional arguments there is reason to believe that the wrinkling stress is independent of the edge conditions and of curvature and that a panel in shear will wrinkle when the compression field equals the critical compressive wrinkling stress.

Sections 4 and 5 of this paper derive by a fairly rigorous analysis formulae for the critical stresses for buckling, as opposed to wrinkling, of simply supported panels in compression which divide into three orders of approximation. The first, which in general is quite inaccurate, is the result of ignoring shear deformation and gives the so-called equivalent plate stress; this, of course, is the same for all forms of filling. The second, which is also independent of the form of filling, provides in effect a correction factor to the equivalent plate stress that is a function of a parameter, denoted by the symbol η , which is a measure of the transverse shear stiffness of the filling in terms of the strength of the whole panel in compression. The third approximation is a more complicated expression giving the failing stress for panels that are weak in shear.

* College of Aeronautics Report No. 25, received 1st September, 1949. ↗

† From recent tests with Dufaylite this is not strictly true for honeycomb material. The transverse shear modulus measured along the direction of a diagonal of a hexagonal cell differs from that measured at right-angles.

This common second approximation is important because it is accurate over a wide range of conditions and because it can also be deduced by a simple allowance for the shear deformation of the filling. Though the latter is a natural and obvious extension of engineer's bending theory, the rather tedious analysis of sections 4 and 5 of this report remains necessary because it establishes the accuracy and range of validity of the approximation. The way is then cleared for applying this simple concept of the behaviour of the filling with a fair hope of comparable accuracy to such problems as clamped edge conditions and shear loading which are difficult by more rigorous methods.

The smaller the parameter η the more marked is the effect of shear deformation with a consequent decrease in both failing stress and buckling wavelength. This applies in all circumstances of long-wave buckling.

The problem of curved panels in compression is treated by an approximate correction of the flat panel solution and also as a check by the method of simple shear allowance. No investigation is made of the stability of the buckled state, so that the apparent gain in stiffness may be illusory.

The critical shear stresses for nearly square and for long narrow panels are calculated by energy methods. Both solutions are approximate particularly the latter. Shear critical stresses appear to fall more rapidly with the filling shear stiffness than the corresponding compressive stresses, and it is difficult to estimate the lower limit of the parameter η for the validity of the formulae.

The formulae derived can only be a guide to the failing stress owing to the simplifying assumptions particularly with regard to edge conditions. A general verification of results is of course necessary but especially of the more approximate formulae such as those for curved panels and panels in shear. Among the many other aspects are two which may be of overriding importance, the effect of non-homogeneous fillings and the effect of initial waviness of the skin. The use of large-mesh honeycomb material is clearly another source of local failure of the skin. Initial waviness will lead to additional bending stresses in the faces that may cause their premature yielding and impose strains that may rupture the adhesive at the interfaces. The implications of initial waviness have been discussed at length by Howard¹ and very similar results to his can be obtained by using the formulae derived in this paper.

2. Notation.

A, C, L, N, F	Elastic constants of filling (equation 1)
E, σ	Young's modulus and Poisson ratio of faces
a, b	Panel length and width
$2h, t$	Filling and face thickness
R	Radius of curvature of panel
f or q	Compressive or shear stress due to applied load
f_c or q_c	Critical compressive or shear stress
X_x, Y_z , etc.	Filling stresses
e_{xx}, e_{yz} , etc.	Filling strains
x, y, z	Geometrical co-ordinates (Fig. 1)
u, v, w	Displacements in x, y, z directions
$N_x, N_{xy}, N_y, Q_x, Q_y$	Stress resultants of faces
M_x, M_{xy}, M_y	Stress couples of faces
$\bar{M}_x, \bar{M}_{xy}, \bar{M}_y$	Stress couples of whole plate
n, m	Number of buckling half-wavelengths along and across the panel.

$$\begin{aligned}
\bar{n} &= nb/a & l^2 &= \frac{n^2\pi^2}{a^2} + \frac{m^2\pi^2}{b^2} & \mu &= 2h/t \\
\delta &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} & \omega &= \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} & \nabla &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \\
\eta &= \frac{L(1 - \sigma^2)b^2}{\pi^2 Eht} & \xi &= \frac{2C - L}{3C} \frac{\pi^2 h^2}{b^2} & \xi^* &= \frac{L}{3C} \frac{\pi^2 h^2}{b^2} \\
D &= \frac{Et^3}{12(1 - \sigma^2)} & \rho &= \frac{2b^2}{\pi^2 R(2h + t)} & \tau &= \frac{E}{(1 - \sigma^2)L} \cdot \frac{t}{R} \\
K &= \frac{L(1 - \sigma^2)}{Et} & \nu &= \frac{C + L}{C - L} & \varepsilon &= 2C/L
\end{aligned}$$

3. *Summary of Formulae.*—(a) *Buckling in Compression of Flat Panel.*—(i) *Approximate Formulae for all Forms of Filling.*

1. *Simply Supported Long Panel.*

$$f_c = \frac{2(\mu + 1)^2 L}{\mu} \cdot \frac{\eta}{(1 + \eta)^2} = \frac{\pi^2(2h + t)^2 E}{(1 - \sigma^2)b^2} \cdot \left(\frac{\eta}{1 + \eta}\right)^2 \dots \dots \dots (26)^*$$

where $\eta/(\eta - 1) < b^2/200h^2$, $\eta = L(1 - \sigma^2)b^2/\pi^2 Eht$
 $\mu = 2h/t$.

2. *Simply Supported Short Panel.*

Where $a/b = 0(1)$ a correction factor to 1 is given at Fig. 10.

Where the panel is very wide compared to its length a modified Euler strut formula is obtained

$$f_c = \frac{\pi^2(h + \frac{1}{2}t)^2 E}{a^2} \left/ \left\{ 1 + \frac{\pi^2 Eht}{(1 - \sigma^2)La^2} \right\} \right. \dots \dots \dots (27)$$

provided $a > 10h$.

3. *Long Panel with Clamped Edges.*

$$\begin{aligned}
f_c &= \frac{(\mu + 1)^2 L}{\mu} \cdot \frac{3 \cdot 49\eta^2}{\eta^3 + 3 \cdot 68\eta^2 + 3 \cdot 6\eta - 1 \cdot 3} \dots \dots \dots (43) \\
&= \frac{17 \cdot 05(2h + t)^2 E}{(1 - \sigma^2)b^2} \cdot \frac{\eta^3}{\eta^3 + 3 \cdot 68\eta^2 + 3 \cdot 6\eta - 1 \cdot 3}
\end{aligned}$$

1 and 3 are plotted at Fig. 8 as correction factors to the equivalent plate stresses. The buckling wavelengths are shown at Fig. 9.

(ii) *More Accurate Formula for Isotropic Filling.*—(Simple Support).

$$\frac{f_c}{L} = \frac{(\mu + 1)^2}{2\mu} \cdot \frac{(\bar{n}^2 + 1)^2}{\bar{n}^2} \left\{ \frac{1}{\bar{n}^2 + \eta + 1 - \xi(\bar{n}^2 + 1)^2} + \frac{1}{3\eta(\mu + 1)^2} \right\} \dots \dots (24)$$

where \bar{n} is chosen to make f_c a minimum and

$$\xi = \frac{2C - L}{3C} \cdot \frac{\pi^2 h^2}{b^2}.$$

* Numbers refer to equation numbers in main text.

f_c/\bar{L} versus μ is plotted for different η at Figs. 2 and 3 for $6\xi\eta/\mu = (1 - \sigma^2)(2C - L)\bar{L}/EC$ equal to 0.001 and 0.0004 respectively.

(iii) *Accurate Formula for Honeycomb Filling.*—(Simple Support).

$$\frac{f_c}{L} = \frac{(\mu + 1)^2}{2\mu} \cdot \frac{(\bar{n}^2 + 1)^2}{\bar{n}^2} \left\{ \frac{1}{\bar{n}^2 + \eta + 1 + \xi^*(\bar{n}^2 + 1)^2} + \frac{1}{3\eta(\mu + 1)^2} \right\} \quad \dots \quad (34)$$

where \bar{n} is chosen to make f_c a minimum and

$$\xi^* = \frac{L}{3C} \frac{\pi^2 h^2}{b^2}.$$

f_c/L versus μ is plotted for different η at Figs. 4 to 7 for values of $6\xi^*\eta/\mu = (1 - \sigma^2)L^2/EC$ of 2.5×10^{-4} ; 10^{-4} ; 2.5×10^{-5} and 10^{-5} respectively.

(b) *Wrinkling of a Simply Supported Flat Panel in Compression.*—(i) *Isotropic Filling.*

$$\frac{f_c}{L} = \left\{ \frac{EC^2}{12(1 - \sigma^2)L(C + L)^2} \right\}^{1/3} \left\{ 1 + 2 \tanh \left[\mu \left(\frac{3(1 - \sigma^2)CL}{2E(C + L)} \right)^{1/3} \right] \right\} + \frac{2L}{C + L}. \quad (30)$$

Approximate condition of validity is $\frac{\mu^3 CL}{E(C + L)} > 2$

This result is plotted at Figs. 2 and 3.

(ii) *Anisotropic Filling.*

$$\frac{f_c}{L} = \left\{ \frac{9EC(\sqrt{AC} - F)(\sqrt{AC} + 2L - F)}{16L^2(1 - \sigma^2)(\sqrt{AC} + L)^2} \right\}^{1/3} + \frac{\sqrt{AC} - F}{\sqrt{AC} + L}. \quad \dots \quad (33)$$

Approximate condition of validity is

$$\frac{\mu^3(\sqrt{AC} - F)^2(\sqrt{AC} + 2L - F)^2}{ECL(\sqrt{AC} + L)} > 100.$$

(iii) *Honeycomb Filling.*

$$\frac{f_c}{L} = \left(\frac{2EC}{3\mu(1 - \sigma^2)L^2} \right)^{1/2} = \frac{1}{3(\xi^*\eta)^{1/2}}. \quad \dots \quad (35)$$

This result is plotted at Figs. 4 to 6; the wrinkling stress is too high to appear on Fig. 7.

(c) *Simply Supported Curved Plates in Compression.*—(i) *Panels.*

$$\frac{f_c}{L} = \frac{(\mu + 1)^2}{2\mu\bar{n}^2} \left\{ \frac{(\bar{n}^2 + 1)^2}{\bar{n}^2 + \eta + 1} + \frac{e^2}{\eta} \right\} \quad \dots \quad (48)$$

where \bar{n}^2 has the nearest value to $\frac{\eta + (\eta + 1)e^2 + \eta[\eta\{\eta + (\eta + 1)e^2\}]^{1/2}}{\eta^2 - \eta - e^2}$

$e = 2b^2/\pi^2 R(2h + t)$ and $\eta > \frac{1}{2}\{1 + (1 + 4e^2)^{1/2}\}$.

The factor of increase of this stress over that for a similar flat panel is plotted at Fig. 11.

by writing u, v, w for the displacements in the x, y, z directions and neglecting second order terms in $1/R$ the well-known equations relating strain and displacement in cylindrical co-ordinates become:—

$$\left. \begin{aligned} e_{xx} &= \frac{\partial u}{\partial x} \\ e_{yy} &= \frac{\partial v}{\partial y} - \frac{1}{R} \left\{ z \frac{\partial v}{\partial y} - w \right\} \\ e_{zz} &= \frac{\partial w}{\partial z} \\ e_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} - \frac{1}{R} \left\{ z \frac{\partial w}{\partial y} + v \right\} \\ e_{zx} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \\ e_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - \frac{z}{R} \frac{\partial u}{\partial y} \end{aligned} \right\} \dots \dots \dots \dots \dots \quad (2)$$

(b) *Equilibrium of the Filling.*—In the same fashion the following equations of equilibrium are obtained:—

$$\frac{\partial X_x}{\partial x} + \frac{\partial Y_x}{\partial y} + \frac{\partial Z_x}{\partial z} - \frac{1}{R} \left\{ z \frac{\partial Y_x}{\partial y} - Z_x \right\} = 0 \quad \dots \dots \dots \quad (3)$$

$$\frac{\partial X_y}{\partial x} + \frac{\partial Y_y}{\partial y} + \frac{\partial Z_y}{\partial z} - \frac{1}{R} \left\{ z \frac{\partial Y_y}{\partial y} - 2Z_y \right\} = 0 \quad \dots \dots \dots \quad (4)$$

$$\frac{\partial X_z}{\partial x} + \frac{\partial Y_z}{\partial y} + \frac{\partial Z_z}{\partial z} - \frac{1}{R} \left\{ z \frac{\partial Y_z}{\partial y} - Z_z + Y_y \right\} = 0. \quad \dots \dots \dots \quad (5)$$

On substituting from equations (1) and (2) into equations (3) and (4), we have:—

$$\begin{aligned} & A \frac{\partial^2 u}{\partial x^2} + N \frac{\partial^2 u}{\partial y^2} + L \frac{\partial^2 u}{\partial z^2} + (A - N) \frac{\partial^2 v}{\partial x \partial y} + (F + L) \frac{\partial^2 w}{\partial x \partial z} \\ &= \frac{1}{R} \left\{ z \frac{\partial}{\partial y} \left[(A - N) \frac{\partial v}{\partial x} + 2N \frac{\partial u}{\partial y} \right] - (A - 2N - L) \frac{\partial w}{\partial x} - L \frac{\partial u}{\partial z} \right\} \quad \dots \quad (6) \end{aligned}$$

and

$$\begin{aligned} & (A - N) \frac{\partial^2 u}{\partial x \partial y} + N \frac{\partial^2 v}{\partial x^2} + A \frac{\partial^2 v}{\partial y^2} + L \frac{\partial^2 v}{\partial z^2} + (F + L) \frac{\partial^2 w}{\partial y \partial z} \\ &= \frac{1}{R} \left\{ z \frac{\partial}{\partial y} \left[(A - N) \frac{\partial u}{\partial x} + 2A \frac{\partial v}{\partial y} + (F + L) \frac{\partial w}{\partial z} \right] - (A + 2L) \frac{\partial w}{\partial y} - L \frac{\partial v}{\partial z} \right\}. \quad (7) \end{aligned}$$

Write $\delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$, $\omega = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

Differentiate equations (6) and (7) with respect to x and y respectively and add, so that:—

$$\begin{aligned} & (A\nabla^2 + L \frac{\partial^2}{\partial z^2})\delta + (F + L)\nabla^2 \frac{\partial w}{\partial z} = \frac{1}{R} \left\{ z \frac{\partial}{\partial y} \left[2A \frac{\partial \delta}{\partial y} - (A - N) \frac{\partial \omega}{\partial x} + (F + L) \frac{\partial^2 w}{\partial y \partial z} \right] \right. \\ & \left. - L \frac{\partial \delta}{\partial z} - (A - 2N + L) \frac{\partial^2 w}{\partial x^2} - (A + 2L) \frac{\partial^2 w}{\partial y^2} \right\}. \quad \dots \dots \dots \quad (8) \end{aligned}$$

Differentiate equations (6) and (7) with respect to y and x respectively and subtract, so that:—

$$\begin{aligned} \left(NV^2 + L \frac{\partial^2}{\partial z^2} \right) \omega &= \frac{1}{R} \left\{ z \frac{\partial}{\partial y} \left[2N \frac{\partial \omega}{\partial y} - (A - N) \frac{\partial \delta}{\partial y} - (F + L) \frac{\partial^2 w}{\partial y \partial z} \right] \right. \\ &\quad \left. - L \frac{\partial \omega}{\partial z} + (2N + L) \frac{\partial^2 w}{\partial x \partial y} \right\} \dots \dots \dots \dots \dots \dots \quad (9) \end{aligned}$$

Substituting from equations (1) and (2) into equation (5) gives:—

$$\begin{aligned} (F + L) \frac{\partial \delta}{\partial z} + \left(LV^2 + C \frac{\partial^2}{\partial z^2} \right) w &= \frac{1}{R} \left\{ z \frac{\partial}{\partial y} \left[(F + L) \frac{\partial v}{\partial z} + 2L \frac{\partial w}{\partial y} \right] + (L + A - F) \frac{\partial v}{\partial y} \right. \\ &\quad \left. + (A - F - 2N) \frac{\partial u}{\partial x} - C \frac{\partial w}{\partial z} \right\} \dots \dots \dots \dots \dots \quad (10) \end{aligned}$$

Among the terms in $1/R^2$ neglected in the last equation is a term $(A - F)w/R^2$ which is worthy of note. It arises since e_{yy} contains w/R and is contained in $(Z_x - Y_y)/R$ in equation (5). When the buckling wavelength is of the same order as R , which it might be for a semicircular panel, $V^2 w$ is of the same order as w/R^2 . This would appear to limit the angle of the sector forming the panel for an accurate solution, but however these equations lead to solutions in section 8 which are identical to those obtained by other means.

(c) *Equilibrium of the Faces.*—When considering the equilibrium of the faces it is necessary to make allowance for the fact that the stress resultants in the face and the reaction of the filling are not in the same plane.

Let unprimed quantities refer to values at an interface, $z = \pm h$, and primed quantities to values at the central surface of a face $z = \pm (h + \frac{1}{2}t)$ where $2h$ and t are the thickness of the filling and a face.

We have approximately:—

$$\left. \begin{aligned} u' &= u \mp \frac{1}{2}t \frac{\partial w}{\partial x} \\ v' &= v \mp \frac{1}{2}t \frac{\partial w}{\partial y} \\ w' &= w \pm \frac{1}{2}t \frac{\partial w'}{\partial z} \end{aligned} \right\}$$

where the top sign refers to the face $z = +h$ and the bottom to the face $z = -h$.

$$\text{Hence } \left. \begin{aligned} \delta' &= \delta \mp \frac{1}{2}t V^2 w \\ \omega' &= \omega \end{aligned} \right\}$$

and since the normal stress in a face may be considered to be negligible

$$\sigma \frac{\partial u'}{\partial x} + \sigma \frac{\partial v'}{\partial y} + (1 - \sigma) \frac{\partial w'}{\partial z} = 0$$

$$\text{whence } w' = w \mp \frac{\sigma t}{2(1 - \sigma)} \delta'$$

However since the areal dilation is small compared to the transverse displacement the last correction can be ignored.

Equilibrium in the Surface of a Face.—Neglecting terms in t/R and $1/R^2$ we have:—

$$\left. \begin{aligned} N_x &= \frac{Et}{1-\sigma^2} \left\{ \frac{\partial u'}{\partial x} + \sigma \frac{\partial v'}{\partial y} - \frac{\sigma}{R} \left(\pm h \frac{\partial v'}{\partial y} - w' \right) \right\} \\ N_{xy} &= \frac{Et}{2(1-\sigma)} \left\{ \frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} \mp \frac{h}{R} \frac{\partial u'}{\partial y} \right\} \\ N_y &= \frac{Et}{1-\sigma^2} \left\{ \sigma \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} - \frac{1}{R} \left(\pm h \frac{\partial v'}{\partial y} - w' \right) \right\} \end{aligned} \right\} \dots \dots \dots (11)$$

where N_x , N_{xy} , N_y are the stress resultants in the face and the top and bottom signs refer to face $z = +h$ and $-h$ respectively.

In considering the equilibrium of the stress resultants in the x and y direction account must be taken of the shear forces acting at the interfaces:—

$$\begin{aligned} \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{h}{R} \frac{\partial N_{xy}}{\partial y} &= \pm Z_x = \pm L \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)_{z=\pm h} \\ \text{and } \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} - \frac{h}{R} \frac{\partial N_y}{\partial y} &= \pm Z_y + \frac{Q_y}{R} = \frac{Q_y}{R} \pm L \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \mp \frac{L}{R} \left(h \frac{\partial w}{\partial y} \mp v \right)_{z=\pm h} \end{aligned} \quad (12)$$

where Q_y is the y -stress resultant normal to the face.

Substituting from equations (11) into equations (12) yields:—

$$\begin{aligned} \frac{\partial^2 u'}{\partial x^2} + \frac{1-\sigma}{2} \frac{\partial^2 u'}{\partial y^2} + \frac{1+\sigma}{2} \frac{\partial^2 v'}{\partial x \partial y} &= \pm K \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \\ &\pm \frac{1}{R} \left\{ \frac{1+\sigma}{2} h \frac{\partial^2 v'}{\partial x \partial y} \mp \sigma \frac{\partial w'}{\partial x} + (1-\sigma) h \frac{\partial^2 u'}{\partial y^2} \right\} \end{aligned} \quad (13)$$

$$\begin{aligned} \text{and } \frac{1+\sigma}{2} \frac{\partial^2 u'}{\partial x \partial y} + \frac{1-\sigma}{2} \frac{\partial^2 v'}{\partial x^2} + \frac{\partial^2 v'}{\partial y^2} &= \pm K \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) - \frac{K}{R} \left(h \frac{\partial w}{\partial y} \pm v \right) \\ &\pm \frac{1}{R} \left\{ \frac{1+\sigma}{2} h \frac{\partial^2 u'}{\partial x \partial y} \mp \frac{\partial w'}{\partial y} + 2h \frac{\partial^2 v'}{\partial y^2} \pm \frac{K}{L} Q_y \right\} \end{aligned} \quad (14)$$

where $K = (1-\sigma^2)L/Et$.

Differentiating equations (13) and (14) with respect to x and y respectively and adding, and eliminating the primed quantities, we obtain:—

$$\begin{aligned} (KV^2 + \frac{1}{2}tV^4)w - \left(\pm V^2 - K \frac{\partial}{\partial z} \right) \delta &= \frac{K}{R} \left(\pm h \frac{\partial^2 w}{\partial y^2} + \frac{\partial v}{\partial y} \right) \\ &- \frac{1}{R} \left\{ 2h \frac{\partial^2 \delta}{\partial y^2} - \frac{1+\sigma}{2} h \frac{\partial^2 w}{\partial x \partial y} \right. \\ &\left. \mp \sigma \frac{\partial^2 w}{\partial x^2} \mp \frac{\partial^2 w}{\partial y^2} \pm \frac{K}{L} \frac{\partial Q_y}{\partial y} \right\} \dots \dots \dots (15) \end{aligned}$$

Differentiating equations (13) and (14) with respect to y and x and subtracting, we obtain:—

$$\begin{aligned} \frac{1-\sigma}{2} V^2 w \mp K \frac{\partial w}{\partial z} &= \pm \frac{1}{R} \left\{ (1-\sigma) h \frac{\partial^2 w}{\partial y^2} - \frac{1+\sigma}{2} h \frac{\partial^2 \delta}{\partial x \partial y} \right. \\ &\left. \pm (1-\sigma) \frac{\partial^2 w}{\partial x \partial y} \mp \frac{K}{L} \frac{\partial Q_y}{\partial x} \right\} \dots \dots \dots (16) \end{aligned}$$

Equilibrium Normal to a Face.—With the same approximations the stress couples, M_x , M_{xy} , M_y , are given by:—

$$\left. \begin{aligned} M_x &= -D \left\{ \frac{\partial^2 w}{\partial x^2} + \sigma \frac{\partial^2 w}{\partial y^2} \mp \frac{\sigma}{R} \left(2h \frac{\partial^2 w}{\partial y^2} \pm \frac{\partial v}{\partial y} \right) \right\} \\ M_{xy} &= -(1 - \sigma) D \left\{ \frac{\partial^2 w}{\partial x \partial y} \mp \frac{1}{R} \left(h \frac{\partial^2 w}{\partial x \partial y} \pm \frac{\partial v}{\partial x} \right) \right\} \\ M_y &= -D \left\{ \sigma \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \mp \frac{1}{R} \left(2h \frac{\partial^2 w}{\partial y^2} \pm \frac{\partial v}{\partial y} \right) \right\} \end{aligned} \right\} \dots \dots (17)$$

where $D = Et^3/12(1 - \sigma^2)$.

In considering the equilibrium of the shears and couples account must be taken of the distributed moment of the shear forces at the interfaces about the centre of the faces:—

$$\left. \begin{aligned} Q_x &= \frac{\partial M_x}{\partial x} + \left(1 \mp \frac{h}{R} \right) \frac{\partial M_{xy}}{\partial y} + \frac{1}{2} t Z_x \\ Q_y &= \frac{\partial M_{xy}}{\partial x} + \left(1 \mp \frac{h}{R} \right) \frac{\partial M_y}{\partial y} + \frac{1}{2} t Z_y \end{aligned} \right\} \dots \dots \dots (18)$$

Similarly in considering the equilibriums of the shears and normal forces account must be taken of the tension the filling exerts on the faces:—

$$\frac{\partial Q_x}{\partial x} + \left(1 \mp \frac{h}{R} \right) \frac{\partial Q_y}{\partial y} = ft \frac{\partial^2 w}{\partial x^2} + \frac{N_y}{R} \pm Z_x$$

where f is the stress in the faces resulting from the applied load.

On substituting from equations (17) and (18) into the last equation we obtain:—

$$\begin{aligned} ft \frac{\partial^2 w}{\partial x^2} + D \nabla^4 w - \frac{1}{2} t L \left(\nabla^2 w + \frac{\partial \delta}{\partial z} \right) \pm C \frac{\partial w}{\partial z} \pm F \delta \\ = \frac{D}{R} \left\{ \pm 4h \nabla^2 \frac{\partial^2 w}{\partial y^2} + (2 - \sigma) \frac{\partial^3 v}{\partial x^2 \partial y} + \frac{\partial^3 v}{\partial y^3} \right\} \\ \mp \frac{1}{2} \frac{tL}{R} \left\{ 2h \frac{\partial^2 w}{\partial y^2} + h \frac{\partial^2 v}{\partial y \partial z} \pm \frac{\partial v}{\partial y} \right\} + \frac{F}{R} \left\{ h \frac{\partial v}{\partial y} \mp w \right\} \\ - \frac{Et}{(1 - \sigma^2)R} \left\{ \sigma \frac{\partial w}{\partial x} + \frac{\partial v}{\partial y} + \frac{w}{R} \right\} \dots \dots \dots (19) \end{aligned}$$

It will be noted that the last term in the last bracket of this equation is of order $1/R^2$. This term is retained because it is not a derivative and because E is so much greater than the other elastic constants; other possible terms that might be retained are either derivatives which are necessarily smaller for long-wave buckling, or contain elastic constants of the filling.

5. *Simply Supported Flat Panel in Uniform Compression.*—The conditions of equilibrium are given by putting $1/R = 0$ in equations (8), (9), (10), (15), (16) and (19) and they are:—

$$\left. \begin{aligned} (A \nabla^2 + L \frac{\partial^2}{\partial z^2}) \delta + (F + L) \nabla^2 \frac{\partial w}{\partial z} = 0 \\ (F + L) \frac{\partial \delta}{\partial z} + (L \nabla^2 + C \frac{\partial^2}{\partial z^2}) w = 0 \end{aligned} \right\} \dots \dots (20)$$

$$\left. \begin{aligned} (NV^2 + L \frac{\partial^2}{\partial z^2})\omega &= 0 \\ \left(\frac{1-\sigma}{2} V^2 \mp K \frac{\partial}{\partial z}\right)\omega &= 0 \end{aligned} \right\} \text{at } z = \pm h \dots \quad (21)$$

$$\left. \begin{aligned} (KV^2 + \frac{1}{2}tV^4)w - \left(\pm V^2 - K \frac{\partial}{\partial z}\right)\delta &= 0 \\ ft \frac{\partial^2 \omega}{\partial x^2} + DV^4 w - \frac{1}{2}tL \left(V^2 w + \frac{\partial \delta}{\partial z}\right) \pm C \frac{\partial w}{\partial z} \pm F\delta &= 0 \end{aligned} \right\} \text{at } z = \pm h \dots \quad (22)$$

If we assume w to be of the form $\Sigma W_{mn}(z) \sin(n\pi x/a) \sin(m\pi y/b)$ (n, m integers), δ will be a similar series while ω will be a double cosine series.

Since the equations containing ω do not contain w or δ , the coefficients in the series for ω will be independent of those for w and δ . In each coefficient of ω there will be two constants of integration which must be chosen to satisfy the second equation of (21) at $z = +h$ and $-h$. It is clear that the only possible solution is $\omega = 0$.

Each pair of corresponding terms in the two series for w and δ must satisfy equations (20) and (22) separately. In solving equations (20) each pair of coefficients will contain four constants of integration, which must be chosen to satisfy the four equations of (22). The latter requirement will determine the value of f , but f will now be a function of m and n , so that the equations cannot be satisfied for all m, n at the same time. Hence w will consist of one term only, instead of a series, corresponding to one pair, m and n , which must be chosen to make f least.

For a panel with edges at $x = 0, a$ and $y = 0, b$ we will have $w = 0$ at the edges. The bending moments of the plate as a whole will also be zero at the edges since in addition to $\frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial y^2} = 0$ we have $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} = 0$, because $\delta = 0$ at the edges and $\omega = 0$ everywhere, making the direct stresses as well as the bending moments in the faces vanish at the boundaries. Hence the conditions of simple support are satisfied apart from small displacements in the plane of the panel.

Isotropic Filling.—For an isotropic filling $A = C, N = L, F = C - 2L$.

The solution of equations (20) is:—

$$\begin{aligned} w &= (W_1 c + W_2 lzs + W_1' s + W_2' lzc) \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b} \\ \delta &= -l \{W_1 s + W_2 (lzc + vs) + W_1' c + W_2' (lzs + vc)\} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b} \end{aligned}$$

where $c = \cosh lz, \quad s = \sinh lz$

$$l^2 = \frac{n^2 \pi^2}{a^2} + \frac{m^2 \pi^2}{b^2}$$

$$v = (C + L)/(C - L)$$

$W_1, W_2, W_1', W_2' =$ constants of integration.

Now w and δ are the sums of even and odd functions of z : to the even part of w corresponds the odd part of δ and *vice versa*. It will be seen from inspection of equations (22), taking due account of the alternative signs, that the odd and even parts of w will form independent solutions.

Let us first consider the even part of w (i.e., $W_1' = W_2' = 0$) corresponding to the buckling of the panel as a whole or 'in-phase buckling'.

On substituting for w and δ in equations (22), we have:—

$$\frac{W_1}{W_2} = - \frac{\{K(1 + \nu) + l^2h\}c + \{2Kh + \nu - \frac{1}{2}thl^2\}ls}{(2K - \frac{1}{2}tl^2)c + ls}$$

$$\begin{aligned} \text{and } \frac{n^2\pi^2}{a^2} ft &= Dl^4 + \frac{L\{tlc + 2s\}lW_1 + \{L(2h + \frac{1+\nu}{2}t)lc + (C - \nu F + thl^2L)s\}lW_2}{W_1c + W_2lhs} \\ &= Dl^4 + \frac{lL \left\{ \begin{aligned} &[\{2(C - L)th + \frac{1}{2}Ct^2\}l^2 - 4Kh(C - L)]lc^2 \\ &+ 2[2K(C - L) + tl^2L]cs \\ &+ 2[2Kh(C - L) + C - (C - L)thl^2]ls^2 \end{aligned} \right\}}{[2KC + (C - L)l^2h]c^2 + (C + L)lcs - (C - L)hl^2s^2} \quad \dots \quad (23) \end{aligned}$$

In order to minimise the expression for f it is necessary to make an approximation. Consider first long-wave buckling in which lh is small, say < 0.7 , so that approximately $\tanh lh = lh - \frac{1}{3}l^3h^3$. Neglecting higher powers of lh the equation reduces to:—

$$\frac{n^2\pi^2}{a^2} ft = l^4 \left\{ \frac{(h + \frac{1}{2}t)^2 + \frac{1}{3}h^3 \left(4K \frac{C - L}{C} - \frac{L}{C} tl^2 \right)}{K + l^2h - \frac{1}{3}l^4h^3 \left(2 - \frac{L}{C} \right)} L + D \right\}.$$

In general the second bracket of the numerator of the fraction on the right-hand side is small, so the equation reduces approximately to:—

$$\frac{f}{L} = \frac{(\mu + 1)^2}{2\mu} \frac{(\bar{n}^2 + m^2)^2}{\bar{n}^2} \left\{ \frac{1}{\eta + \bar{n}^2 + m^2 - \xi(\bar{n}^2 + m^2)^2} + \frac{1}{3\eta(\mu + 1)^2} \right\} \quad \dots \quad (24)$$

$$\begin{aligned} \text{where } \mu &= 2h/t & \eta &= \frac{Kb^2}{\pi^2 h} = \frac{(1 - \sigma^2)Lb^2}{\pi^2 Eht} \\ \xi &= \frac{2C - L}{3C} \frac{\pi^2 h^2}{b^2} & \bar{n} &= nb/a. \end{aligned}$$

Now $\frac{\partial f}{m \partial m} - \frac{\partial f}{\bar{n} \partial \bar{n}} = \frac{2}{\bar{n}^2} f > 0$, so that, if there exist m and n for which $\partial f / \partial n$ vanishes, the least f occurs at the least possible value of m , which in this case is unity.

The minimum value f_c/L of this expression is plotted against μ for a range of η of 0.2 to 20 for $6\xi\eta/\mu = (1 - \sigma^2)(2C - L)L/EC$ equal to 0.001 at Fig. 2 and equal to 0.0004 at Fig. 3. In computing these curves it is assumed that the panel is long so that $\bar{n} = nb/a$ can take up its optimum value.

For $\eta > 1$ a further approximation may be made:—

$$\frac{f}{L} = \frac{(\mu + 1)^2}{2\mu} \frac{(\bar{n}^2 + 1)^2}{\bar{n}^2(\bar{n}^2 + 1 + \eta)} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (25)$$

This gives for a long panel, $\bar{n} = \left(\frac{\eta + 1}{\eta - 1} \right)^{1/2}$ and:—

$$\frac{f_c}{L} = \frac{2(\mu + 1)^2}{\mu} \frac{\eta}{(\eta + 1)^2} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (26)$$

This expression requires a correction, when the panel is not long, to allow for \bar{n} being restricted by virtue of n being an integer. The correction is plotted at Fig. 10.

The last approximation is tantamount to substituting lh for $\tanh lh$ so that a reasonable condition for validity is $\frac{\eta}{\eta - 1} < \frac{b^2}{200h^2}$.

When b is large compared to a , $n = 1$ so equation (25) becomes:—

$$f_c = \frac{(\mu + 1)^2 L}{2\mu} \frac{[(b/a)^2 + 1]^2}{(b/a)^2 [(1 + \eta) + (b/a)^2]}$$

$$\simeq \frac{(\mu + 1)^2 L b^2}{2\mu a^2 \eta} \left/ \left(1 + \frac{b^2}{a^2} \eta \right) \right.$$

so $f_c = \frac{\pi^2 (h + \frac{1}{2}t)^2 E}{a^2} \left/ \left(1 + \frac{\pi^2 E h t}{(1 - \sigma^2) L a^2} \right) \right. \dots \dots \dots (27)$

which is a modified Euler strut formula.

Now consider the opposite approximation in which lh is large, say > 2 , so that $\tanh lh \simeq 1$ corresponding to short-wave buckling, or wrinkling of the faces. This rather drastic approximation implies that the two faces fail independently.

With $\tanh lh = 1$ equation (23) yields:—

$$\frac{n^2 \pi^2}{a^2} ft = D l^4 + \frac{L \{ 4K(C - L) + 2Cl + 2tLl^2 + \frac{1}{2} C t^2 l^3 \}}{2KC + (C + L)l}$$

As before $m = 1$; also $l^2 = \frac{\pi^2}{b^2} + \frac{n^2 \pi^2}{a^2} > \frac{4}{h^2}$ by our assumption, but $4/h^2$ is in general large compared to π^2/b^2 so that $n\pi/a$ can be replaced by l .

In addition $K/l > hl/2tE > 0.01$ say, so that terms in K can be neglected. The ratio of the fourth term of the numerator of the fraction to the third, which in itself is small, is $\frac{1}{4} \left(\frac{L}{E} \right)^{1/3}$ approximately and may also be neglected. Therefore:—

$$ft = D l^2 + \frac{2CL}{(C + L)l} + \frac{2tL^2}{C + L} \dots \dots \dots (28)$$

Hence:—

$$\frac{f_c}{L} = \left\{ \frac{9EC^2}{4(1 - \sigma^2)L(C + L)^2} \right\}^{1/3} + \frac{2L}{C + L}$$

for which $\frac{n}{a} = \frac{2}{\pi t} \left\{ \frac{3(1 - \sigma^2)CL}{2E(C + L)} \right\}^{1/3}$.

This makes $lh \simeq \frac{2h}{t} \left(\frac{L}{E} \right)^{1/3}$ which may well be as low as 1, in which case the approximation made would not be acceptable. From an inspection of equation (23) it would appear that a better approximation in place of equation (28) might be:—

$$ft = D l^2 + \frac{2CL}{C + L} \frac{\tanh lh}{l} + \frac{2tL^2}{C + L} \dots \dots \dots (29)$$

Using the previous value of l we have:—

$$\frac{f_c}{L} = \left\{ \frac{EC^2}{12(1 - \sigma^2)L(C + L)^2} \right\}^{1/3} \left\{ 1 + 2 \tanh \left[\left(\frac{2h}{t} \frac{3(1 - \sigma^2)CL}{2E(C + L)} \right)^{1/3} \right] \right\} + 2L/(C + L) \quad (30)$$

This second approximation will be inaccurate if $lh < 3/2$, so $\mu^3 CL/E(C + L) > 2$ is an approximate condition of validity.

It should be noted that $\eta = 1$ is not the threshold between long-wave buckling and wrinkling, but only the boundary of validity of expression (26). Though indeed the wavelength shortens as η tends to zero, the expression $2b \left(\frac{\eta - 1}{\eta + 1} \right)^{1/2}$ for the wavelength no longer holds as η approaches one.

We will now show that out-of-phase buckling modes do not occur; that is modes with w as an odd function of z . By repeating the same process as for in-phase buckling an equation like (23) will be derived except that $\cosh lh$ and $\sinh lh$ will be interchanged.

Considering long-wave buckling first, put $\tanh lh = lh - \frac{1}{3}l^2h^2$ as before, giving:—

$$\frac{n^2\pi^2}{a^2} ft = Dl^4 + L \frac{C + 4Kh(C - L) - hl^2[(C - L)t + \frac{8}{3}Kh^2(C - L)]}{h[L + KhC + \frac{1}{3}l^2h^2(C - 2L - KhC)]} \quad (31)$$

As a first approximation write:—

$$\frac{n^2\pi^2}{a^2} ft = Dl^4 + C/h$$

so that for a minimum $(\pi n/a)^4 = (\pi/b)^4 + C/Dh$ whence $(hl)^4 > (\pi nh/a)^4 > Ch^3/D = 12(1 - \sigma^2) (h/t)^3 C/E$ which in general gives a greater value of lh than the approximation will stand. Now the full expression (31) will yield an even greater value of lh , since its second term on the right-hand side is a decreasing function of l instead of a constant as first assumed. It is therefore concluded that long-wave out-of-phase buckling does not arise.

With regard to short-wave out-of-phase buckling we will obtain the same expression as for in-phase buckling on putting $\tanh lh = 1$ for the first approximation. However for the second approximation the result of interchanging $\cosh lh$ and $\sinh lh$ will be the counterpart to equation (29) with $\coth lh$ for $\tanh lh$, clearly leading to a greater critical stress.

Aeolotropic Filling.—The solution to the equilibrium equations (20) for the filling are:—

$$\left. \begin{aligned} w &= \{W_1c_1 + W_2c_2 + W_1's_1 + W_2's_2\} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b} \\ \delta &= \{A_1s_1 + A_2s_2 + A_1's_1 + A_2's_2\} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b} \end{aligned} \right\}$$

where $c_1 = \cosh k_1 lz$, $s_1 = \sinh k_1 lz$ etc.

$$l^2 = \frac{n^2\pi^2}{a^2} + \frac{m^2\pi^2}{b^2}$$

$$k_1, k_2 \text{ are the roots of } LCk^4 - (AC + 2LF - F^2)k^2 + AL = 0$$

$$W_1/A_1 = W_1'/A_1' = (F + L)k_1/(L - k_1^2C) \text{ etc.}$$

Proceeding as for isotropic filling and considering only the even part of w , we have on substituting for w and δ into equations (22):—

$$W_1 c_1 (K - \frac{1}{2} t l^2) - \Delta_1 (l s_1 + K k_1 c_1) = -W_1 c_2 (K - \frac{1}{2} t l^2) + \Delta_2 (l s_2 + K k_2 c_2)$$

$$\text{and } \frac{n^2 \pi^2}{a^2} f t = D l^4 + \frac{\left\{ l [(C k_1 W_1 + F \Delta_1) s_1 + (C k_2 W_2 + F \Delta_2) s_2] \right\} + \frac{1}{2} t L l^2 [(W_1 - k_1 \Delta_1) c_1 + (W_2 - k_2 \Delta_2) c_2]}{W_1 c_1 + W_2 c_2}$$

whence:—

$$\frac{W_1}{W_2} = -\frac{k_1}{k_2} \frac{[K F - \frac{1}{2} t l^2 (F + L) + K k_2^2 C] k_2 c_2 + (k_2^2 C - L) l s_2}{[K F - \frac{1}{2} t l^2 (F + L) + K k_1^2 C] k_1 c_1 + (k_1^2 C - L) l s_1}$$

$$\text{and } \frac{n^2 \pi^2}{a^2} f t = D l^4 + \frac{l L \left\{ [C(k_1 W_1 s_1 + k_2 W_2 s_2) + F(W_1 s_1 / k_1 + W_2 s_2 / k_2)] \right\} + \frac{1}{2} t l [C(k_1^2 W_1 c_1 + k_2^2 W_2 c_2) + F(W_1 c_1 + W_2 c_2)]}{W_1 c_1 + W_2 c_2} \quad (32)$$

As an approximation for long-wave buckling (lh small) put $c_1 = c_2 = 1$, $s_1 = k_1 lh$ and $s_2 = k_2 lh$ so that:—

$$\frac{k_1^2 W_1 + k_2^2 W_2}{W_1 + W_2} = -\frac{K F - \frac{1}{2} t l^2 (F + L) - l^2 h L}{(K + l^2 h) C}$$

$$\begin{aligned} \text{and } \frac{n^2 \pi^2}{a^2} f t &= \frac{(h + \frac{1}{2} t) L l^2}{F + L} \left\{ \frac{k_1^2 W_1 + k_2^2 W_2}{W_1 + W_2} C + F \right\} \\ &= \frac{(h + \frac{1}{2} t)^2 l^4 L}{K + l^2 h} \end{aligned}$$

i.e., $\frac{f}{L} = \frac{(\mu + 1)^2}{2\mu} \cdot \frac{(\bar{n}^2 + m^2)^2}{\bar{n}^2(\bar{n}^2 + m^2 + \eta)}$, which is the same expression as derived for an isotropic filling (equation 25) by a comparable approximation.

For short-wave buckling (lh large) put $c_1 = c_2 = s_1 = s_2$ so that equations (32) become:—

$$\frac{W_1}{W_2} = -\frac{k_1}{k_2} \frac{[K F - \frac{1}{2} t l^2 (F + L) + K k_2^2 C] k_2 + (k_2^2 C - L) l}{[K F - \frac{1}{2} t l^2 (F + L) + K k_1^2 C] k_1 + (k_1^2 C - L) l}$$

$$\begin{aligned} \text{and } \frac{n^2 \pi^2}{a^2} f t &= D l^4 + \frac{l L \left\{ C(k_1 W_1 + k_2 W_2) + F \left(\frac{W_1}{k_1} + \frac{W_2}{k_2} \right) \right\} + \frac{1}{2} t l (k_1^2 W_1 + k_2^2 W_2) + F(W_1 + W_2)}{W_1 + W_2} \\ &= D l^4 + \frac{l L \left\{ K F^2 + F C(k_1^2 + k_2^2) + k_1^2 k_2^2 C + l C(F + L)(k_1 + k_2) \right. \\ &\quad \left. + \frac{1}{2} t l^2 [L C(k_1^2 + k_2^2) - C^2 k_1^2 k_2^2 - 2C(F + L)k_1 k_2 - F^2] \right. \\ &\quad \left. + \frac{1}{2} t l^3 C(F + L)k_1 k_2 (k_1 + k_2) \right\}}{l(C k_1 k_2 + L) + K C k_1 k_2 (k_1 + k_2)} \end{aligned}$$

For similar reasons to those given for short-wave buckling with an isotropic filling, put $n\pi/a = l$ and neglect terms in K and $t^2 l^3$, so that:—

$$f t = D l^2 + \frac{L C (k_1 + k_2)}{l(C k_1 k_2 + L)} + \frac{\frac{1}{2} t L [L C (k_1^2 + k_2^2) + 2(F + L) C k_1 k_2 - k_1^2 k_2^2 C^2 - F^2]}{(F + L)(C k_1 k_2 + L)}$$

which is at least for $n\pi/a = l = \left\{ \frac{LC(k_1 + k_2)}{2D(Ck_1k_2 + L)} \right\}^{1/3}$ so that

$$\frac{f_c}{L} = \left\{ \frac{9EC(\sqrt{AC} - F)(\sqrt{AC} + 2L + F)}{16L^2(1 - \sigma^2)(\sqrt{AC} + L)^2} \right\}^{1/3} + \frac{\sqrt{AC} - F}{\sqrt{AC} + L} \dots \dots \dots (33)$$

on substituting for k_1, k_2 from the quadratic in k^2 .

This formula only holds when k_1lh and k_2lh are large, say greater than 2, so that an approximate condition of validity is:—

$$\frac{\mu^3(\sqrt{AC} - F)^2(\sqrt{AC} + 2L + F)^2}{ELC(\sqrt{AC} + L)} > 100$$

since $(k_1lh)^3$ or $(k_2lh)^3 \simeq \left[\frac{(k_1 + k_2)lh}{2} \right]^3 = \frac{h^3LC(k_1 + k_2)^4}{16D(Ck_1k_2 + L)}$.

Honeycomb Filling.—It is assumed that this type of filling has no stiffness in the plane of the plate, so that $A = N = F = 0$. The solution to the equations of equilibrium of the filling now become:—

$$w = \{W_1 + W_2l^2z^2 + W_2'lz\} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}$$

$$\delta = l\{W_1lz - W_2(\varepsilon lz - \frac{1}{3}l^3z^3) + W_1' + \frac{1}{2}W_2'l^2z^2\} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}$$

where $\varepsilon = 2C/L$.

Proceeding as in the previous two cases for in-phase buckling we obtain on substituting for w and δ in equation (22)

$$\frac{W_1}{W_2} = \frac{\varepsilon(K + Hl^2) - (\frac{1}{3}h + \frac{1}{2}l)h^2l^4}{(h + \frac{1}{2}l)l^2}$$

and
$$\frac{n^2\pi^2}{a^2} ft = Dl^4 + \frac{LW_2l^2\varepsilon(h + \frac{1}{2}l)}{W_1 + W_2l^2h^2}$$

$$= l^4 \left\{ \frac{(h + \frac{1}{2}l)^2}{K + l^2h + \frac{L}{3C}l^4h^3} L + D \right\}$$

whence
$$\frac{f}{L} = \frac{(\mu + 1)^2}{2\mu} \cdot \frac{(\bar{n}^2 + m^2)^2}{\bar{n}^2} \left\{ \frac{1}{\eta + \bar{n}^2 + m^2 + \xi^*(\bar{n}^2 + m^2)^2} + \frac{1}{3\eta(\mu + 1)^2} \right\}$$

where
$$\xi^* = \frac{L}{3C} \frac{\pi^2 h^2}{b^2} \dots \dots \dots (34)$$

This is of the same form as obtained for an isotropic filling (24) and approximates to precisely the same expression (25) obtained for the other two types of material for long-wave buckling when $\eta > 1$.

When \bar{n}^2 is large compared to $(1 + \eta)$, an alternative approximation may be made:—

$$\frac{f}{L} \simeq \frac{(\mu + 1)^2}{2\mu} \left\{ \frac{1}{1 + \xi^*\bar{n}^2} + \frac{\bar{n}^2}{3\eta(\mu + 1)^2} \right\}$$

which on eliminating $\left(\frac{\partial Z_x}{\partial x} + \frac{\partial Z_y}{\partial y}\right)$ gives

$$f \left(\nabla^2 - \frac{\pi^2 \eta}{b^2} \right) \frac{\partial^2 w}{\partial x^2} = \frac{L(\mu + 1)^2}{2\mu} \nabla^4 w. \quad \dots \dots \dots (40)$$

If $w = W \sin(n\pi x/a) \sin(m\pi y/b)$ then from equation (38) it follows that $\partial Z_x/\partial x$ and $\partial Z_y/\partial y$ are also proportioned to $\sin(n\pi x/a) \sin(m\pi y/b)$. Hence by equations (37) $\bar{M}_x = \bar{M}_y = 0$ as well as $w = 0$ at the edges $x = 0, a$ and $y = 0, b$ of the panel, satisfying the boundary conditions for a simply supported panel.

Substituting for w in equation (40) gives

$$\frac{f}{L} = \frac{(\mu + 1)^2}{2\mu} \cdot \frac{(\bar{n}^2 + m^2)^2}{\bar{n}^2(\bar{n}^2 + m^2 + \eta)}$$

that is $\frac{f_c}{L} = \frac{(\mu + 1)^2}{\mu} \cdot \frac{2\eta}{(1 + \eta)^2}$, which result is the same as (26), and demonstrates that the approximation leading to the common formula for the long-wave buckling stress is equivalent to accepting the simple concept of shear deformation enunciated earlier in this section.

7. *Long, Flat Panel with Clamped Edges under Uniform Compression.*—We will define clamping of the edges by the boundary condition $w = \partial w/\partial y = 0$ holding throughout the thickness of the plate. This implies that the filling as well as the faces is fully constrained, a condition not likely to be attained in practice, but it is of interest to investigate the effect of maximum constraint.

Consider long-wave buckling, for which the approach outlined in the previous section is suitable.

Equation (40) states:— $f \left(\nabla^2 - \frac{\pi^2 \eta}{b^2} \right) \frac{\partial^2 w}{\partial x^2} = \frac{L(\mu + 1)^2}{2\mu} \nabla^4 w.$

Assume that $w = W(y) \sin(\lambda\pi x/b)$, which, when substituted into the above equation, gives:—

$$\frac{d^4 W}{dy^4} - \frac{\lambda^2 \pi^2}{b^2} (2 - \phi) \frac{d^2 W}{dy^2} + \frac{\lambda^4 \pi^4}{b^4} \left[1 - \phi \left(1 + \frac{\eta}{\lambda^2} \right) \right] = 0 \quad \dots \dots (41)$$

where $\phi = \frac{2\mu f}{(\mu + 1)^2 L}.$

The failing stress will be least for the simplest wave-form across the panel, so that W must be an even function of y , if the edges are taken to be at $y = \pm b/2$. Therefore put

$$W = A \cosh \frac{2\alpha y}{b} + B \cos \frac{2\beta y}{b}.$$

The boundary conditions $w = \partial w/\partial y = 0$ at $y = \pm b/2$ reduce to :—

$$\left. \begin{aligned} A \cosh \alpha + B \cos \beta &= 0 \\ \alpha A \sinh \alpha - \beta B \sin \beta &= 0 \end{aligned} \right\}$$

Hence $\alpha \tanh \alpha + \beta \tan \beta = 0.$

Also $\alpha^2 - \beta^2 = \frac{\lambda^2 \pi^2}{4} (2 - \phi)$

and $\alpha^2 \beta^2 = \frac{\lambda^4 \pi^4}{16} \left[\phi \left(1 + \frac{\eta}{\lambda^2} \right) - 1 \right]$

since W satisfies equation (41).

$$\dots \dots \dots (42)$$

These last three equations are solved numerically by finding the least value of η corresponding to a given value of ϕ . It is found that the following expression fits the numerical results within $\frac{1}{2}$ per cent:—

$$\frac{f_c}{L} = \frac{(\mu + 1)^2}{\mu} \cdot \frac{3 \cdot 49 \eta^2}{\eta^3 + 3 \cdot 68 \eta^2 + 3 \cdot 6 \eta - 1 \cdot 3} \quad \dots \quad (43)$$

alternatively $f_c = \frac{\eta^3}{\eta^3 + 3 \cdot 68 \eta^2 + 3 \cdot 6 \eta - 1 \cdot 3} \cdot f_e$

where f_e is the equivalent plate critical stress for the clamped-edge condition.

Equations (42) break down when η passes through unity with the wavelength tending to zero in a similar fashion to the approximation in the simply supported case.

It will be noted that the critical stresses for both cases tend to the common value of $L(\mu + 1)^2/2\mu$ as η tends to 1, presumably due to the lessening influence of boundary conditions with decreasing wavelength.

The correction factors to the equivalent plate stresses for both cases are plotted at Fig. 8 and the buckling half-wavelengths are plotted against η at Fig. 9.

8. *Simply Supported Curved Plates under Uniform Compression.*—In the first instance we shall regard the problem of curved plates under long-wave buckling as an extension of that of flat plates as treated in section 5. We will assume that w and δ can be expanded in powers of h/R , viz.:—

$$w = w_0 + \frac{hw^*}{R} + \frac{h^2w^{**}}{R^2} + \dots$$

$$\delta = \delta_0 + \frac{h\delta^*}{R} + \frac{h^2\delta^{**}}{R^2} + \dots$$

where w_0, w^*, w^{**} may be expected to be of the same order. Clearly w_0, δ_0 will be the displacements of hypothetical flat plate of the same dimensions constrained to buckle with the same wavelengths (*i.e.*, the same m and n) and are therefore known functions.

Substitute these expansions for w and δ into the equilibrium equations (8), (10), (15) and (19) and neglect second order terms. In effect all terms in $1/R^2$ other than the term $Et w_0/(1 - \sigma^2)R^2$ arising in equation (19) are ignored. It should be noted that such terms as Fhw^*/R^2 that arise are small since Fh is small compared to Et , while all derivatives are small.

Now since w_0 and δ_0 are a solution to the equations obtained when the terms in $1/R$ are ignored, the equations of equilibrium of the filling (8) and (10) on the substitution of the expansions, reduce to

$$\left(AV^2 + L \frac{\partial^2}{\partial z^2} \right) \delta^* + (F + L) V^2 \frac{\partial w^*}{\partial z} = F_1(u_0, v_0, w_0) \quad \dots \quad (44)$$

$$(F + L) \frac{\partial \delta^*}{\partial z} + \left(LV^2 + C \frac{\partial^2}{\partial z^2} \right) w^* = F_2(u_0, v_0, w_0) \quad \dots \quad (45)$$

where F_1, F_2 are known functions of z independent of w^* and δ^* .

Put $w^* = w_1 + w_2 + w_3$

$\delta^* = \delta_1 + \delta_2 + \delta_3$

where w_1 and δ_1 form the particular solution of equations (44) and (45) and w_2 and w_3 are the odd and even parts of the complementary solution; δ_2 and δ_3 correspond to w_2 and w_3 and are even and odd functions of z respectively.

First suppose that w_0 is an even function of z and therefore δ_0 an odd function. From equations (8) and (10) it will be seen that F_1 and F_2 are even and odd respectively, so that w_1 and δ_1 are odd and even respectively, the opposite of w_0 and δ_0 .

In a similar fashion equation (15) for the equilibrium of the faces reduces to:—

$$(K\nabla^2 + \frac{1}{2}t\nabla^4)w^* - \left(\pm \nabla^2 - K \frac{\partial}{\partial z}\right)\delta^* = F_3(u_0, v_0, w_0) \quad \dots \quad (46)$$

where F_3 is an odd function of z , and the top and bottom signs refer to the faces $z = \pm h$.

Adding the two forms of this equation corresponding to the two faces together we obtain:—

$$(K\nabla^2 + \frac{1}{2}t\nabla^4)w_3 - \left(\nabla^2 - K \frac{\partial}{\partial z}\right)\delta_3 = 0.$$

Now w_0 and δ_0 are also a solution of this equation and in addition w_0, δ_0 and w_3, δ_3 are both complementary solutions of equations (44) and (45) and both w_0 and w_3 are even functions and δ_0 and δ_3 odd functions. Since both pairs of functions only contain two constants of integration apiece, they must therefore be identical except for a common factor. Hence the addition of hw_3/R to w_0 makes no difference to the solution and we may put $w_3 = \delta_3 = 0$.

Consider the equation of equilibrium (19) for the normal forces acting on the faces. Since w_0, δ_0 are a solution to the left-hand side of this equation taken alone, when $f = f_0$ the failing stress of the hypothetical flat plate, this equation now reduces to

$$\begin{aligned} (f - f_0)t \frac{\partial^2 w_0}{\partial x^2} + \frac{h}{R} \left\{ ft \frac{\partial^2 w^*}{\partial x^2} + D\nabla^4 w^* - \frac{1}{2}tL \left(\nabla^2 w^* + \frac{\partial \delta^*}{\partial z} \right) \pm C \frac{\partial w^*}{\partial z} \pm F\delta^* \right\} \\ = F_4(u_0, v_0, w_0) - \frac{Et w_0}{(1 - \sigma^2)R^2} \quad \dots \quad (47) \end{aligned}$$

where F_4 is an odd function of z .

As we have shown w^* to be an odd function of z and δ^* an even function, on adding the two forms of the last equation corresponding to the faces, we obtain:—

$$(f - f_0) \frac{\partial^2 w_0}{\partial x^2} = - \frac{E w_0}{(1 - \sigma^2)R^2}$$

that is
$$\frac{f}{L} = \frac{(\mu + 1)^2}{2\mu} \left\{ \frac{(\bar{n}^2 + m^2)^2}{\bar{n}^2(\bar{n}^2 + m^2 + \eta)} + \frac{e^2}{\eta \bar{n}^2} \right\} \quad \dots \quad (48)$$

where $e = 2b^2/\pi^2 R(2h + t)$.

For a panel $m = 1$, and \bar{n}^2 has the nearest value to $\frac{\eta + (\eta + 1)e + \eta\{\eta[\eta + (\eta + 1)e^2]\}^{1/2}}{\eta^2 - \eta - e^2}$ provided $\eta > \frac{1}{2}[1 + (1 + 4e^2)^{1/2}]$ for η less than this value f has no minimum, so that the approximation fails. The ratio of this stress to that of the corresponding flat panel is plotted at Fig. 11.

The odd functions w_1 and w_2 are not zero; they are determined by the two equations that are obtained by subtracting the forms of equations (46) and (47) corresponding to opposite faces from one another. Hence for curved plates true in-phase buckling does not occur.

9. *Simply Supported Flat Panel under an Uniform Shear Load.*—(a) *Short Panels.*—For short panels in shear energy methods will be used employing the approximations developed in section 6.

Putting $w = \sum_{mn} W_{mn} \sin(n\pi x/a) \sin(m\pi y/b)$, we have from equations (38)

$$Z_x = \frac{2h+t}{2h} L \Sigma \left\{ \frac{\bar{n}^2 + m^2}{\bar{n}^2 + m^2 + \eta} \cdot \frac{n\pi}{a} W_{mn} \cos \frac{n\pi x}{a} \sin \frac{m\pi y}{b} \right\}$$

$$Z_y = \frac{2h+t}{2h} L \Sigma \left\{ \frac{\bar{n}^2 + m^2}{\bar{n}^2 + m^2 + \eta} \cdot \frac{m\pi}{b} W_{mn} \sin \frac{n\pi x}{a} \cos \frac{m\pi y}{b} \right\}.$$

Clearly the boundary conditions are met for edges at $x = 0, a$ and $y = 0, b$.

From equations (36) therefore:—

$$u = \frac{\pi\eta}{b} (h + \frac{1}{2}t) \Sigma \left\{ \frac{\bar{n}W_{mn}}{\bar{n}^2 + m^2 + \eta} \cos \frac{n\pi x}{a} \sin \frac{m\pi y}{b} \right\}$$

$$v = \frac{\pi\eta}{b} (h + \frac{1}{2}t) \Sigma \left\{ \frac{mW_{mn}}{\bar{n}^2 + m^2 + \eta} \sin \frac{n\pi x}{a} \cos \frac{m\pi y}{b} \right\}.$$

Neglecting the contribution from the bending of the faces, the strain energy of the panel is:—

$$\begin{aligned} & \int_0^a \int_0^b \left\{ \frac{Et}{1-\sigma^2} \left[\left(\frac{\partial u}{\partial x} \right)^2 + 2\sigma \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} + \left(\frac{\partial v}{\partial y} \right)^2 + \frac{1-\sigma}{2} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \right] + \frac{h}{L} [Z_x^2 + Z_y^2] \right\} dx dy \\ &= \frac{ab}{4} \left\{ \frac{Et}{1-\sigma^2} \frac{\pi^4}{b^4} (h + \frac{1}{2}t)^2 \eta^2 \Sigma \frac{(\bar{n}^2 + m^2)^2 W_{mn}^2}{(\bar{n}^2 + m^2 + \eta)^2} + \frac{L}{h} \frac{\pi^2}{b^2} (h + \frac{1}{2}t)^2 \Sigma \frac{(\bar{n}^2 + m^2)^3 W_{mn}^2}{(\bar{n}^2 + m^2 + \eta)^2} \right\} \\ &= \frac{\pi^2}{8} \frac{at}{b} \frac{(\mu + 1)^2}{\mu} L \Sigma \frac{(\bar{n}^2 + m^2)^2 W_{mn}^2}{\bar{n}^2 + m^2 + \eta}. \end{aligned}$$

The work done by the shear force $2qt$ on buckling is:—

$$\begin{aligned} & 2qt \int_0^a \int_0^b \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} dx dy \\ &= 16qt \Sigma_{\substack{(m-p) \& (n-q) \\ \text{both odd}}} \left\{ \frac{nm pq W_{mn} W_{pq}}{(m^2 - p^2)(q^2 - n^2)} \right\}. \end{aligned}$$

Hence

$$\frac{q}{L} = \frac{\pi^2}{128} \frac{a}{b} \frac{(\mu + 1)^2}{\mu} \left\{ \frac{\Sigma \frac{(\bar{n}^2 + m^2)^2 W_{mn}^2}{n^2 + m^2 + \eta}}{\Sigma \Sigma \frac{nm pq W_{mn} W_{pq}}{(m^2 - p^2)(q^2 - n^2)}} \right\}.$$

The critical stress is that value of q which makes the following system of equations compatible:—

$$\sum_{p,q} \frac{mnpq(\bar{n}^2 + m^2 + \eta)W_{pq}}{(m^2 - p^2)(q^2 - \bar{n}^2)(\bar{n}^2 + m^2)^2} = \lambda W_{mn}, \text{ for all } m, n \dots \dots \dots (52)$$

where
$$\lambda = \frac{L \pi^2 a (\mu + 1)^2}{q 64b \mu}$$

If we order the double sequence of terms W_{mn} in some manner so that it can be represented by a single sequence $\Omega_i = W_{mn}$ then the last equation can be written as

$$\sum_{j=1}^{\infty} a_{ij} \Omega_j = \lambda \Omega_i, \quad i = 1 \text{ to } \infty .$$

An approximate answer for λ is obtained by considering a system of equations for the form:—

$$\sum_{j=1}^k a_{ij} \Omega_j = \lambda \Omega_i, \quad i = 1 \text{ to } k .$$

This is tantamount to ignoring all $a_{ij}, i > k$ with $j \leq k$, and assuming that λ and $\Omega_i, i = 1$ to k , found from the last equation together with $\Omega_i = 0, i > k$, is a reasonable approximation. This is unlikely to be the case if any $a_{ij}, i > k, j \leq k$ are not small though the condition for them to be small is sufficient rather than necessary. When $p = m + 1, q = n + 1$ the coefficients of equation (52), on dividing through by η , are approximately $\frac{mn}{4} \cdot \frac{1 + [(\bar{n}^2 + m^2)/\eta]}{(\bar{n}^2 + m^2)^2}$ for m and n not small. As m and n increase together this quantity will not be small when $(\bar{n}^2 + m^2)$ is of the order of η . Therefore an accurate solution cannot be expected unless η is reasonably large: the minimum value of η will depend on the ratio a/b , for as this departs from one more equations have to be considered since \bar{n} moves more slowly as n increases. For $a = b$ it is suggested that η should be greater than 10.

The equations (52) split into two systems, for since $(m - p)$ and $(q - n)$ are both odd numbers, $(p + q)$ is odd or even whenever $(m + n)$ is odd or even, so that terms W_{ij} with $(i + j)$ both even and odd do not occur in the same equations. The system with $(m + n)$ even may be expected to give the least value of q_c , since it contains terms corresponding to the longest wavelength.

If terms involving suffices m and n greater than three are ignored, the characteristic determinant of (52) reduces to:—

$$\begin{vmatrix} \frac{\lambda}{C_{11}} & \frac{4}{9} & 0 & 0 & 0 \\ \frac{4}{9} & \frac{\lambda}{C_{22}} & \frac{4}{5} & -\frac{4}{5} & \frac{36}{25} \\ 0 & \frac{4}{5} & \frac{\lambda}{C_{13}} & 0 & 0 \\ 0 & -\frac{4}{5} & 0 & \frac{\lambda}{C_{31}} & 0 \\ 0 & \frac{36}{25} & 0 & 0 & \frac{\lambda}{C_{33}} \end{vmatrix} = 0$$

hence
$$\frac{q}{L} = \frac{(\mu + 1)^2}{8\mu mn} \left[\frac{\{n^2 + (1 + m)^2\}^2}{n^2 + (1 + m)^2 + \eta} + \frac{\{n^2 + (1 - m)^2\}^2}{n^2 + (1 - m)^2 + \eta} \right].$$

To find m, n to make q least, put $n = 2/r$ and $m = s/r$ and differentiate giving:—

$$\begin{aligned} & \frac{2(r + s)\{1 + (r + s)^2\}}{1 + (r + s)^2 + \eta r^2} - \frac{\{1 + (r + s)^2\}^2 \{(r + s) + \eta r\}}{\{1 + (r + s)^2 + \eta r^2\}^2} \\ & + \frac{2(r - s)\{1 + (r - s)^2\}}{1 + (r - s)^2 + \eta r^2} - \frac{\{1 + (r - s)^2\}^2 \{(r - s) + \eta r\}}{\{1 + (r - s)^2 + \eta r^2\}^2} = 0 \end{aligned}$$

and
$$\begin{aligned} & \frac{4s(r + s)\{1 + (r + s)^2\}}{1 + (r + s)^2 + \eta r^2} - \frac{2s(r + s)\{1 + (r + s)^2\}s^2}{\{1 + (r + s)^2 + \eta r^2\}^2} - \frac{\{1 + (r + s)^2\}^2}{1 + (r + s)^2 + \eta r^2} \\ & - \frac{4s(r - s)\{1 + (r - s)^2\}}{1 + (r - s)^2 + \eta r^2} + \frac{2s(r - s)\{1 + (r - s)^2\}s^2}{\{1 + (r - s)^2 + \eta r^2\}^2} - \frac{\{1 + (r - s)^2\}^2}{1 + (r - s)^2 + \eta r^2} = 0 \end{aligned}$$

Approximate by expanding the latter pair of expressions in terms of $1/\eta$ and neglecting second and higher order terms:—

$$\begin{aligned} (r^2 - s^2 - 1)(r^2 + s^2 + 1) &= \frac{1}{\eta r^2} \left\{ r^6 - 3r^2(1 + 6s^2 + 5s^4) - 2(1 + s^2)^3 \right\} \\ \text{and } s^2(6r^2 + 3s^2 + 2) - (1 + r^2)^2 & \\ &= \frac{1}{\eta r^2} \left\{ s^2(3 + 18r^2 + 9s^2 + 15r^4 + 45r^2s^2 + 5s^4) - (1 + r^2)^3 \right\}. \end{aligned}$$

For η very large we find $r = (3/2)^{1/2}$ and $s = (1/2)^{1/2}$ and as a second approximation we obtain:—

$$\begin{aligned} r &= \sqrt{\frac{3}{2}} \left(1 - \frac{5}{3\eta} \right) \\ s &= \sqrt{\frac{1}{2}} \left(1 + \frac{1}{\eta} \right). \end{aligned}$$

To give the correct stress for very large η the constant in the above equation should be 2.69 instead of 2.83: this discrepancy is the same as that found for the homogeneous panel in Ref. 3 and is due to the initial assumptions. It is likely that the other numerical constants in equation (62) will have the same order error.

As η decreases the buckling wavelength shortens as in the other cases; also the angle the trough in the wave form makes with the sides sharpens slightly.

At Fig. 13 are plotted the correction factors to the equivalent plate stresses for the three panels for which formulae have been derived.

If a parabolic interpolation of the form $K_1 - (K_1 - K_2)(b/a)^2$ is made between the curves in Fig. 13 for the square and the long panel, a good fit is obtained for the curve for $a : b = 5 : 4$, which suggests that such interpolations may give reasonable results for all values of $a : b$.

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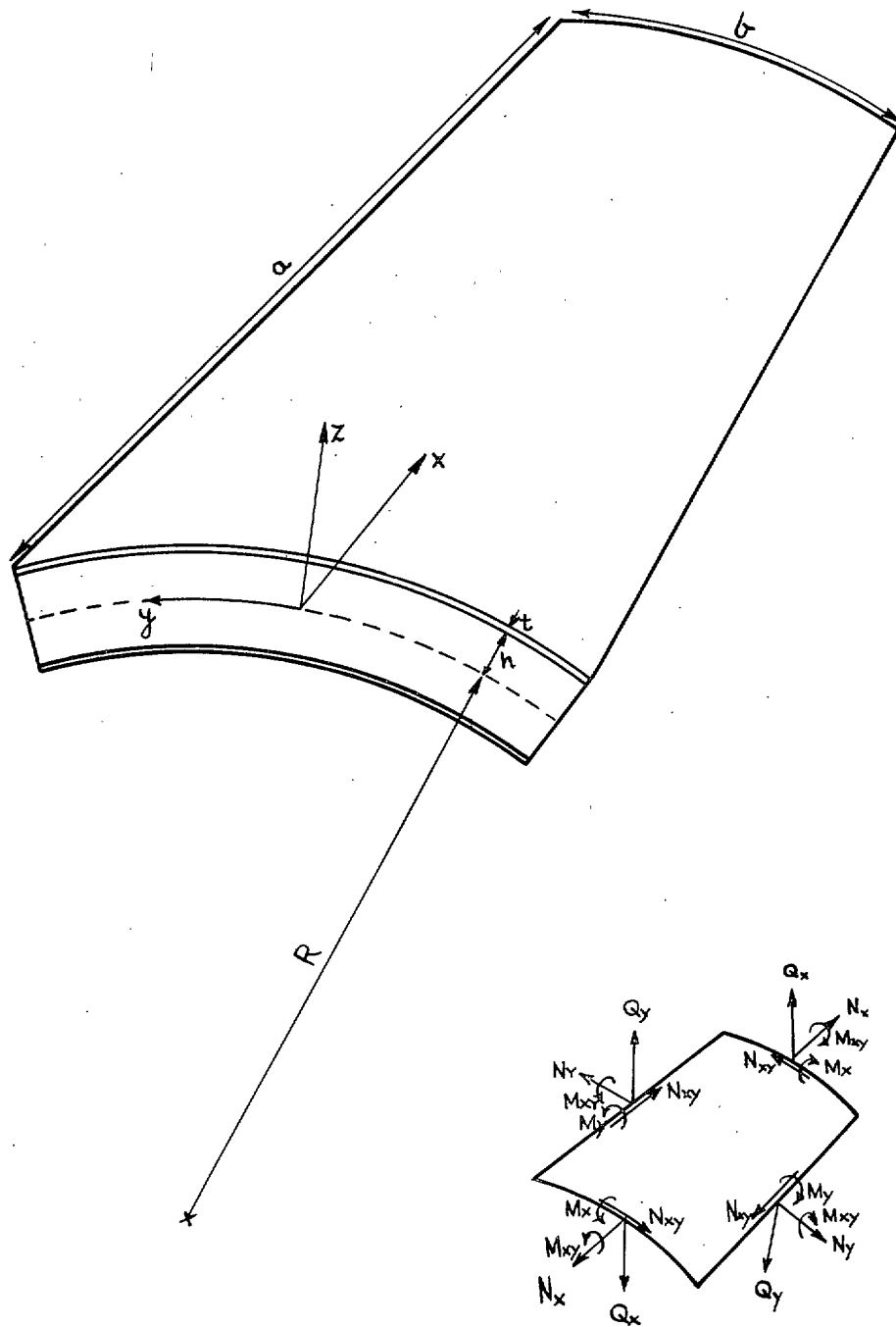


FIG. 1. Co-ordinate system.

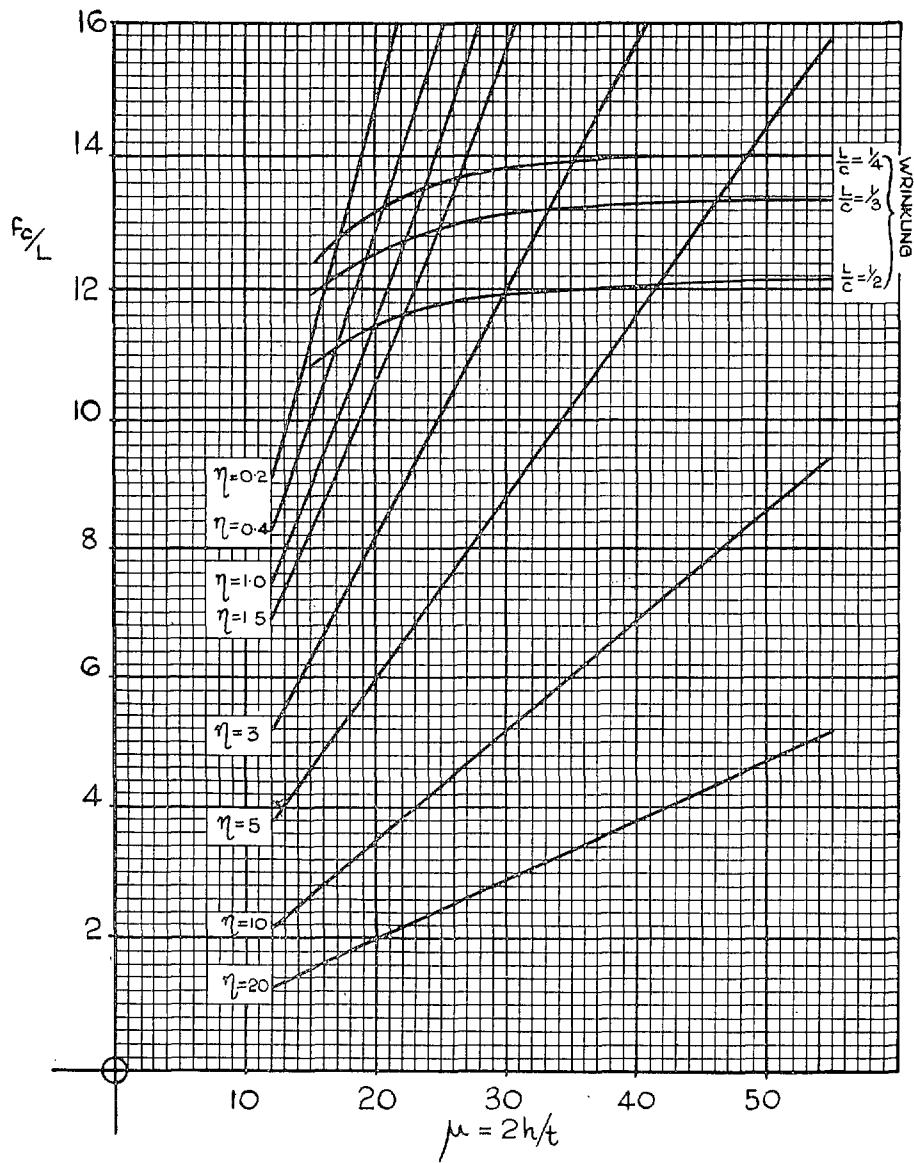


FIG. 2. Critical compressive stress for simply supported flat panel with isotropic filling.

$$\frac{L(2C - L)(1 - \sigma^2)}{EC} = 0.001$$

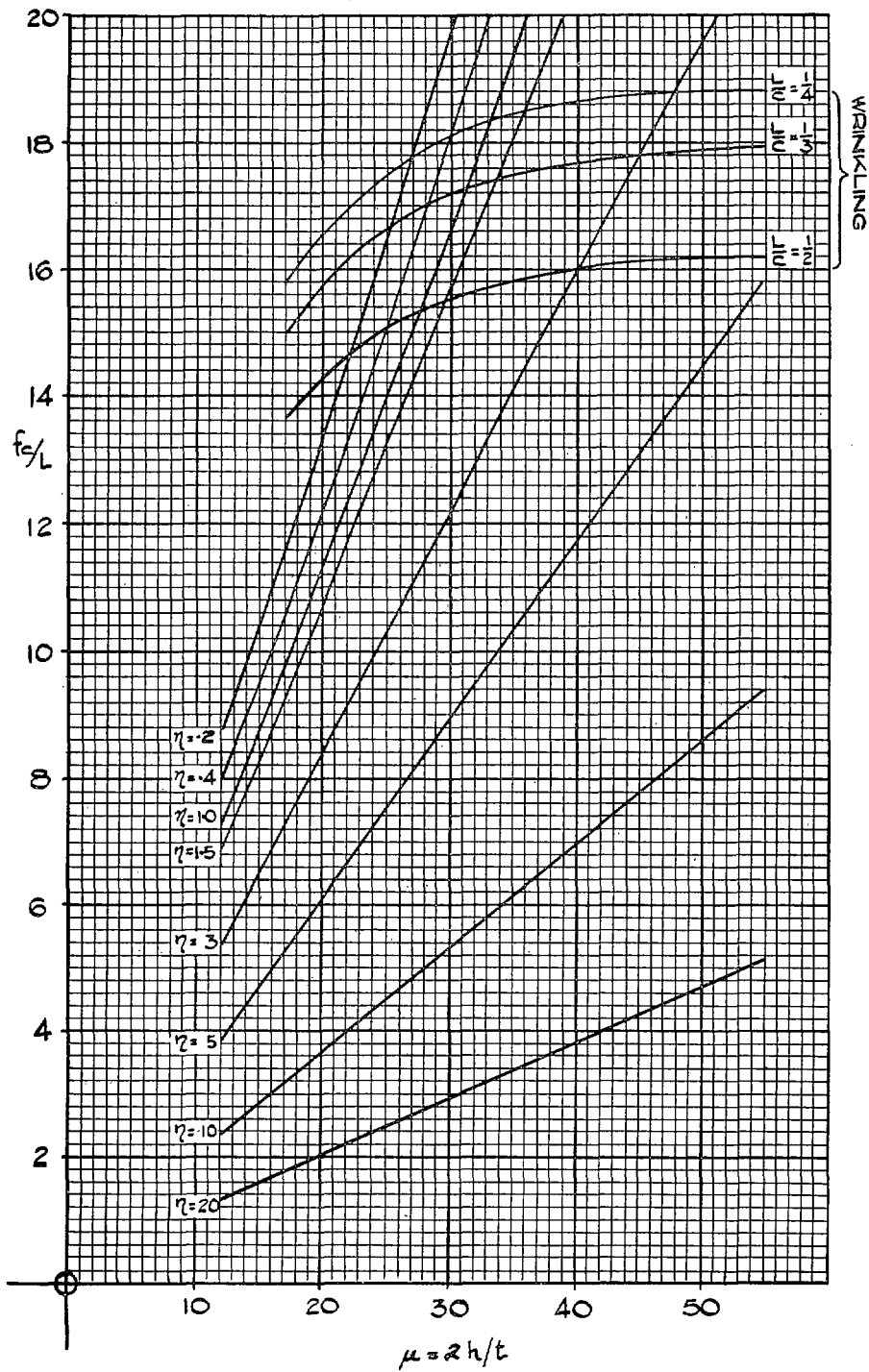


FIG. 3. Critical compressive stress for simply supported flat panel with isotropic filling.

$$\frac{L(2C - L)(1 - \sigma^2)}{EC} = 0.0004$$

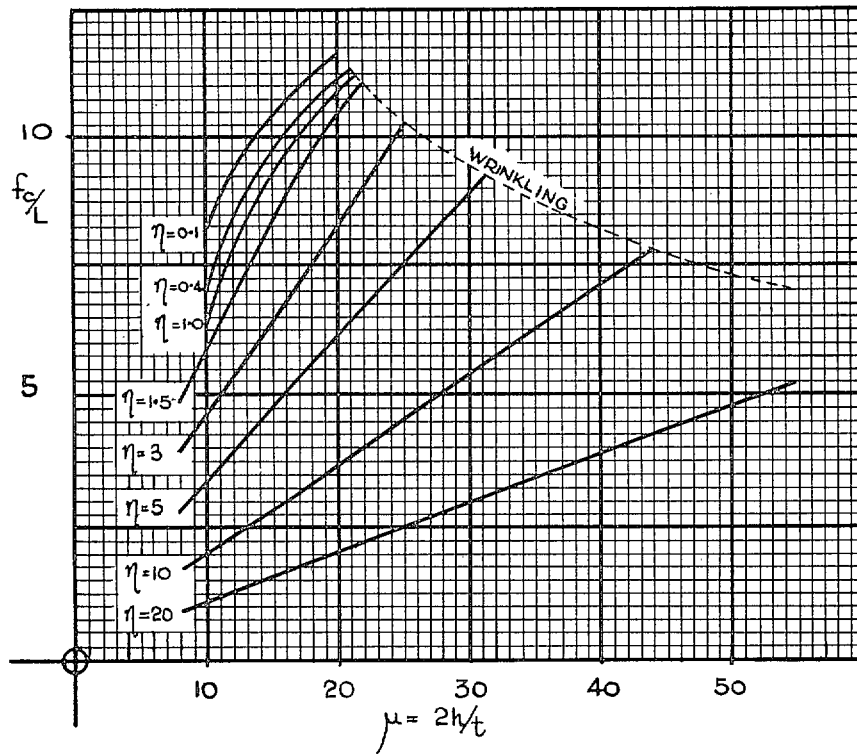


FIG. 4. Critical compressive stress for simply supported flat panel with honeycomb filling.

$$\frac{L^2(1 - \sigma^2)}{EC} = 2.5 \times 10^{-4}$$

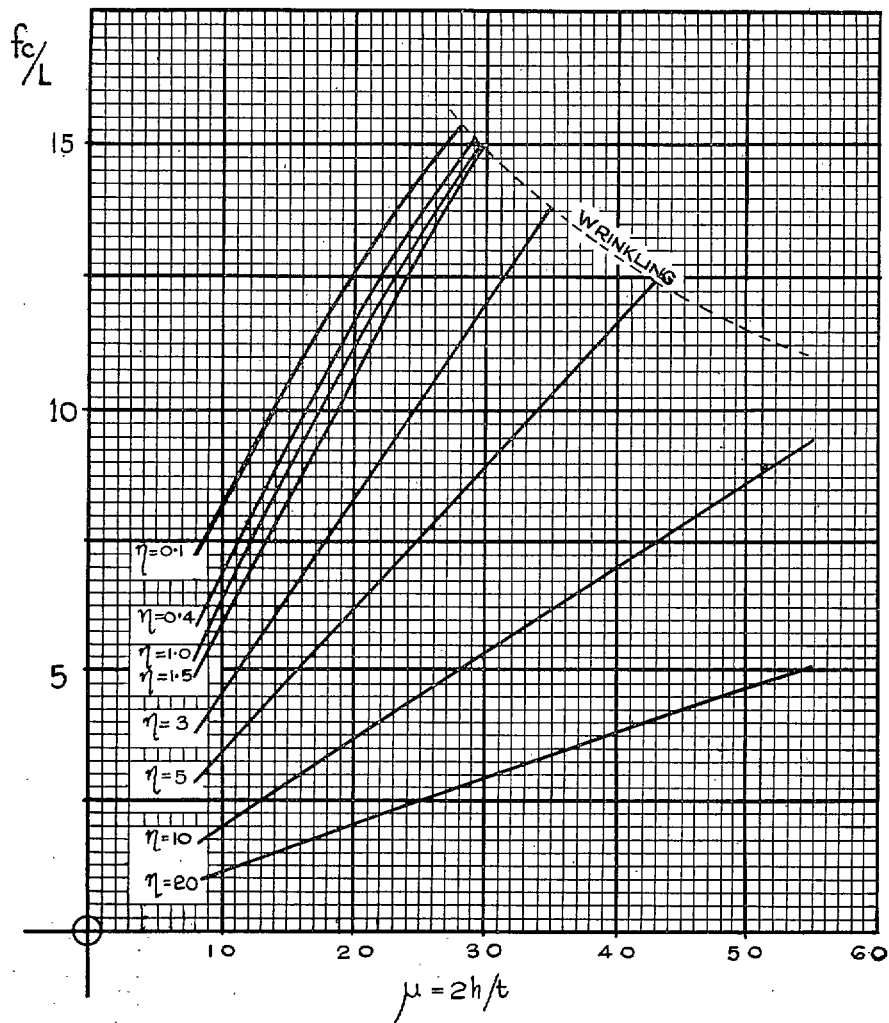


FIG. 5. Critical compressive stress for simply supported flat panel with honeycomb filling.

$$\frac{L^2(1 - \sigma^2)}{EC} = 1.0 \times 10^{-4}$$

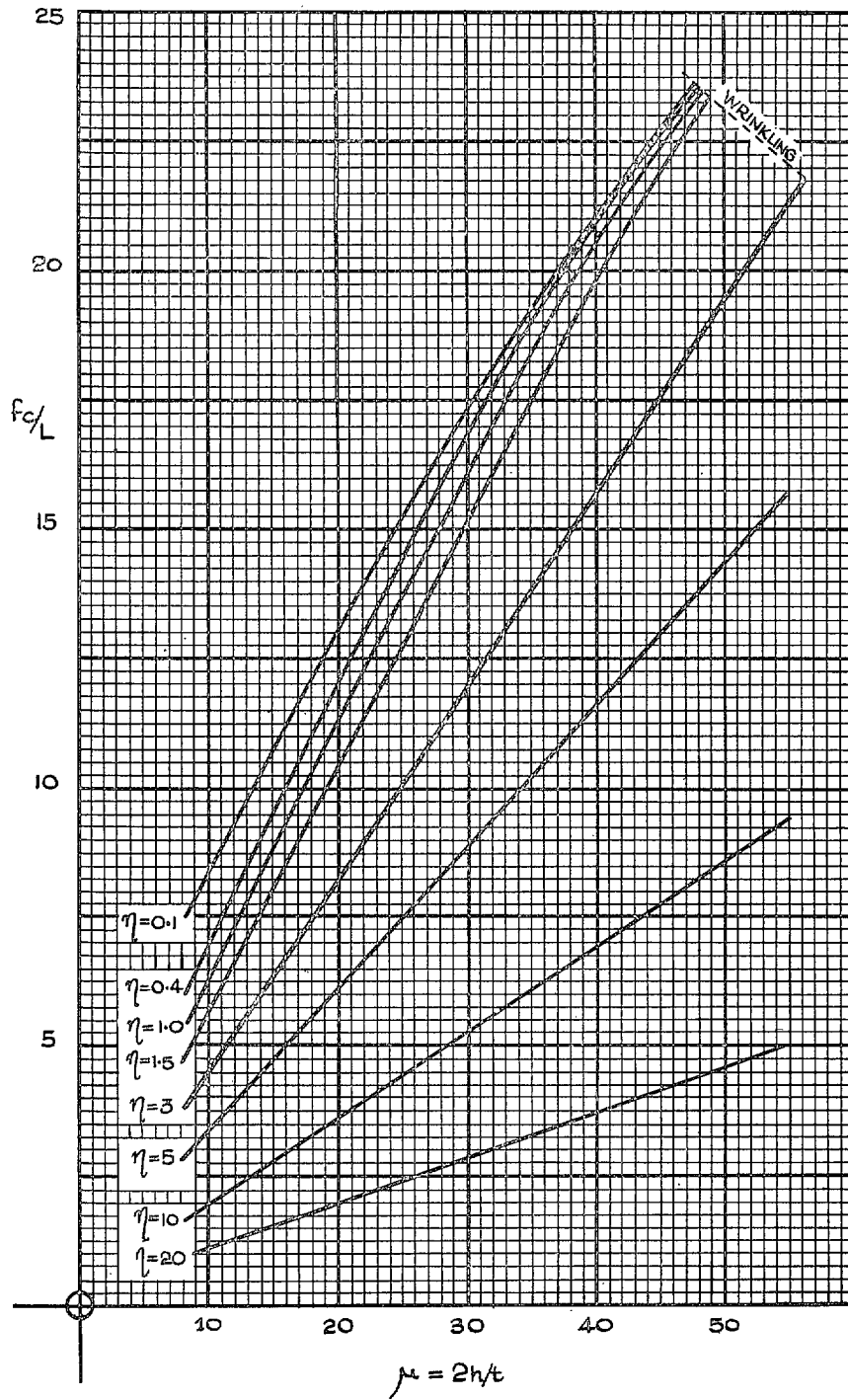


FIG. 6. Critical compressive stress for simply supported flat panel with honeycomb filling.

$$\frac{L^2(1 - \sigma^2)}{EC} = 2.5 \times 10^{-4}$$

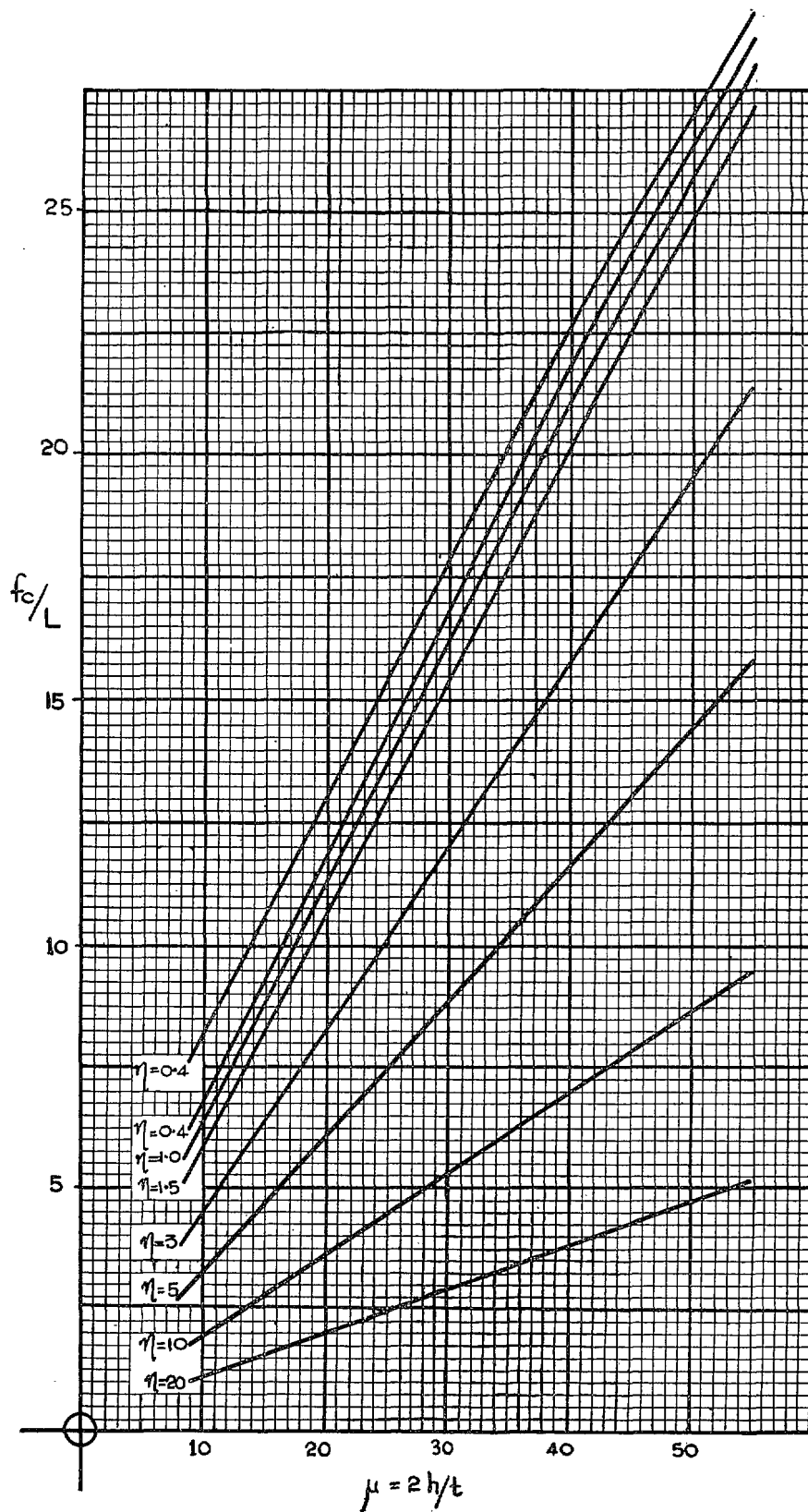


FIG. 7. Critical compressive stress for simply supported flat panel with honeycomb filling.

$$\frac{L^2(1 - \sigma^2)}{EC} = 1.0 \times 10^{-5}$$

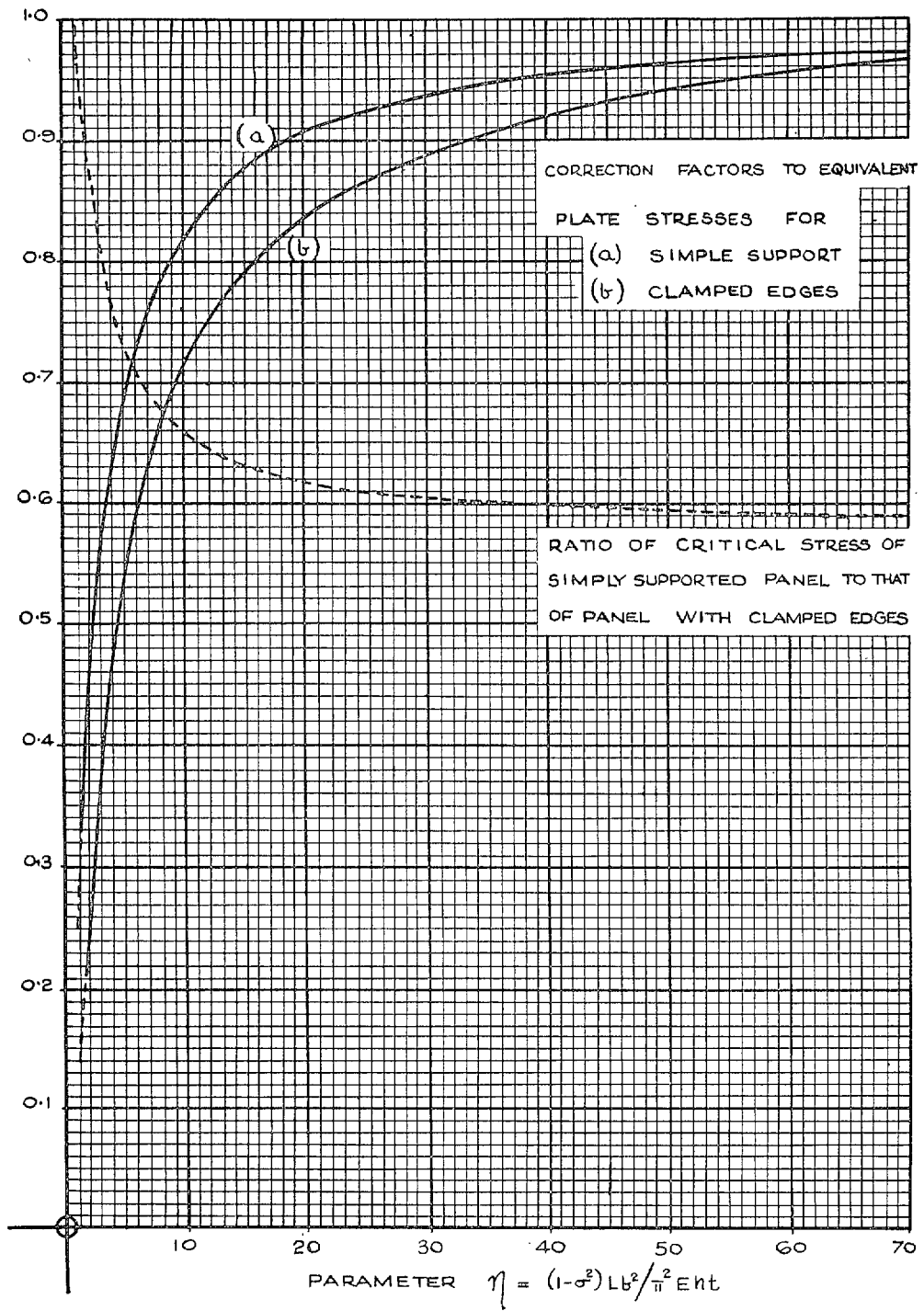


FIG. 8. Critical stresses of long flat panels in compression.

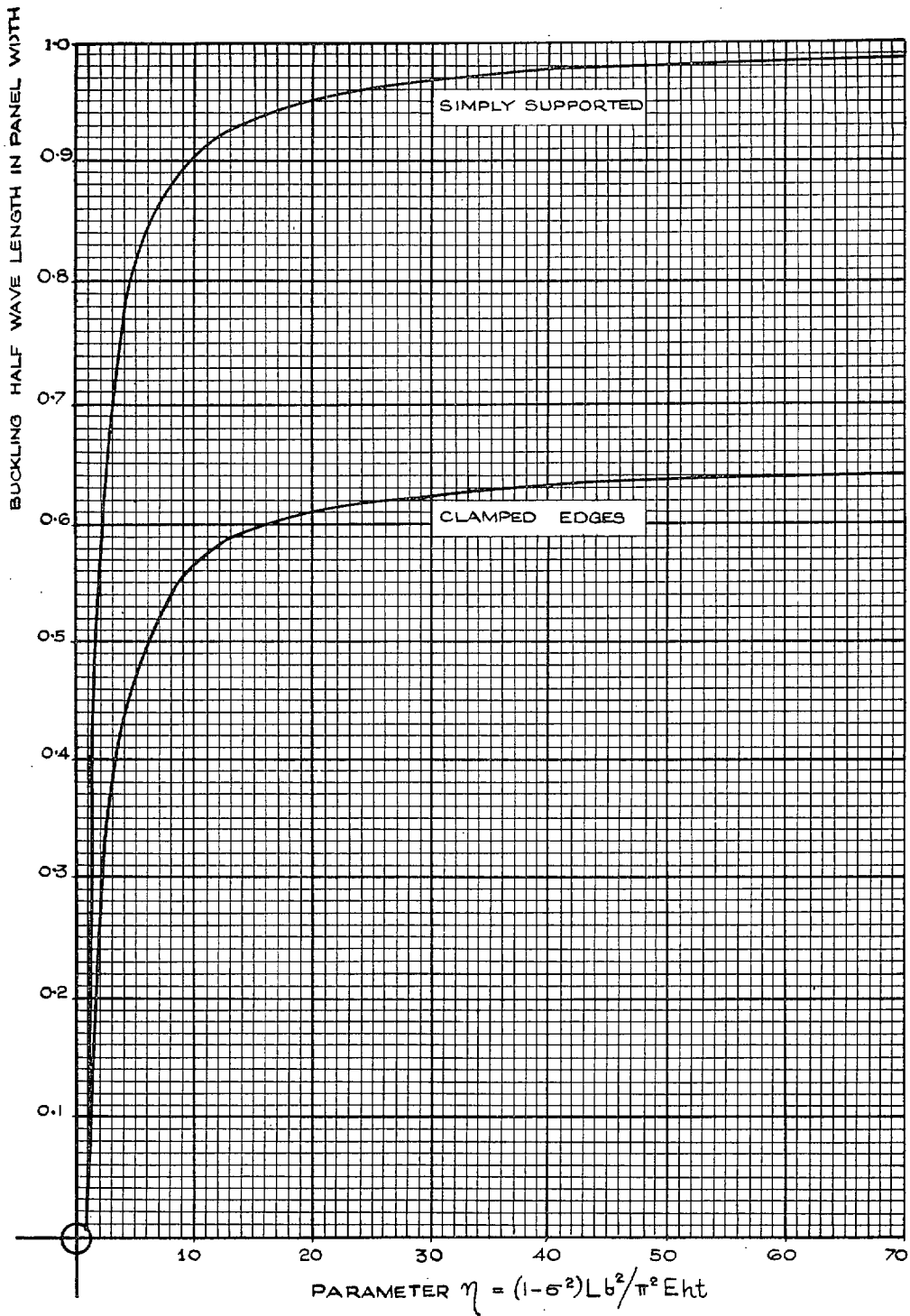


FIG. 9. Buckling half-wavelength for long flat panels in compression.

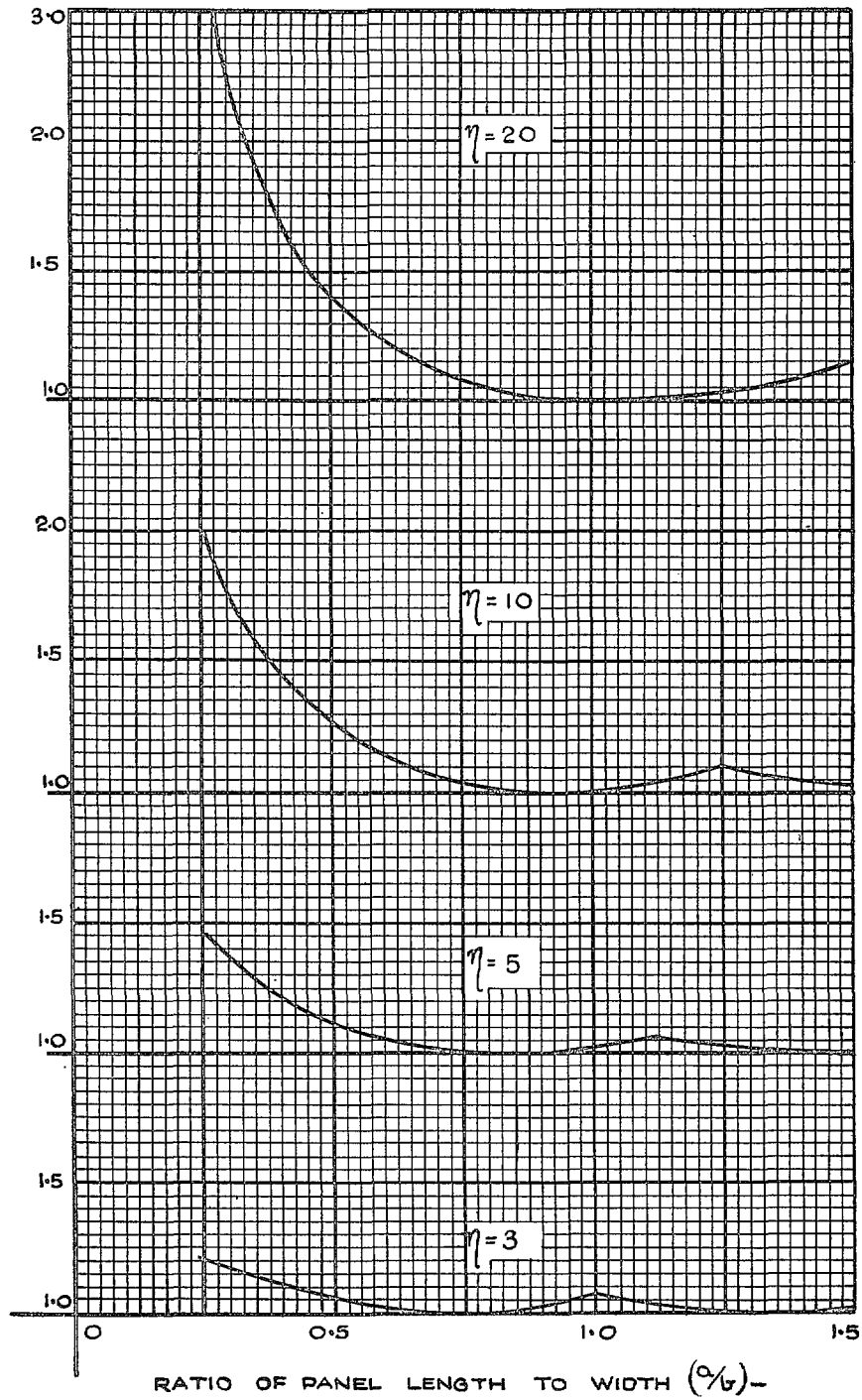


FIG. 10. Factor of increase in critical compressive stress for simply supported panels of short length.

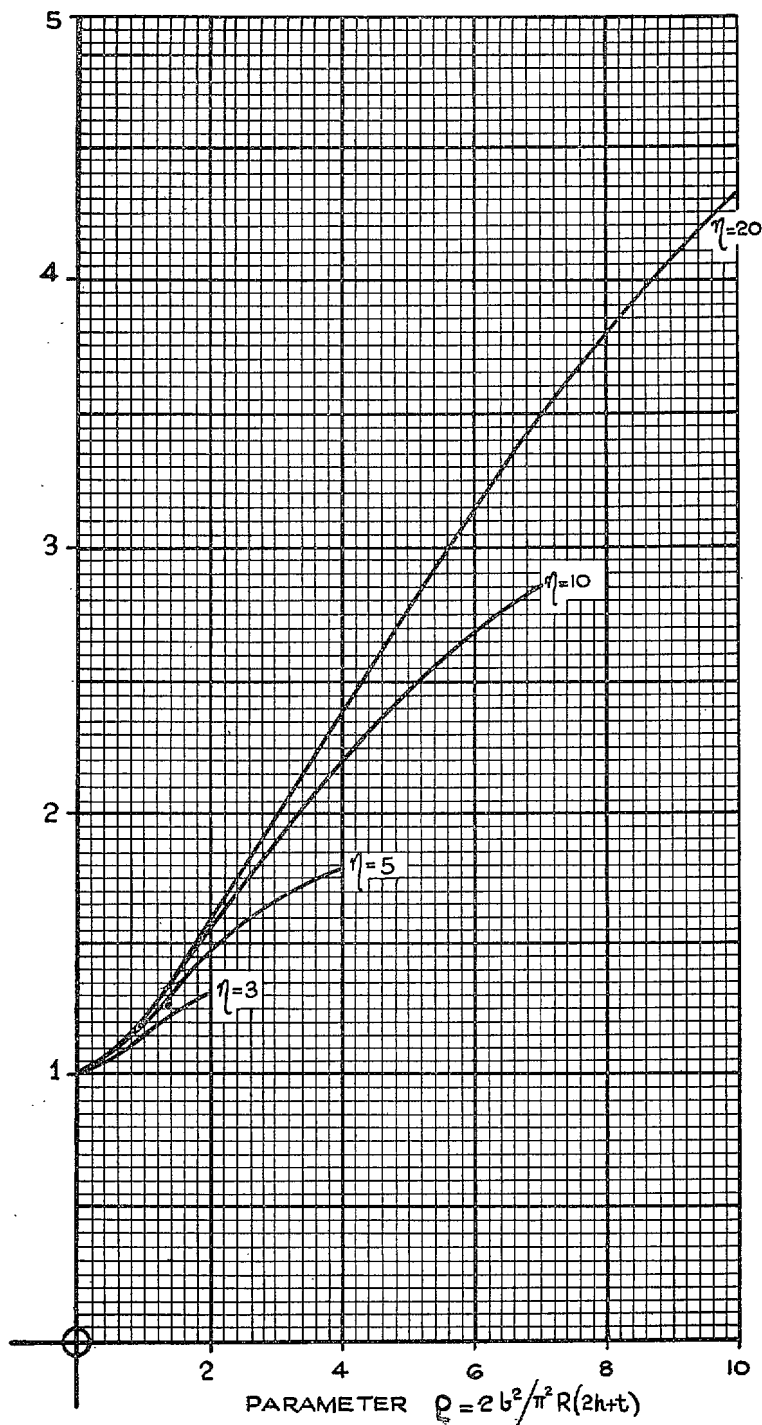


FIG. 11. Factor of increase in critical compressive stress due to curvature.

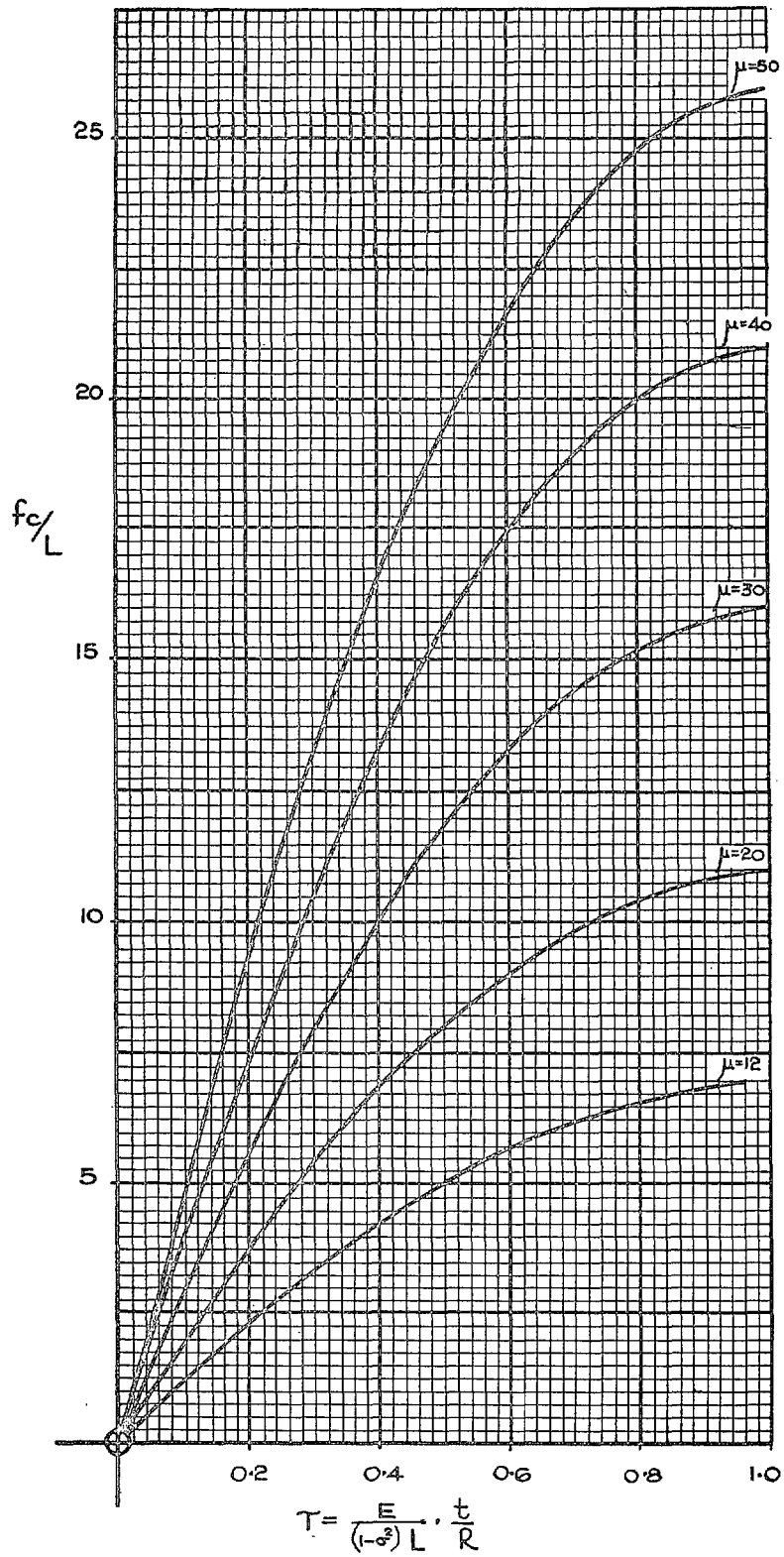


FIG. 12. Critical stress for cylinder under axial compression.

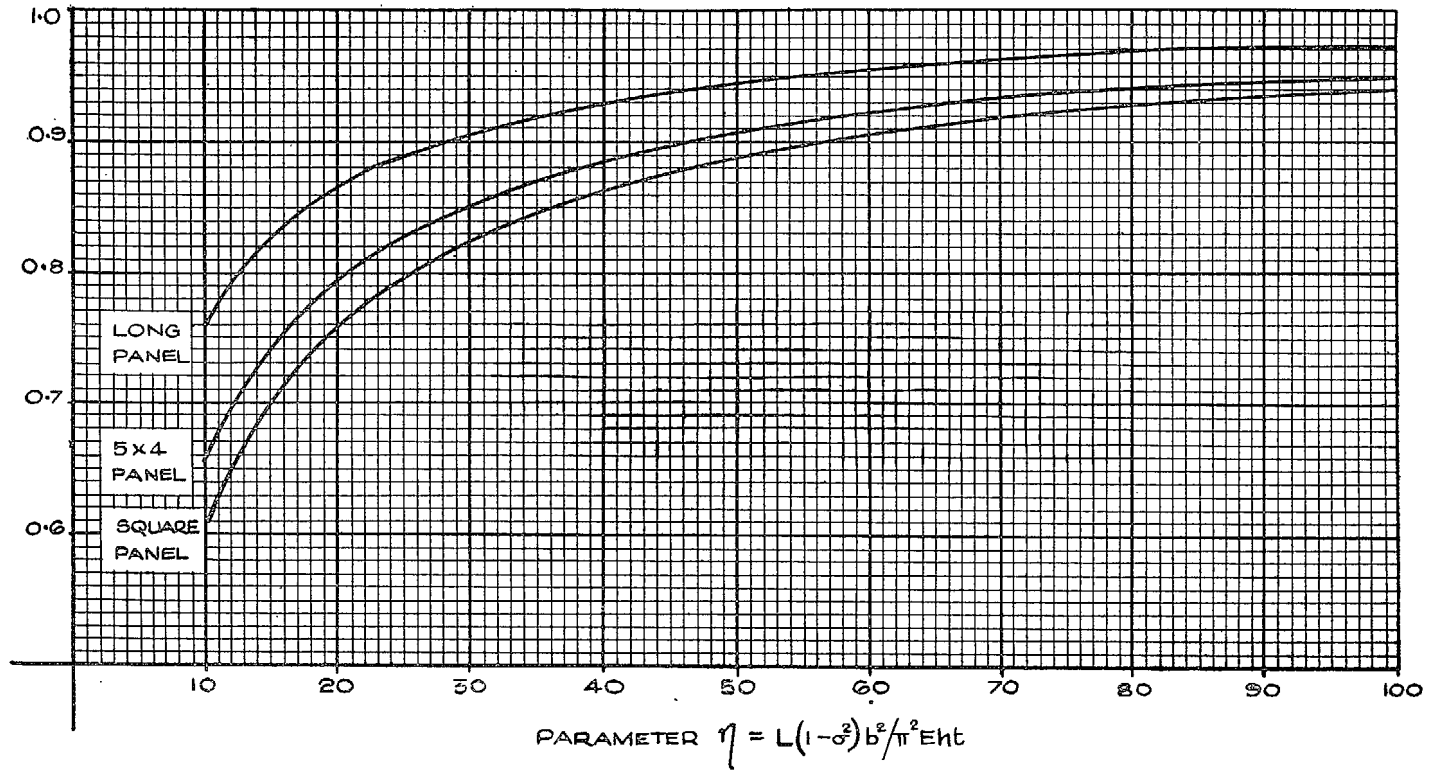


FIG. 13. Correction factor to equivalent plate critical shear stress for flat panel.

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