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Natural Frequencies and Modes of a Model Delta Aircraft

By

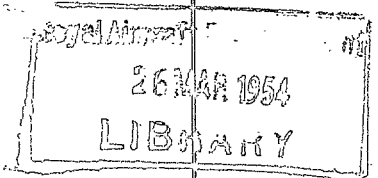
D. R. GAUKROGER, M.A.

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Natural Frequencies and Modes of a Model Delta Aircraft

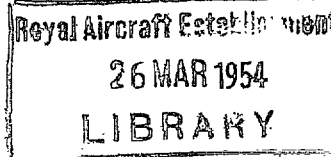
By

D. R. GAUKROGER, M.A.

COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR),
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Summary.—Resonance tests on a model delta wing are described. Consideration is given to the effect of inertia distribution on the first three normal symmetric modes of vibration. The mean centre of gravity position, fuselage pitching moment of inertia and wing inertia axis have been independently varied, and the effect of concentrated tip masses has been examined.

Results are given which are intended as a general guide in flutter calculations.

1. *Introduction.*—A knowledge of the natural frequencies and modes of vibration of an aircraft is required for the investigation of various problems, *e.g.*, flutter, mechanical vibration, buffeting, gust and landing effects. These data are generally obtained for aircraft in the project design stage, either by a theoretical approach or by dynamic models, and in the prototype stage by ground resonance testing.

In this report an investigation is described which is intended to give a general indication of the natural modes of vibration of delta wings, and particularly their dependence on inertia distribution. For this investigation a model of a delta-winged aircraft was constructed and resonance tested. The elastic properties of the model were reasonably representative of future design and were kept constant; the inertia properties on the other hand, could be varied to cover a wide range of distributed and concentrated masses. The tail unit and wing controls were not represented and symmetric vibration only was considered.

The results show that the natural frequencies for the various cases lie within three narrow frequency bands. The corresponding modes differ to some appreciable extent but it is possible to describe them in general terms. The fundamental mode is mainly wing flexure with pitching or twisting of the centre-section. The second mode is a flexural overtone and the third mainly torsional.

Tests are proceeding on models and full-scale delta aircraft and comparison will be made between the results from these tests and those of the present report. Further work on research models of various structural layouts will also be considered.

2. *Details of Model*—2.1. *Structural Details.*—The model is a Duralumin structure of approximately 11 ft span and 5 ft 'root chord' or fuselage length, with other leading dimensions as shown in Fig. 1. The wings are supported internally by five ribs on each side running in an

* R.A.E. Report Structure 75, received 13th September, 1950.

aerodynamic chordwise direction. The fuselage, which is the space between the two inboard ribs, is hollow, but is designed with a constant cross-section to enable wood blocks, fitted internally, to be moved to any desired position. These blocks form attachment points for the fuselage loads and for the exciter, which could not be attached to the Duralumin sheeting alone. Wing masses are represented by lead strips fitted along the four outboard ribs of each wing to both top and bottom surfaces.

2.2. *Design Considerations.*—The chief aims in the design of the model were to attain a representative spanwise flexural-torsional wing-stiffness ratio, to keep reasonably close to a full-scale type of construction and to make the construction of the model as simple as possible. The stiffness ratio chosen was based on figures available for a full-scale delta wing aircraft (a projected fighter), and at 0.7 span the ratio for the model was:—

$$\frac{\text{Flexural stiffness}}{\text{Torsional stiffness}} \frac{l_{\phi}}{m_0} = \frac{2.5}{1}.$$

To check that this ratio was achieved in the model the ratio between fundamental flexural and torsional frequencies was calculated from the design stiffness ratio and compared satisfactorily with the test results. It will be seen from Fig. 2 that the leading edge of the wing of the model is, in effect, a spar and may be considered the equivalent of the front spar of a full-scale wing. Similarly the trailing edge forms a rear spar. The construction therefore corresponds to normal construction except that the two spars are further apart than they would be in full-scale practice. With the exception of local stiffening panels the model is in 20 s.w.g. gauge (0.036 in.) Duralumin (including the ribs, spars, and top and bottom skins); the skin therefore provides considerable flexural stiffness equivalent to that provided by stringers in full-scale design. By the use of a rectangular section simplicity of construction was achieved. Although only inertia variations were made in the tests, this type of model is suitable for stiffness variations since both ribs and spars can be changed without difficulty.

2.3. *Standard Inertia Condition.*—For convenience in expressing values of moments of inertia, a 'standard' condition of the model was decided upon, and the variations of fuselage and overall pitching moments of inertia are expressed as percentages of the standard. The following table gives the details of the standard condition where \bar{x} is the distance of the c.g. of a component from the model c.g.

| | M (lb) | \bar{x} (in.) | $M\bar{x}$ | $M\bar{x}^2$ | Mk^2 | $M(\bar{x}^2 + k^2)$ |
|----------------|----------|-----------------|------------|--------------|--------|----------------------|
| Bare structure | 62 | 6.25 | 390 | 2,420 | 35,400 | 37,820 |
| Rib 1 | 100 | 7.0 | 700 | 4,900 | 20,420 | 25,320 |
| Rib 2 | 76 | 13.5 | 1030 | 13,800 | 8,900 | 22,700 |
| Rib 3 | 48 | 18.5 | 890 | 16,400 | 2,400 | 18,800 |
| Rib 4 | 22 | 25.5 | 560 | 14,300 | 230 | 14,530 |
| Exciter | 41 | — 24.0 | — 980 | 23,620 | — | 23,620 |
| Fuselage mass | 101 | — 24.0 | — 2420 | 58,200 | — | 58,200 |
| Wood block 1 | 27 | — 24.0 | — 650 | 15,500 | — | 15,500 |
| Wood block 2 | 27 | 17.5 | 470 | 8,280 | — | 8,280 |

The centre of gravity is at 50 per cent of the root chord (which equals the fuselage length) and \bar{x} in an aft direction is taken as positive. The total pitching moment of inertia $M(\bar{x}^2 + k^2)$ is 224,770 lb in.²; of this 111,900 lb in.² is fuselage moment of inertia, the remainder being wing inertia. Throughout the report pitching moments of inertia (about the mean c.g.) are referred to as percentages of these figures. The wing inertia axis is at 50 per cent of the chord for the standard condition. Throughout the test the fuselage and wing total masses are kept constant.

3. *Method of Test and Presentation of Results.*—3.1. *Test Rig.*—The model was suspended by rubber bungee from points at the nose and tail of the fuselage, to give a low frequency support. Excitation was by means of an inertia vibrator fitted to the fuselage on the model centre-line, to give a vertical (symmetric) exciting force.

Amplitudes of the structure were measured with velocity type pick-ups stationed along the ribs. These were slung vertically above the model and the pick-up rods were attached to similar rods fixed to the structure by using a sleeve of small internal diameter rubber drawing the two rods together. It was found that this method of attachment is the best yet devised for pick-ups of the non-seismic type, since it permits movement of the structure without damage to, and without affecting the operation of, the pick-up. Eighteen pick-ups were used on the port wing, and two on the starboard wing provided checks of amplitude and phase. Fig. 2 shows the model slung in the test frame.

Pick-up outputs were measured on single-channel equipment and phases were obtained relative to a master signal at the port wing tip by normal resonance test methods (R. & M. 2155²).

3.2. *Mode Presentation.*—A combination of the displacement curve and the contour line methods of mode presentation has been used. As the contour line method is not widely known a short description of it and its uses is given in Appendix I.

The modes shown in Figs. 5 to 41 include the contour line diagram for the port wing, and in addition displacements along chordwise and spanwise sections are shown. Amplitude *vs.* frequency curves for the wing tip are given in Fig. 3.

4. *Range of Tests.*—4.1. *General.*—The tests made were in four stages:—

- (a) Variation of mean c.g. position keeping the total mass, pitching moment of inertia and wing inertia distribution 'standard' (or as near as possible in the case of pitching moment of inertia).
- (b) Variation of fuselage pitching moment of inertia with standard c.g., total mass, and wing mass distribution.
- (c) Variation of wing inertia axis with standard total mass and c.g.
- (d) The effect of outboard masses (such as wing-tip fuel tanks) on the standard model.

4.2. *Variation of Mean C.G. Position.*—By suitable positioning of the fuselage masses three positions of the c.g. were obtained, at 47.75, 50 and 55 per cent of the root chord. Unfortunately, without stiffeners, which would have added to the mass, it was not possible to bring the pitching inertia up to 100 per cent in the 47.75 per cent case. The variation, however, is not large and for the three cases is:—

| C.G. (per cent) | Total pitching moment of inertia (per cent) |
|--------------------|--|
| 47.75 | 97.6 |
| 50 | 100.0 |
| 55 | 99.0 |

4.3. *Variation of Fuselage Pitching Moment of Inertia.*—Again it was impossible without adding to the structure, to vary the fuselage moment of inertia over a sufficiently wide range keeping the c.g. at 50 per cent. With the c.g. at 53.7 per cent a 36 per cent variation of the fuselage moment of inertia could be obtained, and three cases were tested as follows:—

| C.G. (per cent) | Fuselage pitching moment of inertia (per cent) | Total pitching moment of inertia (per cent) |
|--------------------|--|---|
| 53.7 | 100 | 96.0 |
| 53.7 | 118 | 104.9 |
| 53.7 | 136 | 113.7 |

4.4. *Variation of Wing Inertia Axis.*—Two positions of the wing inertia axis have been examined in addition to the standard case: (i) 45 per cent chord, (ii) 40 per cent chord. In both these cases the overall c.g. position was adjusted to 50 per cent, though in neither case was it possible to obtain the standard pitching moment of inertia. The values of total pitching and fuselage pitching moments of inertia are given below:—

| <i>Wing C.G. Axis</i> (per cent) | <i>Total pitching moment of inertia</i> (per cent) | <i>Fuselage pitching moment of inertia</i> (per cent) |
|-------------------------------------|---|--|
| 45 | 97.0 | 104.3 |
| 40 | 82.9 | 111.1 |

4.5. *The Effect of Wing-Tip Masses.*—The fitting of fuel tanks at wing tips is common on many aircraft. Percentage weights for tank and fuel up to 20 per cent (10 per cent on each tip) of the all-up weight of the aircraft are not unknown; one of the requirements for such tanks is that they shall be as near the c.g. axis of the aircraft as possible. With a delta-wing design this would mean fitting the tanks well forward of the wings, and as from strength considerations this would be impossible on the model, lead weights to represent tanks were attached at the leading edges of rib 3. Two sets of tests were made, firstly with an additional load of 4 per cent of the all-up weight, and secondly with 8 per cent.

5. *Test Results.*—5.1. *General.*—Examination of the amplitude *vs.* frequency curves (Fig. 3) shows that the natural frequencies for the various cases lie within three narrow frequency bands at 20, 40 and 62 c.p.s. approximately. The modes in these bands are summarised in Fig. 4. In addition to these, smaller peaks are evident, occurring in one or two cases only. Of these, some are anti-symmetric modes probably excited on account of some slight asymmetry in the model or in the exciter position. The remainder are symmetric modes and these have been measured and are included in the mode diagrams. For a number of the cases tested the main resonance in the 40 c.p.s. region is followed by a smaller peak in the amplitude *vs.* frequency curve at a slightly higher frequency; this apparent resonance was often found to have a similar mode shape to the main resonance and where this occurred it has not been shown separately. In order to simplify examination of the results, Table 1 has been drawn up, which shows all the peaks in the amplitude *vs.* frequency curves and indicates, in the 'comments' column, into which of the above categories they fall. Detailed comparison of the modes is made easier if the corresponding modes of the various cases are considered together, rather than all the modes of a particular case, and this is the method which has been adopted in the subsequent sections.

5.2. *Modes in 20 c.p.s. Region.*—The fundamental wing modes for each case are shown in detail in Figs. 5 to 14. For convenience they are summarised together with nodal line diagrams in Fig. 4.

5.2.1. *Mode shape.*—Considering first the three cases in which the mean c.g. position only has been varied (Figs. 5, 6 and 7) it will be seen that the movement aft of the c.g. reduces the amount of torsion or pitching in the mode. For the 47.75 and 50 per cent c.g. cases the mode is of 'horseshoe' shape with pitching rather than torsion occurring over the centre section; this can be deduced from the contour line shape in Figs. 5 and 6. In the 55 per cent c.g. case (Fig. 7) the mode more nearly resembles a fundamental mode for an unswept wing; bending of chordwise sections is less for the 55 per cent case except at the forward end of the fuselage rib, but it should be noted that the latter may be due entirely to local distortion near the exciter.

The effect of wing c.g. axis variation with constant mean c.g. position is seen to have considerable effect on the mode shape (Figs. 6, 11 and 12). With the wing c.g. axis at 45 per cent of the chord a horseshoe shape nodal line is evident (Fig. 12) though the 'corners' of the horseshoe have moved until they are forward of the wing. If the wing c.g. axis is moved still further forward to 40 per cent (Fig. 11) the mode becomes simple wing flexure. When the fuselage

pitching moment of inertia is varied with mean c.g. position constant (at 53.7 per cent) the nodal line moves such that with decreasing inertia the point of intersection with the trailing edge moves inboard. (Figs. 8, 9 and 10). When the fuselage inertia is 100 per cent (the minimum of the three cases tested) the nodal line never reaches the trailing edge but takes on an 'inverted horseshoe' shape (Fig. 10). It may be noted that the intermediate of the three cases (118 per cent fuselage inertia Fig. 9) corresponds closely in fuselage and pitching moments of inertia to the 55 per cent c.g. variation case (Fig. 7), the only difference being in the c.g. position. It would be expected that the nodal line would be intermediate between that of the standard case and the 55 per cent c.g. case; inspection shows that this is so—the nodal line for the 118 per cent fuselage moment of inertia case showing a slight tendency towards horseshoe pattern.

The addition of outboard mass to the standard model has an important effect on the mode (Figs. 13 and 14); the addition of tip masses representing together 4 per cent of the all-up weight gives a mode which is predominantly wing flexure (Fig. 14) whilst a further addition gives a nodal line almost parallel to the trailing edge at about 70 per cent of the root chord (Fig. 13).

5.2.2. *Frequency variation.*—The effects on frequency may be summarised as follows:—

(a) The movement aft of the mean c.g. position from 47.75 per cent to 55 per cent corresponds to a 5 per cent reduction in frequency, approximately.

(b) The movement forward of the wing inertia axis from 50 per cent to 40 per cent corresponds to a 22.5 per cent reduction in frequency; the variation is almost linear over the range tested.

(c) Addition of wing-tip weights reduces the frequency.

There are some inconsistencies in the frequencies measured, notably in the differences between the mean c.g. position variations and the fuselage moment of inertia variations. It is difficult to see why the 55 per cent c.g. case with total pitching moment of inertia 99 per cent and fuselage 117.9 per cent should differ so greatly from the 53.7 per cent c.g. case with total pitching moment of inertia 104.9 per cent and fuselage 118 per cent, the frequencies being 20.2 and 19.5 c.p.s. respectively; the variation is in the wrong direction, *i.e.*, one would expect the frequency for the latter case to be the higher of the two, and consequently one would expect all the fuselage inertia variation cases to be of higher frequency than the c.g. variation cases. It may be, however, that the adjustment of fuselage masses has some effect on the fuselage stiffness, and since there is fuselage bending in all six cases a stiffness variation may affect the frequency.

The addition of wing-tip weights reduces the frequency initially but a further addition causes a slight increase. The relative values of the frequencies of the higher modes of the two cases cast some doubt on the frequencies for the mode under discussion though it is difficult to see how such an error could have arisen.

5.3. *Modes in 40 c.p.s. Region.*—It is not proposed to deal with these and higher modes in such detail as with the fundamental modes.

5.3.1. *Mode shape.*—The general shape of these modes is of a wing flexural overtone with two nodal lines running in a fore-and-aft direction across the wing (Fig. 4). Very little torsion is present in any of the modes, and none are likely to be dangerous from the flutter aspect. The mean c.g. position is the only variable appreciably affecting the mode shape (Figs. 15, 16 and 17); with c.g. forward of the nodal line becomes continuous across the planform, giving torsion of the wings and pitching of the fuselage (Fig. 15). With the c.g. at 50 per cent the nodal line is no longer continuous but has divided into three parts (for the complete planform), the centre part being a horseshoe in itself, similar to the fundamental mode for this particular case (Fig. 16). The remainder of the cases give modes of very similar general shape except for the minimum fuselage pitching moment of inertia case where there is a central nodal line of thin horseshoe shape (Fig. 20). It should be noted that wing-tip masses eliminate this central horseshoe.

5.3.2. *Frequency variation.*—The frequency variations show the same general trends as for the fundamental modes except that for fuselage moment of inertia cases the variation in frequency is of the order of 2.5 per cent compared with 15 per cent for the fundamental mode, and for the wing c.g. axis cases 12 per cent compared with 22.5 per cent. An inconsistency is apparent if the frequencies for the standard model and standard with 4 per cent tip mass are compared, but an error of 0.1 c.p.s. may be explained by the difficulty of obtaining consistently accurate frequency readings, particularly above 30 c.p.s. where a gear box was used between the motor and the exciter. Again there is an inconsistency in the wing-tip mass cases, the frequency for the mode with tip mass 4 per cent being higher than the standard case, and with 8 per cent lower.

5.4. *Modes in 62 c.p.s. Region.*—5.4.1. *Mode shape.*—The most notable characteristics of all the modes in this region is their similarity of shape, the only change being in the amount of flexure present (Fig. 4). There is little or no variation of the mode shape for change of mean c.g. position (Figs. 25, 26 and 27) or for variation of wing inertia axis position (Figs. 26, 31 and 32). Increase of fuselage pitching moment of inertia reduces the flexure over the outer sections of the wing, and with the fuselage pitching moment of inertia a maximum (136 per cent), (Fig. 28), the nodal line follows closely the 50 per cent chord line with only very slight indications of flexure.

5.4.2. *Frequency variations.*—The variation in frequency for all the cases tested is less than 7 per cent and for the variation of any particular parameter is considerably less than this. It is noticeable, however, the frequency variations no longer follow the general trends of the lower modes except for the variation of fuselage pitching moment of inertia where the frequency increases with the increase of inertia. Movement aft of the mean c.g. position gives mode frequencies of 60.9, 61.9 and 61.5 c.p.s.; movement aft of the wing inertia axis gives frequencies of 63.0, 60.7 and 61.9 c.p.s. whilst with increasing wing-tip masses the frequencies are 61.9 (no tip mass), 62.4 and 60.9 c.p.s. respectively. Even allowing a margin for experimental error it is obvious that the frequency variations are not as well defined as for the two lower modes already considered.

5.5. *Remaining Modes.*—Two modes occurred in the region of 26 c.p.s. which were not anti-symmetric; one at 24.2 c.p.s. (Fig. 35) was for the 40 per cent wing c.g. axis case and the second at 28.4 c.p.s. (Fig. 36) was with 4 per cent wing-tip masses added. Both modes were of similar shape with nodal lines resembling over the outer wing that of the fundamental mode with c.g. at 53.7 per cent and fuselage pitching moment of inertia 100 per cent (Fig. 10). The existence of symmetric modes at frequencies near that of the anti-symmetric fundamental mode gives rise to the possibility that they may have been present in many, if not all, of the cases tested, but have been impossible to detect owing to the anti-symmetric amplitudes obtained. It should be remembered, however, that with a full-scale aircraft asymmetry of the structure is less likely than with a simple model owing to greater accuracies in manufacture, and thus the excitation of anti-symmetric modes under conditions of (apparently) symmetric excitation will be rare. Even so there is obviously a need for some means of separating such modes, and it is felt that a vector method of analysis on the lines suggested by Kennedy and Pancu³ would be a valuable weapon in tackling this problem.

A mode in the region of 45 to 50 c.p.s. occurs in five cases (Figs. 37 to 41). Three of these are the mean c.g. position variations where, with the c.g. at 47.75 per cent and 55 per cent, there is appreciable wing torsion (Figs. 37 and 39); for the intermediate case (Fig. 38) a resonance is scarcely apparent from the amplitude *vs.* frequency curve but has nevertheless been recorded; its shape bears no resemblance to the modes for the other two cases and it seems certain that a well defined mode in the two cases does not exist in the third. The two remaining modes in this region (Figs. 40 and 41) were obtained with the addition of wing-tip masses; both are of similar general shape with flexure over the centre section of the model and torsion over the outer wing sections.

5.6. *Modes above 65 c.p.s.*—The frequency range was extended up to 188 c.p.s. for the 100 per cent fuselage pitching moment of inertia case with mean c.g. position 53.7 per cent. This case was chosen for its convenience for high-frequency excitation where as short a length as possible of flexible drive between the motor and vibrator was desirable. Four modes were obtained above 65 c.p.s., at 87.0, 93.0, 107.5 and 115 c.p.s.: Considerable low frequency movement of the model took place, particularly above 90 c.p.s. with the result that pick-up readings were difficult to obtain accurately, and it has not been thought worth while to show the chordwise and spanwise displacement curves. Fig. 42 shows the four modes (together with the lower modes obtained for the same case) and the amplitude *vs.* frequency curve.

6. *Discussion.*—The tests as a whole indicate that over a wide range of variations in inertia loading the general resonance test picture is not drastically altered. This is particularly so in the case of mode frequencies, as is indicated by the fact that it has been possible to group the main modes in frequency bands. For the higher modes there is not a very wide variation in mode shape and it is only in the fundamental mode that important variations occur. These indicate that moving the mean c.g. after or the wing inertia axis forward reduces the amount of torsion or pitching mainly over the inboard sections. A reduction in fuselage pitching moment of inertia can produce an inverted horseshoe mode provided the mean c.g. is sufficiently far aft. The fitting of wing tip masses is likely to have a considerable effect on mode shape; the alterations caused by the addition of mass in two stages are so great that any attempt to predict how the mode will alter with tip loads up to 20 per cent of the all-up weight is impossible. Too much reliance should not be placed upon the curves of chordwise displacements; owing to the construction of the model these do not in themselves indicate that in general chordwise bending can be expected; the arrangement of spars makes the model somewhat unrepresentative of present practice from this aspect.

7. *Conclusions.*—(a) Movement of the mean c.g. position aft reduces the torsion or pitching in all modes, but more particularly in the fundamental mode.

(b) Forward movement of the wing inertia axis has the same effect as (a) above and in addition produces a marked frequency reduction in the fundamental mode.

(c) The reduction of fuselage pitching moment of inertia has an effect on the fundamental mode nodal pattern and is accompanied by a reduction in frequency.

(d) The addition of a concentrated mass at the wing tip causes considerable change in mode shape and frequency particularly in the fundamental mode.

8. *Further Developments.*—8.1. Flutter calculations will be made on the modes given in this report in order to obtain more specific evidence of the effect of inertia changes on the flutter characteristics. The results of the calculations will be given in a later report.

8.2. The modes obtained from the tests will be compared with resonance test results of various delta projects, in order that the value of the information derived can be assessed in the light of full scale tests. This will provide a useful guide to the value of model tests in flutter work.

8.3. Consideration is being given to the use of a model to provide similar data to that given in the report, when the elastic properties are varied. Parameters which could be varied are flexural and torsional stiffnesses and flexural axis; tests on these lines will only be made, however, if the flutter calculations and test comparisons mentioned in sections 8.1 and 8.2 indicate that further work would be of value.

Acknowledgements.—Acknowledgement is made to Mr. W. G. Molyneux for the design of the model, and for the frequency calculations and other design details mentioned in section 2.2.

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APPENDIX I

The Contour Method of Mode Presentation

1. *Introduction.*—The usual method of presentation of modes in aircraft resonance testing (R. & M. 2155²) is to show the displacement of each major component of the aircraft structure as a separate figure, and in addition to include a wing and perhaps fuselage nodal line diagram. In order that all the major structural component deformations may be viewed simultaneously the contour method has been adopted, in which the nodal line diagram is extended to cover the whole aircraft and to include lines of constant amplitude as well as lines of no amplitude (nodal lines).

2. *Construction of a Contour Line Diagram.*—The construction of a contour line of any particular amplitude is exactly similar to the construction of a nodal line. For a wing on which, say, two points have been measured for a chordwise section (normally front and rear spars) points of a particular amplitude may be marked on the front and rear spars, these points having been obtained from the spar displacement curves, and, by interpolation and extrapolation points of equal amplitudes may also be marked on any required chordwise section. The line joining the points is the contour. To enable the contour diagram to be quickly interpreted, contour lines should be drawn at equal increments of amplitude and some distinguishing marks added to denote displacements of opposite sign.

3. *Advantage of a Contour Line Diagram.*—In addition to the advantage, mentioned in section 1, of the method showing at a glance the relative movements of all major components of the aircraft simultaneously, there are two other advantages:—

(a) Contour lines indicate immediately whether torsion or pitch of a component is present.

(b) For delta planforms (and other surfaces of low aspect ratio) a contour diagram combines bending in two directions.

As far as (a) is concerned, consider an untapered aircraft wing in which, for a given mode, the nodal line follows the half-chord line. It is impossible to say whether this represents wing torsion or bending, without further evidence; but if the contour lines are added they will run parallel to the nodal line if pitch alone occurs, and will taper towards the tip (or root) if torsion is present.

Figs. 5 to 41 of this report demonstrate bending in both chordwise and spanwise directions represented on contour line diagrams; it should be noted, however, that it is necessary to measure displacements at more than two points on a chordwise section in order to determine bending of the section and therefore the drawing of chordwise as well as spanwise displacement curves is a necessary preliminary to the construction of a contour line diagram. In spite of this there is no doubt that the contour diagram presents a much more readily understood picture of the mode than a large number of displacement curves.

TABLE 1
Amplitude vs. frequency curve maxima

| Wing c.g. axis per cent | Mean c.g. position per cent | Total pitch inertia per cent | Fuselage pitch inertia per cent | Frequency c.p.s. | Comments |
|---|-----------------------------------|------------------------------------|---------------------------------------|---------------------|-----------------|
| 50 | 47.75 | 97.6 | 85.7 | 21.2 | Recorded Mode |
| 50 | 47.75 | 97.6 | 85.7 | 42.1 | " " |
| 50 | 47.75 | 97.6 | 85.7 | 47.4 | " " |
| 50 | 47.75 | 97.6 | 85.7 | 60.9 | " " |
| 50 | 50 | 100 | 100 | 21.0 | " " |
| 50 | 50 | 100 | 100 | 40.7 | " " |
| 50 | 50 | 100 | 100 | 49.5 | " " |
| 50 | 50 | 100 | 100 | 61.9 | " " |
| 50 | 55 | 99 | 117.9 | 20.2 | " " |
| 50 | 55 | 99 | 117.9 | a 25.0 | Antisymmetric |
| 50 | 55 | 99 | 117.9 | 37.1 | Recorded Mode |
| 50 | 55 | 99 | 117.9 | 48.0 | " " |
| 50 | 55 | 99 | 117.9 | 52.2 | " " |
| 50 | 55 | 99 | 117.9 | 61.5 | " " |
| 50 | 53.7 | 113.7 | 136 | 20.1 | " " |
| 50 | 53.7 | 113.7 | 136 | a 27.0 | Antisymmetric |
| 50 | 53.7 | 113.7 | 136 | 41.4 | Recorded Mode |
| 50 | 53.7 | 113.7 | 136 | a 44.0 | Similar to 41.4 |
| 50 | 53.7 | 113.7 | 136 | 64.7 | Recorded Mode |
| 50 | 53.7 | 104.9 | 118 | 19.5 | " " |
| 50 | 53.7 | 104.9 | 118 | a 26.0 | Antisymmetric |
| 50 | 53.7 | 104.9 | 118 | 41.0 | Recorded Mode |
| 50 | 53.7 | 104.9 | 118 | a 44.0 | Similar to 41.0 |
| 50 | 53.7 | 104.9 | 118 | 64.3 | Recorded Mode |
| 50 | 53.7 | 96 | 100 | 17.5 | " " |
| 50 | 53.7 | 96 | 100 | 40.1 | " " |
| 50 | 53.7 | 96 | 100 | a 43.0 | Similar to 40.1 |
| 50 | 53.7 | 96 | 100 | 63.8 | Recorded Mode |
| 40 | 50 | 96 | 100 | 16.3 | " " |
| 40 | 50 | 96 | 100 | 24.25 | " " |
| 40 | 50 | 96 | 100 | 34.8 | " " |
| 40 | 50 | 96 | 100 | 38.3 | Similar to 34.8 |
| 40 | 50 | 96 | 100 | 63.0 | Recorded Mode |
| 45 | 50 | 96 | 100 | 19.0 | " " |
| 45 | 50 | 96 | 100 | a 26.0 | Antisymmetric |
| 45 | 50 | 96 | 100 | 37.1 | Recorded Mode |
| 45 | 50 | 96 | 100 | a 47.0 | Antisymmetric |
| 45 | 50 | 96 | 100 | 60.1 | Recorded Mode |
| 8% Tip Tanks | 50 | 100 | 100 | 19.6 | " " |
| 50 | 50 | 100 | 100 | a 24.0 | Antisymmetric |
| 50 | 50 | 100 | 100 | 40.3 | Recorded Mode |
| 50 | 50 | 100 | 100 | 43.2 | " " |
| 50 | 50 | 100 | 100 | 60.9 | " " |
| 4% Tip Tanks | 50 | 100 | 100 | 19.5 | Recorded Mode |
| 50 | 50 | 100 | 100 | a 25.0 | Antisymmetric |
| 50 | 50 | 100 | 100 | 28.4 | Recorded Mode |
| 50 | 50 | 100 | 100 | 40.9 | " " |
| 50 | 50 | 100 | 100 | 45.8 | " " |
| 50 | 50 | 100 | 100 | 62.4 | " " |
| Subsidiary Test to High Exciting Frequencies | | | | | |
| 50 | 53.7 | 96 | 100 | 87.0* | Recorded Mode |
| 50 | 53.7 | 96 | 100 | 93.0* | " " |
| 50 | 53.7 | 96 | 100 | 107.5* | " " |
| 50 | 53.7 | 96 | 100 | 115.0* | " " |

* To the nearest 0.5 c.p.s.

a Approximate value.

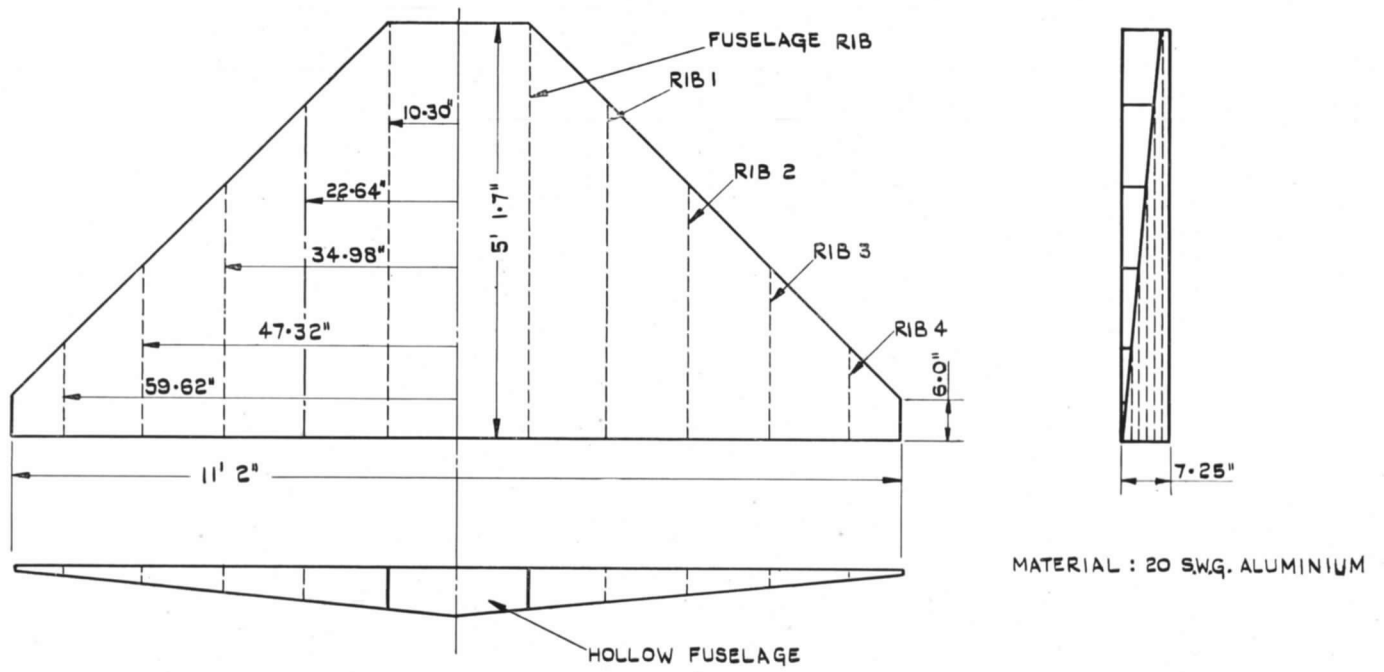


FIG. 1. Model delta.

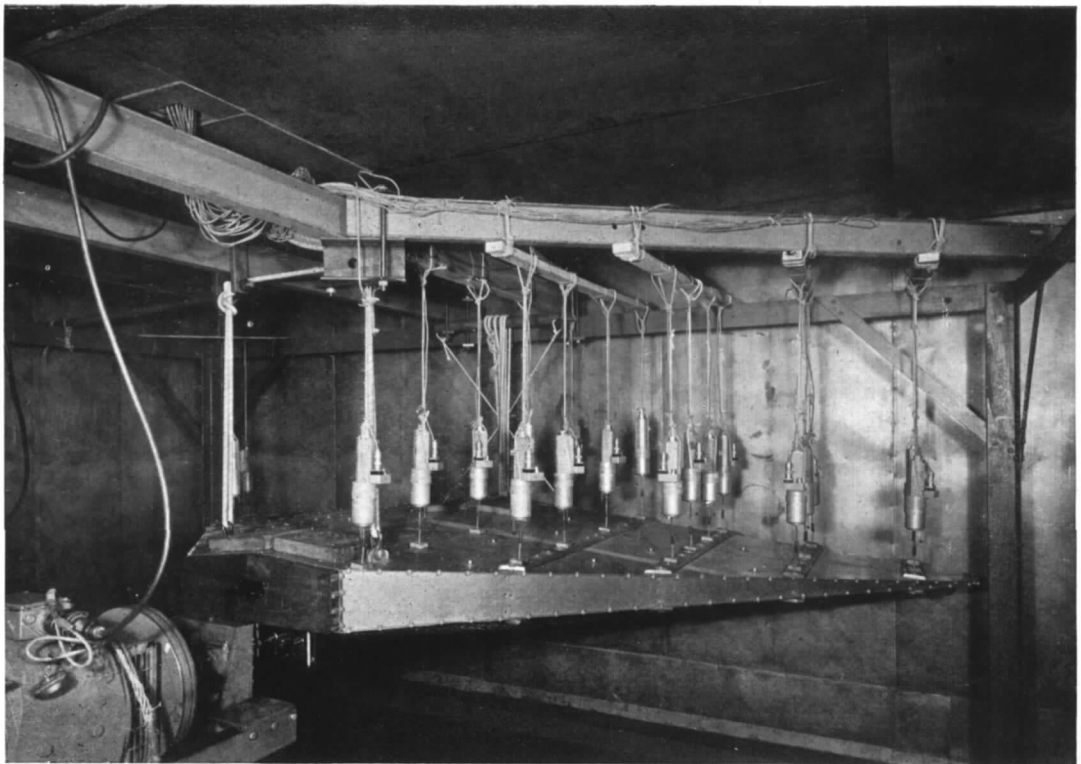


FIG. 2. Model rigged for test.

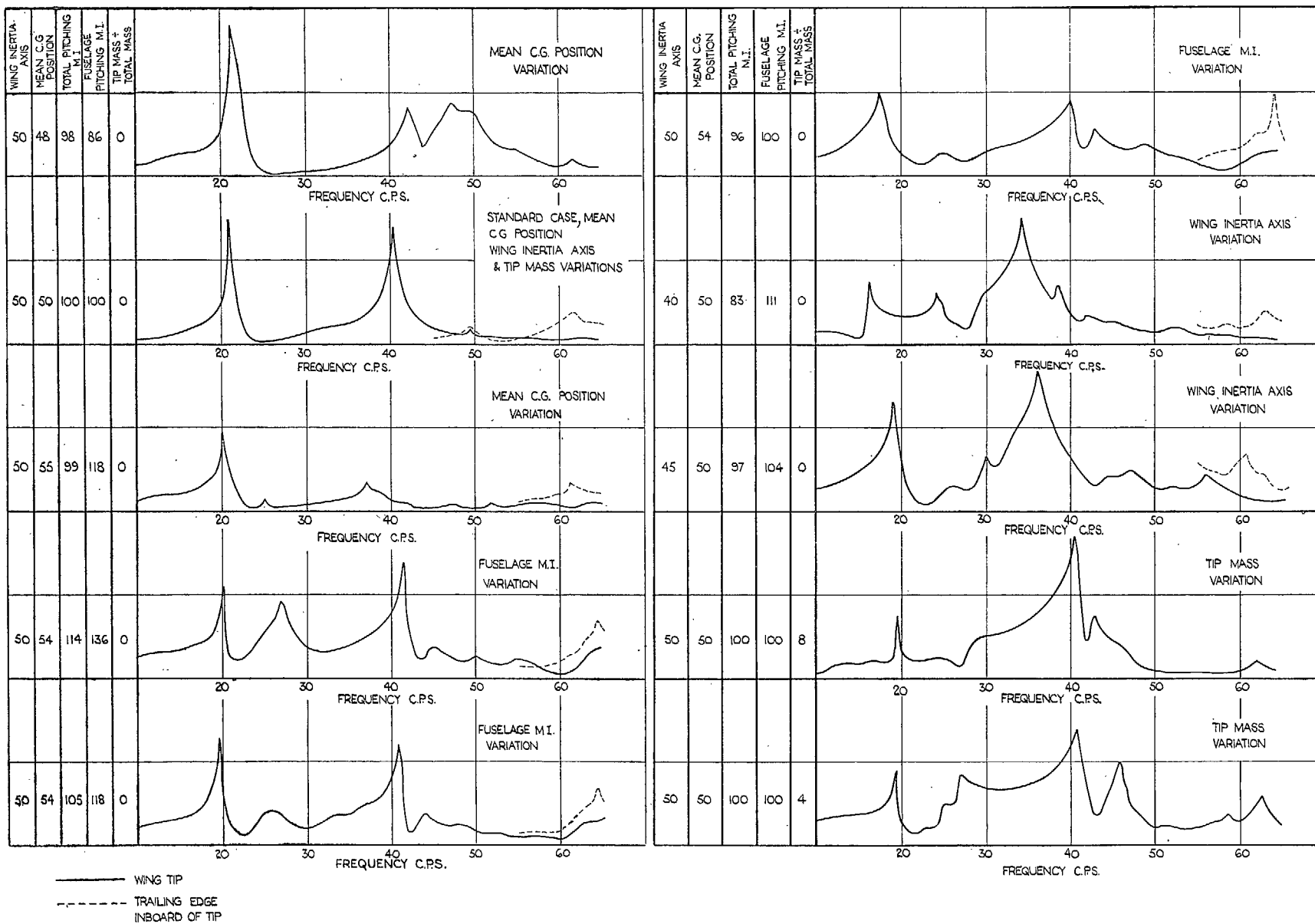


FIG. 3. Amplitude vs. frequency curves.

| INERTIA PARAMETERS | | | | | NATURAL FREQUENCIES & MODES OF VIBRATION | | |
|--------------------|--------------------|---------------------|------------------------|---------------------|--|------------------------------------|-------------------------|
| WING INERTIA AXIS. | MEAN C.G. POSITION | TOTAL PITCHING M.I. | FUSELAGE PITCHING M.I. | TIP MASS TOTAL MASS | FIRST MODE "FUNDAMENTAL" | SECOND MODE "FLEXURAL OVERTONE" | THIRD MODE "TORSION" |
| 50 | 48 * | 98 | 86 | 0 | 21.2 (5) | 42.1 (15) | 60.9 (25) |
| 50 | 50 * | 100 | 100 | 0 * | 21.0 (6) | 40.7 (16) | 61.9 (26) |
| 50 | 55 * | 99 | 118 | 0 | 20.2 (7) | 37.1 (17) | 61.5 (27) |
| 50 | 54 * | 114 | 136 * | 0 | 20.1 (8) | 41.4 (18) | 64.7 (28) |
| 50 | 54 * | 105 | 118 * | 0 | 19.5 (9) | 41.0 (19) | 64.3 (29) |
| 50 | 54 * | 96 | 100 * | 0 | 17.5 (10) | 40.1 (20) | 63.8 (30) |
| 40 * | 50 | 63 | 111 | 0 | 16.3 (11) | 34.8 (21) | 63.0 (31) |
| 45 * | 50 | 97 | 104 | 0 | 19.0 (12) | 37.1 (22) | 60.7 (32) |
| 50 | 50 | 100 | 100 | 8 * | 19.6 (13) | 40.3 (23) | 60.9 (33) |
| 50 | 50 | 100 | 100 | 4 * | 19.5 (14) | 40.9 (24) | 62.4 (34) |

THE FIGURE ON THE LEFT OF EACH DIAGRAM GIVES THE FREQUENCY IN CYCLES PER SECOND. THE FIGURE ON THE RIGHT IN BRACKETS REFERS TO THE FIGURE NUMBER OF THE DETAILED DIAGRAM
* DENOTES THE PARAMETER VARIATION CONCERNED.

FIG. 4. Summary of modes.

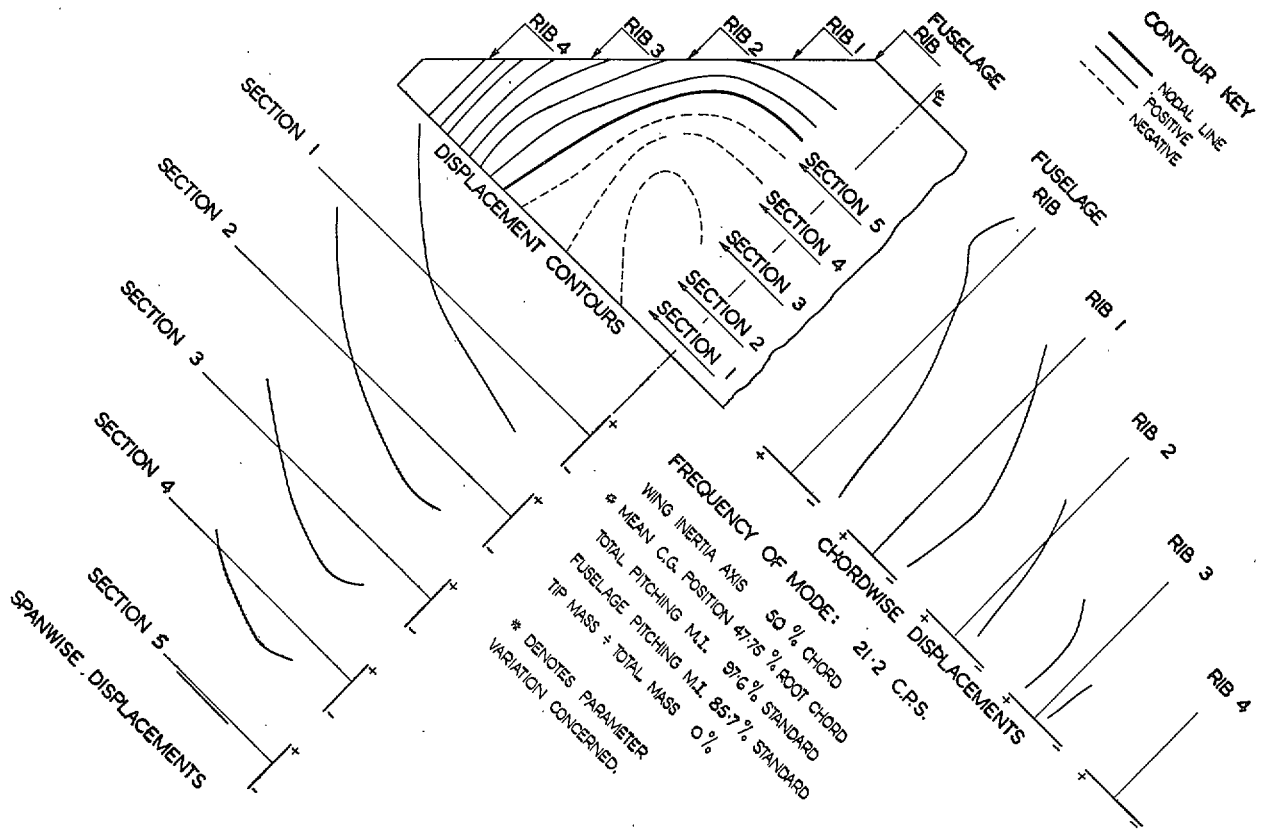


FIG. 5.

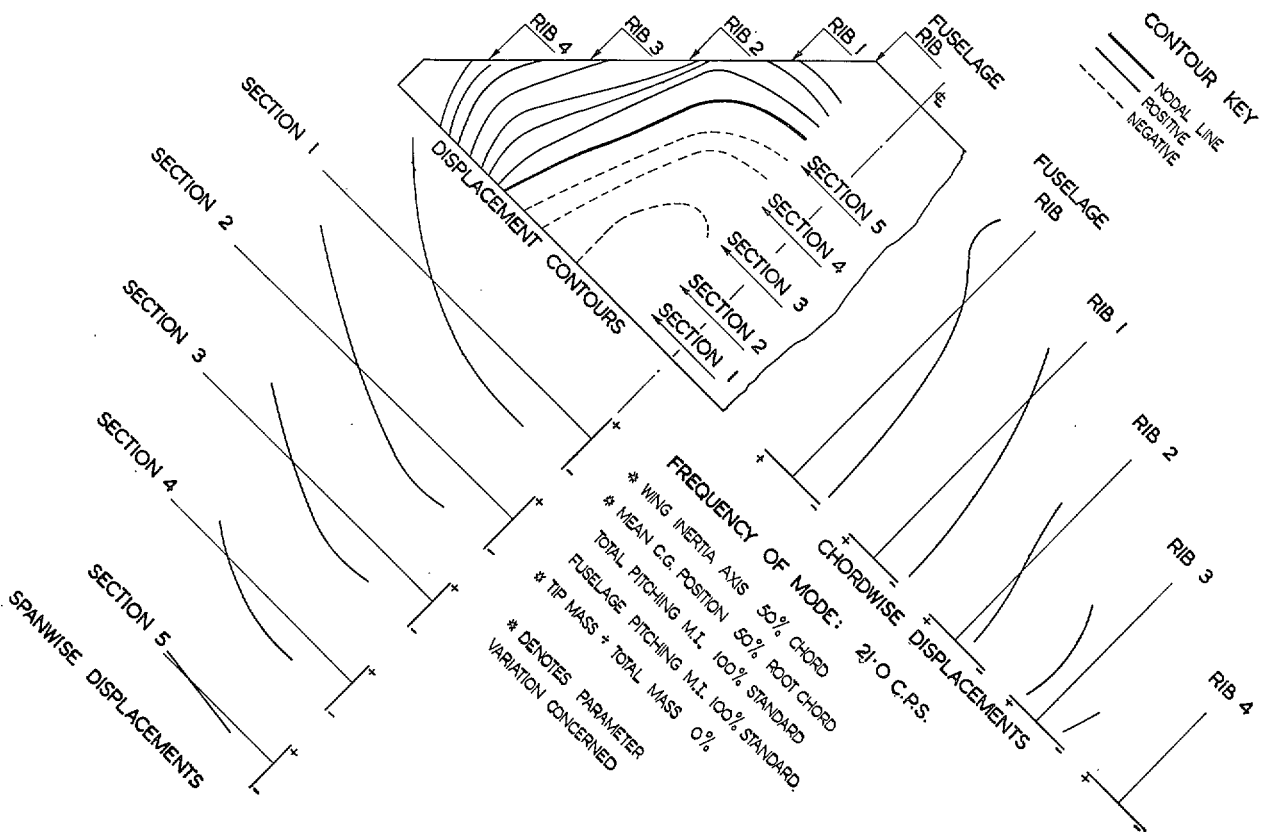


FIG. 6.

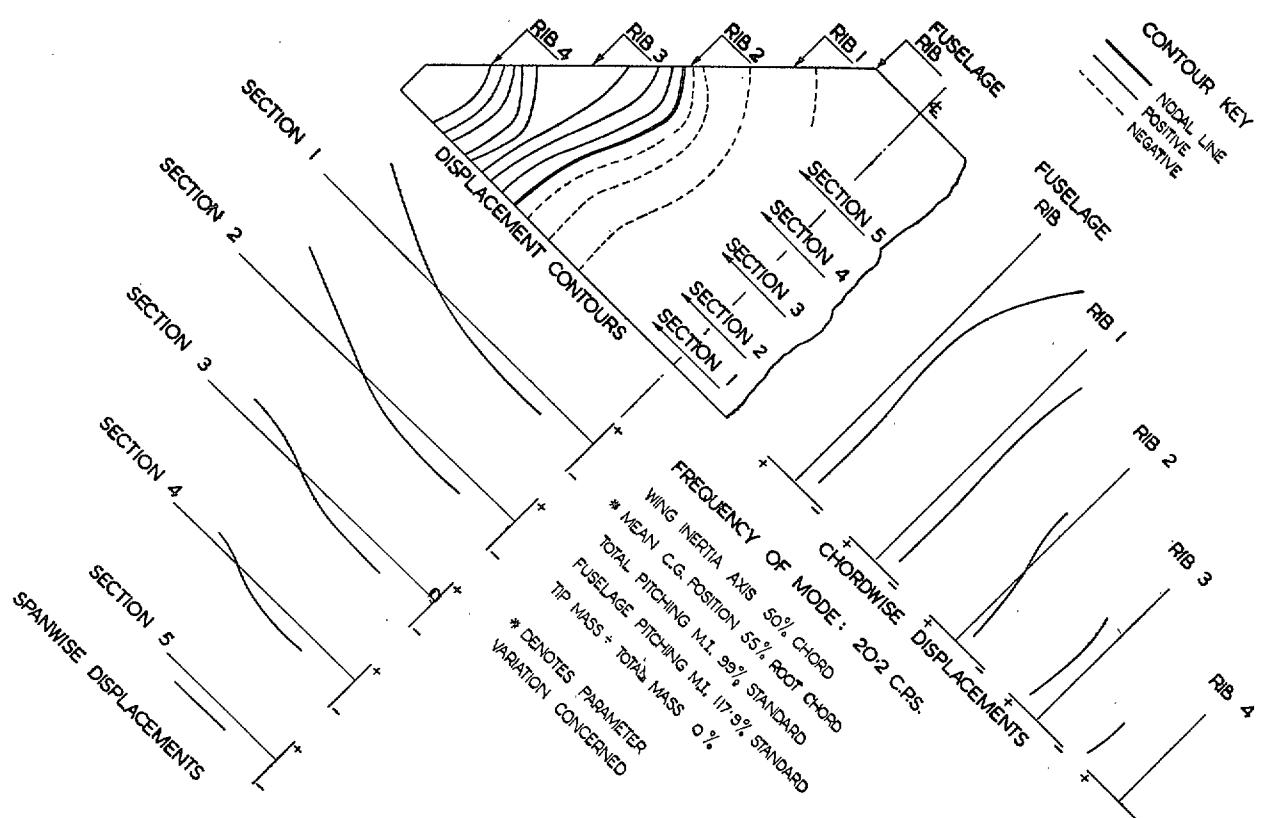


FIG. 7.

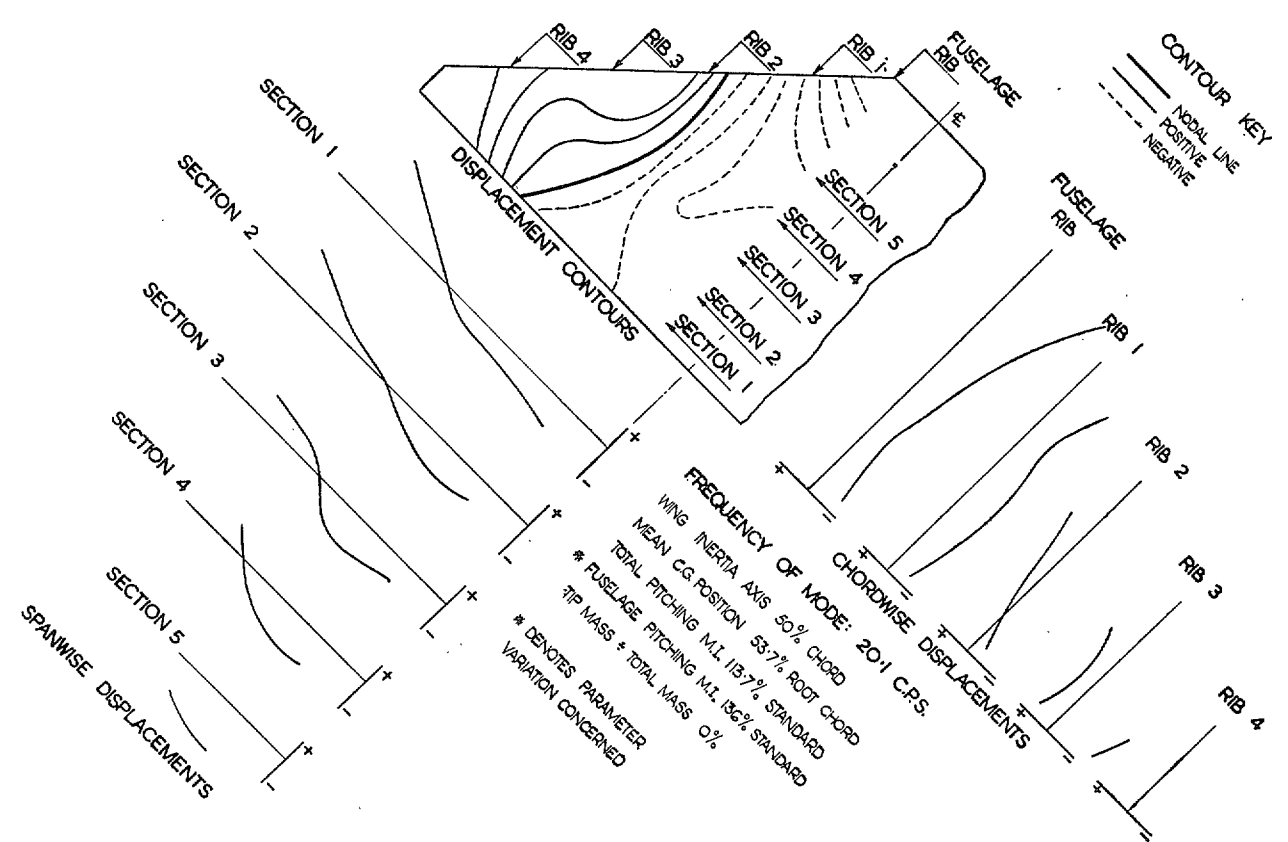


FIG. 8.

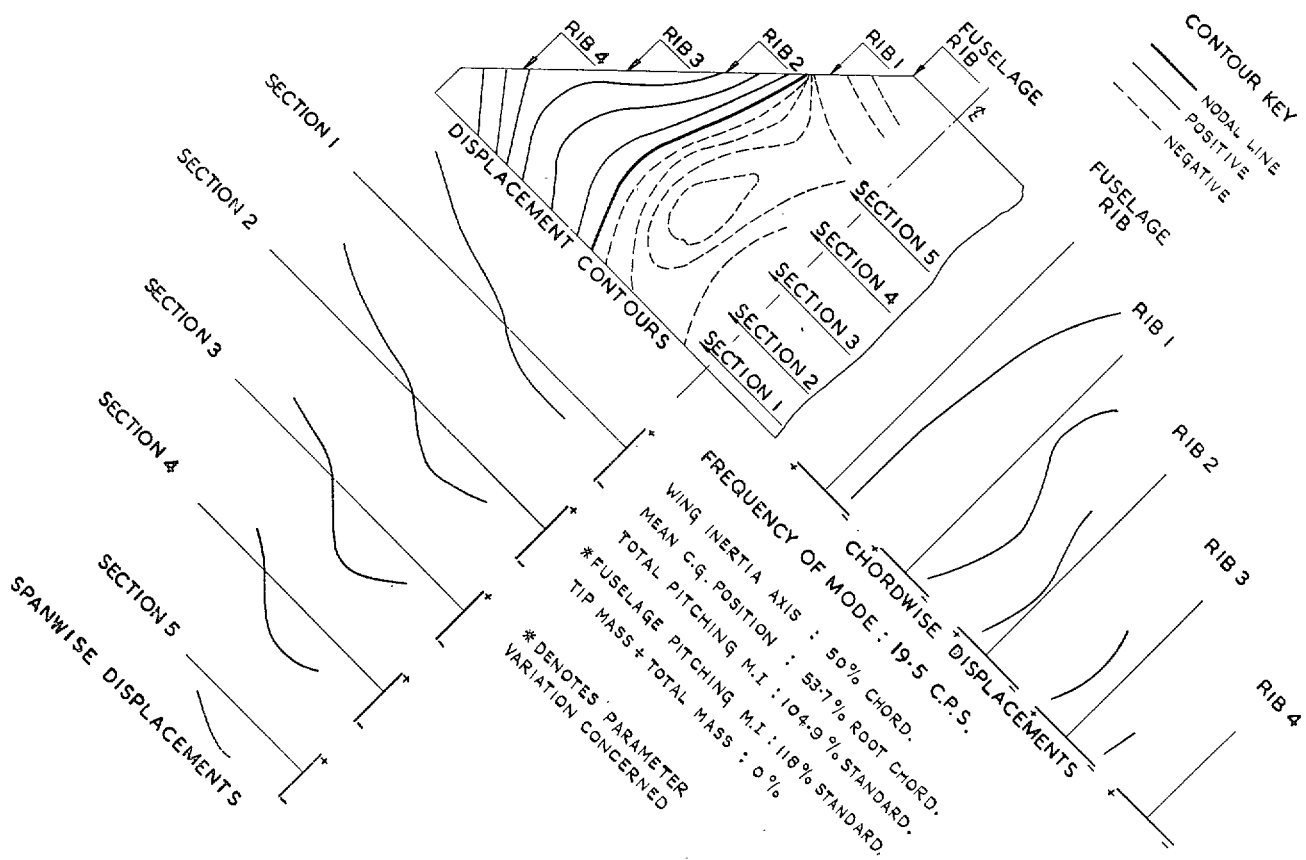


FIG. 9.

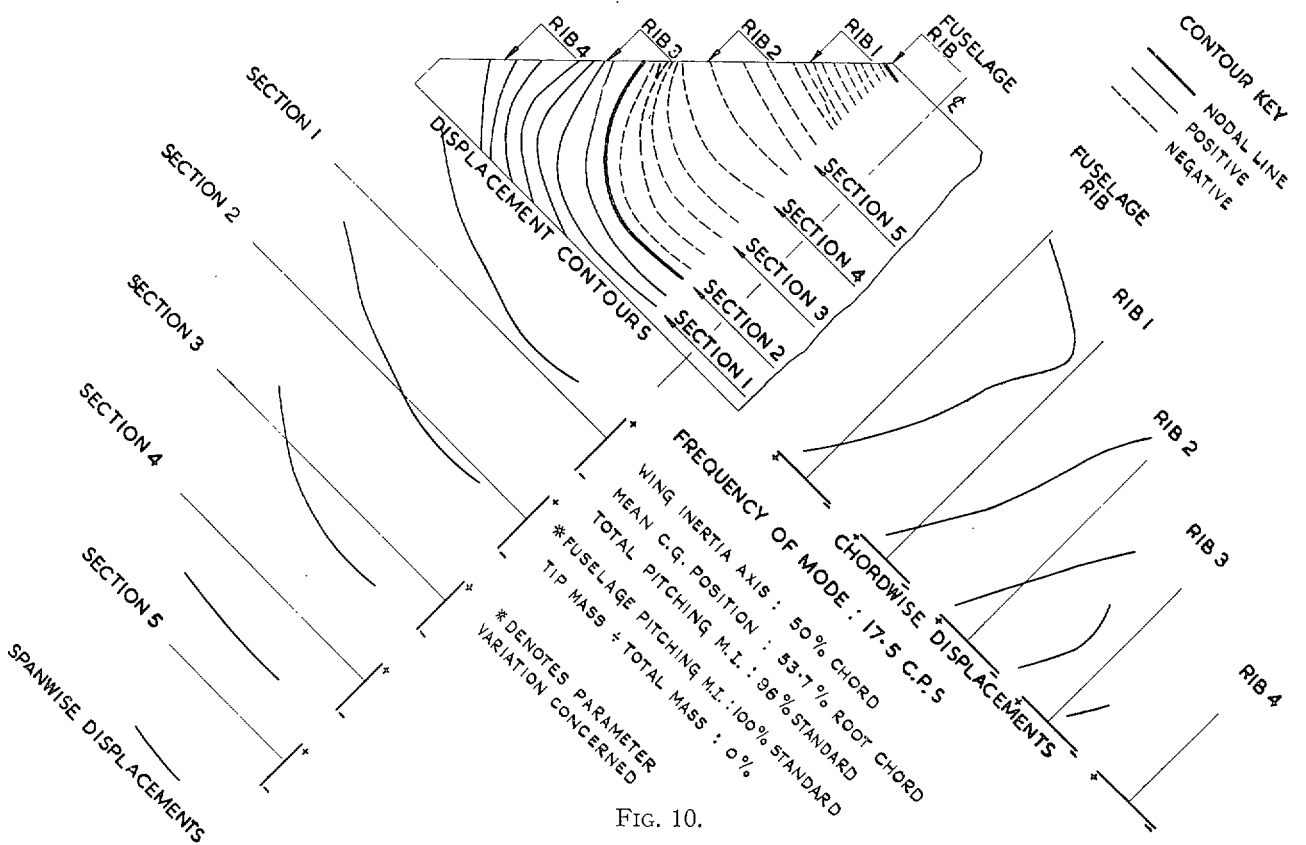


FIG. 10.

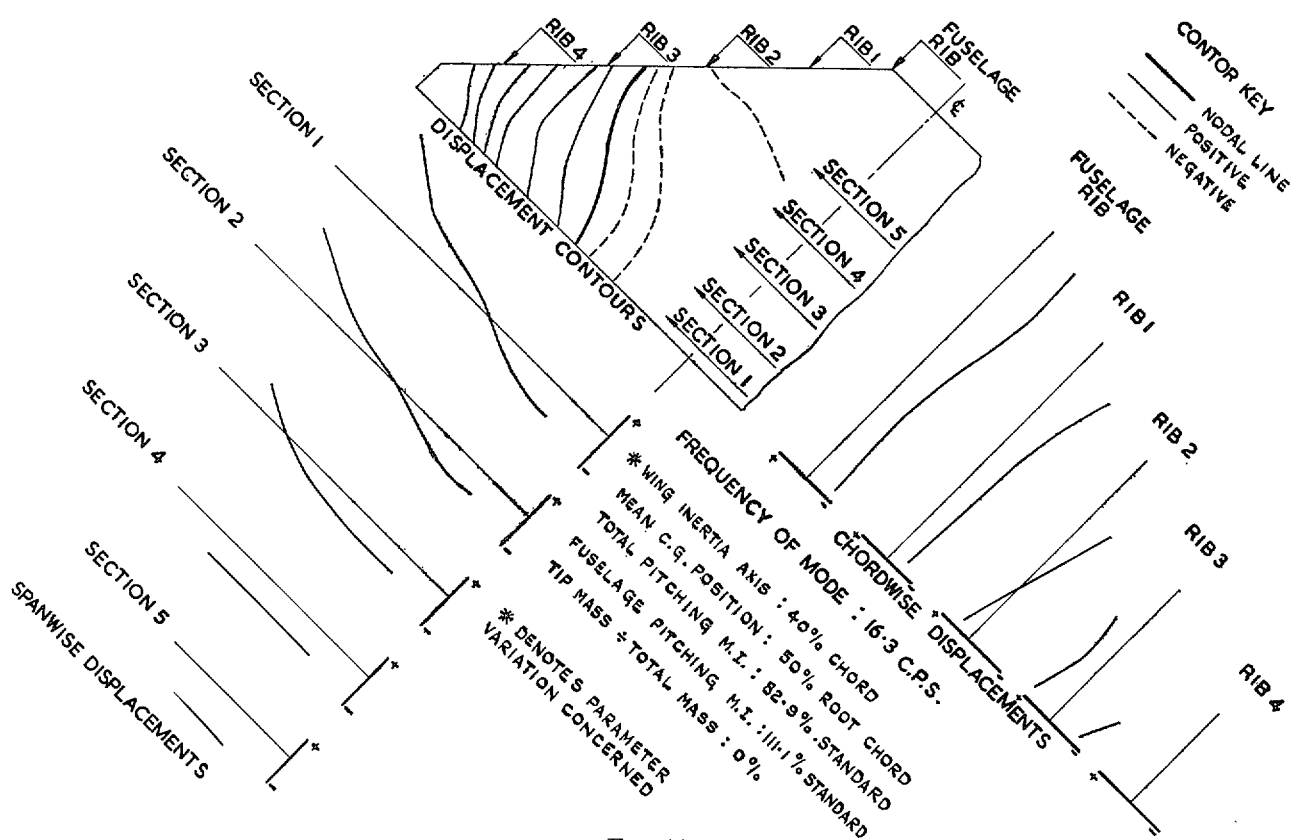


FIG. 11.

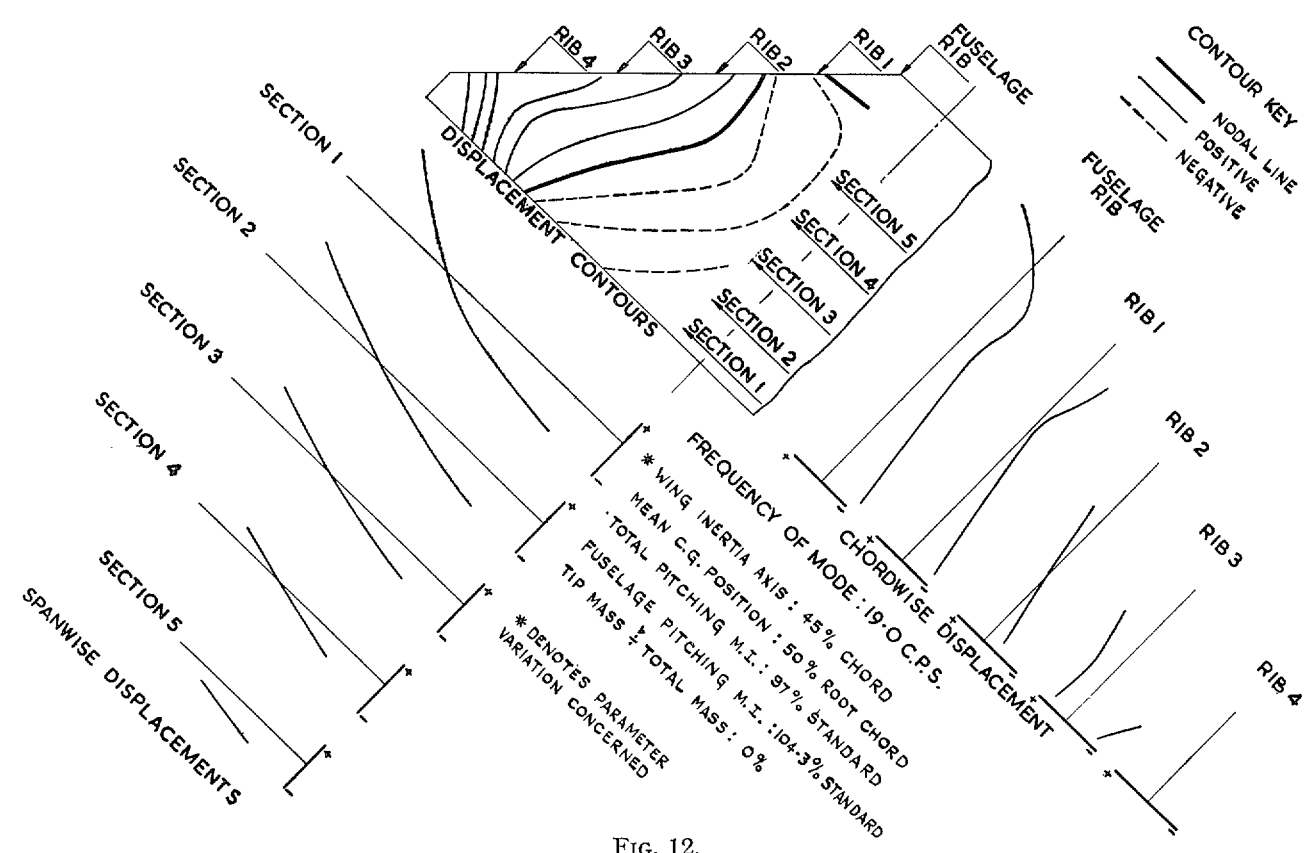


FIG. 12.

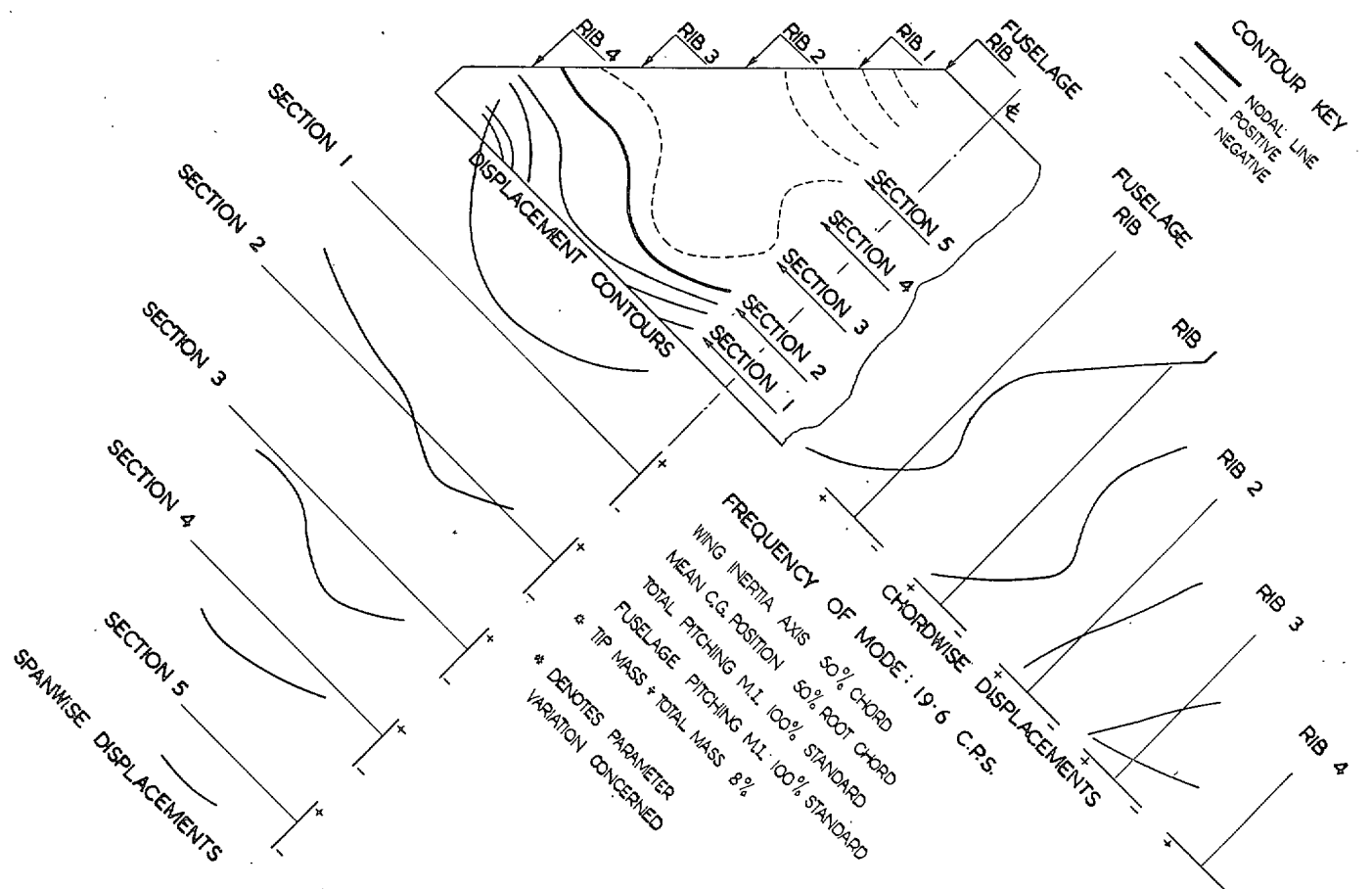


FIG. 13.

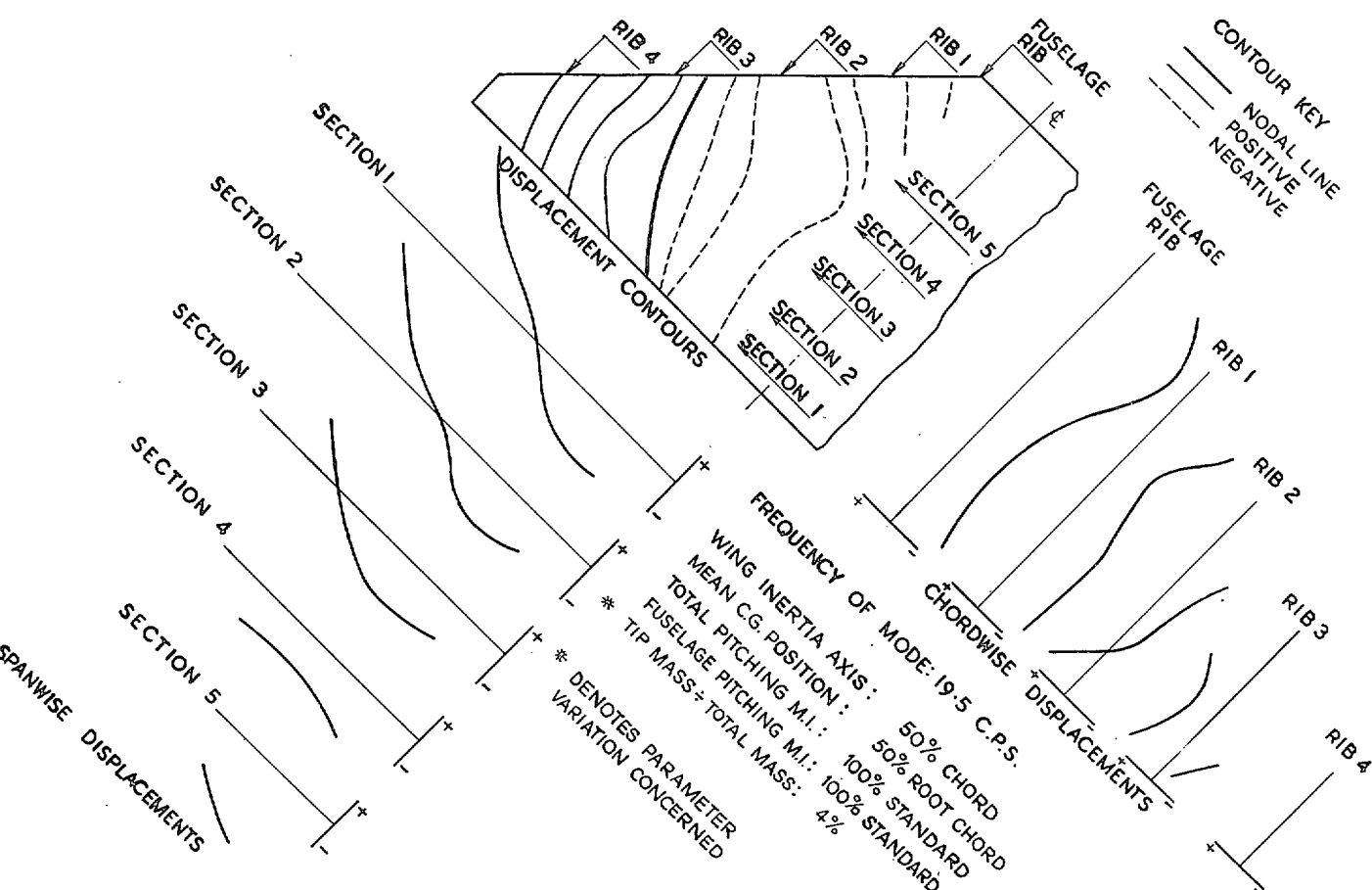


FIG. 14.

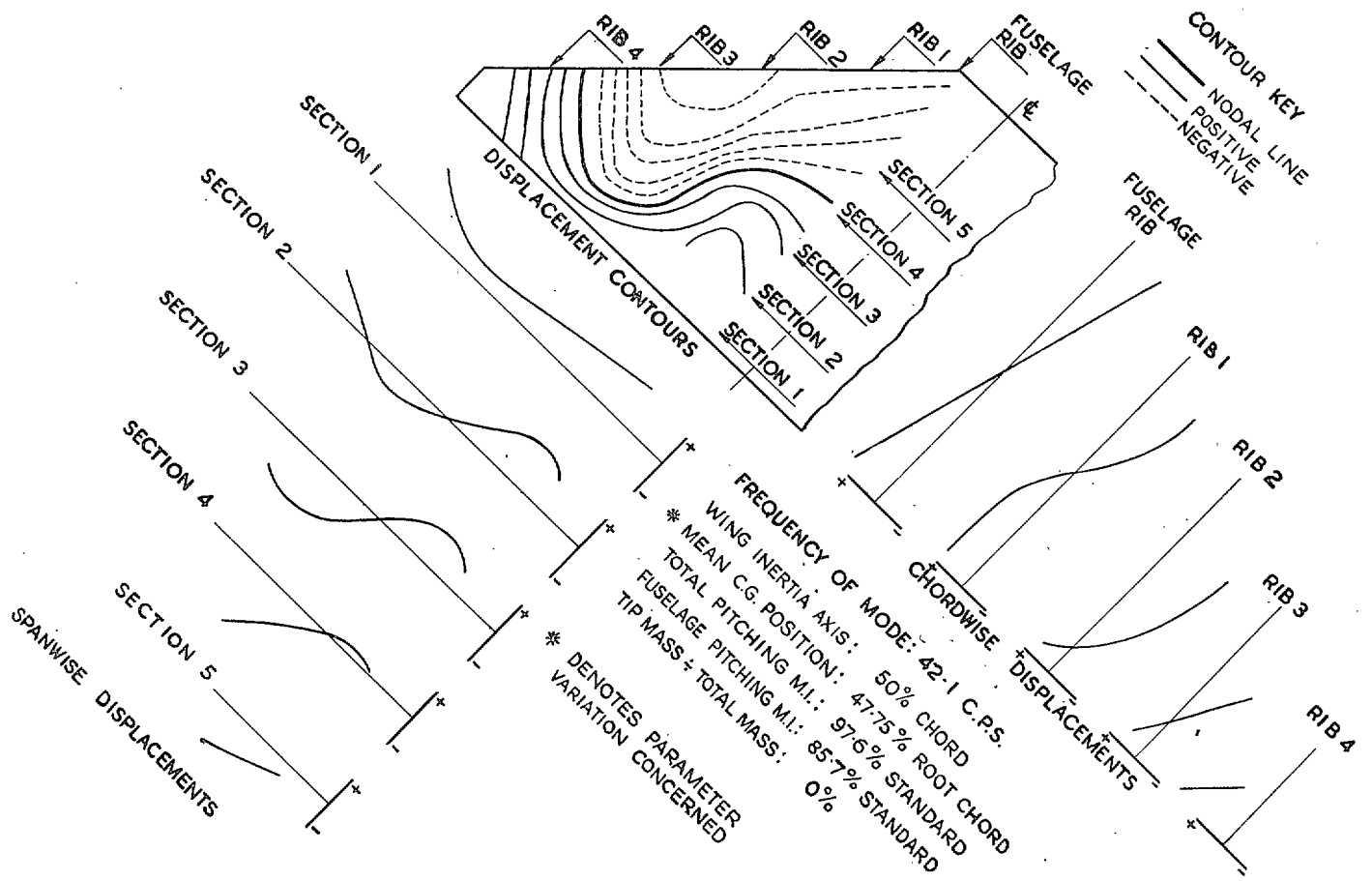


FIG. 15.

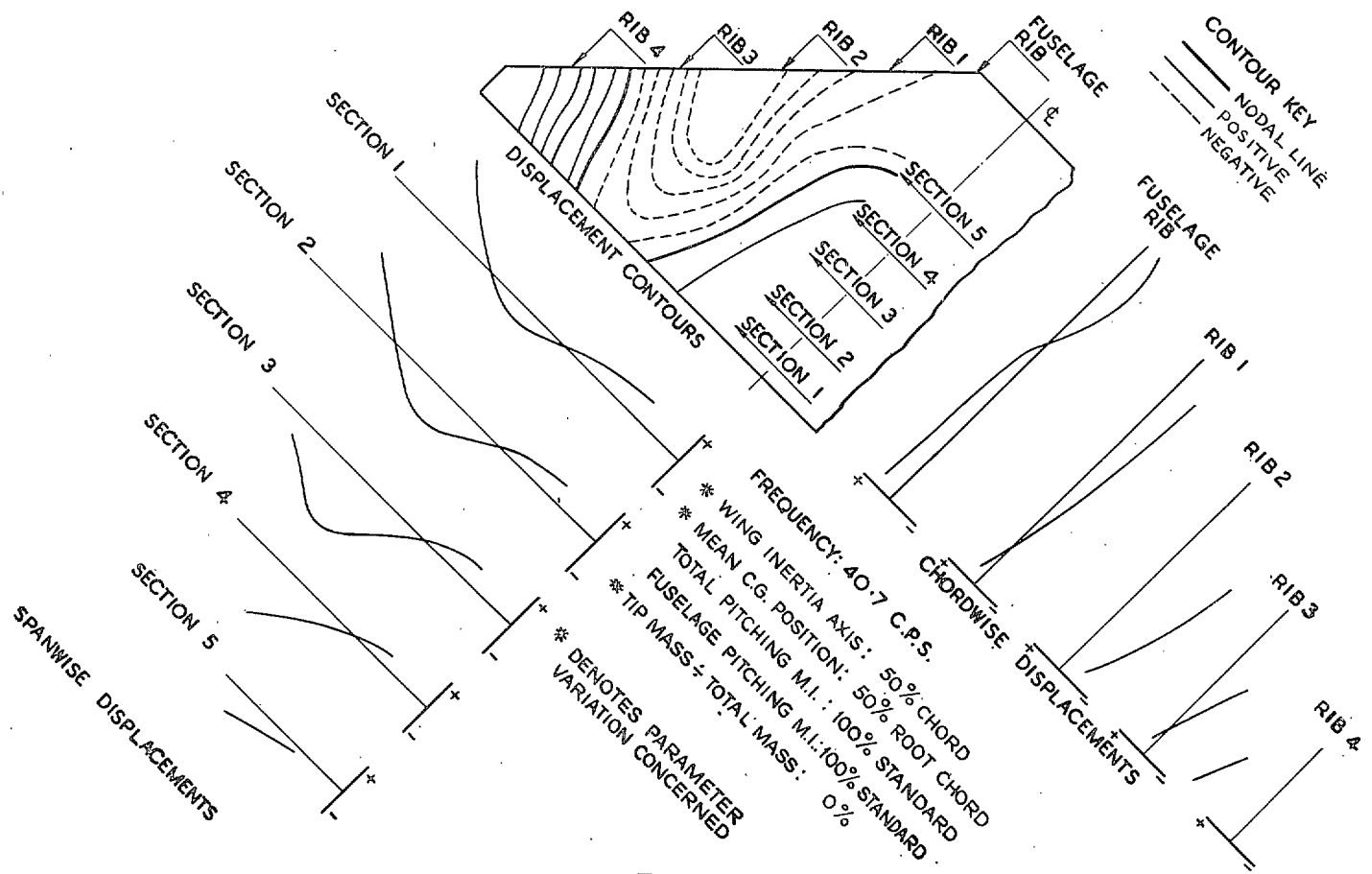


FIG. 16.

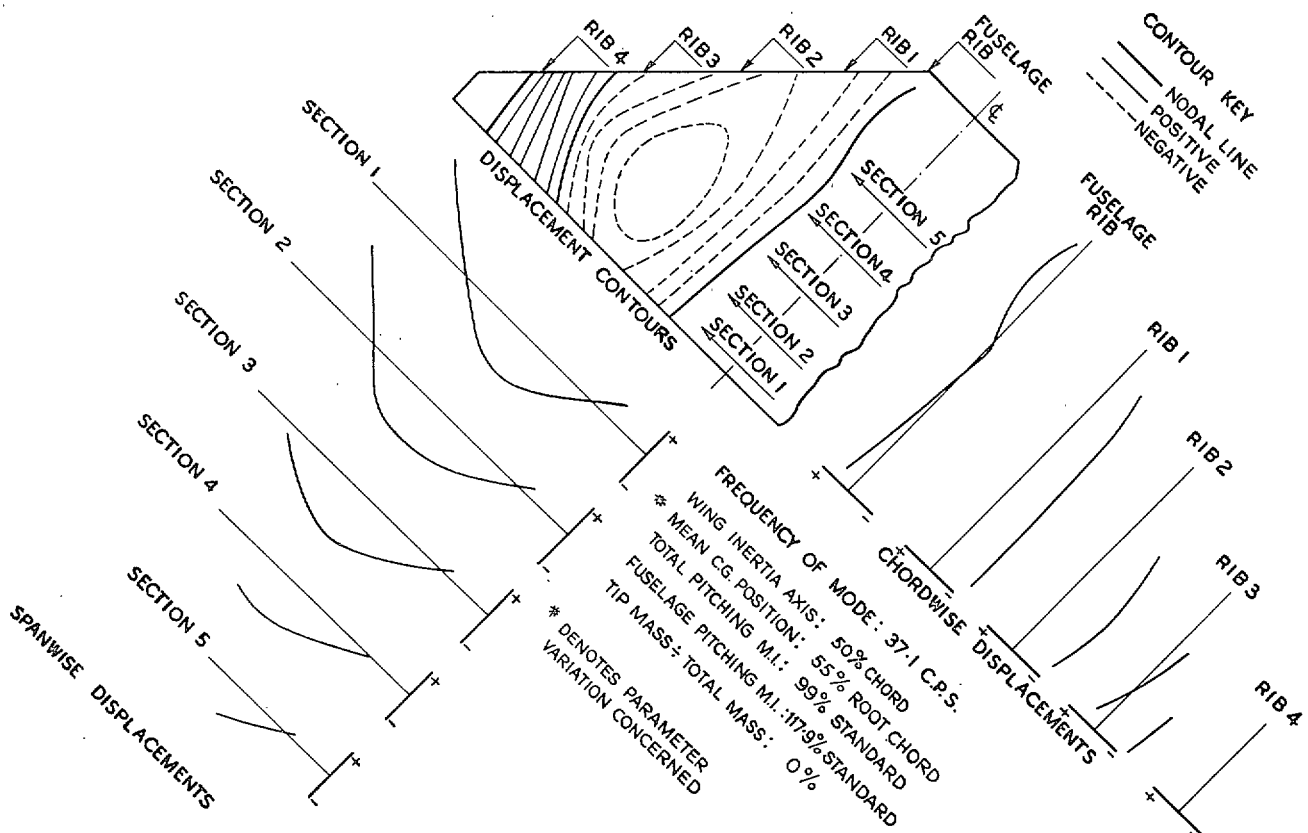


FIG. 17.

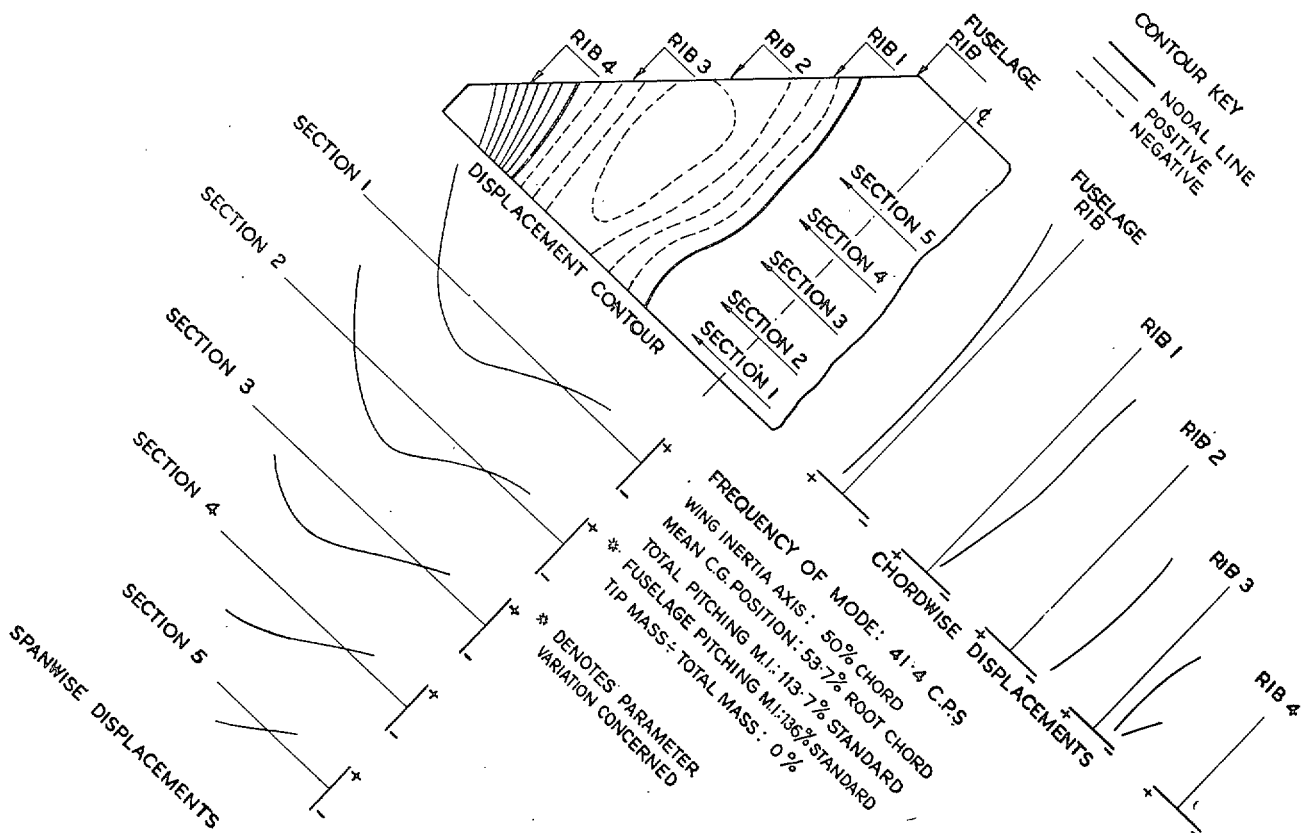


FIG. 18.

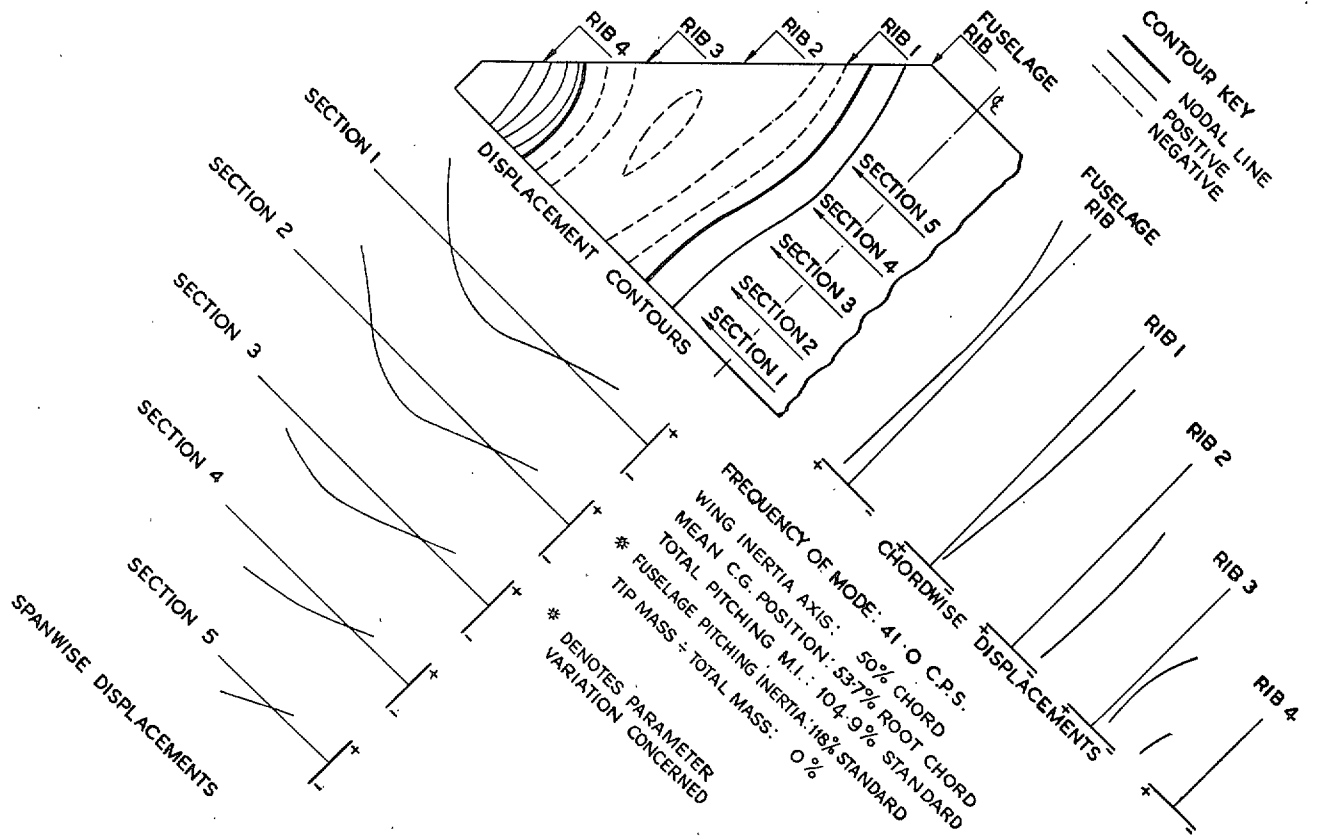


FIG. 19.

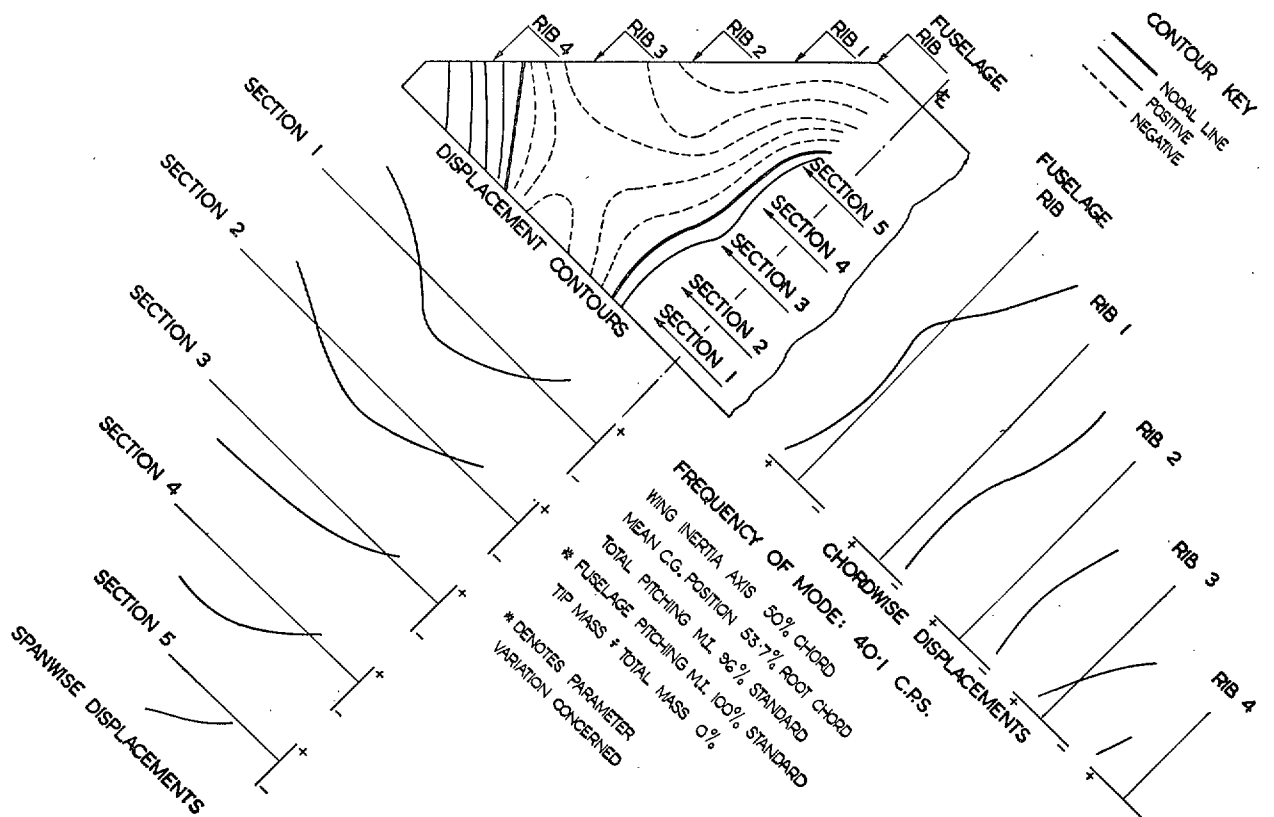


FIG. 20.

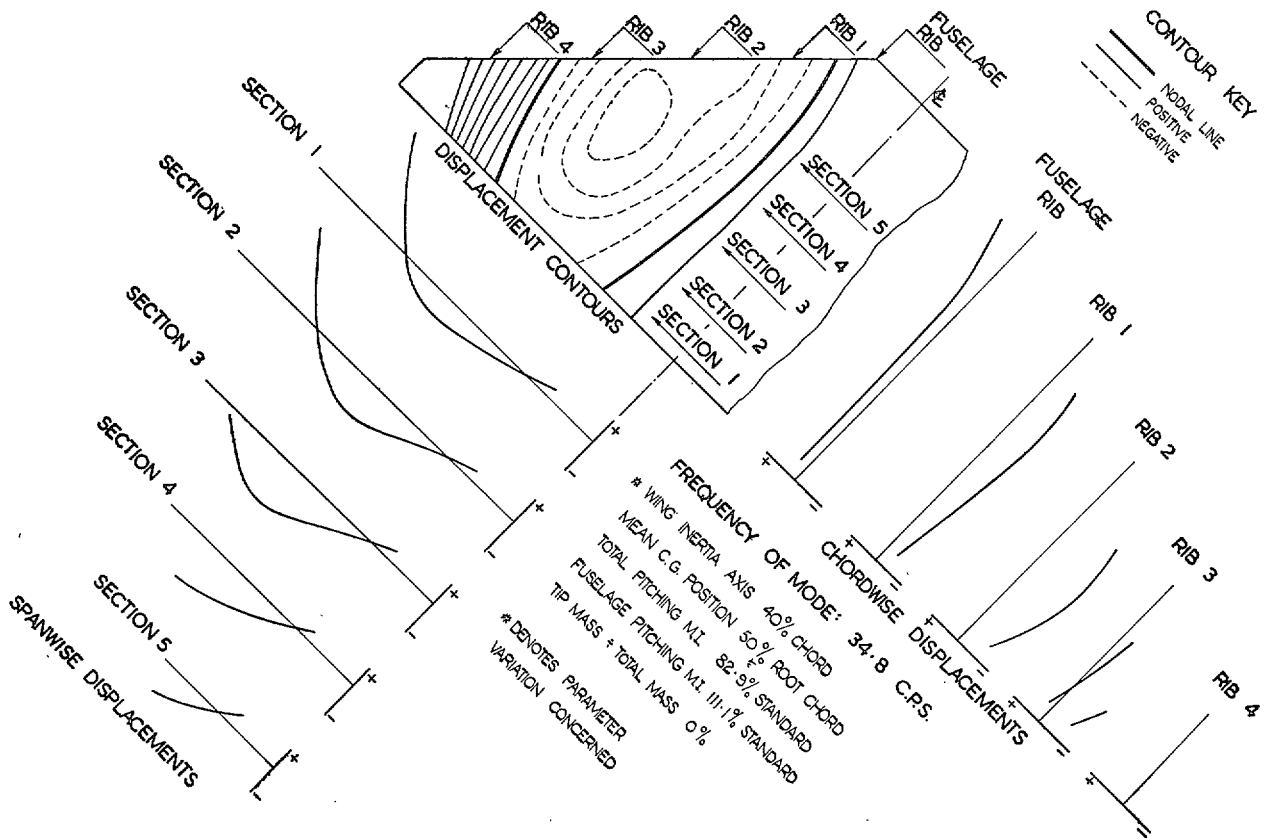


FIG. 21.

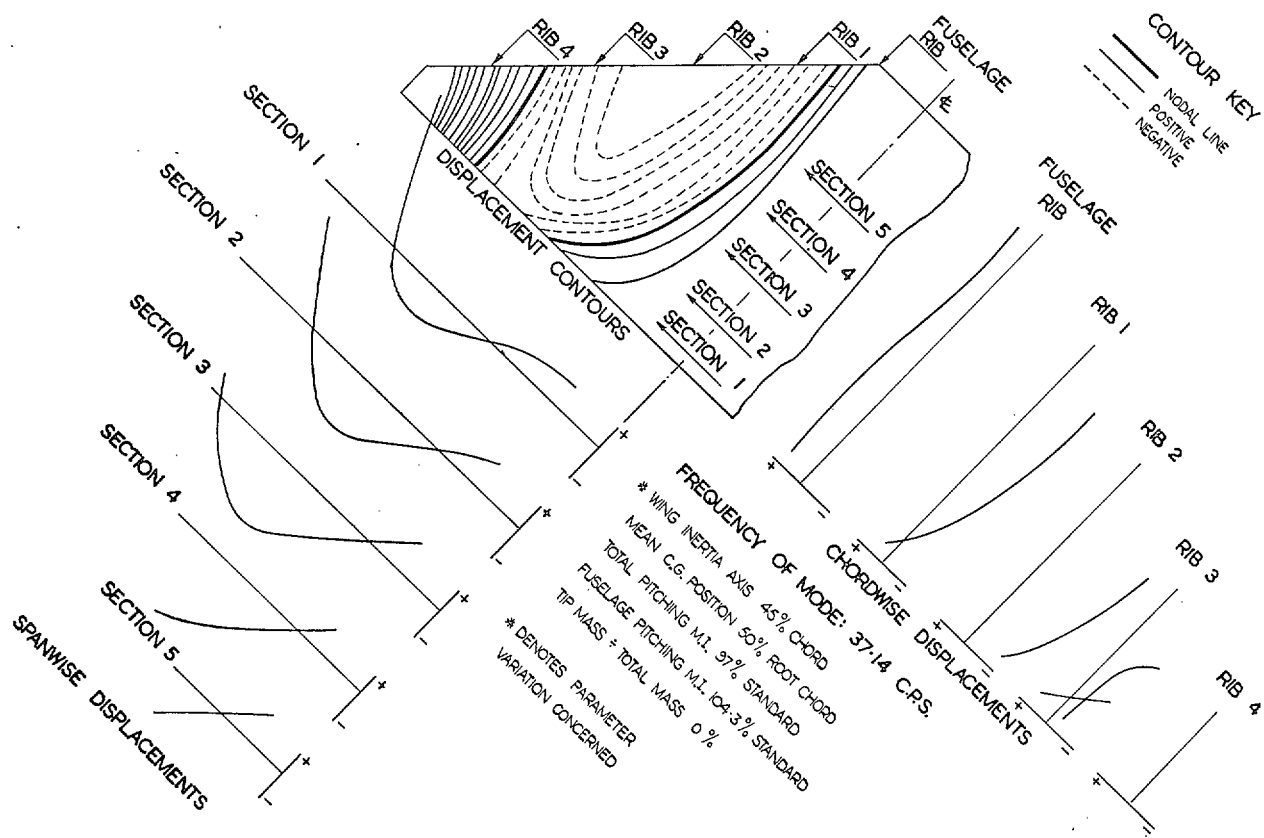


FIG. 22.

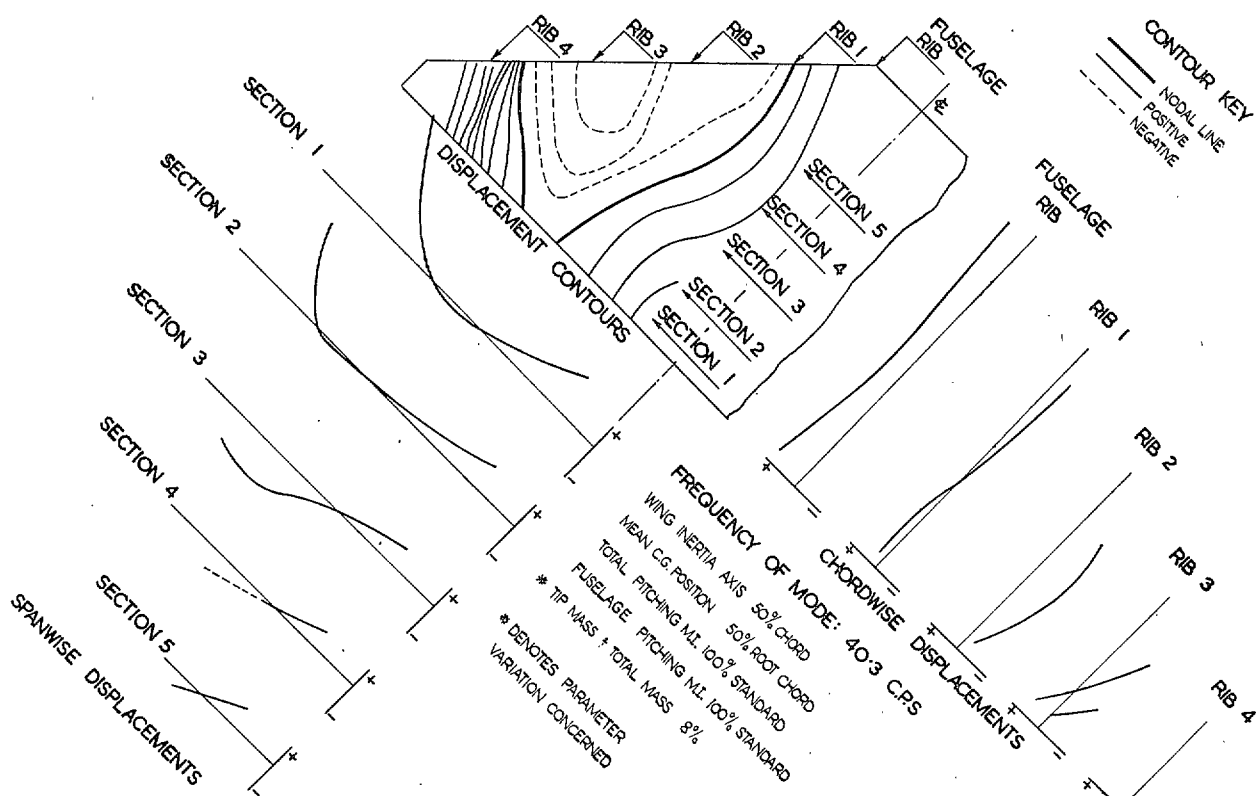


FIG. 23.

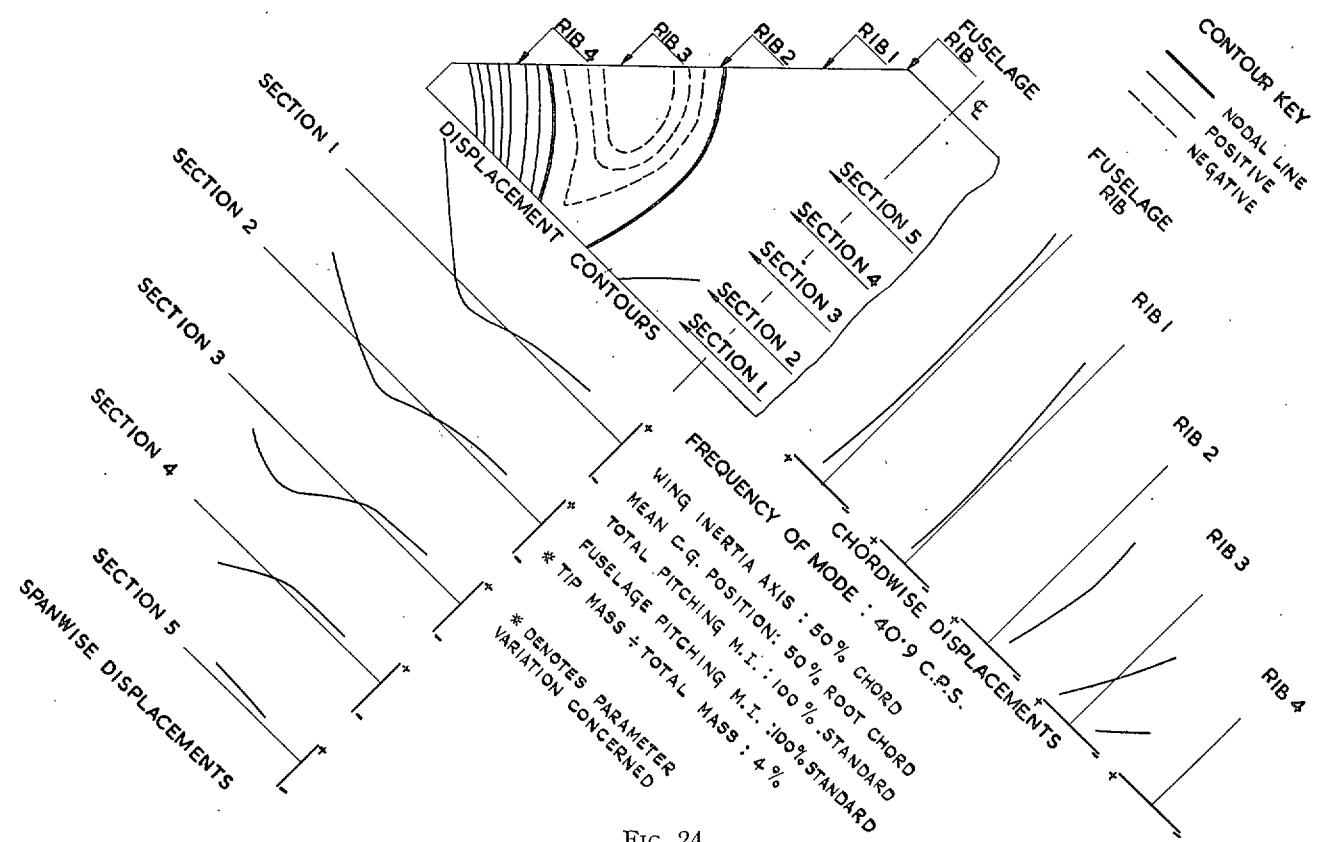


FIG. 24.

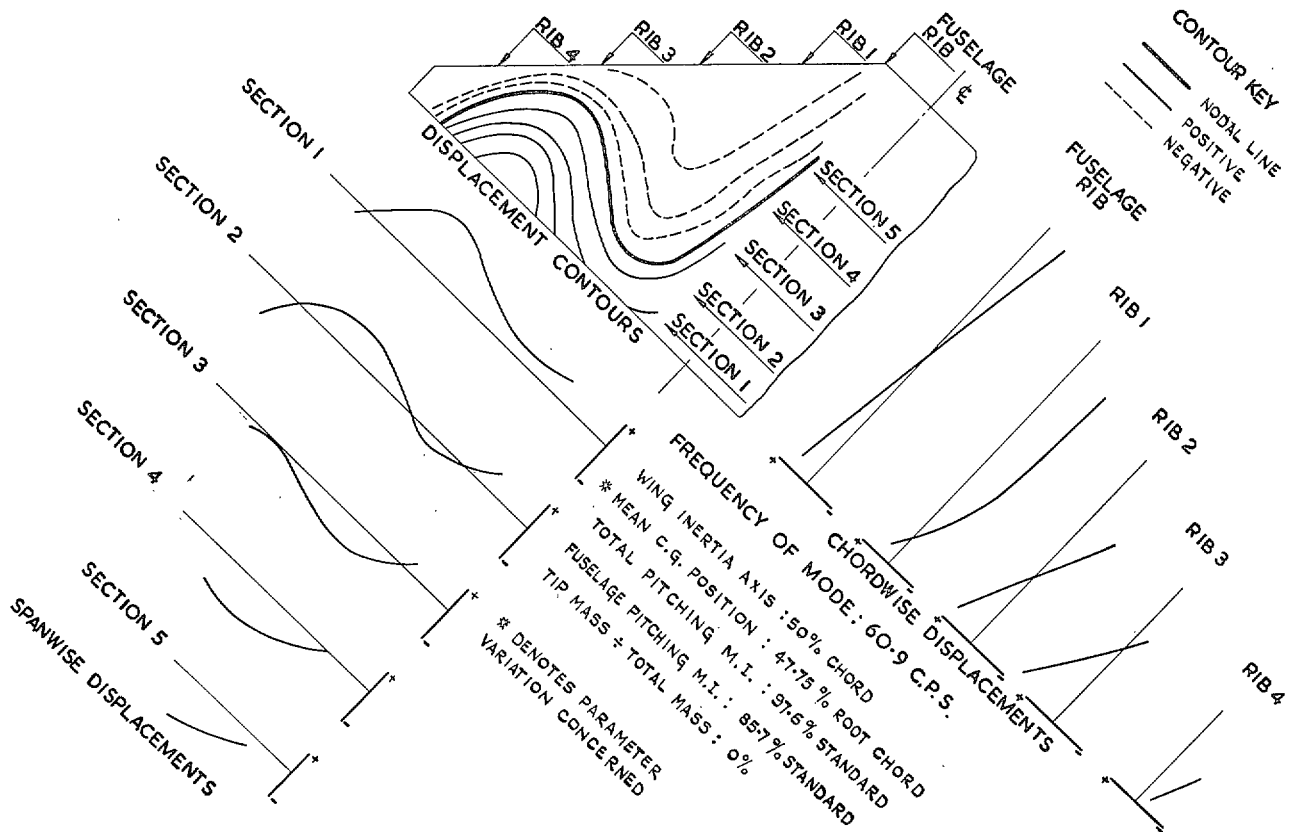


FIG. 25.

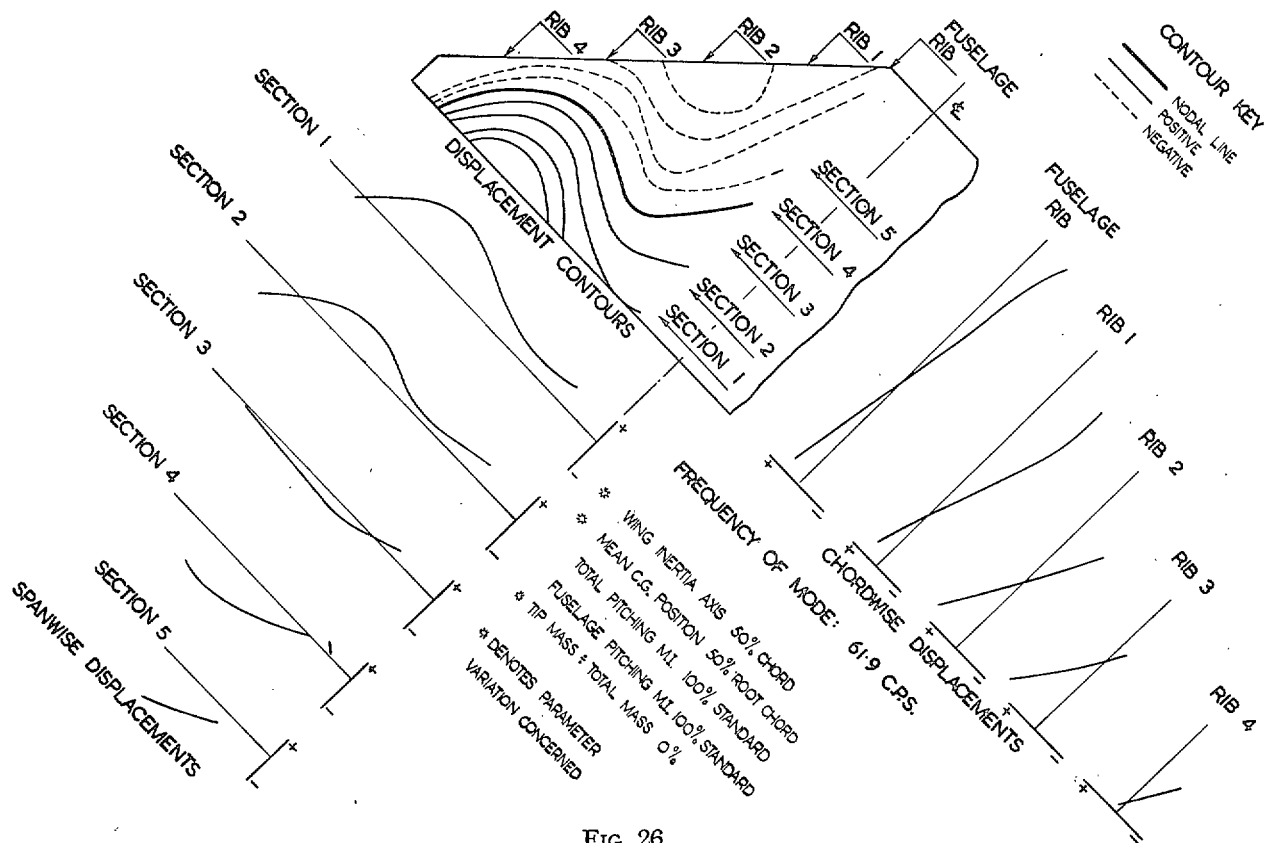


FIG. 26.

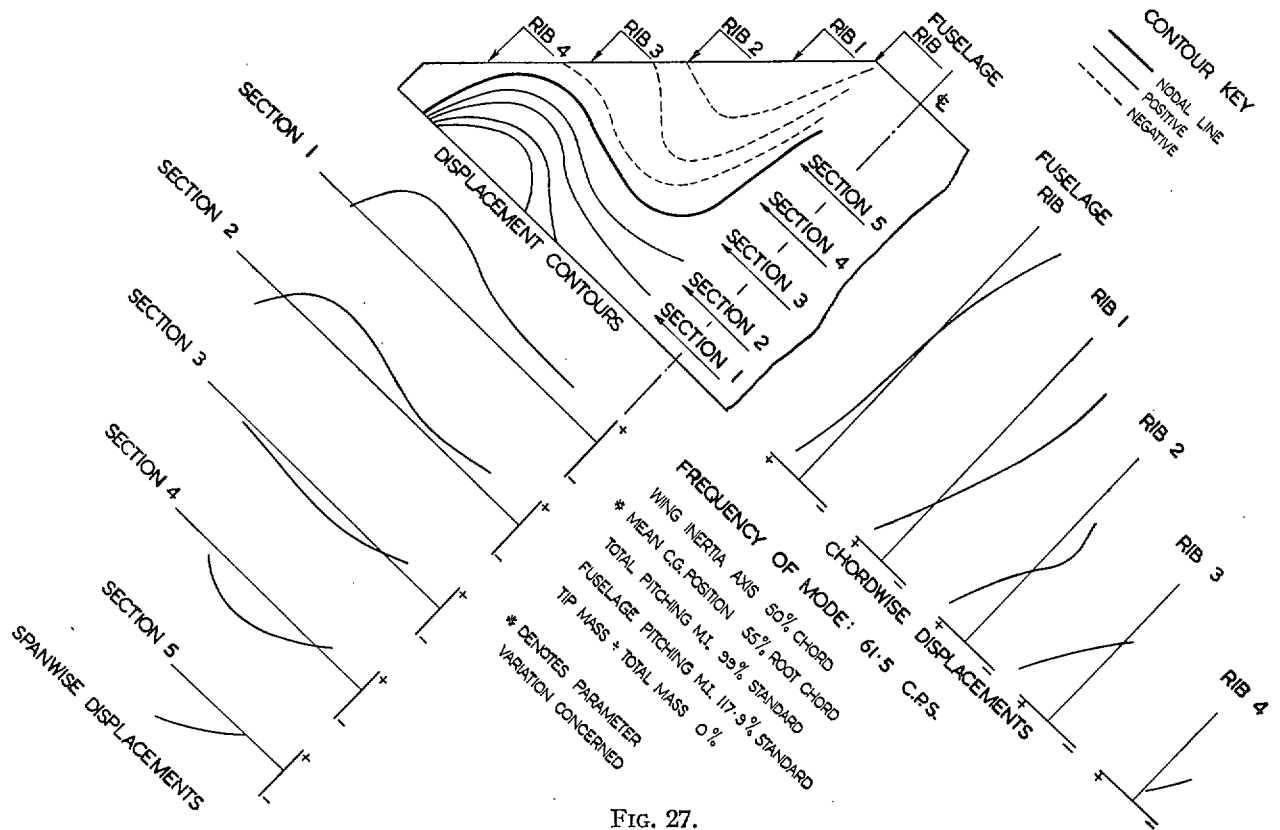


FIG. 27.

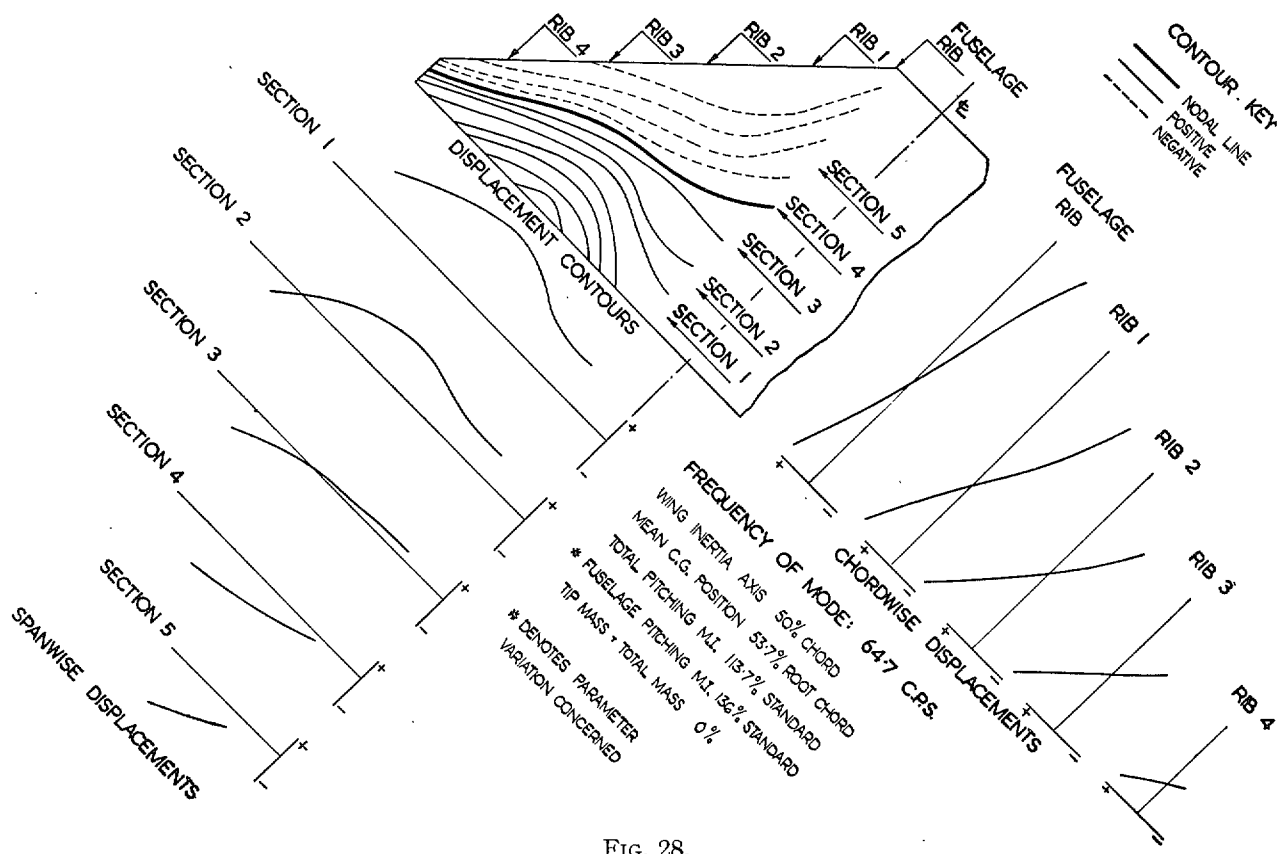


FIG. 28.

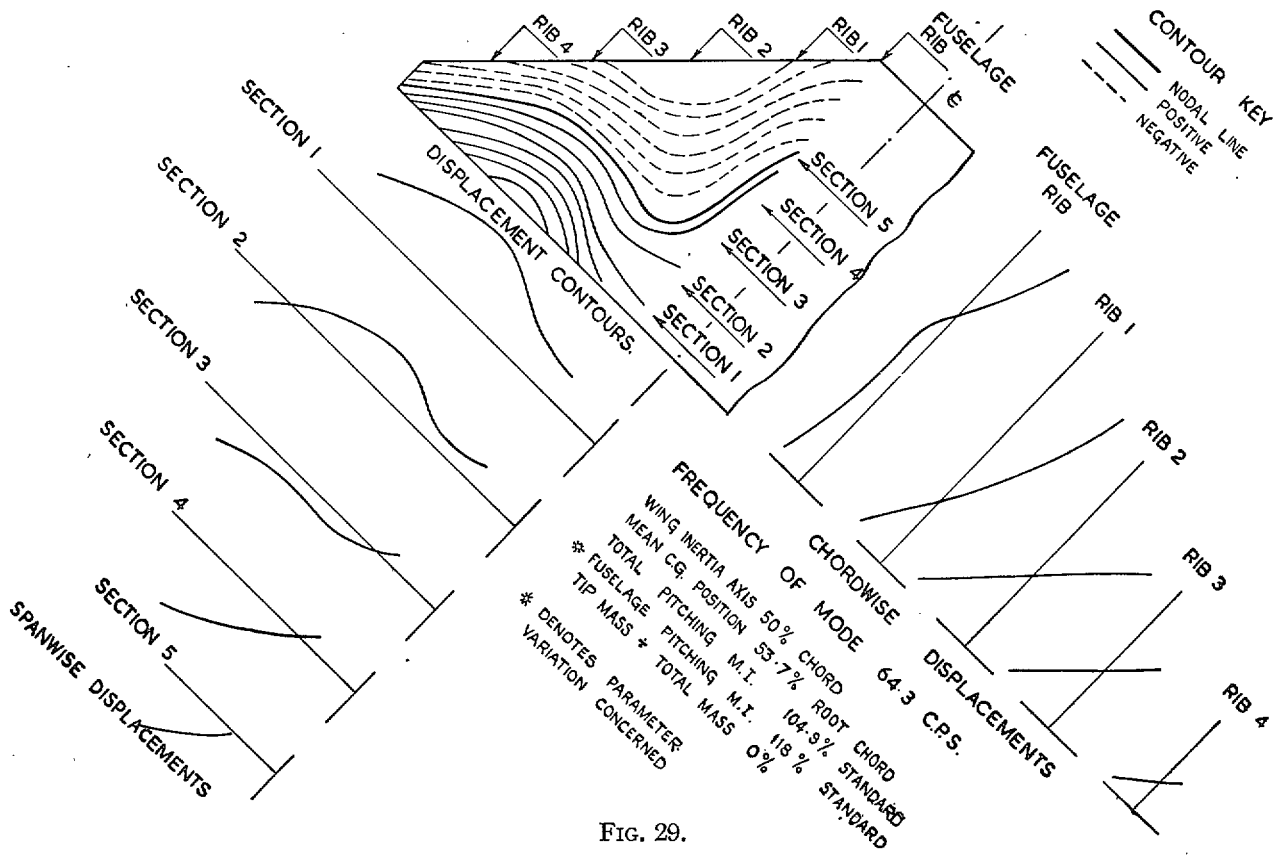


FIG. 29.

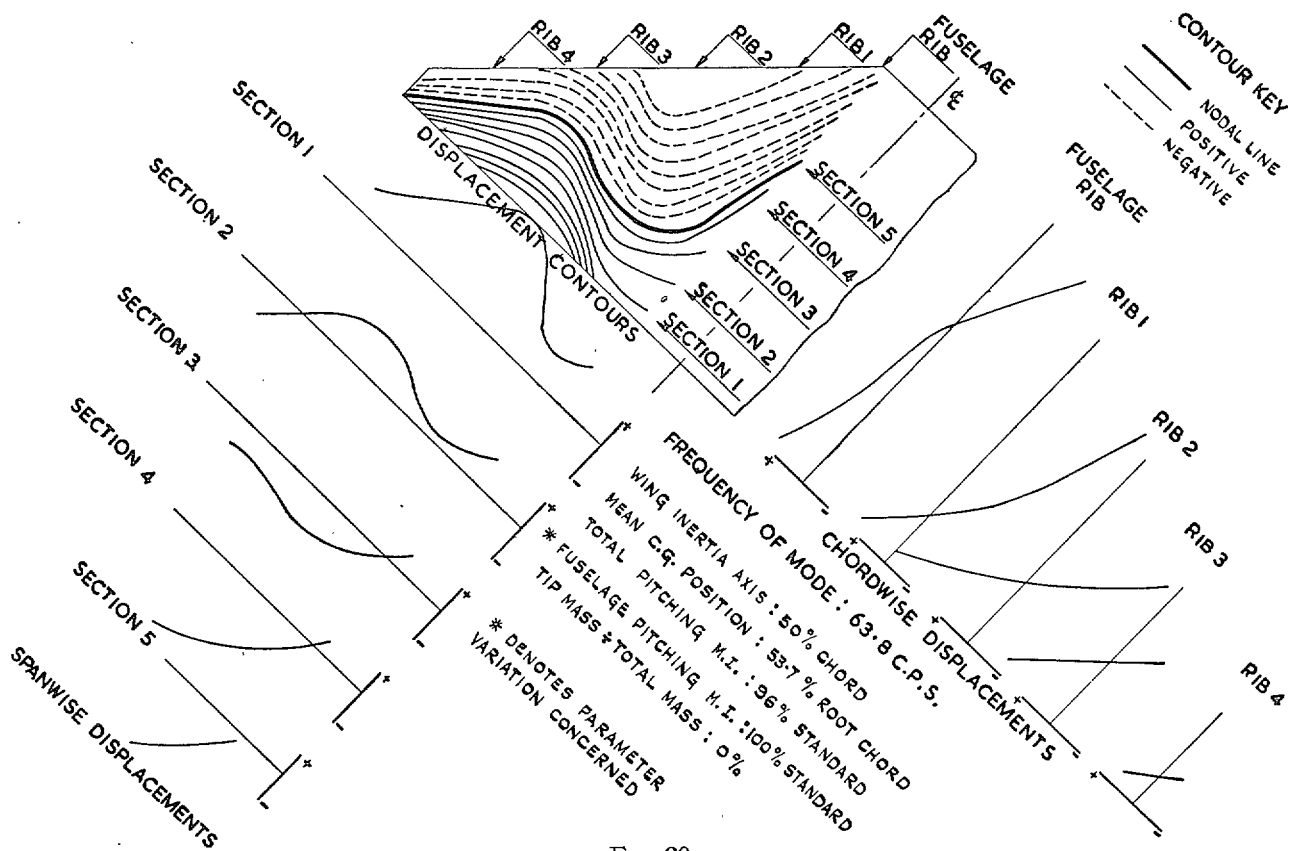


FIG. 30.

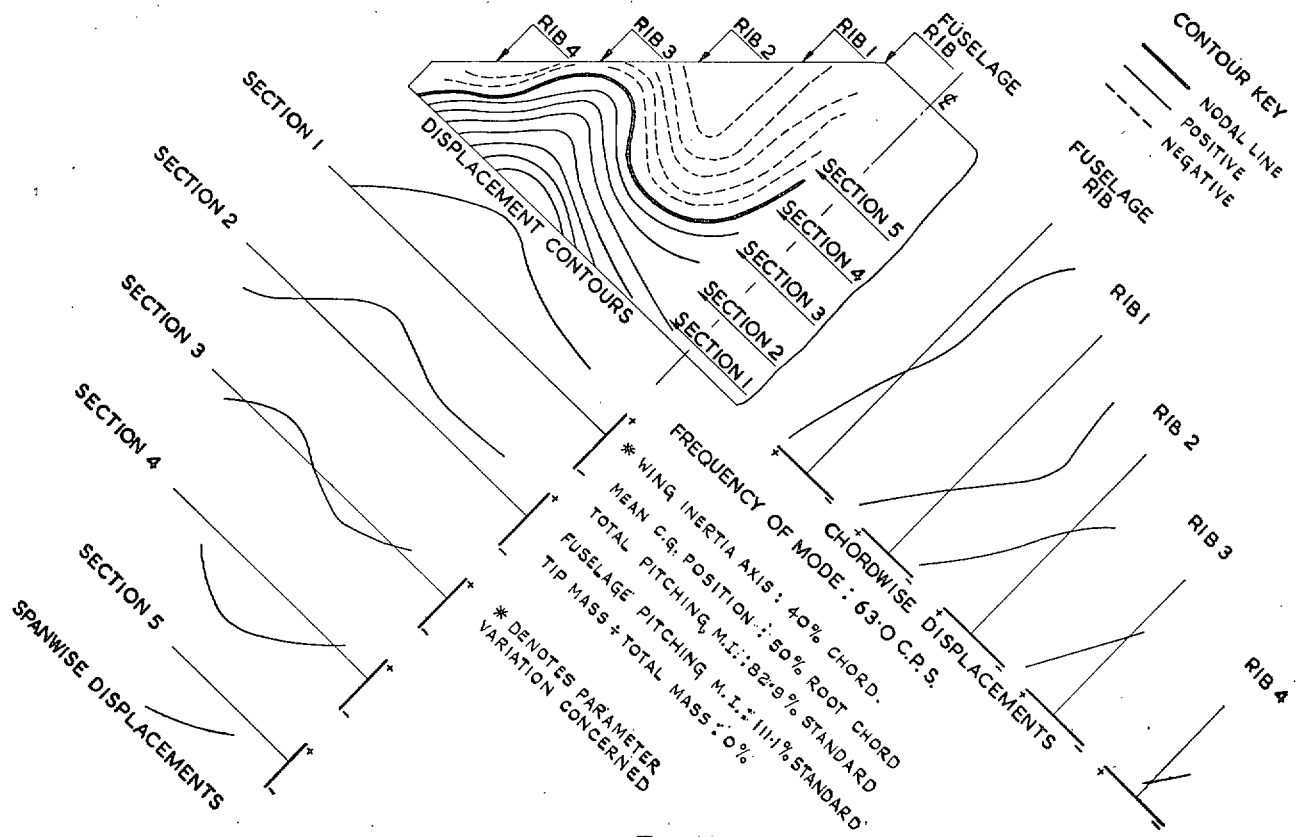


FIG. 31.

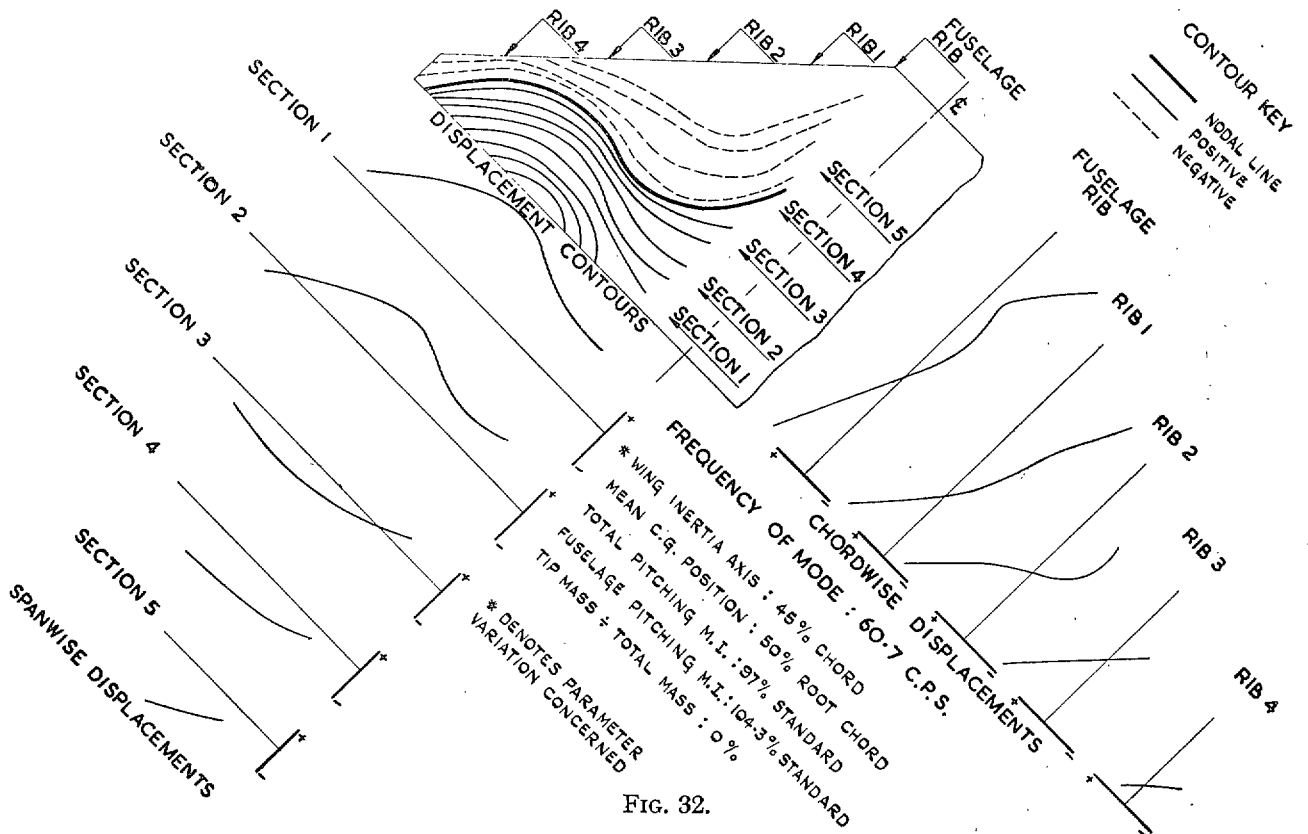


FIG. 32.

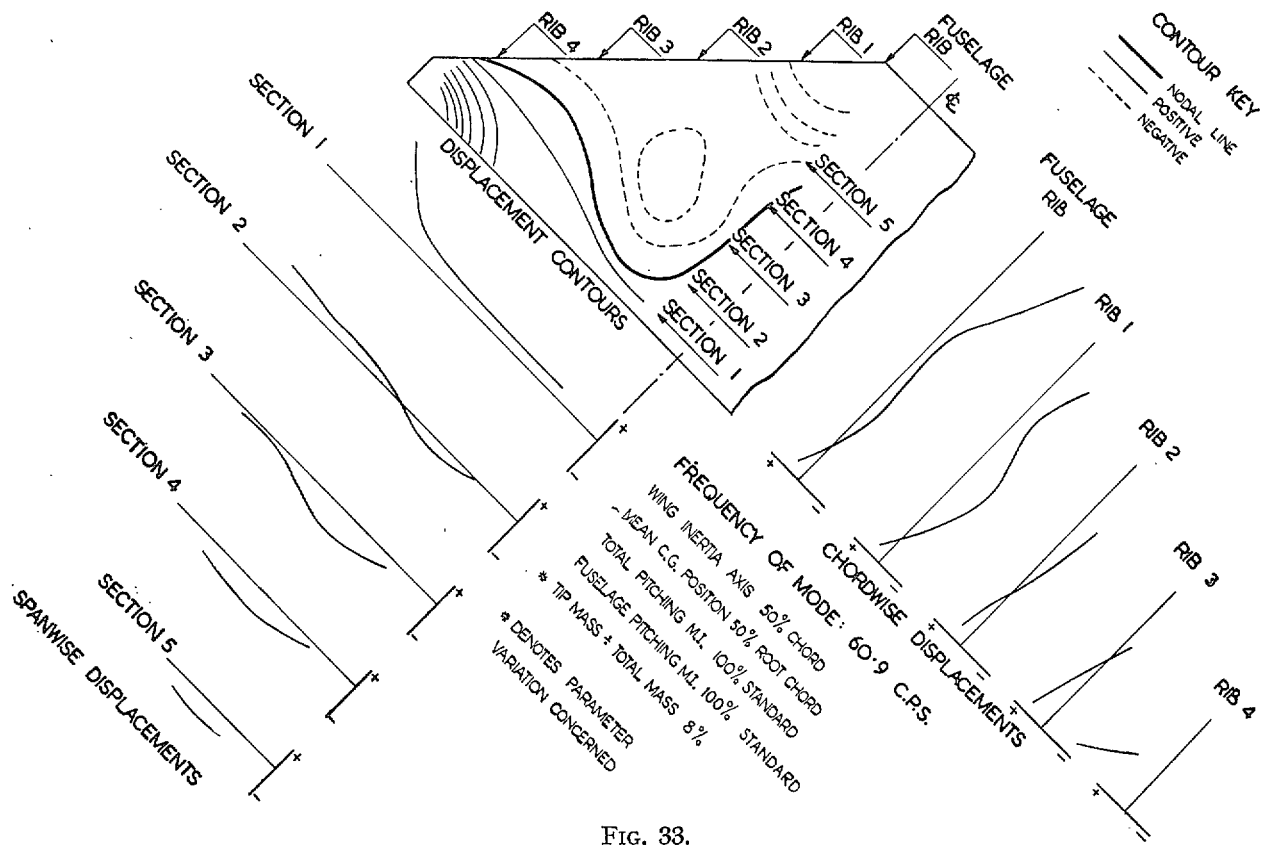


FIG. 33.

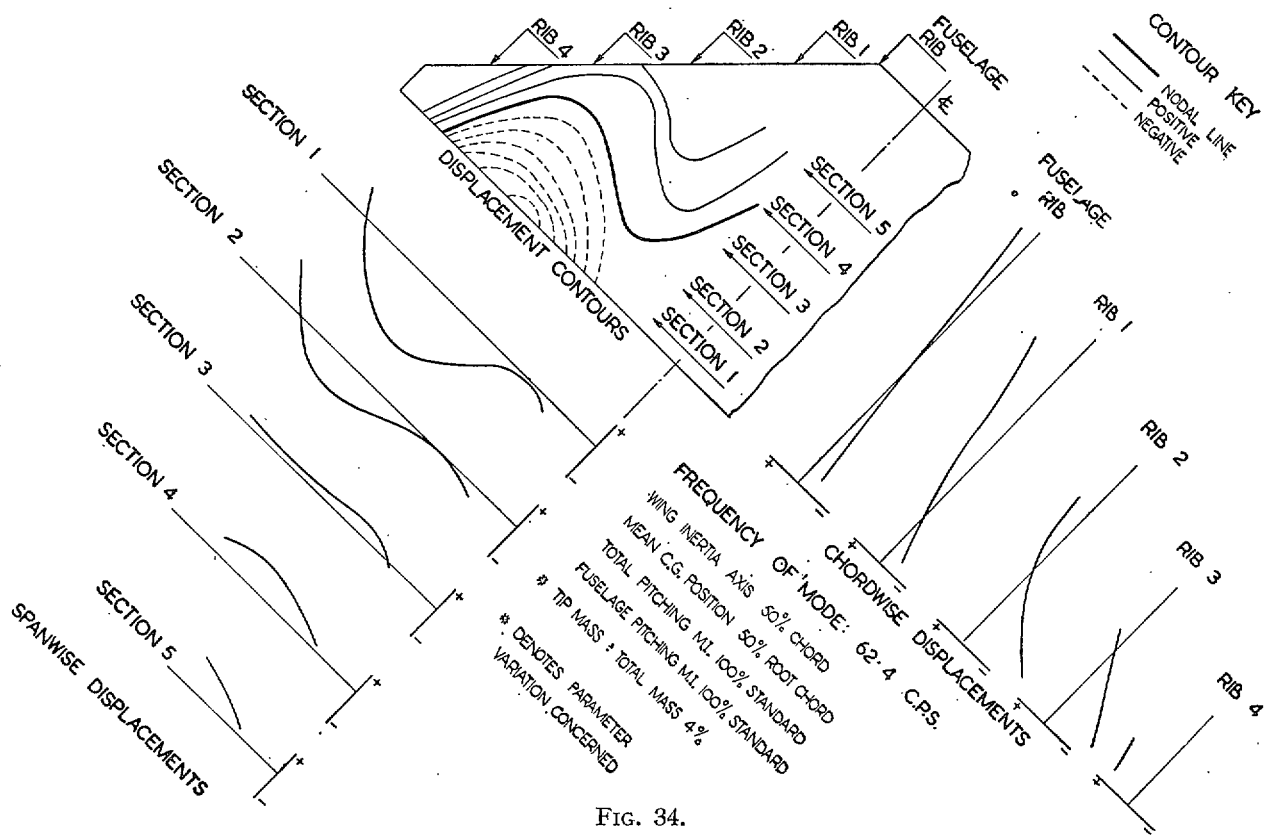


FIG. 34.

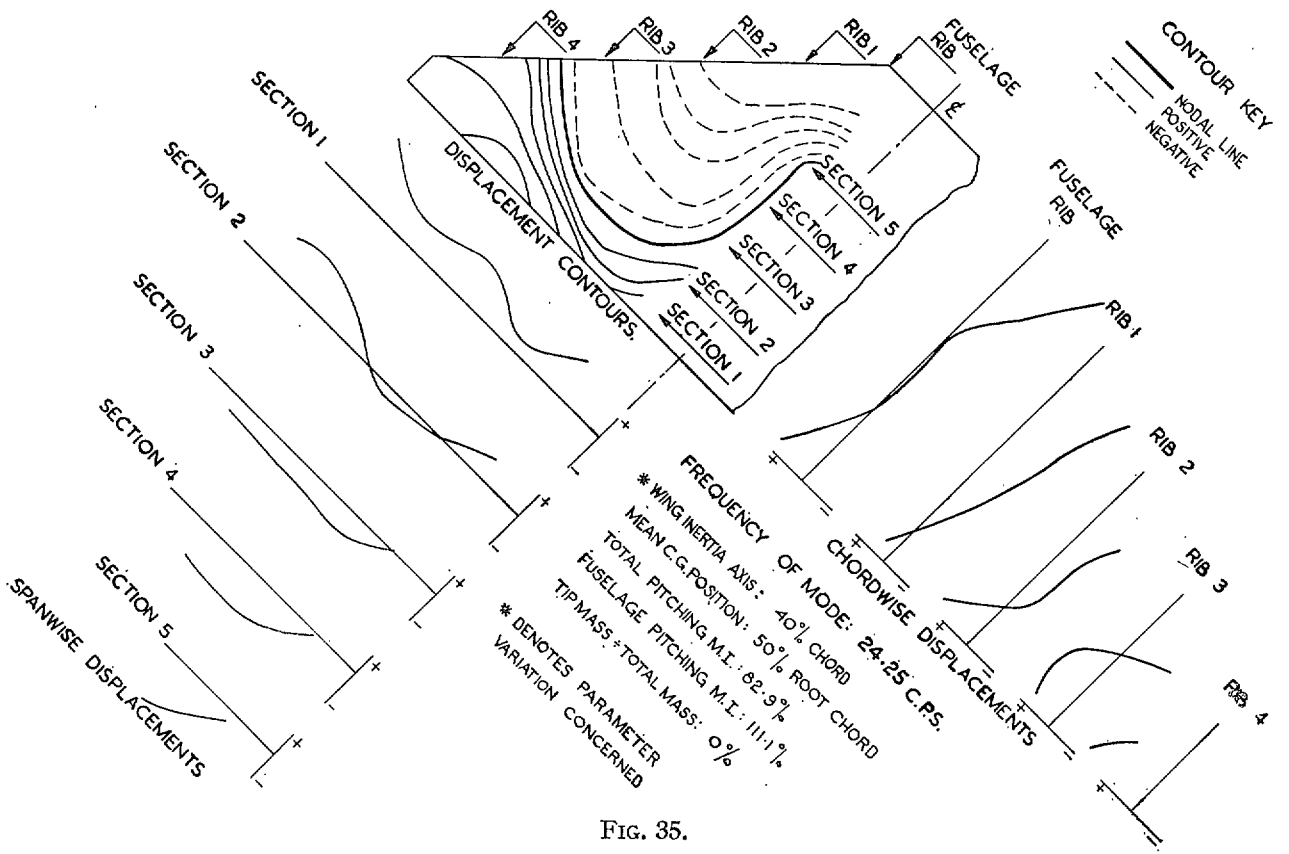


FIG. 35.

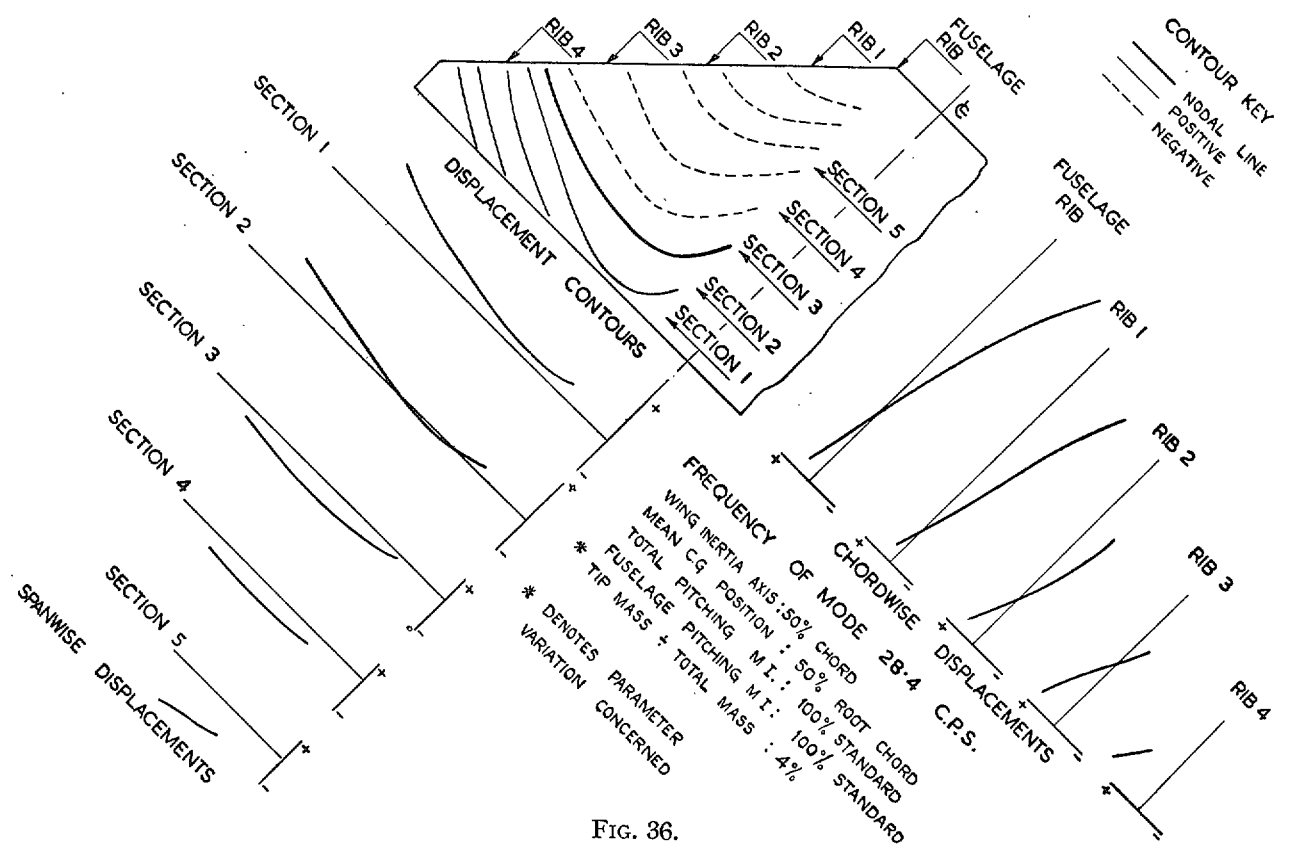


FIG. 36.

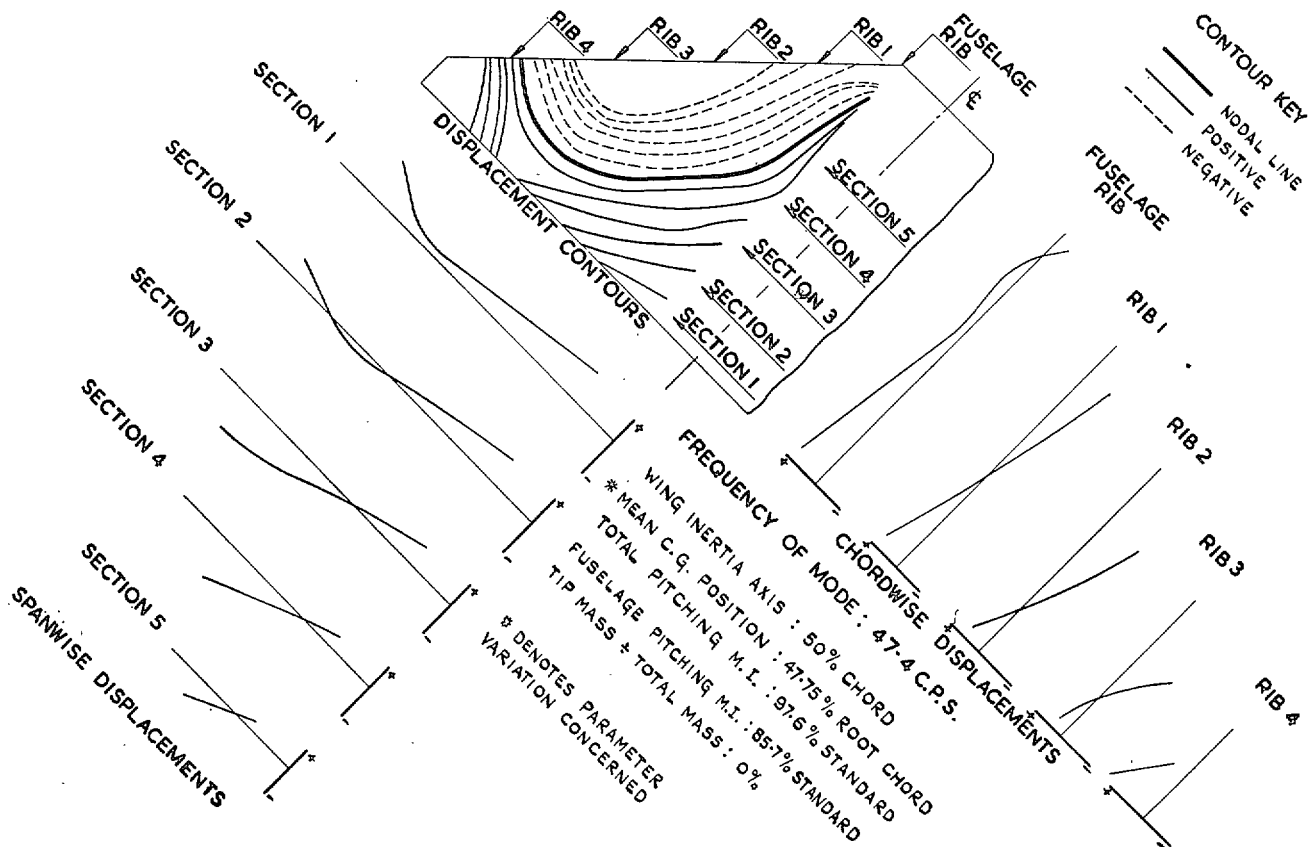


FIG. 37.

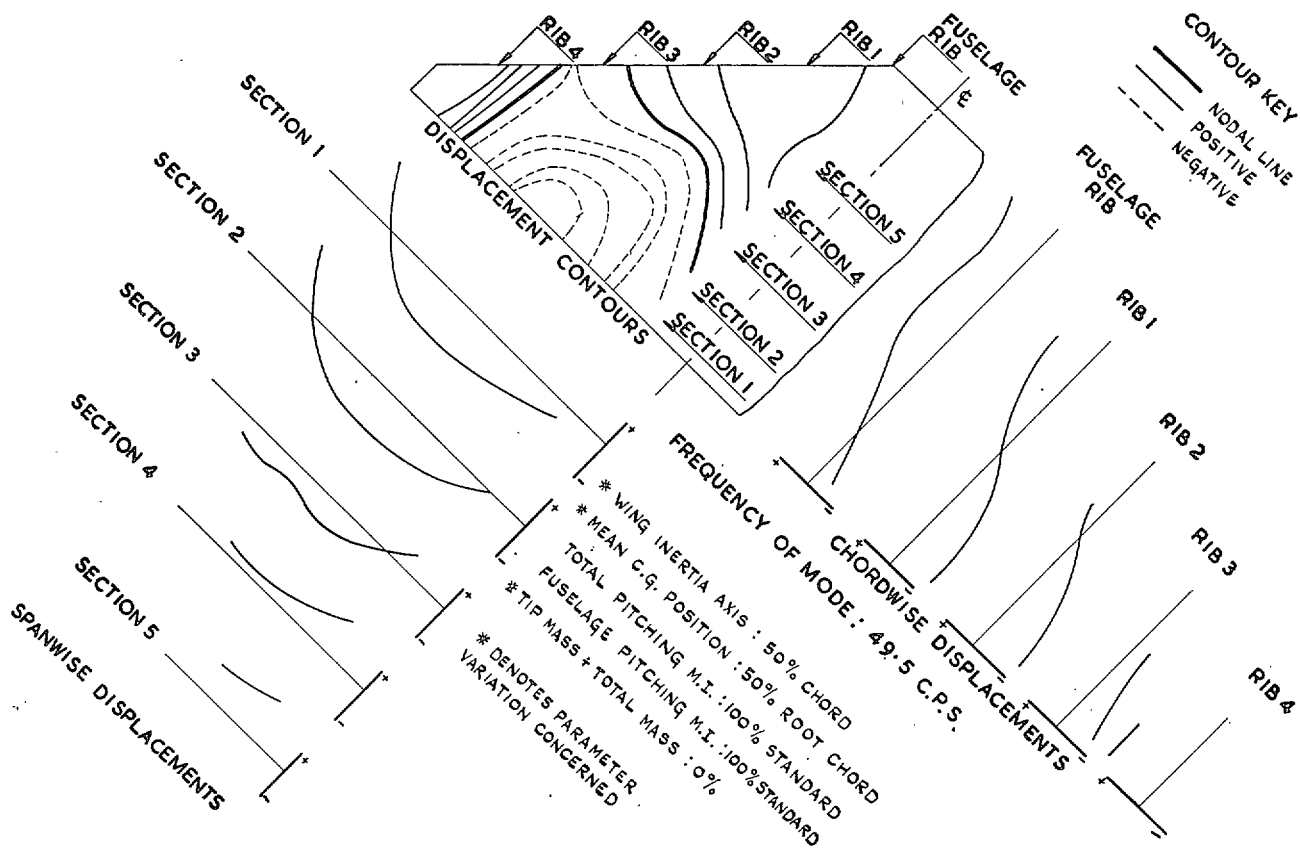


FIG. 38.

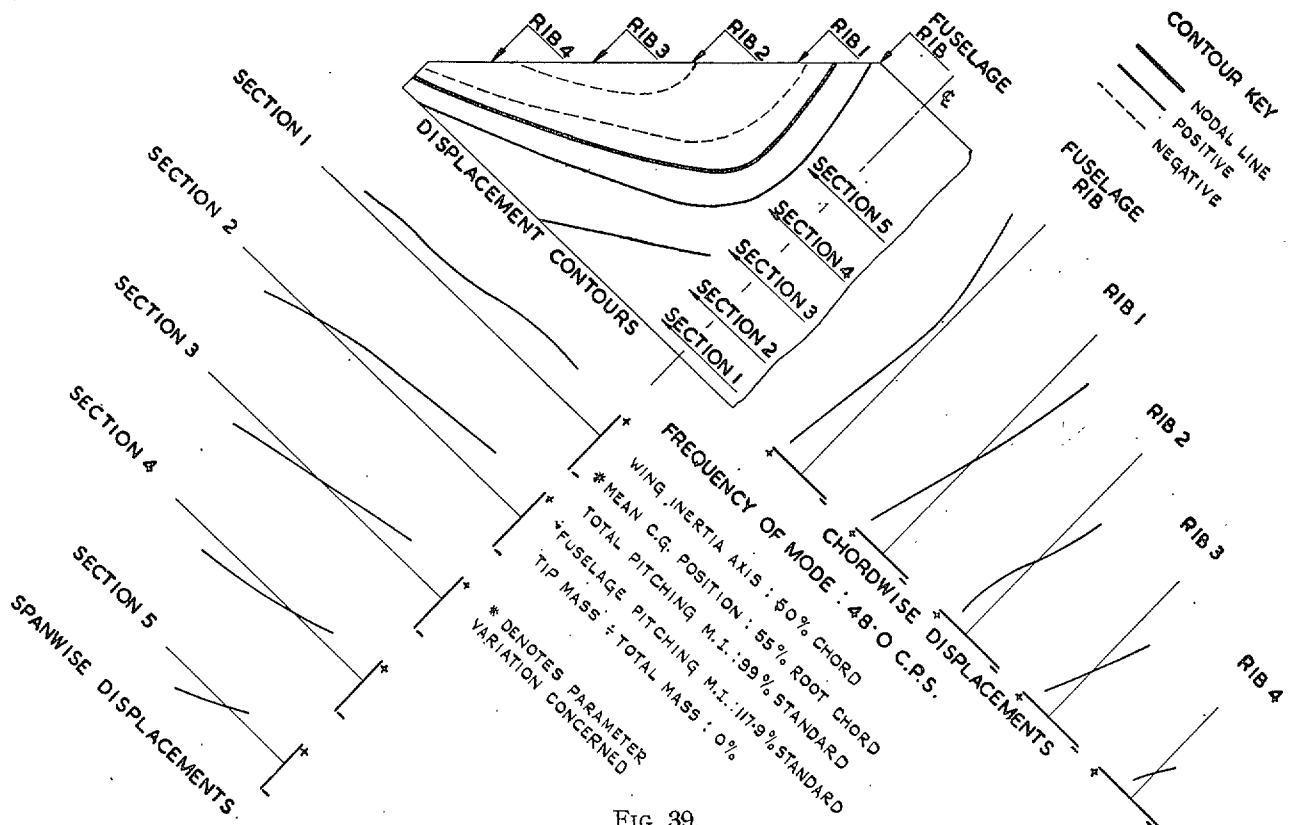


FIG. 39.

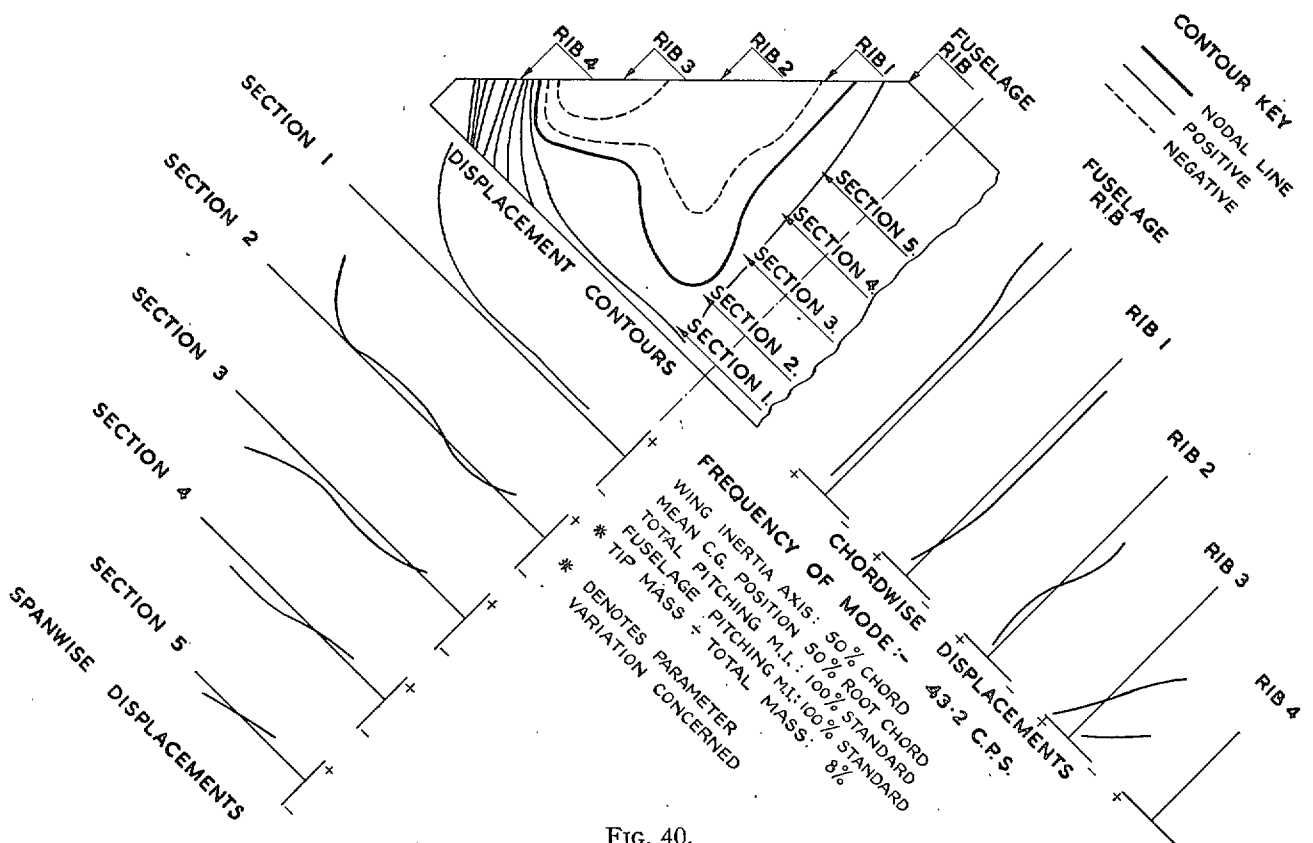


FIG. 40.

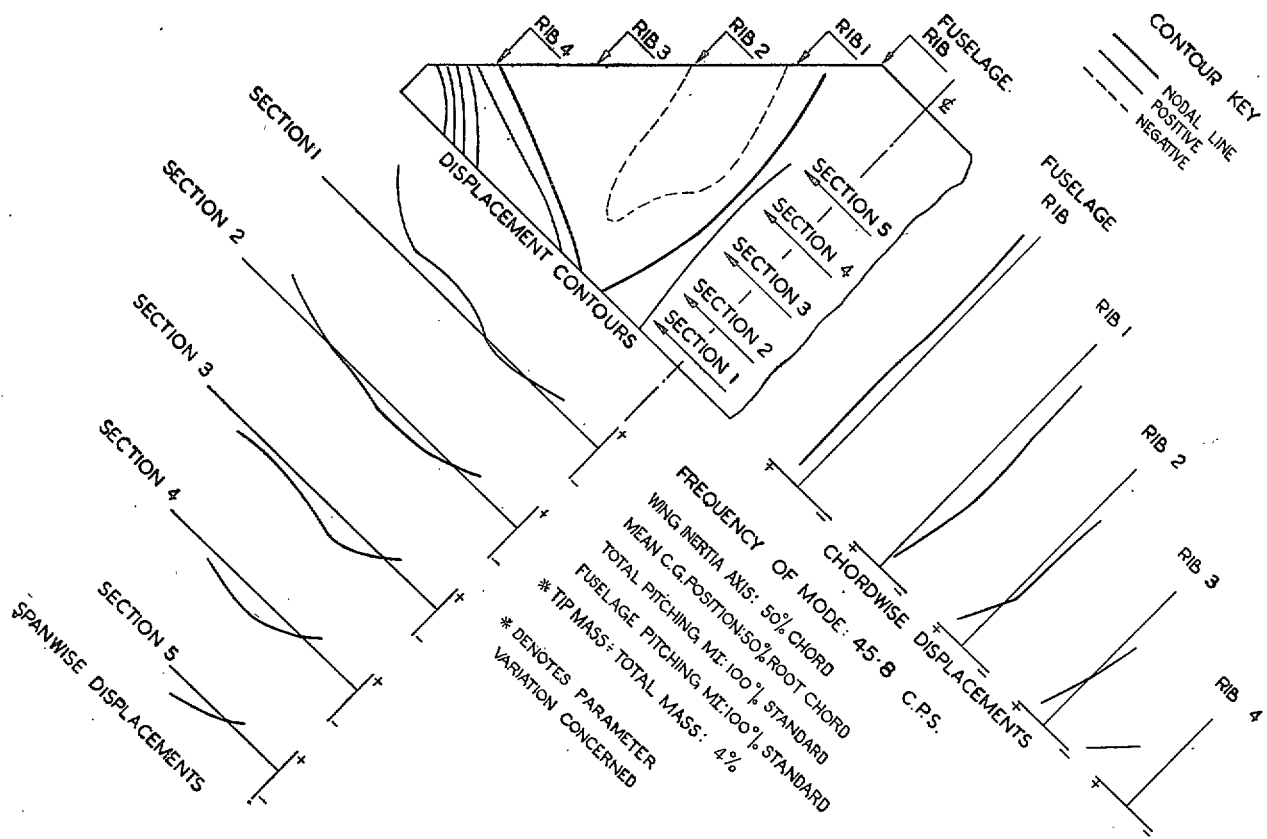


FIG. 41.

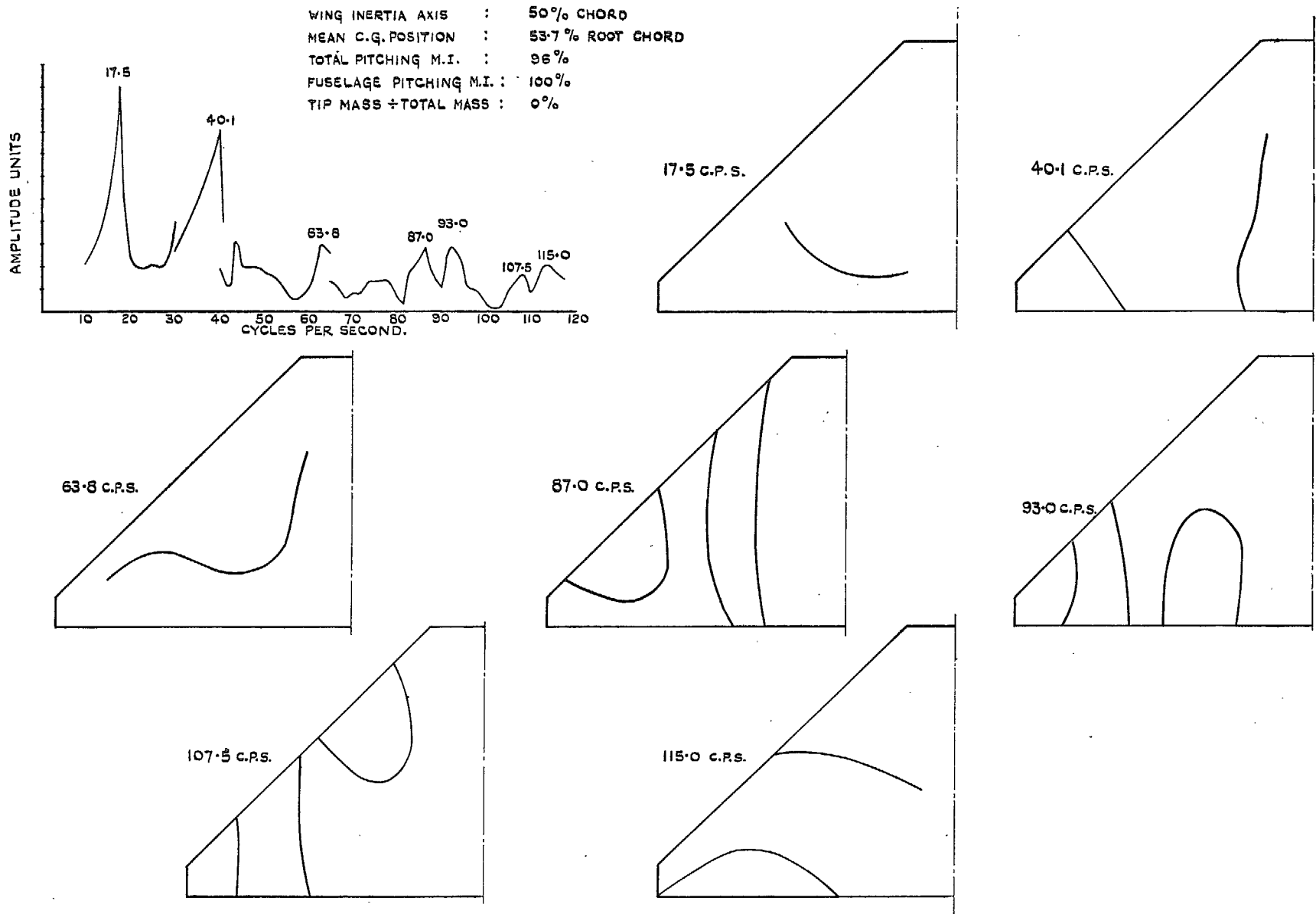


FIG. 42. Modes up to 118 c.p.s.

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