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# The Loss in Climb Performance, Relative to the Optimum, Arising from the use of a Practical Climb Technique

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# The Loss in Climb Performance, Relative to the Optimum, Arising from the use of a Practical Climb Technique

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COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR), MINISTRY OF SUPPLY



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*Summary.—Reasons for Enquiry.*—A practical climb technique will not in general comply with the condition for optimum climb performance and will give an inferior climb. An assessment of the loss of performance involved is, therefore, desirable.

*Scope of Investigation.*—A practical climb technique is considered which is defined by a fixed relation between equivalent air speed (or Mach number) and pressure altitude, and a rough estimate made of the loss in performance involved in using such a technique with a turbine jet aircraft over a range of air temperature, engine speed, thrust, or aircraft weight. An approximate method of calculating a suitable relation is given in an Appendix.

*Conclusions.*—If the technique for optimum climb is not fixed by compressibility effects, use of such a practical climb technique will result in a loss of performance, relative to the optimum, less than the greater of 1 per cent and $\frac{1}{2}$ ft/sec in rate of climb over a wide range of aircraft weight or a moderate range of air temperature, engine speed or thrust. Approximate limits are quoted in Table 2. More precise limits may be estimated for any particular aircraft.

If the technique for optimum climb is determined by compressibility effects such a practical climb technique can give optimum performance over a wide range of weight, air temperature and engine speed.

1. *Introduction.*—It was shown in R. & M. 2557[1] that optimum climb performance is obtained if, at each value of the total energy, the rate of change of the total energy of the aircraft is as great as possible. As, however, the pilot cannot with present instrumentation judge whether he has complied with this condition he must use a climb technique defined in terms of data available to him. For practical reasons such a technique will in general not comply with the condition for optimum performance and an assessment of the loss of performance involved is, therefore, desirable.

2. *Scope of Investigation.*—A technique of climb is considered which is defined by a fixed relation between equivalent air speed (or Mach number) and pressure altitude and an approximate method of calculating a suitable relation, in the absence of compressibility effects, is given in an Appendix.

An estimate is made of the loss in climb performance, relative to the optimum, involved in using such a technique with a turbine-jet aircraft over a range of air temperature, engine speed, thrust or aircraft weight.

The theory given can also be applied to propeller driven aircraft.

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3. *Use of a Technique Based on Air Speed and Height.*—As already noted, the pilot cannot judge directly whether his rate of change of total energy is the greatest possible. This condition for optimum climb does not, therefore, lead directly to a practical piloting technique.

Of the instruments conventionally available to the pilot only the air-speed indicator or the Machmeter, in combination with the altimeter, are easily used as basis for a climb technique at constant engine settings. It is, in fact, customary to define climb techniques by stating a relation between pressure altitude and the reading of the air speed indicator, which is a more sensitive and accurate instrument than the Machmeter. The latter may, however, be the more convenient on aircraft which encounter compressibility effects on the climb, as a constant Mach number may be desirable over part of the climb.

In this report consideration is given specifically to the use of a climb technique defined by a fixed relation between the equivalent air speed (which is very closely related to the air-speed indicator reading) and pressure altitude. The conclusions drawn are, however, equally applicable to a technique based on Mach number and pressure altitude, since at constant pressure altitude the Mach number is proportional to the equivalent air speed.

4. *Proposed Method of Estimating the Loss in Performance Involved.*—It will be assumed that when a fixed relation between equivalent air speed and height is used on the climb it is so chosen as to give optimum climb performance under some standard conditions, the problem being to assess the penalty which results from the use of the same relation under conditions which are different, but not drastically different, from standard.

To do this it is proposed to estimate how the equivalent air speed at a given pressure height on the optimum climb varies with air temperature, aircraft weight, engine output and so on, and hence by how much the constant speed to be used at that height may deviate from that required for optimum climb. An examination of the shape of curves of excess power against air speed will then indicate what loss in performance will be suffered.

5. *Variation of Speed with Height in Optimum Climb: the Optimum Speed.*—For a given aircraft with given engine settings and atmospheric conditions, optimum climb between any specified end conditions requires a certain relationship between air speed and pressure altitude. This relationship will vary with the end conditions, but their influence does not penetrate far into the main body of a long climb and it is only of importance for a small proportion of the total time of climb. As a result the technique used near the beginning and end of the climb is not important; what is importance is the technique used over the rest of the climb.

For a given aircraft flying in a given atmosphere at a stated engine rating we may write (R. & M. 2557[1])

$$\frac{dH_e}{dt} = f(H_e,V) = \chi(H,V)$$

where $V$ is true speed of the aircraft

$H$ height of the aircraft above some arbitrary datum

$t$ time

$H_e$ 'energy height' (R. & M. 2557[1]) of aircraft, $= H + \frac{1}{2}\frac{V^2}{g}$ .

With this notation the air speed required for optimum climb is, excluding end conditions, such that

$$\frac{\partial f}{\partial V} = 0 . \qquad (1)$$

This equation shows how $V$ is related to $H_e$, and hence to $H$, on the optimum climb. The value of $V$ at given $H$ (or $H_e$) on such a climb will be referred to as the optimum value at that $H$ (or $H_e$).

6. *Estimation of the Optimum Speed.*—It is not easy to express $\partial f/\partial V$ in terms of familiar quantities. It is, however, relatively easy to do so for $\partial \chi/\partial V$.

It can be shown that

$$\frac{\partial \chi}{\partial V} = \frac{\partial f}{\partial V} + \frac{V}{g}\frac{\partial \chi}{\partial H} \qquad (2)$$

so that equation (1) can be written

$$\frac{\partial \chi}{\partial V} = \frac{V}{g}\frac{\partial \chi}{\partial H} \qquad (1a)$$

$\frac{V}{g}\frac{\partial \chi}{\partial H}$ is actually small and negative, and the implication of equation (1a) is that at any height the true optimum speed is a little higher than the 'quasi-optimum' speed at which $\partial \chi/\partial V$ is zero. The difference is usually about 5 per cent if no compressibility effects are present. A typical case is illustrated in Fig. 1.

An analytical expression for the quasi-optimum speed in the absence of compressibility effects on drag is derived in Appendix I and is plotted in Fig. 4. A routine for estimating this speed, and hence of estimating the optimum speed, is given in Appendix II.

7. *Variation of Optimum Speed with Other Parameters.*—7.1. *No Compressibility Effects.*—To estimate the variation of the optimum speed $V_{ic}$ with any parameter $x$ we will assume that the ratio of the optimum to the quasi-optimum speed is constant, so that

$$\frac{x}{V_{ic}}\frac{\partial V_{ic}}{\partial x} = \frac{x}{V_{iQ}}\frac{\partial V_{iQ}}{\partial x}. \qquad (3)$$

It is shown in Appendix I that if

$$\tau = \left(\frac{T}{D_{\min}}\right)\left(1 + \frac{V}{T}\frac{\partial T}{\partial V}\right) \qquad (4a)$$

$$= \left(\frac{T}{W}\right)\left(\frac{L}{D}\right)_{\max}\left(1 + \frac{V}{T}\frac{\partial T}{\partial V}\right) \qquad (4b)$$

then in the absence of compressibility effects on drag

$$\frac{x}{V_{iQ}}\frac{\partial V_{iQ}}{\partial x} = \frac{1}{2}\frac{x}{W}\frac{\partial W}{\partial x} + \frac{1}{2}\frac{\tau}{(\tau^2 + 3)^{1/2}}\frac{x}{\tau}\frac{\partial \tau}{\partial x}$$

$$= \mathrm{E} + \mathrm{F}\frac{\tau}{2(\tau^2 + 3)^{1/2}} \text{ say.} \qquad (5)$$

The function $\frac{\tau}{2(\tau^2 + 3)^{1/2}}$ is plotted against $\tau$ in Fig. 3. It increases with $\tau$ from $\frac{1}{4}$ for $\tau = 1$ (corresponding to a jet aircraft near its ceiling) towards an asymptotic value of $\frac{1}{2}$.

The values or ranges of E and F depend on what variable $x$ is under consideration. If it is the weight then it is clear from equations (4b) and (5) that E is equal to $\frac{1}{2}$ and F to $-1$. For changes in engine speed or air temperature E is zero, but F, *i.e.*, $\left(\frac{x}{\tau}\frac{\partial \tau}{\partial x}\right)$, is not, since changes in engine speed or in air temperature affect $T$ and $\left(1 + \frac{V}{T}\frac{\partial T}{\partial V}\right)$, both of which contribute to $\tau$.

We have

$$\frac{N}{\tau}\frac{\partial\tau}{\partial N} = \frac{N}{T}\frac{\partial T}{\partial N} + \frac{N}{1+\frac{V}{T}\frac{\partial T}{\partial V}}\,\frac{\partial\left(1+\frac{V}{T}\frac{\partial T}{\partial V}\right)}{\partial N}\,.$$

The rate of change of thrust with engine speed $\left(\frac{N}{T}\frac{\partial T}{\partial N}\right)$ varies between engine types; for a given type it is independent* of engine speed but increases with aircraft Mach number and lies between about 3 and about 5. The variation of $\left(1+\frac{V}{T}\frac{\partial T}{\partial V}\right)$ and engine speed appears to be about one fifth of this. Hence for variation in $N$, F will lie roughly in the range from 3·5 to 6.

For turbine jet engines

$$\frac{\theta}{\tau}\frac{\partial\tau}{\partial\theta} = -\frac{1}{2}\frac{N}{\tau}\frac{\partial\tau}{\partial N}\,.$$

so that for variation in $\theta$, F will lie in the range from − 1·7 to − 3 approximately.

For variation in thrust, with $\frac{V}{T}\frac{\partial T}{\partial V}$ constant, it will be seen (equations (4a) and (5)) that E is zero and F unity.

The values of E and the values or probable ranges of F are given in Table 1 below, together with the corresponding values of ranges of $\frac{x}{V_{ic}}\frac{\partial V_{ic}}{\partial x}$, for $\tau$ = 1, 5 and 10 — corresponding to an aircraft at its ceiling, with moderate available thrust, and with very large available thrust respectively.

TABLE 1

| Variable $x$ | Value of E | Range of F | Ranges of $\frac{x}{V_{ic}}\frac{\partial V_{ic}}{\partial x}$ | | |
|---|---|---|---|---|---|
| | | | $\tau = 1$ | $\tau = 5$ | $\tau = 10$ |
| $W$ | 0·5 | − 1 | 0·25 | 0·025 | 0·01 |
| $N$ | 0 | 3·5 to 6 | 0·9 to 1·5 | 1·7 to 2·8 | 1·7 to 3·0 |
| $\theta$ | 0 | − 1·7 to − 3 | − 0·4 to − 0·8 | − 0·8 to − 1·4 | − 0·8 to − 1·5 |
| $T$ (with $\frac{V}{T}\frac{\partial T}{\partial V}$ constant) | 0 | 1 | 0·25 | 0·47 | 0·49 |

It will be seen that $\frac{x}{V_{ic}}\frac{\partial V_{ic}}{\partial x}$ changes little with $\tau$ for values of above 5.

7.2. *With Compressibility Effects.*—If, at a given pressure height, the optimum or the best practicable speed for climb is determined by compressibility effects on the drag or handling characteristics of the aircraft it will correspond to a fixed Mach number, sensibly independent of air temperature, engine speed and output, and aircraft weight. As the Mach number is proportional to equivalent air speed (at constant pressure height), the equivalent air speed for best climb will thus be sensibly independent of these variables.

* Observed variation of $\frac{N}{T}\frac{\partial T}{\partial N}$ at climb rating is small but may be more marked at lower engine speeds. The range of values quoted are appropriate to a centrifugal engine over a range of engine speeds and Mach numbers.

8. *The Loss in Performance.*—8.1. *No Compressibility Effects.*—For moderate departures from the optimum the loss in rate of increase of energy will be approximately proportional to the square of the departure from the optimum speed.

Precise generalisation about the amount of the loss, or the range of any variable over which the loss is tolerable, is not possible. If a precise estimate is required in any particular case an individual analysis should be made, using the engine parameters and calculated or experimental performance curves appropriate to the particular aircraft. A rough idea of the loss, sufficient to indicate whether a detailed investigation is needed in any particular case, may, however, be gained by examining the sample curves of $\frac{dH_e}{dt} = f(H_e, V)$ against $V$ given in Fig. 3. If, for example, we take the greater of $\frac{1}{2}$ ft/sec and 1 per cent in $\frac{dH_e}{dt}$ as the maximum acceptable loss it will be seen that the air speed should be within about 7 per cent of the optimum at low altitude and within about 3 per cent near the ceiling. Using the mean values of $\frac{x}{V_{ic}} \frac{\partial V_{ic}}{\partial x}$ given in Table 1 for $\tau = 5$ and $\tau = 1$ respectively we may deduce corresponding limits for various parameters $x$, as given below:—

TABLE 2

| Parameter | Approximate limits of variation | |
|---|---|---|
| | Low altitude | Near ceiling |
| Weight | Large | $\pm$ 12% |
| Engine speed | $\pm$ 3% | $\pm$ 3% |
| Air temperature | $\pm$ 6% ($\pm$ 16° C) | $\pm$ 5% ($\pm$ 11° C) |
| Thrust (with $\frac{V}{T}\frac{\partial T}{\partial V}$ constant) | $\pm$ 15% | $\pm$ 12% |

It will be seen that a fixed climb technique may be used over a wide range of aircraft weight and over a considerable range of air temperature or thrust without incurring a loss of climb performance exceeding the greater of 1 per cent and $\frac{1}{2}$ ft/sec in rate of climb. The permissible range of engine speed is not large, but it is of the order of the difference between climb and combat limitations.

8.2. *With Compressibility Effects.*—If the optimum speed is determined by compressibility effects on the drag or the handling characteristics of the aircraft it will be sensibly independent of air temperature, engine output and aircraft weight. It is, therefore, then possible to obtain optimum climb performance with a fixed climb technique over a wide range of these variables and no question of loss of performance should arise.

9. *Conclusions.*—If the technique for optimum climb is not determined by compressibility effects on drag or handling, use of a practical climb technique defined by a fixed relation between equivalent air speed (or Mach number) and pressure height will in general result in a loss of performance relative to the optimum. This loss will not, however, exceed the greater of 1 per cent and $\frac{1}{2}$ ft/sec in rate of climb over a wide range of aircraft weight or a moderate range of air temperature ($\pm$ 10 or 15 deg C), thrust ($\pm$ 10 or 15 per cent), or engine speed ($\pm$ 3 per cent). A more precise estimate of these limits may be made for any particular aircraft.

If the technique for optimum climb is determined by compressibility effects it is possible to obtain optimum climb performance with a fixed relation between equivalent air speed (or Mach number) and pressure height over a wide range of these variables.

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|---|---|---|
| 1 | K. J. Lush .. .. .. .. | A Review of the Problem of Choosing a Climb Technique, with Proposals for a New Climb Technique for High Performance Aircraft. R. & M. 2557. June, 1948. |


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## LIST OF SYMBOLS

| *Symbol* | *Definition* |
|---|---|
| $A$, $B$ | Constants in the drag equation (Appendix I, section 2) |
| $D$ | Drag of aircraft |
| E, F | Constants defined in section 7.1 (equation 5) |
| $f(H_e, V)$ | $\frac{dH_e}{dt}$ |
| $H$ | Height of aircraft above an arbitrary datum |
| $H_e$ | $H + \frac{1}{2}\frac{V^2}{g}$ |
| $(L/D)_{\max}$ | Maximum lift/drag ratio |
| $N$ | Engine speed |
| $T$ | Nett thrust |
| $t$ | Time |
| $V$ | True air speed |
| $V_i$ | Equivalent air speed |
| $V_c$, $V_{ic}$ | Optimum speeds for climb |
| $V_Q$, $V_{iQ}$ | ' Quasi-optimum ' speeds (section 6) |
| $V_{md}$, $V_{imd}$ | Speeds for minimum drag |
| $W$ | Aircraft weight |
| $\gamma$ | Angle of flight path to horizontal |
| $\theta$ | Air temperature (absolute) |
| $\Lambda$ | Aspect ratio |
| $\lambda$ | $V/V_{md}$ |
| $\lambda_Q$ | $V_Q/V_{md}$ |
| $\tau$ | $\frac{T}{D_{\min}}\left(1 + \frac{V}{T}\frac{\partial T}{\partial V}\right)$ |
| $\chi\,(H, V)$ | $\frac{dH_e}{dt}$ |

## APPENDIX I

*Expressions for $V_{iQ}$ and for $\frac{x}{V_{iQ}}\frac{\partial V_{iQ}}{\partial x}$ in the Absence of Compressibility Effects*

1. *Introduction.*—$\frac{x}{V_{iQ}}\frac{\partial V_{iQ}}{\partial x}$ is a measure of the rate of variation of the quasi-optimum speed $V_{iQ}$ (section 6 of main text) at which $\partial\chi/\partial V$ is zero, varies with any parameter $x$.

Symbols used below will be defined as they occur. A list of symbols, with their definitions, is given at the end of the main test of the report.

2. *The Climb Equation.*—We wish to examine (section 6 of main text) the conditions under which

$$\frac{\partial\chi}{\partial V} = 0$$

where, $\chi(H, V) = \frac{dH_e}{dt}$

$V$ true speed of the aircraft

$H$ height of the aircraft above some arbitrary datum

$t$ time

and $H_e = H + \frac{1}{2}\frac{V^2}{g}$ .

Now,
$$\chi = \left(\frac{T}{W} - \frac{D}{W}\right) V \qquad \text{(A1)}$$

where $W$ is weight of aircraft

$T$ net thrust

$D$ drag of aircraft

and in the absence of compressibility effects the drag of an aircraft in straight flight is given very closely by a relationship of the form

$$D = AV_i^2 + B\frac{W^2}{V_i^2}\cos^2\gamma$$
$$= AV_i^2 + B\frac{W^2}{V_i^2} - B\frac{W^2}{V_i^2}\sin^2\gamma$$

where, $\gamma$ is angle of flight path to horizontal

$V_i = V(\sigma)^{1/2}$ (the 'equivalent' air speed)

and $A$, $B$ are constants for the aircraft, and are positive.

With jet aircraft climbing at air speeds near the optimum the term in $\sin^2\gamma$ is very small, and we may write

$$D = AV_i^2 + B\frac{W^2}{V_i^2} \quad . \qquad \text{(A2)}$$

There is a minimum value of $D$ ($D_{min}$, say), occurring at speed $V_{i\,md}$, such that

$$\left.\begin{aligned} D_{min} &= 2W\sqrt{(AB)} \\ \text{and} \quad V_{i\,md} &= \sqrt[4]{\frac{B}{A}}\cdot\sqrt{W}. \end{aligned}\right\} \qquad \text{(A3)}$$

It is convenient for present purposes to use these relations to put the variation of drag with speed into the general form

$$\frac{D}{D_{\min}} = \frac{1}{2}\left\{\left(\frac{V_i}{V_{i\,md}}\right)^2 + \left(\frac{V_{i\,md}}{V_i}\right)^2\right\}$$

$$= \tfrac{1}{2}\,(\lambda^2 + \lambda^{-2}) \text{ say} . \qquad \text{(A4)}$$

Substituting in equation (A1) we have

$$\chi = V\left\{\frac{T}{W} - \frac{1}{2}\,\frac{D_{\min}}{W}\,(\lambda^2 + \lambda^{-2})\right\} .$$

As $\gamma$ is not large, $W/D_{\min}$ is nearly equal to the maximum ratio $(L/D)_{\max}$ of lift to drag, which is a constant of the aircraft. Hence we may re-write the above equation in the form

$$\frac{\chi\sqrt{\sigma}}{V_{i\,md}}\,\frac{L}{D_{\max}} = \lambda\cdot\left\{\frac{T}{D_{\min}} - \tfrac{1}{2}\,(\lambda^2 + \lambda^{-2})\right\} \cdot \qquad \text{(A5)}$$

since $V = \lambda V_{i\,md}/\sqrt{\sigma}$.

3. *The Quasi-Optimum Speed.*—Of the variables in equation (A5) $V_{i\,md}$ and $D_{\min}$ are, for a given aircraft, determined only by the aircraft weight. If we consider a particular engine setting (*e.g.* a particular rotational speed, with present simple jet engines) then the thrust $T$ is a function of air temperature, air pressure (or 'pressure height') and air speed. $\sigma$ is a function of air temperature and pressure; $\lambda$ is a function of air temperature, air pressure, air speed and aircraft weight.

By differentiating both sides of equation (A5) partially with respect to $V$ it can be shown that the condition $\partial\chi/\partial V = 0$ is equivalent to

$$0 = \frac{V}{\lambda}\,\frac{\partial\lambda}{\partial V}\left\{\frac{T}{D_{\min}} - \tfrac{1}{2}\,(\lambda^2 + \lambda^{-2})\right\} + \frac{T}{D_{\min}}\,\frac{V}{T}\,\frac{\partial T}{\partial V} - \lambda^2 + \lambda^{-2}$$

*i.e.*, as $\dfrac{V}{\lambda}\,\dfrac{\partial\lambda}{\partial V}$ is equal to unity,

$$3\lambda^2 - \lambda^{-2} = \frac{2T}{D_{\min}}\left(1 + \frac{V}{T}\,\frac{\partial T}{\partial V}\right)$$

$$= 2\tau \text{ say} .$$

Hence if $\lambda_Q$ is the quasi-optimum value of $\lambda$

$$\lambda_Q = \left\{\frac{\tau + (\tau^2 + 3)^{1/2}}{3}\right\}^{1/2} \qquad \text{(A6)}$$

$$= \left(\frac{\tau}{3}\right)^{1/2}\left\{1 + (1 + 3/\tau^2)^{1/2}\right\}^{1/2} \qquad \text{(A7)}$$

(since the real and positive solution is the relevant one). Thus $\lambda_Q$ is expressible as a function of $\tau$ only. The function is plotted in Fig. 4.

4. *An expression for* $\dfrac{x}{V_{iQ}}\,\dfrac{\partial V_{iQ}}{\partial x}$.—

As $\lambda_Q$ is a function of $\tau$ only we may write

$$\frac{x}{V_{iQ}}\,\frac{\partial V_{iQ}}{\partial x} = \frac{1}{2}\,\frac{x}{W}\,\frac{\partial W}{\partial x} + \frac{x}{\tau}\,\frac{\partial\tau}{\partial x}\,\frac{\tau}{\lambda_Q}\,\frac{d\lambda_Q}{d\tau} \qquad \text{(A8)}$$

where we have from equation (A6), by differentiation,

$$\left.\begin{aligned}\frac{\tau}{\lambda_Q}\frac{d\lambda_Q}{d\tau} &= \frac{1}{2}\frac{\tau}{(\tau^2+3)^{1/2}} \\ &= \frac{1}{2}\frac{1}{\{1+(3/\tau^2)\}^{1/2}}\,.\end{aligned}\right\} \quad \ldots \quad \text{(A9)}$$

It may be noted that $\frac{x}{W}\frac{\partial W}{\partial x}$ is zero except when $x$ is itself the weight, when it is unity.

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## APPENDIX II

*Approximate Estimation of the Optimum Speed for Climb (No Compressibility Effects)*

It was noted in section 6 that the optimum speed for climb is only a little (roughly 5 per cent, in the absence of compressibility effects) higher than the 'quasi-optimum' speed. An approximate estimate of the optimum speed may, therefore, be made by estimating the quasi-optimum speed and adding 5 per cent to it.

The quasi-optimum speed $V_Q$ may be deduced from Fig. 4 if the following are known:—

(a) thrust ($T$)

(b) variation of thrust with air speed $\left(\frac{V}{T}\frac{\partial T}{\partial V}\right)$

(c) maximum lift/drag ratio $\left(\frac{L}{D}\right)_{max}$

(d) speed for minimum drag ($V_{md}$)

(e) the aircraft all-up weight ($W$)

*Maximum lift/drag ratio*.—This is given by

$$\left(\frac{L}{D}\right)_{max} = \frac{1}{2}\left(\frac{\pi e A}{C_{DZ}}\right)^{1/2}$$

where $C_{DZ}$ is the intercept on the $C_D$ axis of a rectilinear curve of $C_D$ against $C_L^2$, $1/\pi e A$ is the slope of this curve, $A$ the aspect ratio and $e$ the N.A.C.A. 'efficiency factor.'

The following table gives data obtained from flight tests, for a number of aircraft.

| Aircraft | Piston or Jet | $C_{DZ}$ | $\pi e A$ | $\left(\frac{L}{D}\right)_{max}$ |
|---|---|---|---|---|
| *Meteor* 3 | Jet | 0·018 | 11·0 | 12·3 |
| *Meteor* 4 (short span) | ,, | 0·0175 | 8·3 | 10·9 |
| *Vampire* 1 | ,, | 0·014 | 17·6 | 17·7 |
| *Lancaster* 1 | Piston | 0·029 | 22·1 | 13·8 |
| *Lancaster* 2 | ,, | 0·037 | 26·5 | 13·4 |
| *Mosquito* 2 | ,, | 0·023 | 15·2 | 12·8 |

*Speed for minimum drag.*—The equivalent air speed $V_{i\,md}$ for minimum drag is given by

$$V_{i\,md} = \left(\frac{2W}{\rho_0 S}\right)^{1/2} 4\left(\frac{1}{\pi e A C_{DZ}}\right)^{1/2}$$

$$= \left(\frac{W}{\rho_0 S C_{DZ}}\right)^{1/2} \Big/ \left(\frac{L}{D}\right)_{\max}$$

where $S$ is the wing area and consistent units are used.

*Thrust and* $\frac{V}{T}\frac{\partial T}{\partial V}$.—If flight measurements of thrust under the required conditions are not available for the particular installation the makers power curves must be used. These are not normally in a form which gives $\frac{V}{T}\frac{\partial T}{\partial V}$ directly; net thrust must be plotted against air speed over the relevant range and $\frac{V}{T}\frac{\partial T}{\partial V}$ deduced.

*Routine for estimation.*—The estimate must be made by a process of successive approximation, but the second estimate should be sufficiently accurate. The following routine may be used:—

(a) make a rough guess at the optimum speed

(b) find $T$ and $\frac{V}{T}\frac{\partial T}{\partial V}$ at that speed

(c) find $\frac{T}{D_{\min}} = \frac{T}{W}\left(\frac{L}{D}\right)_{\max}$

(d) calculate $\frac{T}{D_{\min}}\left(1 + \frac{V}{T}\frac{\partial T}{\partial V}\right)$

(e) from Fig. 4 find $\lambda_Q$ and hence an estimate of $V_{i\,Q}$

(f) if the value of $V_{i\,Q}$ so obtained differs widely from the value initially guessed, repeat the process using the values of $T$ and $\frac{V}{T}\frac{\partial T}{\partial V}$ appropriate to the new estimate of $V_{i\,Q}$.

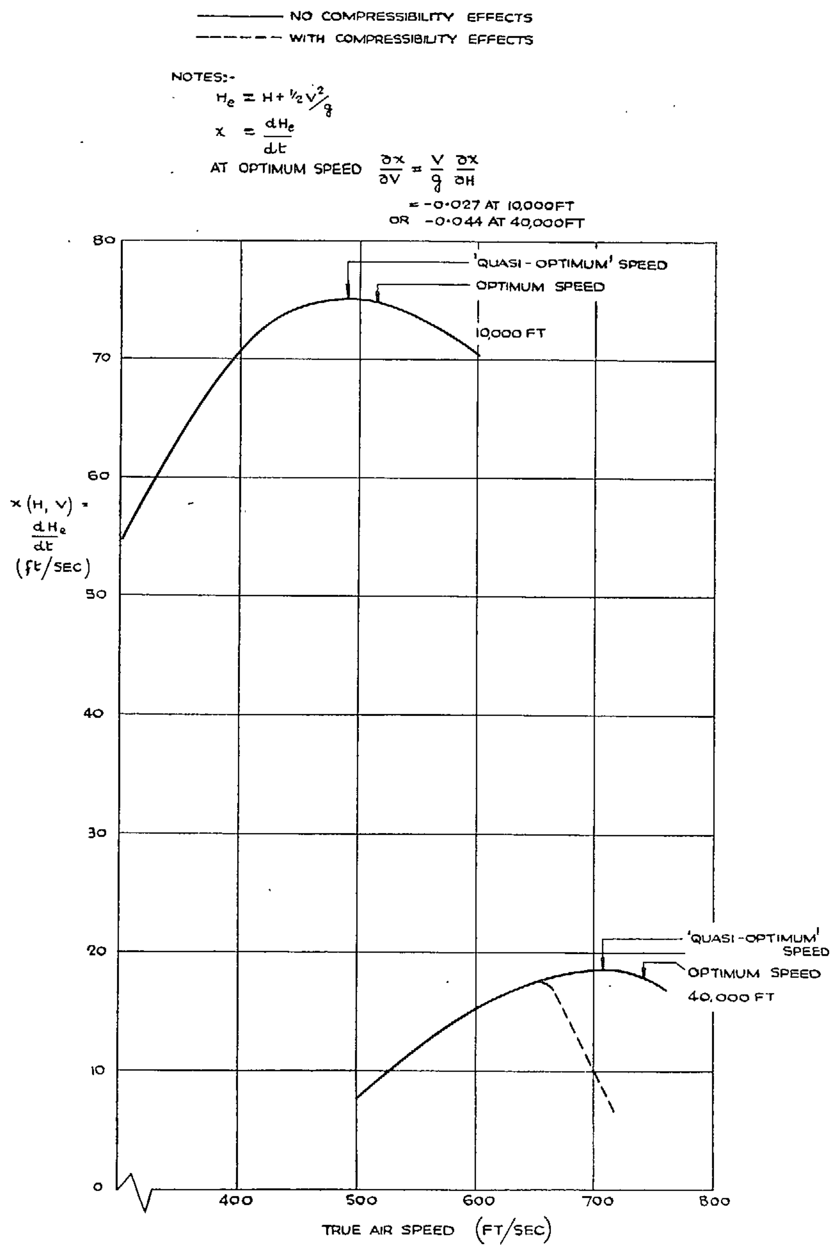


FIG. 1. Typical curves of $\chi$ $(H, V)$ against $V$.

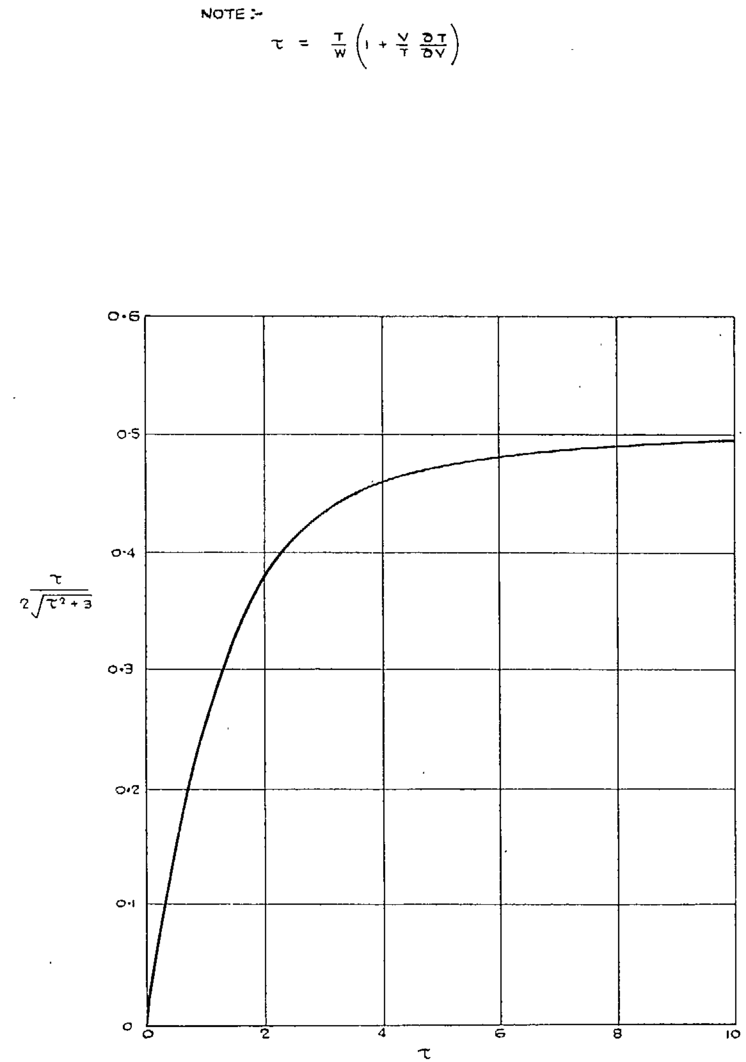


FIG. 2. Variation of $\frac{\tau}{2(\tau^2 + 3)^{1/2}}$ with $\tau$.

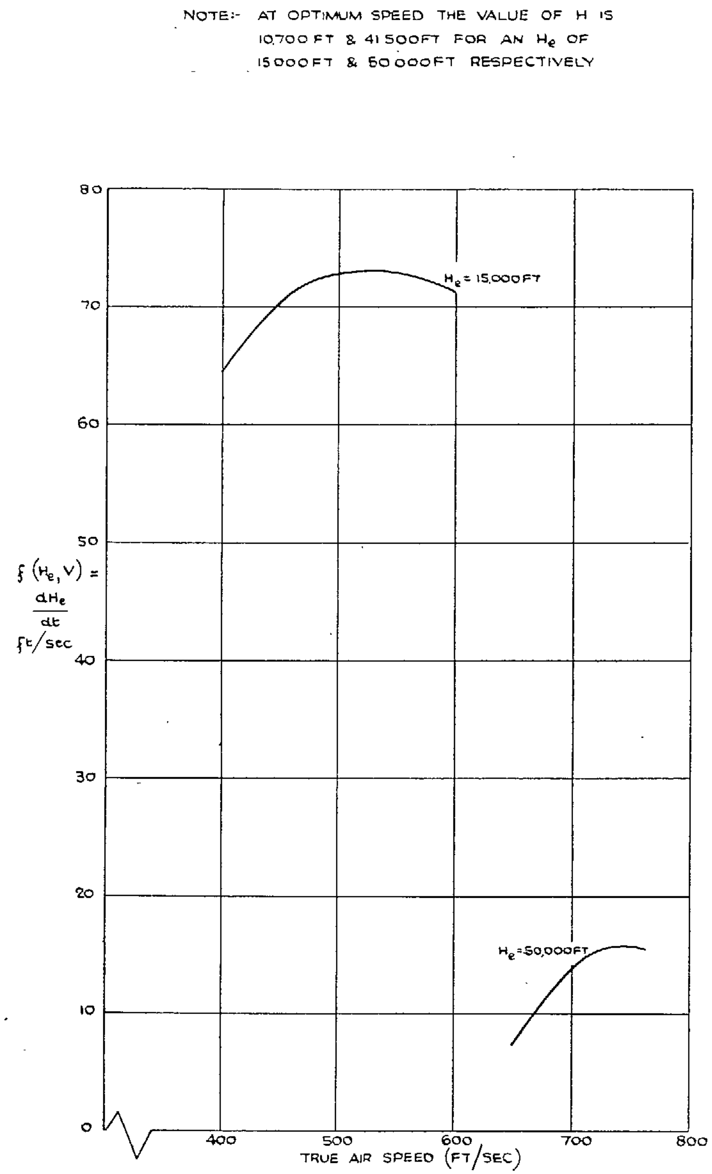


FIG. 3. Typical curves of $f\ (H_e,\ V)$ against $V$.

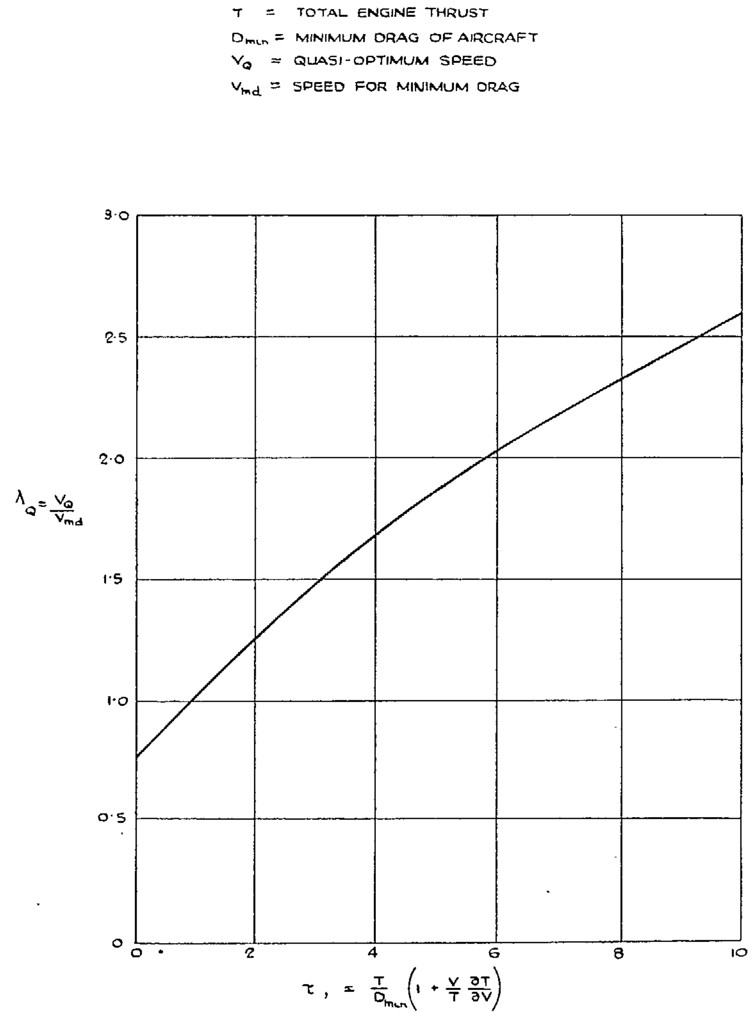


FIG. 4. 'Quasi-optimum' speed: Variation with $\tau$.



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