C.P. No. 316
(18,302)
A.R.C. Technical Report

C.P. No. 316
(18,302)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
CURRENT PAPERS

Experiments on the Growth of Vortices in Turbulent Flow

By

I. M. Titchener, B.Sc.(Eng.), A.C.G.I.
and
A. J. Taylor-Russell, B.Sc.(Eng.).

LONDON : HER MAJESTY'S STATIONERY OFFICE
1957
THREE SHILLINGS NET

C.P. No. 316

Experiments on the Growth of Vortices in Turbulent Flow

- By -
I. M. Titchener, B.Sc.(Eng.), A.C.G.I.
and
A. J. Taylor-Russell, B.Sc.(Eng.).

Department of Aeronautics
Imperial College
Paper No.31

Communicated by P.D.S.R.(A), Ministry of Supply.

21st March, 1956

SUMMARY

Measurements of the flow in turbulent line vortices along the centre of a pipe have been made to determine the growth of trailing vortices in the wake of an aeroplane. It is found that the rate of growth is small and of the same order as for a laminar line vortex.

CONTENTS

1. Introduction

The dominant feature of the wake of an aeroplane is the pair of trailing vortices. In order to assess the influence of this wake on another aircraft passing through it, an estimate of the rate of spread of a vortex in turbulent flow must be made. A suggested theoretical solution[1] to this problem requires knowledge of the eddy viscosity, which may be found from measurements on vortices. Some measurements have been made behind aeroplanes in flight[2,3] but have not led to a conclusive result except in so far as they do show that the effects of the two separate vortices are still detectable after the vortices have been in existence for half a minute or more, at a distance of several thousands of feet behind the aeroplane causing them.

Apparatus to give the required data has been designed and built[4] at the Imperial College. Results obtained from this apparatus are presented in this report.

2. Apparatus

The original apparatus was modified in some details so as to improve the flow and to increase the accuracy of the measurements. A sketch of the layout is given in Fig. 1. The vortex strength could be varied by altering the settings of the swirl vanes in the intake or by changing the axial speed. The velocity and direction of the air flowing in the pipe were measured with a Conrad yawmeter, a total head tube and a static tube. Radial position was measured to ± 0.001 inch with a micrometer head, angular position to ± 0.1 degree with a pointer on a scale. Brief explorations of the flow were also made with a hot wire anemometer and using paraffin 'smoke' injected through the tip of the fairing from which the vortex core originated.

3. Scope of Tests

The test programme consisted of:-

(a) flow visualisation and turbulence investigation,

(b) traverses at intervals along the pipe, measuring direction, total head and static pressure.

Section (a) was intended to verify that the flow in the vortex core was turbulent.

The tests made in Section (b) were as follows:-

Test number	Swirl-vane angle (degrees from radial)	Approx. axial velocity (feet/second)	Axial extent of traverse (inches)
1	5	190	42.5
2	10	190	42.5

If θ is the angle of pitch at radius r and u is the axial velocity (assumed to be independent of r), the circumferential velocity at radius r is $ur \tan \theta$. Hence the vortex strength is given by

$$K = 2\pi u\,(r \tan \theta)_{MAX}. \qquad \ldots(1)$$

and so if $(r \tan \theta)$ is plotted as a function of r (Fig. 6) then K can be deduced.

The size of the vortex (D) is defined in terms of the area between this curve and the value of $(r \tan \theta)_{MAX}$. We take

$$D = \int_{-\infty}^{\infty} \left[1 - \frac{r \tan \theta}{(r \tan \theta)_{MAX}} \right] dr. \qquad \ldots(2)$$

As the vortex develops it would be expected that D would increase. [It may be noted that $(r \tan \theta)$ is a measure of the angular momentum of unit mass; this definition of size may be compared with that of momentum thickness for a boundary layer].

The experimental results are presented as curves showing the variation of K and D for each vortex examined, arbitrarily choosing the zero of time to correspond to the first observation point along the length of the pipe.

An "effective viscosity" ϵ may be deduced from the values of K and D, if the circumferential velocity is assumed to be of the same form (1) as for a laminar vortex, that is

$$(r \tan \theta) = (r \tan \theta)_{MAX} \left[1 - \exp.\left(\frac{-r^2}{4\epsilon(t + t_0)} \right) \right] \qquad \ldots(3)$$

where t_0 is an unknown constant. This distribution of velocity is a good representation of the observed distribution.

We find by substitution from (3) into (2)

$$D^2 = 4\pi\epsilon(t + t_0). \qquad \ldots(4)$$

The effective viscosity ϵ can then be found by plotting D^2 against t.

5. Results

Photographs of smoke filaments admitted to the vortex centre under laminar and turbulent conditions are reproduced in Fig. 2. The hot wire anemometer showed that the turbulence level was of order 3% in the core; the results of the turbulence measurements, which are plotted in Fig. 9, are discussed in the Appendix.

Typical sets of measurements of pitch angle, total pressure and static pressure are plotted against the observed radial position of the traversing instrument in Figs. 3, 4 and 5. Fig. 6 shows $(r \tan \theta)$ plotted against r for the data given in Fig. 3. Vortex strength and size are given in Table I and plotted in Figs. 7 and 8.

Corrections have been applied for the following:-

(1) Displacement of the yaw-meter head when rotated,

(2)/

(2) An effect due to the stem of the instrument not passing exactly through the centre of the vortex,

(3) The change of axial velocity along the pipe due to the growth of the wall boundary layer and

(4) The associated contraction of the stream tubes in the middle of the pipe.

As shown in Fig. 6, the corrections under (1) and (2) were considerable when the pitch angle was large.

The measurements seemed to indicate that the axial velocity at the centre of the pipe was rather above the axial velocity outside the vortex, but this probably arose because of errors in the reading of the static tube in this region.

6. Discussion

The flow observations using smoke (Fig. 2) and the brief investigation with a hot wire anemometer showed that the flow was turbulent, but a fuller hot wire exploration than was actually made would be required to describe quantitatively the turbulent velocity fluctuations. Because the smoke was slightly heavier than air it tended to collect at the edge of the rotating core of the vortex and it may be seen that the radius of the region so defined did not increase appreciably along the pipe. The initial size of the vortex was fairly large, being determined by the boundary layer on the entry.

The strength of each vortex was sensibly constant along its length (Fig. 7).

The outstanding feature of the results is that the measured rates of growth (Fig. 8) were extremely small. This was not due to the presence of the pipe which contained the vortex because the experimental conditions were chosen so that the pipe wall boundary layer and the vortex core did not join.

From (4) we find that the effective viscosity is given by

$$\epsilon = \frac{D}{2\pi} \frac{dD}{dt} \qquad \ldots(5)$$

It was expected that ϵ would be very much greater than the kinematic viscosity ν; also it seems certain that, apart from experimental errors, the minimum rate of growth is given by replacing ϵ by ν in (4). The rate of growth of a laminar vortex of the same initial size is shown in Fig. 8 and it will be seen that this is of the same order as the observed rate of growth.* This rate is so small that it is not possible to measure it accurately with the present apparatus.

7. Comparison with Flight Tests

The American flight tests were not sufficiently detailed for analysis but they indicate that the trailing vortices grow very slowly. The R.A.E. flight tests were primarily intended to determine the amount of disturbance to an aircraft flying through the trailing vortices. They

were/

---

*The results of test 1 suggest at first sight a considerable rate of growth but this indication depends on two or three experimental points and these values may be in error.

were also analysed to determine the vortex size, but the method of measurement, which consisted of observations of aileron displacement of the following aircraft, is not sufficiently sensitive to give reliable results for the growth of the vortices.

8. Conclusion

The laboratory tests described in this report indicate that the rate of growth of a turbulent line vortex is small and of the same order as for a laminar vortex.

Acknowledgements

This work was carried out under the supervision of Professor H. B. Squire and Mr. C. Jackson.

References

No.	Author(s)	Title, etc.
1	H. B. Squire	The growth of a vortex in turbulent flow. Communicated by P.D.S.R.(A), Ministry of Supply. A.R.C.16,666 - 18th March, 1954.
2	D. R. Andrews	A flight investigation of the wake behind a Meteor aircraft, with some theoretical analysis. R.A.E. Tech. Note Aero.2283 C.P. 282. December, 1954.
3	C. C. Kraft	Flight measurements of the velocity distribution and persistence of the trailing vortices of an aeroplane. N.A.C.A. Tech. Note 3377. (March, 1955).
4	D. G. Mabey	Imperial College Thesis (1953).

Notation

K	Circulation
D	Mean width of curve of $(r \tan \theta)$ vs. r.
r	Radius from vertex centre
t	Time measured from arbitrary origin
t_Q	Constant depending on choice of origin for t
u	Component velocity parallel with vortex axis
ϵ	Effective viscosity in turbulent flow
ν	Kinematic viscosity
θ	Pitch angle of streamline
$(\ )_{MAX}$	Denotes conditions when r is large

TABLE I/

TABLE I

Measurements of Strength and Size of Four Vortices

Test Number	Axial Distance (inches)	Axial Velocity u (ft/sec)	Time $t \times 10^3$ (seconds)	Vortex Strength K (ft^2/sec)	Vortex Size D (inches)
1	0	180.6	0	2.84	0.256
	5	182.8	2.29	2.79	0.249
	10	185.0	4.56	2.88	0.282
	15	187.1	6.80	3.35	0.330
	17½	188.0	7.92	3.23	0.370
	20	189.2	9.02	2.98	0.311
	22½	190.2	10.12	3.70	0.369
	25	191.3	11.21	2.82	0.295
	27½	192.3	12.29	3.39	0.409
	32½	194.4	14.45	3.46	0.403
	37½	196.5	16.58	3.21	0.445
	42½	198.5	18.67	3.48	0.384
2	0	179.9	0	7.63	0.309
	5	182.3	2.30	7.32	0.289
	10	185.0	4.57	6.84	0.270
	15	187.5	6.81	7.02	0.278
	17½	188.9	7.91	6.44	0.266
	20	190.0	9.01	7.07	0.286
	22½	191.3	10.10	7.13	0.297
	25	192.7	11.18	7.22	0.298
	27½	193.9	12.26	6.55	0.283
	32½	196.4	14.40	6.77	0.284
	37½	198.9	16.50	6.95	0.323
	42½	201.5	18.57	7.07	0.317
3	0	170.1	0	15.48	0.338
	5	172.3	2.43	15.17	0.358
	10	174.5	4.84	15.17	0.378
	15	176.7	7.21	15.54	0.400
	17½	177.9	8.38	15.08	0.336
	20	178.9	9.55	16.41	0.421
	22½	180.0	10.71	15.21	0.367
	25	181.0	11.87	15.75	0.370
	27½	182.0	13.02	15.04	0.345
	32½	184.1	15.29	15.25	0.378

TABLE I contd.

Measurements of Strength and Size of Four Vortices

Test Number	Axial Distance (inches)	Axial Velocity u (ft/sec)	Time $t \times 10^3$ (seconds)	Vortex Strength K (ft^2/sec)	Vortex Size D (inches)
4	0	283.1	0	24.60	0.296
	5	286.5	1.46	24.45	0.312
	10	290.0	2.91	24.60	0.345
	15	293.6	4.34	26.13	0.403
	17½	295.2	5.04	26.00	0.403
	20	297.0	5.75	24.76	0.318
	22½	298.8	6.44	24.31	0.382
	25	300.5	7.14	24.45	0.321
	27½	302.2	7.83	24.67	0.341
	32½	305.6	9.20	23.86	0.351
	37½	309.0	10.56	21.19	0.305
	42½	312.3	11.90	24.59	0.392

APPENDIX/

APPENDIX

Turbulence Measurements

The longitudinal component of turbulence, u', was measured by vertical traverse across the vortex, for the following conditions:-

I Vane angle : 10°

Axial distance from beginning of cylindrical section of pipe : 2 ft.

Axial velocity U (ft/sec)	77	114	159
Corresponding time (sec)	0.0267	0.0176	0.0126

II Vane angle : 10°

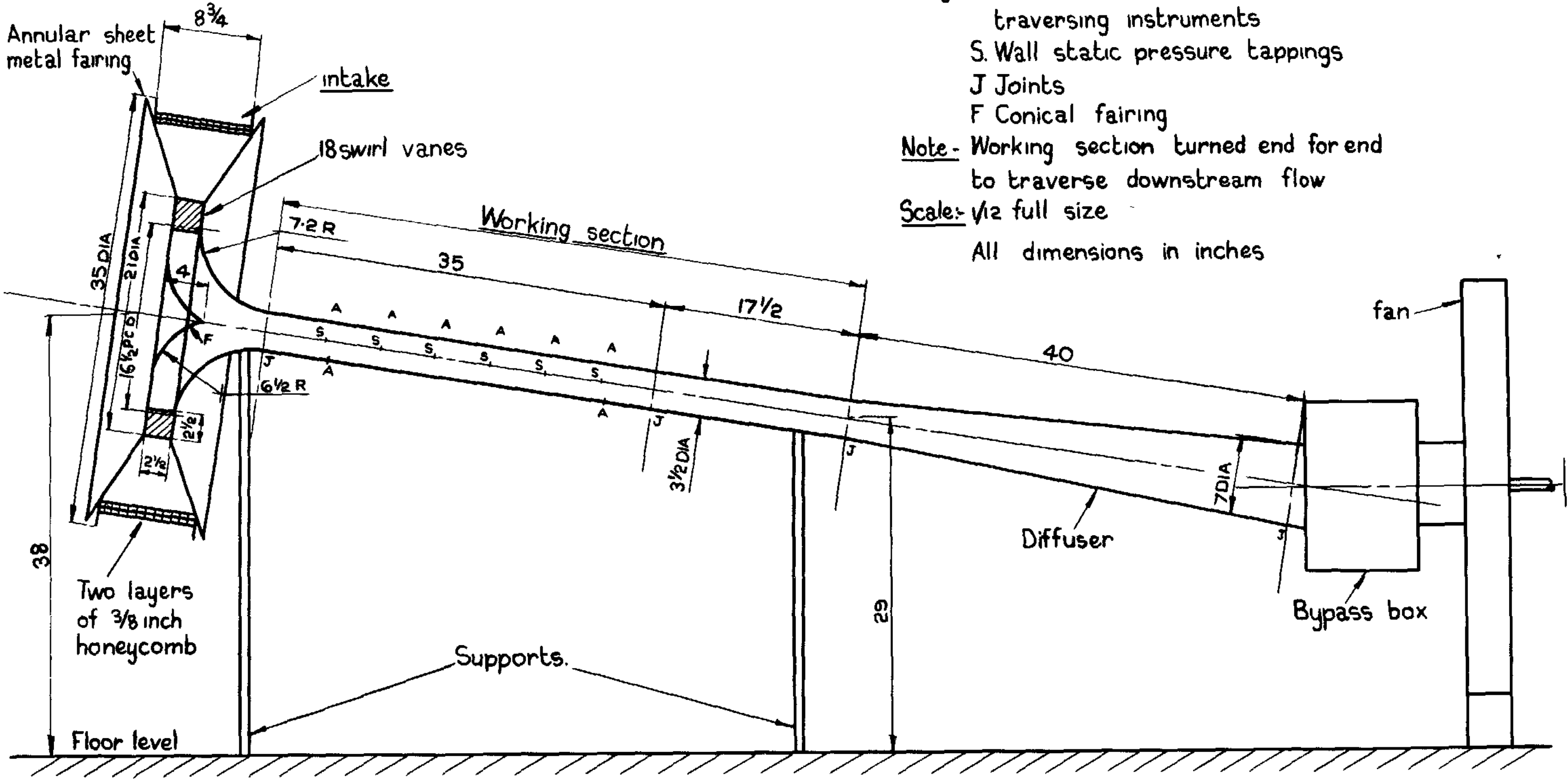
Axial velocity : 87 ft/sec

Distance from beginning of cylindrical section (ft)	1	2
Corresponding time (sec)	0.0115	0.0230

The results are given in Fig. 9 in the form of 100 u'/U plotted against distance from the axis of the tube. This shows (1) that the turbulence level in the vortex core increases with increase of speed and (2) that the effect of distance downstream on the turbulence level is small at a speed of 87 ft/sec.

No definite conclusions can be drawn from these results. It is possible, (1) that the vortex growth is little affected by turbulence level, because of the stabilising influence of the swirl and/or (2) that the turbulence level is affected by transition position on the entry fairing.

SC



Key. A. Access holes for traversing instruments
S. Wall static pressure tappings
J Joints
F Conical fairing

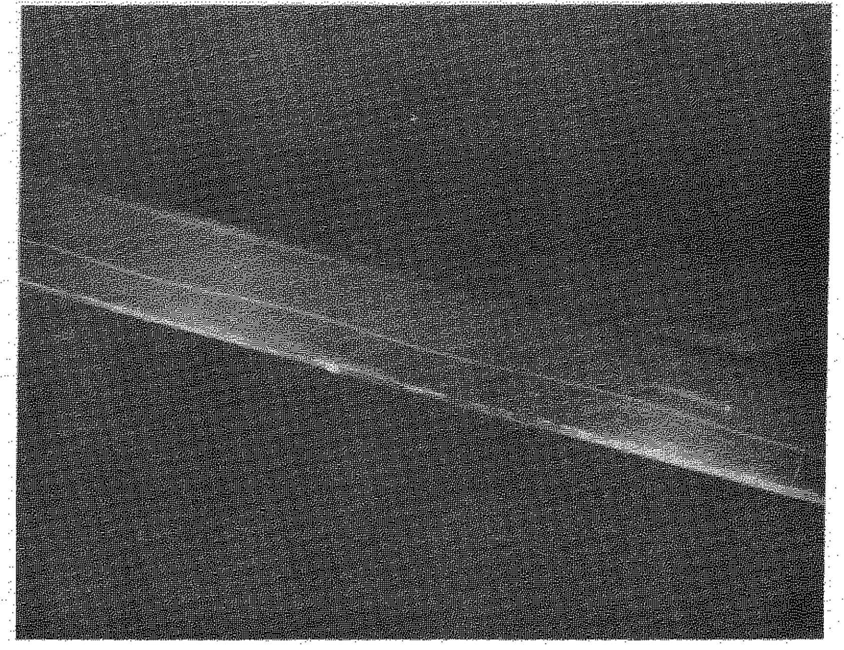
Note - Working section turned end for end to traverse downstream flow

Scale:- 1/12 full size

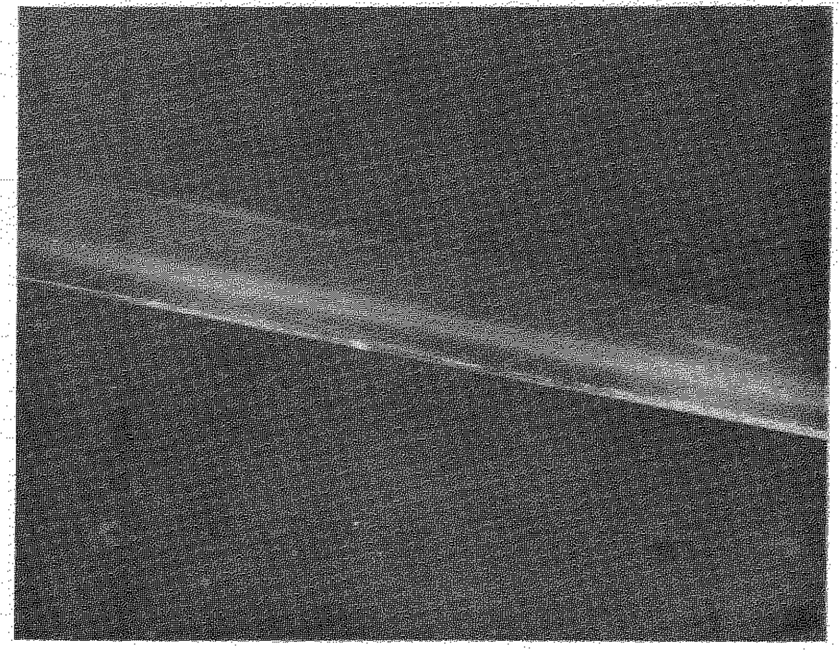
All dimensions in inches

Layout of vortex decay apparatus

FIG. 1

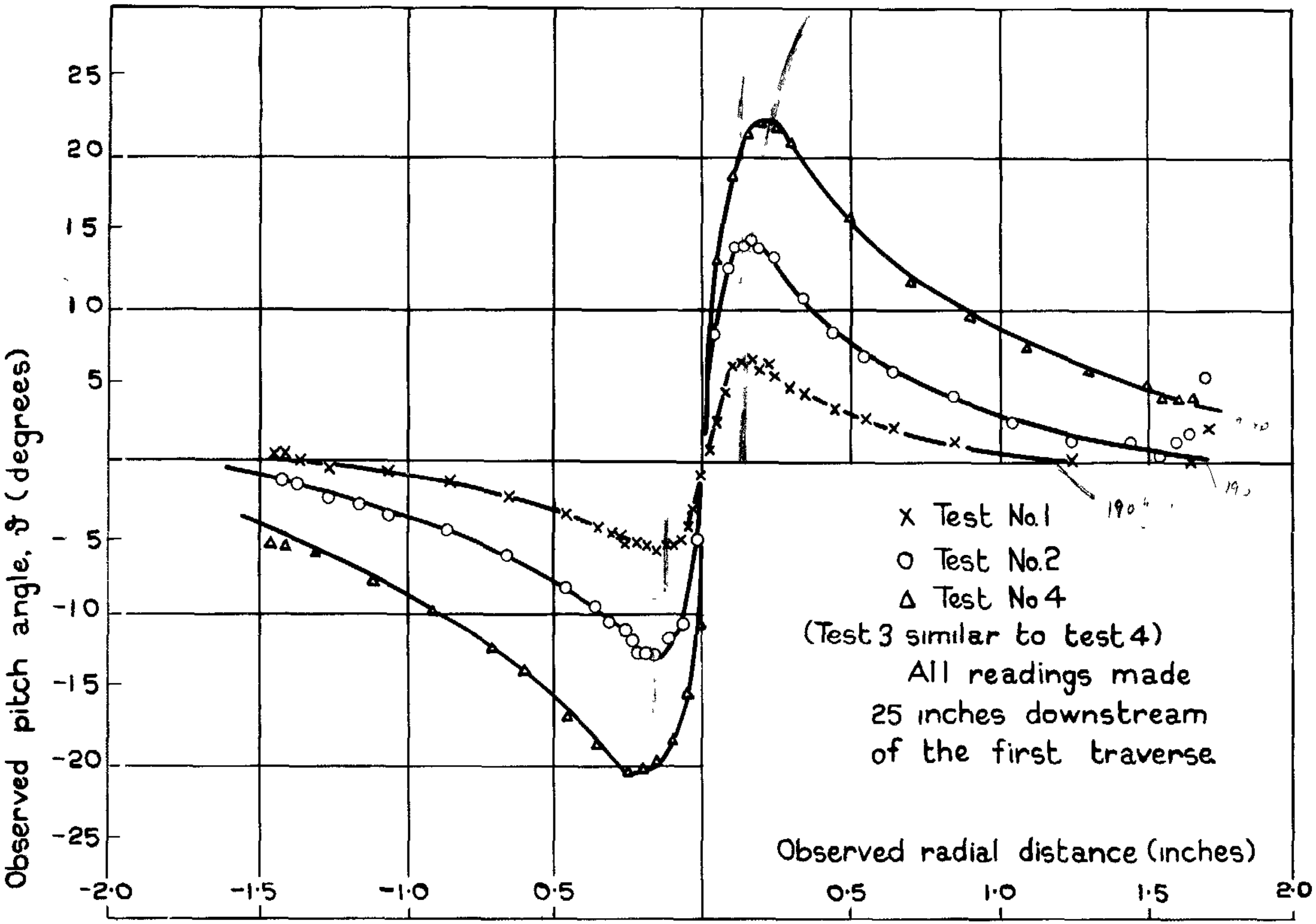


A. Flow (from L to R) at about 10 feet/second indicating laminar flow. The smoke has assumed a uniform thread-like appearance which is maintained along the length of the pipe.



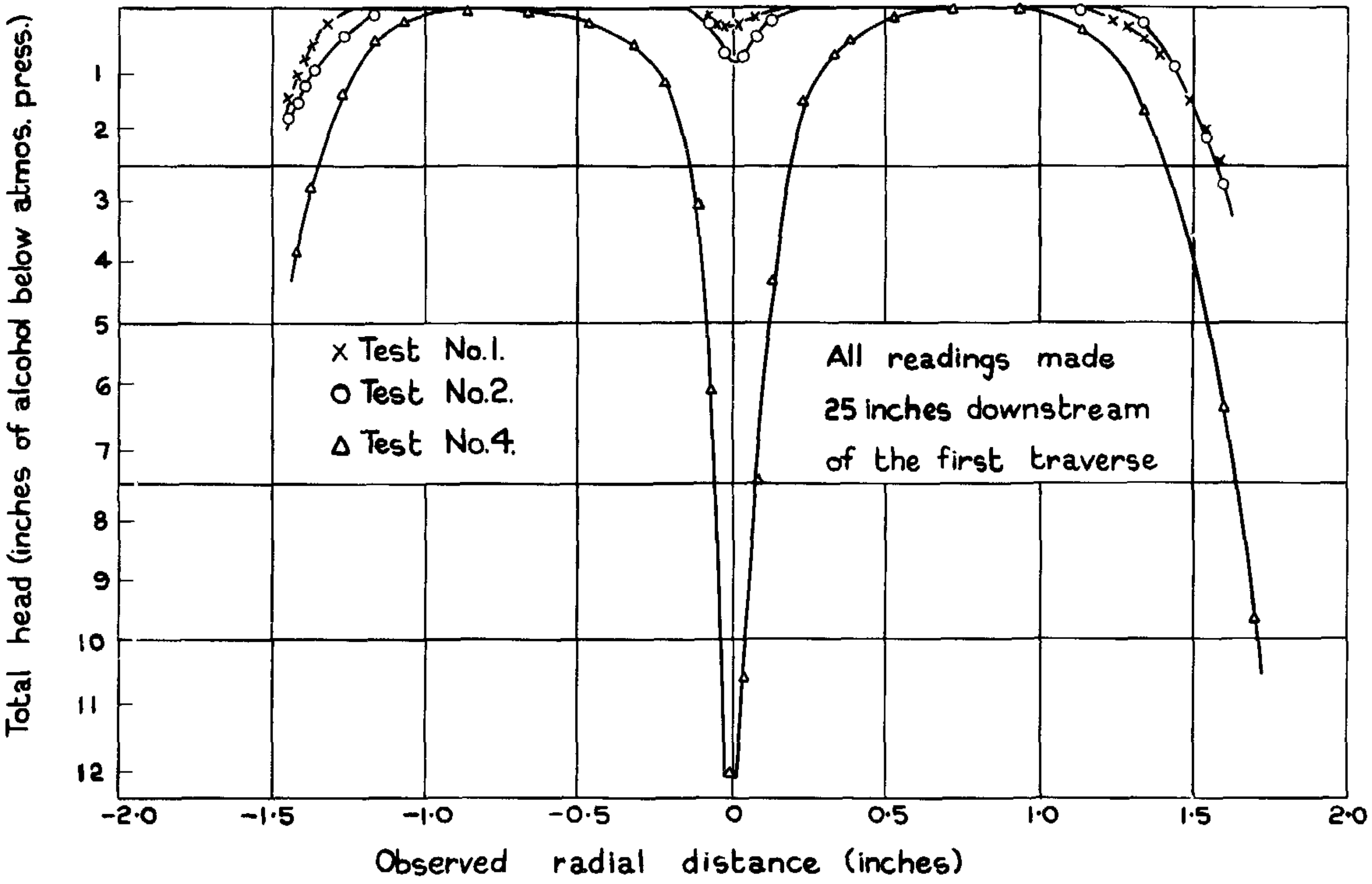
B. Flow at about 185 feet/second indicating turbulent flow. The smoke appears as a diffuse cloud situated at the edges of the core of the vortex.

NOTE. In both photographs the lower limit of the illuminated pipe is not the wall of the pipe but a screen used solely for photographic



Typical pitch angle curves

FIG 3.

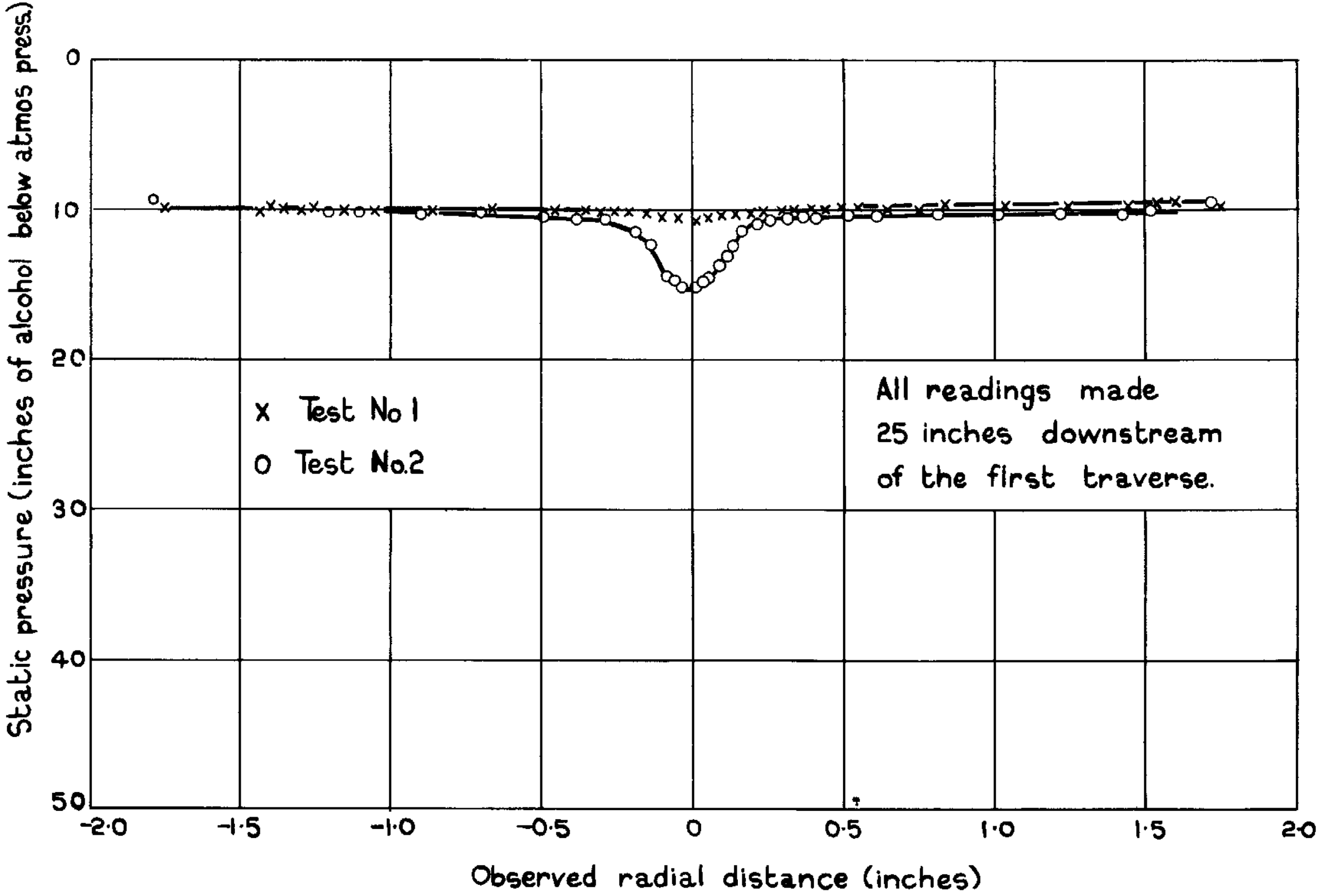


All readings made 25 inches downstream of the first traverse

Typical total head curves

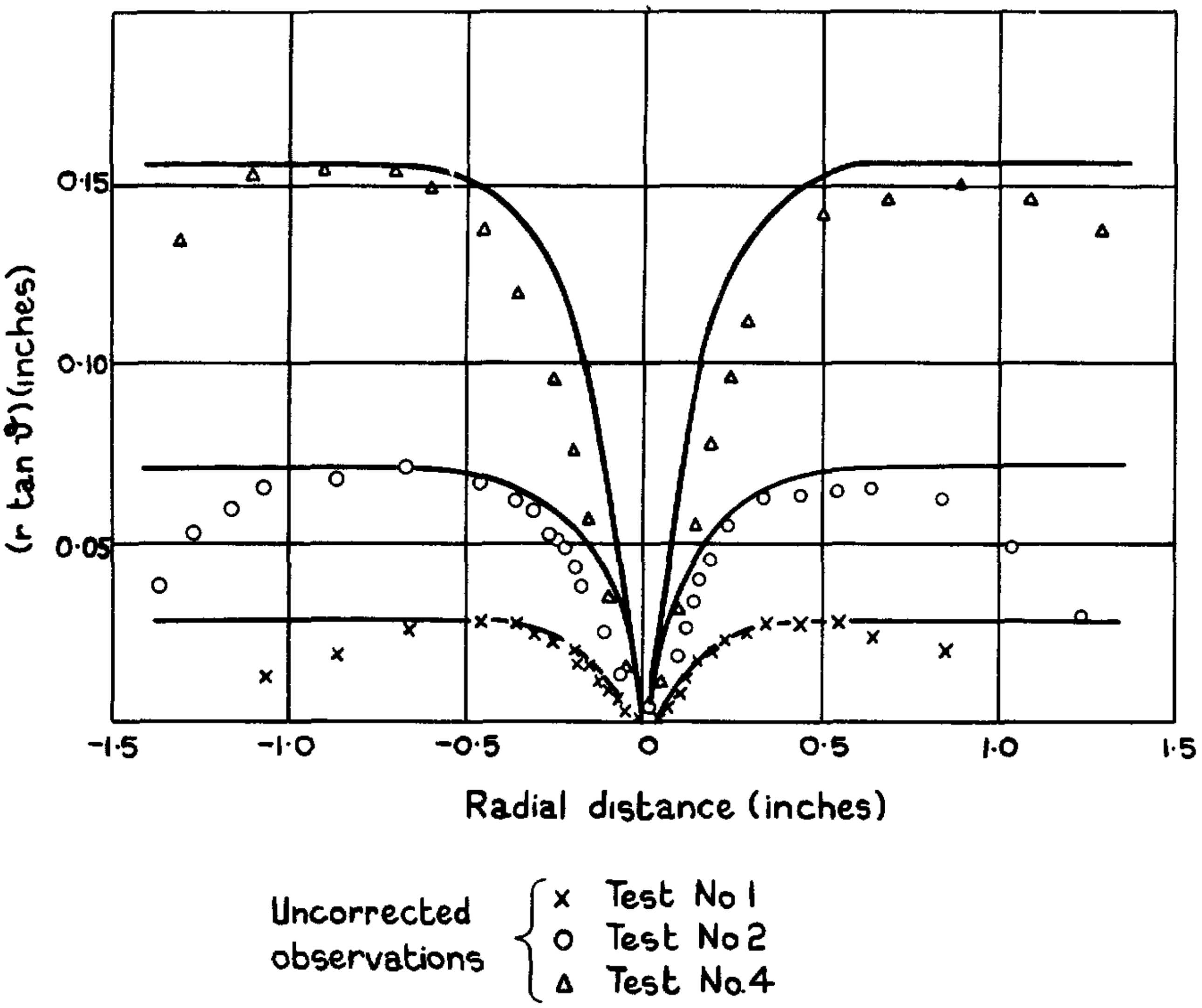
FIG.4

FIG 5



Typical static pressure curves.

FIG 6.

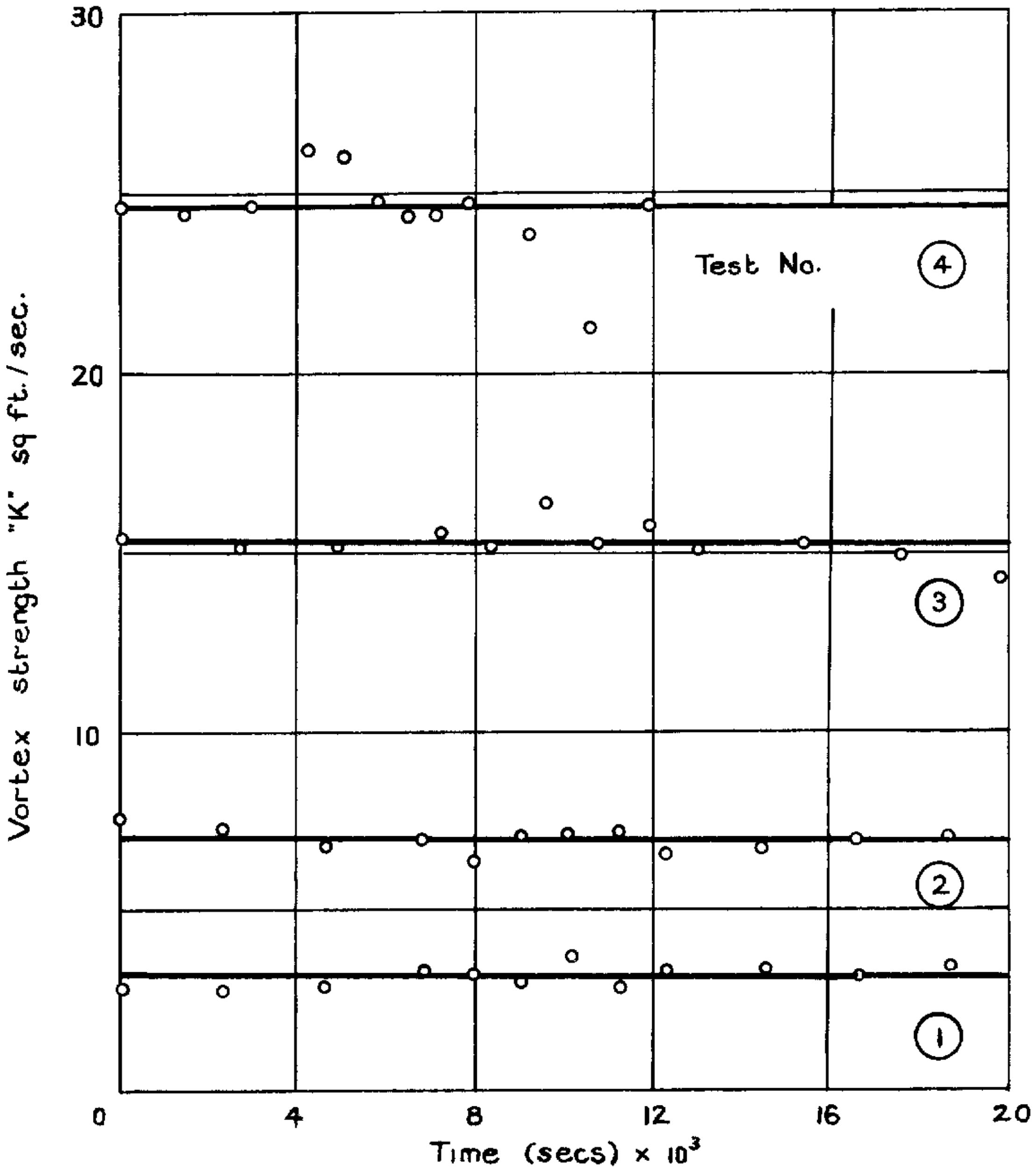


(Test 3 similar to test 4)

All readings made 25 inches downstream of the first traverse.

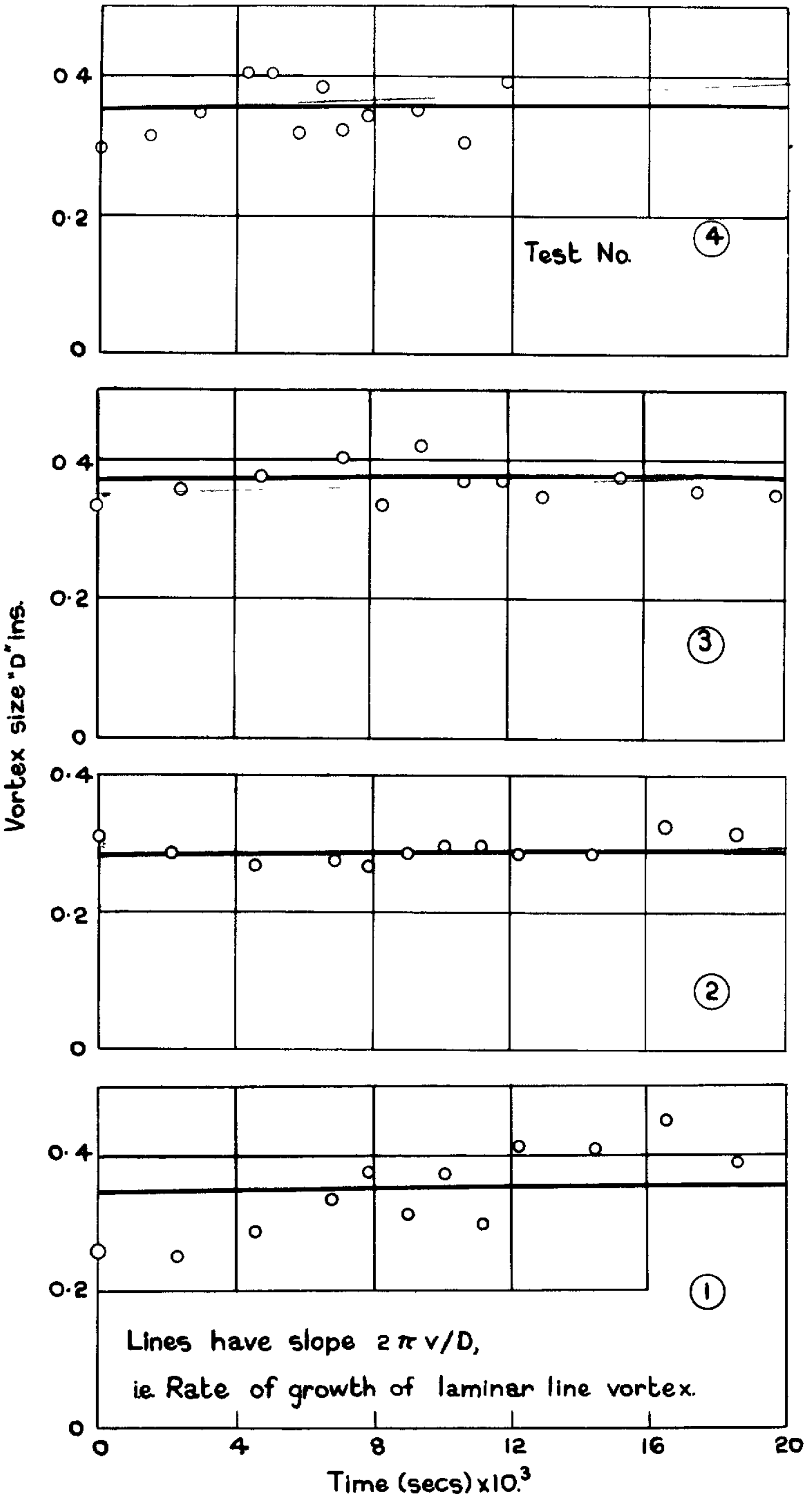
The curves in this figure incorporate corrections (1) & (2) (see para. 5 page 5)

FIG 7.



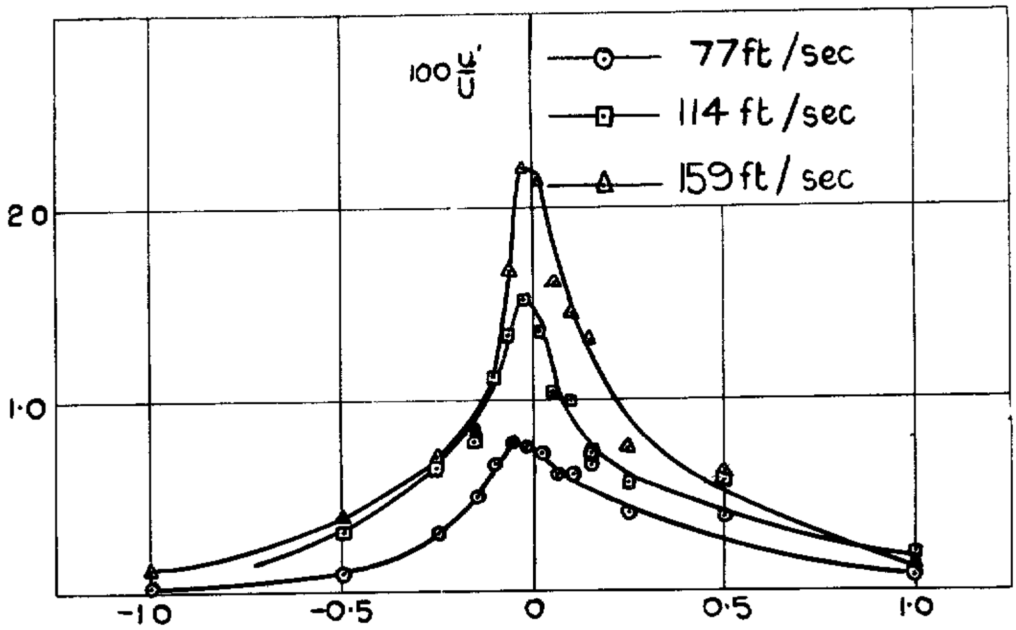
Measurements of vortex strength

FIG 8



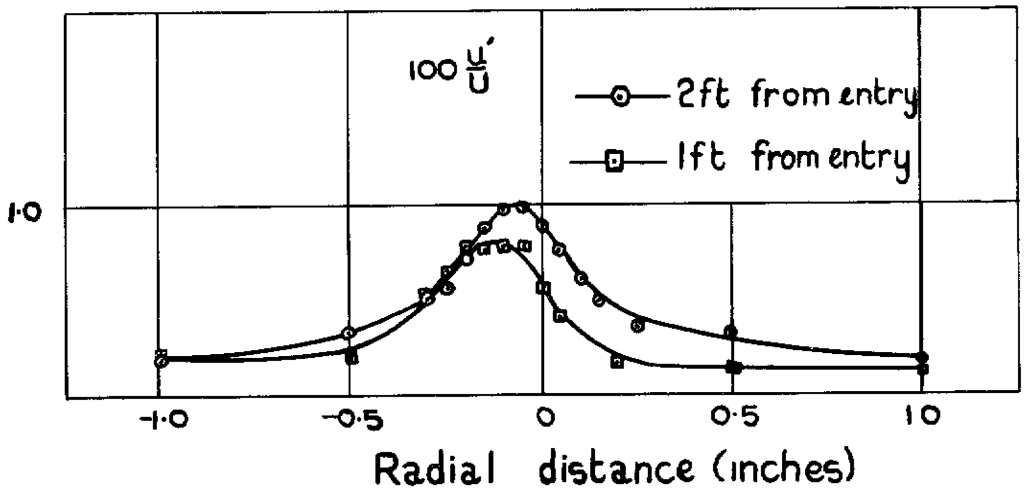
Measurements of vortex size

FIG 9



Variation for three axial velocities

Distance 2 ft from entry



Variation for two axial positions

Velocity 87 ft/sec

(Horizontal scales have arbitrary zero)

Variation of longitudinal turbulence across vortex.

C.P. No. 316

(18,302)

A.R.C. Technical Report

Printed and published by
HER MAJESTY'S STATIONERY OFFICE
To be purchased from
York House, Kingsway, London W.C.2
423 Oxford Street, London W.1
P.O. Box 569, London S.E.1
13A Castle Street, Edinburgh 2
109 St. Mary Street, Cardiff
39 King Street, Manchester 2
Tower Lane, Bristol 1
2 Edmund Street, Birmingham 3
80 Chichester Street, Belfast
or through any bookseller

Printed in Great Britain

S.O. Code No. 23-9010-16

C.P. No. 316