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General Performance Calculations for Gas Turbine Engines

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Summary.—In this report an attempt is made to summarise the theoretical work carried out during the past few years aimed at discovering the potentialities of the gas turbine as a power plant in many fields of application, but especially as an aircraft power unit. To do this the performance of the various modifications of the ideal gas turbine cycle is considered in some detail, and the works of various authors are then combined and edited in order to depict the performance attainable by practical engines. The influence of component efficiencies on this latter performance is examined and the effects of modifications, such as reheating the gas after partial expansion or introducing a heat exchanger, are compared with the effects predictable from the ideal cycle calculations.

The association between the gas turbine and jet reaction as a means of aircraft propulsion is considered and the probable performance of several simple jet engines estimated over a speed range from 0 to 1,500 m.p.h. The influence of forward speed and altitude on the output and efficiency of the gas turbine is obtained and combined with the influence of varying operating conditions upon the propulsive efficiency of the jet to give the overall performance of a jet-turbine combination.

Finally a method of estimating the performance of a simple jet engine from the non-dimensional characteristics of its components is detailed and the results of an example employing this method are used to illustrate the influence of several factors, such as propelling nozzle size, upon the equilibrium running conditions of such an engine.

1. *Introduction.*—During the war years the gas turbine may be said to have come into its own, both as an engine already accepted and in operation in jet-propelled aircraft, and also as a potential power plant in many other fields of service. It is desirable, therefore, to summarise the main features and characteristics of gas turbine engine performance known at present, and to give short descriptions of the methods which may be adopted to determine this performance. Such is the object of this report.

2. *The Cycle.*—2.1. *The Plain Cycle.*—The gas turbine operates on what is usually referred to as the 'constant pressure cycle' the implication being that the working fluid remains at constant pressure during the period in which heat is being supplied. The ideal cycle is shown in Fig. 1a on an Enthalpy-Entropy chart. Ideally air, the working substance, is taken in at the State 1 and compressed without change of entropy to State 2. Then, keeping constant pressure, heat is added, increasing the temperature and the volume until the air is at State 3, when it is allowed to expand, again isentropically, to State 4. The air standard efficiency of such a cycle, based on

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the assumption of constant specific heats at all temperatures, may easily be assessed. For if k_p is the specific heat at constant pressure, the heat taken in is $k_p(T_3 - T_2)$ and the heat rejected is $k_p(T_4 - T_1)$ so that

$$\text{Air Standard Efficiency} = \frac{\text{Heat taken in} - \text{Heat rejected}}{\text{Heat taken in}}$$

$$= \frac{k_p(T_3 - T_2) - k_p(T_4 - T_1)}{k_p(T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2}.$$

If the pressure ratio in compression equals the pressure ratio in expansion, both being denoted by R, then the temperature ratios T_2/T_1 and T_3/T_4 are both equal to $(R)^{(\gamma-1)/\gamma}$, where γ = ratio of specific heats, and

$$\text{Air Standard Efficiency} = 1 - \left(\frac{1}{R}\right)^{(\gamma-1)/\gamma}. \qquad \text{(1)}$$

The variation of this efficiency with pressure ratio is shown in Fig. 2a using $\gamma = 1{\cdot}4$ for air.

The increase in efficiency with pressure ratio is seen to be rapid at first but the curve is finally assymptotic to 100 per cent. Note should be made of the fact that for gas turbines it is more usual to work in terms of pressure ratio and not with the volume ratio which is usually employed in work on internal combustion reciprocating engines.

It is clear too that, if it were possible to carry out the compression and expansion of the gas adiabatically the efficiency of a gas turbine engine of constant pressure ratio would be unaffected by variations in the maximum temperature of the cycle. The work output will, however, increase with increasing maximum temperature, T_3, if intake temperature, T_1, is assumed to remain constant. For work output per lb of working fluid

$$= k_p\{[T_3 - T_2] - [T_4 - T_1]\}$$

$$= k_p\left[T_3 - T_1 . \frac{T_2}{T_1} - T_3 . \frac{T_4}{T_3} + T_1\right]$$

$$= k_p\,[T_3(1 - 1/c) - T_1(c - 1)] \qquad \text{(2)}$$

where $c = (R)^{(\gamma-1)/\gamma} = T_2/T_1$.

The variation of the quotient, work/k_pT_1, is plotted in Fig. 2b as a function of pressure ratio and T_3/T_1. The work, like the efficiency, increases rapidly at first with increasing pressure ratio, but unlike the efficiency it reaches an optimum and then decreases. The pressure ratio at which the optimum work occurs increases with increasing temperature ratio and is, in fact, that which makes $c^2 = T_3/T_1$ (by differentiating the equation (2)) and in consequence makes T_2 and T_4 equal.

2.2. *Modifications to Plain Cycle.*—2.2.1. *Improvement of efficiency : heat exchange.*—If we assume that by the use of a perfectly efficient heat exchanger we may remove heat from the air at the end of expansion and supply it to the compressed air as part of the heating process we may improve the efficiency of the cycle under certain conditions. In the limiting case of a perfectly efficient contra-flow heat exchanger the temperature of the exhaust air may be lowered to that at the end of compression, and the temperature of the compressed air raised to that of the air at the end of expansion. It is clear, therefore, that such a heat exchanger is only of use if T_4 is greater than T_2 in the normal cycle. This implies that mechanical or other considerations prevent the attaining of the pressure ratio which yields maximum work output for a given temperature ratio T_3/T_1 for under such conditions T_2 and T_4 are equal. Fig. 1b illustrates the cycle. States 1 to 4 are as in the normal cycle but State 5 is introduced to indicate the point where the air passes from heat exchanger to heater and State 6 gives the state at which the air leaves the engine. We are concerned at present only with the case when $T_5 = T_4$ and $T_6 = T_2$, that of a perfectly efficient exchanger.

$$\text{Efficiency} = \frac{\text{Heat in} - \text{Heat out}}{\text{Heat in}}$$

$$= \frac{k_p(T_3 - T_5) - k_p(T_6 - T_1)}{k_p(T_3 - T_5)}$$

$$= \frac{(T_3 - T_4) - (T_2 - T_1)}{T_3 - T_4}$$

$$= 1 - \frac{T_1}{T_4}$$

or

$$= 1 - c \, . \, \frac{T_1}{T_3} \qquad \qquad (3)$$

because

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = c \, .$$

The ideal efficiency is now seen to be dependent both on pressure ratio and also the ratio of the temperature before expansion to the initial temperature T_1. The equation (3) also indicates that if there is an upper limit to the maximum temperature ratio T_3/T_1 which may be employed in an engine with a heat exchanger, maximum efficiency will be obtained by employing the lowest pressure ratio compatible with obtaining the necessary work.

The ideal efficiency will exceed that of the engine without heat exchanger as long as T_4 is greater than $T_1 \times R^{(\gamma-1)/\gamma}$, *i.e.*, than T_2.

The heat exchanger does not effect the work output per lb of working substance, which is

$$k_p \{(T_3 - T_5) - (T_6 - T_1)\}$$

$$= k_p \{T_3 - T_2 - T_4 + T_1\}$$

$$= k_p \left\{ T_3 \left(1 - \frac{1}{c}\right) - T_1(c - 1) \right\}, \text{ as before.}$$

Fig. 2b also represents, therefore, the work output from the cycle with heat exchanger as well as that from the normal cycle.

The curves of Fig. 2c show the efficiency variations quite clearly. The efficiency of the cycle incorporating a heat exchanger and using a constant temperature ratio, is large at low pressure ratios and decreases with increasing pressure ratio up to a point where it equals that of the normal cycle.

By comparison between Figs. 2a, 2b and 2c it is made clear that the pressure ratio at which the efficiencies of the two cycles are equal is also the pressure ratio of optimum work for the given temperature ratio. Under these conditions $T_2 = T_4$ and the heat exchanger shows no advantage. At lower pressure ratios, however, the advantage can be appreciable. Consider $T_3/T_1 = 4$. At a pressure ratio of 11 : 1 the air standard efficiency is 50 per cent and the work output per lb/sec is ideally a maximum. A heat exchanger has no effect on these values. At a pressure ratio of 5 : 1, the work output per lb is reduced by 11 per cent from the maximum, but the air standard efficiency of the cycle with a heat exchanger, is now 60 per cent. The normal cycle operating at 5 : 1 pressure ratio would have an air standard efficiency of only 37 per cent.

2.2.2. *Improvement of output : reheat.*—Supposing again that there is an upper limit to T_3 an increase in output per lb of working substance over the plain cycle may be obtained by allowing the expansion to take place in parts and by re-heating the air to the upper limit T_3 between each part. Fig. 1e shows the new cycle if the expansion is split into two such parts. State 5 is now at the end of the first expansion and state 6 is reached after re-heating, so that $T_6 = T_3$. The work output per lb is now

$$k_p\,[(T_3 - T_2) + (T_6 - T_5) - (T_4 - T_1)]\,.$$

If we denote T_3/T_5 by b, having already used $T_2/T_1 = c$, then $T_6/T_4 = c/b$ because $P_2/P_1 = (P_3/P_5)(P_5/P_4)$ and therefore $\dfrac{T_2}{T_1} = \dfrac{T_3}{T_5} \times \dfrac{T_6}{T_4}$ since $P_5 = P_6$.

$$\text{Therefore Work } W = k_p\,[2T_3 - T_2 - T_5 - T_4 + T_1]$$

$$= k_p\,[2T_3 - cT_1 - T_3/b - bT_3/c + T_1]$$

$$\text{Therefore} \qquad \frac{dW}{db} = k_p T_3 \left[\frac{1}{b^2} - \frac{1}{c}\right].$$

This equals zero and the work is a maximum if $b = \sqrt{c}$. This is when the temperature ratios and consequently the pressure ratios across the two parts of the expansion are equal. It can be proved that the work from this cycle is greater than that from the plain cycle for all values of b between 1 and c, *i.e.*, in all practical cases, but it is also true that the maximum ideal efficiency occurs when $b = 1$, *i.e.*, in the plain cycle with one expansion only.

Figs. 2g and 2h show, respectively, the efficiency and the work output parameter of the new cycle, as a function of pressure ratio and temperature ratio. The best division of temperature drop between the two parts of the expansion is assumed. The pressure ratios for optimum work output are seen to be higher than the corresponding figures in the normal cycle. The efficiency, which is always lower than the efficiency of the normal cycle at similar pressure ratios, is also now a function of temperature ratio and decreases with decreasing temperature ratio.

2.2.3. *Improvement of output : intercooling.*—Results comparable to those found for the reheated cycle are obtained if the compression is divided into two parts and the air cooled to its original temperature T_1 between the two compressors (*see* Fig. 1c). The output is increased, reaching a maximum when the pressure ratios in the two compressors are equal, but the ideal efficiency of the cycle is reduced.

At low pressure ratios and between fixed temperature limits T_3 and T_1, however, though intercooling increases the work output of a cycle of fixed pressure ratio somewhat less than does reheat, the lowering of the air standard efficiency is also somewhat smaller. Figs. 2d and 2e illustrate these features, the curves occupying an intermediate position between those for the plain cycle and those for the reheat cycle over the range of pressure ratio shown.

2.3. *Value of Ideal Cycle Estimates.*—The foregoing in no way represents all the variations which may be made to the ideal cycle. It is intended only to show the effects which may be expected from three types of modification which it is relatively easy to employ on a gas turbine engine in its present general form. Although the inefficiencies of the various components make the formulae and values of the air standard efficiencies quoted above matters of academic interest only, they remain, nevertheless, the limits to which the realised performance of a turbine engine will tend to approach with all-round improvements in component efficiencies.

2.4. *Multiple Modifications.*—Before passing on to more detailed consideration of realised performance it is of interest to note the effect in the ideal case of incorporating in the plain cycle combinations of reheat, of intercooling and of heat exchange.

2.4.1. *Reheat and heat exchange.*—The introduction of the heat exchanger into the cycle with reheat (*see* Fig. 1f) does not affect the work output of the cycle any more than did its introduction into the plain cycle, and the optimum work for a given T_3/T_1 still occurs when the three temperatures, T_2, T_5 and T_4, are equal, the condition at which a heat exchanger has no advantage. At pressure ratios below that giving the optimum work for a given temperature ratio, the influence of the heat exchanger is more marked than in the case of the plain cycle with heat exchanger. Fig. 2i indicates this result, the efficiency again being that corresponding to the best division of the temperature drop between the two parts of the expansion (actually equal temperature drop in both parts).

The introduction of the heat exchanger has the effect that this division of temperature drop is now not only that for optimum work output but also for optimum cycle efficiency. Without the heat exchanger the efficiency at fixed temperature and pressure ratios was at an optimum when no reheat was introduced. The heat exchanger makes the two optima coincide at the same division of temperature drop.

Considering again $T_3/T_1 = 4$ as in section 2.2.1. Fig. 2h shows that the inclusion of reheat in the cycle of a pressure ratio of 11 : 1 will increase the work output per lb/sec by 34 per cent on that of the plain cycle (Fig. 2b) and that this is still slightly below the optimum obtainable with $T_3/T_1 = 4{:}0$ at pressure ratios of approximately 20 : 1. Figs. 2g and 2i show that the air standard efficiency at 11 : 1 pressure ratio would be 42 per cent with reheat alone and 58 per cent with a heat exchanger and reheat. The 5 : 1 pressure ratio cycle previously considered would have a work output 6 per cent greater with reheat than the optimum for the plain cycle, whilst the air standard efficiency of 33 per cent with reheat alone, would be increased to 64 per cent with the addition of a heat exchanger.

2.4.2. *Intercooling and heat exchange.*—The influence of a heat exchanger on the cycle with intercooler is comparable with its influence upon the reheat cycle. The work output remains unaltered, but the efficiency is appreciably increased at low pressure ratios (Fig. 2f).

2.4.3. *Intercooling and reheat.*—Interesting results are obtained from a study of the cycle incorporating both reheat and intercooling shown in Fig. 1g. The equation for the work output per lb/sec of working substance when the compression and expansion temperature changes are equally divided by the intercooler and re-heater respectively is

$$\frac{\text{Work}}{k_p T_1} = 2\left[\frac{T_3}{T_1}\left(1 - \frac{1}{\sqrt{c}}\right) - (\sqrt{c} - 1)\right].$$

Comparison with equation (2) shows that at a pressure ratio of $(c)^{\gamma/(\gamma-1)}$ the new cycle gives, ideally, just twice the work of the plain cycle of pressure ratio $(\sqrt{c})^{\gamma/(\gamma-1)}$, using the same temperature ratio T_3/T_1. This always represents an increase in work over the plain cycle at the same pressure ratio $(c)^{\gamma/(\gamma-1)}$ and temperature ratio T_3/T_1. Fig. 2k illustrates that the work output is, in fact, greater than that from any cycle previously considered but the air standard efficiency Fig. 2j is lower than for all the others.

2.4.4. *Intercooling, reheat and heat exchange.*—The addition of a heat exchanger to this last cycle, however, finally associates the maximum output per lb for a given temperature ratio, with maximum air standard efficiencies. The efficiency of the cycle shown in Fig. 1h is

$$1 - \frac{T_1}{T_3}\sqrt{c}$$

and equals, therefore, the efficiency of the plain cycle with heat exchange at pressure ratios equal to the square root of those employed in the new cycle. Fig. 2i shows the efficiency of this cycle to be higher than the corresponding efficiency for any other cycle considered here.

2.5. *Example.*—Enough has been said to show that both the output and efficiency of a plain cycle can be considerably increased by the modifications of reheat or intercooling and heat exchange used in conjunction with each other, it being assumed that the temperature ratio, T_3/T_1 represents a fixed limit to the various cycles. The investigation has been restricted to the introduction of one stage only, of reheat and intercooling, in order to show the effects that might be obtained by reasonably practicable modifications to the cycle.

In order to summarise these effects the results of the example already discussed are completed and tabulated below.

Temperature Ratio $= 4:1$.

Cycle	Pressure Ratio					
	5:1			11:1		
	Work Output Parameter	Air Standard Efficiency		Work Output Parameter	Air Standard Efficiency	
		Without heat exchange	With heat exchange		Without heat exchange	With heat exchange
Plain	0·89	0·37	0·60	1·00	0·50	0·50
Plain + Intercooling	0·96	0·35	0·66	1·17	0·45	0·59
Plain + Reheat	1·06	0·33	0·64	1·34	0·42	0·58
Plain + I.C. + R.H.	1·12	0·32	0·68	1·50	0·40	0·65

3. *The Cycle with Losses.*—The discussion of the ideal cycles leads to results which, though valuable indications of the relative importance of temperature ratio, pressure ratio and various modifications on the constant pressure cycle, are, nevertheless, unattainable in practice. In practice, it is impossible to carry out either the compression or expansion of the working substance isentropically, and the heating process is likely to introduce small losses in pressure. Modifications such as reheating, intercooling and heat exchange are also accompanied by pressure losses. Complete exchange of heat between hot exhaust gases and the compressed gas is improbable, and contrary to the assumption so far made, the specific heat of the gas is not constant with changes in temperature, so that the ratio of temperature change to heat change is not constant in all parts of the cycle. These are some of the reasons why figures obtained for the ideal cycle are not quantitative estimates of the performance of a practical engine operating at the same temperature and pressure ratios.

3.1. *Specific Heat Changes.*—The performance of the ideal cycle has been assessed on the assumption that the specific heat of the working fluid is constant at $0·24$ ($\gamma = 1·4$) at all points in the cycle, this being the specific heat of air under N.T.P. conditions. In actual fact, however, the specific heat of pure air increases with temperature, and in addition the admixture of the products of combustion of any fuel which may be burnt in the air stream, increases the specific heat still further. Fig. 3a shows the variation of the true specific heat of air with temperature and in passing shows also the influence of the products of combustion of a typical hydrocarbon fuel. To allow for the true specific heat at all points in the cycle necessarily makes any performance calculation a somewhat laborious business. As an approximation it is possible to use values of mean specific heat for the compression, heating and expansion parts of the cycle estimated from the appropriate temperature change. A further approximation which is sometimes used for added simplicity, is the adoption of average figures for the mean specific heat in compression and expansion which are used independent of the actual temperature changes. Figures of 0·24 for compression and 0·276 for expansion are typical values for these average mean specific heats.

Figs. 3b and 3c show the variation of air standard efficiency and of work output with cycle pressure ratio at a temperature ratio of 4 when these various assumptions are made regarding specific heat. The work output is shown as C.H.U. lb of air which would be given if the inlet temperature of the cycle were 288 deg K and it corresponds to the difference between the heat drop in expansion and the heat rise in compression. Curves are drawn using the following different assumptions.

(1) Constant specific heat of 0·240 C.H.U./lb/deg C.

(2) Constant specific heat of 0·276 C.H.U./lb/deg C.

(3) Appropriate mean specific heat in compression, heating and expansion.

(4) Appropriate mean specific heat in heating but constant mean specific heat of 0·240 in compression and of 0·276 in expansion.

(5) Actual total heats.

In the three latter cases allowance is made for the small increase in the mass of working fluid due to adding fuel to the air stream. It is seen from Fig. 3 that the approximations 3 and 4 above, both give curves lying very close to the curve 5 using actual total heats, and that they fall between the curves for constant specific heats. At the lower pressure ratios there is a tendency for the three curves to approach the curve for a constant value of 0·276, but at higher pressure ratios they occupy a more intermediate position between the curves for specific heat of 0·240 and 0·276 over the whole cycle. This is not altogether an effect of changing specific heat, but is due in part to the influence of the mass of the fuel (2 per cent of the mass of air at pressure ratio = 2, 1·3 per cent at pressure ratio of 20). In all cases, however, the air standard efficiency is lower than that which would be obtained if air had, at all temperatures, the specific heat of 0·240 occurring at N.T.P.

3.2. *Component Efficiencies*.—When practical considerations of component losses are taken into account the cycle on which an engine operates in the simplest case, the plain cycle, is similar to that shown in Fig. 4, changes in specific heat being taken into account in plotting the constant pressure lines. The temperature after compression, T_2, is greater than the temperature, T_2', which would have been reached in isentropic compression over the same pressure ratio. More work has had to be done to obtained a given pressure ratio than in the ideal case of isentropic compression. The pressure P_3 is lower than the pressure after compression, P_2, by the amount of the pressure loss incurred in the heating process. Inefficiencies in the expansion process, cause the temperature drop and consequently the work done in the expansion to be less than if the expansion were isentropic. The final temperature, T_4, is therefore greater than the temperature T_4' corresponding to isentropic expansion. It will be seen that logical definitions of the efficiencies of compression and expansion are therefore:

$$\text{Efficiency of compression, } \eta_{12} = \frac{\text{Isentropic Temperature Rise}}{\text{Actual Temperature Rise}}$$

$$= \frac{T_2' - T_1}{T_2 - T_1}$$

$$= \frac{T_1[R^{(\gamma-1)/\gamma} - 1]}{T_2 - T_1}$$

(where R is again the pressure ratio).

$$\text{Efficiency of expansion, } \eta_{34} = \frac{\text{Actual temperature drop}}{\text{Isentropic temperature drop}}$$

$$= \frac{T_3 - T_4}{T_3 - T_4'}$$

$$= \frac{T_3 - T_4}{T_3\left[1 - \left\{\frac{1}{R}\right\}^{(\gamma - 1)/\gamma}\right]}.$$

3.3. *Static and Stagnation Conditions.*—If, as is often the case, the working substance is not at rest at any stage in the cycle which it is necessary to consider, distinction must be made between the static and stagnation conditions of the gas at that stage. The static pressure and temperature of the gas are those which would be recorded by an observer moving freely in the gas stream at the same velocity as the gas, whilst the stagnation, or total head, temperature and pressure are those which would be observed if the gas were brought to rest instantaneously and without loss.

The total head temperature T_t of the gas is greater than the static temperature T, by the temperature equivalent of the gas velocity, V,

$$T_t = T + \frac{V^2}{2gJk_p}$$

and the total head pressure, P_t, may be related to the static pressure, P, in a compressible fluid, by the relationship

$$P_t = P\left\{\frac{T_t}{T}\right\}^{\gamma/(\gamma - 1)}.$$

At low gas velocities this approximates to the formula for an incompressible fluid,

$$P_t = P + \tfrac{1}{2}\rho V^2$$

where ρ is the density of the gas, but at the velocity of sound in air, P_t is some 11 per cent greater than the value which would be obtained by assuming air to be incompressible.

In defining component efficiencies in an engine where the working substance is in motion, therefore, it is clearly of importance to state whether total head or static conditions are being referred to and this is a point which will receive further attention in the following sections where the particular case of the constant pressure cycle gas turbine engine will be considered in more practical detail.

4. *Gas Turbine Engines.*—At the present time all engines which may be classified as gas turbines have in common three major components, a compressor, a combustion chamber and a turbine, and of these the only component which might suffer a radical change without altering the classification of the engine, is the combustion chamber. In other words the present practice of introducing heat to the working fluid by burning fuel in it, might be replaced by a method which supplies heat from an external source, but so long as the engine incorporates some form of compressor and some form of turbine as major components it may rightly be regarded as a gas turbine.

4.1. *Present Engines.*—Confining the discussion to gas turbines in their present form we may note how the various processes in the cycle are performed, the working substance in all cases being air.

A. *Compression.*—This may be done by

(a) a centrifugal compressor, single or double-sided depending on design, single or multi-staged, depending on pressure ratio required,

(b) an axial compressor, invariably multi-staged,

(c) a combination of centrifugal and axial compressors,

(d) a diagonal-flow design of compressor intermediate between the centrifugal and axial,

(e) a displacement compressor.

B. *Heating.*—In all known gas-turbine engines and projects*, heating of the air is carried out by burning liquid fuel (paraffin or petrol) or gaseous fuel in the air and allowing the heat generated to be communicated to the whole mass of the air.

C. *Expansion.*—A proportion of the work obtained from the expansion of the hot gases, is required to drive the compressor and this work is extracted by the component which gives its name to the whole engine, the turbine. To date, all engines have incorporated single- or multi-staged axial turbines. No radial-flow turbines have been used. Further expansion beyond that required to provide the work to drive the compressor may be carried out in a further turbine or in later stages of the first turbine and the power absorbed in the most convenient way (*e.g.*, by an electric generator, or in the case of aircraft applications, by a propeller). In the particular case of aircraft jet propulsion, the final expansion is carried out in the propelling nozzle and imparts a high velocity to the air which leaves the engine as a propulsive jet. It is clear that in engines of this type the air is never at rest but flows in a steady stream through the engine, and further that no engine for use with jet propulsion can operate on a closed cycle, using the same charge of air continuously, but must inspire fresh air to replace that being exhausted from the engine.

4.2. *Definitions of Efficiencies.*—As at all stages in a gas-turbine engine the air is in motion it becomes necessary to distinguish between total head and static conditions in defining component efficiencies. In general the following conventions are adopted.

4.2.1. *Compressor efficiency.*—Compressor efficiency is defined using total-head conditions at both inlet and outlet. This is a convenient arrangement because the total-head temperature rise in the compressor is a measure of the work being absorbed. Further it is generally easier to measure total-head pressures than static ones, so that compressor tests yield more reliable estimates of total-head efficiency than of static efficiency.

4.2.2. *Combustion pressure loss and combustion efficiency.*—As the combustion process is far from being an adiabatic change it is not possible to relate the changes of pressure and temperature taking place in combustion by any form of adiabatic efficiency. Instead the small loss in pressure in the combustion chamber is usually referred to in absolute units of combustion chamber pressure loss and the term 'combustion efficiency' is reserved for the ratio of the actual temperature rise to the temperature rise which would be obtained if all the fuel were completely burned.

4.2.3. *Turbine efficiency.*—Again it is generally more convenient to use total-head to total-head efficiency when considering the turbine, for the same reasons as in the compressor. In some cases, however, for example when the turbine is exhausting to atmosphere, it is more convenient to relate the actual temperature drop to the isentropic drop corresponding to the ratio of the total-head inlet pressure to static outlet pressure. The actual temperature drop is now the sum of the total-head temperature drop (the work) and the temperature equivalent of the outlet velocity. This total-head to static efficiency is somewhat higher than the corresponding total-head to total-head efficiency.

4.2.4. *Jet-pipe efficiency.*—In the special case of jet propulsion, an efficiency has to be applied to the last expansion in the jet pipe and nozzle. Since, apart from a negligible loss of heat due to radiation, no heat is removed in the jet pipe the total-head temperature is constant at all stages along the pipe. The only way an efficiency can be applied therefore is by relating the actual and isentropic temperature changes between total-head conditions at inlet and static conditions at outlet.

4.2.5. *Combined efficiencies.*—In certain general calculations it is convenient to apply a combined efficiency to two or more components. An example of such an efficiency is an overall expansion efficiency covering both the temperature drops in the turbine or turbines and the temperature

* In 1946.

equivalent of the exhaust velocity. Occasionally, when it may be assumed that the exhaust velocity from a turbine will be relatively small, it is often useful to assume an efficiency which expresses the total-head temperature drop in a turbine as a percentage of the isentropic temperature drop between total-head inlet and static outlet conditious. Finally the pressure loss in the combustion system may be charged against the compressor to give a combined efficiency easy to use in work of a general nature. Thus

$$\eta_{13t} \quad \begin{matrix}\text{Combined efficiency}\\ \text{of compressor and}\\ \text{combustion chamber}\end{matrix} = \frac{\text{Isentropic rise corresponding to total-head pressure ratio across both components}}{\text{Total-head temperature rise in compressor}}$$

5. *Performance Calculations on Cycles with Losses.*—Having enumerated the various ways in which losses are incurred in a gas turbine and defined the ways in which component efficiencies may be expressed, it is now possible to investigate the influence of the inefficiencies of any engine upon the ideal performance for the cycle given in section 2.

It will be assumed that the type of engine considered is one in which power is extracted by a turbine, the engine being regarded as a stationary power plant operating under normal sea-level atmospheric conditions at inlet.

5.1. *Effect of Varying Compressor and Turbine Efficiencies.*—5.1.1. *Assumptions.*—

(a) Efficiency of power turbine = constant = 80 per cent. (assuming negligible leaving velocity)

(b) Combustion efficiency = 98 per cent.

(c) Mechanical efficiency of transmission of work from turbine to compressor = 99 per cent.

(d) The compressor and turbine efficiencies are all total-head to total-head quantities and the compressor efficiency includes combustion chamber pressure loss.

(e) Mean specific heat of air at constant pressure = 0·240.

(f) Mean specific heat of hot gases at constant pressure = 0·276.

5.1.2. *Specimen calculation.*—To determine the output and efficiency of the cycle operating at a turbine inlet total-head temperature of T_{3t} and a pressure ratio across compressor and combustion chamber of P_{3t}/P_{1t};

Isentropic temperature rise from inlet temperature, T_{1t}, of 15 deg C (288 deg K) corresponding to pressure ratio P_{3t}/P_{1t}

$$= 288\left[\left(\frac{P_{3t}}{P_{1t}}\right)^{(\gamma-1)/\gamma}_{\text{air}} - 1\right].$$

Therefore, the actual temperature rise in the compressor, $T_{2t} - T_{1t}$, is

$$\frac{288}{\eta_{13t}}\left[\left(\frac{P_{3t}}{P_{1t}}\right)^{0\cdot285} - 1\right].$$

The work in C.H.U./lb/sec, required for this compression is

$$k_{p\ \text{air}}\left[T_{2t} - T_{1t}\right].$$

If the masses of air and gas are regarded as equal (mass of fuel neglected*) the temperature drop in the turbine driving the compressor $T_{3t} - T_{4t}$, is given by the work equation

$$k_{p\,\text{gas}}\,[T_{3t} - T_{4t}] = \frac{k_{p\,\text{air}}}{\eta_{\text{mech}}}\,[T_{2t} - T_{1t}]$$

$$T_{3t} - T_{4t} = \frac{0\cdot240}{0\cdot276 \times 0\cdot99}\,[T_{2t} - T_{1t}]\,.$$

The isentropic drop, $T_{3t} - T_{4t}'$ in this turbine must be

$$T_{3t} - T_{4t}' = \frac{1}{\eta_{34t}}\,(T_{3t} - T_{4t})$$

and the total-head pressure ratio across it, P_{3t}/P_{4t}, is given by

$$\frac{P_{3t}}{P_{4t}} = \left(\frac{T_{3t}}{T_{4t}'}\right)^{\gamma/(\gamma-1)}_{\text{gas}} = \left(\frac{T_{3t}}{T_{4t}'}\right)^{4\cdot025}$$

The pressure ratio across the power turbine is P_{4t}/P_{1t} if it is assumed that the turbine exhausts to the atmospheric pressure P_{1t}. Since T_{3t} has been assumed, T_{4t}, the total head temperature at inlet to the power turbine, is also known. Hence using the power turbine efficiency of 80 per cent the actual total-head temperature drop in the power turbine is

$$T_{4t} - T_{5t} = 0\cdot80T_{4t}\left[1 - \left(\frac{P_{1t}}{P_{4t}}\right)\frac{1}{4\cdot025}\right].$$

The work done by the engine is, therefore, equal to

$$0\cdot276\,(T_{4t} - T_{5t})\ \text{C.H.U. per lb/sec of air}$$

or

$$\frac{0\cdot276\,(T_{4t} - T_{5t}) \times 1400}{550}\ \text{b.h.p. per lb/sec.}$$

The burning of the fuel causes a temperature rise of $T_{3t} - T_{2t}$ and therefore Q_F', the minimum fuel required per lb/sec of air is given by

$$Q_F' \times C_F = k_p(T_{3t} - T_{2t})\ \text{lb/sec}$$

where C_F = calorific value in C.H.U./lb,

and k_p = mean specific heat during combustion.

Allowance must be made for the unburnt fuel so that the actual fuel consumption Q_F is given by

$$Q_F = \frac{1}{\eta_{\text{comb}}}\cdot Q_F' = \frac{1}{0\cdot98}\,Q_F'\ \text{lb/sec for lb/sec airflow.}$$

The cycle efficiency, therefore, is

$$\eta = \frac{\text{Work done C.H.U./lb/sec}}{\text{Heat added C.H.U./lb/sec}}$$

$$= \frac{0\cdot276\,(T_{4t} - T_{5t})}{k_p(T_{3t} - T_{2t})} \times 0\cdot98\,.$$

* If the mass of fuel is not negligible the work equation should contain a factor $(1 + q)$ in the right-hand side, q being the mass of fuel burnt per sec for every lb/sec of airflow.

5.1.3. *Results.*—By carrying out this calculation for a range of pressure ratios and inlet temperatures and for three different values of both compressor and turbine efficiency an estimate may be made of the influence of the two main component efficiencies on the overall efficiency of the cycle with losses. The results using compressor efficiencies of 75 per cent, 82·5 per cent and 90 per cent, turbine efficiencies of 82 per cent, 87 per cent and 92 per cent and turbine inlet temperatures of 900 deg K, 1000 deg K and 1100 deg K are shown in Fig. 5. To illustrate the relationship of the new curves to those of the ideal cycle, curves for the latter are also plotted, the work output parameter being converted to the more practical form of b.h.p./lb/sec airflow, (specific b.h.p.) and made to correspond to an inlet temperature of 288 deg K and a mean specific heat for the cycle of 0·276 since for pressure ratios between 1 and 10 this approximates more closely to reality than does the value of 0·24.

5.1.4. *Discussion.*—The introduction of losses is seen to cause an appreciable diminution in both the output and the efficiency obtained from a given cycle.

5.1.4.1. *Effect on efficiency.*—Considering a typical pressure ratio of 5:1 Fig. 5c shows that with the highest combination of efficiencies considered—compressor 90 per cent, turbine 92 per cent (power turbine efficiency 80 per cent)—the cycle efficiency is reduced from the air standard value of 33 per cent to 23·8 per cent when the maximum temperature is 1100 deg K or 23·3 per cent at 900 deg K. The maximum temperature has mainly a secondary effect on the cycle efficiency provided that pressure ratios are reasonably low and component efficiencies fairly good. As pressure ratios increase, however, the temperature changes in the turbine and compressor increase and the cycle efficiency becomes more susceptible to changes in maximum temperature, and also to changes in the efficiency of the two components. Thus at 10: 1 pressure ratio and maximum efficiencies, a lowering of T_{3t} from 1100 deg K to 900 deg K causes a drop in cycle efficiency from 30·5 to 28·5 per cent. At this pressure ratio a drop in turbine efficiency from 92 to 82 per cent still using 90 per cent compressor efficiency lowers the cycle efficiency from 30·5 per cent quoted for 1100 deg K to 26·5 per cent whilst the figure of 28·5 per cent for 900 deg K is reduced to 20·5 per cent. At 5: 1 pressure ratio the effect of lowering the turbine efficiency is less drastic (23·8 to 21·5 per cent at 1100 deg K and 23·2 to 19·5 per cent at 900 deg K).

Figs. 5c, 5e and 5g illustrate that the effect of falling compressor efficiency is comparable to that of falling turbine efficiency. At the values of $T_{3t} = 1000$ deg K and turbine efficiency of 87 per cent, mean values of their respective ranges, a fall of compressor efficiency from 90 to 75 per cent lowers the cycle efficiency from 22 to 17·5 per cent for the 5 : 1 pressure ratio cycle and from 28·5 to 18 per cent for a pressure ratio of 10 : 1.

5.1.4.2. *Effect on output.*—The diminution in the power output of the cycle due to component efficiencies is of comparable size to the diminution in cycle efficiency. As in the ideal case, however, the maximum temperature of the cycle is of major importance in defining the work output. At a pressure ratio of 5 : 1 and a maximum temperature of 1100 deg K the specific output is 105 b.h.p./lb/sec instead of 155·5 b.h.p. given by the ideal cycle. This reduction amounting to 32·5 per cent is somewhat bigger than the reduction in cycle efficiency under the same conditions, which is 28 per cent. This is due to the fact that inefficiency in compression means that, to obtain a given pressure ratio, the compressor delivery temperature must be higher than when compression is isentropic so that the fuel then required to raise the temperature of the air to a given maximum is reduced. The fall in cycle efficiency is consequently not so great as the loss in output. (The figures quoted may slightly over emphasise this effect because the change in output is augmented and the change in efficiency is reduced to a small extent by the variations in specific heat already considered. However, at the same time the efficiency of the cycle with losses assumes 2 per cent of the fuel is unburnt and this increases the apparent change in efficiency.) The percentage loss in thrust of a cycle with fixed pressure ratio increases both with decreasing maximum temperature and, as would be expected, with decreasing compressor and turbine efficiencies. At the same time the pressure ratio which gives optimum work for a given maximum temperature is reduced.

5.1.4.3. *Pressure ratios for zero work.*—The latter effects can best be followed if the curves of specific output are considered as approximately similar curves all cutting the axis of zero specific output at a pressure ratio of approximately 1 and again at a pressure ratio depending upon the component efficiencies and the maximum temperature. The closer this second pressure ratio is to 1 the lower is the maximum output and also the pressure ratio at which that maximum occurs. The pressure ratio at which the work output is zero can be derived from the work equation already used by considering the case where all the work done in expansion is required for compression. Under these conditions

$$k_{p\,\text{gas}}\,(T_{3t} - T_{4t}) = \frac{k_{p\,\text{air}}}{\eta_{\text{mech}}}\,(T_{2t} - T_{1t})$$

and

$$T_{2t} - T_{1t} = \frac{T_{1t}}{\eta_{13}} \times [(R)^{(\gamma-1)/\gamma}_{\text{air}} - 1]$$

and

$$T_{3t} - T_{4'} = \eta_{34} T_{3t} \times \left[1 - \frac{1}{R^{(\gamma-1)/\gamma}_{\text{gas}}}\right] \text{ where } R = \frac{P_{3t}}{P_{1t}}\,.$$

Using the same k_p's as before

$$\eta_{34} T_{3t} \eta_{13} = \frac{0{\cdot}24 T_{1t}}{0{\cdot}276 \times 0{\cdot}99} \times \left[\frac{R^{0\cdot285} - 1}{1 - \left(\frac{1}{R}\right)^{0\cdot248}}\right]$$

In Fig. 6, this value of $\eta_{13} \times \eta_{34}$ is plotted against the pressure ratio R for different values of T_{3t} assuming $T_{1t} = 288$. Thus, if the maximum temperature of the cycle is 900 deg K and the component efficiencies both 0·80, work may be obtained from the cycle at pressure ratios up to 13 : 1 but if the product of the efficiencies falls below 0·32 no work is obtainable at any pressure ratio. The pressure ratio for zero work decreases with decreasing maximum temperature and decreasing component efficiencies. Fig. 6 also illustrates why, for any given values of η_{13}, η_{34} and T_{3t}, the performance of the cycle deteriorates more from that of the ideal cycle at high pressure ratios than at low. The horizontal lines shown are for $\eta_{13} = 90$ per cent, $\eta_{34} = 92$ per cent ($\eta_{13} \times \eta_{34} = 0{\cdot}828$) and for the ideal cycle when $\eta_{13} = \eta_{34} = 100$ per cent. At any given pressure ratio the work output will increase with increasing $\eta_{13} \times \eta_{34}$ from zero on the appropriate temperature line to 80 per cent (power turbine efficiency) of that of the ideal cycle when $\eta_{13} \times \eta_{34} = 1$. At a pressure ratio of 2, therefore, the output corresponding to $\eta_{13} = 90$, $\eta_{34} = 92$ would be expected to be a closer approximation to ideal cycle output than at a pressure ratio of 20 when the value of $\eta_{13} \times \eta_{34}$ for zero output is so much higher. Fig. 5d confirms that such is the case.

5.1.4.4. *Pressure ratios for optimum efficiency.*—Finally the form of the curves for cycle efficiency in Fig. 5 can also be related in general terms to the information given in Fig. 6. For when losses occur in the components, the cycle efficiency must become zero at the same conditions of temperature and pressure ratio at which the work output is zero. This is not the case in the ideal cycle, for there, at constant maximum temperature, as the work tends to zero with increasing pressure ratio, the cycle efficiency continues to increase because the heat to be added is also tending to zero. But when losses occur heat has still to be added to the cycle to counterpart these losses, even at conditions which give no work. The efficiency with which this heat is used is then zero. It is clear, therefore, that there must be, for any given values of η_{13}, η_{34} and T_{3t} a value of pressure ratio giving optimum cycle efficiency as well as one giving optimum work output. The pressure ratio for the former is always greater than that for the latter, a result which is in agreement with the conception that the introduction of losses causes a bending over (or ' wilting ') of the performance curves for the ideal cycle (Figs. 5a and 5b), when it is remembered that the output of the ideal cycle has a maximum at some finite pressure ratio but the air standard efficiency only reaches its maximum at infinite pressure ratio.

Plotted also in Figs. 5c and 5d are the appropriate curves for the most optimistic component efficiencies but with 1500 deg K maximum temperature. They illustrate the large increase in output obtained by using high turbine temperatures, but also shows that at constant pressure ratio as the temperature is increased the cycle efficiency becomes almost independent of maximum temperature providing that component efficiencies remain unchanged.

5.1.5. *Probable trend of component efficiencies with pressure ratio : polytropic efficiencies.*—It is likely that as the pressure ratio of a type of compressor is increased the adiabatic efficiency with which that pressure ratio is attained will decrease. It is probable that the small stage, or polytropic efficiency of a type of compressor will be more independent of pressure ratio than is the adiabatic. This efficiency $\eta_{\infty 12}$, is defined for a compressor as

$$\eta_{\infty 12} = \frac{\gamma - 1}{\gamma} \times \frac{n}{n - 1}$$

where the actual temperature ratio, $\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{(n-1)/n}$.

In expansion the polytropic efficiency becomes

$$\eta_{\infty 34} = \frac{\gamma}{\gamma - 1} \times \frac{n - 1}{n} .$$

Fig. 7 shows the variation of adiabatic efficiency with pressure ratio corresponding to constant polytropic efficiencies of 85 per cent in compression and expansion. At a pressure ratio of 20 : 1 the corresponding adiabatic efficiencies are 78 and 89 per cent respectively. Theoretically, the rise in adiabatic efficiency of expansion with an increase in the number of turbine stages, is as likely to occur as the fall in the adiabatic efficiency of compression with increased pressure ratio. In practice secondary effects such as boundary-layer thickening will modify these results. However the assumption of constant polytropic efficiency provides a way of approximating to reality when it is too cumbersome to work with a range of adiabatic efficiencies.

Fig. 8 shows the variation of efficiency and specific output with pressure ratio for cycles using polytropic efficiencies of 85 per cent in compression and expansion. This figure for expansion covers both compressor and power turbines and is based on total-head conditions at inlet and static conditions at outlet, but a small deduction is made from the possible work to allow for the temperature equivalent of the leaving velocity. A combustion pressure loss varying between 2 and 4 per cent of the compressor delivery pressure is also assumed. The curves from Fig. 5 for $\eta_{13} = 82{\cdot}5$ per cent, $\eta_{34} = 87$ per cent and $\eta_{13} = 75$ per cent, $\eta = 92$ per cent are replotted and extended to provide comparison with the results using polytropic efficiencies. As would be expected, the former curve provides the closest agreement at low pressure ratios and the latter at pressure ratios approaching 20 : 1.

5.2. *Effect of Heat Exchange on the Performance of the Cycle With Losses.*—The assumption of 85 polytropic efficiency of components provides a reasonable basis for a study of the effect of introducing a heat exchanger into the cycle, and such an assumption will therefore be made.

5.2.1. *Pressure loss.*—The perfect heat exchanger, assumed in section 2 must now be replaced by something more practical. Firstly, the passage of the gas through the body of the heat exchanger causes a loss in pressure. A given absolute loss is more important in the hot exhaust gases than in the compressed air as its influence upon the effective pressure ratio across the expansion is so much greater. In this investigation, therefore, a heat exchanger will be assumed to cause a loss of 1 lb/sq in. pressure in the exhaust gas, thus increasing the back pressure of the turbines by this amount. The loss of pressure in the compressed air will be regarded as part of the combustion pressure loss.

5.2.2. *Thermal ratio.*—A practical exchanger is not capable of carrying out the complete exchange of heat between turbine exhaust gases and compressed air. A heat exchanger efficiency or thermal ratio must therefore be assumed and this is usually defined by

$$\text{Thermal ratio} = \frac{\text{Temperature rise in compressed air}}{\text{Original temperature difference between gas and air}}$$

A more exact definition would use changes in total heat, in which case it would be unnecessary to state whether the actual change occurs in the compressed air or in the exhaust gas. Using temperature changes, as is the common practice, it is necessary to state in which medium the actual change is measured, because of the differences in specific heat between the two media due to the presence of products of combustion in the exhaust gas. A thermal ratio of 75 per cent should be attainable in practice using a contra-flow design of heat exchanger.

5.2.3. *Results.*—Claims of higher thermal ratio and lower pressure drop are made for certain experimental designs of exchanger, but the figures quoted, 75 per cent and 1 lb/sq in. loss, will serve to illustrate the practical possibilities of the cycle with heat exchange, and these are used in presenting Fig. 9. The influence of the heat exchanger is similar at all three maximum temperatures considered, so the values for 1000 deg K may be taken as typical. The pressure loss in the heat exchanger reduces the specific output of the 5 : 1 pressure ratio cycle from 72·5 b.h.p./lb/sec which was the optimum for the simple cycle at 1000 deg K to 63 b.h.p./lb/sec. The optimum output of the heat exchanger cycle, however, occurs at 6 : 1 pressure ratio and is 64 b.h.p./lb/sec. The cycle efficiency which previously was at a maximum of 23·2 per cent at 10 : 1 pressure ratio, has now, with the addition of a heat exchanger of 75 per cent thermal ratio an optimum of 27·5 per cent at a pressure ratio of just below 4 : 1. At this pressure ratio the output of the cycle with heat exchange is 61 b.h.p./lb/sec, which is the same as the plain cycle output at 10 : 1 pressure ratio. Summarising we may compare the cycles with and without heat exchanger at their respective optima for both efficiency and output and tabulate the results as under:

Maximum Temperature 1000 *deg K. Polytropic Component Efficiencies* = 85 *per cent*

		Without Heat Exchange	With Heat Exchange
For optimum cycle efficiency	Efficiency	23·2 per cent	27·5 per cent
	Output	61 b.h.p./lb/sec	61 b.h.p./lb/sec
	Pressure ratio	10 : 1	3·8 : 1
For optimum specific output	Output	72 b.h.p./lb/sec	64 b.h.p./lb/sec
	Efficiency	20·5 per cent	25·9 per cent
	Pressure ratio	5 : 1	6 : 1

Thus the impressions gained from the study of the ideal cycle, that the main advantage of a heat exchanger is that it gives higher cycle efficiencies than are obtainable without its use, and that these efficiencies are associated with low cycle pressure ratios are confirmed in practice, although the magnitude of the advantages is reduced.

5.2.4. *Effect of heat exchanger pressure loss.*—In Fig. 10 the influence of varying thermal ratios upon the efficiency of a cycle with typical losses is shown for the single case when the maximum temperature is 1000 deg K and will be discussed in the following section. In this figure, however, the difference between the values on the two curves, showing respectively the performance of the practical cycle with no heat exchanger and with a heat exchanger of zero thermal ratio represents the loss in cycle efficiency caused by the heat exchanger pressure loss of 1 lb/sq in. reducing the specific output of the cycle. At a pressure ratio of 8 : 1 the cycle efficiency is reduced by this loss from 23 to 20·5 per cent.

In Figs. 9 and 10 the curves for the cycle with losses but without heat exchanger show zero efficiency at a pressure ratio of approximately 1·1 as it is not until this pressure ratio is exceeded that the small absolute losses assumed (combustion chamber pressure loss and leaving velocity loss) are overcome and the work output becomes positive. The additional loss in the heat exchanger causes the efficiency of the modified cycle to be zero at a pressure ratio approximately 1·25, if the efficiencies of the main components assumed are still considered valid at such low pressure ratios.

5.2.5. *Effect of thermal ratio.*—Between this latter pressure ratio and that at which a heat exchanger becomes ineffective due to the temperatures of the exhaust gas and compressed air being equal, an improvement in thermal ratio results in an improvement in cycle efficiency. The improvements only become attractive, however, when the thermal ratio is between 75 and 100 per cent. At a pressure ratio of 3 : 1 an improvement of thermal ratio from 75 to 100 per cent causes as big an increment in cycle efficiency as does a change from 0 to 75 per cent.

The optimum efficiency of the cycle occurs at successively lower pressure ratios as the thermal ratio of the heat exchanger is increased. The performance for 100 per cent thermal ratio, as might be expected, tends to that of the ideal cycle with heat exchange more consistently than does the performance with less efficient exchangers and at 100 per cent thermal ratio the optimum efficiency occurs at a pressure ratio of 2 : 1. This optimum efficiency, however, is associated with very low values of specific output which, as shown in Fig. 9, falls off sharply at pressure ratios below 3 : 1.

It is obvious, therefore, that by using pressure ratios of 3 or 4 to 1 a compromise can be struck so that a heat exchanger causes a reasonable improvement in cycle efficiency, without causing too big a fall in output. It is also clear, from Fig. 10, that the thermal ratio of the exchanger must be greater than 60 per cent before the optimum cycle efficiency becomes greater than the optimum obtainable without a heat exchanger in the cycle. Improvements in thermal ratio above this value pay an ever increasing dividend in terms of improved cycle efficiency.

5.3. *Effect of Reheat and Intercooling.*—It has been shown that the influence of a heat exchanger on a cycle with losses is similar to that which a study of its influence on the ideal cycle leads one to expect, but to a modified extent.

It is now of interest to investigate whether reheating and intercooling have the same type of influence on the performance of a practical engine as on the ideal cycle. To do this it is necessary to study the ideal cycle in a little more detail than before. From Figure 2 it appears that the increments in output caused by introducing intercooling into the ideal cycle for a given temperature and pressure ratio is always less than that caused by reheating, both increments being smaller than the improvement gained by using the two modifications in conjunction and that the cycle efficiency decreases as the output is increased by the modifications. This does not hold, however, over the whole of the range of pressure ratio in which it is possible to obtain work from the cycle.

5.3.1. *Ideal cycle.*—Fig. 11 shows the variation of work output and efficiency for the four types of ideal cycle over their complete range of pressure ratio. The low temperature ratio of 2 is chosen so that the pressure ratios are reasonably small, but the figure is typical for all temperature ratios. Above the pressure ratio at which the simple cycle gives zero work intercooling is found to give more work than reheating and still yields a greater cycle efficiency than reheat, although the cycle efficiencies of both decrease with increasing pressure ratio. It would be futile to operate the perfect cycle under the conditions when intercooling yields more work than reheat, as more work and higher efficiencies can be obtained at lower pressure ratios. Moreover, the intercooled cycle would require work to be done to exhaust the gas, whilst the reheat cycle would require cooling and not heating to take place between the compression and expansion.

5.3.2. *Cycle with losses.*—In the cycle with losses (Fig. 12), however, the pressure ratio above which the output with intercooling exceeds that with reheat is much lower. From Fig. 12b it is seen to be about 12 : 1 whereas in the ideal cycle for the temperature of 1100 deg K assumed here the change over would not occur until a pressure ratio of 110 : 1. Moreover in the range of pressure ratio from 12 : 1 to the maximum plotted (20 : 1) the output from the intercooled cycle is appreciably greater than the optimum obtainable from the plain cycle and the efficiency too is slightly higher. Reheating is assumed to return the gas to its maximum temperature and intercooling to cool it to approximately its original inlet temperature.

It has been noted previously that the greater the range of pressure ratio over which a cycle operates ideally, the less drastic are the effects caused by losses introduced into the cycle. The present comparison of reheating and intercooling appears to be further confirmation of this fact. Ideally the intercooled cycle for 1100 deg K maximum temperature and 288 deg K inlet temperature gives work over a range of pressure ratio up to 880 : 1 and the reheat cycle to 350 : 1. With the assumption of 85 per cent polytropic efficiencies of components and small absolute pressure losses in intercoolers, combustion chambers etc. the work output of the cycles becomes zero at approximate pressure ratios of 200 : 1 and 60 : 1 respectively. This represents a proportionally larger change in the reheat cycle than in the intercooled one, and suggests that the performance of the reheat cycle at any intermediate pressure ratio will be more effected by the introduction of losses than will the performance of the intercooled cycle. Consequently although in an ideal cycle reheating appears to be superior to intercooling as a method of increasing the specific output of the cycle, when a more practical cycle is introduced the advantages of reheat are greatly reduced and intercooling appears a more attractive method.

5.3.3. *Reheat and intercooling.*—The introduction of both reheating and intercooling simultaneously, however, presents the most attractive possibilities. Ideally the optimum specific output of the plain cycle and the air standard efficiency obtained can be doubled by using reheat and intercooling and squaring the overall pressure ratio of the cycle. Figs. 12a and 12b show that this is also approximately true in a practical cycle.

The plain cycle shown has a maximum specific output of 95 b.h.p./lb/sec at a pressure ratio of about 6 : 1. The output of the cycle with both reheat and intercooling is still increasing at a pressure ratio of 20 : 1 and will reach an optimum of nearly 190 b.h.p./lb/sec at a pressure ratio of about 30 : 1. The cycle efficiency at both optima is in the neighbourhood of 27 per cent.

5.3.4. *Conditions for maximum output.*—As in the case of the ideal cycle, so in the cycle with losses the optimum work output for given pressure ratio and temperature limits is obtained if the temperature rise in compression is equally divided by the intercooler and/or the temperature drop in expansion is equally divided into the parts before and after reheat. Figs. 12a and 12b show the variation of this optimum output with pressure ratio and also the cycle efficiency with which it may be obtained. At a pressure ratio of 8 : 1 the output with reheat is slightly higher than with intercooling, both being approximately midway between the output of the plain cycle and that of the cycle incorporating both modifications. The efficiency of the plain cycle is higher than that of any of the other three cycles at this pressure ratio. At a pressure ratio of 20 : 1, however, its efficiency is lower than all the others, its specific output still being considerably lower than that obtained by modifications.

Pressure Ratio	Plain Cycle		+ Reheat		+ Intercooling		+ Both	
	Specific output	Cycle efficiency	Specific output	Cycle efficiency	Specific output	Cycle efficiency	Specific output	Cycle efficiency
	b.h.p./lb/sec	per cent	b.h.p./lb/sec	per cent	b.h.p./lb/sec	per cent	b.h.p./lb/sec	per cent
8 : 1	93	25·0	116	22·0	114	23·9	139	21·2
20 : 1	53	23·4	108	24·0	116	27·0	173	26·2

5.3.5. *Conditions for maximum efficiency.*—Whereas in an ideal cycle without a heat exchanger the introduction of intercooling or reheat or both always causes a loss in cycle efficiency, when more practical cycles are considered it is possible for the maximum efficiency to be increased at certain pressure ratios. It is generally found that intercooling after about a quarter or a third of the compression work has been done, or similarly reheating after about a quarter or a third of the expansion work has been completed gives the maximum efficiency for given pressure and temperature ratios. Figs. 12c and 12d indicate the variation of this maximum efficiency with pressure ratio together with the corresponding specific output. The maximum cycle efficiency is from 1 to 4 per cent higher than the efficiency at maximum output, but the specific output at maximum efficiency may be over 20 b.h.p. in 170 b.h.p. lower than the maximum obtainable.

5.3.6. *Introduction of heat exchange.*—When a heat exchanger is introduced the optimum positions for intercooling and reheating to give maximum efficiency and output respectively almost coincide. (In an ideal cycle they are coincident—section 2.4.4.) Fig. 13 shows the performance of cycles with a heat exchanger of 75 per cent thermal ratio. Due to the additional pressure loss in the exchanger the output is less than that of the corresponding cycle without the exchanger. The cycle efficiency, however, is improved over a wide range of pressure ratio. Intercooling gives higher efficiencies than reheat but the best efficiencies of all are obtained with both intercooling and reheat in addition to the exchanger. With this latter cycle an efficiency of 34 per cent is obtainable between pressure ratios of 6 and 20 to 1, and this efficiency would, of course, be exceeded if the thermal ratio were higher or the component losses were less.

5.3.7. *Multiplicity of intercool and reheat stages.*—Since the introduction of one stage of reheat and one stage of intercooling can double the optimum output of a plain cycle, it is logical to suppose that by increasing the number of reheat and intercooling stages the output may be still further increased, it being realised that the optimum output occurs at rapidly increasing pressure ratios as the number of stages is increased. The logical limit to this process is the condition when cooling is continuous through the compression and heating is continuous through the expansion, and both processes are carried out isothermally.

Fig. 14 shows the variation of the work output parameter with pressure ratio for several stages of intercooling and reheat in the ideal cycle and also the output parameter for the ideal 'isothermal' cycle. The temperature ratio used is 3. The improvements in output may be summarised by quoting the values of the work output parameter for a pressure ratio of 20 : 1 with various cycles.

Plain Cycle	0·375 (Optimum 0·535 at 7 : 1)	
+ 1 stage intercooling and reheat =	1·040	Optima all occur at pressure ratios greater than 20 : 1
+ 2 stages intercooling and reheat =	1·280	
+ 3 stages intercooling and reheat =	1·360	
Isothermal compression and expansion =	1·715	

To demonstrate that these increments do not refer to the purely theoretical performance of an ideal cycle, portions of two curves from Fig. 12b relating to the cycle with normal losses are replotted in Fig. 14. In form and position they approximate closely to the ideal cycle curves for the ranges of pressure ratio used, the difference being, however, that the temperature ratio of the cycle with losses is 3·82 compared with 3 for the ideal cycle so that the product of the temperature ratio and the expansion and compression efficiencies is approximately the same in the two cycles.

5.3.8. *Ericsson cycle.*—The efficiencies of the cycles just considered decrease with the addition of further intercooling and reheat stages if no heat exchanger is incorporated. The addition of a perfectly efficient heat exchanger has the opposite effect of increasing the cycle efficiency until in the limiting case of isothermal compression and expansion the maximum possible efficiency is obtained. The cycle is now the same as that suggested by Ericsson as a constant pressure version of Stirling's regenerative air-engine cycle. As all the external heat is taken in at the maximum temperature, T_3, and is rejected at the minimum temperature T_1 the air standard efficiency of such a cycle is

$$1 - \frac{T_1}{T_3}.$$

This is the efficiency attainable by a reversible engine and is by Carnot's principle greater than that of any other heat engine working between the same temperature limits, unless it, too, is reversible.

6. *The Gas Turbine as an Aero-Engine.*—As the gas turbine has first reached large-scale operation as a power plant for aircraft, it is important to examine the special features of this type of application. The present study is not concerned with more than passing interest in the light weight and simplicity of this type of engine, or in its smooth running and freedom from serious lubricating problems associated with reciprocating parts, all of which have contributed to its rapid development as an aero-engine. Instead, interest will be centred on the influence of this type of application upon the performance of the stationary engines so far considered, upon the present association between gas turbines and pure jet-propelled high-speed flight, and upon the performance aspects of the future development of turbine-driven propeller engines.

6.1. *Special Aspects of Aircraft Applications.*—6.1.1. *Forward velocity.*—The most important influence upon the performance of turbine engines in general, due to their use as aircraft power plants is the pre-compression of the air, resulting from the aircraft's forward motion, which may be gained before the air enters the compressor proper. The inlet pressure to the engine's compressor is no longer the static pressure of the air, but the stagnation pressure corresponding to the aircraft's velocity less any pressure loss incurred in drawing the air from outside the aircraft to the front of the compressor. The consequence is that the pressure ratio of the compressor required to achieve a given overall pressure ratio decreases with increasing aircraft speed.

At the same time the temperature of the air entering the compressor is increased, but the efficiency of the ram compression is generally higher than that obtainable in a normal compressor, provided that the intake is well designed, and, for the time being, speeds in excess of 600 m.p.h. are not considered. The work required to compress the air through the latter stages of compression to a given overall pressure ratio is, therefore, somewhat smaller if the first stages are carried out by ram compression rather than in the compressor of a stationary engine, as the temperatures throughout compression are reduced.

With a given maximum temperature and overall pressure ratio, the work available in expansion is independent of forward speed, but the proportion of this work required for compression is reduced by causing the engine to move forward, mainly because the ram compression requires no turbine work, but also because of this small reduction in the temperatures in the compressor proper. It is, of course, necessary to use some of the expansion work to expel the gas with a velocity at least equal to the forward velocity of the engine. As the temperature of the exhaust gas is greater than the intake temperature, the expulsion of the gas requires a lower pressure ratio than the ram pressure ratio created by the forward motion, and so there remains a greater pressure ratio across the turbines than across the compressor. At a given overall pressure ratio and maximum temperature there is, therefore, an increase in the net work output per lb/sec of mass flow, as the forward speed is increased, and this is responsible for an increase in cycle efficiency.

Fig. 15 shows how the pressure ratio and temperature ratio of perfect 'ram' compression due to aircraft speed vary with increasing speed. Because the ratios are simple functions of the Mach number of the aircraft*, two additional scales are added to show the true speed of the aircraft

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* From section 3.3

$$\frac{T_t}{T} = 1 + \frac{V^2}{2gJk_pT} = 1 + \frac{g\gamma R}{2gJk_p}\frac{V^2}{V_{\text{sonic}}^2} = 1 + \frac{\gamma - 1}{2}M^2 \text{ where } M = \text{Mach number} = V/V_{\text{sonic}}.$$

at sea-level and in the stratosphere. It is clear that at present-day aircraft speeds of 500 to 600 m.p.h. the ram compression ratio attainable (approx. $1 \cdot 4$) is small compared with that obtainable at higher supersonic flight speeds. At a Mach number of 2, for example, the ram pressure ratio which might be achieved is 8 : 1, although the flight speed would only be about three times that giving the present typical values of $1 \cdot 4$.

However, a ram ratio of $1 \cdot 35$ corresponding to 500 m.p.h. at sea-level, small though it is in comparison with what may ultimately be attained, still has an appreciable effect on the performance of a gas turbine, as may be seen from a comparison of the appropriate curves in Fig. 16. The compressor pressure ratio required at 500 m.p.h. to give a certain specific output or efficiency is seen to be lower than in the static condition, as would be expected. At the same time, owing to the fact that the ram compression requires no work from the turbine, higher specific output and efficiency are obtainable at 500 m.p.h. than at 0 m.p.h., even when comparison is made at compressor pressure ratios giving the same overall pressure ratio.

6.1.2. *Altitude.*—Because of the freedom of an aircraft to fly at varying altitudes, an aero-gas-turbine is liable to experience a wide range of inlet air temperature variation. If it is assumed that the maximum temperature permissable in the engine is unaltered by altitude, then the temperature ratio of the cycle will be increased by 33 per cent in changing from a sea-level air temperature of 288 deg K to an air temperature of $216 \cdot 6$ deg K at the I.C.A.N. tropopause. Fig. 16 shows that the effect of altitude on the cycle performance is a combination of the effect of changing temperature ratio (section 5.1.4) with the effect that, at the same pressure ratio and temperature ratio, the cycle work output tends to be proportional to the inlet temperature (*cf.* the work parameter work/$k_p T_1$ in section 2.1). It has been shown previously that the performance of the ideal cycle of temperature ratio equal to 3 approximates to the performance of a practical cycle such that the product of the temperature ratio, expansion efficiency and compression efficiency is also equal to 3. With the efficiencies assumed here this means a cycle operating at a maximum temperature of about 1100 deg K under sea-level static conditions. Similarly, the ideal cycle of temperature ratio equal to 4 approximates to the practical cycle of maximum temperature equal to 1100 deg K when the inlet temperature is reduced to that of the tropopause. Fig. 2b shows that for the ideal cycle at a typical pressure ratio of 10 : 1, the work output parameter would be approximately doubled in increasing the temperature ratio from 3 to 4. However, if this is done by decreasing the inlet temperature to three-quarters of its original value the increase in actual work lb/sec mass flow will only be 50 per cent, because actual work is proportional to the parameter multiplied by the inlet temperature. In the practical cycle of 10 : 1 pressure ratio (Fig. 16), the specific output at the tropopause is, in fact, 52 per cent higher than the specific output at sea-level, the actual outputs being dependent, of course, on the ratio of the mass flows in the two conditions.

6.2. *Methods of Propulsion.*—So far the gas turbine engine has been considered solely as a means of obtaining a certain amount of power, expressed as b.h.p./lb of air flowing per second, at a certain efficiency in the use of its fuel. Now that the turbine engine is being considered as an aircraft power plant it becomes necessary to consider the different ways in which the power available may be used to drive the aircraft along. In all forms of aircraft propulsion, the force driving the aircraft forward is equal and opposite to the force with which a working substance (air, except in the case of rockets) is driven backwards. Such a force is proportional to the rate at which the backward momentum of the working substance is being changed. The problem resolves itself, therefore, into a study of the different means of accelerating varying masses of air rearwards past or through the aircraft, with a view to finding the most efficiency method of employing the available power under particular conditions.

6.2.1. *Thrust.*—The propulsive force, or thrust, of a device which when moving forward at a velocity V_a, takes in a mass M of working substance per unit time and then ejects it at a velocity V_J relative to itself in the opposite direction to V_a is, therefore, proportional to $M(V_J - V_a)$ and the thrust horsepower of the device is proportional to $MV_a(V_J - V_a)$.

6.2.2. *Propulsive efficiency.*—The efficiency of propulsion of such a device is the ratio of the thrust horse power generated to the horse power added to the air by the engine and available for propulsion. As the latter, which we may call the air horse power is proportional to $\frac{1}{2}M(V_J^2 - V_a^2)$ so long as the gas has no velocity other than V_J when it leaves the device and there are no losses, the propulsive efficiency η_p may be expressed, for these ideal conditions as:

$$\eta_p = \frac{MV_a(V_J - V_a)}{\frac{1}{2}M(V_{J2} - V_a^2)} = \frac{2V_a}{V_J + V_a}.$$

These equations also ignore the fact that in some cases a small increase in mass occurs during the passage of the working fluid, air, through the engine due to fuel being added.

It will be noted that the propulsive efficiency is unity when $V_a = V_J$ but at this condition the thrust is zero, the horse power absorbed being zero, too. Alternatively if the air horse power is finite, maximum propulsive efficiency can only be obtained if the mass flow of the device becomes infinitely large thus allowing the increase in velocity $(V_J - V_a)$ to be infinitely small.

If a hypothetical example of a device absorbing constant horse power independent of forward speed is considered then the variation of thrust obtained at different forward speeds by employing an increasing mass flow through the device would be as shown in Fig. 17a. This figure shows that with a perfect propulsive device, an increase in thrust per available horse power at a given flight speed can always be obtained by increasing the mass flow of the device, but that the increase becomes increasingly slight at high forward speeds.

Supposing, however, that the effective leaving velocity of the gas is only 90 per cent of its ideally attainable value, the results, as shown in Fig. 17b, lead to a different conclusion. The advantages of using large mass flows at low forward speeds are still apparent, but at a forward speed of 1000 ft/sec (682 m.p.h.) the maximum thrust is obtained when the mass flow is about 25 lb/sec per 1000 h.p. and at higher forward speeds the optimum mass flow rapidly becomes smaller. The figure of 90 per cent for the ratio of the effective and ideal leaving velocities is purely arbitrary, a higher ratio would have shown higher optimum values of mass flow and thrust, and a lower ratio lower values. The reasons why the ideal leaving velocity may not be attained are many and varied and include:

(*a*) Frictional losses in the device.

(*b*) Frictional losses between the air and the device.

(*c*) Losses due to turbulence in the air stream.

(*d*) The presence of rotation in the air stream or of a component of the leaving velocity at right-angles to the flight path.

(*e*) The impossibility of imparting a constant velocity to the whole of the mass.

6.2.3. *Propellers.*—A propeller is a propulsive device which employs a relatively large mass of air to create its thrust. It is subject in some measure to all the forms of loss just enumerated and is therefore essentially for use at relatively low flight speeds, where it is considerably more effective than a device employing smaller mass flows. Moreover, the large increases in losses which are encountered with propellers at their present stage of development, when the flight speed approaches the velocity of sound, tend to restrict the use of propellers still further to subsonic flight speeds.

6.2.4. *Jet propulsion.*—The term jet propulsion is here used in a limited sense to describe propulsion by the exhaust gases of an engine, and in particular of a gas turbine. With such a means of propulsion the mass flow of gas per h.p. is considerably smaller than with a propeller. Jet propulsion is, therefore, a very unsatisfactory way of using the available power of an engine at low flight speeds and especially during the take-off of an aircraft. It is only when high flight speeds are used that it becomes more efficient to employ the relatively low mass flows typical of jet propulsion, rather than the high mass flows of a propeller. The losses incurred in producing a high velocity jet are generally lower than those incurred in extracting power from the engine and transmitting it to the air passing through a propeller, as the only important source of loss is the skin friction between the gas and the jet pipe.

6.2.5. *Ducted fans.*—A ducted fan may be regarded as a multi-bladed propeller installed in a duct, and its characteristics from the view-point of this momentum theory are, therefore, those of a propeller. A ducted fan has a possible advantage over a propeller in that the losses involved in achieving high flight speeds may be smaller, but this appears to be an unprofitable line of development as it leads to flight speeds at which it is more efficient to use smaller mass flows. At all flight speeds the comparison of the relative advantages of propellers and ducted fans is a matter of design, weight and aerodynamic drag, as well as of implicit performance characteristics.

6.3. *Gas Turbines and Jet Propulsion.*—A gas turbine is eminently suitable for use in high-speed jet-propelled flight from speeds of 500 m.p.h. up to and over 1000 m.p.h. because the mass of air required for the turbine can be used directly in the jet, and, by a suitable choice of pressure ratio, it can be made to approximate closely to the optimum mass flow on which the power output of the engine should be expended to give maximum thrust at flight speeds of this order.

6.3.1. *Output in terms of thrust.*—The gas-turbine cycle output may be reconsidered, therefore, in terms of the thrust produced per lb/sec of airflow at various speeds rather than of specific b.h.p. When this is done the results shown in Fig. 18f are obtained for a maximum temperature of 1100 deg K, speeds up to 1500 m.p.h. being considered. By modification Fig. 17a can be replotted as Fig. 18d to show the relationship of specific thrust to the air h.p. absorbed per lb/sec mass flow of the propulsive device. In the case of jet propulsion this latter is equal to the specific horse power which the gas turbine provides for propulsion, as the mass flows through the engine and the propulsive device are equal.

The specific horse power which yields the specific thrust plotted in Fig. 18f is shown in Fig. 18b. The horse power at a given speed and pressure ratio is seen to be higher than that given in Fig. 16, and this illustrates the effect of eliminating the losses incurred in a power turbine and replacing them by the smaller losses incurred in an average jet pipe. The values shown refer to an expansion efficiency of 88 per cent across the power turbine and the jet, in the results of Fig. 16 and an expansion efficiency of 95 per cent across the jet for the pure jet propulsion case (Fig. 18b). It should be noted further that the whole of the power output shown in Fig. 18b is employed in doing useful propulsive work, but that before useful propulsive work may be obtained from the output of a power turbine, further losses due to friction or airstream rotation must be incurred in a propeller or a ducted fan.

6.3.2. *Discussion of variations in specific thrust.*—The values plotted in Fig. 18 refer only to a maximum temperature of 1100 deg K, and represent the attainable output and efficiency of the cycle with reasonable component efficiencies. For supersonic flight speeds the intake losses assumed are those which would occur in a normal shock-wave. It is possible that by designing an intake to give oblique shock-waves these losses may be reduced, so that the high-speed performance given here is in no way optimistic.

The maximum value of specific output provided by the engine for propulsion has been shown in section 6.1.1 to occur at decreasing values of compressor pressure ratio as the forward speed is increased. Fig. 18 shows that under the temperature conditions assumed when the forward speed reaches between 1000 and 1500 m.p.h. the optimum output occurs at a compressor pressure ratio of 1, *i.e.*, when there is no compressor or turbine and the engine becomes a propulsive duct, or ram jet. The maximum output falls off above 1000 m.p.h. due to the increase in the intake losses.

The specific thrust yielded by the engine and jet varies with pressure ratio in much the same way as the horse power of the engine, in that at a given speed maximum thrust occurs at the same cycle pressure ratio as maximum horse power, for it can be seen from Fig. 18d that at constant forward speed an increase in horse power always yields an increase in thrust. Fig. 18d also shows, however, that the specific thrust corresponding to a given specific output decreases rapidly with forward speed. Consequently though the optimum output of the gas-turbine cycle may and does increase with increasing forward speed the optimum values of specific thrust given when this output is used in a propulsive jet, decrease with forward speed. So, although 61·5 lb thrust/lb/sec mass flow may be obtained from a stationary gas-turbine jet engine of 7 : 1 pressure ratio, at 1000 m.p.h. 33·5 lb/lb/sec is the maximum obtainable. This occurs at a pressure ratio of 3·1 where the specific output of the turbine moving at 1000 m.p.h. is about 10 per cent greater than the optimum output of the stationary turbine.

6.3.3. *Discussion of variations in overall efficiency.*—The overall efficiency of a turbine-jet engine is the ratio of the useful propulsive work done by the jet to the heat energy supplied in the gas turbine. It is, therefore, the product of the cycle efficiency of the gas turbine and the propulsive efficiency of the jet, and is consequently only definable when the engine has a forward velocity as otherwise the propulsive efficiency is zero. The optimum cycle efficiency of the turbine like the optimum output, occurs at lower compressor pressure ratios as the forward speed increases. These ratios are, however, greater than the corresponding pressure ratios for maximum work. The maximum cycle efficiency also increases with increasing speed until the intake losses become too great, after which the maximum cycle efficiency falls again.

Fig. 18c, which is a different form of Fig. 18d shows that the propulsive efficiency with which a given power supplied to the jet can be used increases with forward speed. In consequence, the maximum overall efficiencies of the turbine and jet together continue to increase even when the turbine cycle efficiency has begun to fall. Thus a maximum overall efficiency of 28·5 per cent is attainable at 1500 m.p.h., compared with 26·5 per cent at 1000 m.p.h. or 17·5 per cent at 500 m.p.h. The compressor pressure ratios corresponding to these maxima are 3, 6 and 16 respectively.

6.3.4. *Propulsive ducts.*—It is of interest to note that at 1500 m.p.h. a propulsive duct might obtain an overall efficiency of 23 per cent which is only a little lower than that of the 3:1 pressure ratio cycle and at the same time would have a specific thrust 25 per cent greater than the engine incorporating a 3:1 pressure ratio compressor. The elimination of the turbine means further, that the maximum permissible temperature of the cycle may be raised above the maximum for a turbine and would result in even higher values of specific thrust.

6.3.5. *Effect of changes in maximum temperature.*—The temperature of 1100 deg K should not, however be regarded as the maximum temperature at which a turbine may operate. By the development of better high temperature materials or of schemes for cooling turbine blades and discs a higher temperature will be practicable. Fig. 19 shows a comparison between the performance of the practical cycles operating at sea-level with the maximum temperatures of 1400 deg K and 1100 deg K respectively. It is assumed that the turbine efficiency is not reduced at the higher operating temperatures and that if blade cooling is employed it is done without the use of air from the compressor. By the increase in temperature considered the optimum specific thrust can be increased from 61·5 lb/lb/sec to 82·5 lb/lb/sec (34 per cent) at 0 m.p.h. and from 23·5 lb/lb/sec to 38 lb/lb/sec (62 per cent) at 1500 m.p.h. with intermediate percentage increases at intermediate speeds. The corresponding percentage increases in the specific horse power of the engine are actually greater than these, for, as can be seen from Fig. 18d, a given percentage increase in horse power always results in a lower percentage increase in thrust at a given speed, though the difference decreases as the forward speed is increased.

The maximum overall efficiency is also increased by increasing the maximum temperature of the cycle but occurs at higher pressure ratios. At relatively low pressure ratios the overall efficiency for a temperature of 1400 deg K may in fact be 1 or 2 per cent lower than the corresponding efficiency for 1100 deg K. This is caused by lower propulsive efficiency at 1400 deg K assuming pressure ratio and flight speed constant.

6.3.6. *Effect of changes in altitude.*—The effect of changes in altitude is summarised in Fig. 20 which shows the performance of the gas-turbine-jet cycle for 1100 deg K, at the tropopause and at sea-level. The effects can be seen to follow directly from the influence of altitude on the gas turbine's performance discussed in section 6.1.2 and the modifying influence of the jet, just discussed, in that percentage increases in thrust are smaller than the corresponding increases in b.h.p. supplied by the engine.

6.4. *Propeller-Turbine Engines.*—At aircraft speeds of about 400 m.p.h. a propeller may be expected to give a propulsive efficiency in the neighbourhood of 80 per cent which Fig. 18c shows to be approximately double that of a simple jet at the same speed. Even allowing for the loss of h.p. from the engine due to the losses in a power turbine it is clear that at these aircraft speeds a gas turbine driving a propeller is considerably more efficient than a gas turbine expending its power in a propulsive jet.

This does not mean, however, that it would always be more economical to employ a propeller turbine at these speeds. The range of the aircraft has to be great enough for the saving of fuel weight due to the greater efficiency of the propeller engine to outweigh the extra weight of the power plant due to the power turbine, gearing, propeller and the like. The influence of design characteristics upon the plant weight of the two types of engine required to give the same thrust at a given speed makes the problem a complicated one and puts it beyond the scope of the present work. Enough has been said however to make it apparent that the range below which the total plant and fuel weight of a simple jet engine is less than that of a propeller-turbine engine. increases rapidly as the speed at which the aircraft is assumed to fly is increased. Further as an increase in operational height is likely to cause an appreciable reduction in the efficiency of a propeller because the Mach number corresponding to a constant forward speed is increased it is to be expected that such an increase in altitude will swing the comparison in favour of the jet.

6.5. *Choice of Engines for Different Duties.*—The preceding comparison of simple-jet and propeller-turbine engines shows that both plant and fuel weight must be considered before an engine is chosen as the most suitable for a specified duty. Many factors contribute to the complexity of the problem even when the type of aircraft and the desired range are known. A heat exchanger is known to be capable of improving the efficiency of a particular engine but if the weight of the exchanger is excessive the saving in fuel weight will be outbalanced at all except extremely long ranges. Or again, although extra thrust may be obtained by incorporating reheating or intercooling in an engine, the question must be answered as to whether the increase in plant weight might not have been better used in increasing the pressure ratio of the simpler engine.

Even so a comparison of engines on a plant and fuel weight basis is not completely adequate. The drag coefficient of the aircraft may be greater with one type of engine than with another due to larger nacelles or excrescences caused by air coolers and the like. The structure weight of aircraft may also be affected by the type of engine employed, for example a larger under-carriage is often required for a propeller engine than for a jet engine in order to get sufficient ground clearance. The space available in an aircraft may also be an important factor influencing the choice, in which case the advantage of a low plant weight and higher fuel weight is reduced, as the fuel occupies much more space than would the same weight of engine. If fuel costs are an important consideration a similar argument will apply.

Finally, mention must be made of the various yardsticks by which the suitability of different engine-aircraft combinations for a certain duty may be judged. Comparison may be made between the percentage of the payload to the all-up weight in the different cases, or between the product of this percentage payload and the average speed of flight, when allowance may or may not be made for the time taken in loading and unloading. Considerations of serviceability and reliability lie over and above all these.

Military requirements are often much different from civil ones. A bomber aircraft presents a different problem from a civil transport plane of equal weight and payload at take-off and having equal range for the bomber drops its ' payload ' half-way through its flight and is not likely to encounter such high landing wing-loadings as will a loaded civil plane.

In consideration of these various factors it is not possible to state hard and fast rules regarding the type of turbine engine best suited to a particular duty, but from the foregoing a few general conclusions regarding the probable choice may be drawn.

6.5.1. *Fighter aircraft.*—For a small high-speed aircraft of the fighter type it appears that a simple jet-turbine engine is most suitable, because of the superiority of the jet as a means of propulsion at the speeds at which the aircraft would be expected to fly. The pressure ratio of the compressor (sea-level static) will remain small, probably below 4 : 1, in order to approximate to optimum specific thrust conditions at speeds of 600 m.p.h. and over whilst still keeping the plant weight small. As improvements in aircraft design make it possible to achieve supersonic speeds, the fighter aircraft's engine may become simply a propulsive duct, though such a scheme would involve the use of some additional power supply, such as a rocket, to accelerate the plane to speeds at which the duct would operate effectively. Some form of rocket motor appears to be the only serious rival to the jet-turbine or the duct as an engine for this type of aircraft.

6.5.2. *Bomber aircraft.*—At first sight, propeller-turbine engines would seem the best engines for a bomber aircraft as their lower fuel consumption, compared with jet-turbines, would enable greater striking range to be obtained. However, if high speed is also demanded of a bomber a jet-turbine may be more suitable, provided that there is sufficient power available at the take-off to lift a large military load. Some form of boosting or assisted take-off may be necessary.

6.5.3. *Civil passenger and freight aircraft.*—For services operating on ranges below 1000 miles and where there is a plentiful supply of passengers or freight demanding rapid transport, a jet-turbine engine may prove to be superior to any other type of engine for civil aircraft, as its light weight will permit a greater percentage payload than would propeller engines. For ranges of 2000 miles and above, the propeller-turbine engine scores because of its greater efficiency. As there is not likely to be a demand for excessive speeds from transport aircraft, the propeller should continue to be the most efficient means of propulsion under the required conditions.

Because of the weight involved it seems unlikely that sea-level static-pressure ratios greater than 8 : 1 will be employed in aero-gas-turbines, because even with turbine temperatures of 1400 deg K, little or no improvement in specific output is attainable by increasing the pressure ratio above this value.

The introduction of reheating or of intercooling although adding a further component does not necessarily increase the weight of engine required to give a fixed power, for the output per lb/sec of throughput is increased and a smaller mass flow may be used thus giving smaller and lighter components. Reheating could probably be made to involve a smaller increase in weight than intercooling, but would also result in a bigger drop in overall efficiency.

A heat exchanger in addition to being, at its present state of development, a heavy component, reduces rather than increases the specific output of an engine, so that it increases the size of engine required to give a certain power. In compensation, the heat exchanger improves the efficiency of the engine and so reduces fuel weight and fuel cost and increases goods stowage space.

A propeller-turbine engine fitted with a heat exchanger should, therefore, be a suitable engine for a civil transport aircraft operating on trans-ocean ranges of the order of 3000 miles. If, further, the plant weight to power ratio can be reduced by the incorporation also of reheating, the range above which this type of engine then becomes most profitable can also be reduced.

Only a comprehensive study of engine and aircraft design, of aircraft power requirements and engine weight analysis, can yield quantitative answers to the questions arising over the choice of engine for a particular aircraft application, the conclusions drawn above being merely general ones based on the fundamental aspects of the problem.

7. *Performance of a Turbine Engine at Conditions other than Design.*—Sections **5** and **6** of this report have been given over to a study of the performance of turbine engines always operating at their design pressure ratio and maximum temperature. Comparisons which have been made, for example between sea-level and tropopause performance have been comparisons between different engines having similar component efficiencies designed to give the same pressure ratio under the differing conditions. It remains necessary, therefore, to consider how the performance of a particular engine is influenced by changes in operating conditions. These changes can be grouped under two headings.

(*a*) Changes in altitude and forward speed at constant engine rotational speed.

(*b*) Changes in engine rotational speed.

Some generalisations must be made in order that the essential features of the changes in performance are not confused by secondary effects. Moreover, the discussion will be confined to a simple engine with no reheat, intercooling or heat exchanger, and consisting of a single rotor mounting the compressor and its turbine. In general the results will be expressed in terms of the thrust given by a propulsive jet, but this can be related to the horse-power available in a power turbine application.

7.1. *Performance at Constant Engine Rotational Speed.*—The work done by a typical present-day compressor, centrifugal or axial, per lb of air flowing per sec, a measure of which is given by the total-head temperature rise in the compressor, can be regarded as approximately proportional to the square of the rotational speed regardless of entry conditions. At constant rotational speed, therefore, the temperature rise in the compressor is approximately constant. The pressure ratio is, in consequence, a function of the total-head temperature at inlet to the compressor, and as a result, the pressure ratio decreases with increasing forward speed, but increases with altitude. The table below indicates this variation in pressure ratio for a polytropic efficiency of 87 per cent and a specific heat of 0·241. The pressure ratio quoted allows for a loss of pressure in the combustion chamber. A value of 2 lb/sq in. is assumed for the sea-level static conditions, and this is taken to be proportional to the total-head pressure at inlet to the compressor in order to allow for the considerably higher absolute pressures in the chamber at high forward speeds and the lower pressures occurring at altitude.

Compressor Temperature Rise deg C	Compressor Pressure Ratio (including combustion pressure loss)							
	Sea level				36,000 ft			
	0 m.p.h.	500 m.p.h.	1,000 m.p.h.	1,500 m.p.h.	0 m.p.h.	500 m.p.h.	1,000 m.p.h.	1,500 m.p.h.
90	2·16	2·03	1·76	1·51	2·76	2·50	2·02	1·63
120	2·76	2·56	2·15	1·77	3·71	3·30	2·54	1·96
200	4·88	4·40	3·44	2·61	7·25	6·19	4·35	3·01
300	8·73	7·68	5·65	3·97	14·22	11·69	7·57	4·77
400	14·20	12·28	8·63	5·73	24·35	19·71	12·08	7·10
500	21·57	18·41	12·50	7·94	38·63	30·76	18·09	10·07

The pressure ratios shown in any line of this table are those which would be obtained under the various conditions by a compressor designed to give the stated temperature rise running at constant r.p.m. provided that the polytropic efficiency is not altered and that the original assumption of constant temperature rise is correct.

7.1.1. *Variation of specific thrust and efficiency.*—From this table and the cycle performance in terms of pressure ratio it is a simple matter to construct Fig. 21 showing the variation with forward speed at the two altitudes considered, of the specific thrust and efficiency of jet-turbine engines having different values of compressor work. The turbine inlet temperature is again constant at 1100 deg K.

The crossing over of the curves of Fig. 21 illustrates again how the optimum work required in the compressor decreases with increasing forward speed. The figure also shows that, at a given forward speed, the compressor temperature rise for maximum specific thrust is smaller than that for maximum overall efficiency. The improvement of both specific thrust and overall efficiency with increasing altitude is also apparent. The increasingly rapid fall of the specific thrust curves above 1000 m.p.h. is due to the increase in intake loss due to shock-waves, which are here assumed to be normal to the flow.

7.1.2. *Variation of mass flow.*—Whereas the values of overall efficiency shown in Fig. 21 are independent of the mass flow of air per sec through the engine, a knowledge of the variation of this quantity is required before the variation of actual thrust may be determined from the variation of specific thrust with altitude and forward speed.

There are many ways in which the throughput of the engine may be limited, but it is most convenient to assume, initially, that a limit is imposed by the turbine nozzles choking, a condition which usually occurs in practice. We may assume, too, that the turbine inlet temperature remains unaltered by changes in forward speed and altitude provided that the rotational speed is unchanged. This would mean, under certain conditions, that by fitting a variable-area propelling nozzle to the engine, the exhaust conditions of the turbine can be controlled in order to keep the inlet temperature sensibly constant. When choking conditions occur in the propelling nozzle as well as in the turbine nozzles (a condition which will generally hold good in the present study), conditions of constant turbine inlet temperature will correspond closely with constant propelling nozzle area conditions.

If the turbine nozzles are choking and the inlet temperature to the turbine is constant then the mass flow through the nozzles, and consequently through the whole engine is directly proportional to the total-head pressure at inlet to the nozzles and to the nozzle throat area provided. The variation of the inlet total-head pressure may therefore be regarded as the variation of the mass flow of air per sec per unit area of the nozzle throats.

This variation is shown in Fig. 22, again assuming a normal shock-wave at entry, and a constant polytropic efficiency in compression. The figure illustrates the considerable increases in turbine inlet pressure and consequently in mass flow, due to the forward speed and shows also the effect of the reduced ambient pressure and temperature at altitude on these quantities.

7.1.3. *Variation of thrust.*—It is now possible to construct curves showing the variation in thrust with forward speed at two altitudes for four typical gas-turbine jet-propulsion engines (distinguished by the different values of compressor work) subject to the various assumptions which have already been detailed. These curves are shown in Fig. 23, the thrust being expressed as lb/sq in. of turbine nozzle throat area. This area will, of course, remain constant for one particular engine, but it is not necessarily a yardstick for the overall size of the engine. Comparison between the performance of the different engines is, therefore, not satisfactory, unless it is restricted to the manner in which the thrust of the various engines varies with speed at a given altitude.

Such a comparison is made in Fig. 24, for the sea-level case only. All four engines are assumed to give the same thrust at 750 m.p.h. (approximately $M = 1 \cdot 0$) and the thrust at other aircraft speeds is expressed relative to this value. For each engine, as forward speed increases, the thrust falls to a minimum, then rises again to a maximum before finally beginning to fall to zero as the output per lb/sec mass flow of the gas turbine tending to zero predominates over the increasing mass flow. The figure then illustrates clearly the fact that at speeds below 750 m.p.h. at which the thrusts are made equal the engine with the lowest value of compressor work gives least thrust, the limit being reached, of course, with the propulsive duct which gives no thrust until reaching a forward speed providing a ram pressure rise great enough to overcome the losses in the system. Over the range of compressor work shown, the relative thrust at low aircraft speeds increases with compressor work but it will clearly reach a maximum and return to zero as the temperature after compression approaches the maximum temperature of the cycle.

Above the reference speed, the relative thrust at a given speed increases as the compressor temperature rise falls, as does also the aircraft speed at which the engine achieves maximum thrust.

At the constant maximum temperature of 1100 deg K an engine of 90 deg C compressor work is seen to be capable of giving over twice the thrust at 1250 m.p.h. that it provides at take-off and at aircraft speeds up to 400 m.p.h. An engine of 300 deg C compressor work, on the other hand, provides its best thrust at 0 m.p.h. and from 80 to 90 per cent of this thrust at speeds up to 1100 m.p.h. before there is a sudden fall. It must be remembered, of course, that the overall efficiency of the latter engine is greater than that of the former, and that it also requires a smaller mass flow to achieve a given design thrust.

If a higher maximum temperature is considered the overall effects just enumerated are not substantially altered, excepting that the values of forward speed and compressor temperature rise at which the various maxima and minima occur are increased.

7.1.3.1. *Effect of component inefficiencies : intake efficiency.*—Enough has already been said about the influence of compressor and turbine efficiencies upon the output of the turbine engine to make further discussion of these points superfluous at this stage in the present work. It must suffice to point out that the conditions of constant main component efficiencies assumed in this general theoretical study of the performance of a simple jet-engine at full rotational speed will rarely be attained in practice. Some modification of the figures presented should, therefore, be anticipated.

At the supersonic speeds which have been considered, however, intake efficiency begins to have a predominant influence on the performance of the engine. This is because an increasing amount of the compression of the air is being carried out in the intake and therefore the output and overall efficiency of the engine depends more and more on this compression being done efficiently. Further the amount of air inspired by the engine per sec is also dependent upon the intake efficiency.

In the foregoing an intake efficiency based on the formation of normal shock-waves at entry has been assumed in the treatment of supersonic flight speeds. To illustrate the important influence of intake efficiency, however, curves are also included in Fig. 24 which correspond approximately to the assumption of oblique shock fronts and related losses (based on recent German work). The effect is to increase considerably the thrust obtained at 1500 m.p.h. especially in a low pressure ratio engine, *e.g.*, 45 per cent for a compressor temperature rise of 90 deg C. At the same time the forward speed at which the maximum thrust of low pressure ratio engines is obtained is, of course, increased.

7.2. *Performance at Different Engine Rotational Speeds.*—In the preceding sections an approximation to the performance of a simple jet-engine running at constant r.p.m. has been made by employing somewhat sweeping assumptions. These included the constancy of compressor and turbine efficiencies and the independence of the compressor temperature rise on inlet conditions or throughout, whilst the mass flow and maximum temperature of the engine are assumed to be controlled by choking conditions in the turbine nozzles and the propelling nozzle, or by varying the propelling nozzle area when it is not choked.

In considering the performance of the engine at reduced r.p.m. in the same general terms, the temperature rise in the compressor can be considered as approximately proportional to the square of the rotational speed, and as long as the two choking conditions hold good, the maximum temperature of an engine with fixed exhaust area also varies roughly as this quantity. The thrust of a stationary engine, therefore, falls in proportion to the third or fourth power of the rotational speed, in response to both falling throughput and falling specific output. The assumptions regarding compressor performance made as a means to an end in the foregoing can no longer be justified when a simple method for assessing the airflow ceases to be available, *i.e.*, when the speed of the engine is such that choking in the turbine nozzles no longer occurs.

For these conditions turbine characteristics based on the particular design of turbine must be used, and it is clear that when this is done, more realistic compressor characteristics than those so far assumed, should also be employed. The method for determining the performance of the engine now is based on the matching of component characteristics, and is that which must be employed for an assessment of engine performance demanding greater accuracy than the previous general assumptions can yield. This method is detailed, for a simple jet-engine, in the following section of this report.

8. *Performance Estimation by Matching of Component Characteristics.*—In estimating and expressing the performance of a gas turbine engine, widespread use is made of non-dimensional groups of the controlling factors which allow a considerable simplification in both the calculation and presentation of the required performance data. The use of the method probably has its origin in the non-dimensional representation of supercharger characteristics which is now widely accepted. Turbine performance, and indeed complete engine performance, can be expressed by similar methods, and the simple jet-turbine engine is such that the variation of any one performance criterion over the whole of the possible operating range of the engine as an aircraft power plant can be expressed graphically in one simple diagram. In more complex turbine engines where there are more variable controls on the performance, a series of such diagrams may be necessary, but a non-dimensional treatment still permits considerable simplification.

8.1. *Usual Non-Dimensional Groups for a Simple Jet-Engine.*—For a particular design of turbine-jet engine of a given shape, five independent variables may be regarded as completely determining the performance of the design under all flight conditions. They are:

N the rotational speed,

V the forward speed,

P_a the ambient air pressure,

T_a the ambient air temperature,

L a characteristic reference length denoting the scale.

The main performance quantities dependent upon these variables are:

F the thrust,

Q_F the fuel consumption,

Q_A the air consumption.

Instead of expressing the three latter as functions of the five variables, by dimensional analysis it is possible to form groups of non-dimensional quantities and to express the performance by showing the variation of

$$\frac{F}{L^2P_a}, \quad \frac{Q_F}{L^2P_a\sqrt{T_a}} \quad \text{and} \quad \frac{Q_A\sqrt{T_a}}{L^2P_a},$$

as functions of two variables only, namely

$$\frac{NL}{\sqrt{T_a}}, \quad \frac{V}{\sqrt{T_a}}.$$

When considering the performance of one engine only, the length L is constant and is often omitted from the above groups. It should be stressed that L denotes the scale of the engine, and variations in L imply a complete scaling up or down of the engine in all dimensions and not of one component only, such as the propelling nozzle. Altering one component without altering all the rest to the same scale contradicts the original assumption that the engine is of particular design and given shape.

The assumptions also presume that the viscosity of the air has no appreciable effect on the performance of the engine and that average values of specific heat for the various components can be assumed and maintained constant without introducing too large an error.

Finally it should be noted that though Q_A and Q_F are commonly expressed in similar units of mass per unit time, this is not fundamentally correct and Q_F should be regarded as a measure of heat energy released per unit time, the units of mass in which it is commonly expressed being convertible to units of heat by using the calorific value of the fuel. This explains the difference in the non-dimensional forms of these two quantities.

8.2. *Non-dimensional Characteristics of Components.*—8.2.1. *Compressor.*—The performance of a compressor can be expressed by the variation with $Q_A\sqrt{T_{1t}}/P_{1t}$ for given values of $N/\sqrt{T_{1t}}$ of any two of the three quantities: pressure ratio, P_{2t}/P_{1t}, temperature rise ratio, $(T_{2t} - T_{1t})/T_{1t}$, or overall efficiency, η_{12}. (The suffix $_{1t}$ indicates total-head conditions at inlet to the compressor and the suffix $_{2t}$ conditions at outlet.) These variations can be expressed on one diagram, however, as shown in Fig. 25 where lines of constant $(T_{2t} - T_{1t})/T_{1t}$ are superimposed on the pressure ratio diagram. The overall efficiency 'contours' may also be included for interest but are not required for the computation of performance.

8.2.2. *Turbine.*—Similarly the performance of a turbine may be expressed by the variation of $(T_{3t} - T_{4t})/T_{3t}$ and P_{3t}/P_{4t} with $Q_A\sqrt{T_{3t}}/P_{3t}$* and $N/\sqrt{T_{3t}}$, where suffix $_{3t}$ indicates inlet total-head conditions and suffix $_{4t}$ total-head conditions at outlet. As, however, in matching a turbine with its compressor, the independent variable to be chosen so that the work output of the former is sufficient to drive the latter, is the temperature ratio T_{3t}/T_{1t}, it is often more convenient to plot the turbine performance as a function of Q_AN/P_{3t} and $(T_{3t} - T_{4t})/N^2$, thus eliminating a laborious trial and error process to determine T_{3t}/T_{1t}, as will be shown later. The latter method of plotting the characteristics is shown on a small scale in Fig. 26.

8.2.3. *Jet pipe and propelling nozzle.*—It may be assumed that the characteristic of a jet pipe to which propelling nozzles of various sizes are fitted can be obtained from a single curve showing the variation of $Q_A\sqrt{T_{4t}}/A_5P_{4t}$ with P_a/P_{4t}, where A_5 is the effective throat area of the nozzle. This implies that the losses in the pipe are independent of the size of the propelling nozzle fitted over the range of size considered, Fig. 27 shows both this variation of $Q_A\sqrt{(T_{4t})}/A_5P_{4t}$ with P_a/P_{4t} and also the variation of $F_GA_5P_a$, where F_G is the gross thrust created, assuming that the nozzle is a plain convergent one and that when choking the thrust can be considered partly as momentum thrust and partly as pressure thrust applied over the outlet area of the nozzle.

8.2.4. *Combustion chamber.*—The efficiency with which fuel is burnt is ignored in obtaining the equilibrium conditions of a turbine engine as it simply affects the ratio of the *actual fuel consumption* to that ideally necessary. The pressure loss in the combustion chamber does, however, influence the matching of the components as well as influencing the performance. It is often sufficiently accurate to assume that this loss is a fixed proportion of the inlet pressure, P_{2t}, or alternatively the proportion may be assumed to vary as $(Q_A\sqrt{T_{2t}}/P_{2t})^2$, a quantity which is determinable from the compressor characteristics. There should also be an additional expression relating the pressure loss to the temperature ratio obtained by combustion but this is, as yet, little used in performance work.

* It is assumed that the mass flows through turbine and compressor are equal.

8.3. *Linkage of Compressor and Turbine Characteristics.*—For every point on the compressor characteristic the following quantities are known

$$Q_A\sqrt{T_{1t}}/P_{1t}\,,\quad N/\sqrt{T_{1t}}\,,\quad P_{2t}/P_{1t} \text{ and } (T_{2t}-T_{1t})/T_{1t}\,.$$

The relation between $(T_{2t}-T_{1t})$ and $(T_{3t}-T_{4t})$ has been quoted in section 5.1.2 and this enables $(T_{3t}-T_{4t})/N^2$ to be calculated. Further as P_{3t}/P_{2t} may be assessed from the combustion pressure loss data, Q_AN/P_{3t} may also be computed. These quantities are sufficient to determine $(T_{3t}-T_{4t})/T_{3t}$ and P_{3t}/P_{4t} from the turbine characteristics.

Now

$$\frac{T_{3t}}{T_{1t}}=\frac{(T_{3t}-T_{4t})/N^2}{(T_{3t}-T_{4t})/T_{3t}}\times\left(\frac{N}{\sqrt{T_{1t}}}\right)^2,$$

and can, therefore, be determined. Lines for constant values of T_{3t}/T_{1t} may be superimposed on the compressor characteristics, as in Fig. 28, to indicate the temperature ratio required for equilibrium running between these components at any compressor operating point.

8.4. *Addition of the Jet Pipe.*—Each compressor operating point now determines a value of $Q\sqrt{T_{4t}}/P_{4t}$ and of P_{1t}/P_{4t}. If the engine is then assumed to be moving forward at any chosen flight Mach number (*i.e.*, chosen value of $V/\sqrt{T_a}$) so that P_{1t}/P_a and T_{1t}/T_a are known, P_a/P_{4t} is determined and $Q_A\sqrt{T_{4t}}/A_5P_{4t}$ obtained from the jet-pipe characteristic. The propelling nozzle area A_5, required for operation at the given point at the chosen flight Mach number is therefore determined. This value can be used to give F_G/P_a from the appropriate value of F_G/A_5P_a and the net thrust, F_N, is finally assessed by

$$\frac{F_N}{P_a}=\frac{F_G}{P_a}-\frac{Q\sqrt{T_{1t}}}{P_{1t}}\cdot\frac{P_{1t}}{P_a}\cdot\frac{V}{\sqrt{T_a}}\cdot\sqrt{\left(\frac{T_a}{T_{1t}}\right)}\cdot\frac{1}{g},$$

where V has been converted to ft/sec.

Further

$$\frac{N}{\sqrt{T_a}}=\frac{N}{\sqrt{T_{1t}}}\times\sqrt{\frac{T_{1t}}{T_a}}$$

so that F_N/P_a, $N/\sqrt{T_a}$ and A_5 are known for each operating point when $V/\sqrt{T_a}$ is chosen. It follows that if A_5 is caused to remain constant, a unique operating line for each value of $V/\sqrt{T_a}$ is determined on the equilibrium running diagram and for this value of A_5, F_N/P_a may be expressed as a function of $N/\sqrt{T_a}$ and $V/\sqrt{T_a}$, as was stated above.

Similarly, along the operating line for a chosen value of $V/\sqrt{T_a}$, $(T_{3t}-T_{2t})/T_{1t}$ may be calculated and, if specific heat variations can be ignored, this will yield distinct values of q/T_{1t}, where q is the fuel : air ratio and so lead to the evaluation of $Q_F/P_a\sqrt{T_a}$ thus:

$$\frac{Q_F}{\sqrt{(T_a)}\,.\,P_a}=\frac{q}{T_{1t}}\cdot\frac{Q\sqrt{T_{1t}}}{P_{1t}}\cdot\frac{P_{1t}}{P_a}\cdot\sqrt{\left(\frac{T_{1t}}{T_a}\right)}.$$

Unfortunately the relationship between $(T_{3t}-T_{2t})/T_{1t}$ and q/T_{1t} is not simply a function of the calorific value of the fuel, for, because of the inconstancy of the specific heat of air, it is also dependent upon the temperature range, over which the fuel is supplied. A non-dimensional representation for the variation of fuel flow can only be exact for one altitude (defining the temperature range covered) and a maximum error of about 5 per cent may be incurred by assuming the curves are correct for all altitudes. This illustrates the limitations of the non-dimensional representation of jet-turbine engine performance.

8.5. '*Equivalent*' *Conditions.*—It is generally found advantageous to plot the non-dimensional representation of performance thus derived as the variation not of the simple non-dimensional quantities indicated but of the equivalent values at some altitude, usually sea-level. Thus, instead of F_N/P_a, is plotted $F_N \times 14 \cdot 7/P_a$ so that values for thrust at sea-level may be read off directly from the curves. At other altitudes a knowledge of the atmospheric conditions relative to sea-level is required in order to determine the actual thrust. If atmospheric pressure and temperature relative to the respective values at sea-level are denoted by p_a and t_a then equivalent parameters may be derived from the non-dimensional ones by replacing P_a by p_a and T_a by t_a.

8.6. *Example.*—The characteristics of the components taken for this example are those shown in Figs. 25, 26 and 27. At the design speed of 7,000 r.p.m. the compressor gives a pressure ratio of $3 \cdot 6$ to 1 and passes 50 lb/sec of air ($Q \sqrt{(T_{1t})}/P_{1t} = 57 \cdot 72$) under sea-level static conditions. The overall efficiency at the design point is slightly in excess of 84 per cent and the characteristics are intended to be typical for a British axial compressor. The peak compressor efficiency is approximately 88 per cent and the lowest efficiency in the range covered is about 78 per cent.

The turbine characteristics are theoretical ones for a single-stage turbine of nozzle angle 25 deg and blade outlet angle of 35 deg measured from the circumferential direction at the mean blade height. At the design point marked on Fig. 26, the mean blade speed, u, is approximately 909 ft/sec (sea-level static conditions). This means that the diameter of the turbine at mean blade height is about $29 \cdot 8$ in. and to give the required annulus area, S, the inner and outer diameters must be $27 \cdot 6$ in. and $32 \cdot 0$ in. These dimensions are slightly greater than those of the turbine of the Metropolitan-Vickers F2/4 engine, the only comparable British single-stage turbine at present in use. At the greater rotational speeds associated with centrifugal compressors, the turbine diameter for a given blade speed need not be so great. The total-head efficiency of the turbine at the design point is approximately 87 per cent, the inlet total-head temperature being 1100 deg K.

The only point requiring further explanation in Fig. 27 is the form of the mass flow curve. This is a semi-empirical curve which removes the anomaly obtained by assuming constant jet efficiency, that the mass flow parameter, $Q \sqrt{(T_{4t})}/A_5P_{4t}$, reaches a maximum before the velocity of the gas in the throat of the nozzle reaches the sonic value. For the losses assumed, the mass flow parameter reaches a maximum of $0 \cdot 976$ if the maximum isentropic value ($0 \cdot 3896$ for a K_p of $0 \cdot 274$) at a pressure ratio of $0 \cdot 526$, under which conditions the exit velocity would be expected to be sonic.

Fig. 28 shows the compressor and turbine characteristics linked by lines of constant temperature ratio T_{3t}/T_{1t}. These are seen to be almost parallel lines, running into the compressor surge line at their 'lower' ends. At a given mass flow the temperature ratio increases with compressor pressure ratio, the extra work required for compression demanding a higher turbine inlet temperature. At the design point the value of T_{3t}/T_{1t} is, of course, $1100/288 = 3 \cdot 82$.

8.6.1. *Variation of thrust and specific fuel consumption with jet area.*—In Figs. 29 and 30, the values of equivalent thrust and equivalent specific fuel consumption at three values of equivalent aircraft speed, 0, 300 and 600 m.p.h. obtained by working along the lines of constant $N/\sqrt{t_{1t}}$ for the compressor, are plotted against the effective area of the jet required for equilibrium running. The intake efficiency is assumed to be 85 per cent.

With this example, as in virtually all engines so far built, at all combinations of speeds, maximum thrust occurs at the compressor surge line where the turbine operating temperatures are highest and the thrust falls off as the jet area is increased to bring the equilibrium running point further from the surge line. This fall-off is greatest at the highest aircraft speeds.

It will also be noted that the jet area below which surge-free operation is impossible increases as the speed of the compressor is increased. At the higher speeds, variations in aircraft velocity have no influence on the jet area which causes surging at a fixed value of $N/\sqrt{t_{1t}}$. This is due to the propelling nozzle choking, so that the changes in jet pressure ratio due to forward speed cause no alteration in the value of $Q \sqrt{(T_{4t})}/A_5P_{4t}$ (*see* Fig. 27) and hence of A_5 corresponding to a fixed point on the compressor characteristics.

It is, at first sight, surprising to find that the specific fuel consumption also falls, as the operating point leaves the surge line, though in this case a minimum value is reached and at large jet areas the specific fuel consumption is again increasing. Further, if the running lines for a jet area of 141·3 sq in., the design value, are plotted on the compressor characteristics (Fig. 31) comparison between Figs 25, 30 and 31 shows that although the compressor is then made to operate at peak efficiency in the range of speed between 7,000 and 8,000 r.p.m. minimum specific fuel consumption is not being obtained. Instead the minima occur at higher jet areas and are associated with lower turbine operating temperatures, lower pressure ratios and lower values of thrust. Ignoring the r.p.m. of the compressor it can be deduced that at constant pressure ratio also there is an optimum value of temperature ratio T_{3t}/T_{1t} at which minimum specific fuel consumption occurs.

Referring back to Fig. 5 it will be recalled that at constant pressure ratio the cycle efficiency of a gas turbine shows no tendency to reach a maximum at an optimum temperature ratio in the range encountered here, though at high temperature ratios the efficiency is almost independent of the temperature ratio. The specific fuel consumption, however, is a measure of the efficiency of the turbine and the jet together, and when the aircraft is moving at a speed, V,

$$\text{specific fuel consumption} \propto V/\eta_o$$

$$\propto V/\eta_c \eta_p ,$$

where

η_o overall efficiency of turbine and jet,

η_c cycle efficiency of the turbine,

η_p propulsive efficiency of the jet.

At constant forward speed although η_c continues to increase slightly with increasing temperature ratio at constant pressure ratio Fig. 18c shows that η_p will decrease, because the specific output from the engine is increasing, and a jet is only perfectly efficient when it does no work. These two effects opposing each other are responsible for the form of the curves of specific fuel consumption against jet area and explain why optimum consumption does not necessarily occur where the compressor efficiency is highest.

When the aircraft is stationary the quotient V/η_o is indeterminate but the relation between b.h.p. output from the turbine and thrust output from the jet (Fig. 18d) enables a similar argument to be established.

8.6.2. *Performance with fixed propelling nozzle.*—The equilibrium running diagram for the engine fitted with a fixed propelling nozzle is shown in Fig. 31 and the variations of equivalent thrust and equivalent specific fuel consumption with equivalent speed are shown in Figs. 32 and 33. These variations are also given in 'carpet diagram' form in Fig. 34. In these diagrams, the lines of constant $V/\sqrt{t_a}$ are spaced out equidistantly and connected by curves of constant $N/\sqrt{t_a}$, the data being thus plotted and cross-plotted in one and the same figure.

From Fig. 31 it can be seen that near $N/\sqrt{t_{1t}}$ of 7,000 r.p.m. the running lines for all aircraft speeds become coincident due to choking conditions being reached in the jet pipe. Below this speed the lines fan out, the highest forward speeds being associated with higher mass flows. At constant altitude and revolutions per minute the value of $N/\sqrt{t_{1t}}$ decreases with increasing forward speed, and on Fig. 31 are marked the running points corresponding to 7,000 r.p.m. at the three values of $V/\sqrt{t_a}$ at sea-level and in the stratosphere. Because the ram temperature rise is proportional to the square of the forward speed it is not surprising that the difference between the points for 300 and 600 m.p.h. is much greater than between 0 and 300 m.p.h. This also illustrates the feature that a turbine engine is more likely to encounter surging in the compressor when it is flying at low forward speeds, say during climbing, than when flying level at high speeds at the same altitude and revolutions per minute. With the engine considered, surging is just avoided, but from Fig. 29 it can be seen that with a propelling nozzle some 10 per cent under size surge-free running would not be possible at design revolutions per minute even at sea-level.

The curves of thrust and specific consumption require little comment. Those for thrust show that the expected tendency to fall initially with increasing forward speed and then to rise again, though more will be said about the form of these curves in the following section. The carpet diagram for specific fuel consumption (Fig. 34) shows that at sea level, although the minimum fuel consumption occurs below 7,000 r.p.m. when the aircraft is stationary, it occurs close to this speed at high flight velocities. Fig. 34 also makes clear that within the range of equivalent revolutions per minute from 6,000 to 8,000 r.p.m. which covers all cruising and maximum operating conditions at any altitude, engine speed is more important than aircraft speed in determining the thrust, but in determining specific fuel consumption the reverse is true. It must be remembered that complete combustion of the fuel is assumed in presenting these results.

8.6.3. *Effect of intake and compressor efficiencies.*—If reference is now made to Fig. 24 and the curve in Fig. 34 showing the variation of thrust with forward speed assuming sea-level operation at 7,000 r.p.m. is compared with the appropriate portions of the curves previously obtained, it is seen to agree most closely with the curves for compressor temperature rises of 200 deg to 300 deg C although the actual rise at design conditions for the engine is only 150 deg C. As in the engine, both turbine and propelling nozzle are choking, or very nearly so and the maximum temperature is constant at 1100 deg K* this difference is due to differences in the component efficiencies assumed in the two cases. The compressor and turbine efficiencies in the engine now considered are close to those assumed in Fig. 24, though there is now a slight rise in compressor efficiency with forward speed. This would tend to improve the thrust at 600 m.p.h., however, and not result in it being lower than anticipated. This latter effect is, therefore, due to the lower intake efficiency of 85 per cent assumed for the engine compared with 100 per cent for subsonic flight speeds assumed in producing the previous curves, and the example again illustrates the dependence of a high-speed turbine-jet engine on the effectiveness of its air intakes.

The other component efficiency which appears most likely to vary and have an important effect on the performance of the engine is that of the compressor when operating at high values of $N/\sqrt{t_{1t}}$. In the compressor considered here a fall of efficiency from 84 to 78 per cent occurs between the values of $N/\sqrt{t_{1t}}$ of 7,000 and 8,000 r.p.m. This is going to cause a deterioration in the altitude performance of the engine at full revolutions per minute and since the rise of efficiency below $N/\sqrt{t_{1t}} = 7{,}000$ is not so steep as the fall at higher speeds the deterioration will be more important at low aircraft velocities than at high ones. To show the magnitude of this effect, in Fig. 35 the appropriate portion of the carpet diagram for thrust is compared with the diagram corresponding to constant compressor efficiency equal to that at the design point. At $N/\sqrt{t_a} = 8{,}000$ and zero velocity the gain in thrust in the latter case would be some 15 per cent, but at an equivalent velocity of 600 m.p.h. it would only be about 4 per cent.

The curves in the lower part of Fig. 35 show what this means in terms of the actual thrust obtained at 7,000 r.p.m. and 0 and 500 m.p.h. at varying I.C.A.N. altitudes. Although the differences due to falling compressor efficiency appear small when compared with the fall in thrust due to the lowering of atmospheric pressure with altitude, the percentage differences at a given altitude are appreciable. The figures of 15 per cent and 4 per cent quoted above correspond closely to the gain in thrust available in the stratosphere if constant compressor efficiency could be maintained. The effect on the rate of climb of an aircraft, and, to a smaller extent, on the top speed in level flight at high altitude would be noticeable.

The specific fuel consumption of the engine in the example taken was not greatly altered by assuming constant compressor efficiency and is therefore not plotted.

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* *See* point for $V = 600$ m.p.h., on Fig. 31. $T_{3t}/T_{1t} = 3{\cdot}4$ and $T_{1t} = 288 + 36 = 324$ deg K. Therefore $T_{3t} = 1100$ deg K = design value at sea-level static conditions.

8.7. *Confirmation of Theory by Practical Tests.*—A theory for deducing the performance of a mechanical device can only be worth while if the results it predicts are borne out in practice. The theory used to predict the performance of a gas-turbine jet engine in flight cannot yet be said to be fully tested, but it is also true to add that no test has produced a general result of reasonable reliability with which the theory of the matching of component characteristics is not in agreement. The theory does not yet extend to cover the accurate prediction of the component characteristics from design data, but has to assume that these may be obtained experimentally. For example, with probably the simplest component of all, the jet pipe and nozzle, the theoretical assumption of constant adiabatic efficiency is found to have shortcomings and the relationship between the effective area of the propelling nozzle and the actual area is not theoretically predictable.

After the combustion system, whose development to a high standard of efficiency has been largely a matter of testing and re-testing, the major component on which most testing has been done is the compressor. Tests on several centrifugal and axial compressors have been carried out in this country and in America and Germany, but the growth of the theory to predict results in full agreement with those yielded by tests, is still incomplete.

For the characteristics of the other major component, the turbine, we are still largely dependent on theoretical assumptions as yet unconfirmed by tests on the component as a separate unit. Some confirmation may be obtained from tests on the complete engine, but this falls far short of what is desirable. Much experimental work must be done before a completely adequate theory of turbine performance can be put forward.

Tests of engines on the bench, especially tests involving calibrations with different sizes of propelling nozzle, have done much to confirm the method of matching non-dimensional component characteristics and the agreed method of correcting test results to standard atmospheric conditions implies a wide acceptance of this method of treatment.

Tests of engines in flight, though limited by the amount and type of instrumentation which can be carried, and the necessity of estimating thrust indirectly, since no suitable measuring device has been developed, have done nothing to disprove the suitability of the non-dimensional method of expressing the performance of the engine. When operating at the same conditions of equivalent aircraft velocity and revolutions per minute although at different altitudes and consequently at different actual values of velocity and revolutions per minute, the engine performance expressed in non-dimensional form is found to be identical within the limits of experimental accuracy. Changes due to varying specific heat and air viscosity lie within the present limits of this accuracy.

Since the end of the war, British engines have been tested in the German altitude test chambers and it is understood that though the study of these results is not yet complete, they too are in general agreement with the non-dimensional theory.

9. *Conclusion.*—In this report an attempt has been made to outline the extent to which the theoretical possibilities of the gas-turbine cycle have been investigated during recent years. These theoretical investigations have not, of course, been the work of any one person or of any one group, but the author wishes to acknowledge his indebtedness to those from whose work he has drawn freely in preparing this report.

The investigations have been concerned not only with the limits upon performance, inherent in even the ideal cycle, but also with the extent to which the ideal performance may be realised in machines of various component efficiencies. The influence of reheating, intercooling and heat exchange on the performance of the simple cycle has been studied in some detail. Because of the close association between the gas turbine and jet propulsion for aircraft a large proportion of research work has centred on the gas turbine as an aero-engine, and the variations in the performance of such an engine under varying flight conditions have been examined, together with the methods by which this performance may be assessed from a knowledge of the component characteristics.

Many avenues of research, however, remain virtually unexplored or at the best, only lightly trodden. To name but a few of the latter, in which some advances in knowledge have been made, but which have not been mentioned in this report; they include:

(a) The effect of injecting liquids such as water, ammonia or alcohol into the airstream before compression, in order to obtain increased thrust.

(b) The closed-cycle gas turbine, operating at high pressures and with other working substances than air.

(c) The equilibrium running of engines involving a multiplicity of compressors and turbines.

(d) The application of the gas turbine to duties other than aircraft propulsion.

In addition to the necessity for studying these further aspects of the subject of gas-turbine engine performance, there will also be in the future, a constant need to re-survey the ground already covered in the light of practical developments, using the results of more comprehensive experimental engine testing to confirm or amend the assumptions which form the basis of the theoretical approach to the problem, at the present early stage in the history of the gas-turbine power plant.

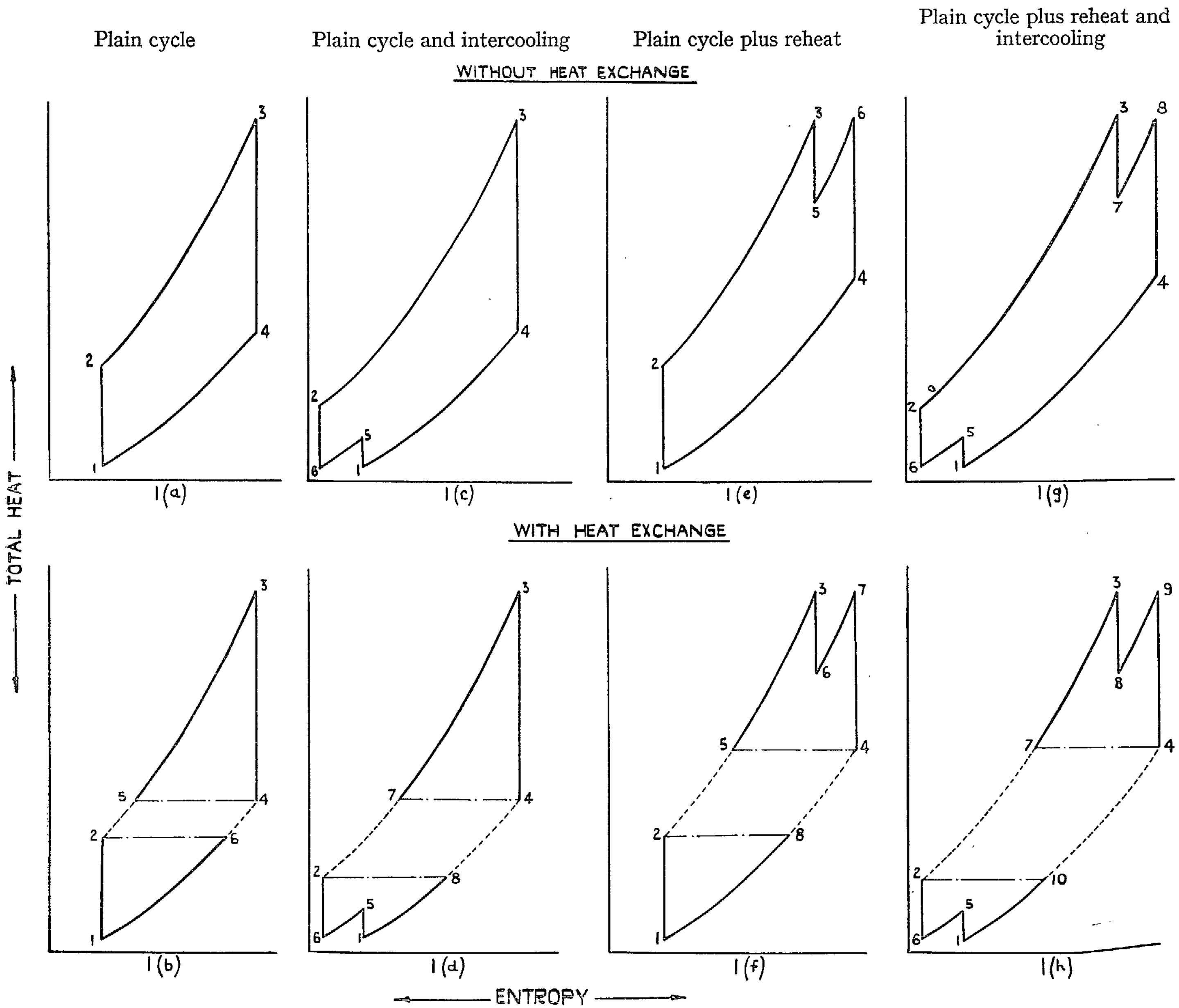


FIG. 1. Total heat-entropy diagrams of ideal cycles.

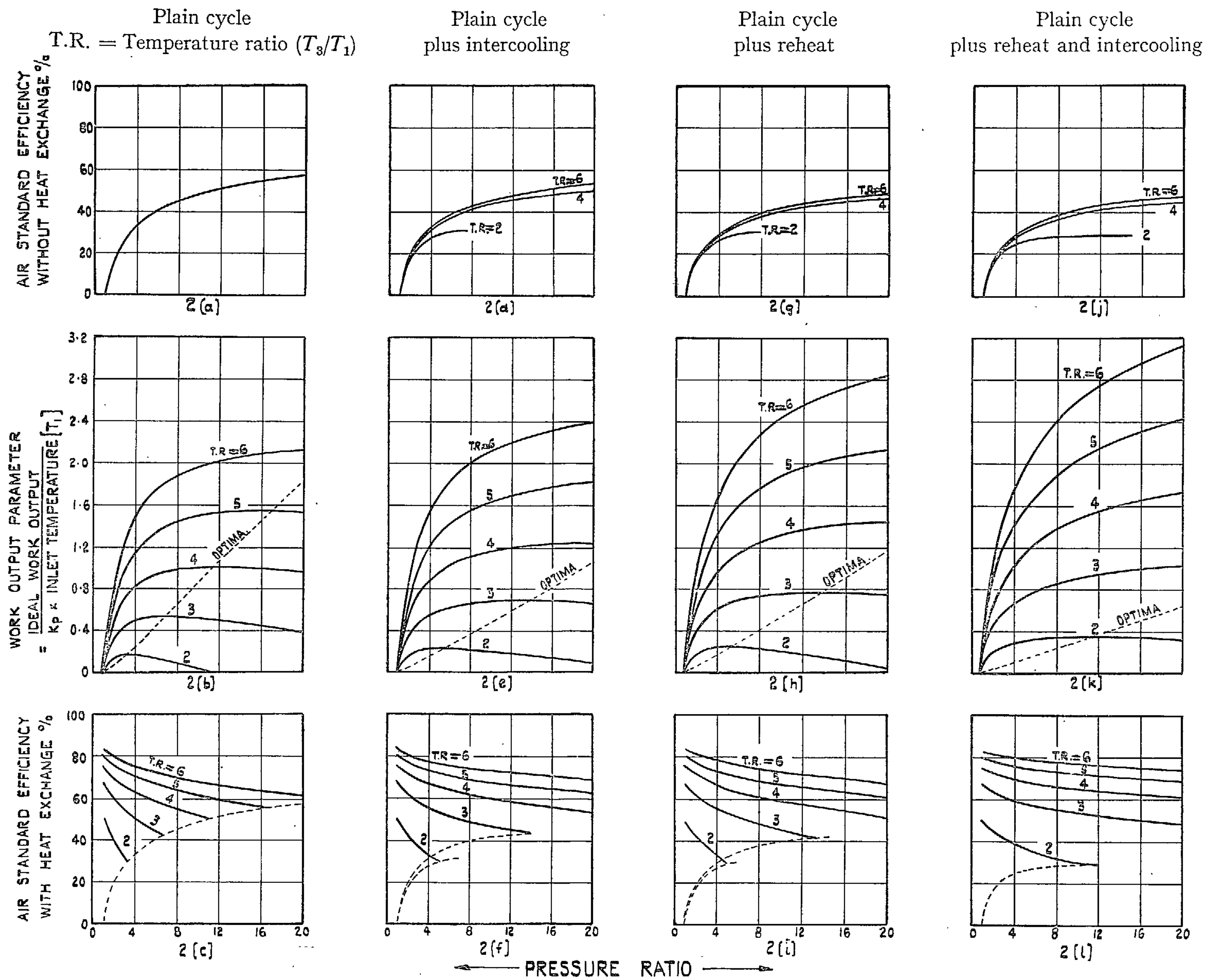


FIG. 2. Performance of ideal cycles.

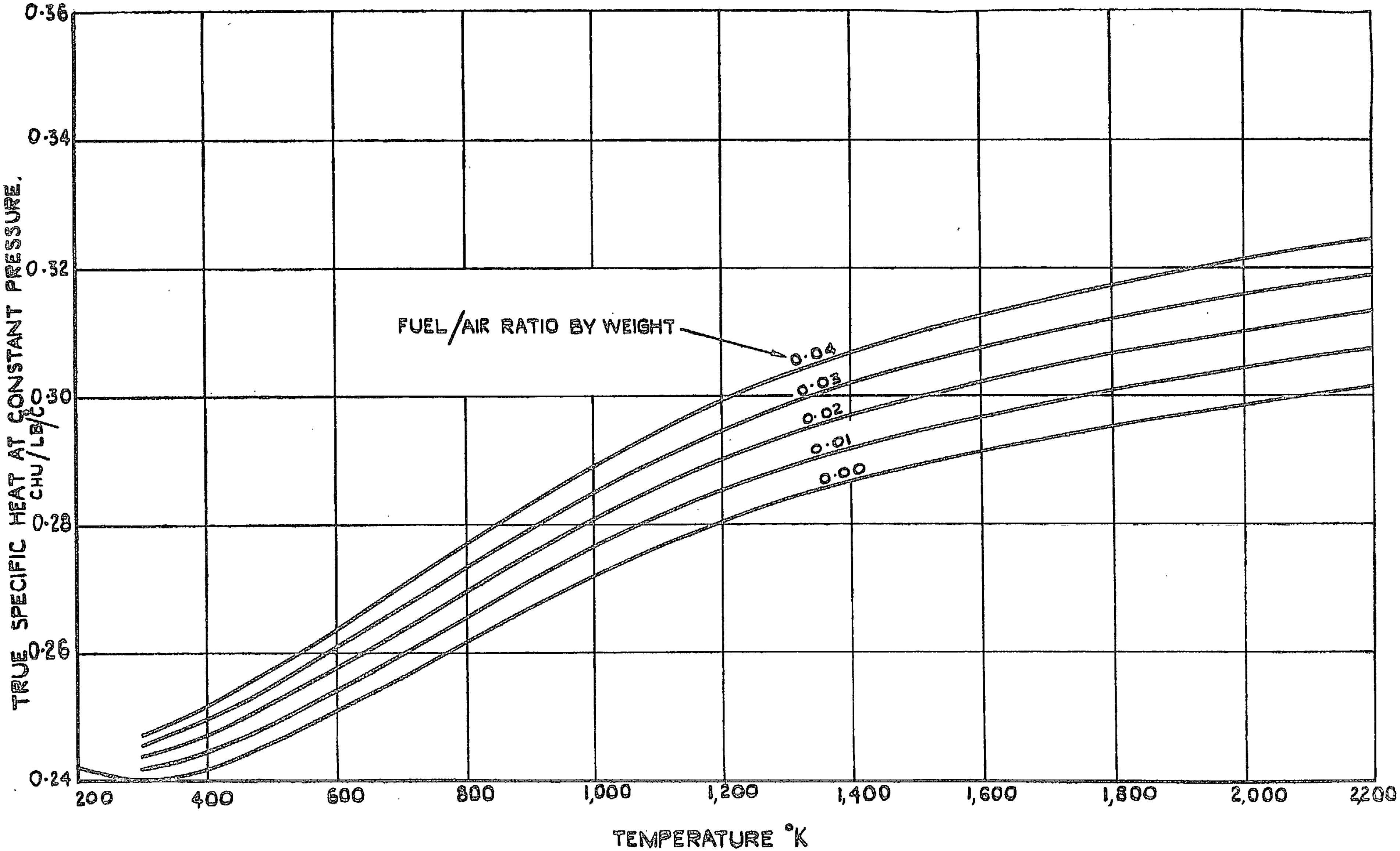
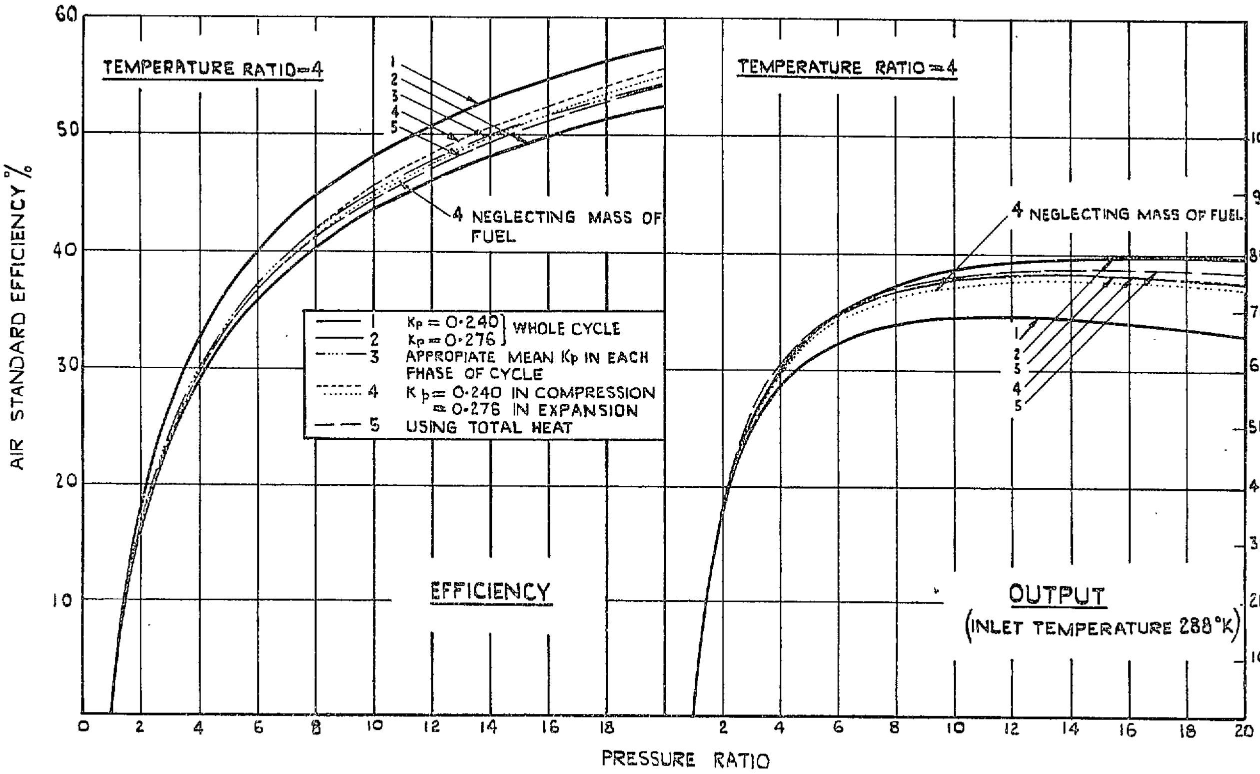


FIG. 3a. Properties of air and combustion gases. Variation of true specific heat with temperature (after H. M. Spiers). Fuel analysis: C = 86·5 per cent. H_2 = 13·5 per cent. Lower calorific value = 10,300 C.H.U./lb.



FIGS. 3b and 3c. Influence of various specific heat assumptions on performance of the ideal cycle. Specific output—C.H.U./lb/sec.

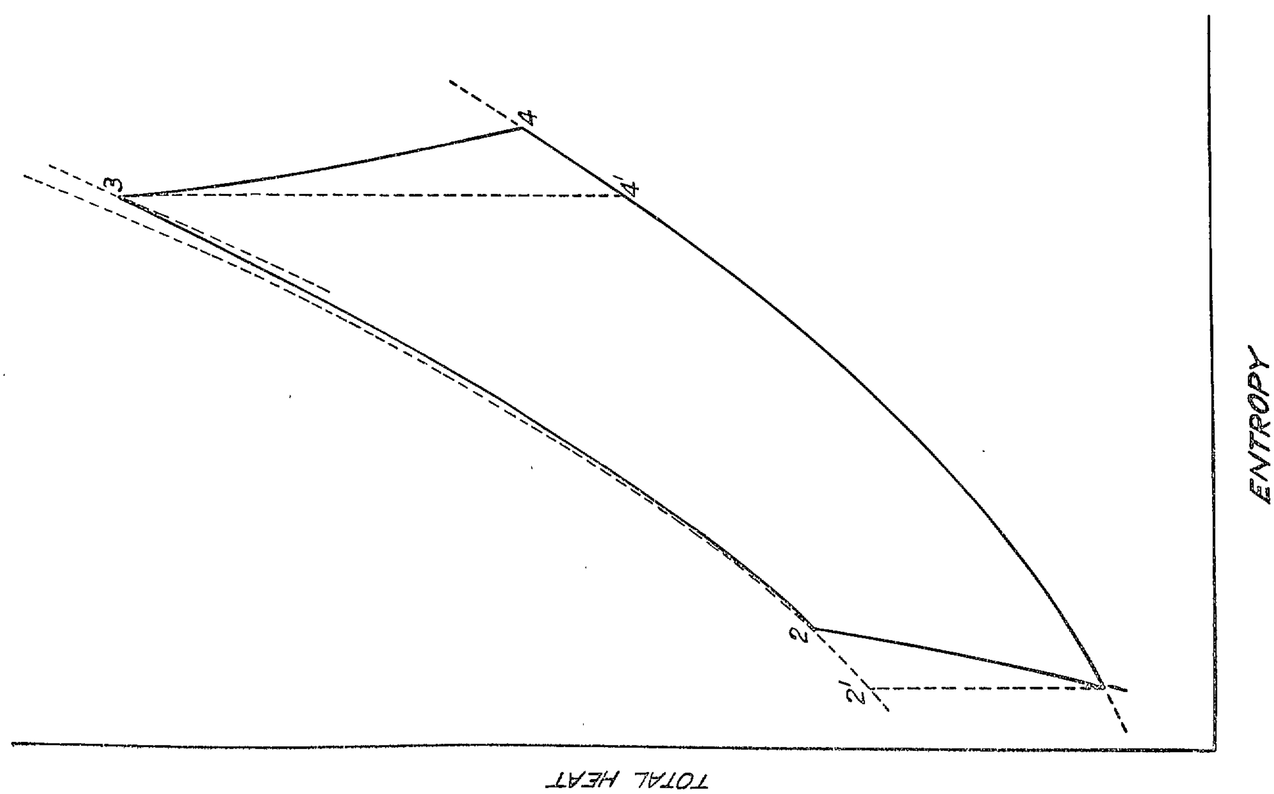


FIG. 4. Total heat *vs.* entropy diagram for plain cycle with losses.

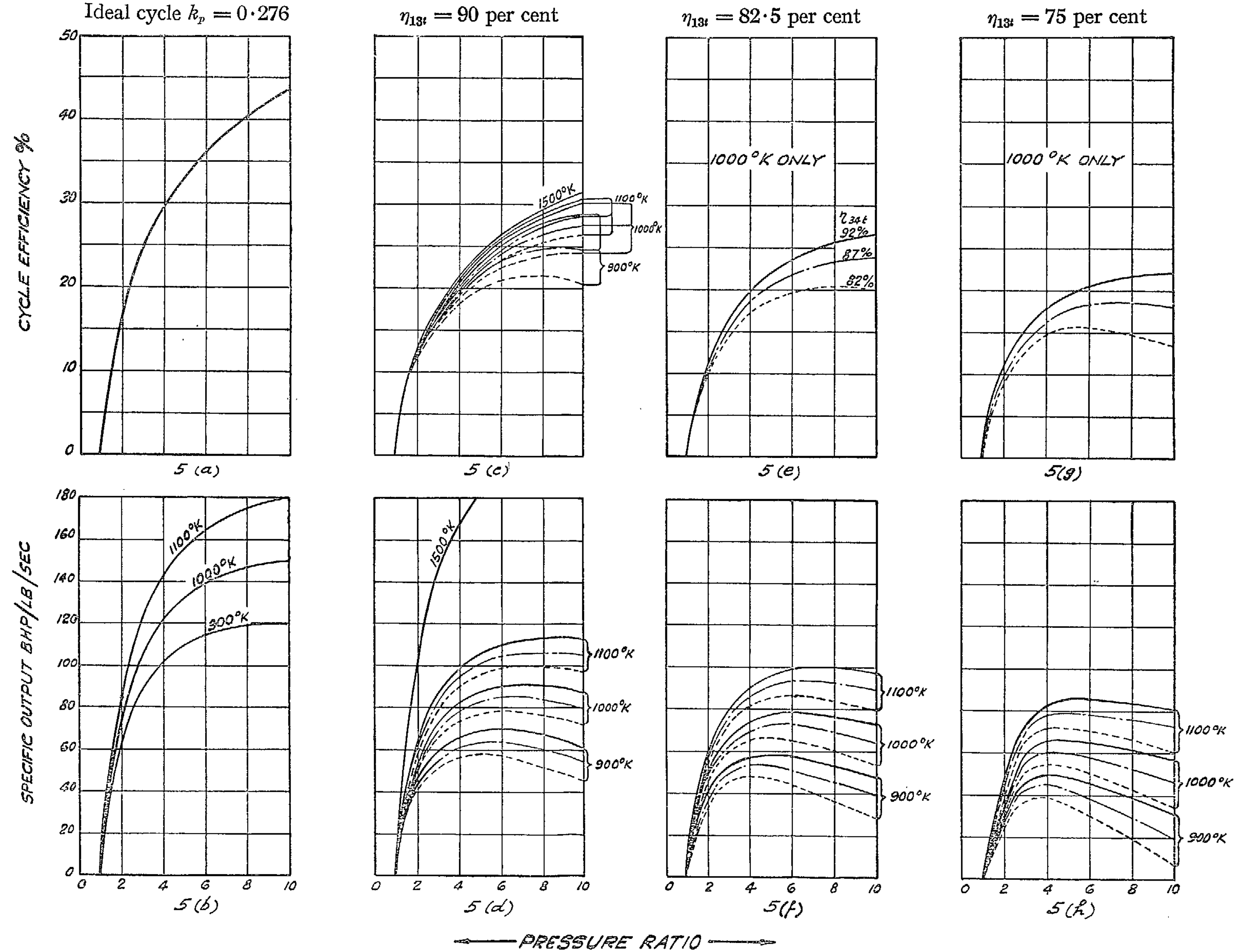


FIG. 5. Performance of the cycle with losses.

η_{13t} = compressor efficiency, η_{34t} = compressor turbine efficiency; power turbine efficiency = 80 per cent

——— $\eta_{34t} = 92$ per cent —·— $\eta_{34t} = 87$ per cent ----- $\eta_{34t} = 82$ per cent

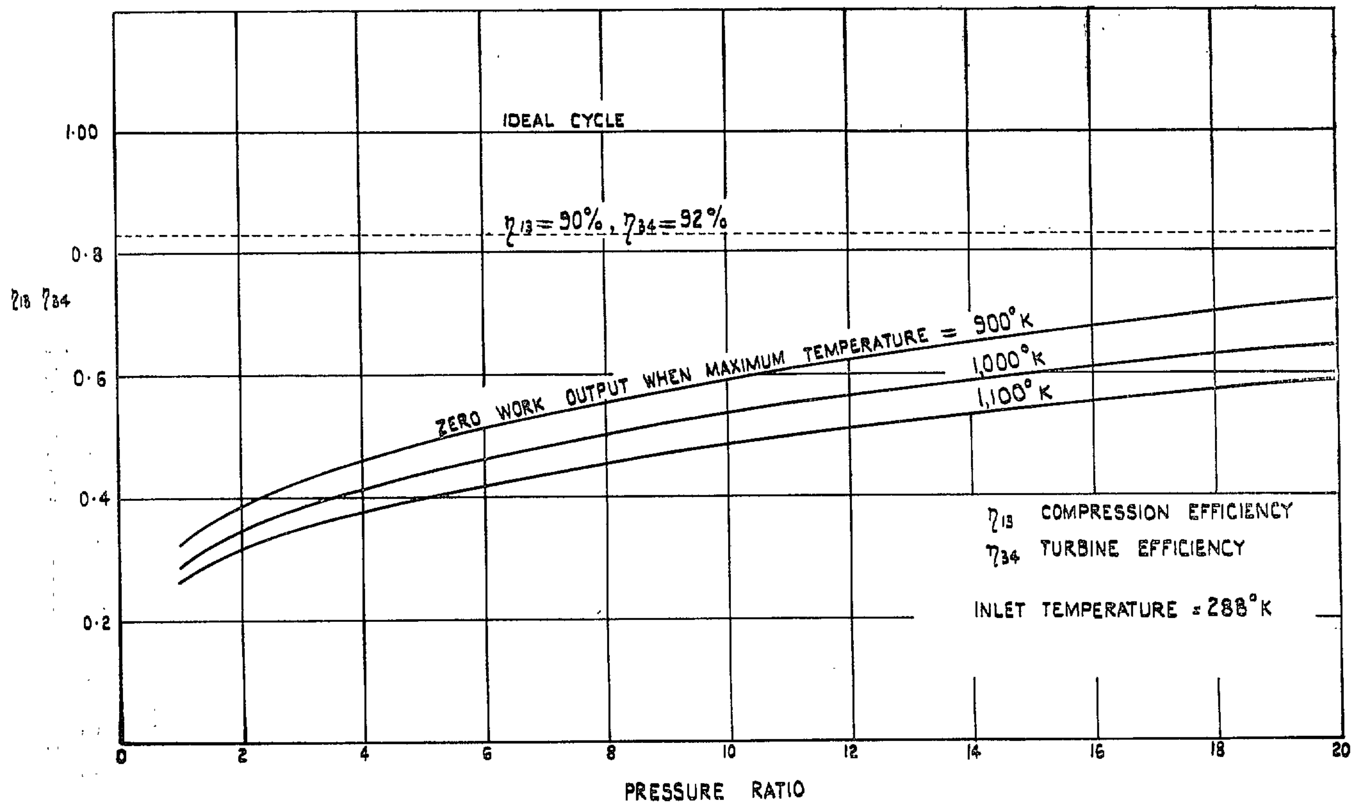


FIG. 6. Influence of component efficiencies and maximum temperature on the range of pressure ratio in which a cycle gives positive work output.

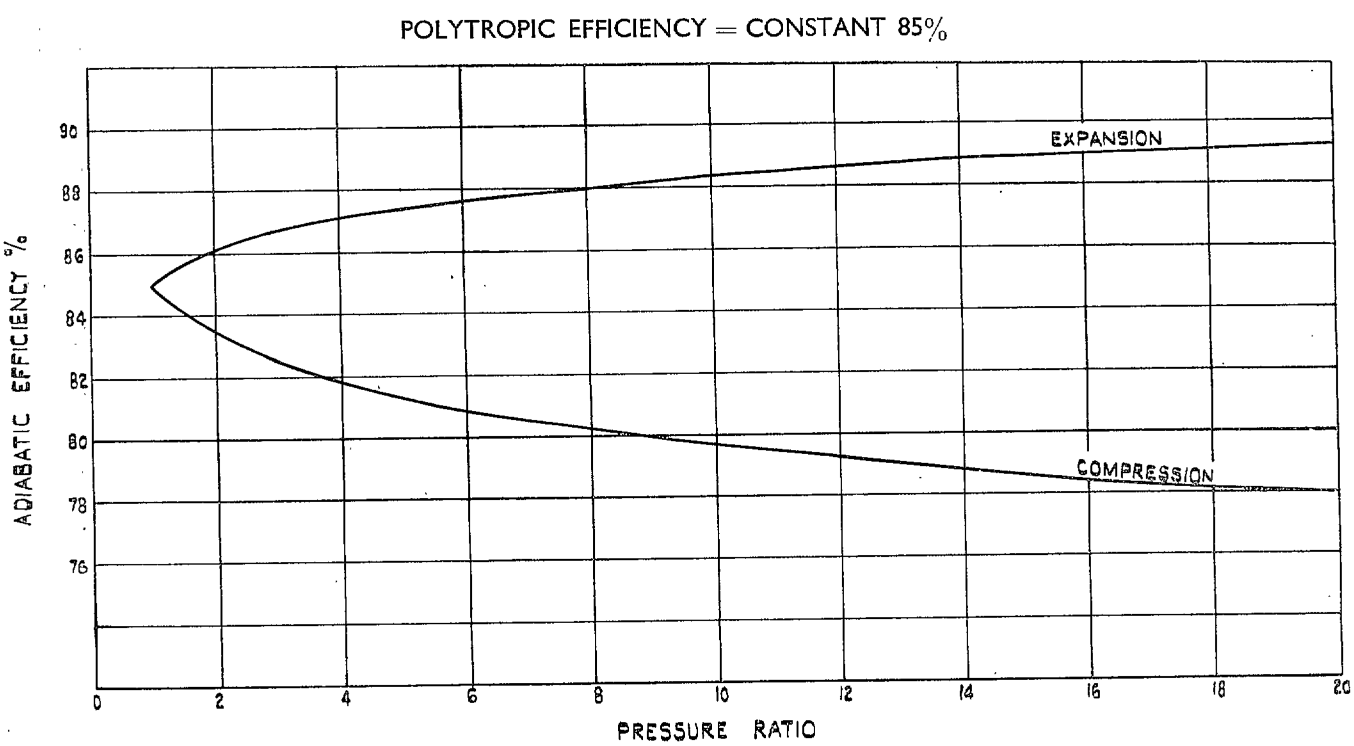


FIG. 7. Relationship between polytropic and adiabatic efficiencies of compression and expansion.

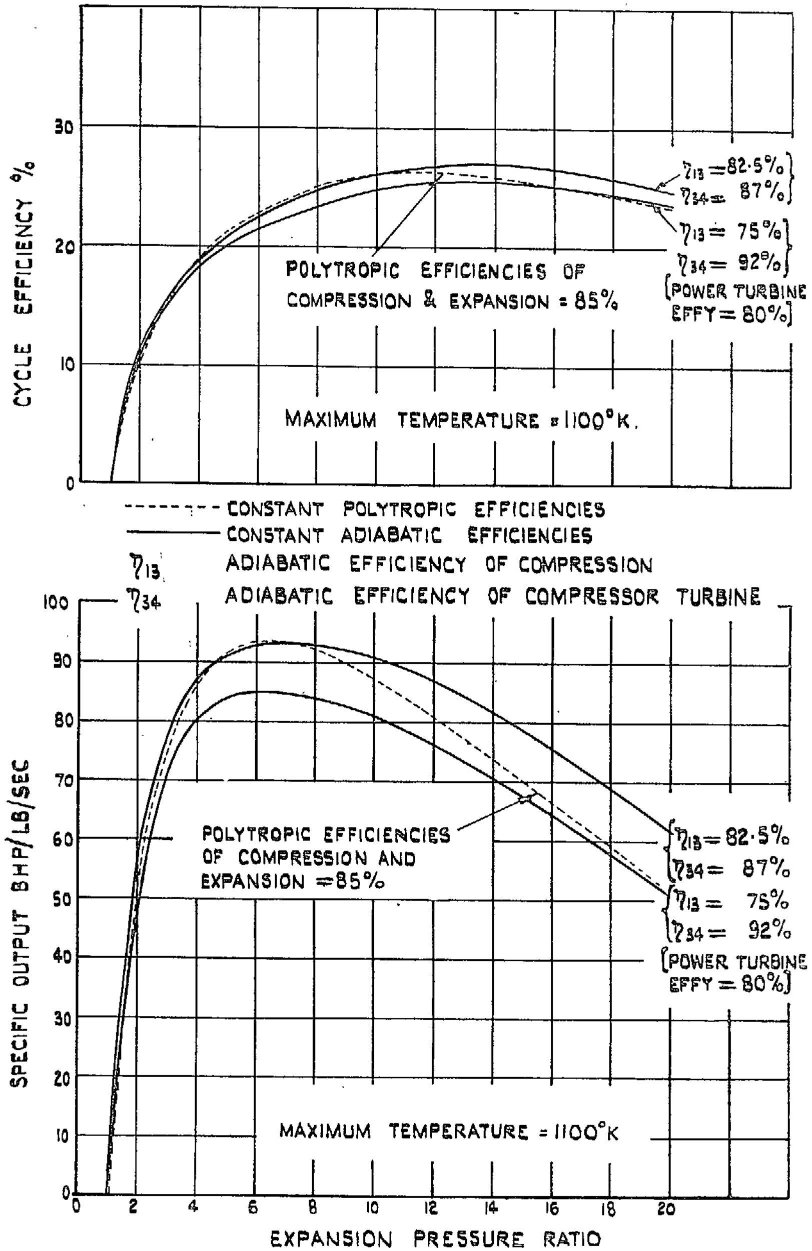


FIG. 8. Performance of the cycle with losses assuming polytropic efficiencies of components.

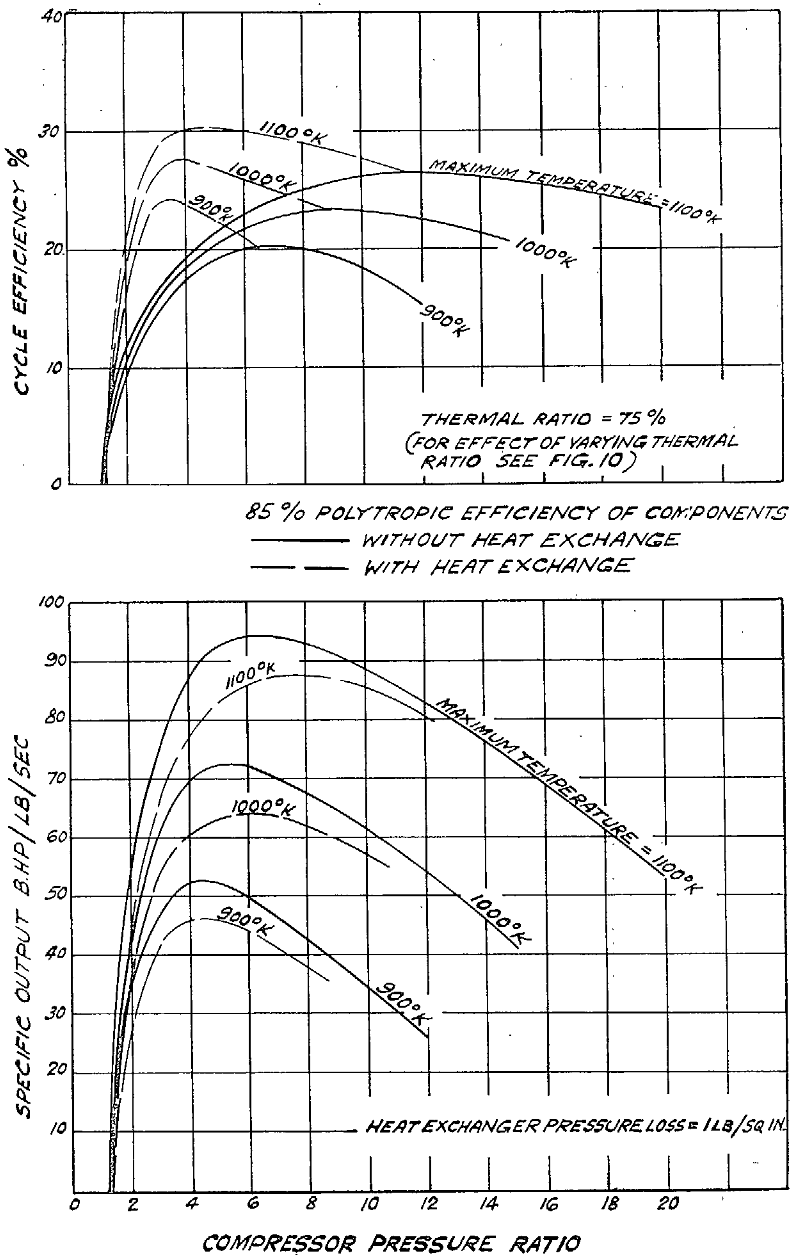


FIG. 9. Effect of heat exchange on the performance of a practical cycle.

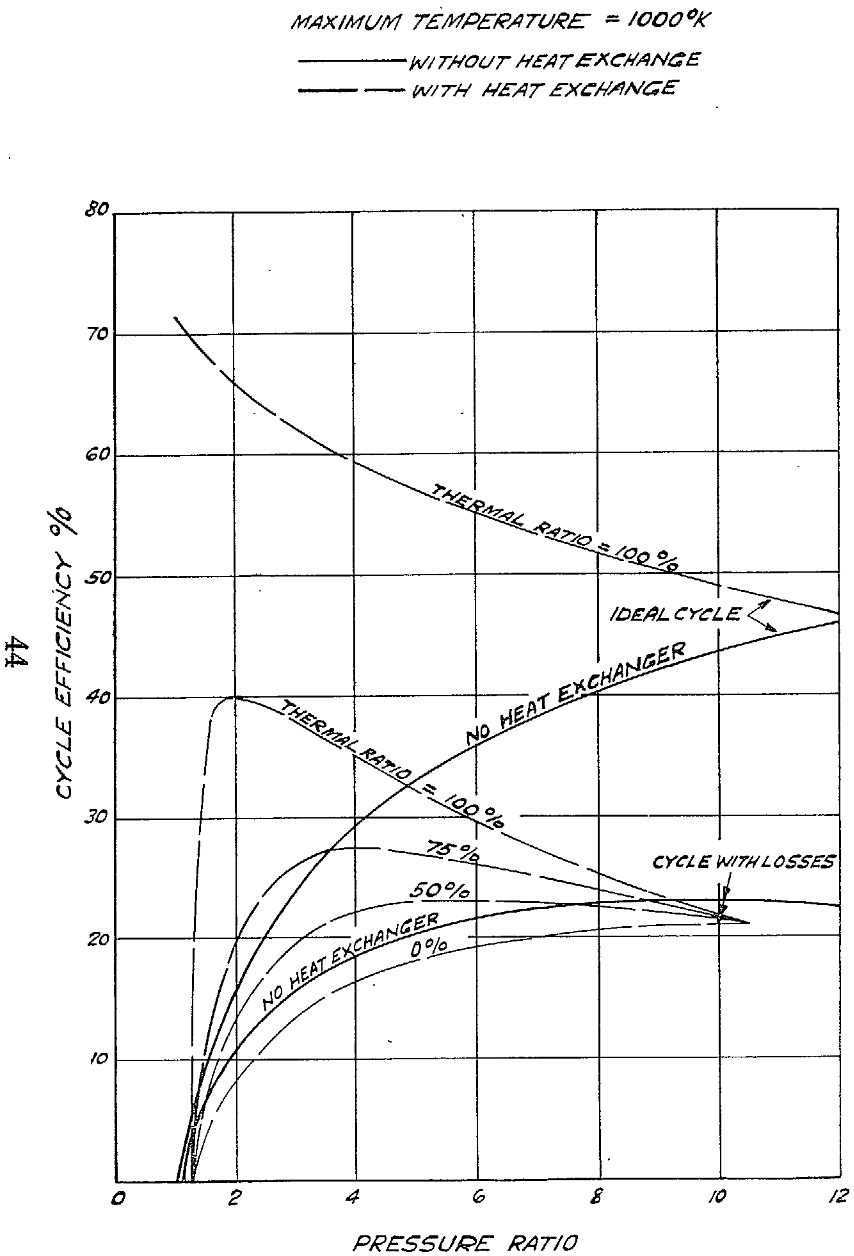


Fig. 10. Effect of heat exchanger thermal ratio on the performance of the ideal cycle and a practical cycle.

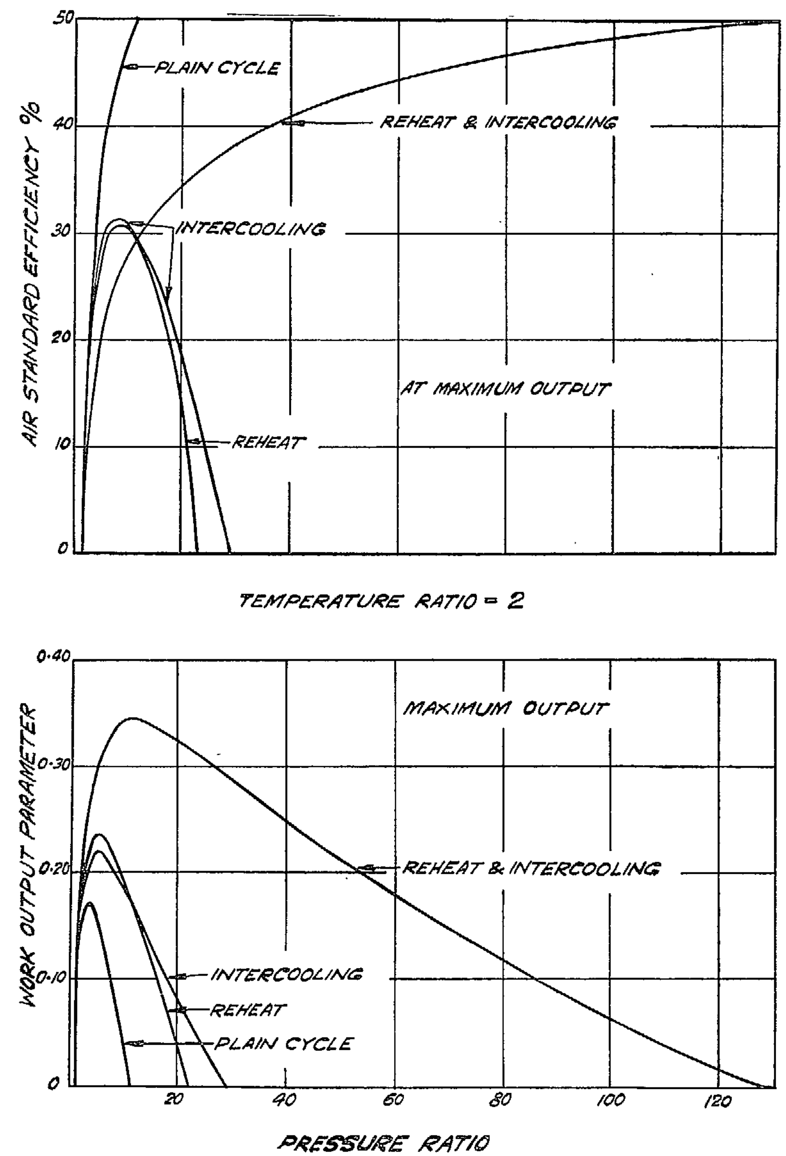


Fig. 11. The ideal cycle. Extension of portions of Fig. 2 to higher pressure ratios to show effect of reheat and intercooling. No heat exchange.

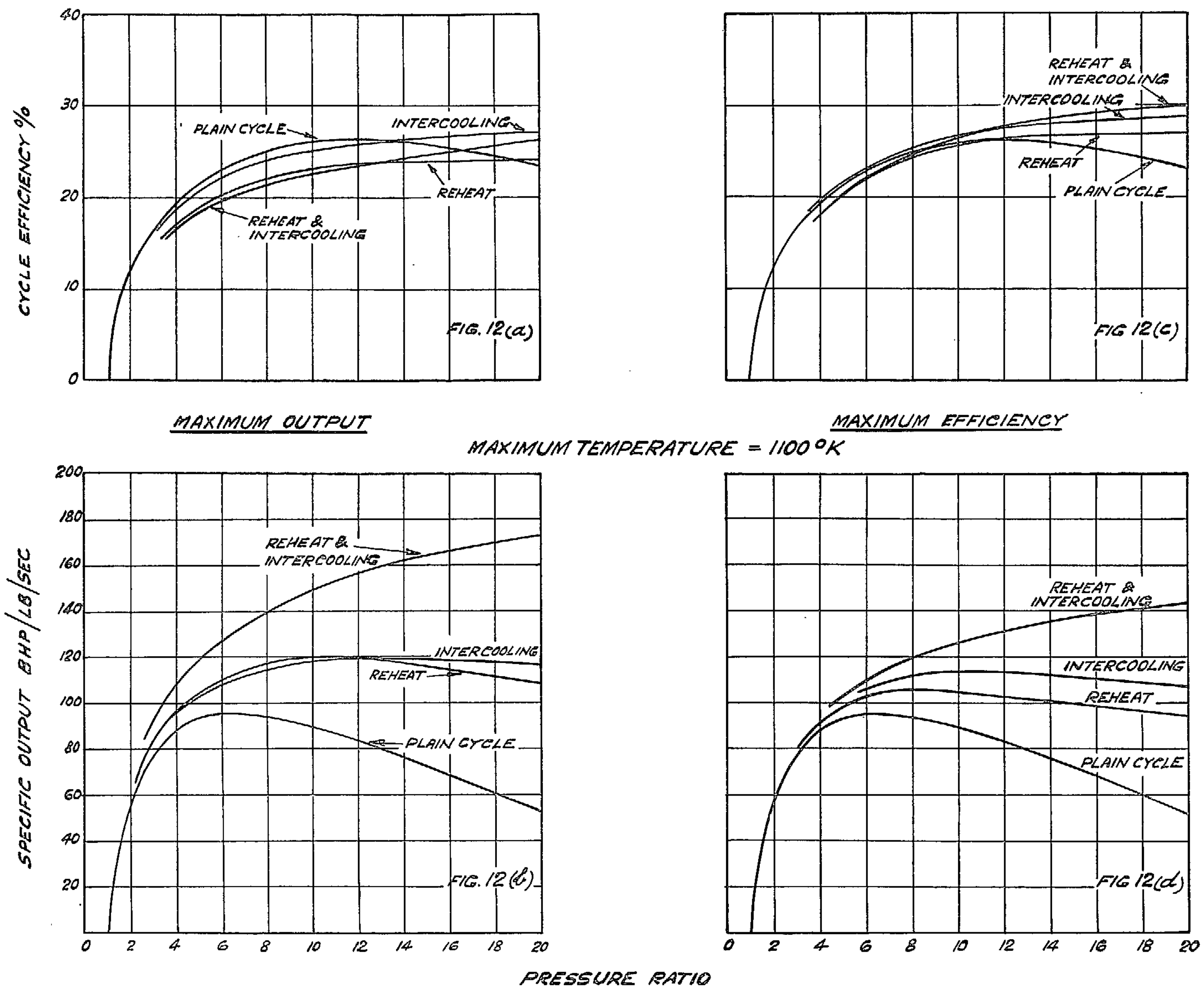


FIG. 12. The effect of reheat and intercooling on the performance of a practical cycle. Reheat and intercooling position chosen to give (1) maximum output, and (2) maximum efficiency. No heat exchange.

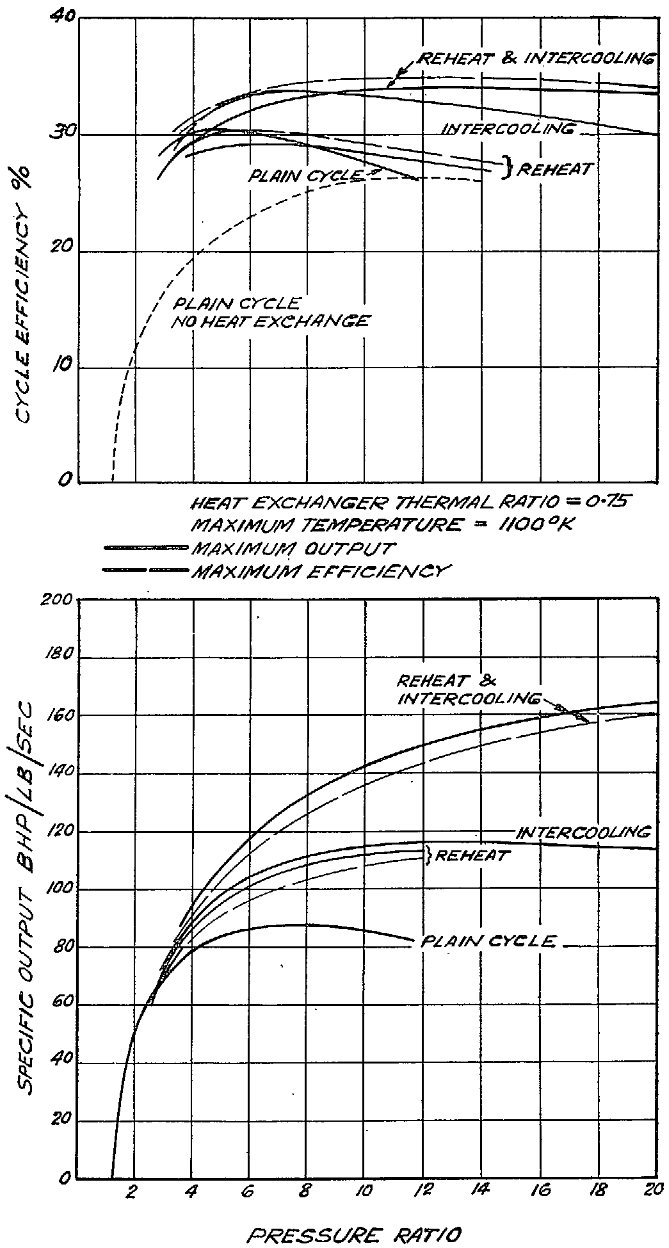


FIG. 13. Effect of heat exchange on the performance of the reheated and intercooled versions of a practical cycle shown in Fig. 12.

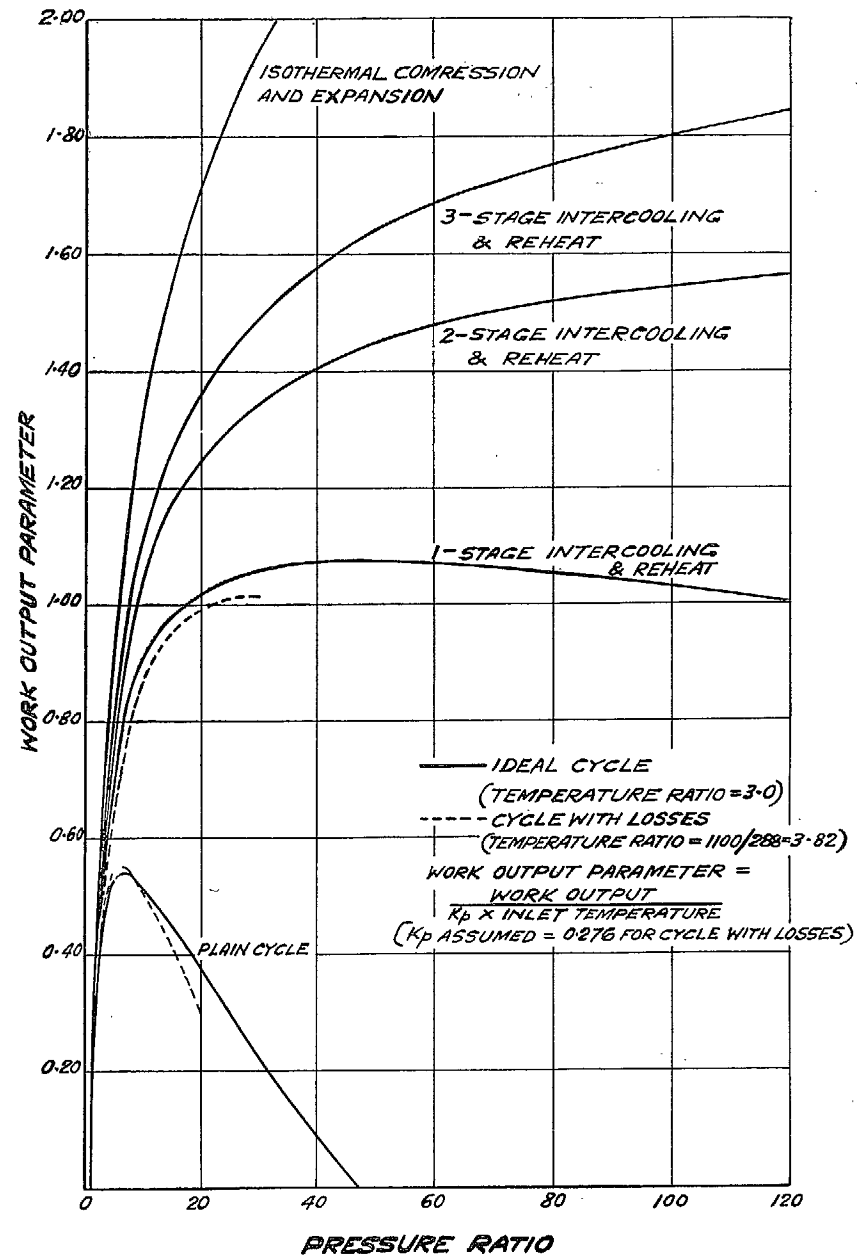


FIG. 14. The output of ideal cycles with reheat and intercooling and of the ideal isothermal cycle.

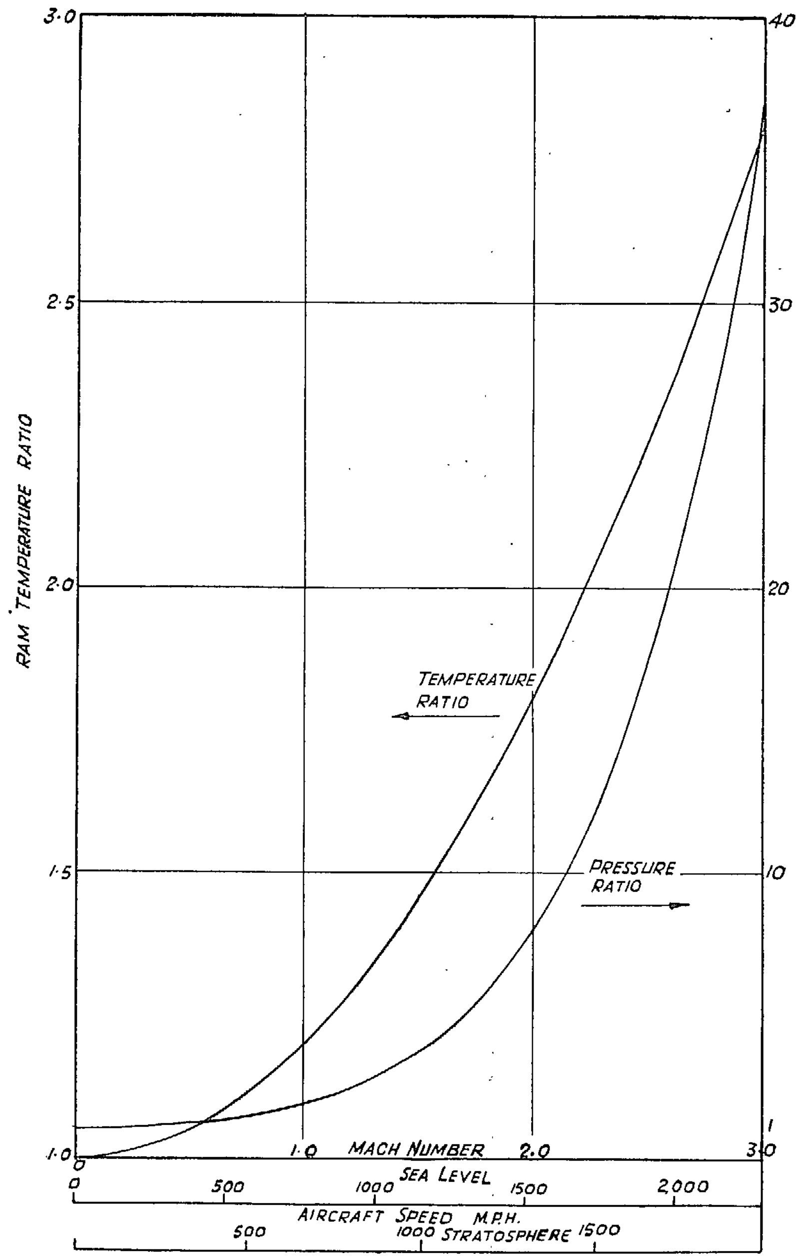


FIG. 15. Variation of ram temperature ratio and of the equivalent isentropic ram pressure ratio with forward velocity.

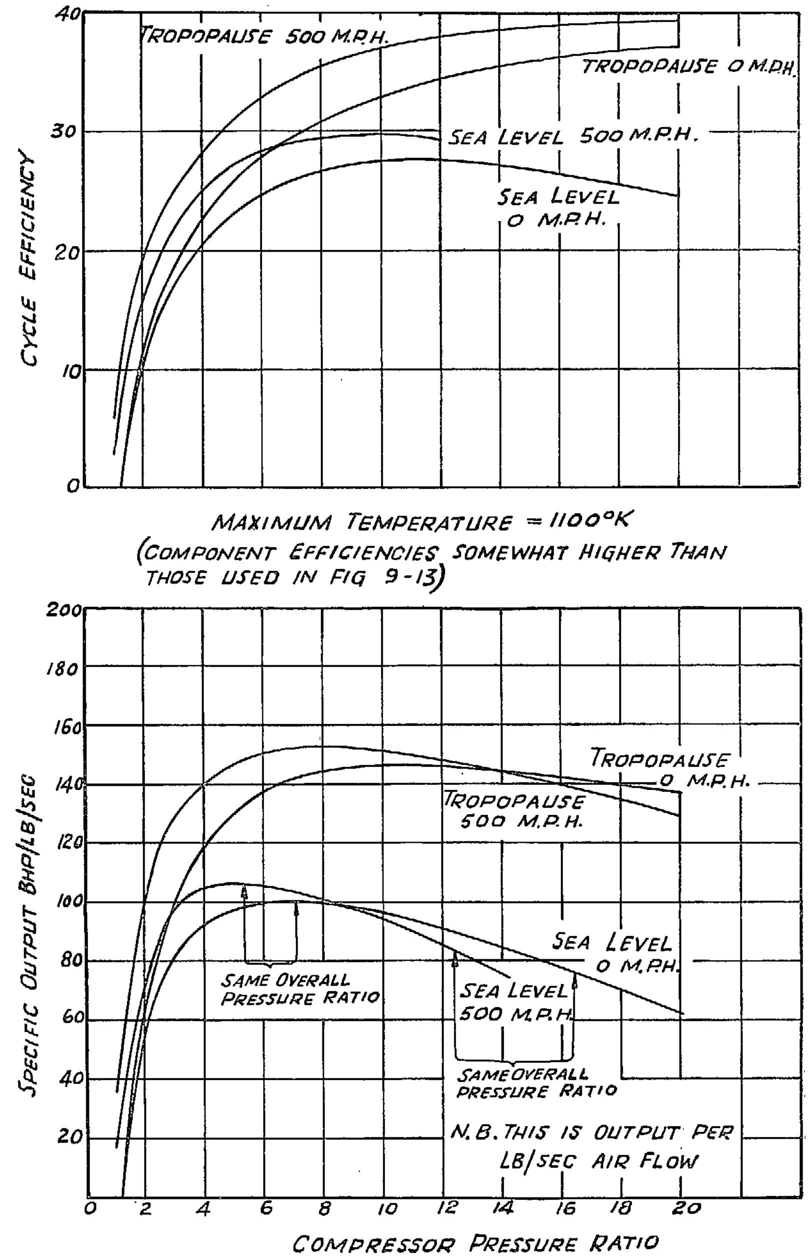


FIG. 16. Effect of forward speed and altitude on the performance of a practical cycle.

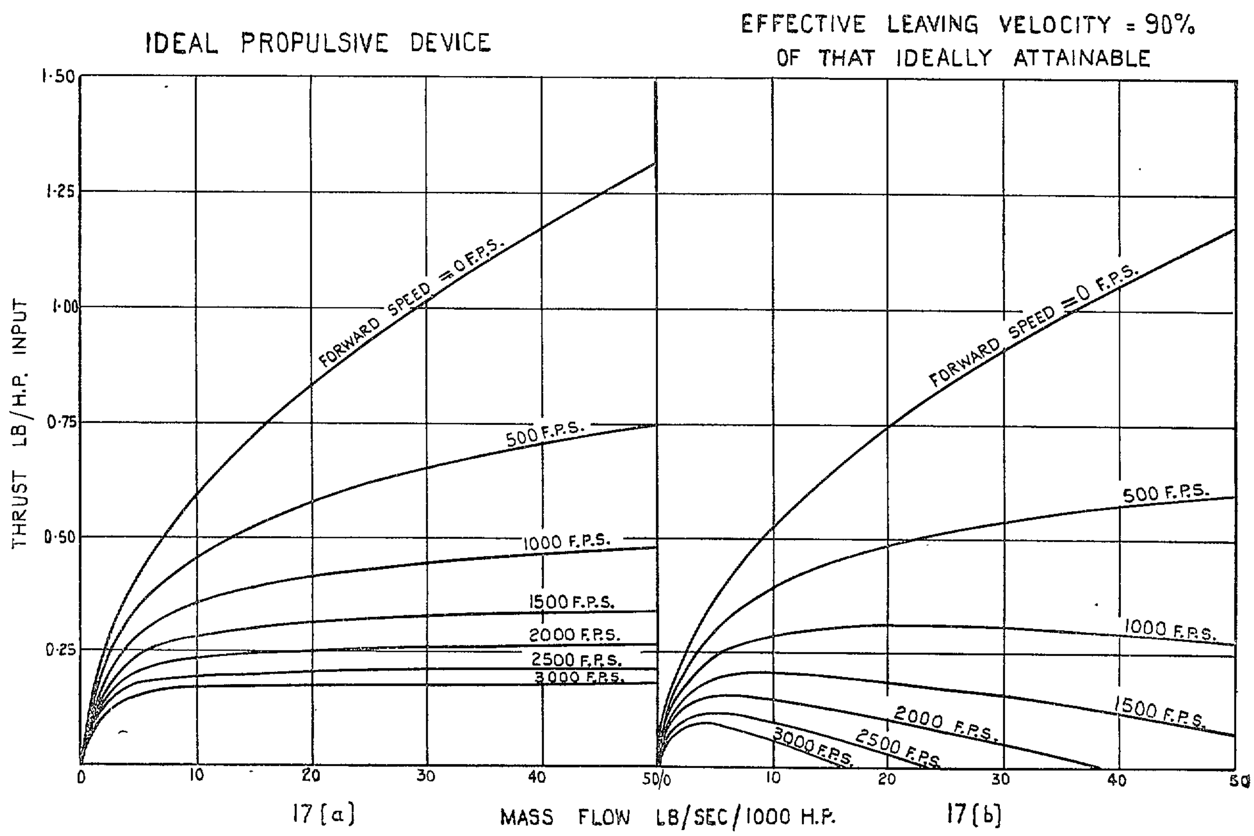


FIG. 17. Variation of thrust from a propulsive device absorbing constant power at all forward speeds but employing varying mass flow of working fluid.

(97023)

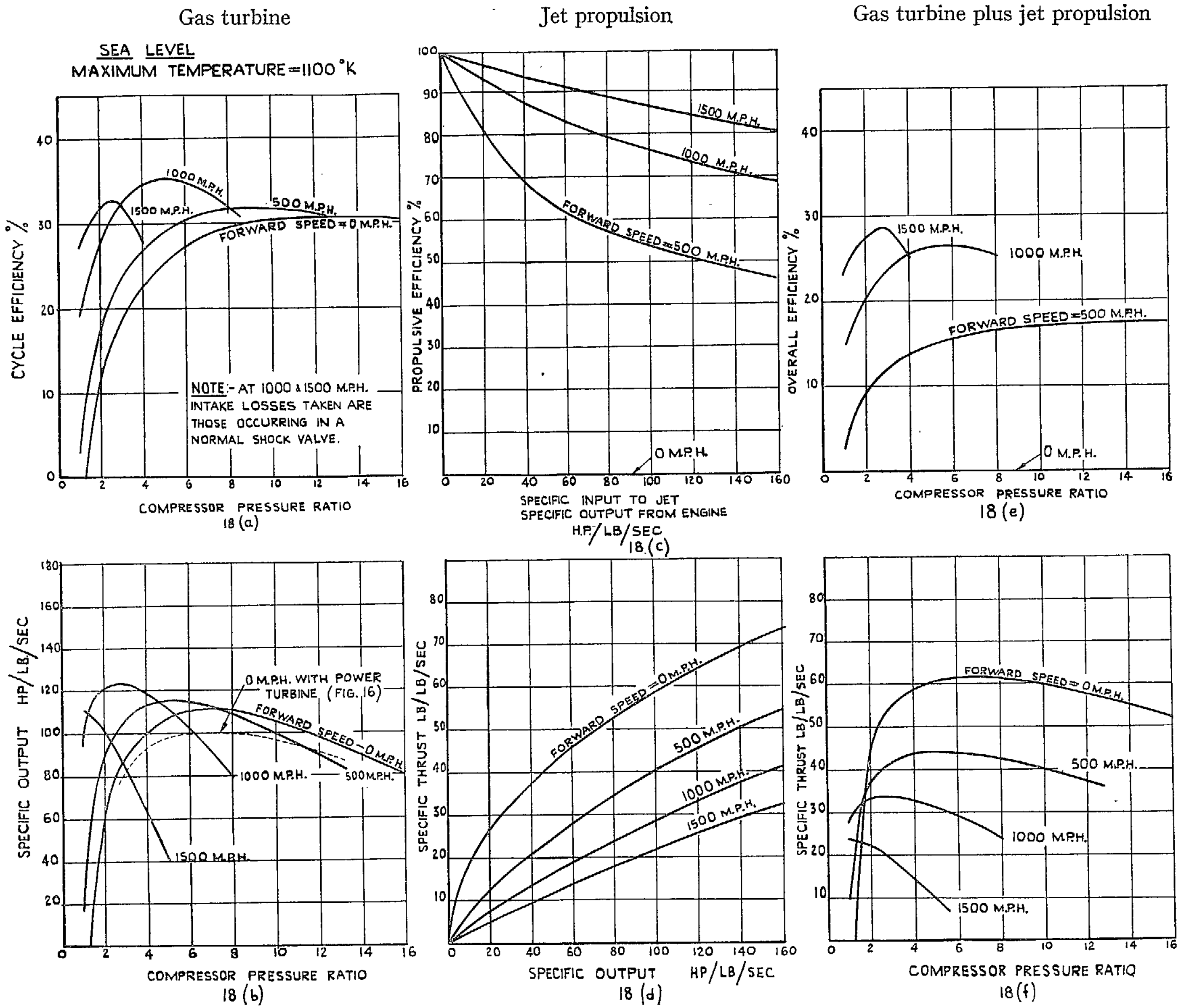


FIG. 18. The marriage of gas turbine with jet propulsion.

D

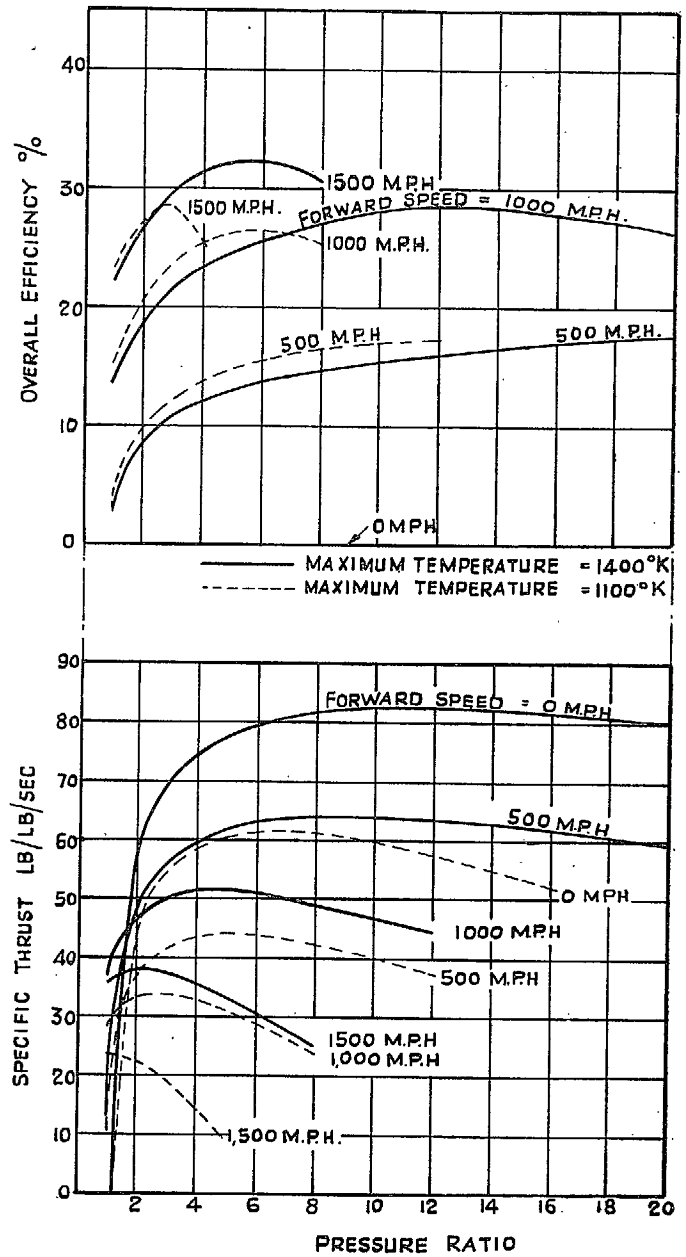


FIG. 19. Influence of maximum temperature on the performance of a practical turbine-jet cycle at sea-level.

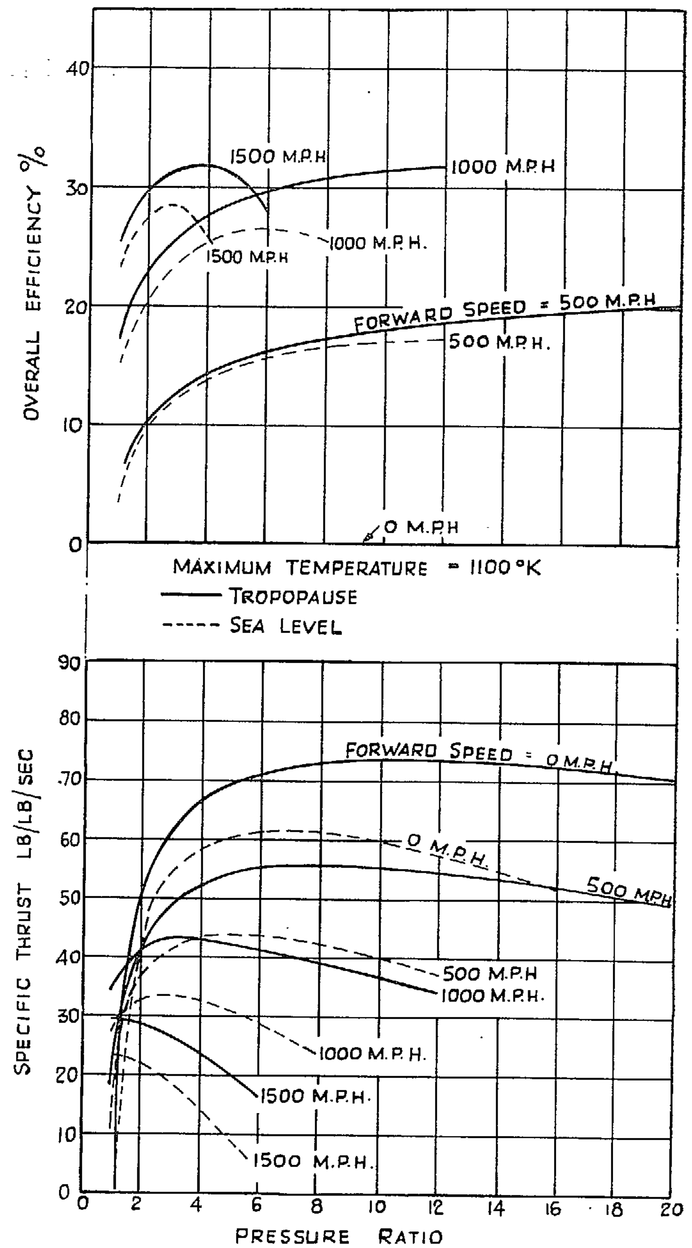


FIG. 20. Influence of altitude on the performance of a practical turbine-jet cycle at 1100 deg K maximum temperature.

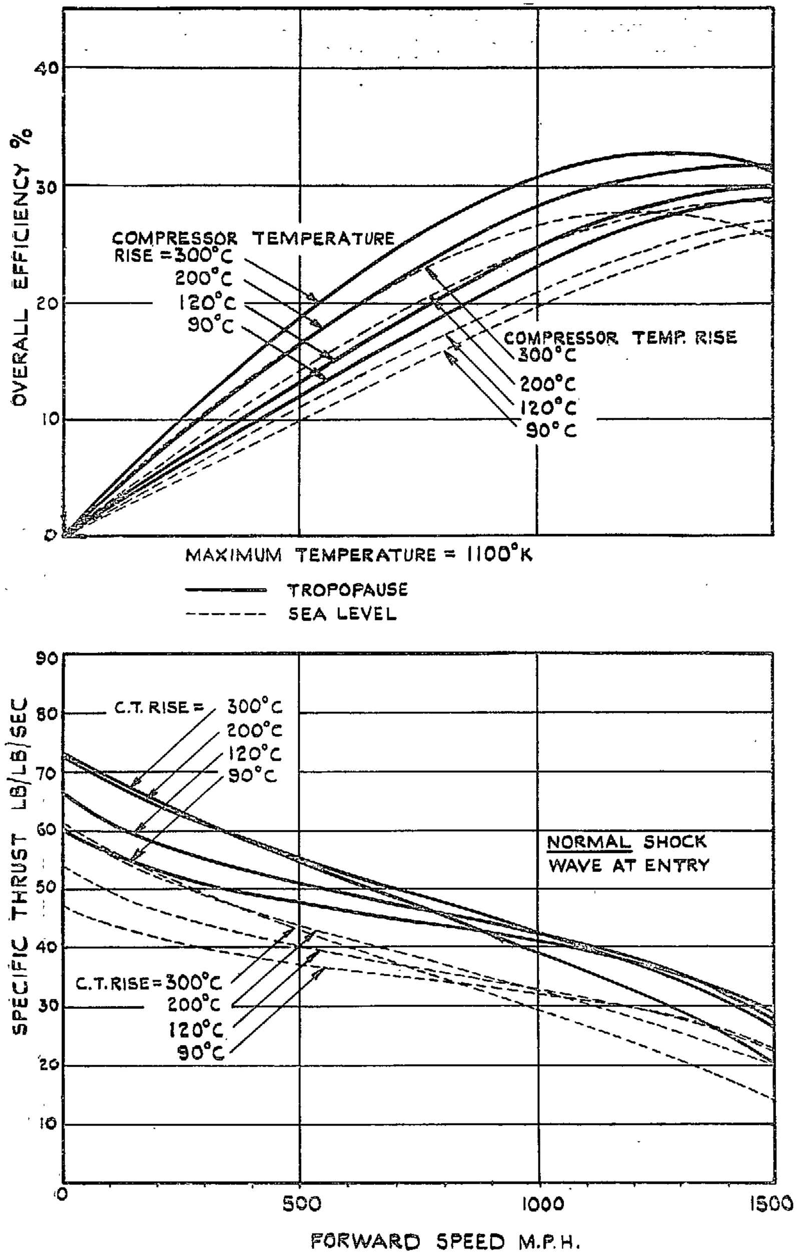


FIG. 21. Variation of specific thrust and overall efficiency of a practical turbine-jet cycle of constant maximum temperature at two altitudes and for four values of compressor work.

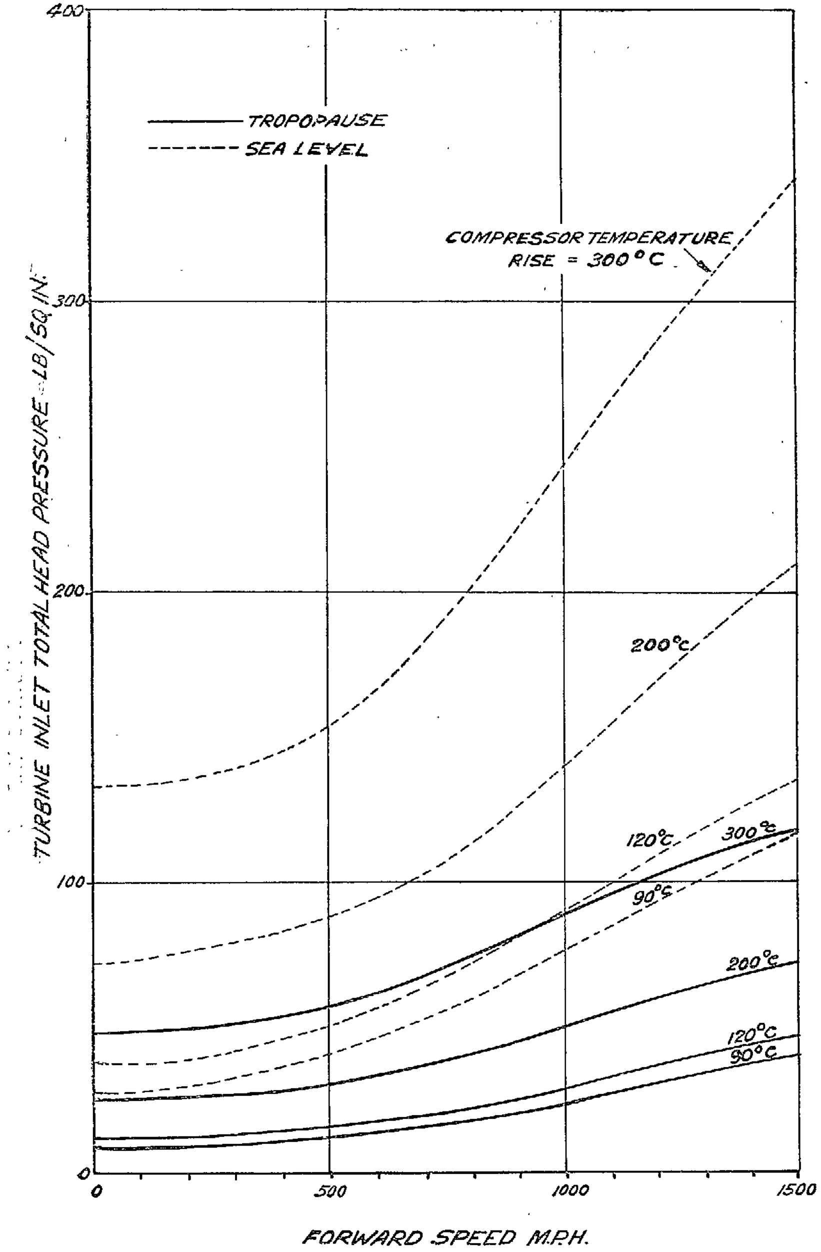


FIG. 22. Variation of turbine inlet total-head pressure with forward speed at two altitudes and for four values of compressor work, normal shock intakes when supersonic.

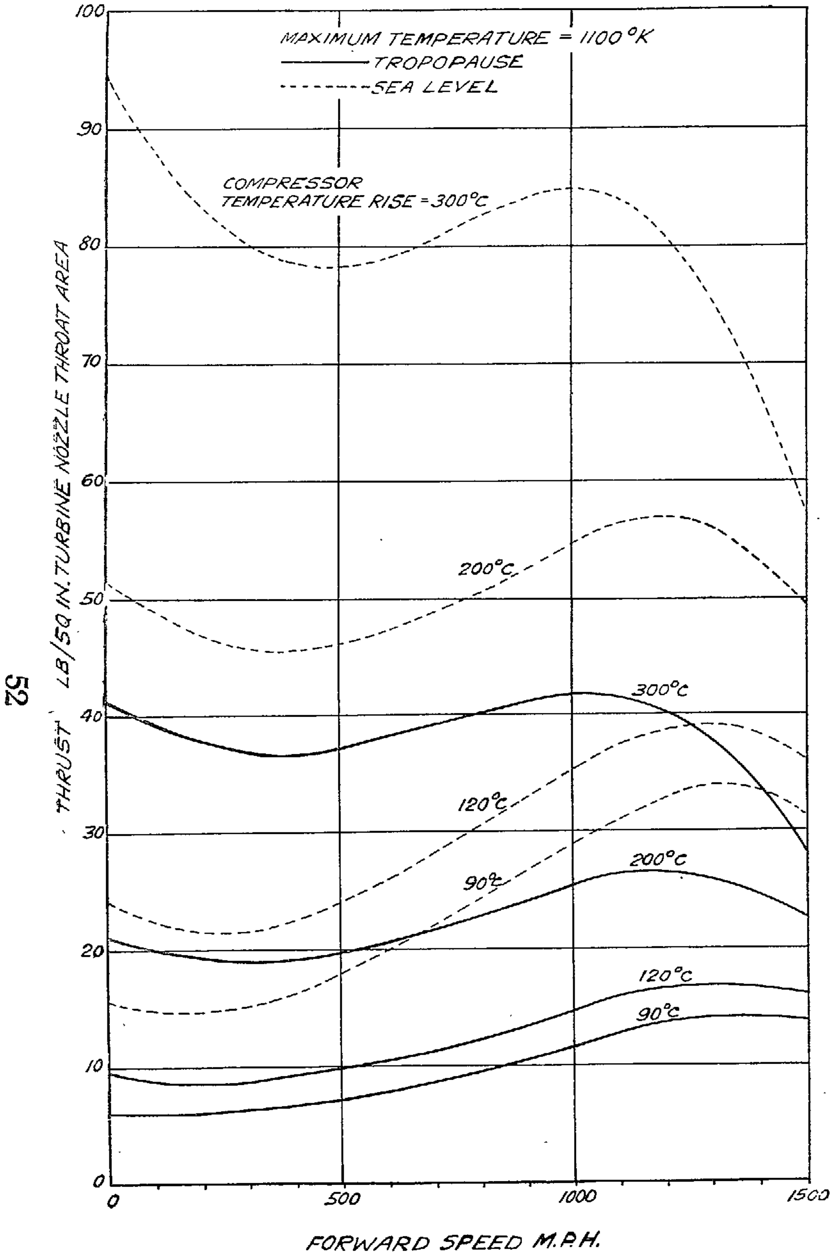


FIG. 23. Variation of thrust with forward speed for practical turbine-jet cycles with four different values of compressor work, compiled from Figs. 21 and 22 assuming choking in the turbine nozzles.

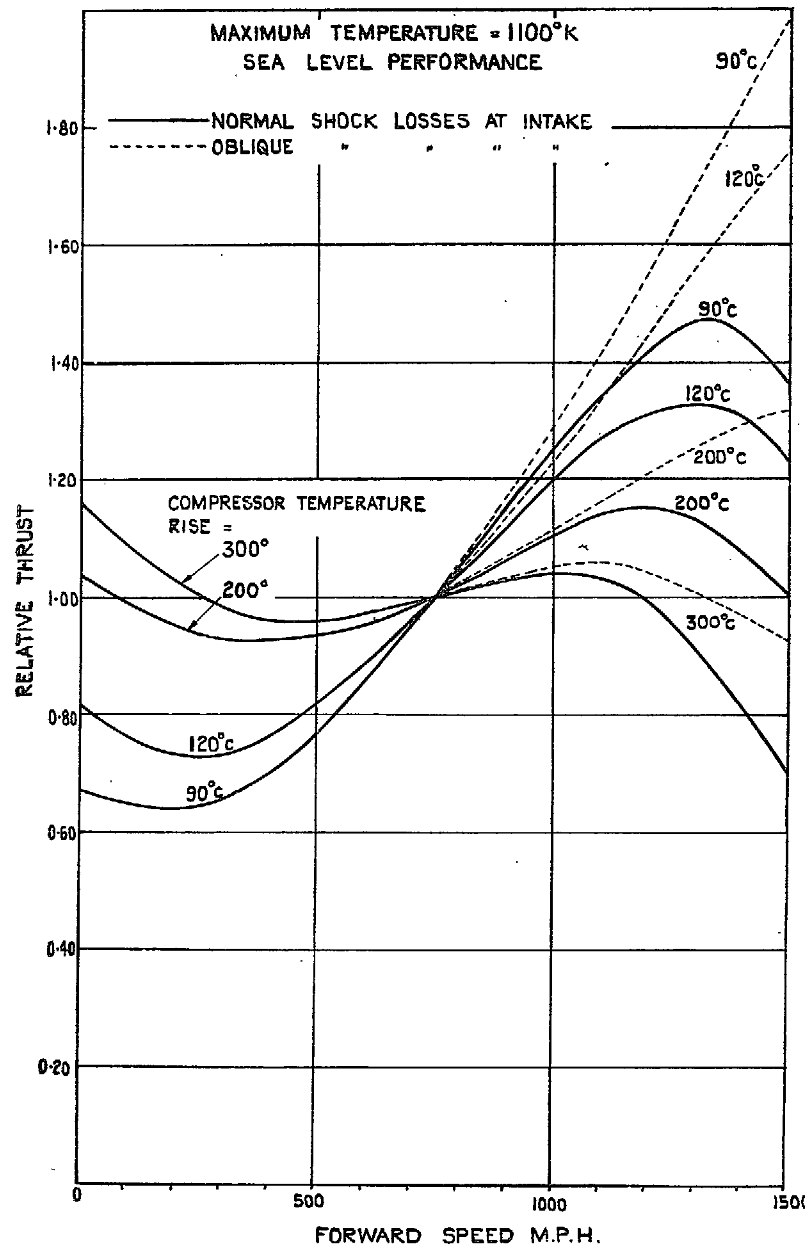


FIG. 24. Curves of Fig. 23 (sea-level) expressed relative to their respective values at 750 m.p.h.

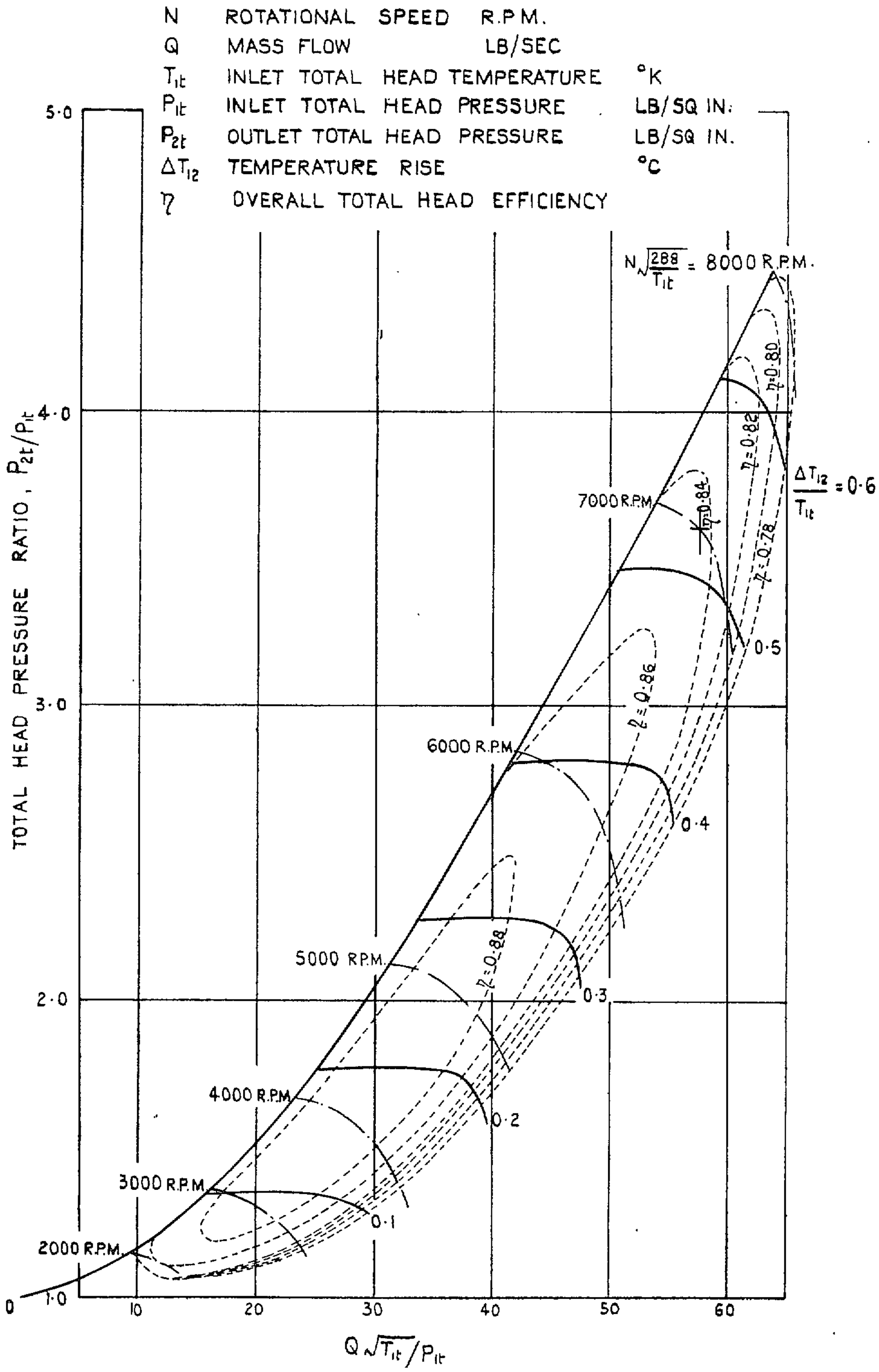


FIG. 25. Typical axial compressor characteristics used in example in the text.

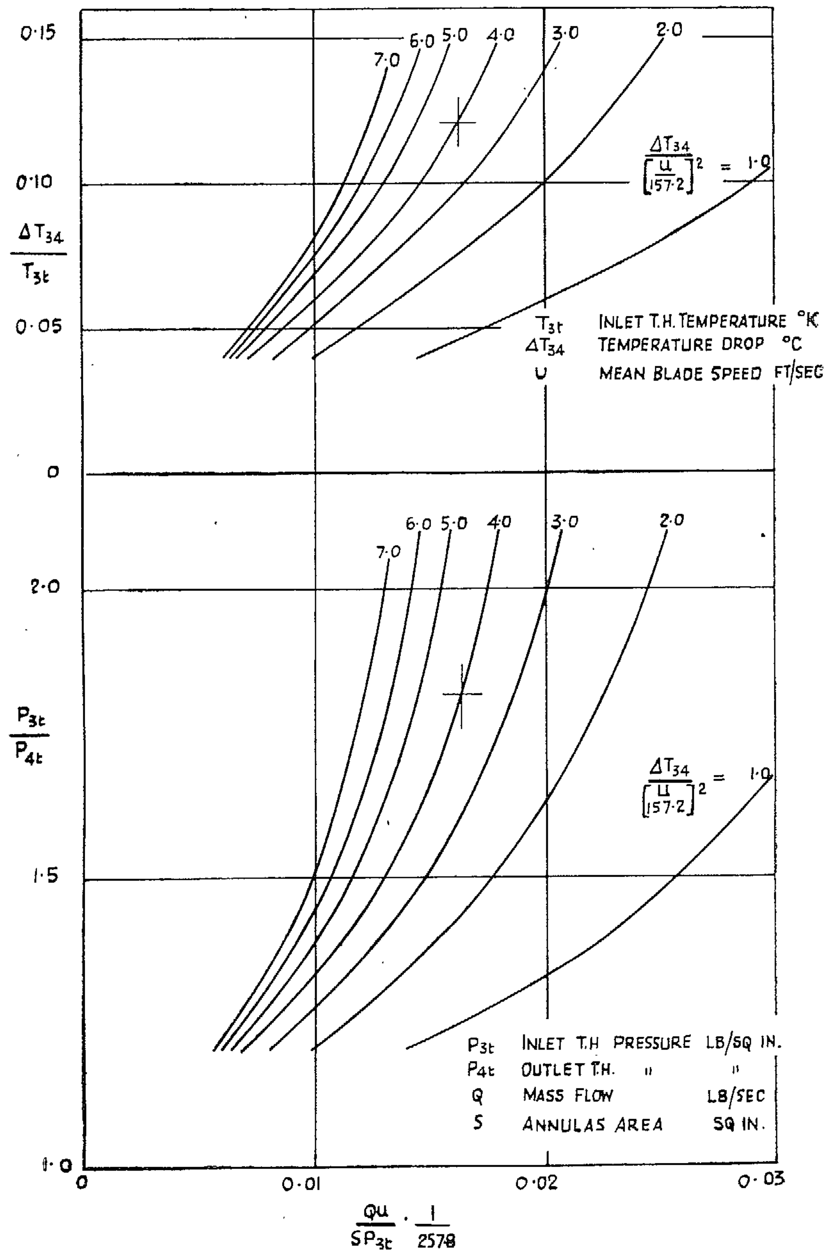


FIG. 26. Theoretical characteristics for a single-stage axial turbine of 25 deg nozzle outlet angle and 35 deg blade outlet angle.

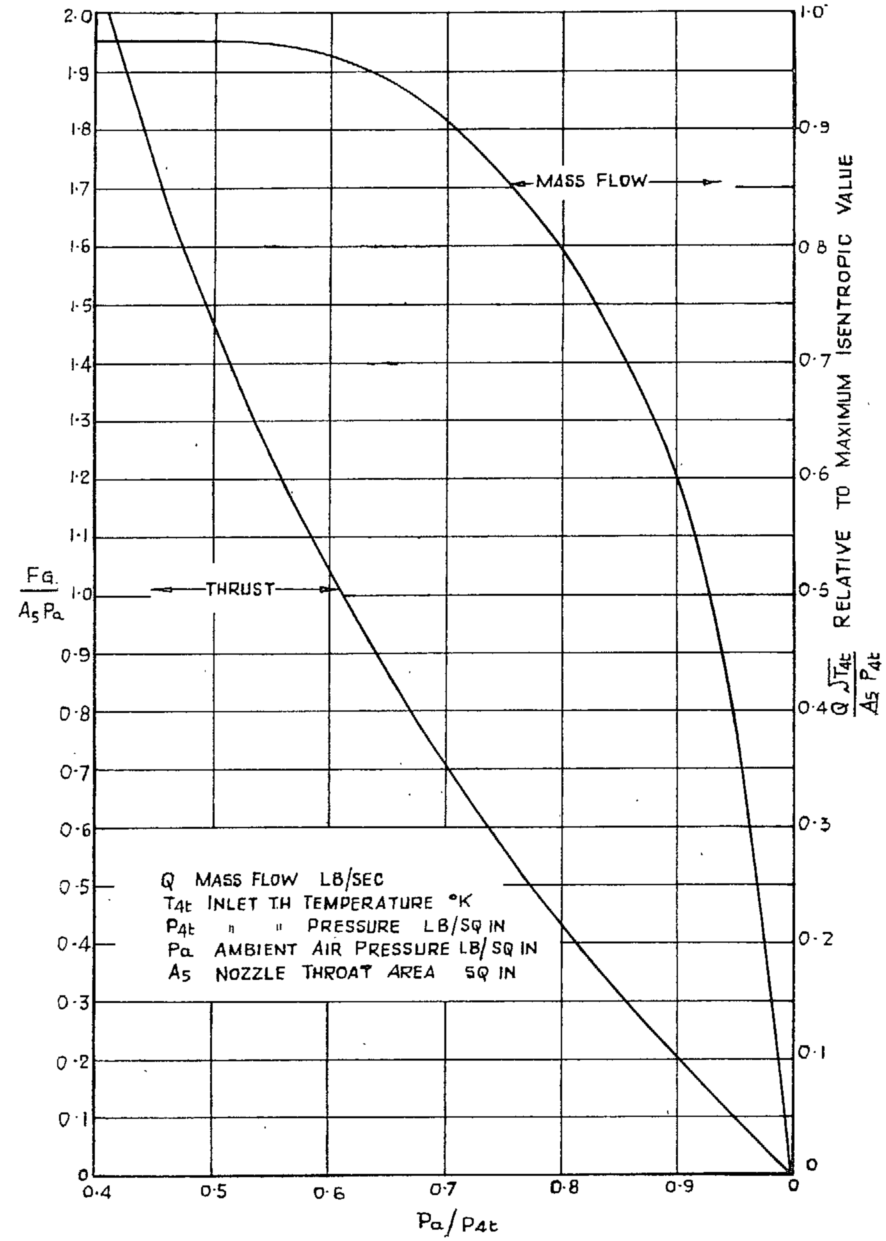


FIG. 27. Typical jet-pipe characteristics.

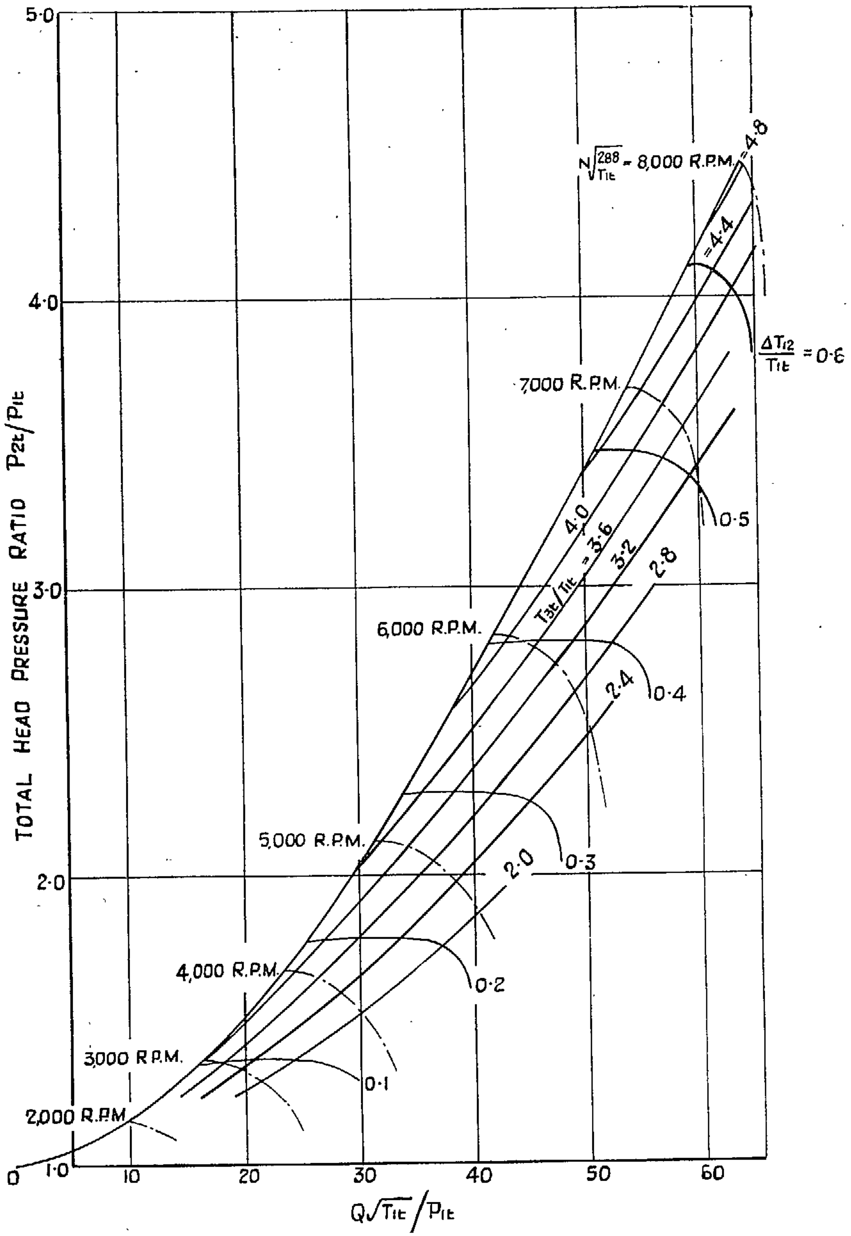


FIG. 28. Linkage of compressor and turbine characteristics (Figs. 25 and 26) by lines of constant inlet temperature ratio, T_{3t}/T_{1t}.

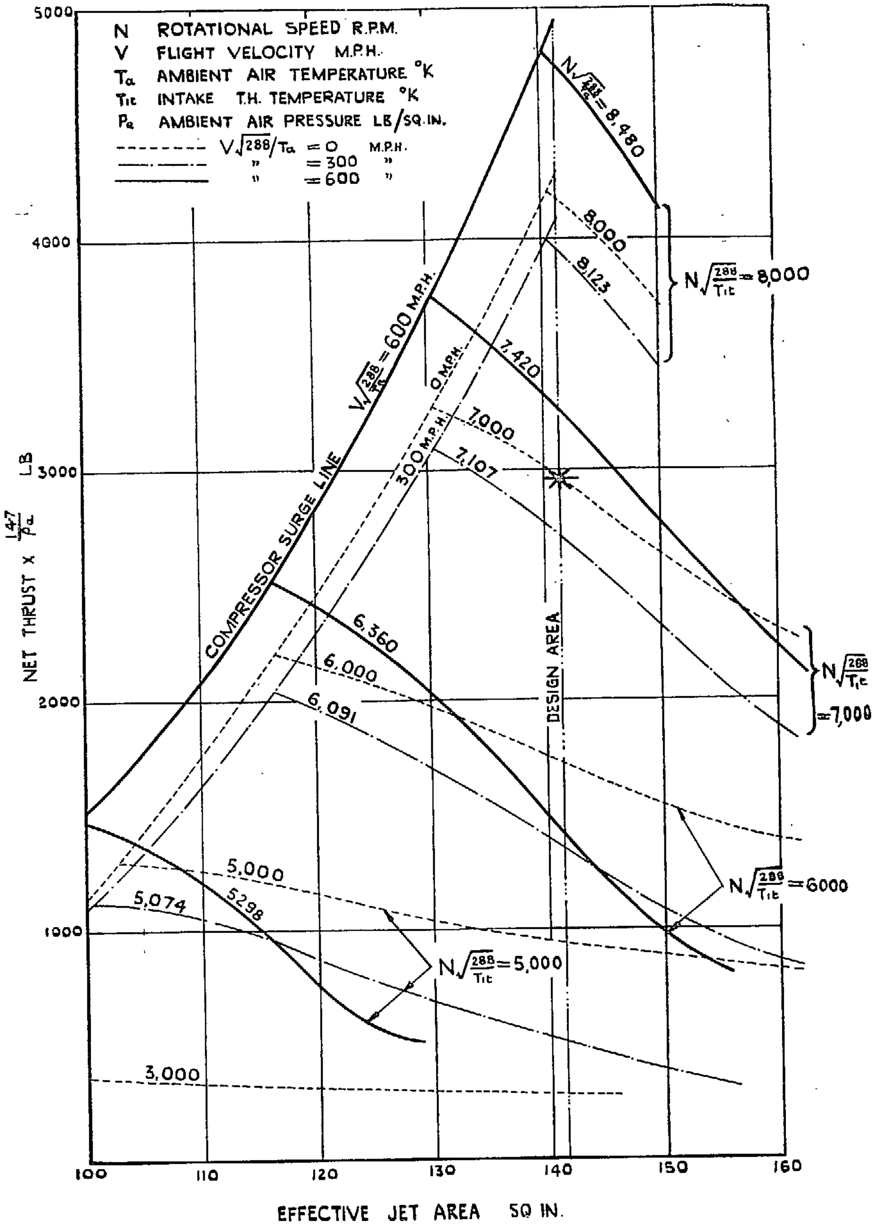


FIG. 29. Example of simple jet-turbine engine. Variation of net thrust with jet area for different rotational speeds and flight velocities.

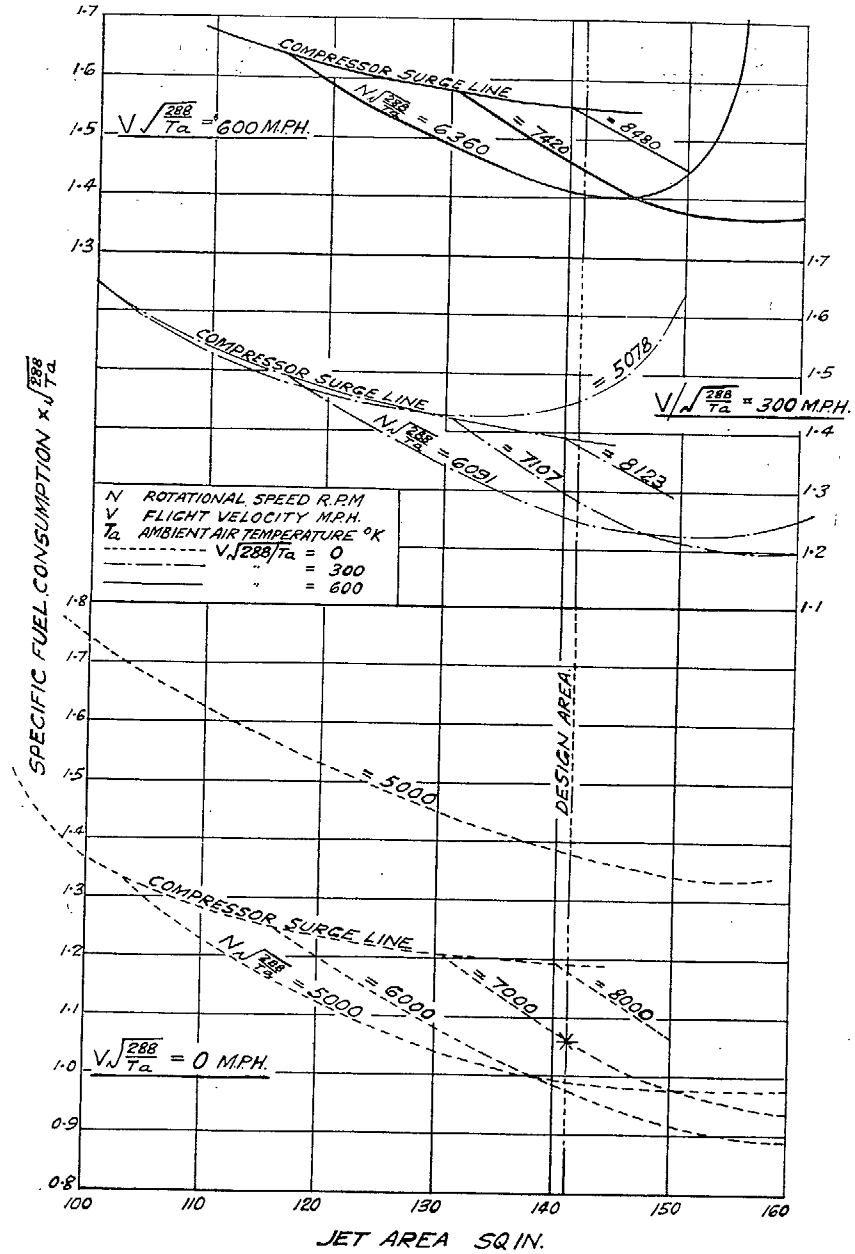


FIG. 30. Example of simple jet-turbine engine. Variation of specific fuel consumption with jet area for different rotational speeds and flight velocities. 100 per cent combustion efficiency assumed.

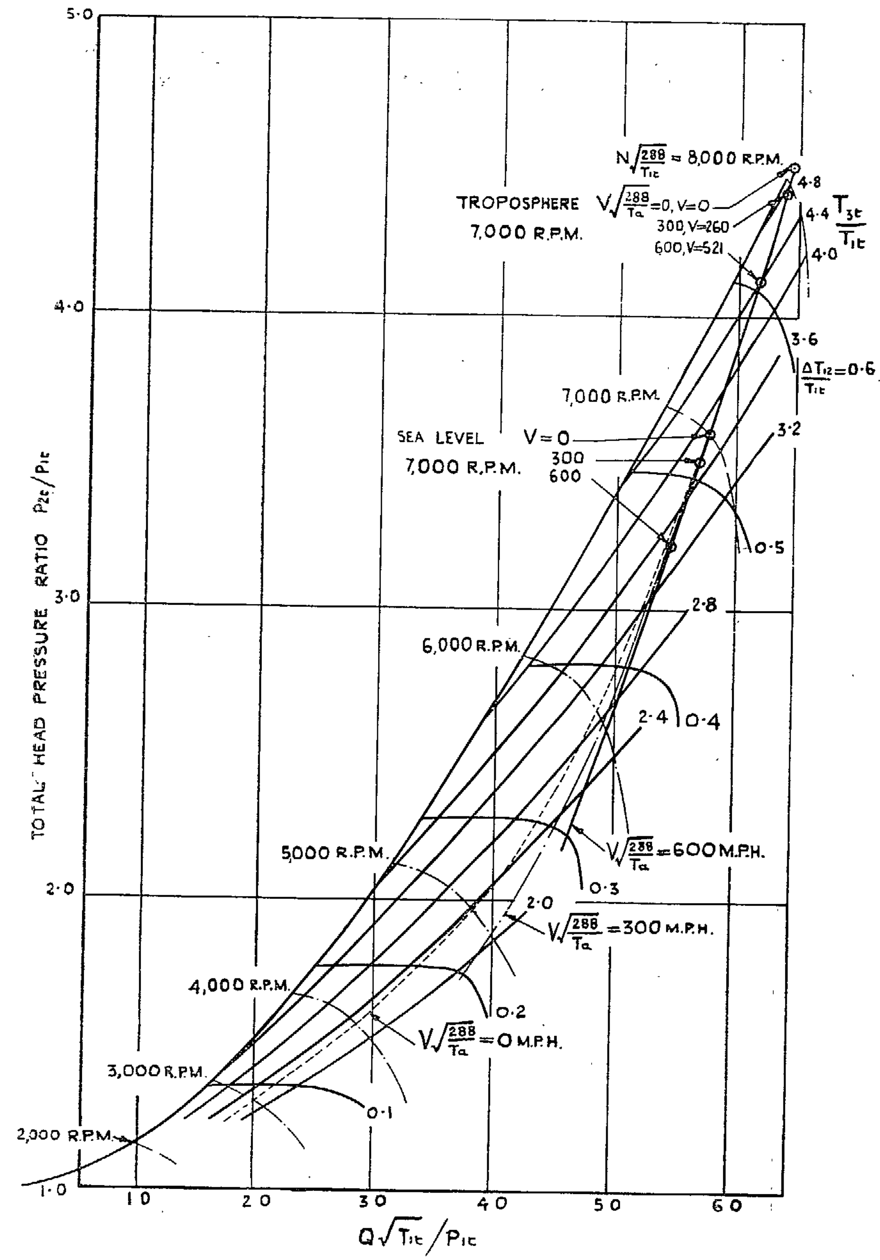


FIG. 31. Example of simple jet-turbine engine. Equilibrium running diagram for fixed jet area.

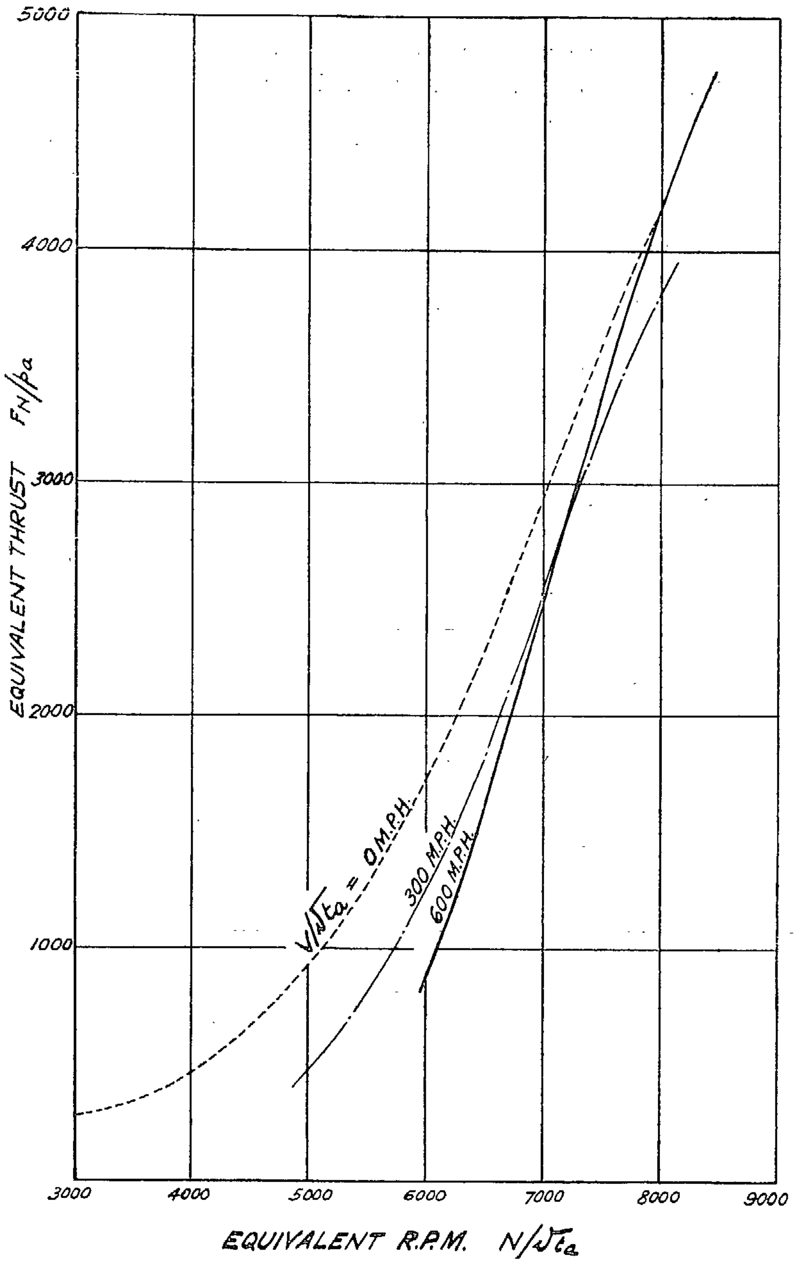


FIG. 32. Example of simple jet-turbine engine. Variation of thrust with rotational speed and flight velocity expressed as equivalent sea-level values. Fixed jet area.

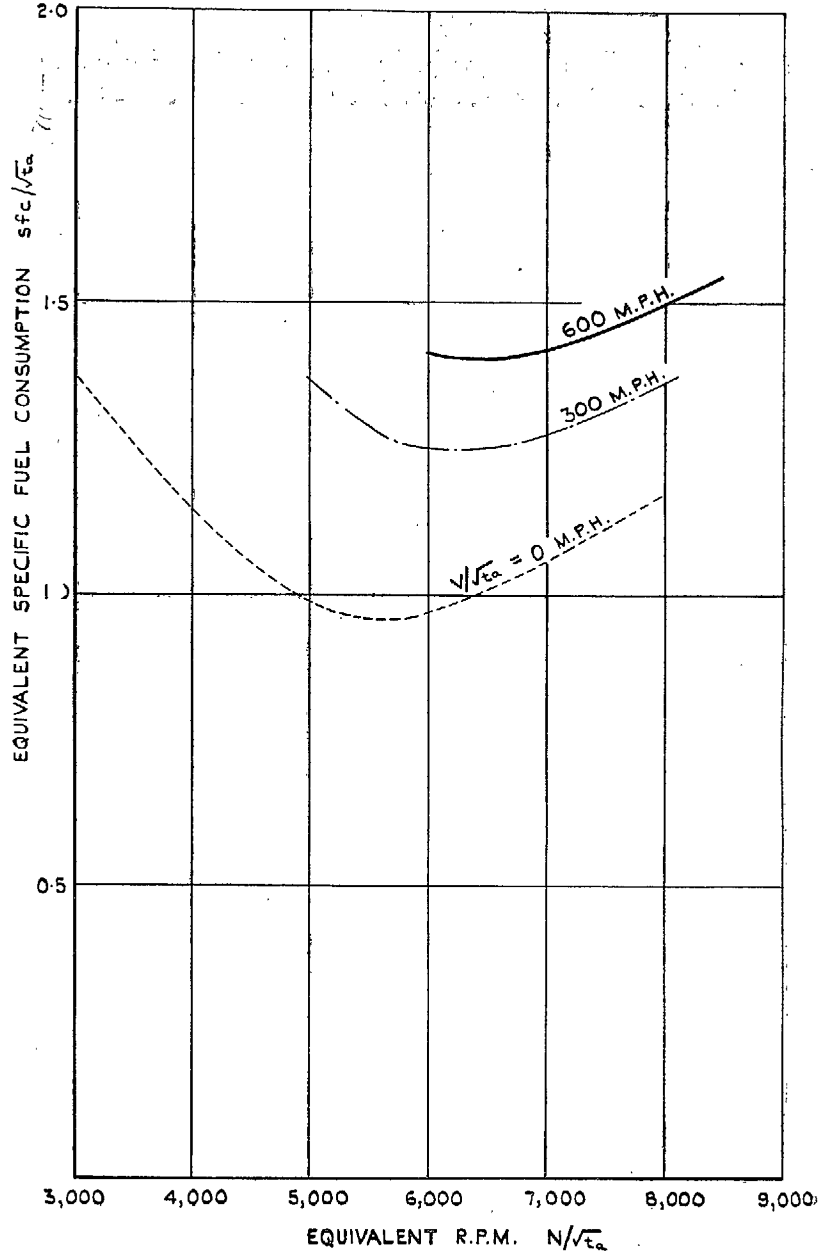


FIG. 33. Example of simple jet-turbine engine. Variation of specific fuel consumption with rotational speed and flight velocity expressed as equivalent sea-level values. Fixed jet area.

(97023) Wt. 14/655 K.5. 1/54 Hw.

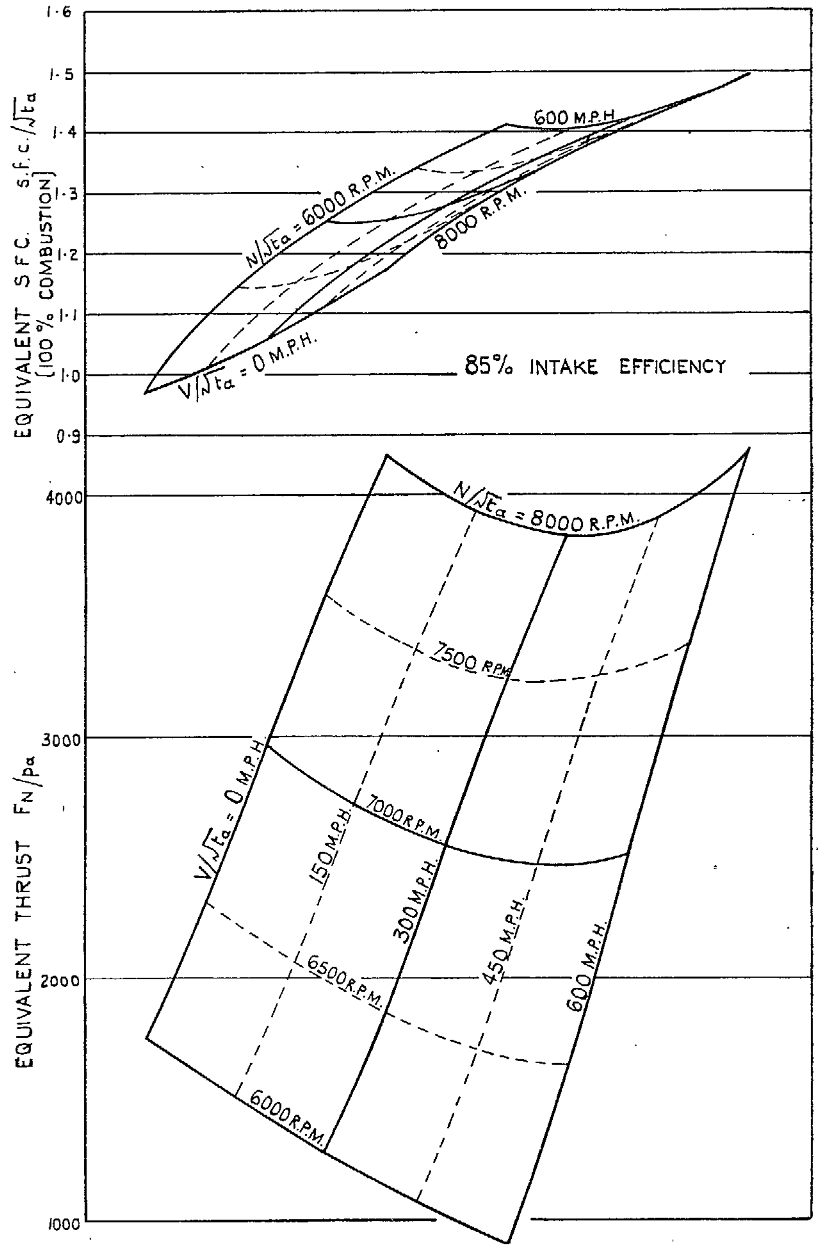


FIG. 34. Example of simple jet-turbine engine. 'Carpet diagrams' corresponding to Figs. 32 and 33.

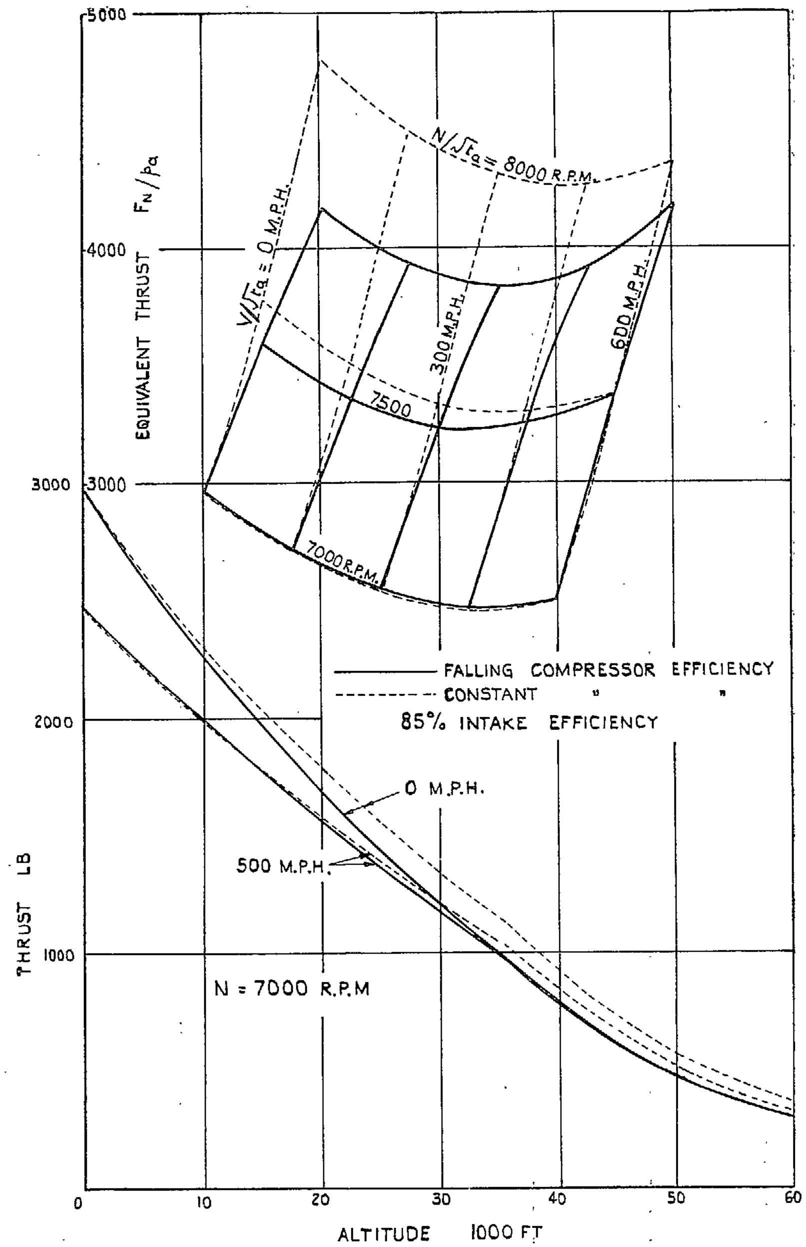


Fig. 35. Example of simple jet-turbine engine. Thrust for full rotational speed at 0 m.p.h. and 500 m.p.h. at altitudes from 0 to 60,000 ft compared with that corresponding to constant component efficiencies.

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