R. & M. No. 2677
(5981)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

Proposal for an Elevator Manœuvrability Criterion

By

S. B. GATES, M.A.

LONDON: HIS MAJESTY'S STATIONERY OFFICE
1951
FOUR SHILLINGS NET

Proposal for an Elevator Manœuvrability Criterion

By

S. B. Gates, M.A.

Communicated by the Principal Director of Scientific Research (Air)
Ministry of Supply

Reports and Memoranda No. 2677*

June, 1942

Summary.—The approximate theory of response to elevator developed by Bryant, Gandy and Gates yields a compact formula for a criterion of manœuvrability Q, the 'stick force per increment in g'; there is an analogous but less useful criterion in terms of stick travel. It is recommended that Q be adopted for designers' use, that its limits of validity be checked by careful tests on one aeroplane, and that more force measurements in pull out from dives be made on a number of aeroplanes in order that numerical standards may be attached to Q. Reference is made to American standards and to experimental work already done in this country.

The rate of growth of acceleration, which is not represented in the criterion, is discussed and illustrated by a numerical example. From this it appears that within limits which probably apply to a pilot's normal control movements :—

(1) The rate of application of force affects the time to reach maximum acceleration but not the value reached.

(2) The acceleration produced by a given stick force is independent of speed if the static margin is fixed, but the time to reach it is inversely proportional to the speed.

(3) The acceleration produced by a given stick force increases with altitude; this effect is the greater the less the static stability. The bearing of this on the difficult control of high altitude fighters near the ceiling is discussed.

The close connection between the problems of manœuvrability and safety is noticed throughout. The inertia weight is not ideal as a deterrent to the production of high acceleration, and more promising variants of this device are referred to.

1. *Introduction.*—The formulation of a criterion of manœuvrability in the vertical plane is at first sight an intricate problem, which can be stated as follows. In departing from straight flight the pilot's typical action, whether it is expressed in terms of control movement or force, is a more or less linearly-graded change to a constant new value. The effect of this typical action is twofold if the aeroplane is stable :—

(1) a quick rise of incidence and angular velocity to maxima, during which the speed remains sensibly constant,

followed by

(2) a slow oscillation in angular velocity, speed and incidence, the angular velocity damping to zero and the speed and incidence to new values for which the aircraft is again in equilibrium.

---

* R.A.E. Report Aero. 1740—received 20th July, 1942.

In terms of stability analysis, the initial motion (1), which is all over in a very few seconds after the action is taken, is the damped quick period oscillation, and the subsequent motion (2) is the phugoid. The initial motion (1) has been analysed variously by Bryant and Gandy[1], by Howard and Owen[2, 3] and by Gates[4]. The discussion which follows is a development of the work of R. & M. 2275 and Ref. 4. The problem of a manœuvrability criterion is to relate the pilot's action to the most significant element of motion in its effect, and to express this relation in a formula which includes the essential aerodynamics of the matter. Evidently we are concerned only with what happens immediately after the action is taken, and our choice of the element of response should be such that it is naturally related not only to the change of path but to the aeroplane's structural strength, for in practice any consideration of what the pilot can make the aeroplane do leads immediately to a question of what the aeroplane can stand. It is suggested, therefore, that the significant element in the effect is the normal acceleration, and that the most rational criterion is that which expresses the maximum normal acceleration in the initial motion (1) following a given pilot's action. This restriction to the constant speed regime is a vital step in the analysis, which can be further simplified by remarking that in the initial motion the changes in normal acceleration arising through incidence change are always large compared with the change in gravity component; and hence, to a fair approximation, gravity can be neglected. This approximation to the initial motion leads, therefore, to a fictitious steady circle, maintained by a normal acceleration which is sensibly equal to the maximum acceleration actually attained. But the 'steady' value is clearly independent of the time of the pilot's action. Hence to this approximation the maximum acceleration depends only on the magnitude but not on the time of the pilot's action, which affects only the time in which the maximum acceleration is reached. This line of argument, therefore, yields a manœuvrability criterion in which both elements of the relation—the action and its effect—are simplified to manageable proportions without losing their essential validity.

2. *Summary of Analysis.*—Details of the response analysis on these assumptions are given in Appendix I, where it is shown that if in steady flight the pilot's pull is increased instantaneously by P lb and held at that value, and if in consequence the normal acceleration rises from $n_0 g$ to $(n_0 + n_{max})\, g$ then Q the criterion of stick force per g is given by

$$Q = \frac{P}{n_{max}} = -\frac{mb_2}{a_2} w \frac{c_\eta S_\eta}{V}\left(H'_n + \frac{a'_1}{2\mu}\bar{V}\right) \qquad \text{(1)}$$

where, in addition to the customary symbols, which are listed later,

H_n' = static margin stick free $(h_n' - h)$

$$a_1' = a_1\left(1 - \frac{a_2 b_1}{a_1 b_2}\right)$$

m elevator gearing (radians/ft)

This formula includes the effect of a weight moment in the elevator circuit.

Similarly if in steady flight the stick is moved back instantaneously a distance x ft and held there, we have for Q_1, the criterion of 'stick travel per g',

$$Q_1 = \frac{x}{n_{max}} = \frac{w}{ma_2\bar{V} \cdot \frac{1}{2}\rho V^2}\left(H_n + \frac{a_1\bar{V}}{2\mu}\right) \qquad \text{(2)}$$

where H_n is the static margin stick fixed

Alternatively in this case if $(\Delta C_L)_{max}$ is the maximum increment in C_L, we have for Q_2, the criterion of 'stick travel per unit C_L', the formula

$$Q_2 = \frac{x}{(\Delta C_L)_{max}} = \frac{1}{ma_2\bar{V}}\left(H_n + \frac{a_1\bar{V}}{2\mu}\right) \qquad \text{(3)}$$

The angular velocity q_{max} and the radius of curvature R_{min} associated with n_{max} are given by

$$q_{max} = \frac{n_{max}g}{V} \quad \ldots \quad (4)$$

$$R_{min} = \frac{V^2}{n_{max}g} \quad \ldots \quad (5)$$

These formulæ are applicable when the pilot's action is not instantaneous, provided that he is not so slow that radical changes in speed occur during the growth of the acceleration.

3. *Choice of a Manœuvrability Criterion.*—The formulæ (1) to (3) give a choice of manœuvrability criteria in compact and simple forms expressed entirely in terms which the designer is accustomed to use in assessing the tail characteristics and the longitudinal stability of his design. The relative importance of stick force (criterion Q) and stick travel (Q_1 or Q_2) in the assessment of manœuvrability is to some extent a matter of opinion. There is no doubt that stick force per g earns its place in any scheme because the pilot's strength is fundamental to the problems both of manœuvrability and safety. Advocates of the importance of stick travel gradient[5] argue that accuracy of manœuvre depends more on this than on stick force gradient, but opinion is divergent as to whether a large or a small gradient should be aimed at. It seems, therefore, that if we are to rely on a single criterion of manœuvrability, the stick force per g criterion Q should be chosen, and it is suggested that this be adopted, leaving Q_1 or Q_2 for consideration if necessary in any particular case.

4. *Discussion of the Criterion Q.*—It may be useful to summarise as follows the main characteristics of the criterion Q, as indicated by the formula (1):—

(a) Manœuvrability increases as Q decreases, but safety increases as Q increases. In fighter design it is essential, and in bomber design it is often necessary, to consider both an upper and a lower limit of Q.

(b) If the stability and the tail characteristics are independent of incidence, Q is independent of speed, and of the inclination of path from which the manœuvre starts. Independence of speed is particularly important, for it runs counter to the common idea that as speed increases the elevator must be lightened (b_2 decreased) to preserve the manœuvrability. On the contrary, if for example the top speed of a given design were doubled, the same force produces the same acceleration with a quarter of the stick travel, but the angular velocity is halved and the radius of curvature is multiplied by four. This does not mean, however, that speed is entirely irrelevant to manœuvrability; as will be seen later it has a vital effect on the time in which the acceleration is produced. It should be noticed that there is in general a variation of static margin and tail characteristics with speed. To this extent, therefore, Q is a function of the speed.

(c) The acceleration is produced by a change of incidence and accompanied by an angular velocity; the first is resisted by the static stability and the second by the damping moment, and Q, as shown by the quantity within the brackets of formula (1), is proportional to the sum of these effects, of which the static moment is represented by H_n' and the damping moment by $\frac{a_1'}{2\mu}\bar{V}$. H_n', the static margin, varies between 0 and about 0·1, and $\frac{a_1'}{2\mu}\bar{V}$ has a range of the same order, since $a_1' \simeq 2$, $\bar{V} \simeq \frac{1}{2}$ and μ the relative density $w/g\rho l$ varies from about 5 for a larger bomber at ground level to about 100 for a fighter at its ceiling. It follows that, in general, manœuvrability is resisted in equal degree by static stability and damping moment, but the influence of the latter increases with size and proximity to ground level.

(d) It follows from the form of μ that the laws of variation of Q, with wing loading w, linear dimension l and air density σ are

$A_1 w + B_1$	for wing loading
$A_2 l^3 + B_2 l^4$	for size (similar aeroplanes)
$A_3 + B_3 \sigma$	for altitude

where A_1, A_2, A_3 are proportional to H_n'. Thus for a statically neutral aeroplane the manœuvrability is independent of wing loading and varies inversely as $l^4\sigma$. The altitude law is of particular interest in view of recent reports of the unsatisfactory behaviour of high altitude fighters near the ceiling. If static margin and damping moment are of equal importance at the ground ($A_3 = B_3$), then at 40,000 ft ($\sigma = \frac{1}{4}$) Q will be decreased in the ratio 5/4 to 2; and if the stability is neutral Q will be divided by 4. The transition from ground level to 40,000 ft, therefore, multiplies the acceleration produced by a given stick force by 1·6 in the first case and 4 in the second.

(e) Some reference is needed to the limits of stability within which the formula (1) is valid. Formally it holds down to

$$H_n' + \frac{a_1'}{2\mu} \bar{V} = 0 ,$$

at which point the quick oscillation becomes unstable. It seems, however, that the approximation on which the analysis is based ceases to be realistic when $H_n' = 0$, for if there is static instability the maximum acceleration calculated by this method may be exceeded in the subsequent divergent motion. The formula should therefore be used with reserve when $H_n' < 0$.

5. *Growth of Acceleration*.—It should be noticed that though Q is probably the most useful single and simple criterion of manœuvrability which can be devised, it is incomplete in the sense that it takes no account of the growth of the acceleration to the maximum which it determines. The growth of acceleration can, however, be obtained by a simple additional calculation in any particular case.

It is shown in the appendix that if ng is the increment in normal acceleration at aerodynamic time τ after the instantaneous application of a constant stick force, then

$$\frac{n}{n_{\max}} = 1 + \frac{\lambda_2 e^{-\lambda_1 \tau} - \lambda_1 e^{-\lambda_2 \tau}}{\lambda_1 - \lambda_2} \quad \ldots \quad (6)$$

where $-\lambda_1$, $-\lambda_2$ are the roots of the quadratic

$$\lambda^2 + \left(\frac{a}{2} + \frac{3}{4}\frac{a_1'c}{li_B}\bar{V}\right)\lambda + \frac{ac}{2li_B}\mu\left(H_n' + \frac{a_1'\bar{V}}{2\mu}\right) = 0 \quad \ldots \quad (7)$$

and the unit of τ is $w/g\rho V$ sec.

Equations (1), (6) and (7) now give n/P, the 'g per pound', as a function of aerodynamic time τ; the limiting value of n/P is the inverse of Q.

6. *Example of Growth of Acceleration for a Fighter*.—I am indebted to Dr. Neumark for an illustration of the growth of acceleration on a fighter of Spitfire size, the characteristics assumed being listed in Appendix II. The results, which are intended to show particularly the effects of static margin H_n', relative air density σ, and speed, are plotted in Figs. 1 to 6. In considering these it should be remembered that the true time to reach an ordinate plotted against aerodynamic time τ is inversely proportional to σV.

In Fig. 1, $n/n_{\max}$ is plotted for a series of values of H_n'/σ. This diagram shows the closeness of the approximation of $n_{\max}$ to the maximum acceleration actually reached in the motion: the curves do not rise appreciably above $n/n_{\max} = 1$ except when the static margin is large.

In Figs. 2, 3, n/P is plotted for two heights $\sigma = \frac{1}{2}$ (22,000 ft) and $\sigma = 1$. Scales of time in seconds for indicated speeds of 200 and 400 m.p.h. are added to indicate the speed effect. These diagrams show in a general way how manœuvrability is increased by decrease of static margin, increase of altitude, and increase of indicated speed, the last being effective only by quickening the motion toward the attainable acceleration.

Fig. 4 is drawn to emphasise the important conclusion that the time to reach a given acceleration under a given force is inversely proportional to the speed, although the acceleration attainable is independent of it. For instance, suppose a pilot pulls back instantaneously with a force of 20 lb when his static margin is 0·01. Fig. 4 shows that if he holds this on he will reach an added acceleration of about $9g$. At 200 m.p.h. he is approaching this high figure in 2 secs.; at 400 m.p.h. with a quarter of the stick movement of the lower speed, he is approaching it in 1 sec. The danger of high speed is that it quickens the growth of acceleration beyond the control of the pilot.

Fig. 5 shows strikingly how manœuvrability increases with altitude, particularly if the static margin is small:—if $H_n' = 0 \cdot 01$, a 20-lb pull produces $6g$ at ground level and about $9g$ at 22,000 ft, where the rate of growth of acceleration is markedly increased. In considering the bearing of this on the reported bad behaviour of high altitude fighters near the ceiling, it should be observed that high manœuvrability, as defined in this analysis, is not necessarily to be equated to good controllability or high accuracy of manœuvre. The outcome of this discussion is that an aeroplane grows more lively to handle on the elevator as the aeroplane rises, but it is equally certain that as the ceiling is approached the pilot gets less lively to handle it without losing height under the constantly decreasing margin of power available for level flight. It is likely that there is no single explanation of the pilots' reports that control at very high altitude amounts to 'balancing on a pin point or a tight rope' and very probable that increase of manœuvrability has a direct bearing on the subject.

Figs. 1 to 5 refer to instantaneous application of force. Fig. 6 (with Fig. 1), which is drawn from the material of R. & M. 2275[1], shows roughly how the growth of acceleration is delayed when the force rises linearly to its maximum at various rates. The curve marked 'instantaneous' in this figure corresponds roughly to $H_n'/\sigma = 0 \cdot 025$ in Fig. 1.

7. *Effect of Weight Moment on Manœuvrability.*—It should be clear from the above discussion that it is sometimes necessary to restrict the manœuvrability in the sense of making it difficult for the pilot to reach easily the aeroplane's breaking load. A note can be made here on the properties of the inertia weight, which is the most obvious safeguarding device. This is simply a weight arranged in the control circuit to push the stick forward and the elevator down. This has three effects :—

(1) If its moment about the elevator hinge is K ft lb, it increases the static margin stick free

by an amount $-\dfrac{a_2\bar{V}}{b_2 w c_\eta S_\eta} K$.

(2) Therefore it inceases the push, to hold straight flight at C_L when trimmed at $C_L + \Delta C_L$, by $mK \dfrac{\Delta C_L}{C_L}$ lb.

(3) In accelerated motion it increases Q, the force per g, by mK lb.

In virtue of (3), therefore, the inertia weight acts as a deterrent to sharp pull outs from a trimmed position. For instance, if K is 10 lb ft and the gearing m is $\frac{1}{2}$, the pilot will be opposed by 25 lb at $5g$. It is, however, equally necessary to safeguard another condition, in which the pilot lets the stick go in a dive when trimmed at a slower speed.* It may be shown as follows that on the assumption of this theory the inertia weight fails in this situation. The stick push P_1 in a dive at C_L when trimmed for $C_L + \Delta C_L$ is given by

$$P_1 = - mwc_\eta S_\eta \frac{b_2}{a_2} \frac{H_n'}{\bar{V}} \frac{\Delta C_L}{C_L}$$

If the stick is now freed instantaneously

$$P_1 = P = n_{\max} mwc_\eta S_\eta \frac{b_2}{a_2 \bar{V}} \left(H_n' + \frac{a_1'}{2\mu} \bar{V} \right)$$

and so we have

$$n_{\max} = \frac{1}{1 + \dfrac{a_1' \bar{V}}{2\mu H_n'}} \frac{\Delta C_L}{C_L}$$

for our approximation to the maximum acceleration after freeing the stick at C_L when trimmed at $C_L + \Delta C_L$.

If n_m is the true maximum of acceleration in the pull out, it can be shown without difficulty that

$n_m = n_{\max}$ if the roots of equation (7) are real and that $\dfrac{n_m}{n_{\max}} = 1 + e^{-p\pi/q}$ if the roots are complex and equal to $-p \pm iq$.

Hence $$n_m = \frac{1}{1 + \dfrac{a_1' \bar{V}}{2\mu H_n'}} \frac{\Delta C_L}{C_L} \quad \text{or} \quad \frac{1 + e^{-p\pi/q}}{1 + \dfrac{a_1' \bar{V}}{2\mu H_n'}} \frac{\Delta C_L}{C_L}$$ in these two cases.

Now consider the effect of an inertia weight on n_m. It increases H_n', and when the roots of equation (7) are complex it increases q, leaving p unchanged. Hence in every case it increases n_m, the maximum acceleration reached on freeing the stick.

This conclusion from an approximate theory needs careful experimental check before it can be accepted. It is not confirmed by one set of tests[7], and further experimental work to clear up the point is in progress.

The simple inertia weight is not ideal as a deterrent to high acceleration, since it can only be made really effective at high g by being too strong at low g. What is wanted is a variant which only comes into action at a moderately high g, and then if possible with increasing strength. Morgan has proposed[6] one such device in which the weight is constrained by a pre-loaded spring and moves so as to increase its leverage about the stick hinge when the pre-set acceleration is exceeded. George Miles has proposed another, in which a small weight moves under a pre-loaded spring to actuate an elevator tab which forces the elevator down, the stick force being proportional roughly to the increment of acceleration above the pre-set value and to the square of the speed. These devices are in process of development.

8. *Practical Considerations in Regard to the Criterion Q.*—The criterion Q is proposed as a convenient framework of analysis for the designer, but to make it effective in design numerical standards are required. It is probably necessary, in fighter design at any rate, to lay down upper and lower limits to Q, an upper for good manœuvrability and a lower for safety. Such considerations, involving both the strength of the pilot and the strength of the aeroplane, are

*Here it should be noted that the degree of stability affects the safety in different ways according to the diving technique. If the aeroplane is trimmed in the dive the aeroplane with low stability is easiest to break in pulling out ; but if the aeroplane is trimmed at a lower speed and pushed into the dive it is the highly stable one which is dangerous.

ultimately faced with the fact that it is always possible for an average pilot to break a small aeroplane in pulling out of a dive unless a strong deterrent device is used in the elevator circuit, and so a compromise must be made. The practical aspects of this question, particularly the use of tabs in recovery from dives, have been reviewed by a Sub-committee of the Joint Airworthiness Committee, extracts from whose report are given in Appendix III. It was recognised in these discussions that measurements of force in pull-outs are required on many aeroplanes before standards can be fixed. Arrangements were made to collect this information, and the results to date are summarised in Table 1 for a few fighters and bombers. The value of Q for these fighters varies between 2 and 11; the lower values were considered exceptionally light, the higher values (between 5 and 10) were acceptable. The value of Q for the bombers varies between about 35 and 100; the higher figure was not objected to, but this may only be because pilots of heavy bombers have not yet been supplied with light operating forces. An exception is the Mosquito, where with centre of gravity back (and presumably with a negative static margin) the force to pull out is reported as negligible.

The evidence on fighters suggests, perhaps, standardisation at an upper limit of about 8, but more evidence on bombers with lighter forces is required before a figure can be even suggested. In this connection it should be noted that the American upper limits for Q are 6 for fighters and 50 for bombers[5], with an over-riding requirement that ' on any airplane a steady pull force of not less than 30 lb should be required to obtain the allowable load factor '.

The analysis of this paper is tentative in some respects, particularly in its assumptions regarding the type of pilot's action. The results of Table 1 are too rough to give anything in the nature of a thorough check of the theory, and it is considered that it would be well worth while to do this by thorough flight tests of one aeroplane whose stability and tail characteristics are known. It is only by precise correlation of records of acceleration, stick force and stick travel that the theory can be well grounded.

9. *Conclusions*.—The threads of this argument may be gathered up as follows :—

(1) A compact formula for a criterion of manœuvrability Q the stick force per g is proposed as a basis of design. The theory needs thecking by careful flight tests on one aeroplane.

(2) The rate of application of force affects the time to reach maximum acceleration, but not the acceleration reached.

(3) If stability and tail characteristics are invariant with speed, then the acceleration produced by a given stick force is independent of speed, but the time to reach it varies inversely as the speed.

(4) Manœuvrability on this definition increases (*i.e.*, Q decreases) with height, the rate of increase being the greater the smaller the stability. It is suggested that this is one factor governing the unsatisfactory behaviour of high altitude fighters near the ceiling.

(5) The simple inertia weight is not ideal as a deterrent to the production of high acceleration, since to be effective its action is too strong in normal manœuvres. Its effect on freeing the stick in a dive when trimmed at a lower speed is being investigated further. Variants such as Morgan's spring weight device and Miles' inertia-operated spring tab are shown to be necessary.

(6) If Q is adopted as a criterion, numerical standards should be attached to it. More measurements are needed of stick force in pulling out of dives, particularly on bombers, before these can be fixed.

REFERENCES

No.	Author	Title, etc.
1	Bryant and Gandy	Response of an Aeroplane to Application of Elevator. R. & M. 2275. May, 1941.
2	Howard and Owen	The Recovery from a Dive. A.R.C. 5137. (Unpublished.)
3	Howard and Owen	Control Forces during Recovery from a Dive. A.R.C. 5138. (Unpublished.)
4	Gates	Note on Prediction of Stick Force and Tail Load in Pulling Out of Dives. B.A. Departmental Note—Performance No. 63. August, 1941.
5	Gilruth	Requirements for Satisfactory Flying Qualities of Airplanes. N.A.C.A. Advance Report A.R.C. 5543.
6	Morgan	Suggestion for a Non-linear Inertia Device. B.A. Departmental Note—Full scale No. 109. October, 1941.
7		P.7280—Diving Trials with Inertia Device Fitted. Fourth Part of A. & A.E.E. Report No. 692e. January, 1941.
8	Bryant and Gates	Nomenclature for Stability Coefficients. R. & M. 1861. October, 1937.

NOTATION

w	Wing loading
$\bar{V}$	Tail volume lS'/cS
q	Angular velocity (radians/sec)
μ	Relative aeroplane density $w/g\rho l$
h_n	Neutral point, stick fixed
h_n'	Neutral point, stick free
H_n	Static margin stick fixed $(h_n - h)$
H_n'	Static margin stick free $(h_n' - h)$
S_η	Elevator area
c_η	Elevator mean chord
$a = dC_L/d\alpha$	
$a_1 = dC_L/d\alpha$	
$a_2 = dC_L'/d\eta$	
$b_1 = dC_H/d\alpha$	
b_2	$dC_H/d\eta$
m	Elevator stick gearing radians/ft
K	Weight moment about elevator hinge
λ	$K/wc_\eta S_\eta$
Q	Stick force per g as defined in equation (1).
P	Pilot's pull (lb) reaching maximum P_{max} } Measured from straight flight
x	Stick travel (ft) reaching maximum x_{max} } Measured from straight flight
η	Elevator angle, reaching maximum η_{max} } Measured from straight flight
ng	Increment in normal acceleration, reaching $n_{max}\, g$
	Suffix max. refers to the steady state of the approximate calculation
τ	aerodynamic time, unit $w/g\rho V$ secs.

TABLE 1

Force Measurements in Pulling Out of Dives

Aeroplane	Place of test	Weight	C.G. aft of datum	A.S.I. m.p.h.	$(1 + n_{max})$ = acceleration/g during recovery	Max. stick force	Criterion Q	Pilots' remarks
Spitfire K.9796	R.A.E.	6,000	7 in. $h = 0 \cdot 33$	300 350 400 450 300 350 400 450	2 2 2 2 4 4 4 4	2·5 2·5 2·5 4 6 9 10 10	2·5 2·5 2·5 4 2 3 3·3 3·3	Rather too light
Hurricane Z.2385	R.A.E.	6,800	58 in. $h = 0 \cdot 33$	300 350 400 300 350 400	2 2 2 4 4 4	8 9 11 20 22 26	8 9 11 6·7 7·3 8·7	Just right
Mohawk AX.882	A.A.E.E.	6,000	21 in. limits 19 to 26	350	4	10	3·3	Exceptionally light and easy to pull out of dive.
Tomahawk AK.176	A.A.E.E.	7,300	22·7 in. 26·2 in. limits 17 to 25	370 370	4 4	28 15	9·3 5	A nice comfortable pull out. Quite light and considerably below normal for fighters.
Whitley Z.6640	A.A.E.E.	26,000	86 in. limits 75 to 94	185	1·9	33 (full throttle) 47 (1/3 throttle)	37 52	Light and easy to recover.
Halifax L.7245	A.A.E.E.	39,200 48,000	38·5 in. 46·5 in. limits 41 to 51	272 270	2 2	87 87	87 87	Fairly heavy but normal for type.
Stirling N.6008	A.A.E.E.	54,000	110 in. limits 102 to 125	245	2	84	84	Considered normal
Wellington	R.A.E.	—	Aft limit	150 200 250	2 2 2	95 95 105	95 95 105	Not considered exceptionally heavy.
Mosquito (small tail short nacelles)	Firm	15,900	$h = 0 \cdot 283$	393	2·0 2·8 3·8 4·0	13 33·5 57 59	13 19 20 20	
		17,800	$h = 0 \cdot 363$	393	—	—	Small	Force too small to measure.

APPENDIX I

Theory of Response to Elevator

The analysis of the initial response to elevator is based on two assumptions:—

(1) The speed remains constant.

(2) Changes in the gravity component are small compared with acceleration changes caused by incidence changes; gravity is, therefore, neglected.

Using the non-dimensional system[8], the variables being α the change in incidence, $\bar{q} = \frac{\mu l}{V} q$ the angular velocity, and η the elevator angle, the equations of lift and pitching moment are respectively:—

$$\frac{d\alpha}{d\tau} - \bar{q} = z_w \alpha \quad \ldots \quad (1)$$

$$\frac{d\bar{q}}{d\tau} = \frac{\mu m_{\dot{w}}}{i_B} \frac{d\alpha}{d\tau} + \frac{\mu m_w}{i_B} \alpha + \frac{m_q}{i_B} \bar{q} + \frac{\mu m_\eta}{i_B} \eta \quad \ldots \quad (2)$$

If P is the pilot's pull on the stick, m the elevator to stick gearing in radians/ft, and K the weight moment of elevator and elevator circuit about the elevator hinge, the hinge moment equation is

$$\frac{P}{m} = \left[b_1 \left\{ \alpha\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{ql}{V} \right\} + b_2 \eta \right] c_\eta S_\eta \tfrac{1}{2} \rho V^2 - \frac{\dot{w} - Vq}{g} K$$

which may be written

$$\frac{P}{m b_2 c_\eta S_\eta \cdot \frac{1}{2} \rho V^2} = \eta + \frac{b_1}{b_2} \left\{ \alpha \left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{\bar{q}}{\mu} \right\} - \frac{2\lambda}{b_2} \left(\frac{d\alpha}{d\tau} - \bar{q} \right) \quad \ldots \quad (3)$$

where $$\lambda = \frac{K}{w c_\eta S_\eta} .$$

The motion under the pilot's action, which may be specified either by η or by P as a function of τ, is given by equations (1) to (3). If the action ultimately reaches a constant value, and if the motion is stable, it will ultimately reach a steady circle in which the normal acceleration is $n_{\max} g$. This steady motion is given by putting

$$\frac{d\alpha}{d\tau} = \frac{d\bar{q}}{d\tau} = 0$$

and solving for the ratios $\alpha : \bar{q} : \eta : P$.

Using the relations

$$z_w = -\frac{a}{2}$$

$$m_w = -\frac{ac}{2l} H_n$$

$$H_n' - H_n = -\frac{a_2 \bar{V}}{b_2} \left\{ \frac{b_1}{a} \left(1 - \frac{d\varepsilon}{d\alpha}\right) + \lambda \right\}$$

$$m_q = -\frac{a_1 c}{2l} \bar{V} ,$$

$$m_\eta = -\frac{a_2 c}{2l} \bar{V} , \quad a_1' = a_1 \left(1 - \frac{a_2 b_1}{a_1 b_2}\right)$$

and using the suffix max to denote the steady state, it may be shown to be:—

$$\left.\begin{aligned}\frac{n_{\max}}{\alpha_{\max}} &= \frac{a}{w}\,\tfrac{1}{2}\rho V^2\\ \frac{\bar{q}_{\max}}{\alpha_{\max}} &= \frac{a}{2}\\ \frac{\eta_{\max}}{\alpha_{\max}} &= -\frac{a}{a_2\bar{V}}\left(H_n + \frac{a_1\bar{V}}{2\mu}\right)\\ \frac{P_{\max}}{n_{\max}} &= -\frac{mb_2wc_\eta S_\eta}{a_2\bar{V}}\left(H_n' + \frac{a_1'}{2\mu}\bar{V}\right)\end{aligned}\right\} \quad \ldots \quad (4)$$

where $H_n = h_n - h$ and $H_n' = h_n' - h$ are respectively the static margins stick fixed and stick free.

The motion is so heavily damped that if the pilot's action is a more or less steady rise to its ultimate value it would be expected that the steady values $\alpha_{\max}$, $n_{\max}$, $q_{\max}$ are sensibly the maxima occurring in the motion. Bryant and Gandy have shown that this is so when the stick is moved at a constant rate to a new position and held there. In all such cases therefore the expressions given above are good approximations to the maxima of incidence and angular velocity, and these are therefore independent of the rate at which the action is taken, which affects only the time taken to reach the maxima.

In two simple cases which give the most rapid growth of acceleration covered by this approximate theory, the complete solution is readily obtained:—

Case I, instantaneous stick movement: $\eta = \eta_{\max}$ throughout, P varying to $P_{\max}$.

Case II, instantaneous application of constant force: $P = P_{\max}$ throughout, η varying to $\eta_{\max}$.

Case I. The solution is

$$\begin{aligned}\alpha &= A_1 e^{-\lambda_1\tau} + A_2 e^{-\lambda_2\tau} + \alpha_{\max}\\ \bar{q} &= B_1 e^{-\lambda_1\tau} + B_2 e^{-\lambda_2\tau} + \bar{q}_{\max}\\ P &= C_1 e^{-\lambda_1\tau} + C_2 e^{-\lambda_2\tau} + P_{\max}\end{aligned}$$

where $-\lambda_1$, $-\lambda_2$ are the roots of

$$\lambda^2 - \lambda\left(z_w + \frac{m_q + \mu m_{\dot{w}}}{i_B}\right) + \frac{z_w m_q - \mu m_w}{i_B} = 0$$

which is sensibly the quick oscillation of the stability analysis stick fixed. This may be written alternatively

$$\lambda^2 + \lambda\left(\frac{a}{2} + \frac{3}{4}\frac{a_1 c}{l i_B}\bar{V}\right) + \frac{ac}{2l i_B}\mu\left(H_n + \frac{a_1\bar{V}}{2\mu}\right) = 0\,, \quad \ldots \quad (5)$$

Inserting the conditions

$$\left.\begin{aligned}A_1(\lambda_1 + z_w) + B_1 &= 0\\ A_2(\lambda_2 + z_w) + B_2 &= 0\\ \alpha = \bar{q} = 0 \text{ when } \tau &= 0\end{aligned}\right\}$$

we have

$$\left.\begin{aligned}\frac{\alpha}{\alpha_{\max}} = \frac{n}{n_{\max}} &= 1 + \frac{\lambda_2 e^{-\lambda_1\tau} - \lambda_1 e^{-\lambda_2\tau}}{\lambda_1 - \lambda_2}\\ \frac{\bar{q}}{\bar{q}_{\max}} &= 1 + \frac{\lambda_2(\lambda_1 + z_w)e^{-\lambda_1\tau} - \lambda_1(\lambda_2 + z_w)e^{-\lambda_2\tau}}{(\lambda_1 - \lambda_2)z_w}\end{aligned}\right\} \quad \ldots \quad (6)$$

for the growth toward the steady condition given by equation (4).

If the stick travel is $x_{\max}$ ft, we have $\eta_{\max} = mx_{\max}$, and a criterion of manœuvrability is conveniently defined as Q_1, the stick travel per g, or

$$Q_1 = \frac{x_{\max}}{n_{\max}} = \frac{w}{ma_2 \,.\, \bar{V} \,.\, \frac{1}{2}\rho V^2}\left(H_n + \frac{a_1 \bar{V}}{2\mu}\right) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

An alternative criterion Q_2 the stick travel per unit $C_{L\max}$ may also be considered

$$Q_2 = \frac{x}{(\Delta C_L)_{\max}} = \frac{1}{ma_2\bar{V}}\left(H_n + \frac{a_1\bar{V}}{2\mu}\right) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

Case II. In this case, substituting for η from equation (3) in (2), we have

$$\frac{d\bar{q}}{d\tau} = \frac{\mu m_{\dot{w}}'}{i_B}\frac{d\alpha}{d\tau} + \frac{\mu m_w'}{i_B}\alpha + \frac{m_q'}{i_B}\bar{q} + \frac{\mu m_\eta}{i_B}\frac{P_{\max}}{mb_2 c_\eta S_\eta \,.\, \frac{1}{2}\rho V^2}$$

where the dashed derivatives now refer to stick free and are given by

$$m_w' - m_w = \frac{c}{2l}\frac{a_2 b_1}{b_2}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\bar{V}$$

$$m_{\dot{w}}' - m_{\dot{w}} = -\frac{c}{l}\frac{a_2}{b_2}\lambda\bar{V}$$

$$m_q' - m_q = \frac{c}{2l}\frac{a_2}{b_2}\bar{V}\,(b_1 + 2\mu\lambda)$$

The solution is formally the same as equation (6) of Case I, but $-\lambda_1$, $-\lambda_2$ are now the roots of

$$\lambda^2 - \lambda\left(z_w + \frac{m_q' + \mu m_{\dot{w}}'}{i_B}\right) + \frac{z_w m_q' - \mu m_w'}{i_B} = 0$$

which is sensibly the quick oscillation of the stability analysis stick free. This may be written alternatively

$$\lambda^2 + \lambda\left(\frac{a}{2} + \frac{3}{4}\frac{a_1'c}{li_B}\bar{V}\right) + \frac{ac}{2li_B}\mu\left(H_n' + \frac{a_1'\bar{V}}{2\mu}\right) = 0 \qquad \ldots \quad \ldots \quad \ldots \quad (9)$$

In this case a criterion of manœuvrability is conveniently defined as Q 'the stick force per g', which is given by

$$Q = \frac{P_{\max}}{n_{\max}} = -\frac{mb_2 w c_\eta S_\eta}{a_2\bar{V}}\left(H_n' + \frac{a_1'}{2\mu}\bar{V}\right) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

APPENDIX II

Constants used in the Calculations of Figs. 1 to 6

$W = 5{,}800$	$\bar{V} = 0\cdot375$	$a_2 = 1\cdot78$
$S = 242$	$i_B = 0\cdot087$	$b_1 = -0\cdot034$
$S' = 33$	$w = 24$	$b_2 = -0\cdot178$
$S_\eta = 13\cdot2$	$\mu = \dfrac{17\cdot4}{\sigma}$	$a_1' = 2\cdot84$
$l = 18$		$m = 0\cdot8$
$c = 6\cdot53$	$a = 4\cdot48$	$\dfrac{a_1'\bar{V}}{2\mu} = 0\cdot031\sigma$
$c_\eta = 1\cdot26$	$a_1 = 3\cdot18$	

APPENDIX III

Extract from Report of Joint Airworthiness Committee Sub-committee on Trimming Tabs

The sub-committee is in agreement with the recommended flight technique which is laid down in general terms in the 'Flying Training Manual' and more specifically in the 'Pilot's Notes' for each particular aeroplane. These handbooks permit the pilot to trim the aeroplane in the dive by use of the trimmer tabs, and recommend strongly that if the tabs are used for manœuvring that they should be used extremely gently. Depending on the known characteristics of the aeroplane the 'Pilot's Notes' allow a certain (and usually very limited) use of the trimming tabs for manœuvring. This system appears to the sub-committee to be a satisfactory procedure for dealing with present aeroplanes.

Turning now to aeroplanes in the design stage it is agreed that efforts should be made to design the elevator control system so that the pilot would not experience the need to operate the trimmer tabs to effect recovery from dives. With this end in view preliminary tests have been made and a flight test has been recommended. This test now appears in A.D.M.295 (Issue II) in paragraph 8.

'8. *Tests.* The following tests are to be made during which an indicating and maximum recording accelerometer and a stick force recorder are to be fitted . . . With the aeroplane trimmed in a dive at 400 m.p.h., or maximum permissible diving speed whichever is the less, a pull-out without roll is to be made until a steady acceleration of $4g$ is reached. The force on the control column, which is not to exceed 30 lb, is to be recorded and the control column movement estimated.'

The suggested test refers only to fighters, wider application being impossible without further flight tests.

It is generally agreed that tests of the type proposed for fighters should be extended to cover bombers and torpedo carrying aeroplanes. For these larger aeroplanes, with their greater C.G. movements, it would also be advantageous to specify minimum as well as maximum stick forces during pull-outs. In order to define suitable standards considerably more information is required. The industry have undertaken some test work the results of which will be circulated as they become available. It is recommended however that a more comprehensive research programme should be undertaken by the Government Establishments and it is suggested that the Aeronautical Research Committee be approached in this connection. Further tests on fighters to include typical night fighters, twin-engined fighters and F.A.A. types, together with tests on bombers and torpedo bombers, are envisaged.

In the course of these discussions on trimming tabs it became clear that the problems were largely bound up with the general longitudinal stability characteristics of the aeroplane and it is felt that further study of stability and increased attention to stability in the early stages of design of aeroplanes would be worth while.

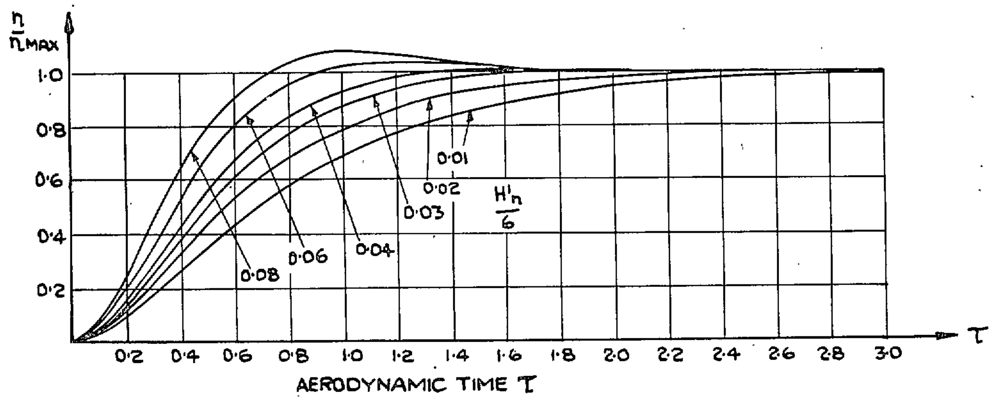

FIG. 1. Effect of static margin on growth of acceleration (instantaneous force)

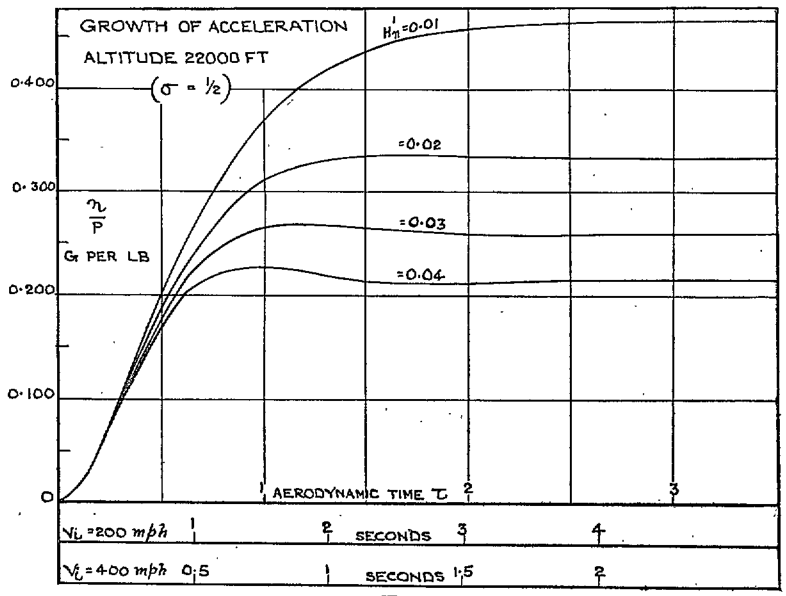

FIG. 2

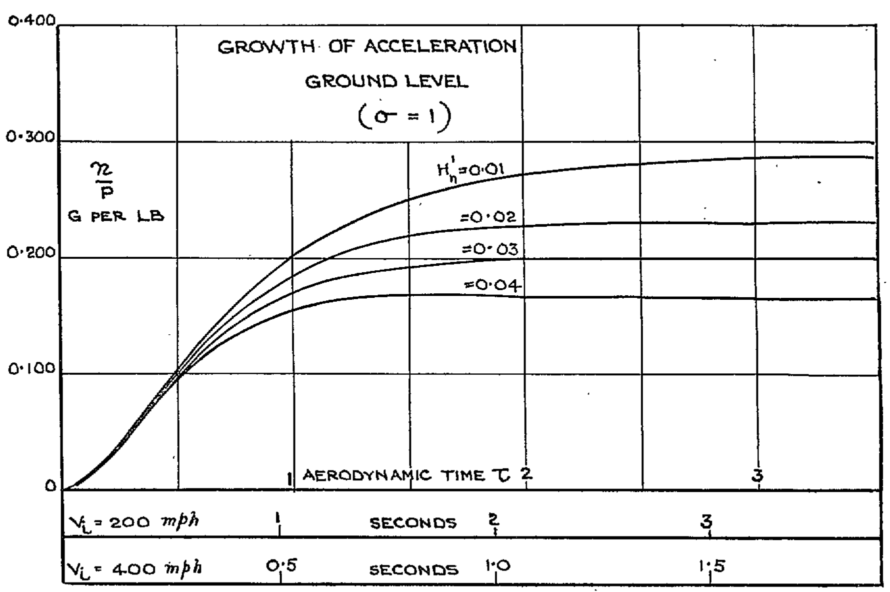

FIG. 3

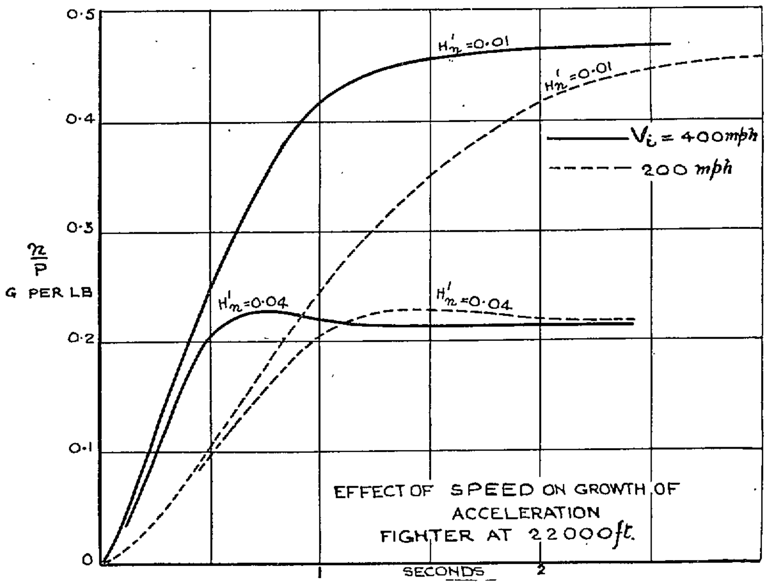

FIG. 4

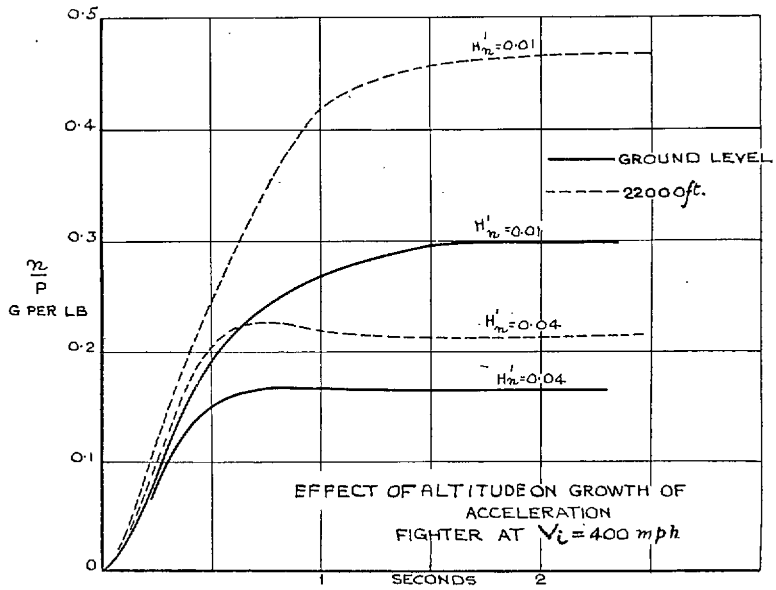

FIG. 5

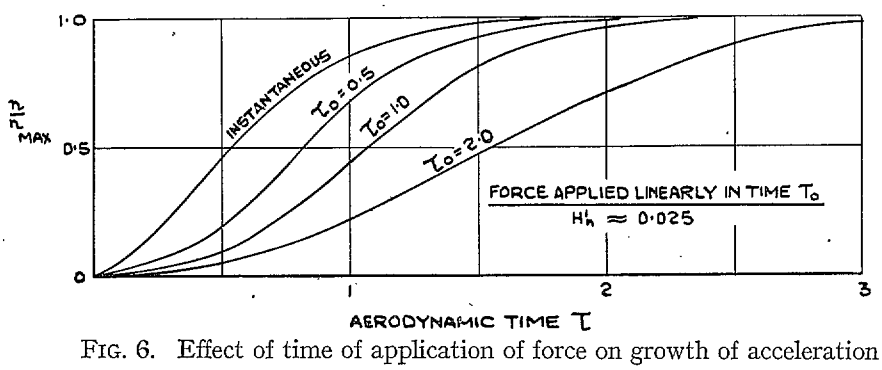

FIG. 6. Effect of time of application of force on growth of acceleration

(96738) Wt. 14/655 K.5 7/51 Hw.

PRINTED IN GREAT BRITAIN

R. & M. No. 2677
(5981)
A.R.C. Technical Report

Publications of the Aeronautical Research Council

ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)—

1934–35 Vol. I. Aerodynamics. *Out of print.*
Vol. II. Seaplanes, Structures, Engines, Materials, etc. 40*s.* (40*s.* 8*d.*)

1935–36 Vol. I. Aerodynamics. 30*s.* (30*s.* 7*d.*)
Vol. II. Structures, Flutter, Engines, Seaplanes, etc. 30*s.* (30*s.* 7*d.*)

1936 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40*s.* (40*s.* 9*d.*)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 50*s.* (50*s.* 10*d.*)

1937 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40*s.* (40*s.* 10*d.*)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 60*s.* (61*s.*)

1938 Vol. I. Aerodynamics General, Performance, Airscrews. 50*s.* (51*s.*)
Vol. II. Stability and Control, Flutter, Structures, Seaplanes, Wind Tunnels, Materials. 30*s.* (30*s.* 9*d.*)

1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50*s.* (50*s.* 11*d.*)
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63*s.* (64*s.* 2*d.*)

1940 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Icing, Stability and Control, Structures, and a miscellaneous section. 50*s.* (51*s.*)

Certain other reports proper to the 1940 volume will subsequently be included in a separate volume.

ANNUAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

1933–34	1*s.* 6*d.* (1*s.* 8*d.*)
1934–35	1*s.* 6*d.* (1*s.* 8*d.*)
April 1, 1935 to December 31, 1936.	4*s.* (4*s.* 4*d.*)
1937	2*s.* (2*s.* 2*d.*)
1938	1*s.* 6*d.* (1*s.* 8*d.*)
1939–48	3*s.* (3*s.* 2*d.*)

INDEX TO ALL REPORTS AND MEMORANDA PUBLISHED IN THE ANNUAL TECHNICAL REPORTS, AND SEPARATELY—

April, 1950 R. & M. No. 2600. 2*s.* 6*d.* (2*s.* 7½*d.*)

INDEXES TO THE TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

December 1, 1936 — June 30, 1939.	R. & M. No. 1850.	1*s.* 3*d.* (1*s.* 4½*d.*)
July 1, 1939 — June 30, 1945.	R. & M. No. 1950.	1*s.* (1*s.* 1½*d.*)
July 1, 1945 — June 30, 1946.	R. & M. No. 2050.	1*s.* (1*s.* 1½*d.*)
July 1, 1946 — December 31, 1946.	R. & M. No. 2150.	1*s.* 3*d.* (1*s.* 4½*d.*)
January 1, 1947 — June 30, 1947.	R. & M. No. 2250.	1*s.* 3*d.* (1*s.* 4½*d.*)

Prices in brackets include postage.

Obtainable from

HIS MAJESTY'S STATIONERY OFFICE

York House, Kingsway, LONDON, W.C.2 429 Oxford Street, LONDON, W.1
P.O. Box 569, LONDON, S.E.1
13a Castle Street, EDINBURGH, 2 1 St. Andrew's Crescent, CARDIFF
39 King Street, MANCHESTER, 2 Tower Lane, BRISTOL, 1
2 Edmund Street, BIRMINGHAM, 3 80 Chichester Street, BELFAST

or through any bookseller.

S.O. Code No. 23-2677