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Aerofoil Oscillations at High Mean Incidences

By

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of the Aerodynamics Division, N.P.L.

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Summary.—The problem of the estimation of the aerodynamic forces acting on two-dimensional aerofoils oscillating at mean incidences below the stall is considered. A method of calculation is suggested which makes use of the steady motion characteristics of the aerofoil. At low frequencies, good agreement with the measured aerodynamic derivatives should be obtained as the method is such that it gives the correct values at zero frequency. A comparison between the estimated and measured values of the pitching-moment derivatives for a particular aerofoil is made, and this shows that the method suggested gives better agreement with experiment than the usual vortex-sheet theory[1].

The method can be extended for the calculation of control-surface derivatives. To some extent, the influence of compressibility could also be taken into account.

1. *Introduction.*—A method for the calculation of aerodynamic derivatives for an oscillating aerofoil is suggested which takes into account mean incidence and thickness/chord ratio effects. The main feature of the proposed scheme is the replacement of the aerofoil at any incidence by an equivalent thin profile which gives, on the basis of linearised theory, the experimentally determined steady motion lift distribution for that incidence. For oscillations of small amplitude about a given mean incidence, it is further assumed that the equivalent profile changes shape instantaneously with incidence. In the estimation of the derivatives such variations in shape are taken into account, and the aerodynamic forces are calculated by the use of the linearised theory for oscillatory motion[1]. It is also supposed that the superposition principle can be applied so that the changing lift distribution due to the oscillation can be separated from the distribution corresponding to steady motion at the mean incidence of the oscillation.

Derivatives calculated on the above basis should perhaps be in better agreement with experiment at the lower than at the higher values of the frequency parameter as in practice the equivalent profile must lag behind the changing incidence. Some comparisons between experiment and theory for a pitching aerofoil are made in Figs. 5 and 6. Measured and calculated values of the pitching moment derivatives for oscillations of 2 deg amplitude at 10 deg mean incidence show good agreement over the lower range of frequency parameter values. At very low frequency, however, the measured stiffness derivative for a 2 deg amplitude oscillation is much lower than the value given by the slope of the pitching moment curve at 10 deg incidence to which it would correspond in the case of very small amplitude oscillations. This is due to the fact that the aerofoil stalls at 11·5 deg incidence which is within the range of the oscillation (*see* Fig. 3). In this case, the effective mean slope of the pitching moment curve over the range 8 deg to 12 deg incidence differs from the slope at the mean incidence. When the frequency of the oscillation is increased, however, the stall is delayed and theory and experiment then show better agreement.

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For the lower range of frequency parameter values the present method is also an improvement on vortex sheet theory for oscillations about zero incidence, since the influence of thickness/chord ratio and the steady motion characteristics of the aerofoil are taken into account.

2. *Steady Motion.*—It is assumed that the field of steady potential flow around an aerofoil at any incidence below the stall can be reproduced by an equivalent thin profile.

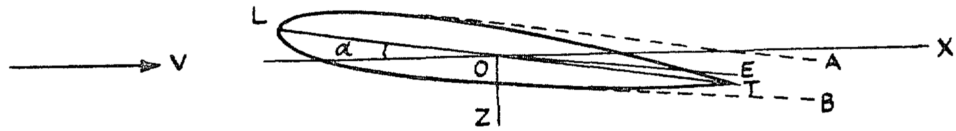

FIG. 1.

Let LE denote such a profile, and let LA and LB represent the limits of the upper and lower boundary layers respectively. The pressure is assumed to be roughly constant across the boundary layers so that their thicknesses can be neglected. If the lift distribution for any incidence α is known, it is then possible on the basis of linearised theory to determine the shape of the equivalent profile LE which gives theoretically the same lift distribution and the same field of flow outside the boundary layers. Under certain conditions, curves of lift and pitching moment plotted against incidence can be used to derive such profiles.

The origin of co-ordinates O is at a distance hc behind the leading edge L, and it lies on the line LT joining the leading edge to the trailing edge point T. The x co-ordinate* of any point is expressed in terms of the angular co-ordinate ϑ by the relation

$$x = \frac{c}{2}(1 - 2h) - \frac{c}{2}\cos\vartheta , \qquad (1)$$

so that $\vartheta = 0, \vartheta = \pi$ correspond to the leading and trailing edges respectively. Let it be supposed next that the lift distribution $l(\vartheta)$ corresponding to incidence α has been measured, and that it can be represented by the general expression

$$l(\vartheta) = \rho V^2[A(\alpha)\bar{\Gamma}_0 + B(\alpha)\bar{\Gamma}_1 + C(\alpha)\bar{\Gamma}_2 + \text{etc.}] , \qquad (2)$$

where A, B, C, etc. are functions of incidence only, and where

$$\begin{aligned} &\bar{\Gamma}_0 \equiv 2\cot\frac{\vartheta}{2} , \quad \bar{\Gamma}_1 \equiv -2\sin\vartheta + \cot\frac{\vartheta}{2} , \\ &\bar{\Gamma}_2 \equiv -2\sin 2\vartheta , \quad \bar{\Gamma}_n \equiv -\sin n\vartheta ; \qquad n \geqslant 2 . \end{aligned} \qquad (3)$$

At any incidence, the coefficients A, B, C, etc., can be chosen by collocation to make equation (2) represent approximately the measured lift distribution. On the basis of linearised theory[1], the lift distribution $l(\vartheta)$ as represented by equation (2) corresponds to a downwash distribution w given by

$$\frac{w}{V} = \frac{\partial z}{\partial x} = A(\alpha) + B(\alpha)(\tfrac{1}{2} + \cos\vartheta) + C(\alpha)\cos 2\vartheta + \text{etc.}, \qquad (4)$$

where z defines the shape of the equivalent profile. By integration, it follows from equation (4) that the equivalent profile with leading edge at L is given by

* It is assumed that second-order terms in α can be neglected.

$$\frac{2z}{c} = A + B - \frac{C}{3} - 2h\alpha + \left(A + \frac{B}{2} - C\right)\xi - \frac{B}{2}\xi^2 + \frac{2C}{3}\xi^3, \quad \ldots \quad (5)$$

where $\xi \equiv -\cos\vartheta$, and only the first three terms in equation (4) have been retained for simplicity.

The lift and pitching moment coefficients referred to O are respectively

$$\left.\begin{aligned}\bar{C}_L &= 2\pi A\,,\\ \bar{C}_M(h) &= \pi\left[2A\left(h - \frac{1}{4} + \frac{B-C}{4}\right]\right.\,.\end{aligned}\right\} \quad \ldots \quad (6)$$

The pressure distributions $\bar{\Gamma}_n(n \geqslant 3)$ in equation (2) make no contribution to the lift or the moment. It seems, therefore, that for symmetrical sections sufficient accuracy might be obtained by the use of three or even two terms only. In the calculation of derivatives for cambered aerofoils and control surfaces, however, a larger number of terms would have to be retained. If $C = 0$ is assumed for the symmetrical aerofoil case, $A(\alpha)$ and $B(\alpha)$ can be derived directly by the use of equations (6) from measurements of lift and pitching moment for each incidence α.

In R. & M. 2064[1], $\bar{C}_M$ curves referred to the half-chord and third-chord axis positions are given for a Joukowski aerofoil. From these results the corresponding values of $A(\alpha)$ and $B(\alpha)$ are derived. They are used in section 3 to estimate the pitching-moment stiffness and damping derivatives for the third-chord axis position. A comparison with the measured values given by Bratt[2] is made in section 4.

3. *Oscillatory Motion.*—As a first approximation it is assumed that the equivalent profile at any incidence α is given by

$$\frac{2z}{c} = A + B - 2h\alpha + \left(A + \frac{B}{2}\right)\xi - \frac{B}{2}\xi^2, \quad \ldots \quad (7)$$

where A and B are now known (*see* Fig. 2). If the aerofoil oscillates about a mean incidence ψ such that

$$\alpha = \psi + \theta(t)\,,$$

where $\theta(t)$ defines the oscillation, it follows that the corresponding downwash at any instant is

$$\begin{aligned} w_t &= \frac{dz}{dt} = \frac{\partial z}{\partial t} + V\frac{\partial}{\partial \xi}\left(\frac{2z}{c}\right),\\ &= \frac{\partial z}{\partial \alpha}\frac{d\alpha}{dt} + V\left[A + \frac{B}{2} - B\xi\right]. \quad \ldots \quad (8)\end{aligned}$$

Let $A' \equiv dA/d\psi$, $B' \equiv dB/d\psi$ and consider the part w of w_t which arises from the oscillation. Since $A(\psi + \theta) = A(\psi) + \theta(dA/d\psi)$ approximately when θ is small, it can be shown that

$$\begin{aligned} w &= w_t - V\left[A(\psi) + B(\psi)(\tfrac{1}{2} + \cos\vartheta)\right],\\ &= V[C_0 + C_1(\tfrac{1}{2} + \cos\vartheta) + C_2\cos 2\vartheta]\,, \quad \ldots \quad (9)\end{aligned}$$

where

$$
\left.\begin{aligned}
C_0 &= \left(\frac{3A'}{2} + B' - 2h\right)\frac{c\dot{\theta}}{2V} + A'\theta\,,\\
C_1 &= B'\theta - \left(A' + \frac{B'}{2}\right)\frac{c\dot{\theta}}{2V}\,,\\
C_2 &= -\frac{B'}{8}\frac{c\dot{\theta}}{V}\,.
\end{aligned}\right\} \qquad \ldots \quad (10)
$$

For a simple harmonic oscillation of frequency $p/2\pi$, $\theta = \theta_0 e^{ipt}$ can be substituted in equations (10). Then, by the linear theory for oscillatory motion, it can be deduced that the lift distribution corresponding to the downwash w of equation (9) is simply

$$
l(\omega', \vartheta) = \rho V^2[C_0\Gamma_0 + C_1\Gamma_1 + C_2\Gamma_2]\,, \qquad \ldots \quad (11)
$$

where, in the usual notation,

$$
\left.\begin{aligned}
\Gamma_0 &= 2C(\omega')\cot\frac{\vartheta}{2} + 2i\omega'\sin\vartheta\,,\\
\Gamma_1 &= -2\sin\vartheta + \cot\frac{\vartheta}{2} + i\omega'\sin\vartheta + \frac{i\omega'}{2}\sin 2\vartheta\\
\text{and}\qquad \Gamma_2 &= -2\sin 2\vartheta + i\omega'\left[\frac{\sin 3\vartheta}{3} - \sin\vartheta\right].
\end{aligned}\right\} \qquad \ldots \quad (12)
$$

The lift function $C(\omega')$ is given in terms of Hankel functions by

$$
C(\omega') = \frac{H_1^{(2)}(\omega')}{H_1^{(2)}(\omega') + iH_0^{(2)}(\omega')}\,, \qquad \ldots \quad (13)
$$

where $\omega' = \omega/2 = pc/2V$ is a reduced frequency parameter.

The lift L and pitching moment $M(h)$ about the h-axis due to the oscillation are given by

$$
\left.\begin{aligned}
C_L &= \frac{2L}{\rho c V^2} = 2\pi\left\{C_0\left[C(\omega') + \frac{i\omega'}{2}\right] + \frac{i\omega'}{4}(C_1 - C_2)\right\}\\
C_M(h) &= \frac{2M(h)}{\rho c^2 V^2} = \frac{\pi}{4}\left\{2C_0C(\omega')\left[1 - 2(1 - 2h)\right] - 2i\omega'(1 - 2h)C_0\right.\\
&\quad \left. + C_1\left[1 + \frac{i\omega'}{4} - i\omega'(1 - 2h)\right] + C_2\left[i\omega'(1 - 2h) - 1\right]\right\}
\end{aligned}\right\} \qquad \ldots \quad (14)
$$

where C_0, C_1 and C_2 are given by equations (10). The above formulae reduce to those of vortex-sheet theory when $A' = 1$ and $B' = 0$ are substituted in the expressions for C_0, C_1 and C_2.

By the use of equations (10) and (14) it can be deduced that the moment coefficient for the third-chord axis position is

$$C_M(\tfrac{1}{3}) = \frac{\pi}{4}\,\theta\,\left\{\frac{2C(\omega')}{3}\left[A' + i\omega'\left(\frac{3A'}{2} + B' - \frac{2}{3}\right)\right]\right.$$

$$\left. + B' - \frac{i\omega'}{3}(5A' + B') + \frac{\omega'^2}{12}\left(11A' + \frac{17B'}{2} - \frac{16}{3}\right)\right\}. \quad \ldots \quad \ldots \quad (15)$$

Similarly, the pitching-moment coefficient referred to the quarter-chord axis position is given by

$$C_M(\tfrac{1}{4}) = \frac{\pi}{4}\,\theta\,\left\{B' + \omega'^2\left(\frac{5A'}{4} + B' - \tfrac{1}{2}\right) - 2i\omega'\left(A' + \frac{B'}{4}\right)\right\}. \quad \ldots \quad \ldots \quad (16)$$

It will be noticed that the damping term in this case is proportional to $A' + B'/4$. Hence the damping will change sign at an incidence ψ_c for which

$$A' + \frac{B'}{4} = 0. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

By equations (6) it follows that when $\alpha = \psi_c$ for steady motion the pitching moment about the three-quarter-chord axis will be a maximum*. On the basis of the present theory, therefore, one degree of freedom oscillations about the quarter-chord axis would occur at incidences greater or equal to ψ_c. In general, the incidence at which the damping changes sign depends on ω and h and it can be deduced from the relation derived by putting the imaginary part of equation (14) equal to zero. It should be remembered that the theory is based on the assumption that the oscillations are of small amplitude.

4. *Experimental Comparisons.*—Measurements of derivatives at high mean incidences have been made by Bratt[1], but for the purpose of comparison with the results of the present theory, the amplitudes of oscillation used in the tests are mostly too large. However, one set of measurements for oscillations of 2 deg amplitude at a mean incidence of 10 deg on an aerofoil which stalls at 11·5 deg incidence was made, and in this case sufficient data is given to determine A' and B'. For this particular aerofoil, pitching-moment curves referred to half-chord and third-chord axis positions are given in Figs. 3 and 4, and by the use of equation (6) it can be shown that

$$\left.\begin{aligned} A' &= \frac{3}{\pi}\left[\frac{d\bar{C}_M(\frac{1}{2})}{d\psi} - \frac{d\bar{C}_M(\frac{1}{3})}{d\psi}\right], \\ \text{and} \quad B' &= \frac{2}{\pi}\left[3\frac{d}{d\psi}\bar{C}_M(\tfrac{1}{3}) - \frac{d}{d\psi}\bar{C}_M(\tfrac{1}{2})\right] \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

Hence the coefficients A' and B' can be deduced when the slopes of the pitching-moment curves for two axis positions are known. It would have been better, however, if the lift and the pitching moment about quarter-chord had been measured as then A' and B' would be given directly by equations (6) and not as differences as in equations (18). In the notation of R. & M. 2064[2]

$$C_M(\tfrac{1}{3}) \equiv (a_1 + ib_1)\theta \equiv 2(m_\theta - \omega^2 m_{\ddot{\theta}} + i\omega m_{\dot{\theta}})\theta.$$

and a comparison between the measured values of a_1 and b_1 given by Bratt and those calculated by equation (15) is made in Figs. 5 and 6. Good agreement is obtained for the lower values of the frequency parameter. When ω is very small, however, the measured value of the stiffness

* $C = 0$ is assumed in equations (6).

derivative a_1 [$\equiv 2(m_0 - \omega^2 m_{\ddot{\theta}})$] shows a marked decrease in the 10 deg mean incidence — 2 deg amplitude case. This is due to the fact that the aerofoil stalls at about 11·5 deg incidence (*see* Fig. 3). It is thought that better agreement would have been obtained if the amplitude of oscillation had been smaller, as in the limit, when $\omega \to 0$, the estimated stiffness derivative corresponds to the slope of the experimental pitching-moment curve, namely, $C_M(\frac{1}{3})$ at the mean incidence. For an oscillation of finite amplitude, the effective mean values of A' and B' should perhaps be taken. In general, they do not correspond exactly to the values at mean incidence, particularly for oscillations near the stall.

REFERENCES

No.	*Author*	*Title, etc.*
1	W. P. Jones	Aerodynamic Forces on Wings in Non-uniform Motion. R. & M. 2117. August, 1945.
2	J. B. Bratt and K. C. Wight	The Effect of Mean Incidence, Amplitude of Oscillation, Profile and Aspect Ratio on Pitching Moment Derivatives. R. & M. 2064. June, 1945.

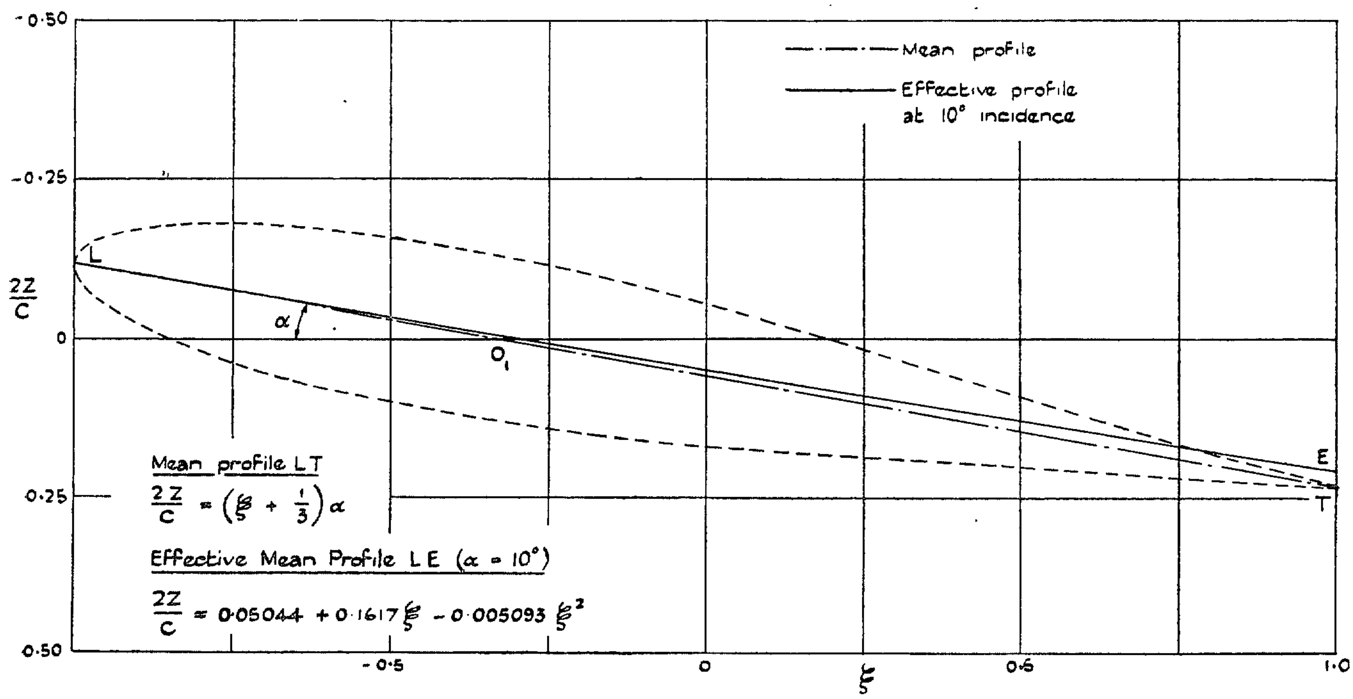

FIG. 2. The effective mean profile at 10 deg incidence.

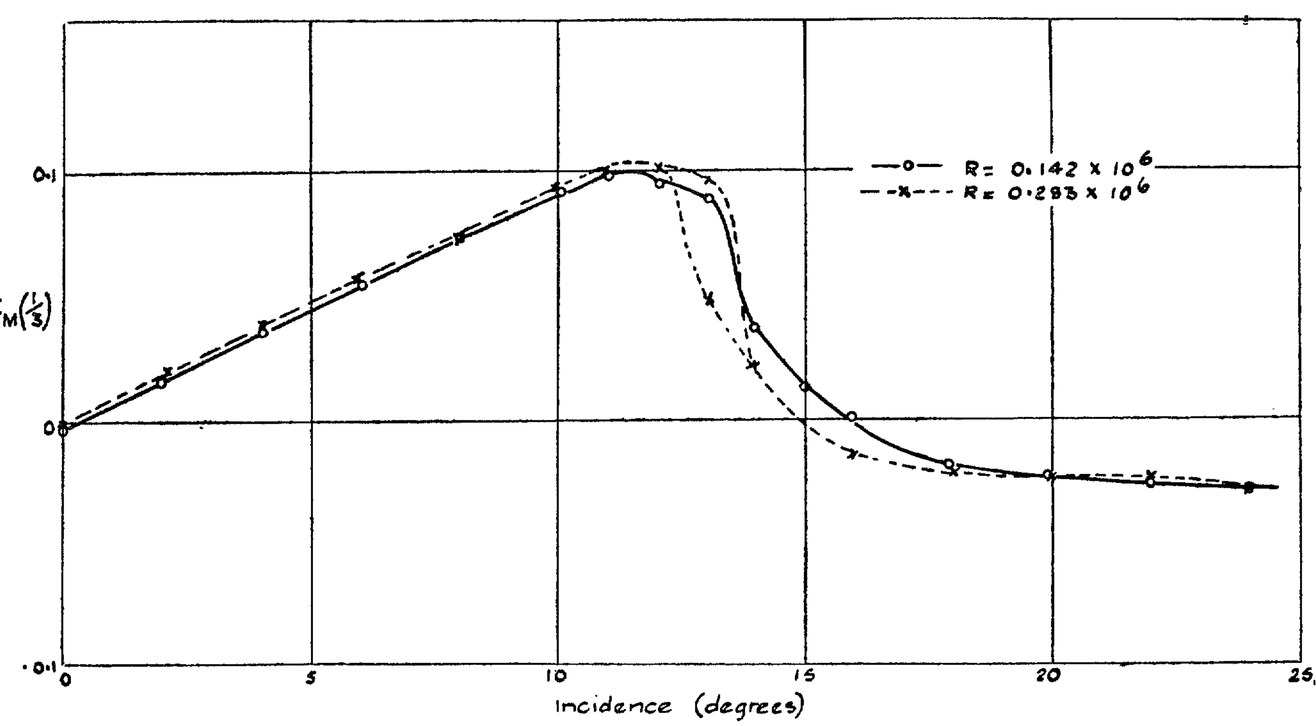

FIG. 3. Static pitching moment coefficient for Joukowski aerofoil.
(Infinite aspect ratio ; third-chord axis.)

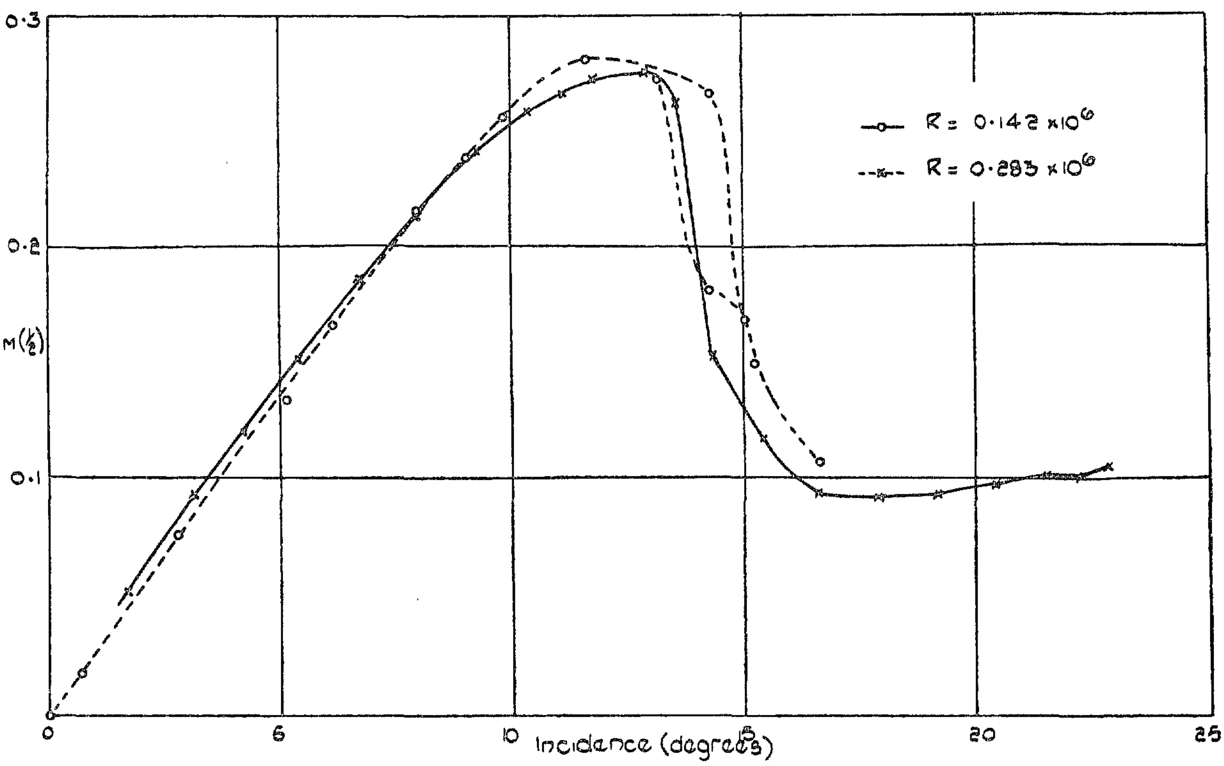

FIG. 4. Static pitching-moment coefficient for Joukowski aerofoil.
(Infinite aspect ratio ; half-chord axis.)

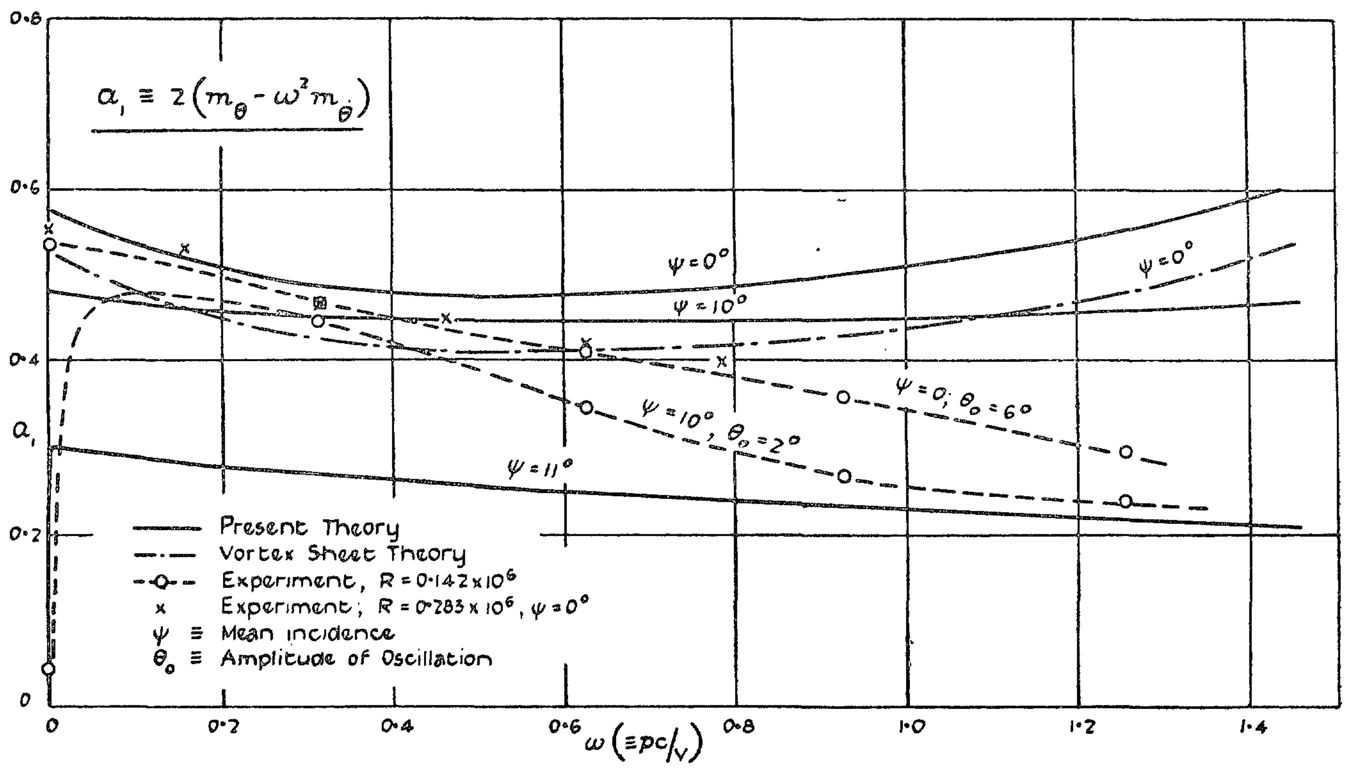

FIG. 5. Pitching-moment stiffness derivative for a Joukowski aerofoil.
(Axis at third-chord.)

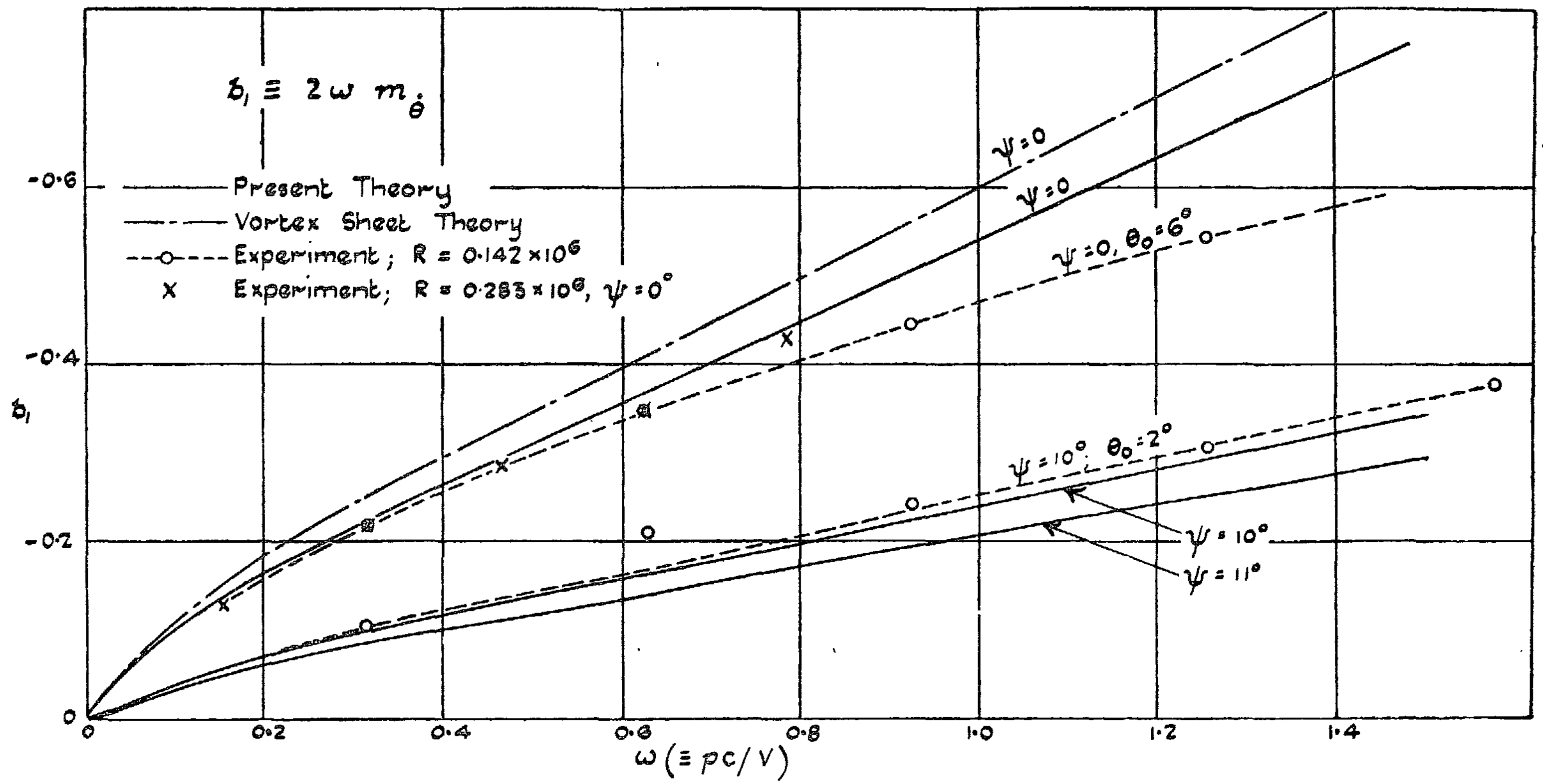

FIG. 6. Pitching-moment damping-derivative for Joukowski aerofoil. (Axis at third-chord.)

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