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SUMMARY

A detailed description is given of a computer program for analysing the inviscid, subsonic compressible pressure distributions around the blades of axial, radial and mixed-flow turbomachines. The flow equations are combined to form a governing, Poisson-type differential equation for the stream function and the numerical solution is obtained by a finite difference matrix method. Solutions for several turbomachines, giving flow patterns and velocity distributions are included.

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CONTENTS

1.0 Introduction

The designer of turbomachinery blading is continually faced with demands for higher duties and improved efficiencies. In order to meet these demands without extensive development testing, sophisticated methods must be used to predict the internal aerodynamics of a design.

In 1952 Wu[1] developed a general theory for the steady, three-dimensional inviscid, subsonic flow through axial, radial and mixed-flow turbomachines. The flow is considered on two intersecting families of stream surfaces, the blade-to-blade (S_1) and hub-to-tip (S_2), Figure 1. The geometry of one family is given by the flow patterns within the other, the complete flow solution being formed by an iterative procedure between the two surfaces. However, the equations are not generally soluble by analytic means and at the time it was impractical to carry out numerical solutions.

With the advent of the high speed digital computer, it has been possible to use numerical techniques to solve Wu's original equations. Marsh[2] programmed a finite difference matrix solution for the flow in the hub-to-tip surface, and Smith[3] adopted a similar approach to solve for the flow in the blade-to-blade surface. Together the two programs provide a solution for the quasi three-dimensional, subsonic, inviscid flow pattern through a blade row. Alternatively they can be used separately to find the flow pattern within any given stream surface.

The purpose of this Report is to present a full outline of the blade-to-blade program established on the computer at the Computer Aided Design Centre, Cambridge. Sections 2.0, 3.0 and 4.0 describe the mathematical model, the finite difference approximations and the solution procedure respectively. This information is not essential for following the instructions on the use of the program given in Section 5.0. The examples in Section 6.0 have been chosen to illustrate the range of problems that the program can solve.

2.0 Mathematical model

2.1 Stream function equation

The equations of motion, continuity, state and first and second laws for the flow within the blade-to-blade stream surface may be combined to give a governing, Poisson-type, differential equation in terms of a stream function. In theory this equation can be solved for any stream surface geometry. However, the S_1 surfaces may be discontinuous at the blade trailing edge and this would produce very complex boundary conditions. Smith[3,4] therefore introduced the assumption that the S_1 surface is a surface of revolution.

This is correct at the hub and the tip of the blade, and is considered to be a close approximation over the rest of the span.

The coordinate system shown in Figure 2 is used in setting up the stream function equation. Any stream surface (which does not turn through an angle of 180^o or more) can be made single valued in y by choosing a suitable value for θ, and hence the same equation may be applied to any type of turbomachine.

The equation for the stream surface then becomes[4]:

$$\nabla^2 \psi = \frac{1}{r^2} \frac{\bar{\partial}^2 \psi}{\partial \phi^2} + \frac{\bar{\partial}^2 \psi}{\partial x^2} = F(\phi, x) \qquad \text{....(1)}$$

where

$$F(\phi, x) = \frac{1}{r^2} \frac{\bar{\partial}}{\partial \phi} (\ln b' \rho) \frac{\bar{\partial} \psi}{\partial \phi} + \frac{\bar{\partial}}{\partial x} (\ln b' \rho) \frac{\bar{\partial} \psi}{\partial x} + \frac{b' \rho}{W_x} \left\{ \frac{W_y}{r} \frac{\bar{\partial}}{\partial \phi} (W_y) + (W_x \sin \theta + W_y \cos \theta) \left(2\omega + \frac{W\phi}{r}\right) \right\}$$

The derivatives $\frac{\bar{\partial}}{\partial x}$ and $\frac{\bar{\partial}}{\partial \phi}$ are those which Smith refers to as special derivatives, taken on the stream surface. The stream function ψ is defined by:

$$\frac{1}{r} \frac{\bar{\partial} \psi}{\partial \phi} = b' \rho W_x \qquad \text{....(2a)}$$

$$\frac{\bar{\partial} \psi}{\partial x} = - b' \rho W_\phi \qquad \text{....(2b)}$$

From Equation (2a) it can be seen that the integrating factor b' is proportional to the local thickness of a thin stream sheet whose mean surface is the S_1 surface considered.

The velocity components W_x and W_y are related by the equation:

$$W_y = - W_x \tan \lambda' \qquad \text{....(3)}$$

This is the geometric condition that the flow follows the stream surface.

2.2 Boundary conditions

The flow field is shown in Figure 3. The blade surfaces and lines of constant ϕ form the upper (ABCD) and lower (EFGH) boundaries, and lines of constant x form the upstream (AE) and downstream (DH) boundaries. The following boundary conditions are imposed:

2.2.1 Upper and lower boundaries

The blade surfaces form streamlines and therefore the stream function along them is constant. For convenience the datum value for the stream function is taken as zero on the lower blade surface and this gives a value on the upper blade surface equal to Q, the mass flow within the stream sheet.

Outside the blade row the flow pattern repeats itself at intervals of one blade pitch. Thus the flow conditions on the upper and lower boundaries are the same for a given value of x for the regions ABFE and CDHG and also:

$$(\psi)_{\text{upper boundary}} = (\psi)_{\text{lower boundary}} + Q \text{ (at constant x)} \qquad \ldots.(4)$$

2.2.2 Up- and downstream boundaries

The flow on the up- and downstream boundaries has been assumed to be uniform. This assumption will not affect the solution near the blades so long as the boundaries are at least one blade pitch away from the blade. If the flow angles are specified, then from Equations (2a) and (2b):

$$\left(\frac{\bar{\partial}\psi}{\partial x}\right)_{\text{boundary}} = \frac{-Q}{r\ \Delta\phi} \tan \alpha_{\text{boundary}} \qquad \ldots.(5)$$

3.0 Numerical techniques

The stream function Equation (1) is not generally soluble by analytic means and so a numerical method has been used. The flow region is covered by a finite difference grid and at each grid point the derivatives are expressed in terms of the values of the stream function at neighbouring points. The resulting finite difference equations form a matrix equation, and this is then solved for the stream function.

3.1 Finite difference grid

The irregular shape of the flow region does not suit the use of a conventional square or rectangular finite difference grid, because the stars on the boundaries would have short limbs and consequently large truncation errors. It is only the boundary conditions which make any problem unique and so these errors are unacceptable. To overcome this problem the flow field is covered by the distorted grid shown in Figure 3. The grid consists of straight lines normal to the x axis and each line has the same number of points equally spaced between the upper and lower boundaries. The spacing of the straight lines may be varied locally to provide a detailed flow pattern in any region (e.g. near the leading edge).

3.2 Finite difference approximations

The stream function equation is defined in terms of special derivatives taken on the stream surface. The finite difference grid also lies in this surface and hence the special derivatives may be treated as normal partial derivatives.

3.2.1 Laplacian operator

When the function $\nabla^2\psi$ is expanded as a two-dimensional Taylor series, it is found that the ten point star shown in Figure 4a is required to give an accuracy of $O[(r\ \delta\phi)^2]$ or $O[\delta x^2]$. However this star includes points on four straight lines and so would give a large bandwidth for the matrix. If the condition that the coefficient of $\frac{\partial\psi}{\partial x}$ must be zero is replaced by the condition that the coefficient of $\frac{\partial^4\psi}{\partial x^2\ \partial\phi^2}$ must be zero, then the star shown in Figure 4b may be used. The stream function equation then becomes:

$$\nabla^2\ \psi + E\frac{\bar{\partial}\psi}{\partial x} = F\ (\phi,x) + E\left(\frac{\bar{\partial}\psi}{\partial x}\right) \qquad \ldots.(6)$$

$$= F'\ (\phi,x)$$

where

$$E = 2\left(\frac{1}{x_{i+1} - x_i} - \frac{1}{x_i - x_{i-1}}\right)$$

3.2.2 The derivative $\frac{\bar{\partial}}{\partial x}$

The first derivative with respect to x is required:

(i) to evaluate the right hand side of the stream function equation (Equation (6))

(ii) to find the component of velocity in the ϕ direction (Equation (2b))

and (iii) to incorporate the up- and downstream boundary conditions (Equation (5)).

Ten point stars of the type shown in Figure 5 are used in the first two cases, giving an accuracy of $O[\delta x^3\ \frac{\partial^4 f}{\partial x^4}]$. However if these stars were used to incorporate the boundary conditions, then the value of the stream function on the boundary would depend on the values at points on the three adjacent straight lines. The result would be an increase in the bandwidth of the matrix operator for points on the straight lines $i = 2$ and $i = m - 1$. To avoid this the value of $\left(\frac{\partial\psi}{\partial x}\right)$ is assumed to be constant near the upstream boundary and to vary linearly with x near the downstream boundary. The errors introduced by this are normally small, but for flows with radial components, it should be noted that angular momentum will not be conserved between the first two and between the last two straight lines.

3.2.3 The derivative $\frac{\bar{\partial}}{\partial\phi}$

The first derivative with respect to ϕ is required:

(i) to evaluate the right hand side of the stream function equation (Equation (6))

(ii) to find the component of velocity in the x direction (Equation (2a)).

The grid spacing is uniform in the ϕ direction and so simple approximations may be used:

General points

$$\frac{1}{r}\left(\frac{\bar{\partial} f}{\partial\phi}\right)_{i,j} = \frac{f_{i,j+1} - f_{i,j-1}}{2r\ (\delta\phi)_{i,j}} + O\ [(r\ \delta\phi)^2] \qquad \ldots.(7)$$

Points on blade surfaces

Lower
$$\frac{1}{r}\left(\frac{\bar{\partial} f}{\partial\phi}\right)_{i,1} = \frac{4f_{i,2} - 3f_{i,1} - f_{i,3}}{2(r\ \delta\phi)_{i,1}} + O\ [(r\ \delta\phi)^2]$$

$$\ldots.(8)$$

Upper
$$\frac{1}{r}\left(\frac{\bar{\partial} f}{\partial\phi}\right)_{i,n} = \frac{f_{i,n-2} - 4f_{i,n-1} + 3f_{i,n}}{2(r\ \delta\phi)_{i,n}} + O\ [(r\ \delta\phi)^2]$$

3.3 Band matrix

The finite difference equations for the operator $\left(\nabla^2 + E\,\frac{\bar{\partial}}{\partial x}\right)$ at each independent (i.e. ψ is not determined by the boundary conditions) grid point form a matrix of order:

$$\left\{(m - 2)\ (n - 1) - (t - \ell - 1)\right\}$$

Equation (6) thus becomes:

$$\underline{M}\ \underline{\psi} = \underline{F} \qquad \ldots.(9)$$

The grid points are arranged in the order:

$$(i,j) = (2,1),\ (2,2)\ \ldots..\ (2,n-1),\ (3,1)\ \ldots..\ (m-1,\ n-1)$$

so that the matrix $\underline{M}$ is banded. The maximum bandwidth occurs for points outside the blade row on the upper and lower boundaries of the calculating region (Figure 6). Using the repeat boundary condition, the maximum difference between the numbers of the centre and another point on the star is $(2n - 3)$, giving a bandwidth of $(4n - 5)$. Only the elements within this band are stored (Figure 7).

4.0 <u>Solution procedure</u>

The matrix Equation (9) is non linear and therefore it is solved by an iterative process:

(i) the right hand side is assumed constant and the linear equation

$$\underline{M}\ \underline{\psi} = \underline{F} = \text{constant} \qquad \ldots.(10)$$

is solved for $\underline{\psi}$

(ii) the solution for the stream function is used to calculate the gas state at each grid point

(iii) a new value for the right hand side is calculated from the gas state

The process is continued until two solutions agree to within a specified tolerance;

$$\frac{1}{Q}\left(\psi_{i,j} - \left(\psi_{i,j}\right)_{p-1}\right)_{max} \leqslant \text{TOL} \qquad \ldots.(11)$$

where p is the current iteration, or the specified number of iterations have been completed.

The first run of any geometry requires an initial guess to start the iteration. For cascade or cylindrical stream surfaces, the flow is assumed to be uniform between two dividing streamlines which are straight up- and downstream of the blade row and follow the blade surfaces within the row. This provides an initial guess for the stream function and so the problem enters the iterative loop at point (ii). For other flows the right hand side is set to zero and the loop is entered at point (i).

The final solution for the stream function is stored on magnetic tape (see Appendix I) so that it can be used as the starting point for subsequent runs of the same geometry (i.e. enter loop at point (ii)). Compressible runs require a fairly accurate first guess if the problem is to converge, and so they are always started from a previous solution.

4.1 Solution of matrix equation

The matrix $\underline{M}$ in Equation (10) is of order $\{(m - 2)\ (n - 1) - (t - \ell - 1)\}$ and bandwidth $(4n - 5)$: typical values of m and n are 90 and 13 respectively, giving an order of $\sim$1000 and bandwidth of 47. This is too large to be kept in the core store of present day computers and therefore use has been made of two matrix solving routines which operate on a small section of the matrix at a time (Figure 7) while the rest is kept on backing stores:

(i) Routine 1 splits the matrix into the product of upper and lower (band) triangular matrices, so that

$$\underline{U}\ \underline{L}\ \underline{\psi} = \underline{F} \qquad \ldots.(12)$$

(ii) Routine 2 solves this equation for $\underline{\psi}$ using a forward and backward substitution process.

To ensure numerical stability, under relaxation is used in calculating the new value of the stream function from this solution:

$$\psi_p = \psi_{p-1} + rf\left(\psi - \psi_{p-1}\right) \qquad \ldots.(13)$$

where ψ is the solution to Equation (10).
The relaxation factor rf lies in the range 0.2 to 0.8, depending on the flow Mach number.

4.2 Calculation of gas state

Given a solution for the stream function throughout the flow field, the products ρW_x, ρW_y and ρW_ϕ may be calculated from Equations (2) and (3), using the finite difference approximations for $\frac{\partial}{\partial\phi}$ and $\frac{\partial}{\partial x}$. The problem is then to calculate the static density ρ.

For isentropic flow:

$$\frac{\rho}{\rho_o} = \left(\frac{h}{H_o}\right)^{\frac{1}{\gamma-1}} \qquad \ldots.(14)$$

This equation for ρ could be solved directly since the stagnation conditions at inlet are known and the local static enthalpy may be found in terms of ρ.

However, Smith has found that a more stable approach is to let the density lag by one iteration.

For the subject blade-to-blade flows, it can be shown[3,4] that the rothalpy is conserved along streamlines:

$$I = H - \frac{2\omega\ r\ V_\phi}{G} = \text{constant} \qquad \ldots.(15)$$

Using the energy equation

$$h = H - \frac{V^2}{G}$$

Equation (15) becomes:

$$h = I + \frac{\omega^2\ r^2}{G} - \frac{W^2}{G} \qquad \ldots.(16)$$

Therefore, substituting in Equation (14):

$$\frac{\rho}{\rho_o} = (a - b)^{\frac{1}{\gamma-1}} \qquad \ldots.(17)$$

where $$a = \frac{I + \frac{\omega^2\ r^2}{G}}{H_o}$$

and $$b = \frac{W^2}{G\ H_o}$$

The term W^2 is found from the relation:

$$W_p^2 = \frac{\left(\rho\ W_x\right)_p^2 + \left(\rho\ W_y\right)_p^2 + \left(\rho\ W_\phi\right)_p^2}{\rho_{p-1}^{\ 2}} \qquad \ldots.(18)$$

This is what is meant by the density lagging by one iteration. As the overall calculation converges, ρ_{p-1} approaches ρ_p.

The velocity components are then calculated using ρ_p.

Wu has shown that the condition for the stream function equation to remain elliptic is that the local relative Mach number is less than unity. Therefore if the local flow becomes supersonic, the density is set to the value corresponding to a Mach number of unity. This artifice enables solutions to be obtained for transonic flows: the solutions are not strictly valid, but they have been found to be useful (see Section 6.7).

The value of velocity for unity Mach number is given by:

$$W^2_{M=1} = (\gamma - 1)\,\frac{G}{2}\,h \qquad \text{....(19)}$$

Substituting for h from Equation (16) and rearranging:

$$W^2_{M=1} = \frac{\gamma - 1}{\gamma + 1}\,(G\,I + \omega^2\,r^2) \qquad \text{....(20)}$$

Hence the complete solution for the static density is:

$$\frac{\rho}{\rho_o} = (a - b)^{\frac{1}{\gamma - 1}} \qquad \text{....(21)}$$

where

$$a = \frac{G\,I + \omega^2\,r^2}{G\,H_o}$$

and

$$b = \frac{W^2}{G\,H_o} \quad \text{for} \quad \frac{W^2}{G\,H_o} \leqslant a\left(\frac{\gamma - 1}{\gamma + 1}\right)$$

$$= a\left(\frac{\gamma - 1}{\gamma + 1}\right) \quad \text{for} \quad \frac{W^2}{G\,H_o} > a\left(\frac{\gamma - 1}{\gamma + 1}\right)$$

4.3 Calculation of new right hand side

Given the values of the stream function, density and components of velocity at each grid point, the new right hand side may be calculated using the finite difference stars for $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial \phi}$.

5.0 The computer program

The matrix blade-to-blade program has been written in segmented form (Appendix II) to economise on core store. For a maximum grid size of m = 90 and n = 13, each segment requires 20K words of core or less. This grid size has proved ample for all types of axial blade rows and for simple radial flow machines. If necessary the segments can be re-dimensioned to allow a finer grid for mixed flow problems, though this will increase the cost.

5.1 Data preparation

The data required by the program falls into three sections: titles, blade and stream surface geometry, and flow data. The specifications of each are given in Appendix III. Particular care should be taken to ensure that the blade data are smooth, since this is a potential flow solution - example 6.4 below illustrates the effect of sudden changes in the blade surface curvature on the solution. Limitations on the grid geometry and suggested spacings for the straight lines are given in Figures 8 and 9.

Two data preparation programs are available to prepare the geometric data for cascade flow problems. The first (Appendix IV) is suitable for blades with blunt leading edges (Figure 10) which are specified by a table of coordinates. The second (Appendix V) produces data for double circular arc compressor blades.

Each new blade geometry and each solution are automatically stored on a master magnetic tape (henceforth called the program tape) (Appendix I). Further runs using the same geometry will then read this section of the data from the program tape (i.e. only the title and flow data sections are required) and they may also read a previous solution to use as a first guess to start the solution procedure. In order to safeguard this information it is advisable to keep at least one copy of this tape, and to update it regularly using the 'COPY' command.

5.2 Running instructions

The job has been split into three parts, to suit the CADC Atlas, but the user sets up only the first; the other two are run automatically.

Typical running instructions are given in Appendix VI.

5.3 Output

The program produces output on both the line printer and the graph plotter. The titles used are defined in the Notation.

5.3.1 Line printer output

(i) Input data. The problem specification and input data are printed out by program 4FIRST.

(ii) Convergence record. The maximum difference (Equation (11)) between two successive solutions for the stream function is printed out by program SOLUTION.

(iii) Results. The results are printed out by program 6RES. If the problem has converged to the specified tolerance, they are produced for the last iteration only: for a problem which has not converged to the specified tolerance they are produced for the last two iterations. If full output is specified then the flow conditions are calculated at each grid point (Appendix VII). Otherwise only the blade surface velocity distribution is printed out.

(iv) Summary of program tape. The program writes out a list of the titles of the arrays stored on the program tape (Appendix I).

5.3.2 Graph plotter output

The graph plotter is used to plot out the blade surface velocity and Mach number (compressible runs only) distributions. The streamline pattern is calculated using linear interpolation for the stream function between the grid points on each straight line, and the streamlines are plotted out in the M-PHI plane.

5.3.3 Diagnostic output

The running instructions are written to the file specified as they are executed, and program warning and error messages will also be written to this file (see Appendix VIII).

6.0 Numerical examples

The matrix program may be used to analyse the inviscid, subsonic compressible flow around the blades of any cascade or turbomachine. The following examples have been chosen to illustrate some of the problems which can be solved. In the first four cases matrix solutions are compared with solutions from other theoretical methods which make the same basic assumptions about the flow. This shows that the matrix program works correctly and demonstrates the overall level of accuracy which is achieved. The fifth example was chosen to show that the program can also deal with relative eddy motion. Finally the sixth and seventh cases give comparisons with experimental results to demonstrate the agreement achieved with real flows.

6.1 Impulse turbine cascade

The first example is for simple, incompressible flow through a two-dimensional cascade. The profile is typical of an impulse type, turbine blade, with 112° camber and the cascade has a pitch/chord ratio of 0.59 and 101° flow deflection. Gostelow[5] has obtained an exact solution for this case, and the matrix program was run using the same flow angles. The two blade surface velocity distributions are compared in Figure 11: they show excellent agreement.

6.2 70° camber blade in cascade

The second example is also for the incompressible flow through a two-dimensional cascade, but at very high negative incidence (-70°). The profile is a turbine blade with 70° camber, and the cascade has a pitch/chord ratio of 0.9.

The matrix solution is compared with an exact solution by Gostelow[5] in Figure 12: the only discrepancy occurs in the region of the blade trailing edge. For this region the exact profile coordinates are a long way apart and it is probable that errors in interpolating for the coordinates for the matrix program have caused the difference. There seems no reasonable doubt that complete agreement would have been obtained if the exact aerofoil shape had been more fully defined.

This example is far more severe than could occur in practice, but it shows that there is no problem in analysing high incidence flows. The streamline pattern calculated by the matrix program is shown in Figure 13 and it may be seen that the leading edge stagnation point is well round on the suction surface.

6.3 Conical flow past stationary and rotating blade rows

The above comparisons are for plane cascade flows. Three-dimensional effects have two main forms; change of stream surface thickness, and change of stream surface radius with the resulting Coriolis forces for rotor rows. To check that the program deals correctly with these aspects it was used to analyse the incompressible flow on a conical stream surface through a turbine row for both stationary and rotating cases. The cone half angle was 45° and the stream surface thickness varied linearly with axial distance, giving an increase of 44.6 per cent beteen the up- and downstream boundaries.

The matrix solutions are compared with the exact Wilkinson/Martensen[6] singularities solutions in Figures 14 and 15. The abscissa ζ is defined as

$\int \frac{1}{r}\, dm$, and gives a conformal transformation of the flow in the meridional stream surface (which generally has double curvature) onto a flat plane in which all angles are preserved. In both the stationary and rotating cases the only significant difference between the two solutions is at the leading edge, and this is probably due to the coarse grid used for the matrix solution. (The spacing at the leading edge was set to 4.2 per cent of an axial chord to avoid the need for interpolation between the data points.)

6.4 Axial compressor stator blades

These comparisons are for the compressible flow past two compressor blade profiles which were designed using a prescribed velocity distribution (PVD) program[7]. In both cases the duty is:

Inlet Mach number	=	0.85
Inlet flow angle	=	50°
Outlet flow angle	=	0°
Pitch/chord ratio	=	0.45

The PVD program calculates the effective shape required to fulfil this duty and produce the specified velocities. The blade surface boundary layers are then estimated and subtracted from the effective shape to give the blade profile.

The blade surface Mach number distributions calculated by the matrix program are compared with those specified as input to the PVD program in Figures 16 and 17. There is generally good agreement, except that the matrix solution is not quite smooth and it predicts lower Mach numbers over the back half of the blades. These differences occur because the matrix program is using the blade profile and not the effective shape found by the PVD program: the matrix solution is therefore for a thinner profile which is not perfectly smooth.

These examples illustrate the ability of the matrix program to produce answers for flows with supersonic patches.

6.5 Radial inflow turbine

The fifth example was chosen to illustrate the ability of the matrix program to deal with relative eddy motion. It is a simplified case of a radial flow machine with radial blades of zero thickness[8]. The flow is incompressible and has no axial component.

The blade relative velocity distributions and the streamline pattern (plotted in the r-$r\phi$ plane) are shown in Figures 18 and 19. Both clearly show the region of reverse flow.

6.6 NASA turbine stator blade

This example compares a matrix solution with the experimental results for the compressible flow through an annular cascade. The profile is the mid section of a NASA turbine stator blade[9] and it is typical of a high temperature section in having blunt leading and trailing edges to allow for cooling passages.

The blade was analysed for the design value of inlet Mach number (0.212) and inlet and exit gas angles (0° and -67°). The flow was assumed to be two-dimensional. The computed distributions of blade surface Mach number are compared with the values calculated from the measured static pressure distribution in Figure 20. There is generally good agreement, the main discrepancies occurring over the last 50 per cent of the blade suction surface. This region is bounded by the trailing edge stagnation streamline and it is considered that the errors are due to the fact that the blade surface boundary layers are neglected in the matrix analysis. The oscillation of the computed distribution in this area is caused by the blade surface being slightly uneven. It is not apparent in the experimental results because of the smoothing effect of the boundary layer.

6.7 Axial turbine stator blade

The last example is for compressible flow through a cascade of turbine blades with flow contraction due to wall boundary layer growth. The blade profile was the mid-span section of the second stage stator of a two stage turbine[10]. The experimental cascade results for blade surface Mach number are compared with two matrix solutions in Figure 21.

The first matrix solution used the outlet angle predicted by the Ainley/Mathieson method (-49.95° compared with the experimental value of -48.56°) and assumed that the stream surface thickness was constant. There is reasonable agreement over the first third of the blade, but the matrix solution then diverges from the experimental results as the boundary layer blockage increases.

For the second solution, the stream surface thickness was varied in order to give the measured value of axial velocity ratio (this required a contraction of 8 per cent), and the experimental value for the outlet angle was used. Since there was no solution available for the S_2 (hub-to-tip) plane, the stream surface thickness within the blade row was assumed to vary linearly with axial distance. Comparison of this solution with the experimental results shows that the predicted Mach number over the first half of the suction surface is too high. Apart from this the agreement is good and the peak Mach number is well predicted, despite the supersonic patch covering nearly half of the suction surface. This suggests that most of the stream surface contraction should in fact be concentrated over the second half of the blade in the region of suction surface diffusion. The changes in slope of the matrix solutions at $M = 1$ are due to the assumption that the density is constant for $M > 1$ (Section 4.2).

This comparison with experimental results illustrates that solutions with transonic flows can be physically meaningful.

7.0 Conclusions

A computer program for analysing the blade-to-blade flow patterns in turbomachines has been described. The theory is based on the earlier work of Wu[1] and the numerical solution is obtained by finite difference approximations to the governing equations.

The program has been demonstrated to produce good estimates for a wide range of flows, including the effects of compressibility, stream tube contraction, change of stream tube radius, Coriolis forces and relative eddy motion.

Although strictly only valid for subsonic flows, the program can produce solutions for flows with supersonic patches, and the results agree well with other theoretical solutions and experimental data.

NOTATION

Coordinate systems (Figure 2)

r, ϕ, z	radial, circumferential and axial coordinates (ϕ is measured in direction of rotation)
x, y	Cartesian coordinates obtained by rotating the r, z axes through an angle θ

Finite difference grid (Figure 3)

m	total number of straight lines
n	number of grid points per straight line
ℓ	number of straight lines upstream of blade row
t	number of straight lines upstream of blade row, plus number of straight lines within blade row, plus one
i	number of a general straight line (starting from upstream boundary)
j	number of a grid point along a straight line (starting at lower boundary)
$\delta\phi$	local grid spacing in ϕ direction
δx	local grid spacing in x direction
$\Delta\phi$	blade pitch in radians (= 2π/number of blades)

Stream surface (Figure 2)

$\underline{n}$	unit vector normal to stream surface
λ	angle of stream surface (in meridional plane) to z axis
b	integrating factor used in defining stream function; proportional to the local radial thickness of the stream surface

Fluid properties

ψ	stream function
W	relative velocity
V	absolute velocity
ω	angular velocity
M	relative Mach number
ρ	static density
ρ_o	stagnation density (absolute) at inlet
H	stagnation enthalpy (absolute)
H_o	stagnation enthalpy at inlet
h	static enthalpy

I	rothalpy
Q	mass flow within stream surface
α	flow angle (relative)
γ	ratio of specific heats
G	constant relating velocity and enthalpy $\left(G = \dfrac{V^2}{H - h}\right)$

Other

rf	relaxation factor
p	number of current iteration
'	primed values are relative to the xy axes

Computer output

X COORD	x coordinate of calculating plane
Y	y coordinate of calculating plane
M COORD	distance of calculating plane from upstream boundary, measured along a meridional streamline
PHI	ϕ coordinate of a grid point (in radians)
LAMBDA	tan λ' (i.e. measured from x axis)
STHICK	stream surface thickness/upstream thickness
BTHICK	blade thickness in radians
VX	component of velocity in x direction
VY	component of velocity in y direction
WU	component of relative velocity in ϕ direction
VU	component of absolute velocity in ϕ direction
W	relative velocity
V	absolute velocity
ABS M	absolute Mach number
REL M	relative Mach number
STR FN	stream function, non dimensionalised by the mass flow Q
STAG H	absolute stagnation enthalpy
STAT H	static enthalpy
STAG PR	absolute stagnation pressure/absolute stagnation pressure at inlet
STAT PR	static pressure/absolute stagnation pressure at inlet
DENSITY	static density
ABS ANG	absolute flow angle $= \tan^{-1}\left(\dfrac{VU}{VX}\right)$

REL ANG — relative flow angle $= \tan^{-1}\left(\frac{WU}{VX}\right)$

V/VD — absolute blade surface velocity/absolute downstream velocity

W/WD — relative blade surface velocity/relative downstream velocity

CP — static pressure coefficient $= \dfrac{p - p_d}{P_d - p_d}$

where p = local static pressure

p_d = downstream static pressure

P_d = absolute downstream total pressure

SL — percentage surface length along blade

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No.	Author(s)	Title, etc.
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APPENDIX I

Program tape

The grid data, band matrix, influence coefficients for $\frac{\overline{\partial}}{\partial x}$ and previous solutions for each geometry are stored on a master magnetic tape, known as the program tape. The information is written in binary form and each array is identified by a title of up to eight alphanumeric characters. Most of the titles are set within the program, but different ones must be specified (in the input data) to identify each set of geometric data (i.e. grid, band matrix, and influence coefficients) and each solution, so that they can be accessed by future runs.

Example

Starting with a new program tape, three problems are run:

(i) First run of geometry A, the geometric data being labelled as 'GEOA' and the result as 'SFØØ1'.

(ii) First run of geometry B, the geometric data being labelled as 'BGEOMETY' and the result as 'SFØ11'.

(iii) Second run of geometry A (NB blade section of input data is not required) producing result 'SFØØ2' (this run may use solution 'SFØØ1' as a first guess).

The arrays stored on the tape will then be:

GEOA

IGV

Y

X

DELX

DELPHI

E

PHI

$$\left.\begin{array}{l}\text{BANDDATA}\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \text{BANDDATA}\end{array}\right\} \text{Band matrix stored in } \left\{\frac{(m-2)\ (n-1) - (t-\ell-1)}{2n-3} + 1\right\} \text{ sections}$$

(see Figure 7)

N1

NN1

N2

N3

N4

24

N5

N6

NN6

IC
.
.
.
.
IC

$(m - 2)$ sections of influence coefficients for finite difference stars for $\frac{\overline{\partial}}{\partial x}$

SFØØ1

BGEOMETY

IGV

Y

X

DELX

DELPHI

E

PHI

BANDDATA
.
.
.
.
BANDDATA

N1

NN1

N2

N3

N4

N5

N6

NN6

IC
.
.
.
.
IC

SFØ11

SFØØ2

ENDFILE

A summary of this form is produced after each run (the titles BANDDATA and IC are written out only once for each set of geometric data). With the present program dimensions ($m \leq 90$; $n \leq 13$), each set of geometric data occupies approximately 130 blocks and each new solution approximately $2\frac{1}{2}$ blocks. There are 4000 blocks on each magnetic tape.

APPENDIX II

Computer program organisation

The matrix program is written in segmented form to minimise the amount of core store required. Disc and magnetic tape backing stores are used to store data and transfer it from one segment to the next (see below).

In order to suit the CADC system, the segments are run in three different jobs, the second and third being set up automatically at the end of the previous job (see Appendix VI).

Job 1

Program File	Description
NATGASØ1/S1/1IND	Reads in all data, and sets up the finite difference grid on the program magnetic tape (first run only).
NATGASØ1/S1/2BAND	Calculates the weighting coefficients for the finite difference approximations for $\frac{\partial}{\partial x}$ and $(\nabla^2 + E\frac{\partial}{\partial x})$. The coefficients are generated in blocks and stored on the program magnetic tape (if this is an update run of a geometry, this information will already be stored on the tape and this segment does nothing).
NATGASØ1/S1/TRANSFER	Transfers the band matrix from the program tape onto the input magnetic tape for the matrix routines and the other data on the program tape onto a disc backing store.

Job 2

NATGASØ1/S1/3RED	Uses the CADC routine REDUCE to split the band matrix into the product of two triangular matrices which are stored on two temporary magnetic tapes (7 and 8). It opens the disc file /MATRIX/INTER to carry details of changes in the matrix (for maximum accuracy) into SOLVE.
NATGASØ1/S1/4FIRST	(i) Writes out the grid data on the line printer (ii) Generates the first guess to start the iterative solution.

Program File		Description
NATGASØ1/S1/SOLUTION	Iterative	Solves the band matrix using CADC routine SOLVE.
NATGASØ1/S1/VELOCITY		Calculates the velocity components and density from the stream function.
NATGASØ1/S1/RHS	Solution	Generates the next right hand side.

NOTE The segments SOLUTION, VELOCITY and RHS are stored in compiled form on files /DUMP/SOLUTION,/DUMP/VELOCITY, and /DUMP/RHS, so that they may be overlaid.

Job 3

NATGASØ1/S1/6RES	(i) Calculates and prints out the line printer results (see Section 5.3). (ii) Writes the final solution for the stream function onto the program magnetic tape.
NATGASØ1/S1/7GRAPH	Plots out graphs of the blade surface velocity and Mach number distributions, and a picture of the streamline pattern.

The following backing stores are used during a run:

Logical Number	Type	Description
2	Magnetic tape	Input tape for matrix solving routines: set up in transfer (Job 1) for use in 3RED (Job 2) and then closed.
2	Temporary disc	Holds solution to stream function equation: set up in SOLUTION (Job 2). (A disc BSD is used to avoid over-writing the band matrix stored on magnetic tape 2.)
3	Temporary disc	Temporary storage for the weighting coefficients of the finite difference stars in 2BAND (Job 1).

Logical Number	Type	Description
3	Temporary disc	Holds next approximation for right hand side of stream function equation: set up in RHS for use in SOLUTION (Job 2).
4)) 5)	Temporary disc	Temporary storage used by matrix solving routines REDUCE AND SOLVE (Job 2).
6	Named disc	Disc file called /MATRIX/INTER (size = 5 blocks). Set up in 3RED and used by SOLUTION (Job 2).
7)) 8)	Magnetic tapes (temporary)	Storage for upper and lower triangular matrices: set up in 3RED for use in SOLUTION (Job 2).
9)) 10)) 11)	Named disc files	Storage for the compiled versions of segments SOLUTION, VELOCITY and RHS.
12	Named disc	Holds data from the program tape which is needed in the second job. It is set up in TRANSFER and deleted after the third job (maximum size = 60 blocks).
13	Temporary disc	Carries information between SOLUTION, VELOCITY and RHS during iterative solution for stream function (Job 2).
14	Magnetic tape	Program tape.
15	Named disc	Carries flow data between all segments: set up in 1IND (maximum size = 10 blocks).

APPENDIX III

Data format for matrix program

(i) Title section of data for matrix blade-to-blade program

Fortran Name	Format	Description
RNAME(I),I=1,1Ø	1ØA4	Job title - up to 40 alphanumeric characters
TITLE(I),I=1,2	2A4	Title identifying geometric data on the program magnetic tape - up to 8 alphanumeric characters (see Appendix I)
ISPEC	I	Control character =Ø for the first run of a problem i.e. blade geometry section included on data tape =1 for an update run, i.e. the finite difference grid, band matrix and influence coefficients have been set up by a previous run and are stored on the program tape under 'TITLE(I),I=1,2'

(ii) Blade geometry section of data for matrix program

NB This section

(i) is only required for ISPEC=0

(ii) can be produced by the data preparation programs - Appendices IV and V

Fortran Name	Format	Description
L	4I	Number of straight lines upstream of blade row = ℓ
IT		Number of first straight line downstream of blade row = t
M		Total number of straight lines = m ($m \leqslant 9Ø$)
N		Number of points per line = n ($6 \leqslant n \leqslant 13$)

Fortran Name	Format	Description
S	2F	Typical blade pitch (used to non-dimensionalise lengths)
THETA		Angle in radians through which x and y axes are turned from r and z axes (see Figure 2 for sign convention)
IORD	5I	Stream surface angle control character =Ø if λ is measured from the x axis =1 if λ is measured from the z axis
NBL		Number of blades in row (=Ø for cascade flow)
IES		Grid spacing control character =Ø for non-uniform spacing =1 for uniform spacing
ICT		Stream surface thickness control character =Ø for varying thickness =1 for uniform thickness
ICR		Stream surface radius control character =Ø for varying radius =1 for cascade or cylindrical flow
SPEED	1F	Rotational speed parameter =Ø for stationary blades =1 for rotating blades
X(I) I=1,M	6F	x coordinate of straight line. Any positions may be chosen so long as skew grids, high aspect ratios and sudden changes in spacing are avoided (see Figure 8). Figure 9 gives typical positions.
Y(1,I)		tan λ (see IORD above for datum and Figure 2 for sign convention)
Y(2,I)		Stream surface thickness b' (measured parallel to y axis)

Fortran Name	Format	Description
PHI(I,1)		ϕ coordinate of lower boundary in radians (measured in the direction of rotation) NB Lines EF and GH in Figure 2 are at constant ϕ.
A		y coordinate of straight line
B		Blade thickness in radians

(iii) Flow data section of data for matrix program

PGA	2F	Tangent of inlet relative gas angle = $\left(\frac{W_\phi}{W_x}\right)_u$
OGA		Tangent of outlet relative gas angle = $\left(\frac{W_\phi}{W_x}\right)_d$
G	6F	Constant relating velocity and enthalpy $\left(\frac{V^2}{H - h}\right)$
GAMMA		Ratio of specific heats GAMMA > 10 - incompressible run, program calculates its own first guess 0 < GAMMA < 10 - compressible run - the actual value for the fluid should be used GAMMA < 0 - incompressible run, starting from the previous result defined by the title 'SF1(I),I=1,2' NB For GAMMA < 10 this must be an update run i.e. ISPEC=1

Fortran Name	Format	Description
SPEED		Angular velocity in radians per second. For a stator blade a typical value should be given since S.SPEED is used to non-dimensionalise velocities within the program.
STH		Absolute stagnation enthalpy at upstream plane
STD		Absolute stagnation density at upstream plane
VELI		Inlet velocity relative to blade row
RF	2F,I	Relaxation factor rf (see Equation (14)). A useful guide is RF = 1 - M for 0.2 < M < 0.8 ($M = 0.5\ (M_{inlet} + M_{exit})$)
TOL		Convergence criterion for stream function, usually between 0.001 and 0.0001.
MAXIT		Maximum number of iterations, usually 20. MAXIT must be chosen such that the second job does not run out of computation time (see Appendix VI)
SF1(I),I=1,2	2A4	Title identifying previous result to be used as a first guess - up to 8 characters (see Appendix I) For GAMMA > 10, a dummy should be used
TRIGGER	I	Line printer output control character =∅ for full output =1 for limited output
SF2(I),I=1,2	2A4	Title to be used to identify the results of this run - up to 8 characters (see Appendix I)

APPENDIX IV

Data preparation program for blades specified by coordinates

This program produces the blade data for the matrix program for cascade or cylindrical flows past blade sections specified by a table of coordinates. It uses a spline fit to interpolate between the points, and the data is produced in the format required by the matrix program.

1. Blade specification

The blade coordinates may be specified with respect to any convenient axes (Figure 10). The spline fit cannot deal with regions of high curvature, and so the program can only be used for blades with blunt leading edges. The limiting condition is given in Figure 10a. At the trailing edge, the blade must be defined by the radius and centre of the trailing edge circle and the two tangent points at which the profile merges into the circle (Figure 10c).

2. Positions of straight lines

The positions of the straight lines may either be specified in the input data, or the standard positions stored in the program may be used (Figure 9). In both cases the program checks the distortion of the grid within the blade row and inserts an extra straight line if the skew is greater than 70 per cent (Figure 8).

3. Stream surface thickness distribution

The stream surface thickness distribution may be either uniform throughout, or three linear variations with x between values specified at the upstream plane, the blade leading edge, the blade trailing edge and the downstream plane.

4. Data input

The data required is as follows:

Fortran Name	Format	Description
TITLE(I),I=1,10	10A4	Job title - up to 40 alphanumeric characters
PITCH MP MS STAGGER LETRIG	F,2I, F,I	Blade pitch Number of points given on pressure surface Number of points given on suction surface Angle between x-y and x'-y' axes (see Figure 10b) in degrees Control character = 0 if leading edge tangent point is not given = 1 if leading edge tangent is specified
XP(I),YP(I), I = 1,MP	2F	Coordinates of points on pressure surface in x'-y' system. Last point is as specified in Figure 10c
XS(I),YS(I), I = 1,MS	2F	Coordinates of points on suction surface in x'-y' system. Last point is as specified in Figure 10c
XCEN YCEN TERAD	3F	Coordinates of centre of trailing edge circle in x'-y' system Radius of trailing edge circle.
If LETRIG = 1 XLE YLE	2F	Coordinates of leading edge tangent point in x-y system
T1,T2,T3,T4	4F	Stream surface thicknesses at: upstream plane, blade leading edge, blade trailing edge and downstream plane
N	I	Number of points on each straight line
NUS	I	Number of upstream calculating planes. If set to zero the upstream planes will be positioned as in Figure 9

Fortran Name	Format	Description
<u>If NUS > 0</u>		
USPL(I),I=1,NUS	10F	Positions of upstream planes as a percentage of axial chord, measured from the leading edge in the upstream direction
NP	I	Number of calculating planes within the blade row (incl leading and trailing edge points). If set to zero the planes will be positioned as in Figure 9.
<u>If NP > 0</u>		
PG(I),I=1,NP	10F	Positions of planes within blade row as a percentage of axial chord, measured from the leading edge in the downstream direction. The first will be 0 and the last 100.
NDS	I	Number of calculating planes downstream of blade row. If set to zero the planes will be positioned as in Figure 9.
<u>If NDS > 0</u>		
DSPL(I),I=1,NDS	10F	Positions of downstream planes as a percentage of axial chord, measured from the trailing edge in the downstream direction.

5. <u>Running instructions</u>

The program is stored on file NATGAS/TKH/PROG, typical running instructions would be:

```
COMMAND(7117NATGASØ3/TKH DATA PREPARATION)
COMP 2 MINS
EXEC 1Ø MINS
LIMSTORE 32 K
LIMOUT 3ØØ
OUTPUT Ø TO FILE (NATGAS/TKH/DIAGS)
NOTES
SWITCH LARGE
```

```
.ASA(S 16K) NATGAS/TKH/PROG
.LIBRARY CAD/GINO/GRAPH CAD/GINOSAL*
.ENTER(I3 NATGAS/TKH/DATA 04 NATGAS/BLADE/DATA
08 PRINTER) S 32K FR 2254
CLOSE
LIST NATGAS/BLADE/DATA
FINISH
```

where NATGAS/TKH/DATA is the input file

and NATGAS/BLADE/DATA is the output file

6. Output

The line printer output gives an echo of the input data, the amount of grid skew and the gradients and curvatures of the surfaces for planes within the blade row, and a listing of the output file. The program also produces a picture of the blade profile and a graph of the blade surface gradients against x. If this graph is smooth then the blade surface velocity distributions produced by the matrix program should also be smooth.

The output file NATGAS/BLADE/DATA contains the blade geometry data for the matrix program (see Appendix III.2). This file is then edited to insert the title and flow data sections (see Appendix III.1 and 3).

APPENDIX V

Data preparation program for double circular arc blades

This program produces the blade data for the matrix program for cascade or cylindrical flow past double circular arc compressor blades.

1. Blade specification

The blade profile is specified by the usual parameters. The program then sets up the equations of the blade surfaces and hence can find the exact coordinates of any point on the blade.

2. Positions of calculating planes

The positions of the calculating planes may either be specified in the input data, or the standard positions stored in the program may be used (Figure 9). In both cases the distortion of the grid within the blade row is checked, and an extra plane inserted if the skew is greater than 70 per cent (Figure 8).

3. Stream surface thickness distribution

The stream surface thickness distribution may be either uniform throughout, or three linear variations with x between values specified at the upstream plane, the blade leading edge, the blade trailing edge and the downstream plane.

4. Data input

The data input is as follows:

Fortran Name	Format	Description
TITLE(I),I=1,10	10A4	Job title - up to 40 alphanumeric characters
STAGGER	6F	Stagger in degrees (positive for a compressor blade)
CAMBER		Camber in degrees
CHORD		Chord length
SC		Pitch/chord ratio
TM		Thickness/chord ratio
RLE		Radius of leading and trailing edges

Fortran Name	Format	Description
T1,T2,T3,T4	4F	Stream surface thicknesses at the upstream plane, blade leading edge, blade trailing edge and downstream plane
N	I	Number of points on each straight line
NUS	I	Number of upstream calculating planes. If set to zero the planes will be positioned as in Figure 9.
<u>If NUS > 0</u> USPL(I),I=1,NUS	1ØF	Positions of upstream planes as a percentage of axial chord, measured from the leading edge in the upstream direction.
NP	I	Number of calculating planes within blade row (incl leading and trailing edge points). If set to zero the planes will be positioned as in Figure 9.
<u>If NP > 0</u> PG(I),I=1,NP	10F	Positions of planes within the blade row as a percentage of axial chord, measured from the leading edge in the downstream direction. The first will be 0 and the last 100.
NDS	I	Number of calculating planes downstream of blade row. If set to zero the planes will be positioned as in Figure 9.
<u>If NDS > 0</u> DSPL(I),I=1,NDS	10F	Positions of downstream planes as a percentage of axial chord, measured from the trailing edge in the downstream direction.

5. Running instructions

The program is stored on file NATGAS/DCA/PROG. Typical running instructions would be:

```
COMMAND (7117NATGASØ3/DCA DATA PREPARATION)
COMP 2MINS
EXEC 8MINS
LIMSTORE 12K
OUTPUT Ø TO FILE (NATGAS/DCA/DIAGS)
NOTES
SWITCH MEDIUM
.ASA NATGAS/DCA/PROG
.LIBRARY CAD/GINO/GRAPH CAD/GINOSAL/*
.ENTER (I1 NATGAS/DCA/DATA O2 NATGAS/BLADE/DATA
O8 PRINTER) S 12K FR 12ØØ
CLOSE
LIST NATGAS/BLADE/DATA
FINISH
```

where NATGAS/DCA/DATA is the input file

and NATGAS/BLADE/DATA holds the blade data for the matrix program

6. Output

The line printer output gives an echo of the input data, the amount of skew of the grid and a listing of the output file. A picture of the blade profile is plotted out, but there is no need to check the smoothness of the surface gradients since the points will be exact.

The output file NATGAS/BLADE/DATA contains the blade geometry data for the matrix program (see Appendix III.2). This file is then edited to insert the title and flow data sections (see Appendix III.1 and 3).

APPENDIX VI

Matrix program - running instructions

The matrix program is run in three separate jobs, to suit the CADC system. Normally the second and third jobs are automatically set up by the previous job, but they can also be restarted by hand if the job has failed due to a system fault. The running commands for each job are kept on master files, which are edited before each run: the edited versions are not kept. Typical examples of the editing instructions and the files produced are given below.

Job 1

Edit Commands	Description
EDIT NATGASØ1/S1/RUNJOB	
G;PROJECT;7117NATGASØ3	Cost centre for job
G;DATA;USER/INPUT/DATA	Input data file
G;/BSD12;USER/BACKING/STORE1	Name of a disc BSD (max size = 60 blocks)
G;/BSD15;USER/BACKING/STORE2	Name of a disc BSD (max size = 10 blocks)
G;PROG;TAP1	Name of a program tape
G;MAT;TAP2	Name of input tape for matrix routines
W	
RUNJOB	

These instructions would produce the running file:

```
COMMAND (7117NATGASØ3/SETUP PROBLEM)
LIMSTORE 16K
LIMOUT 3ØØ
COMP 5 MINS
EXEC 2Ø MINS
TAPE 2 (TAP2)
TAPE 14 (TAP1)
OUTPUT Ø TO FILE(NATGAS/S1/DIAGS)
NOTES
SET NATGASØ1
.EDIT/S1/MASTER WITH H*
G;XX;USER/INPUT/DATA;G;ITAPE;1
```

```
G;BSD12;USER/BACKING/STORE1;G;BSD15;USER/BACKING/STORE2
COMMAND
.EDIT/S1/RUNJOB2 WITH H*
G;12BSD;USER/BACKING/STORE1;G;15BSD;USER/BACKING/STORE2
G;COSTCENTRE;7117NATGASØ3;G;PTAP;TAP1;G;MTAP;TAP2
*
QUEUEJOB Ø
FINISH
```

Note

The expression 'G;ITAPE;1' on line 12 tells the program that it is using an existing program tape. To start a new tape this should be changed to 'G;ITAPE;Ø' and the program will then overwrite the tape from the beginning.

Job 2

The edit commands required are given in lines 16 to 20 of the above file. The running file produced would be:

```
COMMAND (7117NATGASØ3/SOLVE MATRIX)
LIMSTORE 16K
LIMOUT 6ØØ
COMP 15 MINS
EXEC 12Ø MINS
TAPE 2 (TAP2)
TAPE 7 (USE1)
TAPE 8 (USE2)
OUTPUT Ø TO FILE (NATGAS/S1/DIAGS2)
NOTES
SET NATGASØ1
.EDIT/S1/MASTER2 WITH H*
G;BSD12;USER/BACKING/STORE1;G;BSD15;USER/BACKING/STORE2
*
COMMAND
.EDIT/S1/RUNJOB3 WITH H*
G;BSD12;USER/BACKING/STORE1;G;BSD15;USER/BACKING/STORE2
G;PROJECT;7117NATGASØ3;G;PROG;TAP1
*
RUNJOB
FINISH
```

NOTE

The maximum number of iterations (MAXIT) should be chosen such that this job will not run out of computation time

i.e. $2 + 6 \times 10^{-4} \times m \times n \times \text{MAXIT} \leqslant 15$

for $m = 90,\ n = 13,\ \text{MAXIT} \leqslant 18$

Job 3

The edit commands required are given in lines 16 to 20 of the above file. The running file produced would be:

```
COMMAND (7117NATGASØ3/OUTPUT RESULTS)
LIMSTORE 2ØK
LIMOUT 1ØØØ
COMP 4 MINS
EXEC 2Ø MINS
TAPE 14 (TAP1)
OUTPUT Ø TO FILE (NATGAS/S1/DIAGS3)
NOTES
SET NATGASØ1
SWITCH MEDIUM
.ASA (S 16K) /S1/6RES
.LIBRARY /S1/BIN
.ENTER (W15 USER/BACKING/STORE2 K8Ø
R12 USER/BACKING/STORE1 013 PRINTER) S2ØK FR 2315
CLOSE
DELETE USER/BACKING/STORE1
.ASA /S1/7GRAPH
.LIBRARY /S1/BIN CAD/GINO/GRAPH CAD/GINOSAL/*
.ENTER (R15 USER/BACKING/STORE2 K8Ø 013 PRINTER) S2ØK FR 1954
FINISH
```

If the graph plotting routines only are to be re-run, line 6 and lines 11 to 16 in the above file should be deleted.

DUMP files

The iterative solution procedure is carried out by the three segments SOLUTION, VELOCITY and RHS. These are stored in compiled form so that they may be overlaid. The instructions required to recreate the compiled versions from the source programs are:

```
SWITCH MEDIUM
.ASA NATGASØ1/S1/SOLUTION
.LIBRARY APPLIB/NEWRTNS/*
.ENTER(K35 N9 NATGAS/DUMP/SOLUTION) S 16K FR 7ØØ
.ASA (S 16K) NATGASØ1/S1/VELOCITY
.LIBRARY NATGASØ1/S1/BIN
.ENTER (K35 N1Ø NATGAS/DUMP/VELOCITY) S 16K FR1ØØØ
.ASA (S 16K) NATGASØ1/S1/RHS
.ENTER (K35 N11 NATGAS/DUMP/RHS) S 16K FR9ØØ
```

This need only be done if the compiled versions have been deleted, or if they become corrupted.

APPENDIX VII

Matrix program line printer results

The line printer results (5.3.1 c) are arranged in four sets of data, the first 3 being in blocks of n lines per grid line with each block identified by the x coordinate of the grid line.

1. Velocities and Mach numbers

PHI	ϕ coordinate of grid point
VX	Component of velocity in x direction
VY	Component of velocity in y direction
VU	Component of absolute velocity in ϕ direction
WU	Component of relative velocity in ϕ direction
V	Absolute velocity
W	Relative velocity
ABS M	Absolute Mach number
REL M	Relative Mach number

The relative values are only calculated for rotating rows.

2. Gas state

PHI	ϕ coordinate of grid point
STR FN	stream function, non dimensionalised by the mass flow
STAG H	Absolute stagnation enthalpy
STAT H	Static enthalpy
STAG PR	Abs stagnation pressure/Abs stagnation pressure at inlet
STAT PR	Static pressure/Abs stagnation pressure at inlet
DENSITY	Static density

3. Flow angles

PHI	ϕ coordinate of grid point
ABS ANG	Absolute flow angle = $\tan^{-1}$ (VU/VX)
REL ANG	Relative flow angle = $\tan^{-1}$ (WU/VX)

The relative flow angle is only calculated for rotating rows.

4. Blade surface velocity distribution

This is specified in blocks of two lines, the first for the lower boundary and the second for the upper boundary.

	X	x coordinate of grid point
	PHI	ϕ coordinate of grid point
	V/VD	Absolute blade surface velocity/absolute outlet velocity
or	W/WD	Relative blade surface velocity/relative outlet velocity
	CP	Static pressure coefficient $= \dfrac{p - p_d}{P_d - p_d}$
		where p = local static pressure
		P_d = absolute outlet total pressure
		p_d = absolute static pressure
	SL	Percentage surface length

APPENDIX VIII

Matrix program diagnostic output

1. Warning messages (i.e. program does not stop)

(i) CHOKED FLOW AT PLANE I — The local velocity at plane i is supersonic. The local static density will be set to the value for a Mach number of unity.

(ii) REVERSE FLOW AT I,J — $\psi_{i,j+1} < \psi_{i,j}$ i.e. the local axial velocity is negative.

(iii) ITERATIVE SOLUTION HAS DIVERGED RESULTS FOR ITERATION — $\left\{\frac{\psi - \psi_{p-1}}{Q}\right\}_{max} > 1$ after 6 iterations. The program completes the iteration and then produces the results.

(iv) WARNING - STATIC ENTHALPY NEGATIVE — The term $\frac{V^2}{G} > H$

2. Error messages

The following messages will be followed by the system errors Division and Exponent overflow. These are caused deliberately in order to stop the run.

(i) TITLE SECTION DATA ERROR
(ii) BLADE GEOMETRY DATA ERROR
(iii) FLOW DATA ERROR

} The input data is incorrect. The error was detected when reading in the section indicated.

(iv) M GREATER THAN 90
(v) N GREATER THAN 13

} Too large a calculating grid has been specified

(vi) If division or exponent overflow occurs in segment 2BAND in the line after either label 55 or label 66, then the matrix for calculating the weightings for the finite difference approximations to $\frac{\overline{\partial f}}{\partial x}$ or $\left(\nabla^2 f + E \frac{\overline{\partial f}}{\partial x}\right)$ respectively are singular. This is probably caused by two straight lines having the same x coordinate.

This artifice is used to avoid using formatted input and output statements in this segment, and hence to reduce the store required from 20K words to 16K words.

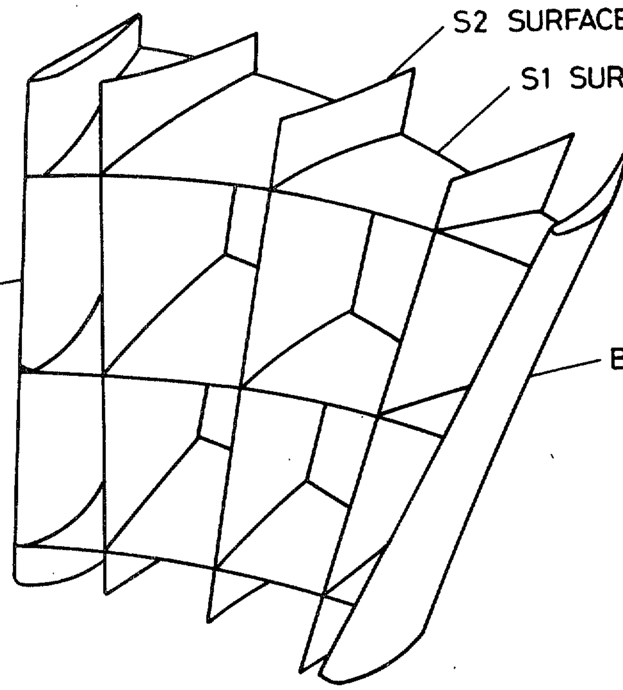
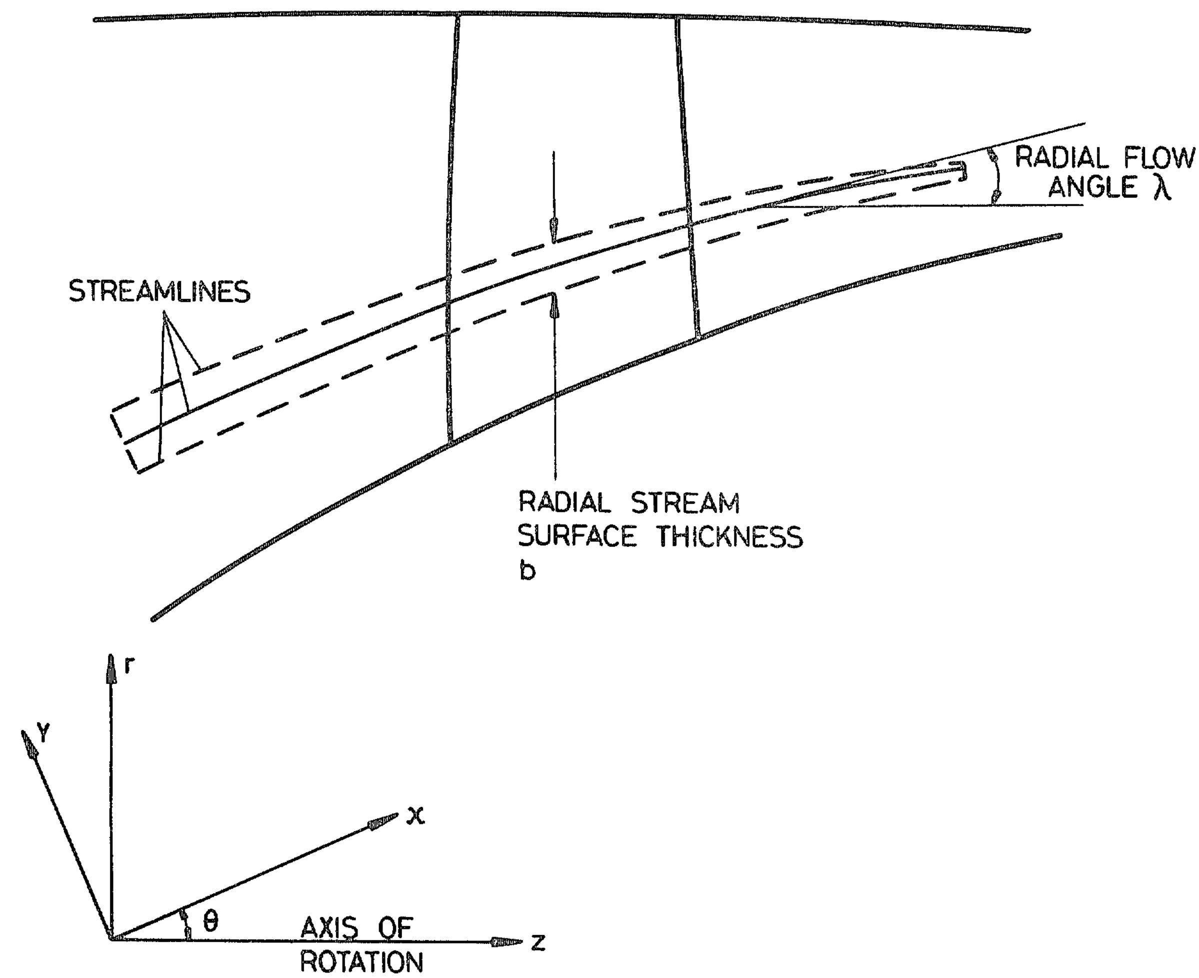


Fig 1 S1 and S2 stream surfaces



SIGN CONVENTION: IN THE ABOVE EXAMPLE $\lambda<0$: $\theta>0$

Fig 2 Coordinate systems

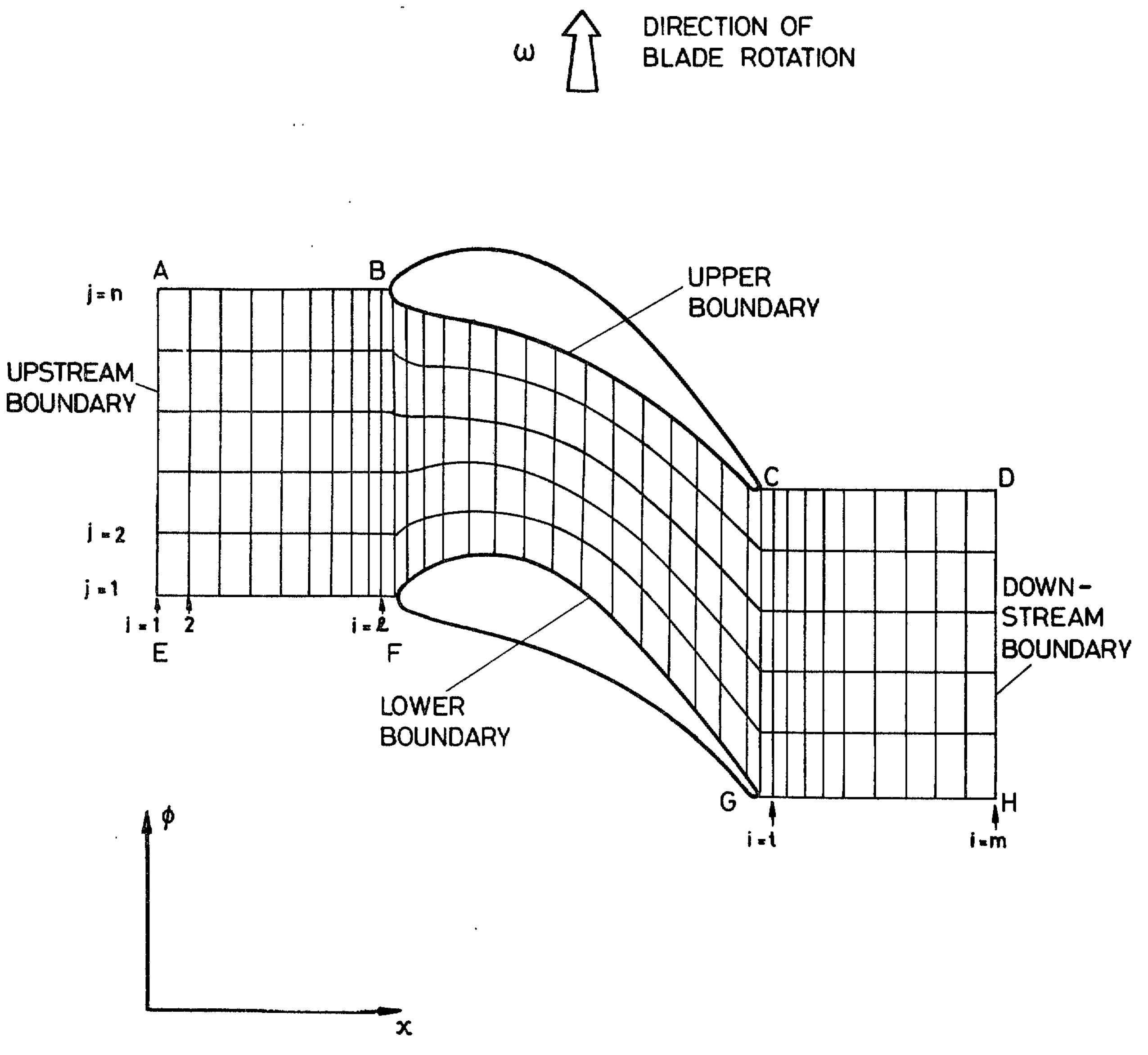


Fig 3 Finite difference grid

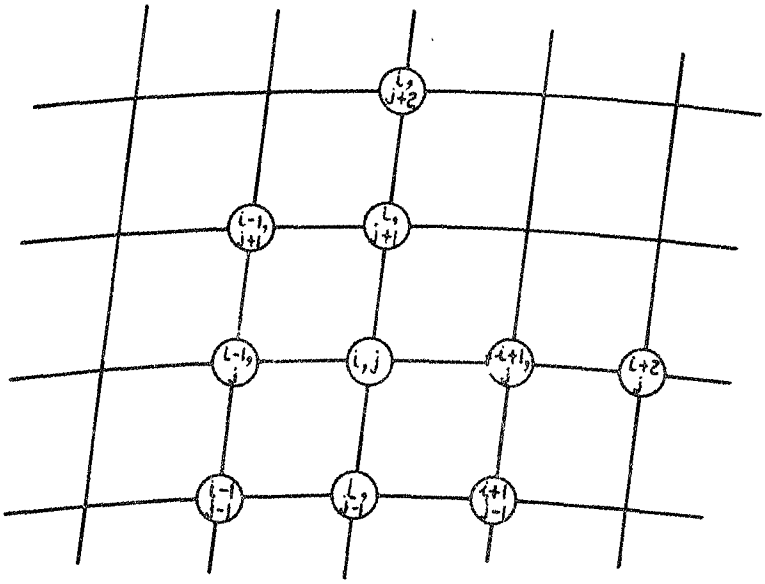


Fig 4a Finite difference star for $\nabla^2\psi$

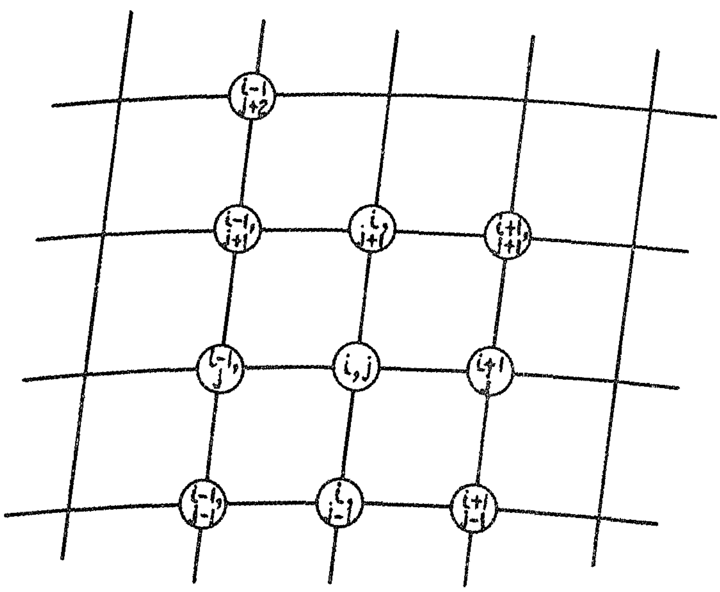
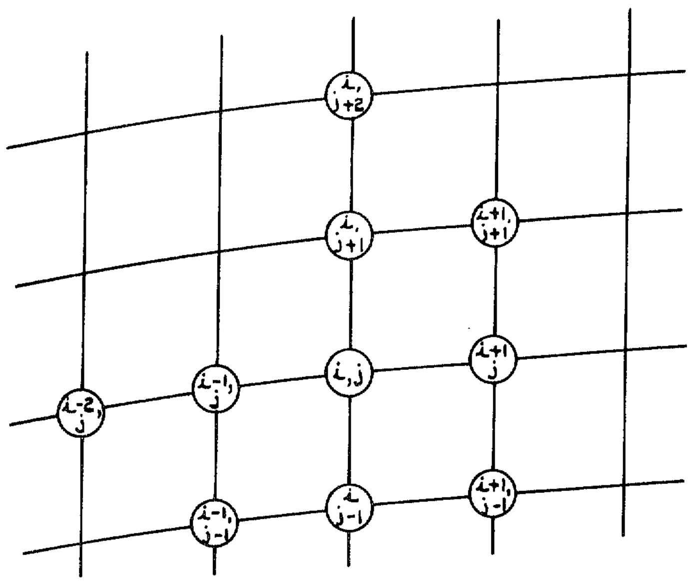
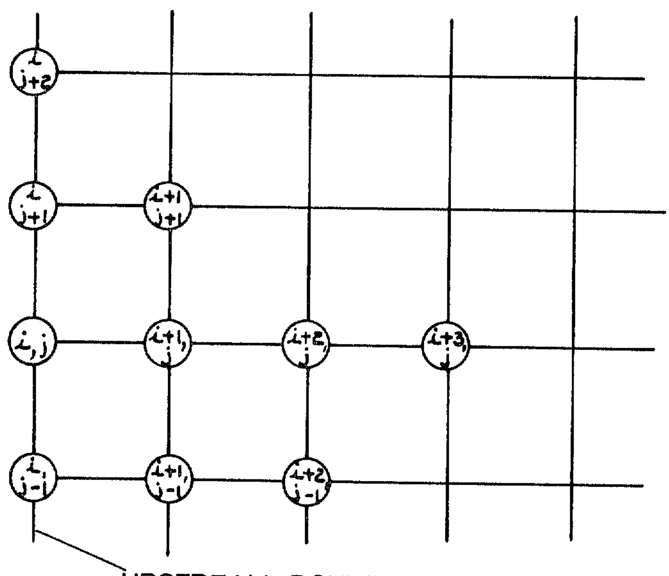


Fig 4b Finite difference star for $\nabla^2\psi + E\,\overline{\frac{\partial\psi}{\partial x}}$

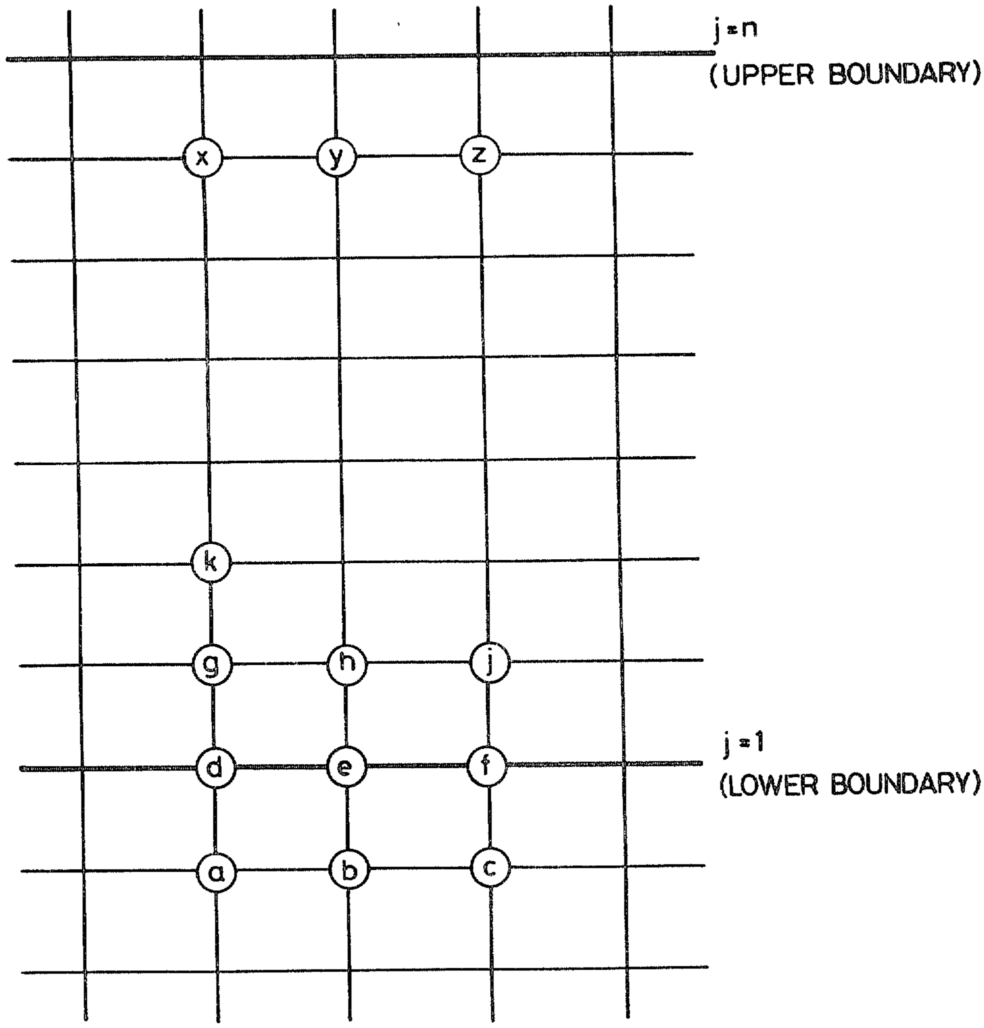


a. GENERAL CASE



b. UPSTREAM BOUNDARY

Fig 5a&b Finite difference stars for $\frac{\bar{\partial}}{\partial x}$



USING THE PERIODICITY OF THE FLOW, THE STREAM FUNCTION AT POINT 'a' IS GIVEN BY:

$$\psi_a = \psi_x - Q$$

WHERE Q IS THE MASS FLOW.

THUS THE POINTS a,b,c OF THE STAR ARE REPLACED BY x,y,z. (THE FACTOR Q x WEIGHTING COEFFICIENT FOR THE POINT IS TRANSFERRED TO THE R.H.S.). THIS RESULTS IN A HALF BANDWIDTH OF (2n-3) BETWEEN THE STAR CENTRE 'e' AND THE POINT 'z'

Fig 6 Maximum bandwidth condition

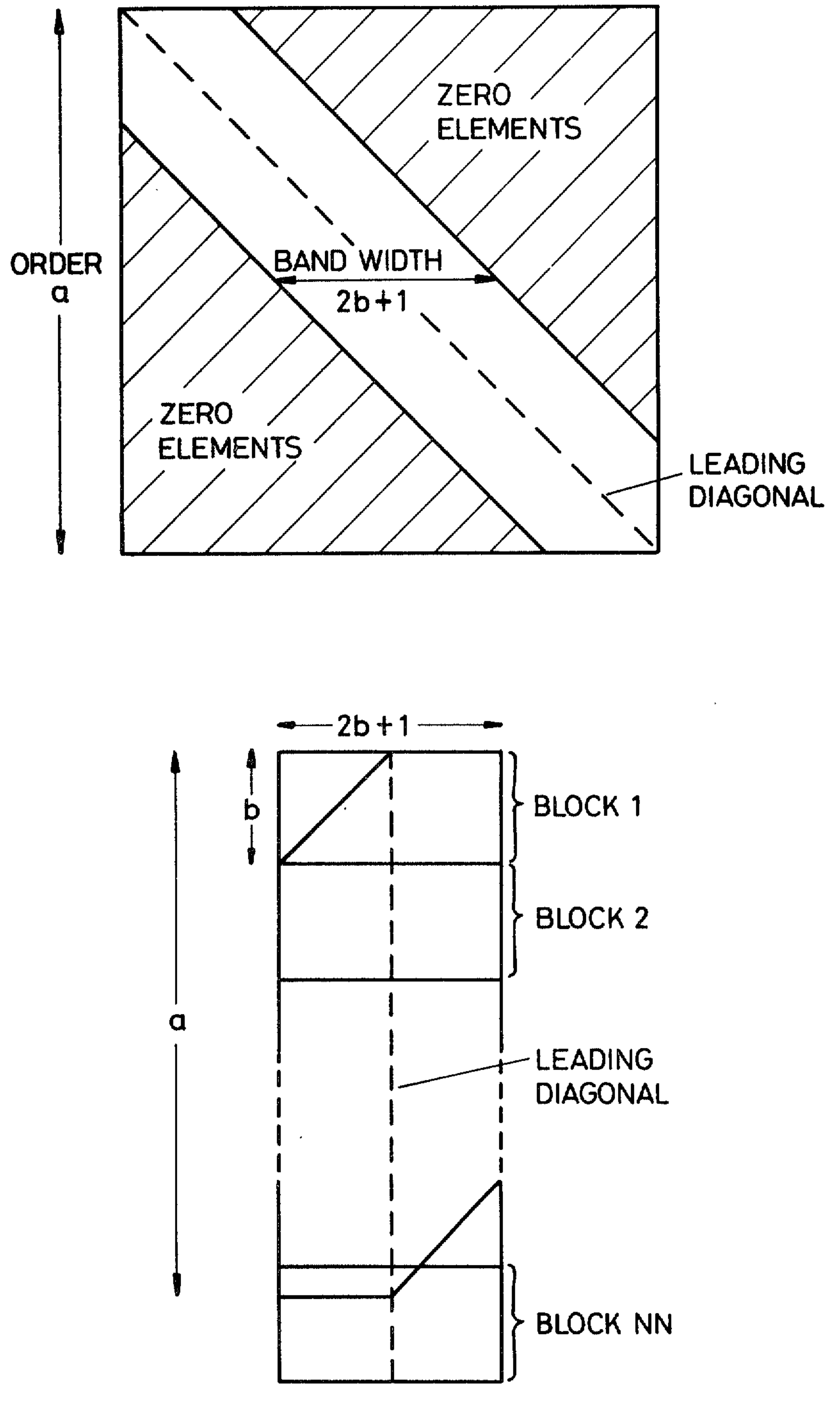
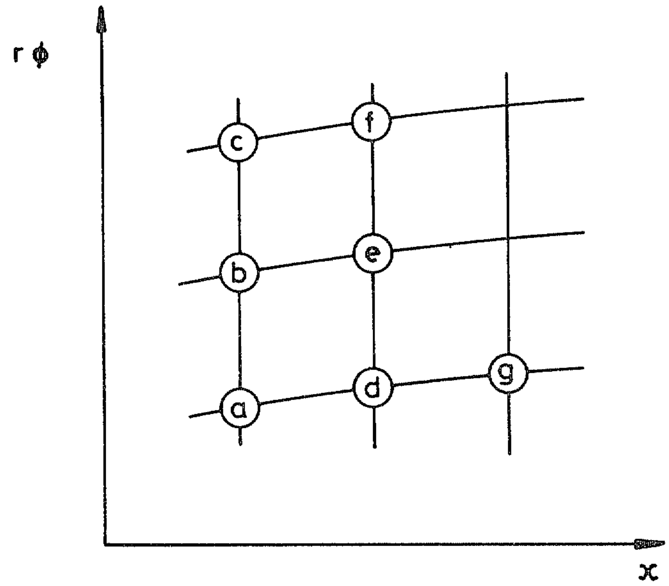


Fig 7 Band matrix storage



i) SKEW GRID

GRIDS FOR WHICH

$$\left| (r\phi)_d - (r\phi)_a \right| > 0.7 \left\{ (r\phi)_b - (r\phi)_a \right\}$$

SHOULD BE AVOIDED

ii) ASPECT RATIO

THIS SHOULD NOT EXCEED 4

i.e. $$\frac{1}{4} \leqslant \frac{(r\phi)_b - (r\phi)_a}{x_d - x_a} \leqslant 4$$

iii) GRID SPACING

THE LOCAL CHANGE IN x SPACING SHOULD NOT EXCEED 2·5

i.e. $$0{\cdot}4 \leqslant \frac{x_d - x_a}{x_g - x_d} \leqslant 2{\cdot}5$$

Fig 8 Finite difference grid limitations

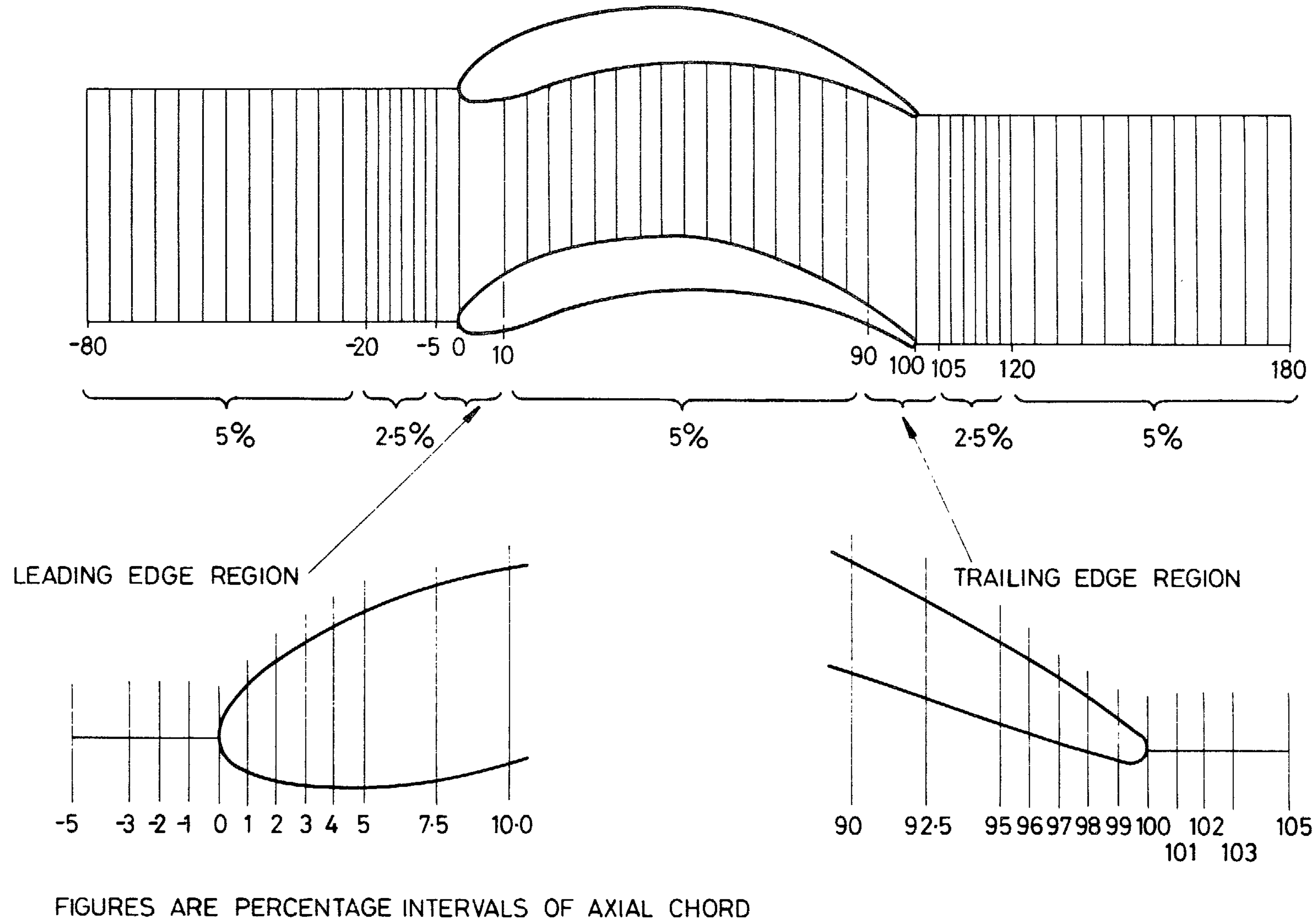
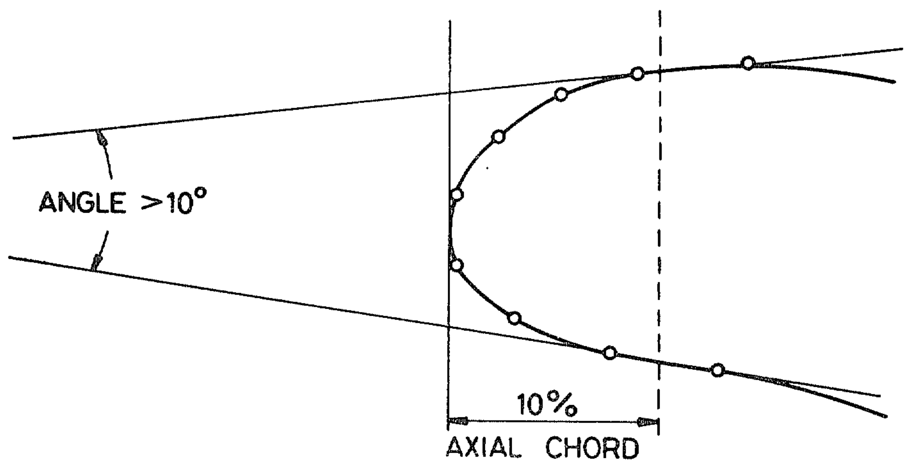
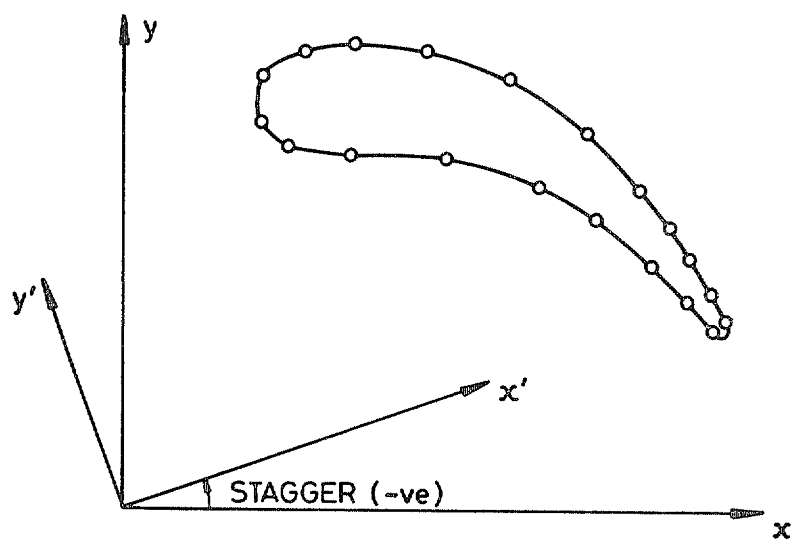


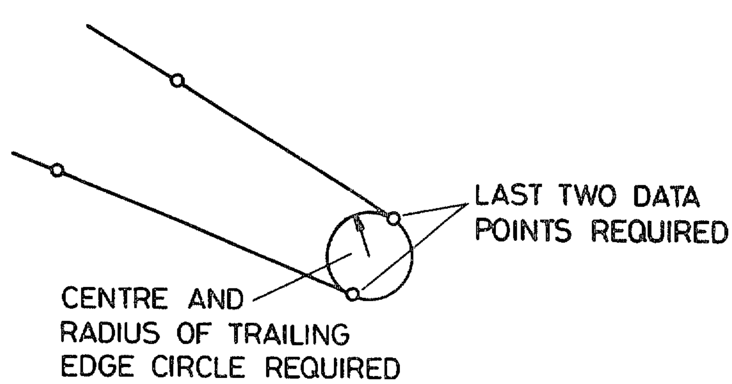
Fig 9 Typical spacings for calculating planes



a. BLUNT LEADING EDGE CONDITION



b. CO-ORDINATE SYSTEM



c. SPECIFICATION OF TRAILING EDGE CIRCLE

Fig 10a-c Data preparation program for blades specified by coordinates

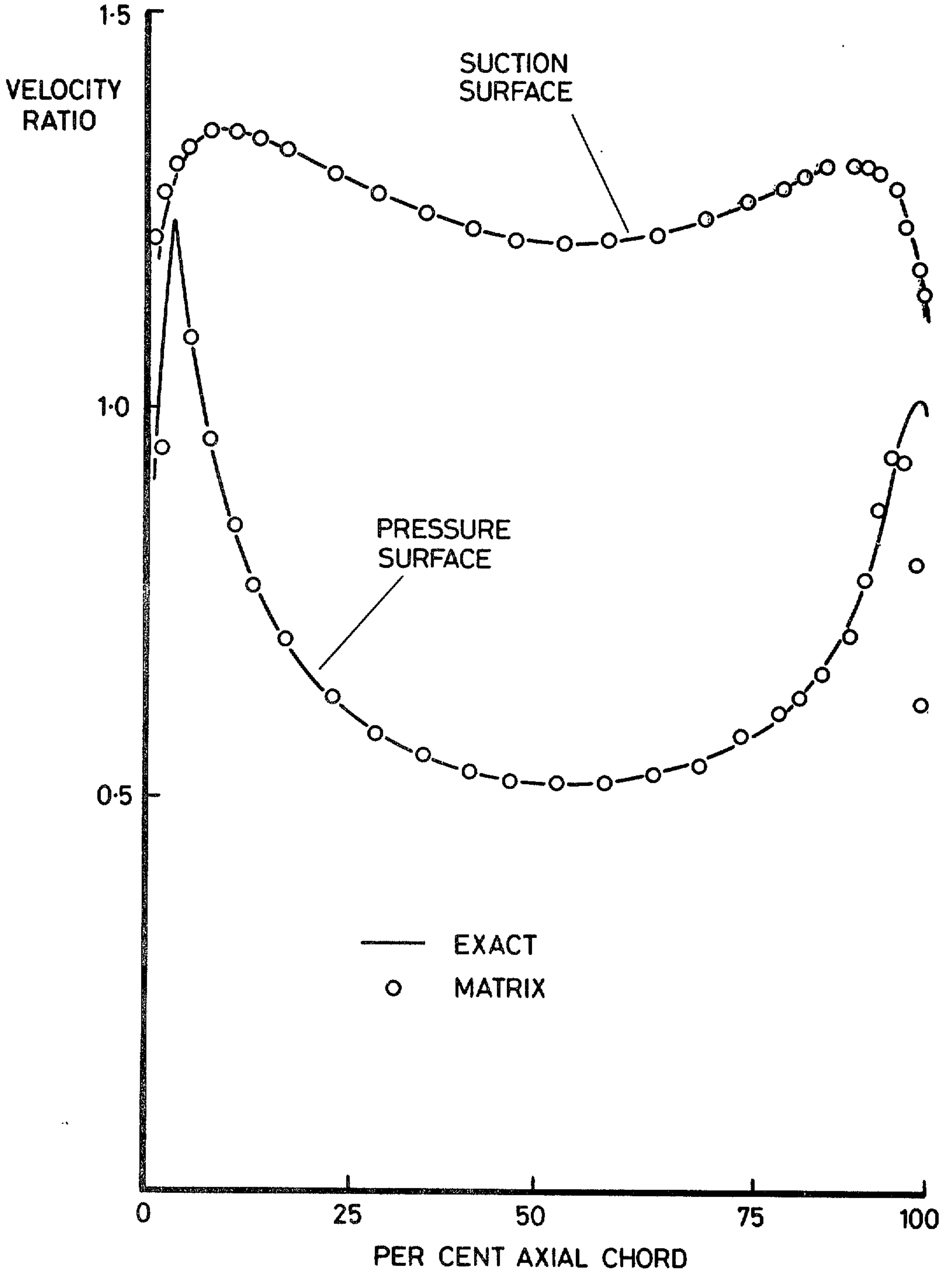


Fig 11 Impulse type blade

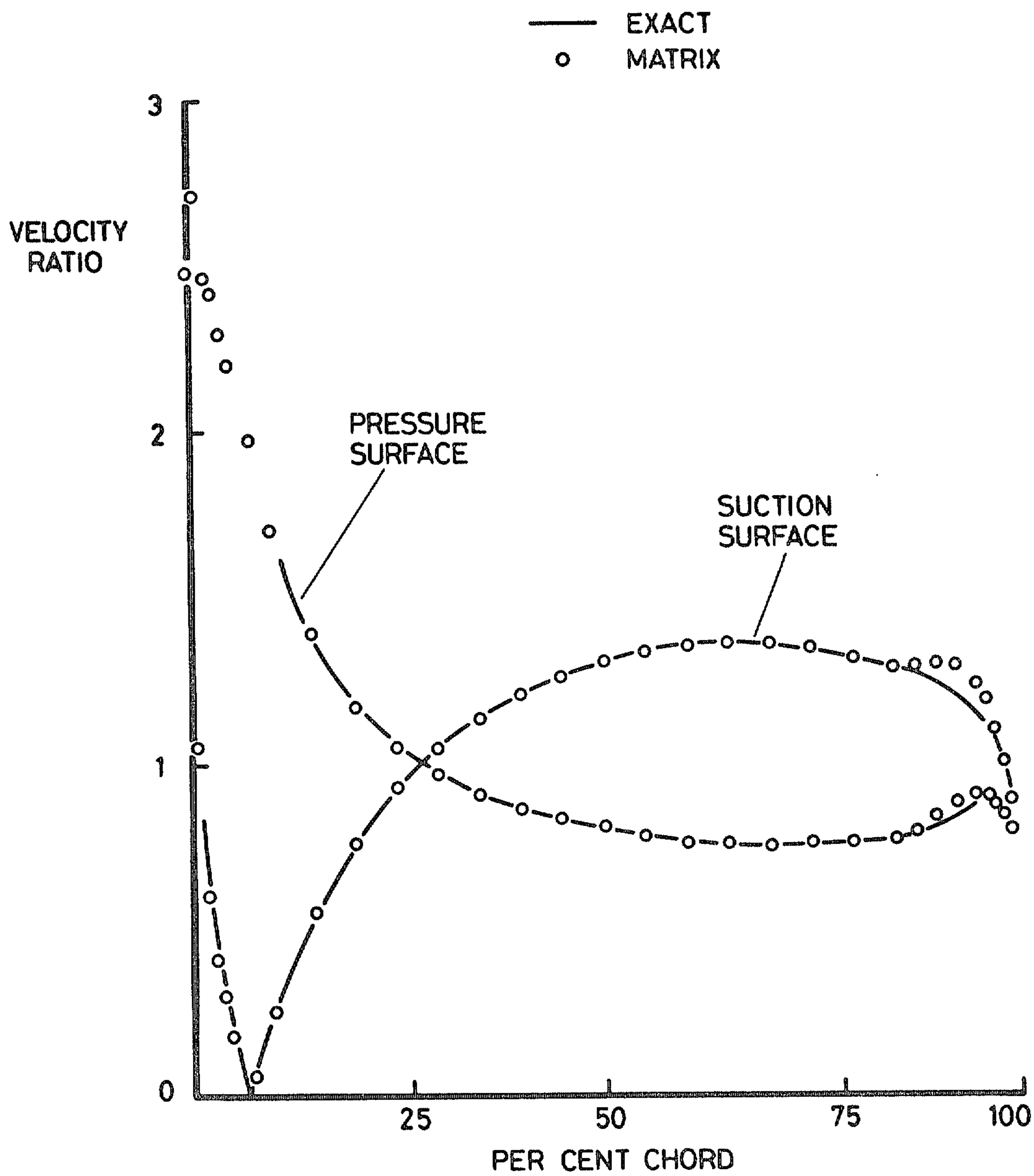


Fig 12 70° camber blade

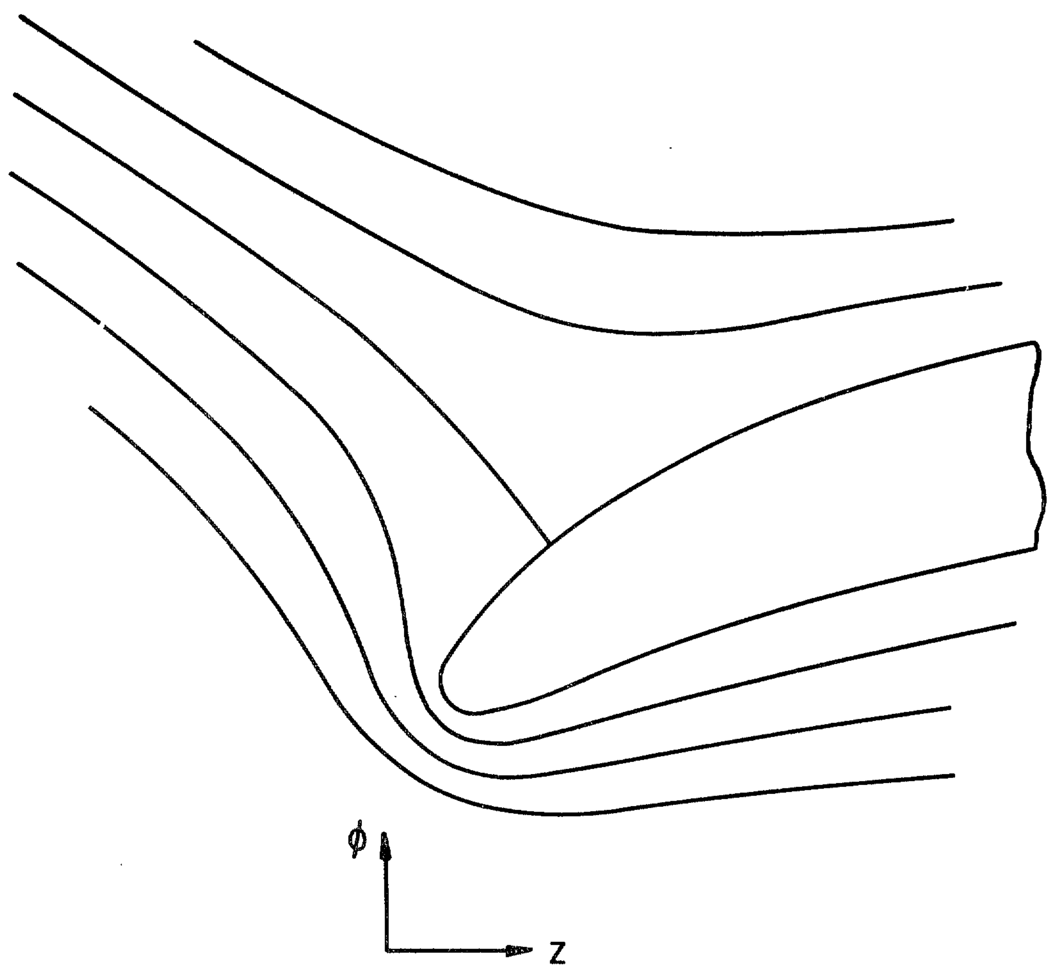


Fig 13 Streamline pattern for leading edge of 70° camber blade

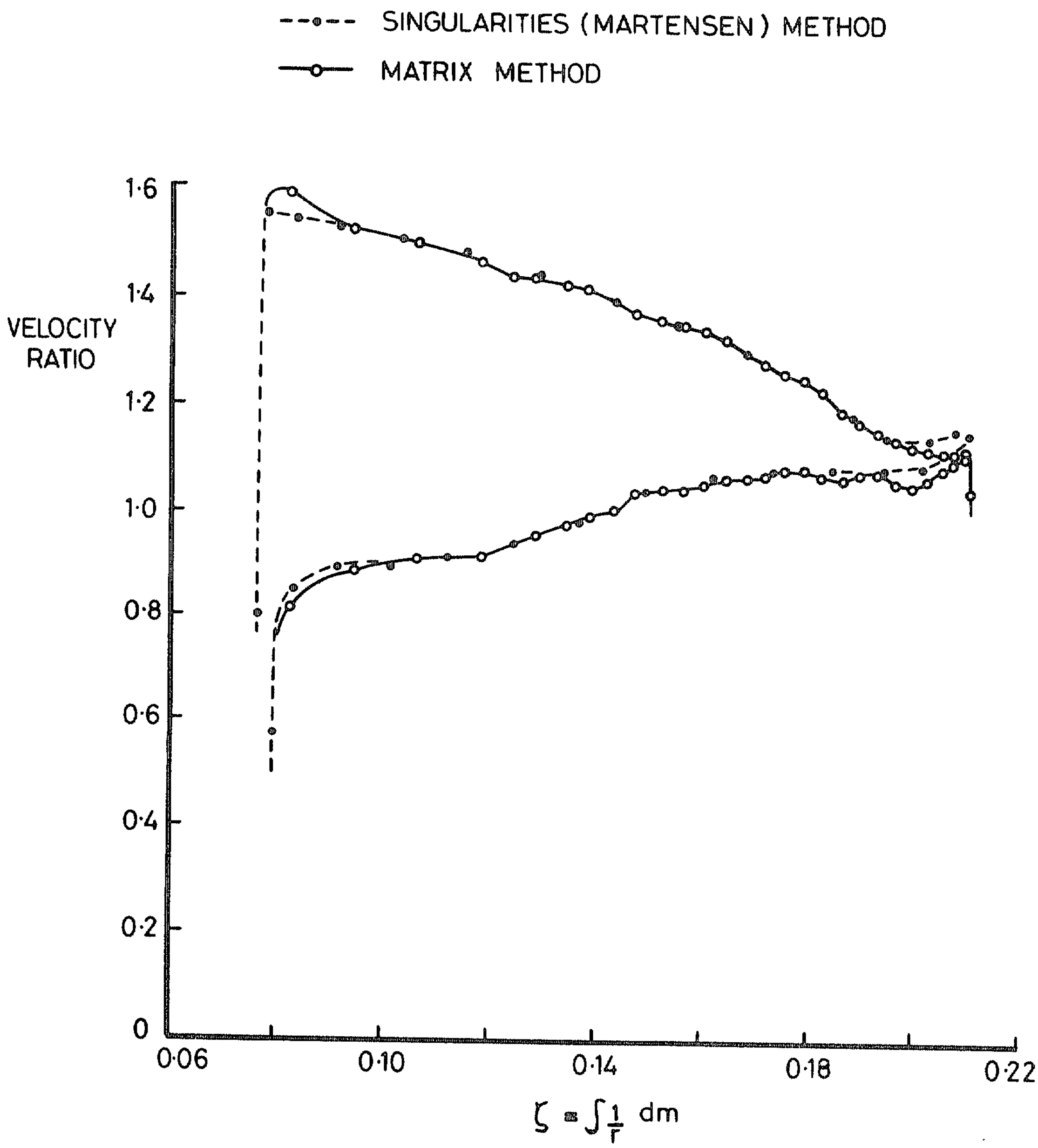


Fig 14 Conical flow past a stator blade

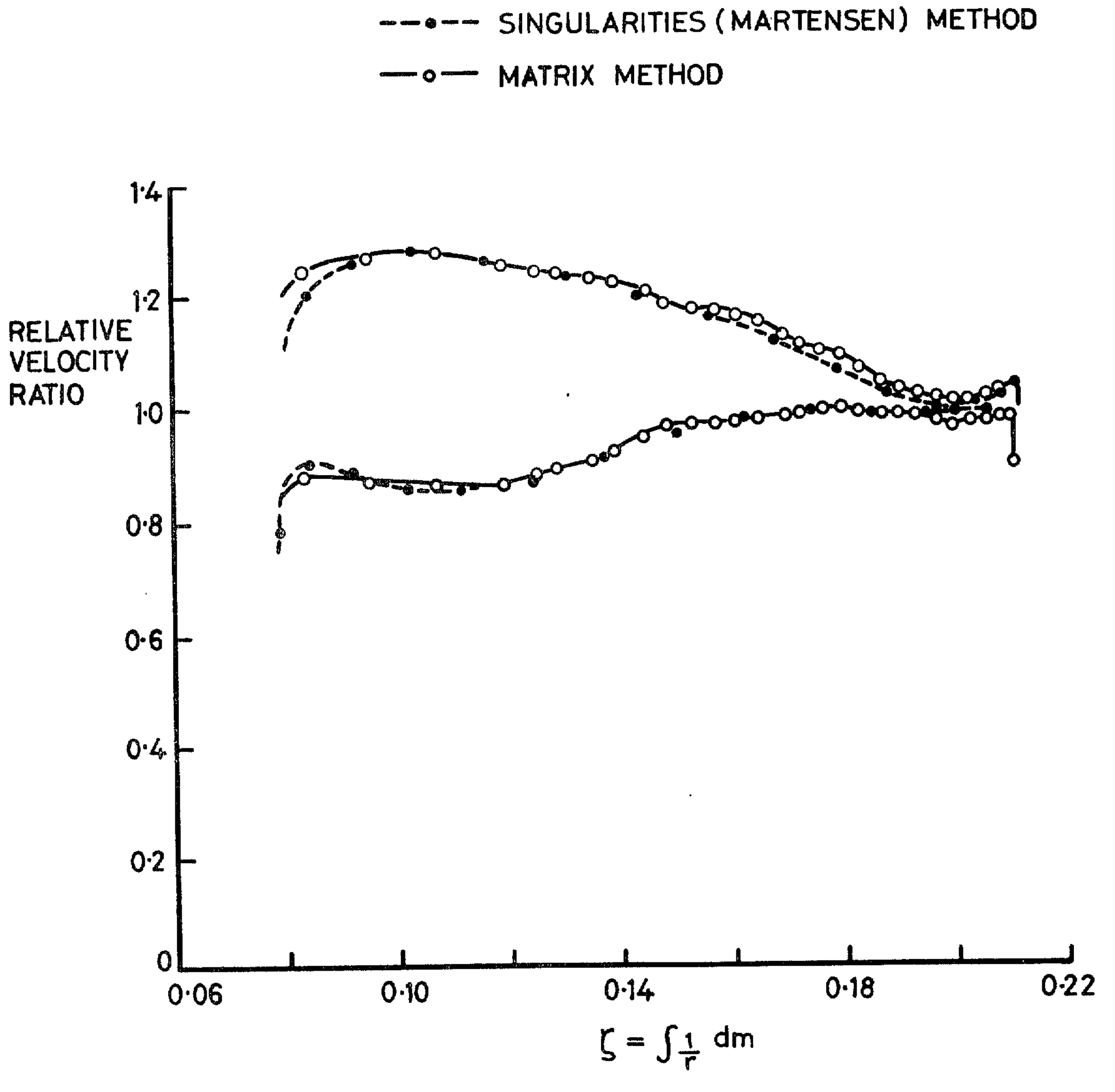


Fig 15 **Conical flow past a rotor blade**

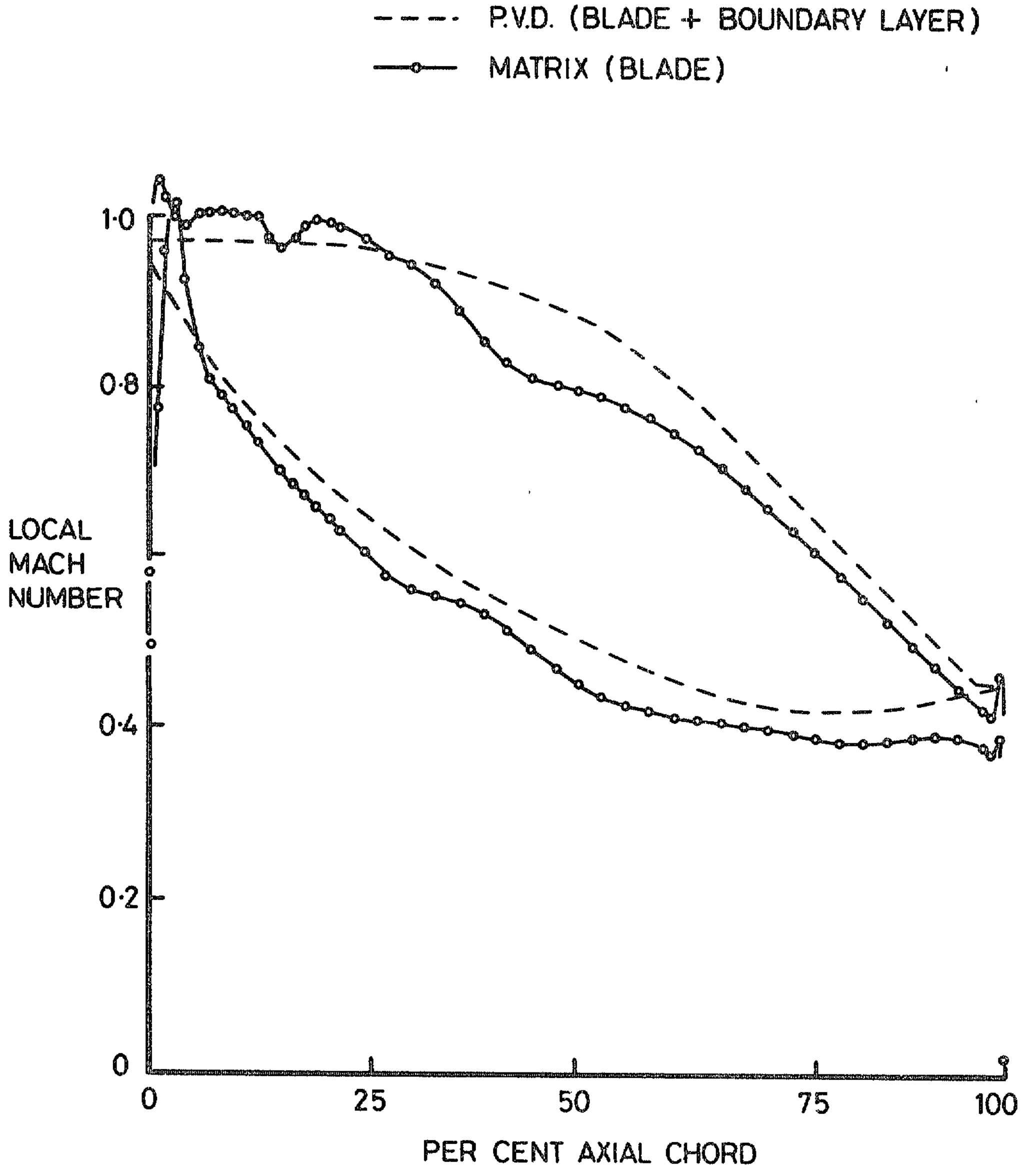


Fig 16 Axial compressor – Stator I

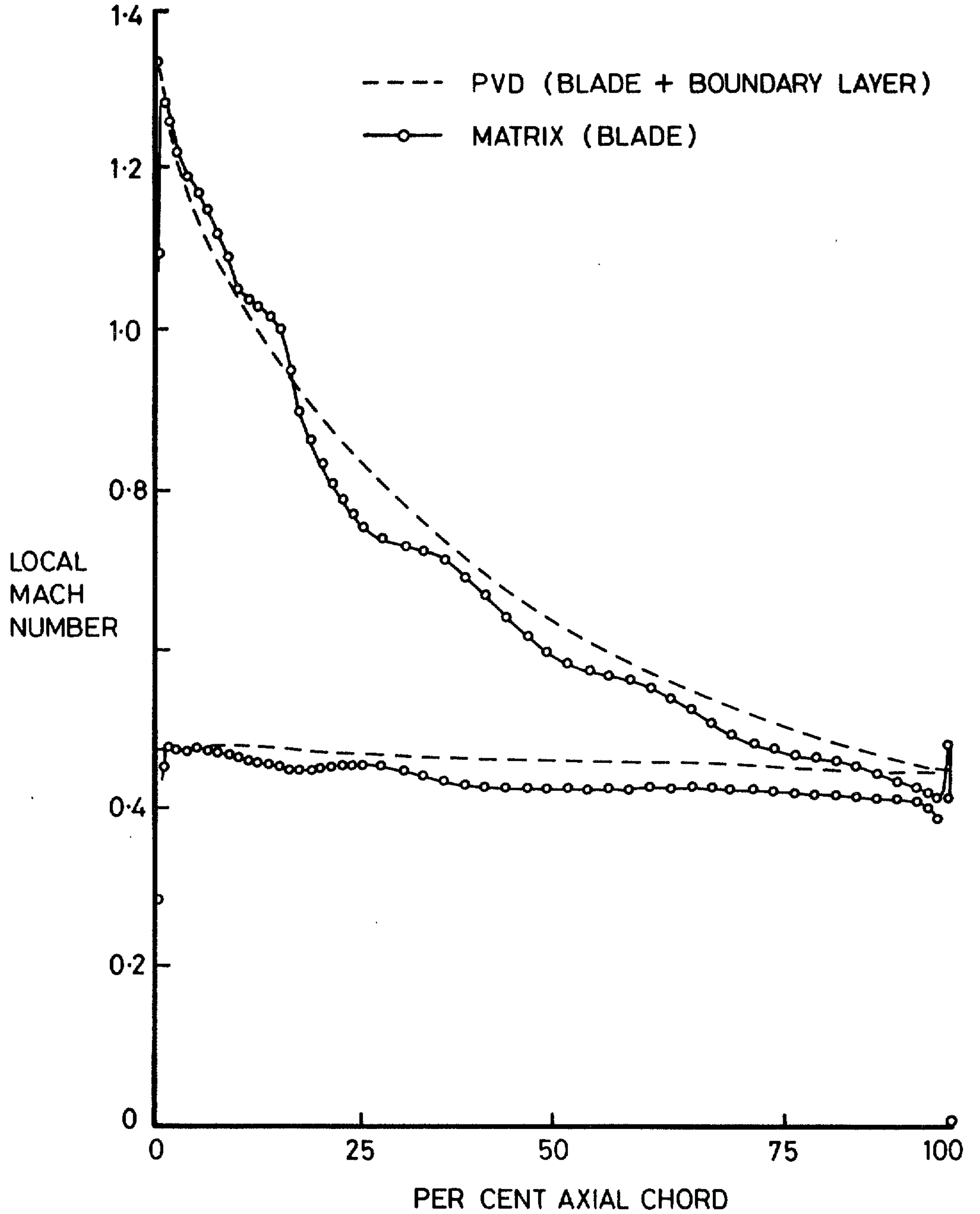


Fig 17 Axial compressor – Stator II

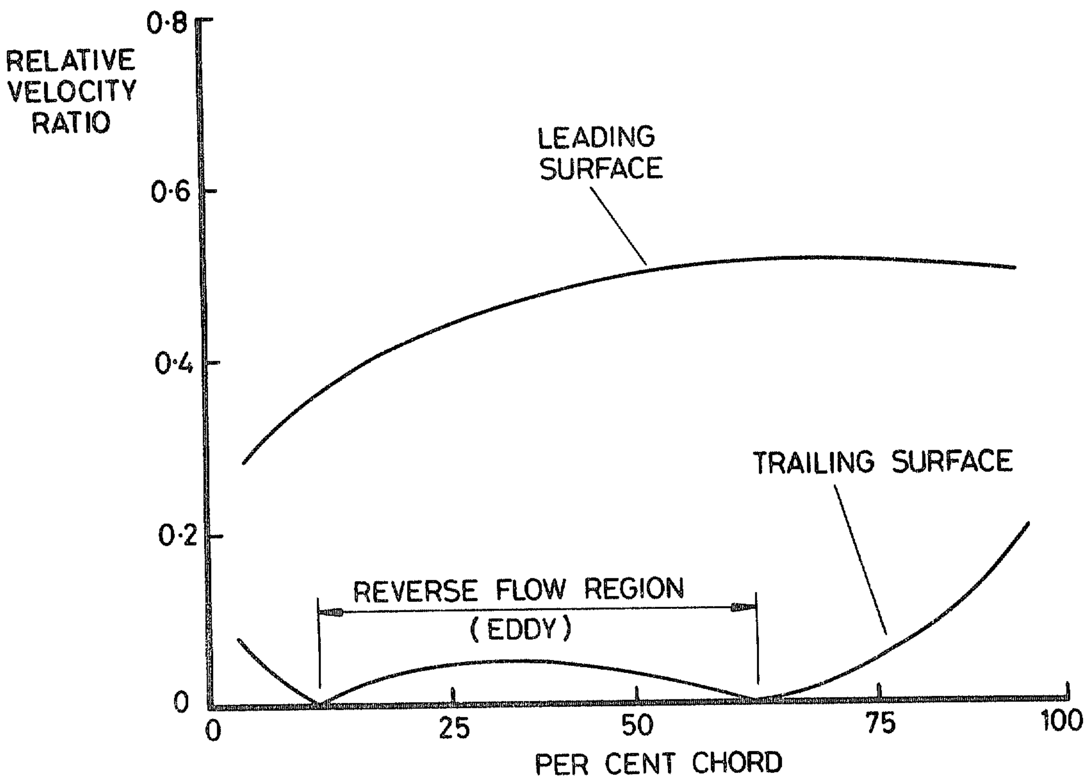


Fig 18 Radial flow rotor

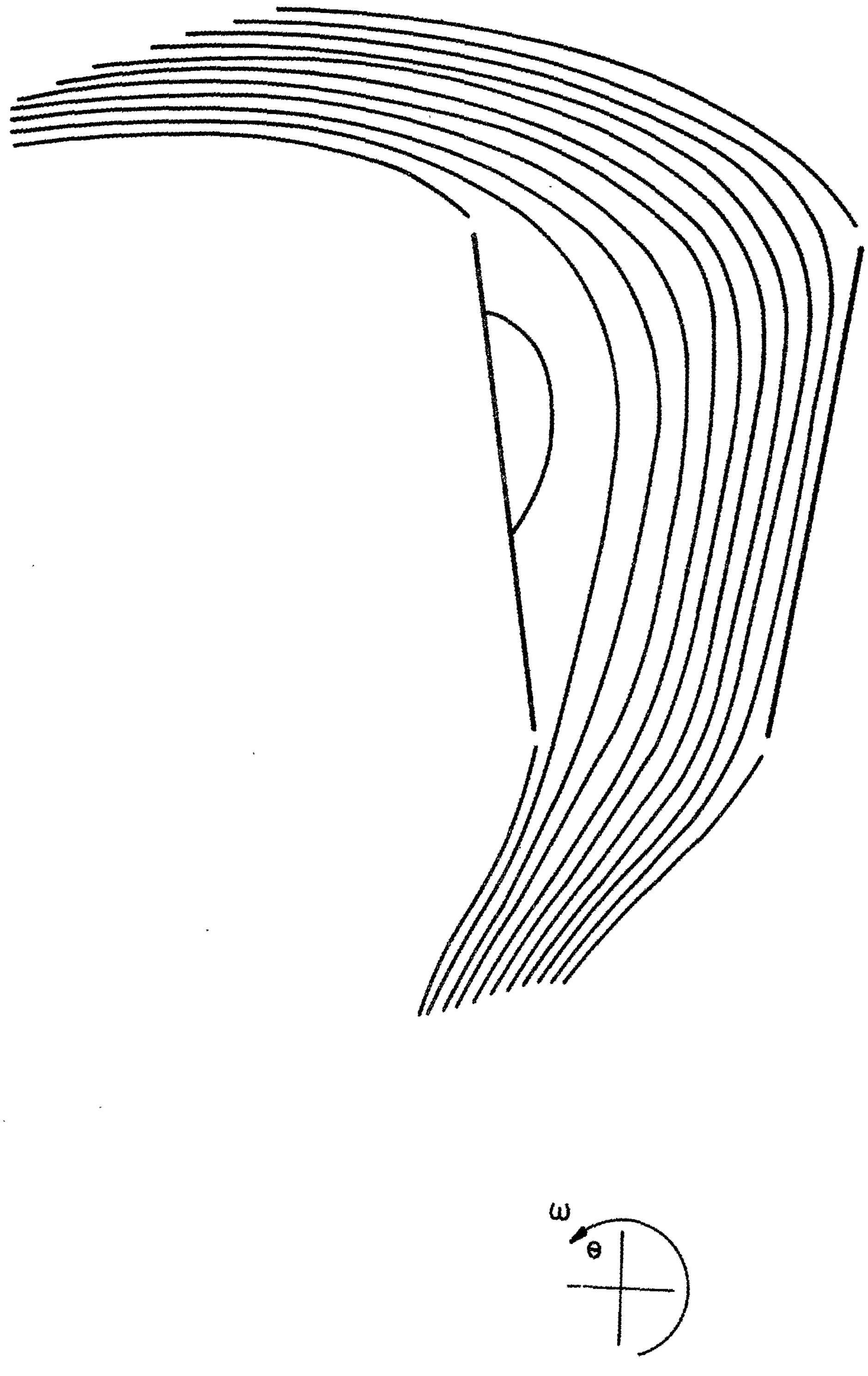


Fig 19 Radial flow rotor — streamline pattern

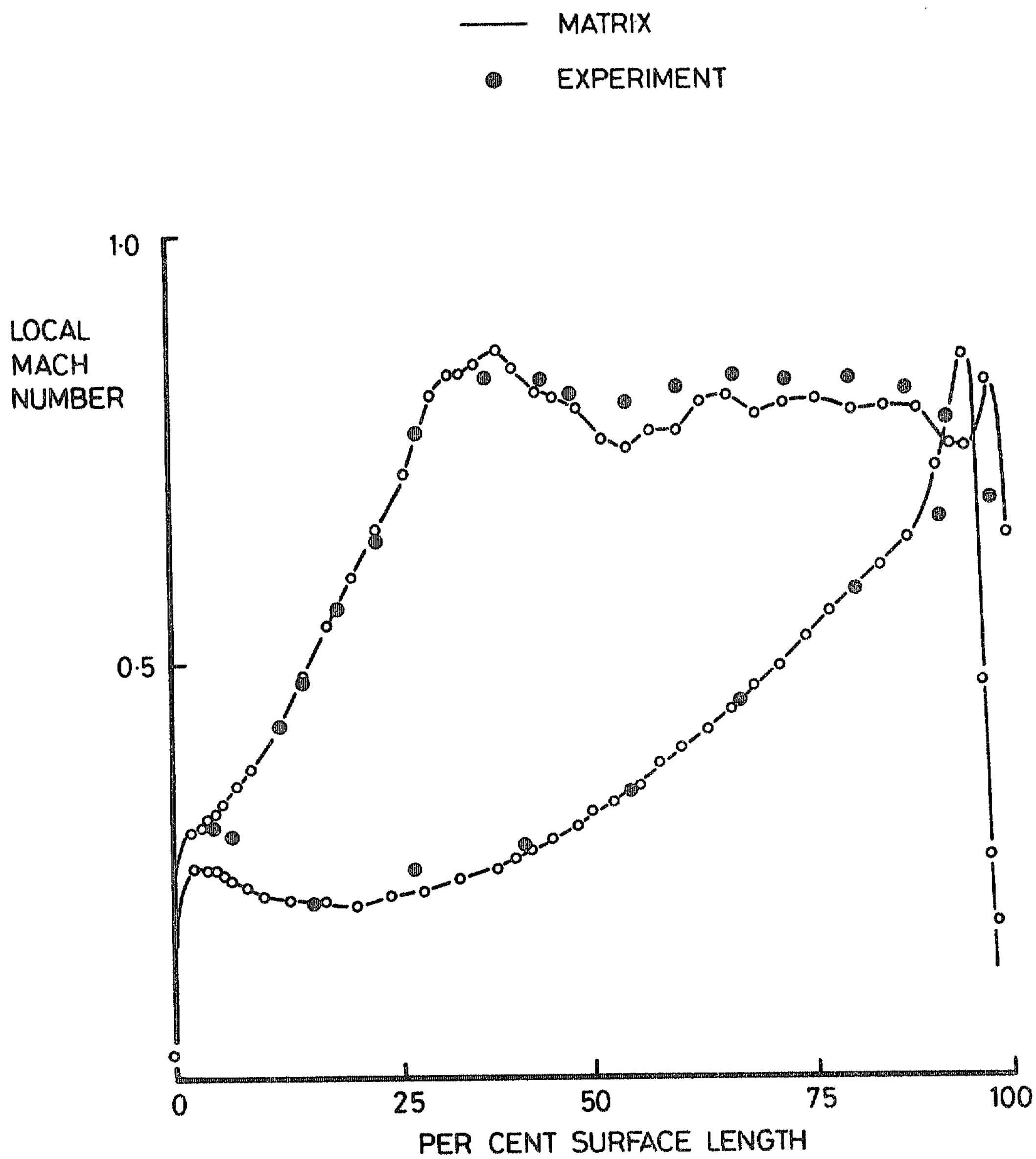


Fig 20 NASA turbine stator

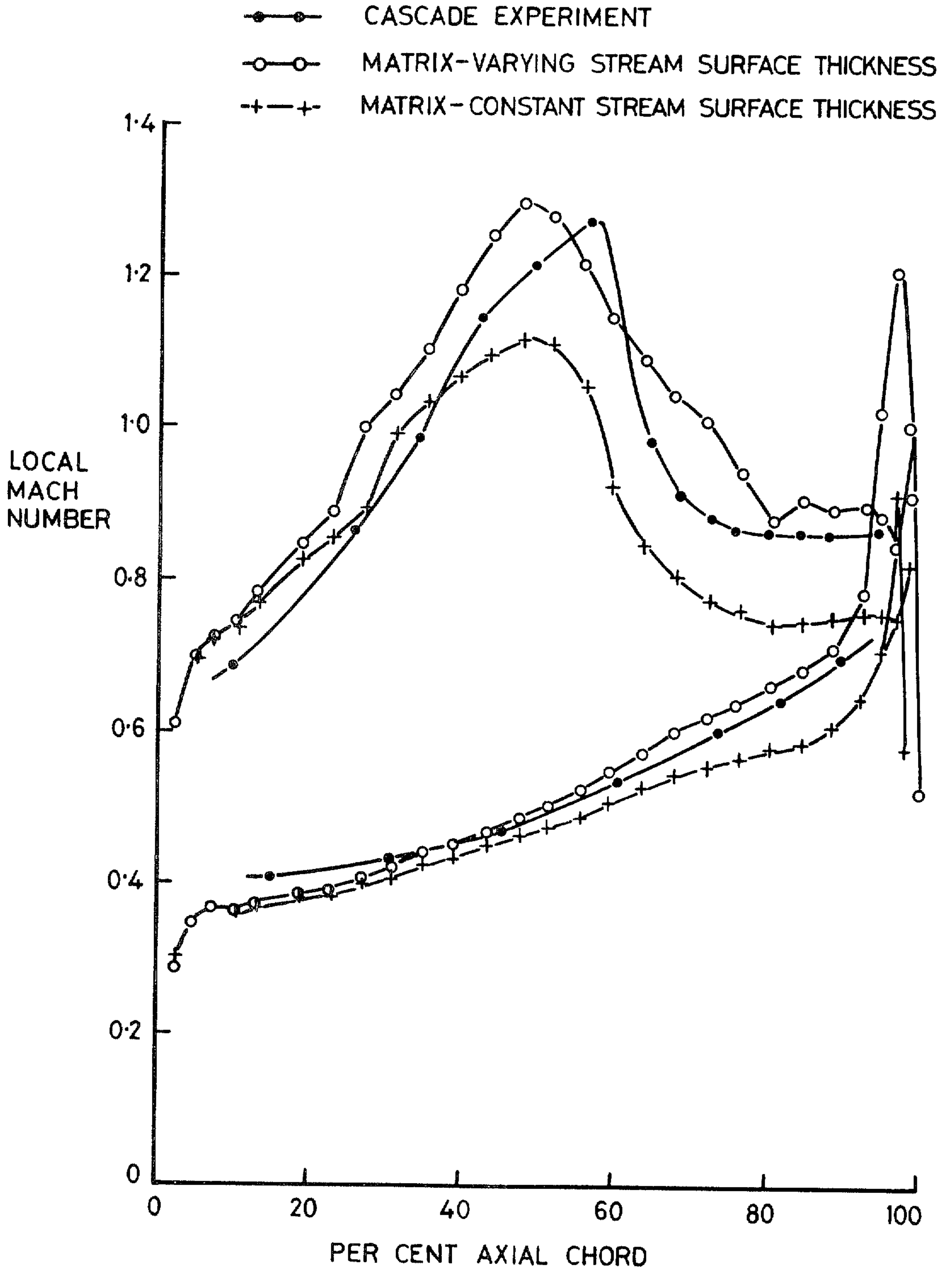


Fig 21 Axial turbine stator

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