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WEIGHT OPTIMISATION WITH FLUTTER CONSTRAINTS

by

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SUMMARY

It is argued that the formulation of the method of applying a flutter constraint in weight optimisation which allows for all the methods of avoiding flutter is so complicated that it is best not to try to include a flutter constraint in a general optimisation program with other constraints but to apply it only if the structure obtained from optimisation with the other constraints alone is flutter-prone. In this way much unnecessary and possibly inappropriate calculation can be avoided.

A method of weight optimisation with only flutter constraints, based on inverse iteration, which can be used in conjunction with any suitable optimisation procedure which requires the values of the objective function and its first derivatives to be calculable is suggested. Its use in association with more than one procedure is described as is the optimisation of a wing of fairly-high aspect ratio.

The question of how detailed a representation of the structure is needed is also discussed.

* Replaces RAE Technical Report 77130 - ARC 37537

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1 INTRODUCTION

When considering weight optimisation with flutter constraints it is important to keep in mind the complexity of the phenomenon being dealt with. In general a surface may be prone to more than one instability within the speed range of interest and each instability might react in a different manner to local structural changes. In some cases measures taken to avoid one type of flutter can introduce another type.

The structural properties that have most effect on flutter can be itemised as overall stiffness, separation of the frequencies of the normal modes in still-air and the shapes of the normal modes themselves. The general characteristics of these last, *ie* those given descriptions such as flexure or torsion, will not change overmuch with changes to structural detail but in some cases small changes in such as the chordwise position of a nodal line or the amount or sign of the incidence in a mode can alter the stability of the surface significantly. One classical method of changing the shapes of the modes is to add mass at appropriate places. In this way flutter can sometimes be avoided without altering the structural stiffness and with a smaller weight penalty than the appropriate change in stiffness would incur.

The variety of ways of achieving stability coupled with the variety of possible forms of instability leads to the conclusion that the flutter constraint in an optimisation process must depend on the flutter characteristics directly rather than on purely structural properties. It also suggests that any optimum found is more likely to be local than global.

Ideally, a complete flutter calculation in which every root at the lower frequencies is examined should be made for every change in the values of the structural parameters. Also the complete speed range should be covered because it is possible for a surface to be stable at its maximum speed whilst being unstable at lower speeds. The need to allow for the changes in the distortion shapes caused by structural changes dictates that the structural and aerodynamic representation should be detailed. Thus the calculations would be of large order as well as extensive in the number of speeds and the number of roots examined. Such a comprehensive approach is too time-consuming to be practical and ways of reducing the amount of calculation to be made must be found.

There are two contexts in which flutter optimisation can be considered; one is as part of the initial design process in conjunction with strength constraints and the other is to find the minimum extra weight needed to improve the flutter characteristics of an existing design. There is an argument for not involving flutter constraints in initial design optimisation on the grounds of lack of definiteness of the objective. In the case of strength the loads the structure has to sustain are fairly well defined and in general local strengthening will not increase the loads on another part of the structure. The equivalent cannot be said in the case of flutter; for an economical cure for an instability cannot be found until the surface suffers from it and an exact description of it is available; local stiffening may have adverse effects overall. Further it is questionable whether a structure optimised with simultaneous strength and flutter constraints will be significantly lighter than one optimised first with strength constraints and subsequently modified in the lightest manner to satisfy any flutter constraints that may have appeared using the 'strength' values of the parameters as minima.

If the weight saved is trivial the complication of optimisation with simultaneous constraints can be avoided and 'flutter' optimisation need only be considered in the context of curing a known instability. It will be assumed that this is so in what follows.

2 ACCOMMODATION OF FLUTTER CONSTRAINT

2.1 Reasons for using inverse iteration

In calculations a system is free from flutter when the real parts of all the roots of the flutter equation (see Appendix A) are negative. When optimising, the task is to change the values of those coefficients in the flutter equation which depend on the structural properties in a way such that either the real parts of the roots remain negative while the weight is reduced to a minimum or, if one or more of the real parts is not negative, all the real parts become negative with the minimum increase in structural weight. In practice the real parts of most of the roots show no tendency to have other than negative values and in general there is one root which can be looked upon as the principal critical root with perhaps one or two other critical roots in the background.

In these circumstances inverse iteration seems likely to provide the most economical method of solving the flutter equation. The principal critical root for the basic structure is found by a calculation in which all the roots are found. Subsequently the variation of this root alone is followed as the structure changes during the optimisation process. At the end of the optimisation process all the roots of the system have to be calculated to reveal whether the real parts of all the roots are still negative up to the required speed. If not the structure must be subjected to the optimisation procedure again with the new principal critical root. It may be possible to prevent reversion to the previous instability by appropriate limitations of the values the variables can take. It cannot be guaranteed that such a reversion will not occur. Neither can it be guaranteed in the general case that a stable root with a value near that of the critical will not be mistaken for the critical root which will, of course, result in the program failing.

For completeness the flutter equation and the method of solving it by inverse iteration used are given in Appendix A. Inverse iteration can be associated with many optimisation routines and some that have been tried are described below.

2.2 Graphical representation of optimisation process

Fig 1 is a graphical aid to the description of what is required of an optimisation process when there are two variable elements and one type of flutter. The sizes, τ_1 and τ_2 , of the elements, in terms of weight, are the axes of the graph and with these axes lines of constant weight are at 45°. The flutter speed varies with the sizes of the elements and contours of constant flutter speed can be drawn. If there is more than one type of flutter there would be more than one set of contours. Optimisation will have been successful in the case of a single flutter instability when the representative point on the contour for the desired flutter speed is the point of contact of the tangent to the contour whose slope is the same as that of the constant-weight lines. If there is more than one type of flutter the achievement of an optimum design is more difficult due to the lack of continuity between the sets of flutter speed contours. Limits on the sizes of the variable elements can be represented by forbidden areas on the graph.

3 WAYS OF USING INVERSE ITERATION

3.1 Optimisation at constant speed

The minimum required by any optimisation routine is that the objective function, in this case the weight, should be calculable for any set of values of the variables. In the problem considered here the calculation of the objective function is trivial; it is the calculation of the values of the variable which satisfy the flutter constraint that is difficult.

The first optimisation routine tried was one based on Powell's conjugate-direction method. When applying this the values of one of the variables were not determined by the optimisation routine but were calculated to be just sufficient for the flutter constraint to be satisfied when the other variables had values determined by the optimisation routine. Thus, referring to Fig 1, starting from the point on the contour at o^1, the routine might require the weight when τ_2 had its value corresponding to α . The flutter speed at α is above that required and the value of τ_1 , the special variable, is reduced from that at α to that at β and the weight calculated is that appropriate to β .

The actual version of Powell's method used was not designed with flutter in mind, did not limit the changes in the variables to small increments and, because of this, the unstable root was often lost. It seemed probable however that, were the program modified to restrict the sizes of the changes, a fairly-efficient program could have been created; notwithstanding the egalitarian criticism that one of the variables was treated differently from the others.

Optimisation with the first routine was abandoned however because a routine which had the advantage of invoking the flutter constraint internally, thus making special treatment of one variable unnecessary, became available. This optimisation routine required the real part of $(\pi - \lambda)$, see Appendix A, for given values of the variables τ .

A sketch of a typical sequence of moves is included in Fig 1. Starting near the point o at the top of one of the contours the flutter contour is found exactly at o (say) and then the slope of the tangent to the contour at o . A step is made along this tangent in the direction of lower weight to a (say). The point a will not, in general, lie on the contour and so a step is made normal to the tangent to reach the contour (at b say) and the weight is calculated. The sequence is continued (through c and d etc) until the point (near D) corresponding to values of τ which result in minimum weight is approximated to.

3.2 Optimum for increased speed

As originally conceived the method of finding the optimum variations of structural elements for increasing the flutter speed was a sequence of two events. First the flutter speed would be increased to the desired level by reliable means such as an overall increase in stiffness and then the new structure would be the structure to be input into the optimisation procedure using the values of the variables pertinent to the original structure as lower limits perhaps.

It turns out that this can be a somewhat uneconomic procedure if the optimum distribution is not close to that chosen for getting the initial increase in flutter speed and if the critical root is close in frequency to other roots so that there is a great danger that the critical root will be lost during the radical changes in mass and stiffness distribution. It was decided therefore that an attempt should be made to find a better method of obtaining increases in speed.

With the encouragement of the comments in Refs 1 and 2 a computer program was written by which an increase in flutter speed was obtained in a specified number of increments, the ratios of the increases in the values of the variables being determined at the beginning of each increment as those that gave the direction of steepest ascent, that is the normal to the flutter speed contours. This does not lead to the structure of least weight even when the steps are small but is likely to be a reliable way of obtaining a good starting point for subsequent optimisation.

The rates of increase of flutter speed with increase in weight for each variable, $\phi_r^{-1}\ (\partial\upsilon/\partial\tau_r)$, are required for the calculation of the direction of steepest ascent and expressions for them are given in Appendix B. The direction is such that the increments in the variables are in the ratios of the speed derivatives with respect to them: for an increase in speed of $\delta\upsilon$ estimates of the increases in the variables were obtained from

$$\delta\tau_r \;=\; \left(\phi_r^{-2}\,\frac{\partial\upsilon}{\partial\tau_r}\,\delta\upsilon\right)\bigg/\left(\sum_{s=1}^{m}\left(\phi_s^{-1}\,\frac{\partial\upsilon}{\partial\tau_s}\right)^2\right) \quad . \qquad (1)$$

The overall factor by which all the estimated increments, $\delta\tau_r$, had to be multiplied to obtain the flutter speed required exactly was obtained by inverse iteration and interpolation. In use this method of increasing the flutter speed was reliable.

An alternative to the above was also programmed. This was to alter only the value of the variable which, from the value of the speed derivative with respect to it, appeared to offer the lightest way of achieving the current speed increment. The process followed is illustrated by the sequence ABCD in Fig 1 although there the actual increments needed are compared rather than the speed derivatives at the lower speed. If none of the starting values of the variables is greater than the value at the optimum and the path to the optimum is smooth, an approximation to the optimum can be obtained quickly and by increasing the number of intervals over which the final increase in speed is made the answer can be found as accurately as desired. The limitation here is the inability to reduce the values of the variables.

In view of the success of the latter method of finding good ways of increasing the flutter speed it was adapted to provide a good method of optimising at constant speed. For this the variable which is least effective in increasing flutter speed is reduced in value so that the flutter speed would fall by a specified amount if it were linear with the variable. The flutter speed is maintained at the required value however by a sufficient increase in the value of the variable most effective in increasing flutter speed. The process is continued until the derivatives of speed with respect to weight of those variables not on limits are sufficiently close to each other in value; for at the exact optimum they are all equal. To start with the weight will always be reduced but the weight will increase as the optimum is passed. If this happens before the structure is near enough to the optimum the implied speed reduction is halved in size.

It might be argued that this method is of no interest because it is only economic if the number of variables is small. It is thought however, that such a program could cover a significant proportion of the flutter optimisation problems that arise in practice. For what is usually wanted is some means of which general regions of the structure need to be stiffened or increased in mass; the exact means of providing the extra stiffness or mass are best left to the detail designer and production engineer. When there are a large number of variables there is the further difficulty of including degrees of freedom adequate, in number and type, to describe all the possible variations without having to make frequent normal-mode calculations; this is discussed further in section 5.

As to the calculation of flutter stability there could hardly be a more economical method than that used here but it is important to keep control over what is asked of it if the unstable root is not to be lost.

4 OPTIMISATION OF SWEPT WING

4.1 Description of wing

The example calculation was done to check that the optimisation programs would work in a simple case based on a preliminary design for a swept-back wing of moderately-high aspect ratio carrying no concentrated masses.

A plan of the wing is shown in Fig 2. It was taken that the aspect ratio was high enough for the structure to behave in a way that could be described using the concept of a flexural axis. This flexural axis was taken to be at 39% chord, the inertia axis at 46% and the radius of gyration was taken to equal 24%

chord. The main hope of increasing the flutter was assumed to lie in increases of skin thickness and it was assumed that these increases would affect flexural and torsional rigidity equally.

The wing was divided into five sections and the distribution of the extra skin thickness was limited to uniform thickening of each of these individual sections. Thus there were five variables whose values could be altered in the basic example. The sections were of equal span apart from the most-outboard one which was almost 50% larger than the others (ill-advisedly as it turned out).

Each of the initial modes involved distortion of only one strip so that the modes followed the discontinuities in the thickness distribution. The modes themselves were based on loadings rather than deflections because it was thought more realistic. The simplest two loadings possible were taken that would represent linear variations of flexural or torsional moments over the strip. One was a uniform moment, which gave flexural curvatures or rates of twist which were proportional to the local $(EI)^{-1}$ or $(GJ)^{-1}$. The second was a moment varying linearly across the strip, orthogonal, with respect to stiffness, to the first mode. In this mode flexural curvature or rate of twist was zero outboard of the strip as well as inboard. Because the moments at the tip must be zero the tipmost strip was allowed only two modes; those whose moment distributions were triangular.

The reference axis for the modes was the flexural axis. As a result of this, the limitation of distortion in each mode to one strip and the orthogonalisation of the modes for any one strip, all the 18 generalised coordinates were orthogonal with respect to stiffness. In the modes, sections outboard of the one that was distorting had deflections due to their carriage as rigid bodies. Inertia and stiffness matrices were calculated for increments in the skin thickness of each section. In calculating the increase in weight it was assumed that the weight of the basic structure was equally divided between structural and non-structural items. Before the optimisation calculation the coefficient matrices were transformed to those for the normal coordinates of the basic wing.

The aerodynamic forces assumed were those obtained from strip theory. The values of the aerodynamic derivatives taken when referred to the leading edge, were

$$\left.\begin{aligned}
\begin{bmatrix} \ell_z & \ell_\alpha \\ -m_z & -m_\alpha \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & \frac{1}{4} \end{bmatrix} \ell_\alpha \\
\begin{bmatrix} \ell_{\dot z} & \ell_{\dot\alpha} \\ -m_{\dot z} & -m_{\dot\alpha} \end{bmatrix} &= \begin{bmatrix} 44 & 16 \\ 11 & 9 \end{bmatrix} \frac{\ell_\alpha}{44} \\
\begin{bmatrix} \ell_{\ddot z} & \ell_{\ddot\alpha} \\ -m_{\ddot z} & -m_{\ddot\alpha} \end{bmatrix} &= \begin{bmatrix} 32 & 16 \\ 16 & 9 \end{bmatrix} \frac{\pi}{128}
\end{aligned}\right\} . \qquad (2)$$

4.2 Description of calculation

The flutter speeds of the wing after uniform increases in skin thickness of 25%, 50% and 100% were found. The optimum distributions to obtain the same flutter speeds were then found by using each of the optimising programs. To test how sensitive the results were to the freedom given the wing the calculations were made allowing 18, 10 and 5 degrees of freedom. The degrees of freedom were the graver normal modes of the basic wing. It was found that the results for 18 and 10 degrees of freedom were almost identical so they are not reported separately. The results with 5 degrees of freedom were somewhat different.

Fig 3 shows the increase in flutter speed against increase in weight with the two optimum distributions, including 10 and five modes, and with uniform distribution. The difference between the increases in flutter speed for a given increase in weight in the case of the optimised wings grows continuously and the increase in flutter speed with five modes is about 37½% for a weight increase of 38% whilst that with 10 modes is about 35¾%. In both cases the increase is over twice that obtained with a uniform thickening.

The difference between the five- and 10-mode cases is more obvious in Fig 4 in which the distributions of the additional thicknesses are given. They are quite different for the two cases and when the thickness distribution which gives 37½% increase in flutter speed with five modes is used with 10 modes the increase is only 29%. Thus although calculations on an overconstrained representation of the structure might give a good upper limit of the increase in flutter speed that can be obtained for a given increase in weight there is a risk that the calculated distribution will give a significantly smaller increase in the flutter speed of the unconstrained structure. In all cases the outboard sections have to be thickened more than the inboard sections for optimum results. This was not anticipated.

The variations of decay rates and frequencies of the roots with speed are shown in Fig 5a&b for the 10 degrees-of-freedom calculation with the highest flutter speed. It can be seen in the optimised case that once the critical flutter speed is exceeded the rate of growth of the flutter oscillation increases abruptly. Whilst this is comforting as an indication that the optimisation procedure has got the last fraction of speed out of the structure, apparently by tuning the overtone bending mode, the gain could be illusory to some extent as the severity of onset of flutter has been changed and it might be thought prudent to have a larger margin between the highest flight speed allowed and the calculated flutter speed in the optimised case. The stability is also likely to be sensitive to the exact frequency and shape of the overtone bending mode.

A further difference between the five-and 10-mode cases was that a local optimum was found in the five-mode case when no thickening of the tip section was allowed to begin with. At the optimum almost all the thickening was to the section immediately inboard of the tip with a trifling amount to the section next inboard. The weight was about 5% lower than that with the other minimum found. In the 10-mode case there was no second optimum and the weight with no thickening at the tip was much greater than the optimum.

The optimisation procedures used were not limited to finding just the critical boundary at which oscillations are maintained but were also able to follow a boundary on which the real part of $(\pi - \lambda)$ was non-zero and which corresponds to growing or decaying oscillations. This ability was used to obtain a structure with a less abrupt change in the rate of decay by having the reference speed in the optimisation below the critical flutter speed and requiring the (positive) decay rate at this subcritical speed to remain unchanged (and greater than that given by the initial optimisation). Fig 5c shows how the severity of the onset of flutter can be reduced by using as reference speed of 1.35 rather than the flutter speed which is 1.448. Fig 4d shows that the increases in thickness are more evenly distributed and Fig 3 shows that the weight increase is about 45½% instead of 42% for the optimum based on critical speed.

The ability to follow a boundary on which the real part of $(\pi - \lambda)$ is non-zero could prove useful in optimising structures whose rates of decay, although never negative, are unacceptably small over a significant range of speed.

The programs should work equally well if the optimum way of increasing the flutter speed is by altering the inertia distribution rather than the stiffness distribution. A calculation was made in which mass balance in the form of masses set ahead of the leading edge between the most outboard section and the one next inboard was used to increase the flutter speed. The optimum distribution was with all the mass added at the position furthest ahead of the leading edge. Unfortunately this solution was much heavier than thickening the skin and it proved impossible to get the two methods of increasing the flutter speed to interact with each other.

4.3 How the increase in flutter speed was obtained

It is instructive to consider which of the factors affecting flutter were most in evidence in the change from the basic wing to the optimum wing with flutter speed increased by 38%. The normal modes in still air of the uniform wing and the optimum wing are given in Fig 6. Two sets of frequencies are given for the uniform wing: the frequencies with twice the original thickness and with the original thickness. The first four modes are shown and it can be seen that the fundamental flexure and torsion frequencies are decreased by the changes whilst the frequencies of the overtone flexure modes are increased.

The changes to the fundamental normal modes do not appear large enough to have a significant effect on binary flexure torsion flutter and when they are scaled so that the inertia terms are equal the optimum wing gains in that its coupling terms in aerodynamic stiffness are smaller by about 20% each but presumably loses because its aerodynamic torsional stiffness is numerically larger and its structural stiffness smaller. The change in structural frequency ratio is insignificant.

However in this particular case it is inappropriate to look for explanations in terms of binary flutter alone and this can be seen from an examination of Fig 5b which gives the variations of the roots of the flutter equation with speed for the wing optimised on critical speed. It would appear from the behaviour of the mode which is first overtone flexure mode in still air that part of the optimisation process is to 'tune' it so that its decay rate falls as the flutter speed is approached at the same time as the decay rate of the mode that goes unstable falls. Thus there are two modes with low decay rates instead of one with a higher decay rate and one with a negative decay rate, *ie* growing in amplitude.

To evaluate the effect of the overtone mode the binary and ternary flutter speeds were found for the uniformly thickened and both the optimum wings. In the case of the uniformly thickened wing the removal of the overtone degree of freedom led to an increase in flutter speed of ½%. In the case in which minimum decay rate was the constraint the flutter speed decreased by about 3% when the overtone mode was removed and in the case of critical speed the decrease was almost 6%. Hence although the tuning of the overtone is apparently effective in increasing the stability it cannot be a major factor in the increase achieved.

Another effect which is not so immediately obvious is the effect of the fourth mode on the stability of the wing with uniform increase in thickness. From the behaviour of the root with airspeed (Fig 5) it would not appear to be having any effect. However the binary flutter speed is about 17½% greater than the speed with 10 degrees of freedom and almost three-quarters of this increase comes on the elimination of the fourth mode. The fourth modes of the optimum wings have negligible effect on their flutter stability.

Thus a substantial part of the increase in stability in this case comes from altering the ratios of the overtone frequencies to those of the fundamentals.

5 STRUCTURAL REPRESENTATION

The normal modes in still air are commonly used in flutter calculations because generally they lead to convergence with fewer coordinates than other choices. During the optimisation process the mass and stiffness are changed and hence the normal modes change. It is inconvenient to calculate the normal modes after every change and in practical cases it is inconvenient to include enough normal modes of the original structure to ensure that a precise description of the effects of possible detailed changes to it is always available. The description of the deflections of a structure in terms of a limited number of semi-rigid modes puts constraints on those deflections and these constraints can make the results obtained nonsense. In general this does not happen because the dangers are obvious but during optimisation the structure is constantly changing and any inappropriateness of the original modes that may arise might not be noticed. Thus a favourable combination of deflections may continue due to the limitations of the modes after the mass and stiffness distribution which gave rise to them have been changed significantly.

The choice of simple changes to the overall skin thickness in the example meant that these difficulties could be kept in the background but by restricting the number of normal modes included in the flutter calculation the effect of overconstraint could be investigated. As has been mentioned in section 4.2 it was found that the first 10 normal modes gave results practically identical to the full set but differences were noticeable when the modes were limited to five.

A further indication of the effects overconstraint can have was obtained by observing how the flutter speed increased with increase in thickness of the tip section alone when 5 and when 10 degrees of freedom were allowed. These rates of increase were almost identical to begin with but differed, albeit slightly, before the speed had been increased by 10%. The flutter speed could not be increased much more than 20% when 10 degrees of freedom were allowed but continued to increase when there were only 5 degrees of freedom although the rate was reduced when the increase in speed passed 25%.

For complete reliability of the results obtained it would seem necessary to have a generalised coordinate for each variable element of the structure so that the effects of changes to the structure on its oscillatory characteristics can be traced accurately. Thus if there are a large number of variable elements it seems likely that the mathematical model should be large in size if it is not to suffer from overconstraint. In this case it might be prudent to identify those general areas of the wing it is most profitable to alter first, the detail of the alterations being determined by subsequent calculations.

It is not easy to choose generalised coordinates which allow adequately for the effects of changes of element stiffness for what are really wanted are generalised coordinates in which the elements are separately given one or more predetermined realistic patterns of strains and the rest of the structure conforms to these strains but is otherwise unloaded. What can be achieved in this direction is very dependent on the structure being considered. If mass balance is being considered however, discrete-load modes[3] provide the complete answer.

6 CONCLUDING REMARKS

The objective of optimisation with flutter constraints has been considered and it has been concluded that, in view of the complicated nature of the phenomenon, it should be confined to the finding of the lightest way of increasing the flutter speed of a wing already designed using other criteria. In this case the critical eigenvalue of the flutter equation is known and inverse iteration provides an economic way of following it as the structure changes. The optimisation routine should only change the structure gradually to minimise the risk of losing the critical eigenvalue.

The biggest problem associated with weight optimisation with flutter constraints is adequate structural representation. It is believed that for complete reliability each variable structural member should have a separate degree of freedom in the calculations to represent each of its possible distortions. Thus calculations with large numbers of variable elements are either expensive or unreliable and to avoid these the numbers of variables should be kept to a minimum. Overconstraint in the mathematical model may not lead to a gross underestimate of the extra weight needed but will probably give an inaccurate distribution which means that the increase in flutter speed will not be achieved in practice.

Work is needed on a method suitable for describing the effect of stiffening local parts of a structure on the modes of the complete structure.

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Appendix A

SOLUTION OF FLUTTER EQUATION USING INVERSE ITERATION

Take the flutter equation to be

$$\left[A\lambda^2 + (B\upsilon + D)\lambda + C\upsilon^2 + E\right]q = 0 \qquad \text{(A-1)}$$

where A,D,E are real square matrices of the structural inertia, damping and stiffness coefficients

B,C are real matrices and the aerodynamic force coefficients are given by $(B\lambda + C\upsilon)\upsilon$

λ is a scalar whose imaginary part is proportional to frequency and whose real part is proportional to growth

υ is a scalar proportional to airspeed.

For convenience write equation (A-1) as

$$\left[A\lambda^2 + \bar{B}\lambda + \bar{C}\right]q = 0 \quad . \qquad \text{(A-2)}$$

Equation (A-2) has a linear equivalent

$$\begin{bmatrix} A\lambda + \bar{B} & \bar{C} \\ -I & \lambda I \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad . \qquad \text{(A-3)}$$

Rearranging equation (A-3)

$$\begin{bmatrix} \bar{B} & \bar{C} \\ -I & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = -\lambda \begin{bmatrix} A & \\ & I \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \quad . \qquad \text{(A-4)}$$

Adding $\pi\lceil A, I \rfloor \{p,q\}$ to each side leads to

$$\begin{bmatrix} \pi A + \bar{B} & \bar{C} \\ -I & \pi I \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = -(\lambda - \pi) \begin{bmatrix} A & \\ & I \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \quad . \qquad \text{(A-5)}$$

π here represents a scalar constant, purely imaginary in value, and iteration based on

$$\begin{bmatrix} \pi A + \bar{B} & \bar{C} \\ -I & \pi I \end{bmatrix} \begin{bmatrix} p_{r+1} \\ q_{r+1} \end{bmatrix} = -(\lambda - \pi) \begin{bmatrix} A & \\ & I \end{bmatrix} \begin{bmatrix} p_r \\ q_r \end{bmatrix} \quad . \qquad \text{(A-6)}$$

gives the latent root and vector nearest π .

Equation (A-6) can be replaced by the two equations

$$(\pi A + \bar{B})p_{r+1} + \bar{C}q_{r+1} = -(\lambda - \pi)Ap_r \tag{A-7}$$

$$\pi q_{r+1} = p_{r+1} - (\lambda - \pi)q_r \quad . \tag{A-8}$$

Multiplied by π and with q_{r+1} substituted for from equation (A-8), equation (A-7) becomes

$$(\pi^2 A + \pi\bar{B} + \bar{C})p_{r+1} = -(\lambda - \pi)(\pi A p_r - \bar{C}q_r) \quad . \tag{A-9}$$

In the flutter equation used in optimisation

$$\left.\begin{aligned} A &= A_0 + \sum_{r=1}^{m} \tau_r A_r \\ E &= E_0 + \sum_{r=1}^{m} \tau_r E_r \end{aligned}\right\} \tag{A-10}$$

and the weight which is the objective function is

$$\phi = \phi_0 + \sum_{r=1}^{m} \tau_r \phi_r \quad . \tag{A-11}$$

Appendix B

FLUTTER-SPEED AND -FREQUENCY DERIVATIVES

The flutter equation can be written

$$\left[A\lambda^2 + (B\upsilon + D)\lambda + C\upsilon^2 + E\right]q = \{0\} \tag{B-1}$$

or

$$p'\left[A\lambda^2 + (B\upsilon + D)\lambda + C\upsilon^2 + E\right] = \lfloor 0 \rfloor \quad . \tag{B-2}$$

Let the matrices A and E be functions of τ_r and let differentiation with respect to τ_r be denoted by a dot. Then the derivative of equation (B-1) is

$$\left[\dot{A}\lambda^2 + 2A\lambda\dot{\lambda} + B\lambda\dot{\upsilon} + (B\upsilon + D)\dot{\lambda} + 2C\upsilon\dot{\upsilon} + \dot{E}\right]q$$
$$+ \left[A\lambda^2 + (B\upsilon + D)\lambda + C\upsilon^2 + E\right]\dot{q} = \{0\} \tag{B-3}$$

Premultiplying by p' and rearranging we get

$$p'\left[\dot{A}\lambda^2 + \dot{E} + (2A\lambda + B\upsilon + D)\dot{\lambda} + (B\lambda + 2C\upsilon)\dot{\upsilon}\right]q = 0 \quad . \tag{B-4}$$

This can be rewritten as

$$\alpha + \beta\dot{\lambda} + \gamma\dot{\upsilon} = 0 \tag{B-5}$$

with $\alpha = p'(\dot{A}\lambda^2 + \dot{E})q$

$\beta = p'(2A\lambda + B\upsilon + D)q$

and $\gamma = p'(B\lambda + 2C\upsilon)q$.

Separating real and imaginary parts of equation (B-5) and using a notation with which, typically, $\alpha = \bar{\alpha} + i\bar{\bar{\alpha}}$ we get

$$\left.\begin{aligned} \bar{\alpha} + \bar{\beta}\bar{\dot{\lambda}} - \bar{\bar{\beta}}\bar{\bar{\dot{\lambda}}} + \bar{\gamma}\dot{\upsilon} &= 0 \\ \bar{\bar{\alpha}} + \bar{\bar{\beta}}\bar{\dot{\lambda}} + \bar{\beta}\bar{\bar{\dot{\lambda}}} + \bar{\bar{\gamma}}\dot{\upsilon} &= 0 \end{aligned}\right\} \quad . \tag{B-6}$$

If we take the rate of change of the real part of the root with the variable to be zero, *ie* $\bar{\dot{\lambda}} = 0$ and hence the time rate of decay is constant, we can write a rearranged equation (B-6) in matrix form as

$$\begin{bmatrix} -\bar{\bar{\beta}} & \bar{\gamma} \\ \bar{\beta} & \bar{\bar{\gamma}} \end{bmatrix} \begin{bmatrix} \bar{\bar{\dot{\lambda}}} \\ \dot{\upsilon} \end{bmatrix} = - \begin{bmatrix} \bar{\alpha} \\ \bar{\bar{\alpha}} \end{bmatrix} \quad . \tag{B-7}$$

The solution of this is

$$\begin{bmatrix} \bar{\bar{\dot{\lambda}}} \\ \dot{\upsilon} \end{bmatrix} = (\bar{\beta}\bar{\gamma} + \bar{\bar{\beta}}\bar{\bar{\gamma}})^{-1} \begin{bmatrix} \bar{\bar{\gamma}} & -\bar{\gamma} \\ -\bar{\beta} & -\bar{\bar{\beta}} \end{bmatrix} \begin{bmatrix} \bar{\alpha} \\ \bar{\bar{\alpha}} \end{bmatrix} \tag{B-8}$$

from which the rates of change of flutter frequency and speed with the variables can be obtained.

Thus to obtain the values of the derivatives analytically a knowledge of the coefficient matrices, A, B, C, D, the derivative matrices, $\dot{A}$, $\dot{E}$, the critical eigenvalue and the corresponding right and left hand eigenvectors is necessary.

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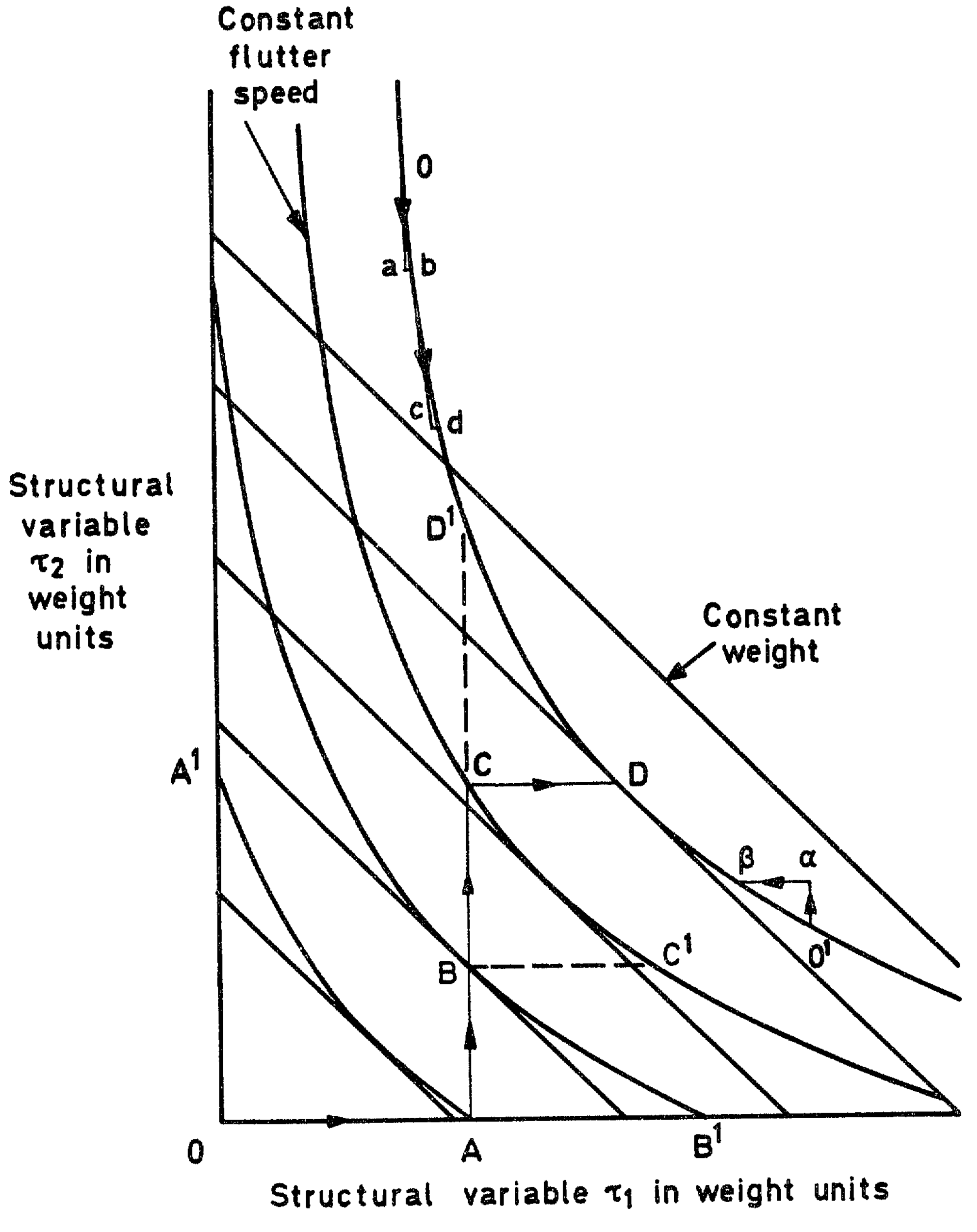

Fig 1 Optimisation processes

Fig 2

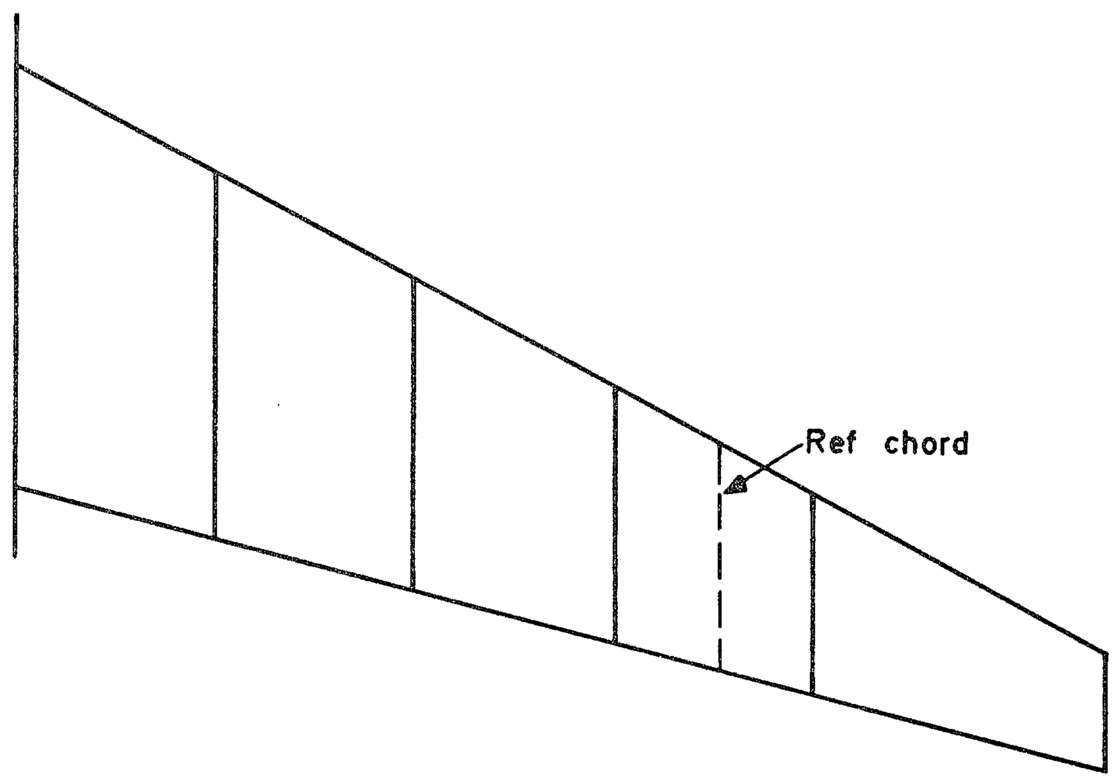

Fig 2 Plan of wing

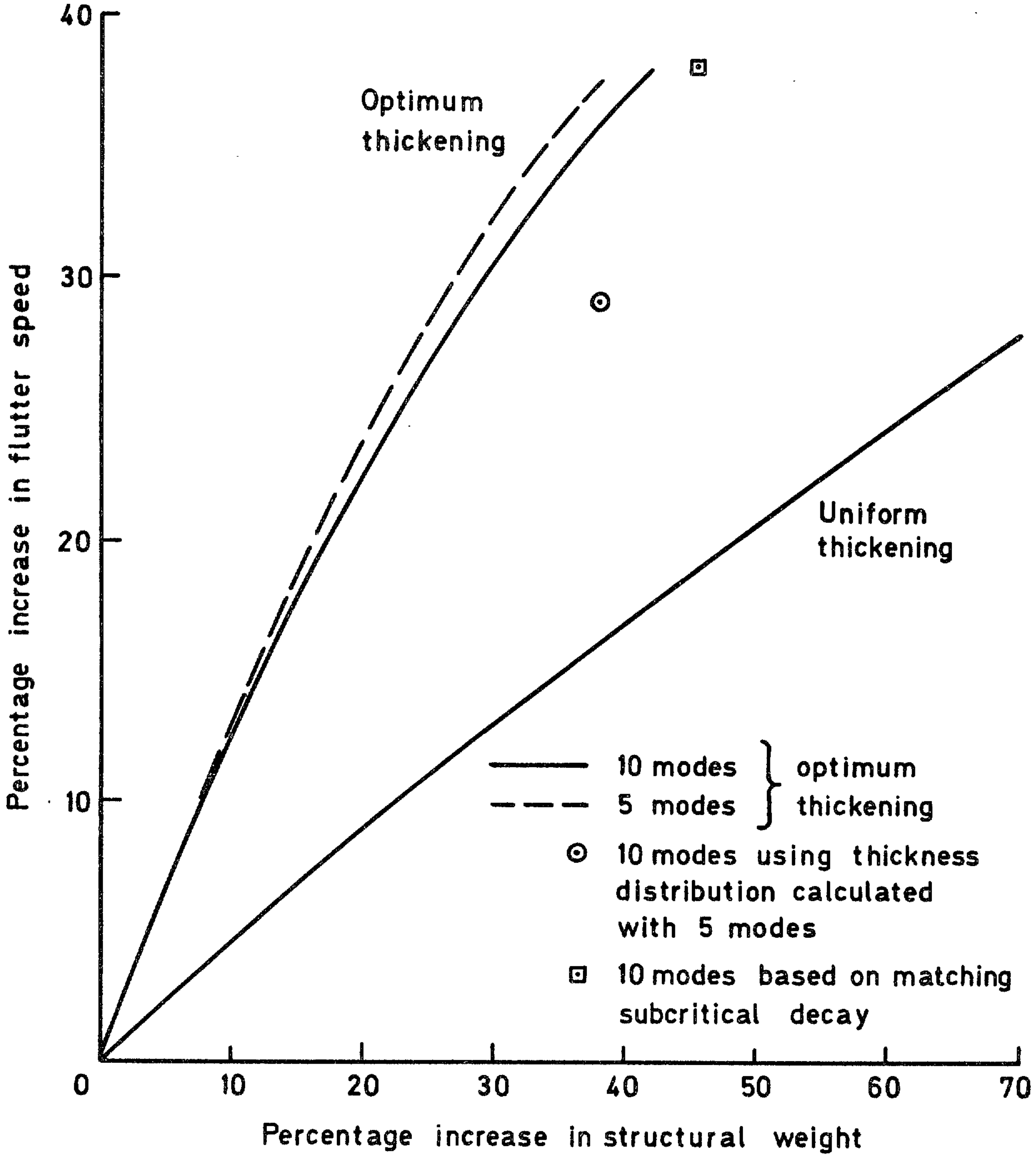

Fig 3 Increase in flutter speed v increase in weight

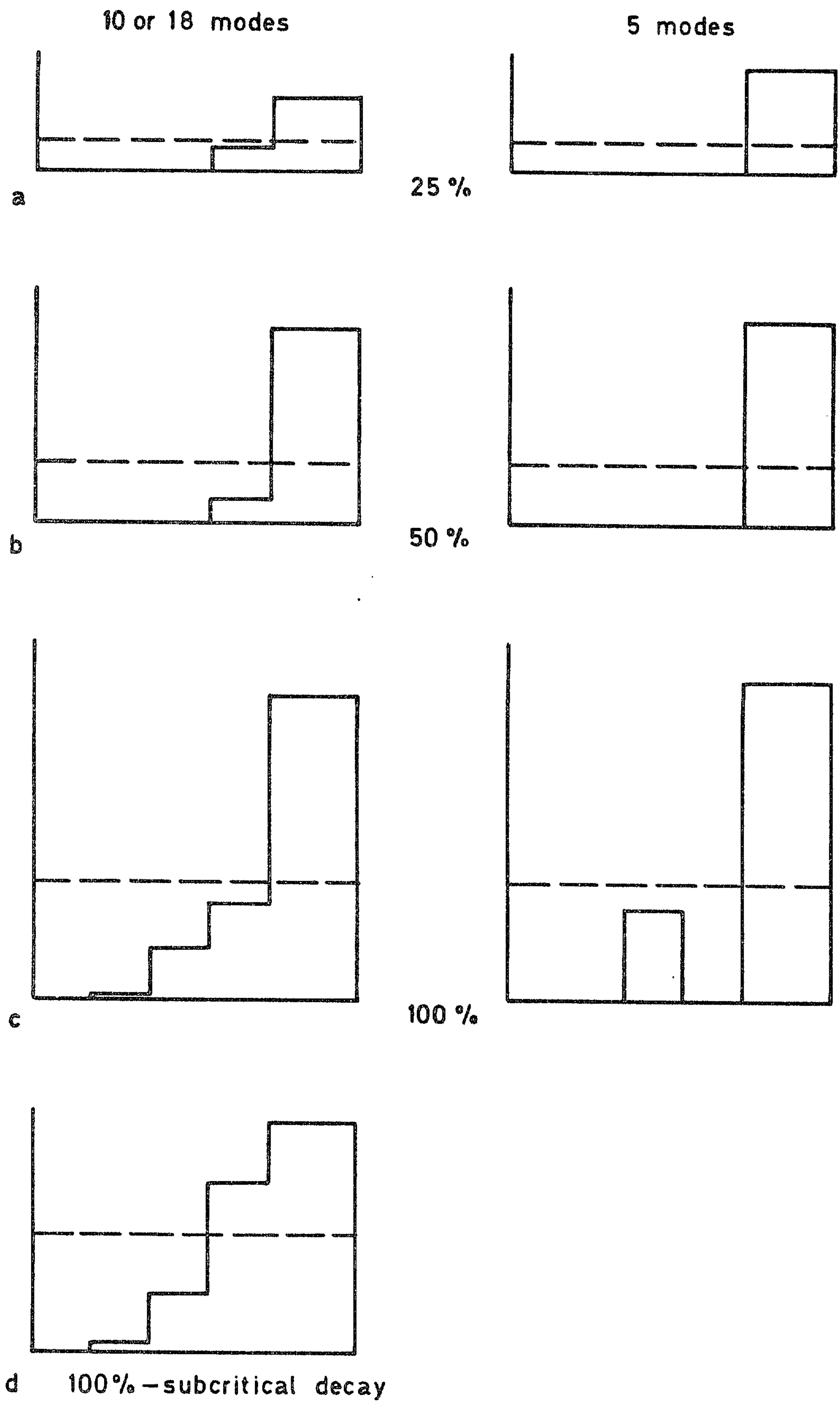

Fig 4 Optimum distribution of additional thickness for same flutter speed as for overall percentage thickening – 5, 10 and 18 modes

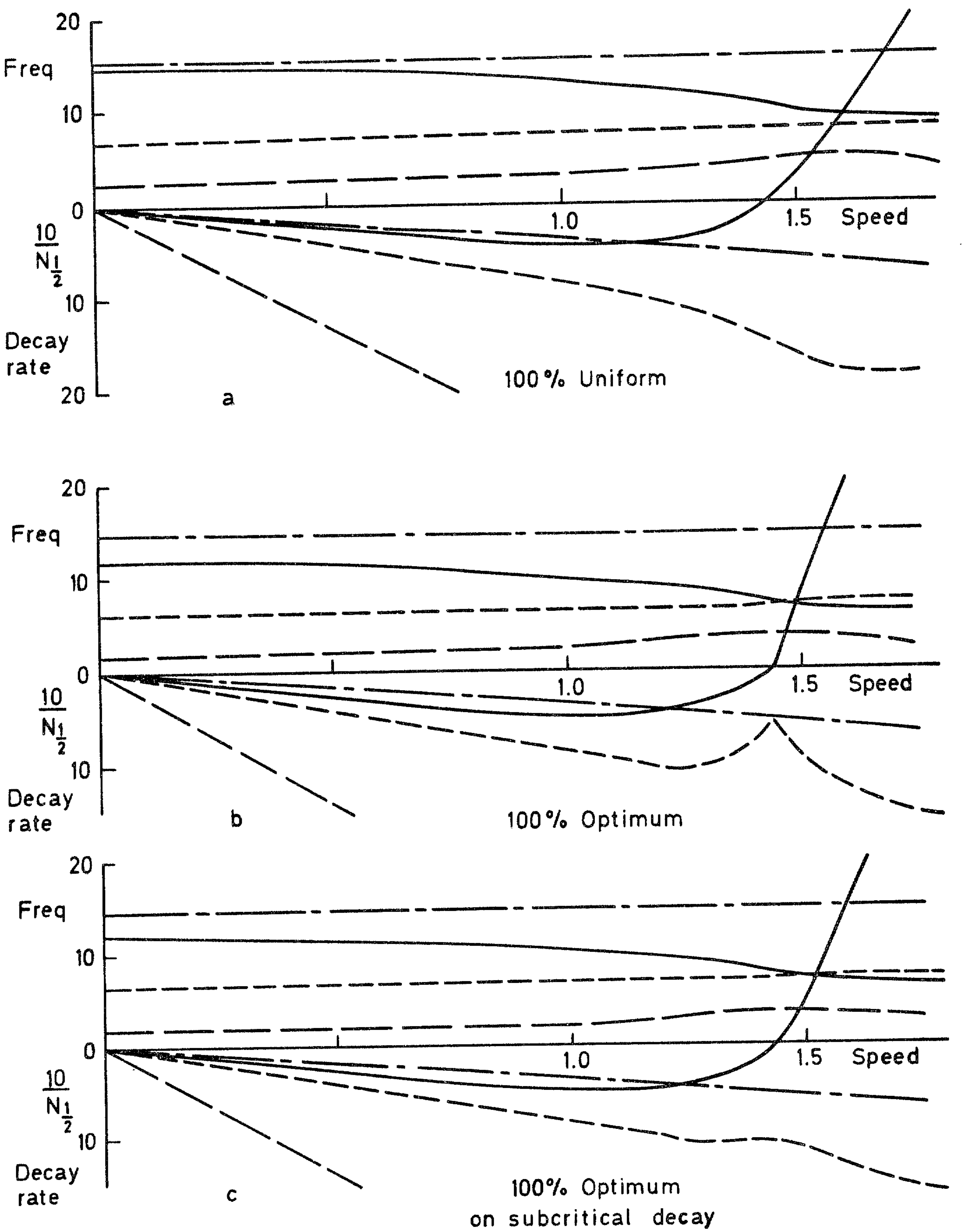

Fig 5 Four gravest roots v speed — 10 degree of freedom

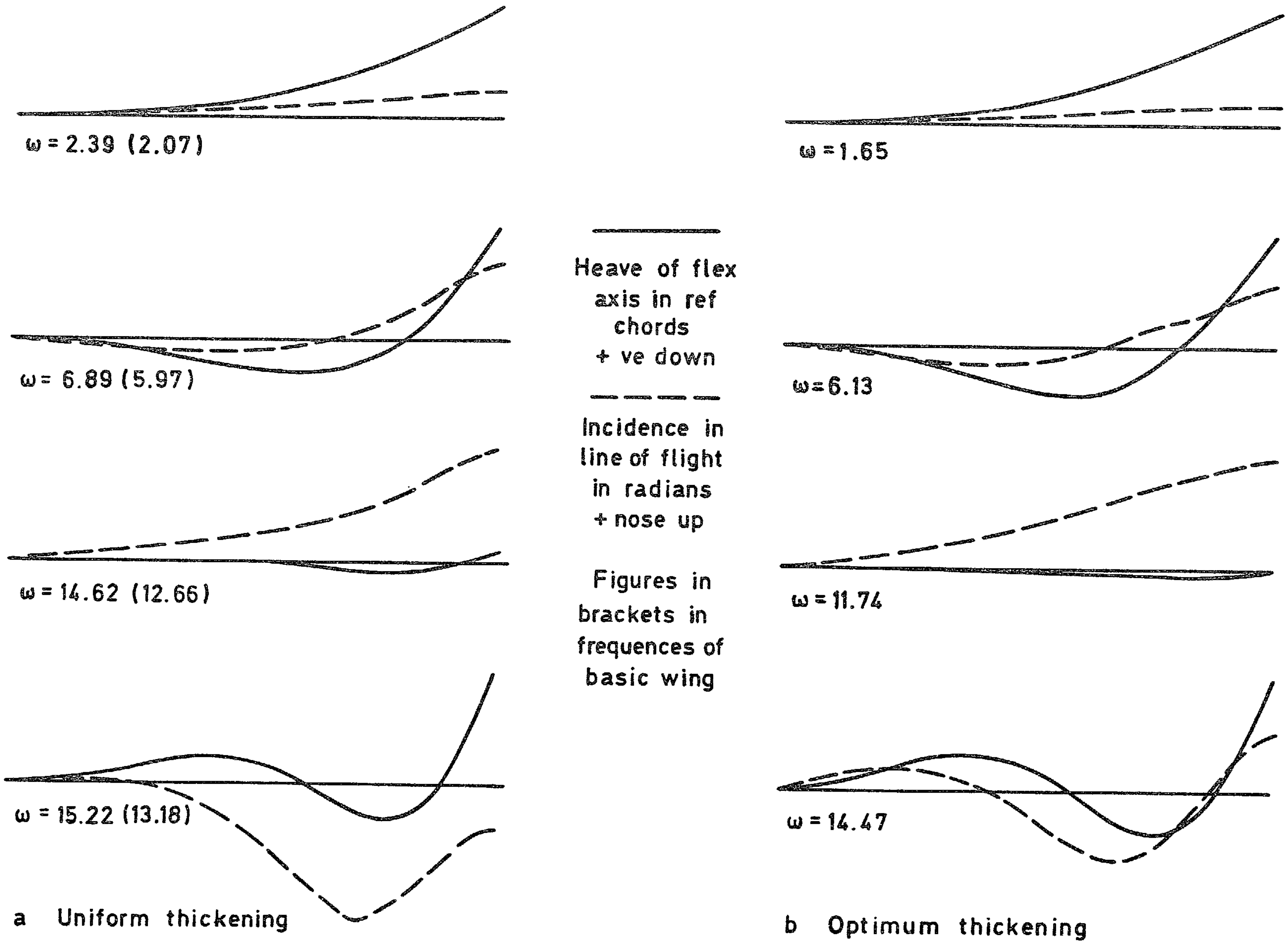

Fig 6 Normal modes in still-air. Wings with highest flutter speed

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