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STRENGTH AND ELASTIC RESPONSE OF SYMMETRIC ANGLE-PLY CARBON FIBRE REINFORCED PLASTICS

by

M.B. Snell

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Structures Department, RAE Farnborough

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SUMMARY

Comparisons between experimentally determined strengths and the theoretical predictions using four simple failure criteria are presented for symmetric angle-ply carbon fibre reinforced plastics laminates under tensile and compressive loads. The effects of non-linearity and failing strain are considered, and a correlation is given between the non-linear behaviour of unidirectional material subjected to shear and the non-linear elastic response of balanced angle-ply test specimens under tension.

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CONTENTS

1 INTRODUCTION

The weight-savings obtainable from the use of carbon fibre reinforced plastics (CFRP) combined with a longer-term potential for reducing manufacturing and in-service costs have prompted UK and USA aircraft manufacturers to consider its use in significant quantities both in current aircraft and in future projects. In the General Dynamics YF16, for example, extensive use in the empennage of CFRP laminated skins yields weight savings over traditional materials of 30%.

The stiffness of multi-directional CFRP laminates is adequately determined using lamination theory[1] and it is relatively straightforward to design a laminate having the best required combination of Young's moduli, shear modulus and Poisson's ratios. In contrast, strength estimation for multi-directional CFRP laminates has hitherto been largely uncertain, although the recent use of various failure criteria has yielded reasonable agreement with experimental data for the strength of other composites, particularly glass/epoxy[2]. The shapes of the stress/strain curves and consequent strains to failure are also largely unknown for CFRP angle-ply laminates.

Several theories which predict failure of orthotropic material have been recently proposed[3-8]; some[3-5] are generalisations of the Von Mises distortional energy criterion, extended to allow for both orthotropy and different strengths in tension and compression, while others[6,7] are simple statements that the material will fail when the limiting stress or strain in any of the natural co-ordinate directions is reached. This Report considers four of the more relevant failure criteria and their application to CFRP laminates in angle-ply form. A comparison is made between the four theories and experimental strength data from tensile and compressive tests on CFRP balanced angle-ply laminates. Also reported is a correlation between elastic non-linearity of angle-ply test coupons in tension and the non-linearity which exists in the elastic response of unidirectional material under shear stress, and angle-ply elasticity equations are derived which relate the two. The variation of strain-to-failure with orientation for angle-ply laminates is given and a discussion deals with possible reasons for differences between the theoretically predicted values and the observed strengths of angle-ply laminates in which the ply orientations relative to the direction of the applied load are small.

2 FAILURE THEORIES

2.1 Quadratic interaction formulae

2.1.1 The Hill/Tsai criterion

An isotropic medium reinforced with straight unidirectionally-aligned fibres can be considered to possess isotropy of stiffness and strength about an axis parallel with the fibres. Azzi and Tsai[4] used this fact in deriving a reduced form of Hill's 'plastic potential'[8], proposed in 1948 to predict the yield stress in anisotropic metals. The reduced form, known as the Hill/Tsai criterion, is expressed as

$$(\sigma_1^2 - \sigma_1\sigma_2)/F_1^2 + \sigma_2^2/F_2^2 + \sigma_3^2/F_3^2 = 1 \qquad (1)$$

where subscript 1 refers to directions parallel to the fibres,

subscript 2 refers to directions normal to the fibres, but in the plane of the laminate,

subscript 3 refers to shear in directions parallel to the fibres, $F_{1,2}$ are the breaking strengths, and are either tensile or compressive according to the stress quadrant

and F_3 is the breaking strength in shear.

Equation (1) represents an ellipsoidal fracture surface in σ_1, σ_2, σ_3 space, composed of four quarter ellipsoids disposed about the σ_3 axis. Sections parallel to the σ_1, σ_2 plane appear as four quarter ellipses coincident at their points of intersection with the σ_1 and σ_2 axes but with discontinuous slope at these points. Representation in two axes is obtained by considering the $(\sigma_3/F_3)^2$ term as a parameter the sole effect of which is to scale the fracture ellipse so obtained. When the shear stress on the material, σ_3, equals the fracture shear stress, F_3, the ellipse is reduced to a single point at the origin, indicating failure by shear with no other stress components present. At values of $\sigma_3 < F_3$, the theory allows additional stresses σ_1 and σ_2, up to the ordinate and abscissa of the chosen point on the scaled fracture ellipse quadrant, before failure occurs. Thus any point inside the region enclosed by the fracture ellipse represents an allowable stress state and any point on or outside the bounding line represents failure of the material. An example of this criterion, applied to a high strength (HT-S) unidirectional CFRP laminate, is given in Fig.1.

2.1.2 The Hoffman criterion

Another modification to Hill's failure surface was proposed by Hoffman[3] in 1967. To cater for the possibility of different strengths in tension and compression, Hoffman included linear terms in σ_1 and σ_2. This gives a continuous fracture surface with no discontinuity in slope at the axis intersections. For plane stress and transverse isotropy as before, Hoffman's criterion is

$$A1(\sigma_1^2 - \sigma_1\sigma_2) + A2\sigma_2^2 + A3\sigma_1 + A4\sigma_2 + A5\sigma_3^2 = 1 \qquad (2)$$

where $A1 = (F_{1T}F_{1C})^{-1}$,

$A2 = (F_{2T}F_{2C})^{-1}$,

$A3 = (F_{1C} - F_{1T})/(F_{1C}F_{1T})$,

$A4 = (F_{2C} - F_{2T})/(F_{2C}F_{2T})$

$A5 = F_3^{-2}$,

subscripts T and C indicating tension and compression respectively. With no shear stress present (i.e. $\sigma_3 = 0$), and for the same unidirectional material as before, this continuous ellipse is shown on Fig.1.

2.2 Stress and strain limit theories

2.2.1 Theory of maximum strain

In the theory of maximum strain it is simply postulated that a material will fail when certain limiting values of strain are reached independent of the stress existing in the failure plane. This strain could arise due to mechanical or thermal effects. For most CFRP composites, however, little overall thermal strain will be expected and for convenience only mechanical strains are considered here.

The strain limits are obtained from uniaxial tests in the longitudinal (fibre) and transverse directions and from shear tests. These limits can then be applied under complex loading situations when the direct and Poisson's strains summate to give the total natural co-ordinate strains. Since the criterion is a simple statement that failure will occur when any of the natural co-ordinate strains reaches its limiting value, it can be written as

$$\left.\begin{aligned} \varepsilon_1 &= \varepsilon_{1f} \ , \\ \varepsilon_2 &= \varepsilon_{2f} \\ \text{and} \qquad \varepsilon_3 &= \varepsilon_{3f} \ , \end{aligned}\right\} \tag{3}$$

where subscript f indicates failure. The values of ε_{if} , $i = 1, 2, 3,$ may be obtained from uniaxial tensile, compressive and shear tests and, if linear stress/strain behaviour is assumed, the criterion may be written as

$$\varepsilon_{if} = F_i/E_i \ . \tag{4}$$

For a linearly elastic material equation (3) can be expressed in terms of stresses, enabling the failure envelope to be compared with the other criteria on the same graph. The familiar two-dimensional stress/strain equations are

$$\left.\begin{aligned} \varepsilon_1 &= \frac{\sigma_1}{E_1} - \frac{\nu_{12}}{E_1}\sigma_2 \\ \text{and} \qquad \varepsilon_2 &= -\frac{\nu_{12}}{E_1}\sigma_1 + \frac{\sigma_2}{E_2} \ . \end{aligned}\right\} \tag{5}$$

Substituting equation (3) in equation (5) gives

$$\left.\begin{aligned} \varepsilon_{1f} &= \frac{\sigma_1}{E_1} - \frac{\nu_{12}}{E_2}\sigma_2 \\ \text{and} \qquad \varepsilon_{2f} &= -\frac{\nu_{12}}{E_1}\sigma_1 + \frac{\sigma_2}{E_2} \ , \end{aligned}\right\} \tag{6}$$

and substituting equation (4) in the above result yields

$$\left.\begin{aligned} \sigma_1 &= F_1 + \nu_{12}\sigma_2 \\ \text{and} \qquad \sigma_2 &= F_2 + \nu_{12}\frac{E_2}{E_1}\sigma_1 \ . \end{aligned}\right\} \tag{7}$$

Equation (7), which is independent of σ_3 , gives two pairs of parallel lines which intersect to give a stress boundary in the σ_1 , σ_2 plane; this is shown in Fig.1 for unidirectional material.

2.2.2 Theory of maximum stress

As might be expected, it is assumed in the theory of maximum stress that failure occurs when any component of the natural co-ordinate stresses reaches its limiting value. Although for a single ply this is likely to be catastrophic, for multi-directional laminates failure in the transverse direction can occur without overall collapse. The criterion is expressed as

$$\left.\begin{aligned} \sigma_1 &= F_1 \text{ ,} \\ \sigma_2 &= F_2 \\ \text{and} \\ \sigma_3 &= F_3 \text{ .} \end{aligned}\right\} \qquad (8)$$

Using the appropriate tensile or compressive strengths for unidirectional material, equation (8) is plotted on Fig.1, giving a rectangular figure with no interaction between the stresses and which is independent of shear stress, σ_3 .

2.3 Comparison of failure theories

The four theories as applied to a high strength (HT-S) unidirectional laminate have been plotted on Fig.1 for zero shear stress. For the two quadratic interaction formulae, the effect of the shear stress term, σ_3/F_3 , is to reduce the size of the ellipse while maintaining its shape. Ultimately the failure surface becomes a point at the origin, indicating failure by shear. These criteria are therefore of the distortional energy type where all deformations causing a distortion of the material, but not those causing a shape-maintaining expansion or contraction, are considered to contribute to the total strain energy of failure.

By contrast, the two limit criteria express no interaction between the direct and shear stresses and strains. They imply that the direct and shear failure stresses can each be approached independently with no loss of strength due to the presence of the other. In addition, the maximum stress theory expresses no interaction between the direct stresses, implying that the two components of direct stress can be increased to failure independently and without either mutual increase or reduction. The maximum strain theory indicates that

direct stress interaction can either increase the failing stress when Poisson's strains tend to a mutual total strain reduction, or reduce it when they cause a mutual increase.

In Fig.1 it can be seen that, for a typical high strength (HT-S) composite, there are quite large differences in the failure envelopes predicted by the four criteria in all four quadrants. It is, of course, possible that no one theory adequately describes the strength of CFRP laminates under combined stresses. An experimental programme was therefore carried out to measure the strengths of balanced angle-ply CFRP laminates in tension and compression and to compare them with the strengths derived using the four failure criteria.

3 APPLICATION OF FAILURE THEORIES TO CFRP

3.1 Method of application

The strength of multi-ply laminates using the four failure criteria can be determined only from a knowledge of the failure surface for a single ply under stresses acting in its natural co-ordinate directions. Hence it is necessary to find the stresses acting on each ply from a knowledge of its orientation and thickness and of the in-plane laminate loading; if bending stresses are present, a knowledge of ply positions will be required also. Lamination theory (LT) is used to find the laminate compliance matrix, [C] , from which the laminate strains due to a known set of loads can be found using the equation

$$\{e_0\} = [C]\{N\} \tag{9}$$

where $\{e_0\}$ is the column matrix of mid-plane strain components
and $\{N\}$ is the column matrix of stress resultants.

The mid-plane strain, arising from a set of stress resultants in the same ratio as the applied loads but at a level known not to cause failure, is found using equation (9). The strain in each ply natural co-ordinate direction is then found using the transformation

$$\{\gamma_n\} = T\{\gamma\}_c \tag{10}$$

where $\{\gamma\}$ is the column matrix of tensorial strain components,
subscript n indicates natural co-ordinate direction of the ply,
subscript c indicates laminate reference co-ordinates,
[T] is the transformation matrix.

$$\begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix}$$

where $m = \cos\theta$

$n = \sin\theta$ and θ is the angle of orientation of the ply.

The natural co-ordinate ply stresses are then found using the equation

$$\{\sigma\}_n = [Q]\{\varepsilon\}_c \tag{11}$$

where $\{\sigma\}_n$ is the column matrix of natural co-ordinate stresses

and $[Q]$ is the orthotropic Hooke's law matrix for the ply.

The amplitude at failure of the known ratio of the three stress components can now be predicted for the individual ply using the four failure criteria. Under the combined stress system in which

$$\left.\begin{aligned} \sigma_2 &= \alpha\sigma_1 \\ \sigma_3 &= \beta\sigma_1 \end{aligned}\right\}, \tag{12}$$

and

the Hill/Tsai criterion (equation (1)) can be solved for σ_{1f}, the maximum allowable level of this stress component, to give

$$\sigma_{1f} = \left(\frac{1}{F_1^2} - \frac{\alpha}{F_1^2} + \frac{\alpha^2}{F_2^2} + \frac{\beta^2}{F_3^2}\right)^{-\frac{1}{2}} . \tag{13}$$

The other two stress components at failure, σ_{2f} and σ_{3f} are then obtained from equation (12) and the laminate failure loads are found using

$$\{N\}_f = p\{N\} \tag{14}$$

where subscript f indicates failure and $p = \sigma_{1f}/\sigma_1$. Hoffman's criterion, equation (2), is used in the same way. The maximum strain criterion is used by first finding the smallest of the three ratios

$$q_i = \varepsilon_{if}/\varepsilon_i , \quad i = 1, 2, 3 , \tag{15}$$

and then obtaining the failure load from

$$\{N\}_f = q_i\{N\} \tag{16}$$

where q_i is the smallest of the three ratios in equation (15). Similarly, the maximum stress theory is used to find the smallest of the three stress ratios and hence the failure load. Subroutines for each of the four criteria were programmed in FORTRAN and were used with the lamination theory program[2] to predict the laminate strengths. The complete strength prediction program is given in Sheets 1-10 at the end of the main text.

For the case of balanced laminates with plies at several orientations, (i.e. $\pm\theta_1$, $\pm\theta_2$ etc.) the natural co-ordinate stress and predicted failure load must be found for a single ply at each orientation. Plies at $+\theta$ and $-\theta$ have the same direct stress but shear stresses of opposite sign and therefore it is necessary only to predict strength from the ply with the +ve orientation. For example, a laminate consisting of balanced groups of plies at $\pm\theta_1$, 0° , $\pm\theta_2$ would be analysed by considering the failure loads of $+\theta_1$, 0° and $+\theta_2$ plies only. In such cases it would usually be found that the plies in a single orientation will fail before the others. Assumptions must then be made as to the effect of the failed group of plies on the strength of the laminate as a whole. In this Report only simple angle-ply laminates are considered in detail (i.e. $\pm\theta$ only) and the laminate failure load is determined from the failure of any one ply.

3.2 Theoretical strength curves

The four theories were first used to predict the compressive strength of high modulus (HM-S) angle-ply laminates using strength and stiffness data for uni-directional material obtained from the 0°, 90° and $\pm45^\circ$ strength results of the present series of compression tests and a previously measured value of in-plane transverse tensile strength of $50MN/m^2$, and curves were prepared which show the variation of compressive strength with orientation (see Fig.2). Similar curves were drawn to predict tensile strength of high strength (HT-S) angle-ply laminates (see Fig.7). As can be seen, the differences in the predicted strengths from the four theories are large, especially in the region between $\pm10^\circ$ and $\pm35^\circ$. Furthermore, in the region between $\pm20^\circ$ and $\pm40^\circ$, where there is considerable shear acting, the differences between the quadratic interaction

formulae and the limit theories are greatly accentuated since the latter take no account of the effect of shear stress on failure. This was effectively demonstrated by running the Hill/Tsai subroutine with the shear interaction term deleted which, between $\pm 25^{\circ}$ and $\pm 35^{\circ}$, gave results close to those of the maximum strain theory.

In addition to the predictions of failure load, the two limit theories indicate the modes of failure. On Fig.3, which shows the variation of compressive strength with orientation according to the maximum strain theory, four distinct modes of failure can be identified. Mode 1 is compressive failure in the fibre directions which, perhaps surprisingly, is replaced at orientations of around 15° by a second mode indicating failure due to transverse tension; this is principally a result of the very high Poisson's ratio (>1.5) which exists between $\pm 15^{\circ}$ and $\pm 35^{\circ}$ for this particular material. Mode 3, between $\pm 35^{\circ}$ and $\pm 75^{\circ}$, indicates shear failure and is succeeded by a fourth mode, transverse compression. In practice it is possible to identify only three separate modes from examination of the failure surfaces, since modes 2 and 3 are difficult to differentiate.

The two interaction formulae can give only the distinction between a shear and a non-shear failure and give continuous curves rather than mode segments.

4 EXPERIMENTAL COMPARISON

4.1 Material fabrication

The laminated boards were made from pre-impregnated sheets, nominally 0.25mm cured thickness at 0.6 fibre volume fraction, supplied by Rotorway Components Ltd. The resin system used was ERLA4617/DDM. Both high modulus (HM-S) and high strength (HT-S) laminates were fabricated using a method similar to that used by Collings and Ewins[9] and were cured between the heated plattens of an hydraulic press to a thickness close to that required for a fibre volume fraction of 0.6. The high modulus (HM-S) laminates were of 2mm nominal thickness and were made at 15° intervals from 0° to 90° with the balanced stacking sequence $+\theta, -\theta, +\theta, -\theta, -\theta, +\theta, -\theta, +\theta$. The high strength (HT-S) laminates were made at 5° intervals from 0° to 50° and also at 90°, and were of 1mm thickness and laid in the sequence $+\theta, -\theta, -\theta, +\theta$.

4.2 Specimen design

4.2.1 Tensile specimens

In tensile testing, the main requirements are a uniform stress field in the specimen gauge length, elimination of bending strains and the minimisation of the

three-dimensional edge effects[10] which occur in angle-ply specimens. Specimens were therefore chosen to be long enough to ensure a uniform stress field according to St. Venant's principle; this also reduced the effect of any bending strains induced by the testing apparatus. Sufficient width is required in order that edge effects, extending inwards for approximately one specimen thickness, become an insignificant proportion of the stress field in the failure region. A minimum width of 20mm was considered adequate for the thickness of 1mm. To allow full freedom of action for modes of failure involving shear displacements, some of the specimens had freely pinned ends and were loaded by steel links. This also eliminated any possible bending in the width.

Failure was limited to the specimen centre by symmetrically grinding a waisted profile using a 200mm diameter grinding wheel. The basic strip width of 30mm was reduced to 20mm over a gauge length of 20mm. The tensile specimen geometry is illustrated in Fig.4.

4.2.2 Compressive specimens

In addition to the requirements for the tensile testing of coupon specimens given in section 4.2.1, compressive strength testing introduces the problem of instability which must be eliminated by the choice of a suitable specimen. For maximum resistance to overall buckling, encastré type end fittings were used, which effectively prevented any end rotation. The criterion used for buckling was the Euler equation factored by 0.9 (a 10% reduction) to allow adequately for transverse shear flexibility. The factored Euler equation is

$$P_c = \frac{3.6\pi^2 E_x I}{\ell^2} \qquad (17)$$

where $I = \frac{\omega t^3}{12}$,

ω is the coupon width,

t is the coupon thickness

and ℓ is the length between the end fittings.

The variation in modulus (E_x) with orientation made it necessary to reduce the length from 30mm at 0° to 5mm at $\pm 90^\circ$ orientation according to equation (17). the failure position was controlled by symmetrically grinding a circular arc profile which reduced the width by 5mm to 15mm. For each specimen length the arc radius was chosen so that the profile occupied a distance slightly greater than the built-in length. The compressive specimens are illustrated on Fig.4.

4.3 Testing and results

4.3.1 Instrumentation and test facilities

The complete load/strain history for the tensile specimens was plotted during the tests using an X-Y recorder. The strain-proportional input to the recorder was derived from the out-of-balance signal from a singly-active Wheatstone bridge circuit with a foil strain gauge as the active element. The load-proportional input was obtained from an integral potentiometer, the wiper arm of which was connected to the load indicating needle of the mechanical test machine. Bridge excitation was 6V with a very low ripple component of <0.5mV peak-to-peak to eliminate oscillation of the X-axis pen. Input to the test machine potentiometer was 1.5V. The mechanical tests were carried out on an Avery Universal Testing Machine of 100kN capacity.

4.3.2 Test procedure and results

The specimens were tested by steadily increasing the load up to failure over a period of about one minute. The load/strain curves recorded from the tensile tests were mostly smooth, although for some specimens small step-like irregularities in the curve were recorded due to the slower rate of response of the load sensor compared with that of the strain gauge. The modes of failure of all specimens were identified from visual examination of the failed surfaces. The test results are given in Tables 1 and 2, and representative stress/strain curves are given in Figs.5 and 6.

5 DISCUSSION

5.1 Comparison of experimental and theoretical strength results

Fig.3 gives the variation of compressive strength with orientation for balanced, high modulus (HM-S) angle-ply laminates at a nominal fibre volume fraction of 0.6. Agreement between experiment and theory is best for the maximum strain theory, with the interaction formulae underestimating the strength at orientations up to $\pm 40^{\circ}$. Exact agreement between experiment and all theories at 0° and 90° is obtained since the experimental results at these orientations are used as input data to predict the strength at all other orientations. At $\pm 45^{\circ}$, where strength is equal to twice the in-plane shear strength, agreement is also exact; the reasons for such agreement are dealt with more fully in section 5.3.

The theoretical prediction of the four failure criteria for the tensile strength of high strength (HT-S) laminates is given in Fig.7 and the comparison between the maximum strain theory and experiment is given in Fig.8. Close

agreement cannot be demonstrated, except in the region of shear failures at around $\pm 45^{\circ}$ and of course at the end points (0° and 90°). However, some reasons why good agreement with the failure criteria was obtained from the compressive strength tests but not from the tensile strength tests can be suggested, which result from the practically available tensile specimens being of inadequate design (see section 5.2). Low strength values were obtained from the tensile specimens at small values of θ because, at the high stress levels existing at failure, load was not transmitted adequately into the fibres. This was shown by the failure mode of these specimens which exhibited in-plane and interlaminar cracking without fibre failure. Load transfer from the test-machine wedge-grips into the specimen gauge length is by shear in the resin matrix and at the matrix/fibre interface. This mechanism, compounded with edge effect, is inadequate to transmit the high loads required to fail the high strength (HT-S) fibres of the tensile specimens. By contrast the compressive specimens, being of high modulus (HM-S) fibres in which the fibres fail at about half the stress of the high strength (HT-S) fibres, require only half as much internal stress to achieve the fibre breaking load. Further, the load path for the low-orientation compressive specimens is principally through direct pressure on the fibre ends with a much reduced shear transfer requirement. The types of failure observed for these specimens indicate that the load transfer mechanism was adequate to achieve fibre failure. Agreement with a failure criteria based on single ply strength values was therefore possible and it was found that good agreement was obtained with the strengths and modes given by the maximum strain criterion.

5.2 Tensile strength results, 'scissor action' and edge effect

The measured tensile strength results are shown on Fig.8, together with the theoretical prediction of the maximum strain theory, which was again found to give best agreement. No theory gave good agreement at small angles of orientation between $\pm 5^{\circ}$ and $\pm 15^{\circ}$, indeed at $\pm 10^{\circ}$ only about half of the predicted value was achieved. Between $\pm 20^{\circ}$ and $\pm 50^{\circ}$, in a region largely of shear mode failures, agreement between the predicted modes and strengths and the measured results is good while at 90° agreement is of course exact, this being a value used in the prediction of the strength curve.

Two reasons for the low strength results between $\pm 5^{\circ}$ and $\pm 15^{\circ}$ can however be suggested, which, although they apply to both tensile and compressive specimens, are likely to be more significant for high strength (HT-S) laminates because of the higher internal stresses. When fibres are symmetrically angled

about the loading axis, the material becomes trellis-like in its structure and, under tensile load, rotations about the almost infinite number of intersections of fibres tend to cause a 'scissor action' type of twisting between the plies, so reducing the orientation angle. This is manifest in the very high Poisson's ratios of up to 2.0 which are predicted by lamination theory and are measured in practice. High transverse stresses and strains are inevitably induced which contribute to failure at relatively low laminate strain. Another cause of early failure may be the interlaminar stresses, τ_{xz} , τ_{yz} and σ_z (see Fig.9), which occur in the edge regions. The interlaminar shear stress in the edge region is reported[10] to rise rapidly with orientation and to have reached at least 75% of its maximum value (which occurs at $\pm 35^{o}$) in the region between $\pm 15^{o}$ and $\pm 50^{o}$. Most of the failures at low orientation (i.e. $< \pm 20^{o}$) were of an indeterminate mode, but since considerable interlaminar separation and cracking were present it is probable that interlaminar stresses have adversely affected the measured strength. The edge effect would of course disappear in a specimen without free edges and the experimental tensile strength results between $\pm 5^{o}$ and $\pm 15^{o}$ serve only as a lower bound to the true tensile strength of the material. Nevertheless the results are a useful indicator of the large reductions in strength which can result from the influence of a free edge. It was noticed that the failure stress of specimens twice the width of those described was slightly higher, indicating that edge effects were indeed affecting the results. The upper width limit, beyond which there is no further strength increase, is difficult to find in practice because of both the scale of specimen required to furnish sufficient aspect ratio for uniform stress and the sensitivity of such specimens to widthwise bending. In any case, the upper strength limit will still be influenced by edge effects and probably the only way to find the true strength is to use a specimen without free edges, such as a tube.

5.3 Laminate stiffness, non-linearity and failure strain

The strain gauging of the tensile specimens enabled a check to be made of the theoretical modulus using lamination theory for the various ply orientations and also gave the full stress/strain curves up to failure. The comparison between the theoretical and experimental moduli at low strain is given in Fig.10 and the very good agreement is further evidence of the accuracy of the theory for small strains. Representative stress/strain curves given in Figs.5 and 6 show that from $\pm 30^{o}$ to $\pm 45^{o}$ non-linearity is marked and increasing. At high strain levels linear analyses are therefore inadequate to deal with laminates composed solely of plies at such orientations and deformations are always underestimated

for stresses above the linear region. The variation of the measured failure strain with orientation is given in Fig.11, together with a possible theoretical comparison based on linear elasticity. As can be clearly seen, the non-linear effects greatly increase strains at failure and limit the application of the theory to the initial linear response. It was noticed that, for the four instances in which the load was released before failure took place, the non-linear curves were almost completely elastically recoverable with very little permanent strain. This indicates that the material is non-linearly elastic for the low strain rates considered and not visco-elastic as might be implied from the relatively high degree of non-linearity. It is possible that the non-linear effects are due to the non-linearity in shear and this is considered in detail in the following section.

5.4 Correlation of angle-ply tensile and unidirectional in-plane shear response

Earlier work[11] on unidirectional in-plane shear properties had indicated considerable shear stress non-linearity and, because of the high shear stress acting on planes parallel to the fibre directions of $\pm 45^{\circ}$ angle-ply tensile specimens, it was thought that the tensile and shear non-linearities might be directly connected. A theoretical analysis was carried out which enabled the complete tensile stress/strain curve to be plotted using unvarying values of E_1, E_2 and ν_{12} together with ordinates and abscissae from the experimentally derived non-linear shear stress/strain curve. A computer program, given in Appendix C, was written to calculate the results. Results identical to lamination theory for the linear portion were obtained, providing a check on the angle-ply analysis. A summary of the more important results derived from the analysis follows, although the detailed derivations are given in the Appendix.

For balanced angle-ply laminates the natural co-ordinate stresses can be expressed in terms of natural co-ordinate shear stress and strain by equations of the form

$$\left.\begin{aligned} \sigma_1 &= A_1\tau_{12} + A_2\gamma_{12} \\ \text{and} \qquad \sigma_2 &= B_1\tau_{12} + B_2\gamma_{12} \end{aligned}\right\} \qquad (18)$$

where the coefficients A_1, A_2 etc. are functions both of the orthotropic parameters E_1, E_2 and ν_{12} and of fourth order multiples of $\sin\theta$ and $\cos\theta$ (see the Appendix). On transforming equations (18) the laminate applied load becomes

$$\sigma_x = A_7\tau_{12} + A_8\gamma_{12} \cdot \tag{19}$$

The natural co-ordinate strains can be expressed as

$$\left.\begin{aligned} \varepsilon_1 &= A_3\tau_{12} + A_4\gamma_{12} \\ \text{and} \\ \varepsilon_2 &= B_3\tau_{12} + B_4\gamma_{12} , \end{aligned}\right\} \tag{20}$$

and the laminate strain as

$$\varepsilon_x = A_5\tau_{12} + A_6\gamma_{12} \cdot \tag{21}$$

Thus equations (19) and (21) can be used to predict the tensile stress/strain curve using the coefficients A_1, A_2, B_1, B_3 etc., assumed constant and a measured shear stress/strain curve for unidirectional material.

At $\theta = 45^o$, the stresses cease to be functions of shear strain so that

$$\sigma_1 = \frac{2(\nu_{12}E_2 + E_1)\tau_{12}}{E_1 + E_2 + 2\nu_{12}E_2} \tag{22}$$

and

$$\sigma_2 = \frac{2E_2(1 + \nu_{12})\tau_{12}}{E_1 + E_2 + 2\nu_{12}E_2} , \tag{23}$$

which together yield

$$\sigma_x = 2\tau_{12} \tag{24}$$

and

$$\varepsilon_x = \frac{2(E_1 - \nu_{12}^2E_2)}{E_1(E_1 + E_2 + 2\nu_{12}E_2)} \tau_{12} + \frac{\gamma_{12}}{2} \cdot \tag{25}$$

It is of considerable interest that the above equations can also be used to determine the initial value of G_{12} from the tensile test results. In the linear portion of the shear stress/strain curve, τ_{12} and γ_{12} are related by

$$G_{12} = \tau_{12}/\gamma_{12} \cdot \tag{26}$$

Strain transformation at $\theta = 45^o$ results in

$$\gamma_{12} = -\varepsilon_x + \varepsilon_y \tag{27}$$

and since

$$\varepsilon_y = -\nu_{xy}\varepsilon_x \tag{28}$$

then

$$\gamma_{12} = \varepsilon_x(1 + \nu_{xy}) \ . \tag{29}$$

Substituting equations (24) and (29) in equation (26) yields,

$$G_{12} = \frac{\sigma_x}{\varepsilon_x 2(1 + \nu_{xy})} \tag{30}$$

or

$$G_{12} = \frac{E_x}{2(1 + \nu_{xy})} \ . \tag{31}$$

It should be noted that this remarkable result, also obtained by a different route[12], although applicable throughout the whole elastic region, should only be used for the linear portion in the determination of the unvarying value of G_{12} (equation (26)). Equation (31) enables the in-plane shear modulus, G_{12}, to be simply determined from a tensile test on a $\pm 45^\circ$ balanced specimen with two strain gauges attached for the determination of E_x and ν_{xy}. Although rough checks of equation (31) using existing data have yielded reasonable agreement, rigorous experimental verification remains to be done.

On the assumption that the three orthotropic parameters E_1, E_2 and ν_{12} do not vary with load (an assumption verified in practice), it is also possible to predict the tensile stress/strain curve for angle-ply laminates using the measured shear stress/strain curve for unidirectional material. This was done at $\theta = 35^\circ$ using equations (19) and (21) and at $\theta = 45^\circ$ using equations (24) and (25) and the results are given in Fig.12.

Throughout this analysis it has been assumed that the normal stress, σ_2, and the parallel shear stress, τ_{12}, are independent. Although this is thought to be true as far as stiffness is concerned, evidence[13,14] exists which indicates that normal stress influences shear strength; compression tends to increase and tension to decrease the shear strength. The statement that in-plane shear strength, F_{12}, equals half the tensile strength, F_x, at $\pm 45^\circ$ must therefore be qualified by the fact that normal tensile stress exists on the failure plane given by equation (23) as

$$\frac{\sigma_2}{\tau_{12}} \simeq 0.1 \ .$$

However this low percentage of normal tensile stress appears to have little effect on the shear strength[13] and it seems that the result,

$$F_{12} = F_x/2 \ , \qquad (32)$$

can be used to determine the in-plane shear strength of unidirectional material.

5.5 Other associated results

The accuracy of equation (31) in predicting the initial value of G_{12} would be impaired if G_{12} were to be highly sensitive to changes in the independent variables. The degree of sensitivity was investigated by plotting equation (31) with E_x constant, for a typical high strength (HT-S) angle-ply laminate at $\pm 45^{\circ}$. Fig.13 gives the result which shows that in the region of practical values of G_{12}, when ν_{xy} varies between 0.7 and 0.9, G_{12} varies between 5.3 and 4.7GN/m^2. Accurate measurements of ν_{xy} must therefore be made if accurate values of G_{12} are to be derived.

Further consideration of the complete curve shows that ν_{xy} decreases with G_{12} until, at $G_{12} = 8.9\text{GN/m}^2$, $\nu_{xy} = 0$; for greater values of G_{12}, ν_{xy} becomes negative. It is unlikely that these effects would ever be realised in practice, however, since the combination of high shear stiffness, high fibre stiffness and low transverse direct stiffness which is required would probably never be encountered.

6 CONCLUDING REMARKS

The variation of compressive and tensile strength with fibre orientation has been obtained both experimentally and theoretically for symmetric CFRP angle-ply laminates. It has been shown that, particularly in the case of tensile stress on high strength (HT-S) laminates, there is a large and theoretically unpredicted reduction in strength whenever the fibres are at small angles from the loading axis. The reason for using angle ply construction is to increase the shear and transverse stiffness and strength, but in so doing there is a large sacrifice in longitudinal strength. A better type of construction might be to use a proportion of fibres in the two principal loading directions to carry the direct loads, and a proportion of symmetric layers at $\pm 45^{\circ}$ to carry shear loads.

It has also been shown that non-linearity and failure strain increase with orientation up to $\pm 40^{\circ}$ where they are a maximum and thereafter decrease, the response becoming linear at angles greater than about $\pm 50^{\circ}$.

Shear and tensile effects have been related, and a simple method of determining the shear modulus, G_{12}, has been given. The correlation between the direct stress on symmetric angle-ply laminates with ply orientations of $\pm 45^{o}$ and the unidirectional shear stress should prove useful in shear property characterisation.

Acknowledgments

Thanks are due to my colleagues, Dr. P. Bartholomew for assistance with the angle-ply equations and Mr. D. Purslow for the use of his shear data.

Appendix

DERIVATION OF ANGLE-PLY ELASTICITY EQUATIONS

A.1 Equilibrium

Referring to Fig.9, the following equilibrium equations apply

$$\sigma_x^a = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta + 2\tau_{12} \sin\theta \cos\theta \qquad \text{(A-1)}$$

$$\sigma_y^a = \sigma_1 \sin^2\theta + \sigma_2 \cos^2\theta - 2\tau_{12} \sin\theta \cos\theta \qquad \text{(A-2)}$$

$$\tau_{xy}^a = -\sigma_1 \sin\theta \cos\theta + \sigma_2 \sin\theta \cos\theta + \tau_{12}(\cos^2\theta - \sin^2\theta) \qquad \text{(A-3)}$$

where superscript a applies to the $+\theta$ ply direction.
For the $-\theta$ ply direction (superscript b),

$$\sigma_x^b = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta + 2\tau_{12} \sin\theta \cos\theta \,, \qquad \text{(A-4)}$$

$$\sigma_y^b = \sigma_1 \sin^2\theta + \sigma_2 \cos^2\theta - 2\tau_{12} \sin\theta \cos\theta \qquad \text{(A-5)}$$

and

$$\tau_{xy}^b = \sigma_1 \sin\theta \cos\theta - \sigma_2 \sin\theta \cos\theta - \tau_{12}(\cos^2\theta - \sin^2\theta) \,. \qquad \text{(A-6)}$$

Adding equations (A-1) and (A-4) gives the relationship between the applied load and the ply internal stresses,

$$\sigma_x = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta + 2\tau_{12} \sin\theta \cos\theta \,, \qquad \text{(A-7)}$$

$$\sigma_y = \sigma_1 \sin^2\theta + \sigma_2 \cos^2\theta - 2\tau_{12} \sin\theta \cos\theta = 0 \qquad \text{(A-8)}$$

and

$$\sigma_{xy} = 0 \,. \qquad \text{(A-9)}$$

A.2 Compatibility

Because there is an equal number of plies at $-\theta^o$ as at $+\theta^o$, shear strain γ_{xy} is eliminated and so

$$\gamma_{xy} = 0 \,. \qquad \text{(A-10)}$$

This leads to

$$2 \sin\theta \cos\theta (-\varepsilon_1 + \varepsilon_2) + (\cos^2\theta - \sin^2\theta)\gamma_{12} = 0 \,. \qquad \text{(A-11)}$$

A.3 Elasticity

The orthotropic elasticity equations, relating the natural co-ordinate stresses and strains, can be expressed as

$$\varepsilon_1 = \frac{\sigma_1}{E_1} - \frac{\nu_{21}\sigma_2}{E_2} \qquad \text{(A-12)}$$

and

$$\varepsilon_2 = \frac{\sigma_2}{E_2} - \frac{\nu_{12}\sigma_1}{E_1} \quad . \qquad \text{(A-13)}$$

A.4 Derivation

Substitution of equations (A-12) and (A-13) in equation (A-11) results in

$$-2/E_1(1 + \nu_{12})\sigma_1 SC + 2(\nu_{12}/E_1 + 1/E_2)\sigma_2 SC = -\gamma_{12}(C^2 - S^2) \qquad \text{(A-14)}$$

where S and C are abbreviations for $\sin\theta$ and $\cos\theta$. This equation, together with equation (A-8), enables σ_1 and σ_2 to be expressed in terms of τ_{12} and γ_{12}, the results for which are

$$\sigma_1 = \frac{2SC\tau_{12}}{\frac{(1 + \nu_{12})E_2}{\nu_{12}E_2 + E_1}C^2 + S^2} + \frac{C^2(C^2 - S^2)\gamma_{12}}{\frac{2(1 + \nu_{12})}{E_1}SC^3 + 2\left(\frac{\nu_{12}E_2 + E_1}{E_1E_2}\right)S^3C} \quad . \qquad \text{(A-15)}$$

By setting

$$\frac{E_2(1 + \nu_{12})}{\nu_{12}E_2 + E_1} = A \ ,$$

$$\frac{\nu_{12}E_2 + E_1}{E_1E_2} = B$$

and

$$\frac{1 + \nu_{12}}{E_1} = D \ ,$$

equation (A-15) becomes

$$\sigma_1 = \frac{2SC\tau_{12}}{AC^2 + S^2} + \frac{C^2(C^2 - S^2)\gamma_{12}}{2DSC^3 + 2BS^3C} , \tag{A-16}$$

and further abbreviation yields

$$\sigma_1 = A_1\tau_{12} + A_2\gamma_{12} \tag{A-17}$$

where $A_1 = \frac{2SC}{AC^2 + S^2}$

and $A_2 = \frac{C^2(C^2 - S^2)}{2DSC^3 + 2BS^3C}$.

Similarly,

$$\sigma_2 = \frac{2SC\tau_{12}}{C^2 + S^2/A} - \frac{S^2(C^2 - S^2)\gamma_{12}}{2DSC^3 + 2BS^3C} \tag{A-18}$$

which is abbreviated to

$$\sigma_2 = B_1\tau_{12} + B_2\gamma_{12} \tag{A-19}$$

where $B_1 = \frac{2SC}{C^2 + S^2/A}$

and $B_2 = \frac{S^2(C^2 - S^2)}{2DSC^3 + 2BS^3C}$.

Equations (A-17) and (A-19) are used to express the natural co-ordinate strain components ε_1 and ε_2 in terms of τ_{12} and γ_{12}. Their substitution in equation (A-12) results in

$$\varepsilon_1 = A_3\tau_{12} + A_4\gamma_{12} \tag{A-20}$$

where $A_3 = A_1/E_1 - \nu_{12}B_1/E_1$

and $A_4 = (A_2 - \nu_{12}B_2)/E_1$,

and

$$\varepsilon_2 = B_3\tau_{12} + B_4\gamma_{12} \tag{A-21}$$

where $B_3 = B_1/E_2 - \nu_{12}A_1/E_1$

and $B_4 = B_2/E_2 - \nu_{12}A_2/E_1$.

The two natural co-ordinate direct strains are used to express the laminate strain in terms of τ_{12} and γ_{12} . Strain transformation yields

$$\varepsilon_x = \varepsilon_1 \cos^2\theta + \varepsilon_2 \sin^2\theta + \gamma_{12} \sin\theta \cos\theta \tag{A-22}$$

and

$$\varepsilon_y = \varepsilon_1 \sin^2\theta + \varepsilon_2 \cos^2\theta - \gamma_{12} \sin\theta \cos\theta \quad . \tag{A-23}$$

Hence equations (A-20) and (A-21) are substituted in equation (A-22) to give

$$\varepsilon_x = (A_3CC + B_3SS)\tau_{12} + (A_4CC + B_4SS + SC)\gamma_{12} \tag{A-24}$$

which is abbreviated to

$$\varepsilon_x = A_5\tau_{12} + A_6\gamma_{12} \tag{A-25}$$

where $A_5 = A_3CC + B_3SS$

and $A_6 = A_4CC + B_4SS + SC$.

Similarly, equation (A-23) yields

$$\varepsilon_y = (A_3SS + B_3CC)\tau_{12} + (A_4SS + B_4CC - SC)\gamma_{12} \quad . \tag{A-26}$$

Hence

$$\varepsilon_y = B_5\tau_{12} + B_6\gamma_{12} \tag{A-27}$$

where $B_5 = A_3SS + B_3CC$

and $B_6 = A_4SS + B_4CC - SC$.

Equations (A-25) and (A-27) enable Poisson's ratio for the laminate to be determined. For the straight line portion of the shear stress/strain curve where G_{12} is constant,

$$G_{12} = \tau_{12}/\gamma_{12} . \tag{A-28}$$

Equation (A-28), when substituted in equations (A-25) and (A-27), gives equations for ε_x and ε_y which yield the expression for Poisson's ratio,

$$\nu_{xy} = \varepsilon_y/\varepsilon_x = (B_5G_{12} + B_6)/(A_5G_{12} + A_6) . \tag{A-29}$$

The laminate stress is found from equation (A-4) by substitution of equations (A-17) and (A-19). This results in

$$\sigma_x = A_7\tau_{12} + A_8\tau_{12} \tag{A-30}$$

where $A_7 = A_1CC + B_1SS + 2SC$

and $A_8 = A_2CC + B_2SS$.

Table 1

ANGLE-PLY COMPRESSION STRENGTH (HM-S/ERLA 4617/DDM)

Orientation + - + - - + - + (degrees)	Fibre volume fraction (v_f)	Number of specimens	Average strength (MN/m^2)	Mode of failure
0	All specimens fell between 0.58 and 0.61	3	605	Longitudinal compression
15		3	524	Longitudinal compression
30		3	265	Transverse tension
45		3	140	Shear
60		3	175	Shear
75		3	185	Transverse compression
90		3	152	Transverse compression

Table 2

ANGLE PLY TENSION STRENGTH (HM-S/ERLA 4617/DDM)

Orientation + - - + (degrees)	Thickness (mm)	Number of specimens	Average strength (MN/m^2)	Coeff. of variation %	Response/failure mode
0	1.1	5	1017	8.99	Linear/tensile, shear cracks
5	1.08	7	745	6.64	Linear/tensile, shear cracks
10	1.08	7	536.4	9.70	Linear/brittle, indeterminate
15	1.1	7	509	8.31	Linear/tensile
20	1.1	7	426.8	9.26	Linear/tensile
25	1.1	7	405	2.79	Non-linear/tensile
30	1.12	7	339.7	7.69	Non-linear/tensile
35	1.1	7	310	3.04	Non-linear/tensile
40	1.2	5	140	10.0	Non-linear/shear
45	1.1	7	106.4	4.98	Non-linear/shear
50	1.14	5	70.2	4.77	Non-linear/shear
90	1.1	3	28.6	8.73	Linear/tensile

SYMBOLS

E	Young's modulus
F	breaking stress
G_{12}	shear modulus of unidirectional material
ℓ	specimen length between and fittings
p	failure stress multiplier
q	failure strain ratio
t	strength specimen thickness
v_f	fibre volume fraction
w	strength specimen width
α,β	natural co-ordinate stress ratios
γ_{12}	in-plane, fibre parallel engineering shear strain
ε	engineering strain
θ	angular orientation from the reference direction
ν	Poisson's ratio
σ	stress
τ_{12}	in-plane fibre-parallel shear stress
$[C]$	3 × 3 compliance matrix
$[N]$	3 × 1 column of loads per unit length
$[T]$	3 × 3 co-ordinate transformation matrix
$[Q]$	3 × 3 matrix of orthotropic stiffness components
$[\varepsilon_0]$	3 × 1 column matrix of mid-plane strain components
$[\gamma]$	3 × 1 column of tensional strain components

Subscripts

1	fibre-parallel direction
2	direction normal to the fibres, in the laminate plane
3	refers to fibre-parallel shear
C	compression
c	indicates that terms are in laminate reference co-ordinates
f	failure
n	indicates that terms are in natural co-ordinate (1,2,3) directions
0	refers to conditions at laminate mid-plane
T	tension
x,y,z	laminate reference co-ordinate directions

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Program listing

Strength

```
      MASTER STRENGTH
      DIMENSION E11(100),E22(100),ANU12(100),ANU21(100),G12(100),TH(100)
     1,THETAD(100),THETAR(100),Q11(100),Q12(100),Q22(100),Q66(100),
     2H(100),Q11B(100),Q12B(100),Q16B(100),Q22B(100),Q26B(100),A(3,3),
     3B(3,3),C(6,6),RO(100),D(3,3),AMA(100),AA(3,3),CC(6,6),R1(100),
     4R2(100),R3(100),R4(100),R5(100),R6(100),RM(3,3),AB(3,3),BA(3,3),
     5BB(3,3),DB(3,3),Q66B(100)
      DIMENSION S(3),ES(3),U(100),T(3,3),EN(3),Q(3,3),SN(3)
      DIMENSION X(1000),Y(1000),LX(1000),LY(1000),TP(120),TX(1000),
     1TY(1000)
      COMMON/B1/FT1,FC1,FT2,FC2,F12/B2/AE,HE,BE,GE,FE,CE,HAE,HHE,HBE/
     1B3/PI/B4/SN,S/B5/X,Y,NXY/B6/D5/B7/EN/B8/EE1,EE2,EE3
      READ(1,90) FT1,FC1,FT2,FC2,F12
      WRITE(2,102) FT1,FC1,FT2,FC2,F12
      READ(1,90) S(1),S(2),S(3)
      WRITE(2,102) S(1),S(2),S(3)
      PI = 8.0*ATAN(1.0)
      DP = PI/72
      NXY = PI/DP
      D1 = 1/(FT1*FC1)
      D2 = 1/(FT2*FC2)
      D3 = (FC1-FT1)/(FC1*FT1)
      D4 = (FC2-FT2)/(FC2*FT2)
      D5 = 1/(F12*F12)
      AE = D1
      HE = D1/2
      BE = D2
      GE = D3/2
      FE = D4/2
      HHE = 1/(2*FT1*FT1)
      RC = 8.0*ATAN(1.0)/360.0
      T1 = 0.1E+8
      T2 = 0.1E+10
      AM = 0.0
      DM = 0.0
      READ(1,100) NC
      DO 99 NI = 1,NC
      READ(1,100) N
      DO 4 J = 1,3
      DO 4 K = 1,3
      A(J,K) = 0.0
      B(J,K) = 0.0
      D(J,K) = 0.0
4     CONTINUE
      TMA = 0.0
      TTH = 0.0
      DO 2 J = 1,N
      READ(1,104) E11(J),E22(J),ANU12(J),G12(J),RO(J),TH(J),THETAD(J)
      AMA(J) = RO(J)*TH(J)
      TMA = TMA+AMA(J)
      TTH = TTH + TH(J)
      ANU21(J)=ANU12(J)*E22(J)/E11(J)
      DENOM = 1.0 - ANU12(J)*ANU21(J)
      Q11(J)= E11(J)/DENOM
      Q12(J) = ANU21(J) * E11(J)/DENOM
      Q22(J) = E22(J)/DENOM
      Q66(J) = G12(J)
2     CONTINUE
      H(1) = -TTH/2.0
      DO 1 J = 1,N
      IF(THETAD(J)) 10,10,14
10    Q11B(J) = Q11(J)
      Q12B(J) = Q12(J)
      Q16B(J) = 0.0
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```
      Q22B(J) = Q22(J)
      Q26B(J) = 0.0
      Q66B(J) = Q66(J)
      THETAR(J) = THETAD(J)
      GO TO 16
   13 THETAR(J) = (360.0+THETAD(J))*RC
      GO TO 15
   14 THETAR(J) = THETAD(J)*RC
   15 U1    = 0.125*(3.*(Q11(J)+Q22(J))+2.*Q12(J)+4.*Q66(J))
      U2    = 0.5*(Q11(J)-Q22(J))
      U3    = 0.125*(Q11(J)+Q22(J)-2.0*Q12(J)-4.0*Q66(J))
      U4    = 0.125*(Q11(J)+Q22(J)+6.0*Q12(J)-4.0*Q66(J))
      U5    = 0.125*(Q11(J)+Q22(J)-2.0*Q12(J)+4.0*Q66(J))
      A1 = U2*COS(2.0*THETAR(J))
      A2 = U3*COS(4.0*THETAR(J))
      A3 = 0.5*U2*SIN(2.0*THETAR(J))
      A4 = U3*SIN(4.*THETAR(J))
      Q11B(J) = U1+A1+A2
      Q12B(J) = U4-A2
      Q16B(J) = -A3-A4
      Q22B(J) = U1-A1+A2
      Q26B(J) = -A3+A4
      Q66B(J) = U5-A2
   16 H(J+1) = H(J)+TH(J)
      CTH = H(J+1)**3 -H(J)**3
      A(1,1) = A(1,1)+Q11B(J)*TH(J)
      A(1,2) = A(1,2)+Q12B(J)*TH(J)
      A(1,3) = A(1,3)+Q16B(J)*TH(J)
      A(2,2) = A(2,2)+Q22B(J)*TH(J)
      A(2,3) = A(2,3)+Q26B(J)*TH(J)
      A(3,3) = A(3,3)+Q66B(J)*TH(J)
      IF(N.EQ.1) GO TO 21
      STH = H(J+1)*H(J+1) - H(J)*H(J)
      R1(J) = Q11B(J)*STH
      R2(J) = Q12B(J)*STH
      R3(J) = Q16B(J)*STH
      R4(J) = Q22B(J)*STH
      R5(J) = Q26B(J)*STH
      R6(J) = Q66B(J)*STH
      B(1,1) = B(1,1)+R1(J)
      B(1,2) = B(1,2)+R2(J)
      B(1,3) = B(1,3)+R3(J)
      B(2,2) = B(2,2)+R4(J)
      B(2,3) = B(2,3)+R5(J)
      B(3,3) = B(3,3)+R6(J)
   21 CONTINUE
      D(1,1) = D(1,1)+Q11B(J)*CTH
      D(1,2) = D(1,2)+Q12B(J)*CTH
      D(1,3) = D(1,3)+Q16B(J)*CTH
      D(2,2) = D(2,2)+Q22B(J)*CTH
      D(2,3) = D(2,3)+Q26B(J)*CTH
      D(3,3) = D(3,3)+Q66B(J)*CTH
    1 CONTINUE
      A(2,1) = A(1,2)
      A(3,1) = A(1,3)
      A(3,2) = A(2,3)
      B(2,1) = B(1,2)
      B(3,1) = B(1,3)
      B(3,2) = B(2,3)
      D(2,1) = D(1,2)
      D(3,1) = D(1,3)
      D(3,2) = D(2,3)
      IF(N.EQ.1) GO TO 22
      DO 70 J = 1,N
      IF(ABS(R1(J)).GT.RM(1,1)) RM(1,1) = ABS(R1(J))
      IF(ABS(R2(J)).GT.RM(1,2)) RM(1,2) = ABS(R2(J))
```

```
      IF(ABS(R3(J)).GT.RM(1,3)) RM(1,3) = ABS(R3(J))
      IF(ABS(R4(J)).GT.RM(2,2)) RM(2,2) = ABS(R4(J))
      IF(ABS(R5(J)).GT.RM(2,3)) RM(2,3) = ABS(R5(J))
      IF(ABS(R6(J)).GT.RM(3,3)) RM(3,3) = ABS(R6(J))
70    CONTINUE
      RM(2,1) = RM(1,2)
      RM(3,1) = RM(1,3)
      RM(3,2) = RM(2,3)
22    DO 72 I = 1,3
      DO 72 J =1,3
      IF(ABS(A(I,J)).GT.AM) AM = ABS(A(I,J))
      IF(ABS(D(I,J)).GT.DM) DM = ABS(D(I,J))
72    CONTINUE
      DO 71 I = 1,3
      DO 71 J = 1,3
      IF(A(I,J)) 80,81,79
79    IF(AM/A(I,J).GT.T2) A(I,J) = 0.0
      GO TO 81
80    IF(AM/ABS(A(I,J)).GT.T2) A(I,J) = 0.0
81    IF(N.EQ.1) GO TO 83
      IF(B(I,J)) 82,83,85
85    IF(RM(I,J)/B(I,J).GT.T1) B(I,J) = 0.0
      GO TO 83
82    IF(RM(I,J)/ABS(B(I,J)).GT.T1) B(I,J) = 0.0
83    IF(D(I,J)) 84,71,86
86    IF(DM/D(I,J).GT.T2) D(I,J) = 0.0
      GO TO 71
84    IF(DM/ABS(D(I,J)).GT.T2) D(I,J) = 0.0
71    CONTINUE
      DO 5 J = 1,3
      DO 5 K = 1,3
      D(J,K) = D(J,K)/3.0
      B(J,K) = B(J,K)*0.5
      C(J,K) = A(J,K)
      C(J,K+3) = B(J,K)
      C(J+3,K) = B(J,K)
      C(J+3,K+3) = D(J,K)
5     CONTINUE
      WRITE(2,112)
      DO 3 J = 1,N
      WRITE(2,116) J,E11(J),E22(J),ANU12(J),ANU21(J),G12(J),RO(J),TH(J),
     1THETAD(J)
3     CONTINUE
      WRITE(2,121) TTH
      WRITE(2,120) TMA
      DO 20 I = 1,3
      DO 20 J = 1,3
      IF(B(I,J)) 51,20,51
20    CONTINUE
      IF(A(1,3).EQ.0.0.AND.A(2,3).EQ.0.0) GO TO 50
      GO TO 51
50    A1 = A(1,2)*A(1,2)
      EXX = (A(1,1)-A1/A(2,2))/TTH
      EYY = (A(2,2)-A1/A(1,1))/TTH
      GXY = A(3,3)/TTH
      ANUXY = A(1,2)/A(2,2)
      ANUYX = A(1,2)/A(1,1)
      WRITE(2,110) EXX
      WRITE(2,105) EYY
      WRITE(2,115) GXY
      WRITE(2,111) ANUXY
      WRITE(2,113) ANUYX
51    CONTINUE
      CALL FMOVE(A,AA,9)
      CALL POSDEF MATINV(AA,9)
      CALL MULT(AA,3,,,S,1,ES)
```

```
      CALL WRITE (ES,3,1,5HSTRXY)
      ES(3) = ES(3)/2
      DO 99 J = 1,1
      EE1 = E11(1)
      EE2 = E22(1)
      EE3 = G12(1)
      U(J) = THETAR(J)
      SQ = SIN(U(J))
      CQ = COS(U(J))
      CCQ = CQ*CQ
      SSQ = SQ*SQ
      SCQ = SQ*CQ
      T(1,1) = CCQ
      T(1,2) = SSQ
      T(1,3) = 2*SCQ
      T(2,1) = T(1,2)
      T(2,2) = T(1,1)
      T(2,3) = -2*SCQ
      T(3,1) = -SCQ
      T(3,2) = SCQ
      T(3,3) = CCQ-SSQ
      Q(1,1) = Q11(J)
      Q(1,2) = Q12(J)
      Q(1,3) = 0
      Q(2,1) = Q(1,2)
      Q(2,2) = Q22(J)
      Q(2,3) = 0
      Q(3,1) = 0
      Q(3,2) = 0
      Q(3,3) = Q66(J)
      CALL WRITE(T,3,3,5HT     )
      CALL MULT(T,3,3,ES,1,EN)
      EN(3) = EN(3)*2
      CALL WRITE(EN,3,1,5HSTR12)
      CALL MULT(Q,3,3,EN,1,SN)
      CALL WRITE(SN,3,1,5HSIG12)
      CE = D5*SN(3)*SN(3)-1.0
      CZ = CE
      WRITE(2,102) D1,D2,D3,D4,D5,AE,HE,BE,GE,FE,CE,DS
      CALL FAILURE STRESS
      CALL HILL STRESS
      CALL MAX STRAIN
      CALL MAX STRESS
99    CONTINUE
      STOP
90    FORMAT(6(E0.0),F0.0)
102   FORMAT(1X,11(E9.2,1X),E9.2)
100   FORMAT(10)
101   FORMAT(1H0,6(F14.10,2X))
104   FORMAT(2E0.0,F0.0,3E0.0,F0.0)
112   FORMAT(1H1,6HLAMINA,5X,3HE11,11X,3HE22,11X,4HNU12,10X,4HNU21,10X,3
     1HG12,9X,7HDENSITY,8X,9HTHICKNESS,7X,5HANGLE)
116   FORMAT(3X,I3,1X,8(F12.5,2X))
120   FORMAT(1H0,61HTOTAL MASS/UNIT SURFACE AREA (I.E.DENSITY*TOTAL THIC
     1KNESS) = ,E17.10)
121   FORMAT(1H0,27HLAMINATE TOTAL THICKNESS = ,F12.10)
110   FORMAT(1H0,'LAMINATE LONGITUDINAL STIFFNESS,  EXX = ',E17.10)
105   FORMAT(1H0,'LAMINATE TRANSVERSE STIFFNESS,  EYY = ',E17.10)
115   FORMAT(1H0,'LAMINATE SHEAR STIFFNESS,  GXY = ',E17.10)
111   FORMAT(1H0,'LAMINATE PRINCIPAL POISSONS RATIO,  NUXY = ',F13.10)
113   FORMAT(1H0,'LAMINATE TRANSVERSE POISSONS RATIO,  NUYX = ',F13.10)
      END


      SUBROUTINE MAX STRAIN
      DIMENSION EN(3),S(3),SN(3)
```

```
      COMMON/B7/EN/B8/EE1,EE2,EE3/B4/SN,S/B1/FT1,FC1,FT2,FC2,F12
      E1 = ABS(EN(1))
      E2 = ABS(EN(2))
      E3 = ABS(EN(3))
      ET1 = FT1/EE1
      EC1 = FC1/EE1
      ET2 = FT2/EE2
      EC2 = FC2/EE2
      E12 = F12/EE3
      EM = -0.1E+50
      T1 = 0.1E+10
      T2 = 0.1E-20
      RL = 0.1E+50
      IF(E1.GT.EM) EM = EN(1)
      IF(E2.GT.EM) EM = EN(2)
      IF(E3.GT.EM) EM = EN(3)
      IF(E1-T2) 12,12,13
13    IF(EM/E1.GT.T1) EN(1) = 0.0
12    IF(E2-T2) 14,14,15
15    IF(EM/E2.GT.T1) EN(2) = 0.0
14    IF(E3-T2) 16,16,17
17    IF(EM/E3.GT.T1) EN(3) = 0.0
16    CONTINUE
      CALL WRITE(EN,3,1,5HSTR12)
      IF(EN(1)) 1,2,3
1     R1 = EC1/E1
      GO TO 7
2     R1 = RL
      GO TO 7
3     R1 = ET1/E1
7     CONTINUE
      IF(EN(2)) 4,5,6
4     R2 = EC2/E2
      GO TO 8
5     R2 = RL
      GO TO 8
6     R2 = ET2/E2
8     CONTINUE
      IF(EN(3)) 9,10,9
9     R3 = E12/E3
      GO TO 11
10    R3 = RL
11    CONTINUE
      IF(R1.LT.RL) RL = R1
      IF(R2.LT.RL) RL = R2
      IF(R3.LT.RL) RL = R3
      WRITE(2,102) A1,A2,AS1,AS2,AS3,R1,R2,R3,RL,SM
102   FORMAT(1X,12(E9.2,1X))
      IF(RL.EQ.R1) GO TO 20
      IF(RL.EQ.R2) GO TO 21
      GO TO 22
 20   CONTINUE
      SX = R1*S(1)
 24   CONTINUE
      WRITE(2,390) SX
 390  FORMAT(1X,'FAILURE AT SX = ',E17.10)
      GO TO 100
 21   SX = R2*S(1)
      GO TO 24
 22   CONTINUE
      SX = R3*S(1)
      WRITE(2,391) SX
 391  FORMAT(1X,'SHEAR FAILURE AT SX = ',E17.10)
 100  CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MAX STRESS
      DIMENSION SN(3),S(3)
      COMMON/B4/SN,S/B1/FT1,FC1,FT2,FC2,F12
      RL = 0.1E+50
      T1 = 0.1E-12
      T2 = 0.1E+10
      SM = -0.1E+50
      AS1 = ABS(SN(1))
      AS2 = ABS(SN(2))
      AS3 = ABS(SN(3))
      IF(AS1.GE.SM) SM = AS1
      IF(AS2.GE.SM) SM = AS2
      IF(AS3.GE.SM) SM = AS3
      IF(SM-AS1.GE.T2) SN(1) = 0.0
      IF(SM-AS2.GE.T2) SN(2) = 0.0
      IF(SM-AS3.GE.T2) SN(3) = 0.0
      CALL WRITE(SN,3,1,5HSIG12)
      IF(SN(1)) 1,2,3
1     A1 = FC1
      R1 = A1/AS1
      GO TO 7
2     R1 = RL
      GO TO 7
3     A1 = FT1
      R1 = A1/AS1
7     CONTINUE
      IF(SN(2)) 4,5,6
4     A2 = FC2
      R2 = A2/AS2
      GO TO 8
5     R2 = RL
      GO TO 8
6     A2 = FT2
      R2 = A2/AS2
8     CONTINUE
      IF(SN(3)) 9,10,9
9     R3 = F12/AS3
      GO TO 11
10    R3 = RL
11    CONTINUE
      WRITE(2,102) A1,A2,AS1,AS2,AS3,R1,R2,R3,RL,SM
102   FORMAT(1X,12(E9.2,1X))
      IF(R1.LT.RL) RL = R1
      IF(R2.LT.RL) RL = R2
      IF(R3.LT.RL) RL = R3
      IF(RL.EQ.R1) GO TO 20
      IF(RL.EQ.R2) GO TO 21
      GO TO 22
 20   CONTINUE
      SX = R1*S(1)
 24   CONTINUE
      WRITE(2,390) SX
 390  FORMAT(1X,'FAILURE AT SX = ',E17.10)
      GO TO 100
 21   SX = R2*S(1)
      GO TO 24
 22   CONTINUE
      SX = R3*S(1)
      WRITE(2,391) SX
 391  FORMAT(1X,'SHEAR FAILURE AT SX = ',E17.10)
 100  CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FAILURE STRESS
      DIMENSION SN(3),S(3)
      COMMON/B4/SN,S/B2/AE,HE,BE,GE,FE,CE,HAE,HHE,HBE/B6/D5
     1/B1/FT1,FC1,FT2,FC2,F12
      CALL WRITE(S,3,1,5HSIGXY)
      CALL WRITE(SN,3,1,5HSIG12)
      T1 = 0.1E-20
      IF(ABS(SN(1)).LE.T1.AND.ABS(SN(2)).LE.T1) GO TO 10
3     SL = SN(2)/SN(1)
      SL1 = SN(3)/SN(1)
      QB = -(GE+FE*SL)
      Q1 = AE-2*HE*SL+BE*SL*SL+D5*SL1*SL1
      Q2 = QB*QB+Q1
      IF(Q2) 1,1,2
1     WRITE(2,99)
99    FORMAT(1X,'COMPLEX ROOTS TO QUADRATIC')
      GO TO 322
2     QR = SQRT(Q2)
      IF(SN(1)) 310,310,311
 310  SM1 = (QB-QR)/Q1
      GO TO 312
 311  SM1 = (QB+QR)/Q1
 312  CONTINUE
      RA = SM1/SN(1)
      SMS = RA*SN(3)
      IF(ABS(SMS)-F12) 320,320,321
 321  CONTINUE
      RA1 = F12/ABS(SN(3))
      SX = RA1*S(1)
      WRITE(2,390) SX
 390  FORMAT(1X,'SHEAR FAILURE AT SX = ',E17.10)
      GO TO 322
 320  CONTINUE
      SX = RA*S(1)
      WRITE(2,391) SX
 391  FORMAT(1X,'FAILURE AT SX = ',E17.10)
 102  FORMAT(1X,11(E9.2,1X),E9.2)
 322  CONTINUE
      WRITE(2,102) SL,QB,Q1,QR,SM1,RA,SMS,RA1,SX
10    RETURN
      END


      SUBROUTINE HILL STRESS
      DIMENSION SN(3),S(3)
      COMMON/B4/SN,S/B2/AE,HE,BE,GE,FE,CE,HAE,HHE,HBE/B6/D5
     1/B1/FT1,FC1,FT2,FC2,F12
      CALL WRITE(S,3,1,5HSIGXY)
      CALL WRITE(SN,3,1,5HSIG12)
      T1 = 0.1E-20
      IF(ABS(SN(1)).LE.T1.AND.ABS(SN(2)).LE.T1) GO TO 21
3     SL = SN(2)/SN(1)
      SL1 = SN(3)/SN(1)
      IF(SN(1)) 10,10,11
10    IF(SN(2)) 12,12,13
12    PA = 1/(FC1*FC1)
      PB = 1/(FC2*FC2)
      PH = 1/(2*FC1*FC1)
      GO TO 20
13    PA = 1/(FC1*FC1)
      PB = 1/(FT2*FT2)
      PH = 1/(2*FC1*FC1)
      GO TO 20
11    IF(SN(2)) 14,14,15
14    PA = 1/(FT1*FT1)
```

```
      PB = 1/(FC1*FC1)
      PH = 1/(2*FT1*FT1)
      GO TO 20
1     PA = 1/(FT1*FT1)
      PB = 1/(FT2*FT2)
      PH = 1/(2*FT1*FT1)
20    CONTINUE
      Q1 = PA-2*PH*SL+PB*SL*SL+D5*SL1*SL1
      IF(Q1) 1,1,2
1     WRITE(2,99)
99    FORMAT(1X,'COMPLEX ROOTS TO QUADRATIC')
      GO TO 322
2     QR = SQRT(Q1)
      IF(SN(1)) 310,310,311
310   SM1 = -QR/Q1
      GO TO 312
311   SM1 = QR/Q1
312   CONTINUE
      RA = SM1/SN(1)
      SMS = RA*SN(3)
      IF(ABS(SMS)-F12) 320,320,321
321   CONTINUE
      RA1 = F12/ABS(SN(3))
      SX = RA1*S(1)
      WRITE(2,390) SX
390   FORMAT(1X,'SHEAR FAILURE AT SX = ',E17.10)
      GO TO 322
320   CONTINUE
      SX = RA*S(1)
      WRITE(2,391) SX
391   FORMAT(1X,'FAILURE AT SX = ',E17.10)
102   FORMAT(1X,11(E9.2,1X),E9.2)
322   CONTINUE
      WRITE(2,102) SL,QB,Q1,QR,SM1,RA,SMS,RA1,SX
21    RETURN
      END
```

```
      SUBROUTINE POSDEF MATINV(A,MAMA)                                  INV 0000
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *INV 0010
C  THE SUBROUTINE FINDS THE INVERSE OF THE MA BY MA SYMMETRIC POSITIVE    INV 0020
C  DEFINITE MATRIX [A].  THE SUBROUTINE REQUIRES:                         INV 0030
C     THE REAL ARRAY A(MA,MA) CONTAINING THE COMPLETE MATRIX [A].         INV 0040
C  A IS OVERWRITTEN BY ITS INVERSE. (MAMA=MA*MA)                          INV 0050
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *INV 0060
      REAL A(MAMA)                                                      INV 0070
      INTEGER ROW,COL                                                   INV 0080
      MA=0.1+SQRT(1.0*MAMA)                                             INV 0090
      J1=MAMA-MA+1                                                      INV 0100
      J2=0                                                              INV 0110
      DO 1 COL=1,J1,MA                                                  INV 0120
      J2=J2+1                                                           INV 0130
      DO 1 ROW=1,MA                                                     INV 0140
      J3=ROW+COL-1                                                      INV 0150
      B=0.0                                                             INV 0160
      IF(J2.EQ.1.OR.ROW-J2.LT.0)GOTO4                                   INV 0170
      J4=ROW                                                            INV 0180
      DO 2 M=1,J2-1                                                     INV 0190
      B=B+A(J4)*A(J2+J4-ROW)                                            INV 0200
    2 J4=J4+MA                                                          INV 0210
    4 IF(ROW-J2)1,3,0                                                   INV 0220
      A(J3)=(A(J3)-B)/A(COL+J2-1)                                       INV 0230
      GOTO1                                                             INV 0240
    3 A(J3)=SQRT(A(J3)-B)                                               INV 0250
    1 CONTINUE                                                          INV 0260
      J2=0                                                              INV 0270
```

```
      DO 5 COL=1,J1,MA                                                  INV 0280
      J2=J2+1                                                           INV 0290
      J5=-MA                                                            INV 0300
      DO 5 ROW=1,MA                                                     INV 0310
      J3=ROW+COL-1                                                      INV 0320
      J5=J5+MA                                                          INV 0330
      B=0.0                                                             INV 0340
      IF(ROW-J2.LE.0)GOTO6                                              INV 0350
      J4=J3                                                             INV 0360
      DO 7 M=J2,ROW-1                                                   INV 0370
      B=B+A(J4)*A(COL+M-1)                                              INV 0380
    7 J4=J4+MA                                                          INV 0390
    6 IF(ROW-J2)5,8,0                                                   INV 0400
      A(J3)=-B/A(ROW+J5)                                                INV 0410
      GOTO5                                                             INV 0420
    8 A(J3)=1.0/A(COL+J2-1)                                             INV 0430
    5 CONTINUE                                                          INV 0440
      J2=0                                                              INV 0450
      DO 9 COL=1,J1,MA                                                  INV 0460
      J4=0                                                              INV 0470
      J2=J2+1                                                           INV 0480
      DO 9 ROW=1,J1,MA                                                  INV 0490
      J4=J4+1                                                           INV 0500
      B=0.0                                                             INV 0510
      IF(J4-J2)11,0,0                                                   INV 0520
      DO 10 M=J4,MA                                                     INV 0530
   10 B=B+A(ROW+M-1)*A(COL+M-1)                                         INV 0540
      GOTO12                                                            INV 0550
   11 A(COL+J4-1)=A(ROW+J2-1)                                           INV 0560
      GOTO9                                                             INV 0570
   12 A(COL+J4-1)=B                                                     INV 0580
    9 CONTINUE                                                          INV 0590
      RETURN                                                            INV 0600
      END                                                               INV 0610
      SUBROUTINE WRITE(AM,I,J,DUMMY)
      DIMENSION AM(I,J)
      WRITE(2,200) DUMMY
200   FORMAT(1H0,7H MATRIX,2X,A5)
111   FORMAT(1X,6(E17.10,2X))
      DO 1 K = 1,I
      WRITE(2,111) (AM(K,L),L = 1,J)
1     CONTINUE
      RETURN
      END


      SUBROUTINE MULT(A,I,J,B,K,C)
      DIMENSION A(I,J),B(J,K),C(I,K)
      DO 1 L = 1,I
      DO 1 M = 1,K
      C(L,M) = 0.0
      DO 1 N = 1,J
      C(L,M) = C(L,M)+A(L,N)*B(N,M)
1     CONTINUE
      RETURN
      END
      SUBROUTINE SCMULT(A,I,J,S)
      DIMENSION A(I,J)
      DO 1 K = 1,I
      DO 1 L = 1,J
      A(K,L) = A(K,L)*S
1     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SUB(A,I,J,B,C)
      DIMENSION A(I,J),B(I,J),C(I,J)
      DO 1 K = 1,I
      DO 1 L = 1,J
      C(K,L) = A(K,L)-B(K,L)
1     CONTINUE
      RETURN
      END
      FINISH
```

Program listing

Angle-ply-theory coefficient

```
0010                MASTER NON LIN LAM
0011                E1 = 0.165 E+12
0012                E2 = 0.93 E+10
0013                Q = 0.33
0014                G = 0.49 E+10
0015                AL = 0.7853981633
0016                T = 0.8726646259 E-01
0017                DT = 0.8726646259 E-01
0018          3     S = SIN(T)
0019                C = COS(T)
0020                SC = S*C
0021                CC = C*C
0022                SS = S*S
0023                A = E2*(1+Q)/(Q*E2+E1)
0024                B = (Q*E2+E1)/(E1*E2)
0025                D = (1+Q)/E1
0026                A1 = (2*SC)/(SS+A*CC)
0027                A2 = CC*(CC-SS)/(2*B*SS*SC+2*D*CC*SC)
0028                B1 = 2*SC/(SS/A+CC)
0029                B2 = -SS*(CC-SS)/(2*B*SS*SC+2*D*CC*SC)
0030                A3 = 1/E1*(A1-Q*B1)
0031                A4 = 1/E1*(A2-Q*B2)
0032                B3 = B1/E2-Q*A1/E1
0033                B4 = B2/E2-Q*A2/E1
0034                A5 = A3*CC+B3*SS
0035                A6 = A4*CC+B4*SS+SC
0036                B5 = A3*SS+B3*CC
0037                B6 = A4*SS+B4*CC-SC
0038                SX = A5*G+A6
0039                R = (B5*G+B6)/SX
0040                A7 = A1*CC+B1*SS+2*SC
0041                A8 = A2*CC+B2*SS
0042                PX = A7*G+A8
0043                EX = PX/SX
0044                WRITE(2,10)SX,R,PX,EX,A5,A6,A7,A8
0045                WRITE(2,10)A1,A2,B1,B2,A3,A4,B3,B4
0046          10    FORMAT(8(1X,E14.6))
0047                IF(AL-T)1,1,2
0048          2     T = T+DT
0049                GO TO 3
0050          1     STOP
0051                END

END OF SEGMENT, LENGTH   274, NAME  NONLINLAM
```

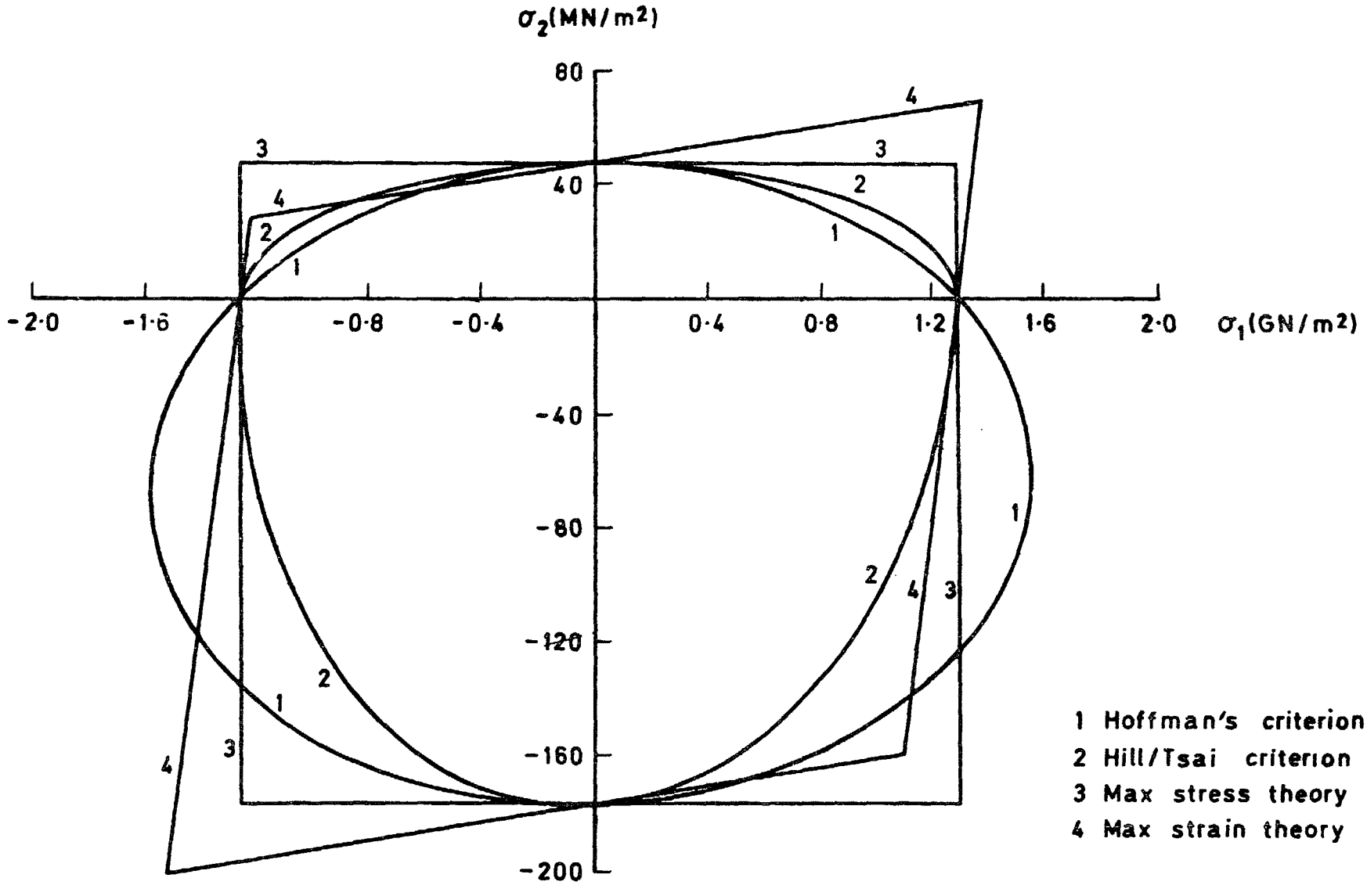

Fig 1 Four failure theories as applied to a high strength (HT-S) unidirectional laminate

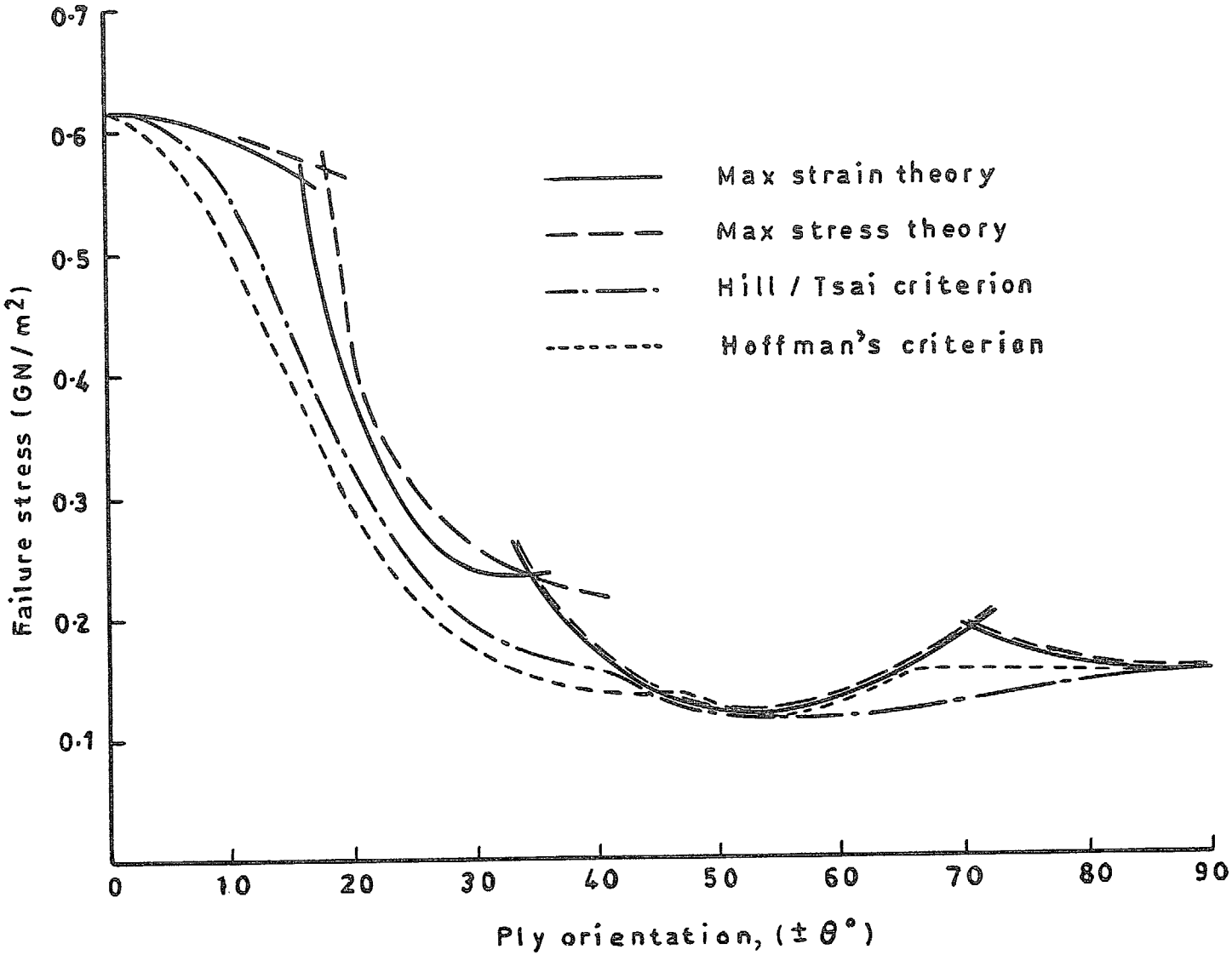

Fig 2 Variation of compressive strength with orientation according to four theories for high modulus (HM-S) angle-ply laminates

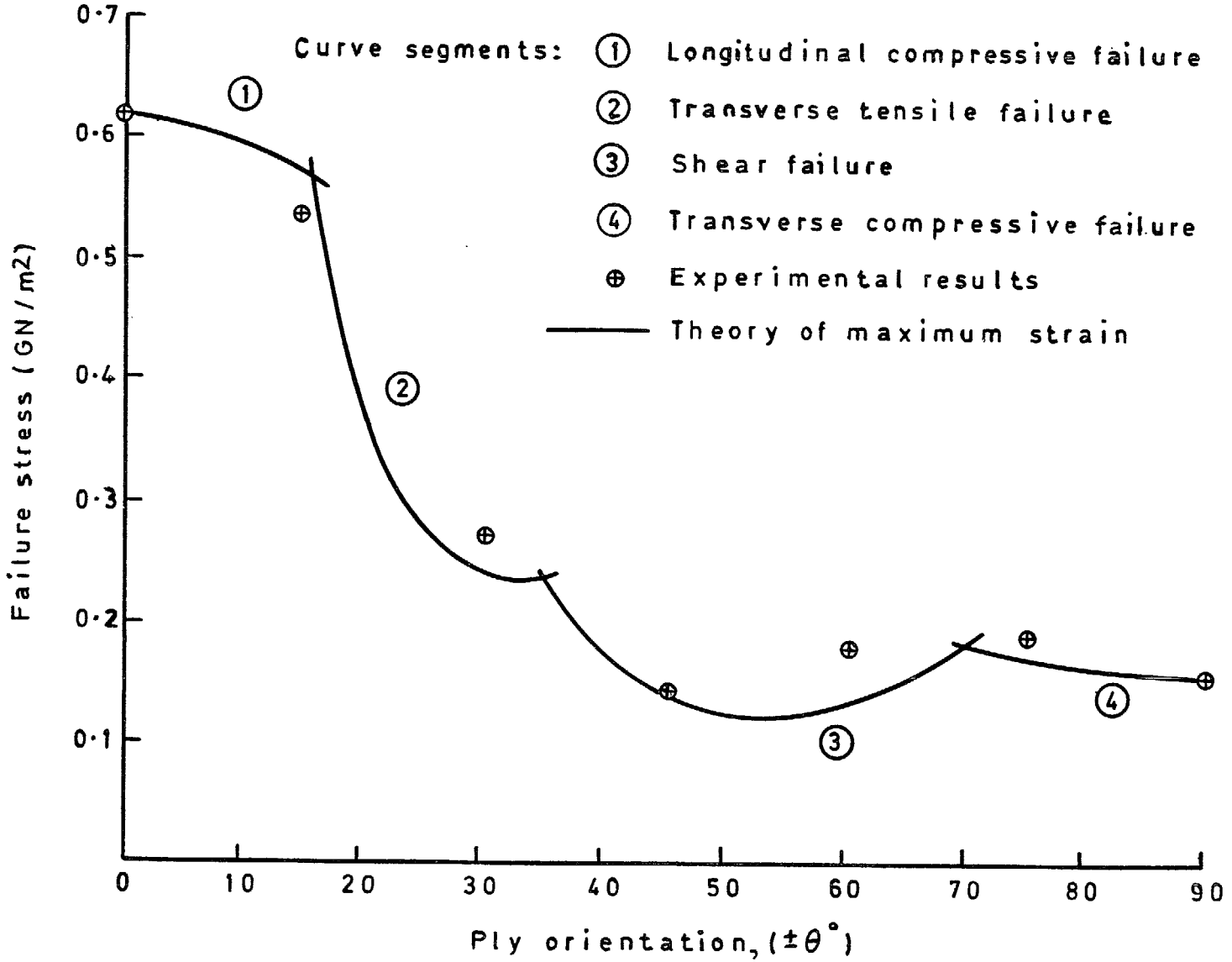

Fig 3 Comparison between experiment and theory of max strain for compressive strength for high modulus (HM-S) angle-ply laminates

Fig 4

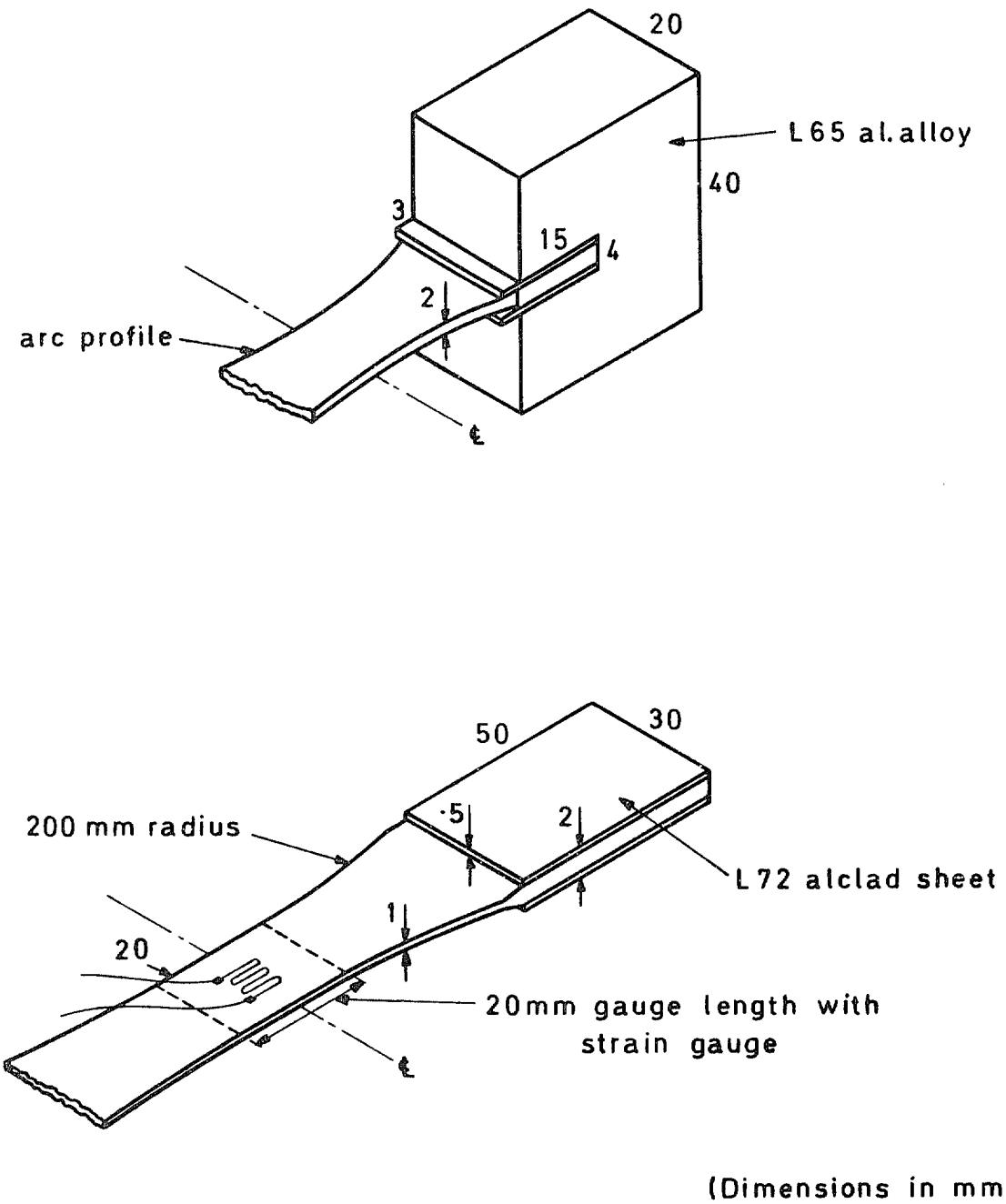

Fig 4 Compressive and tensile specimen design

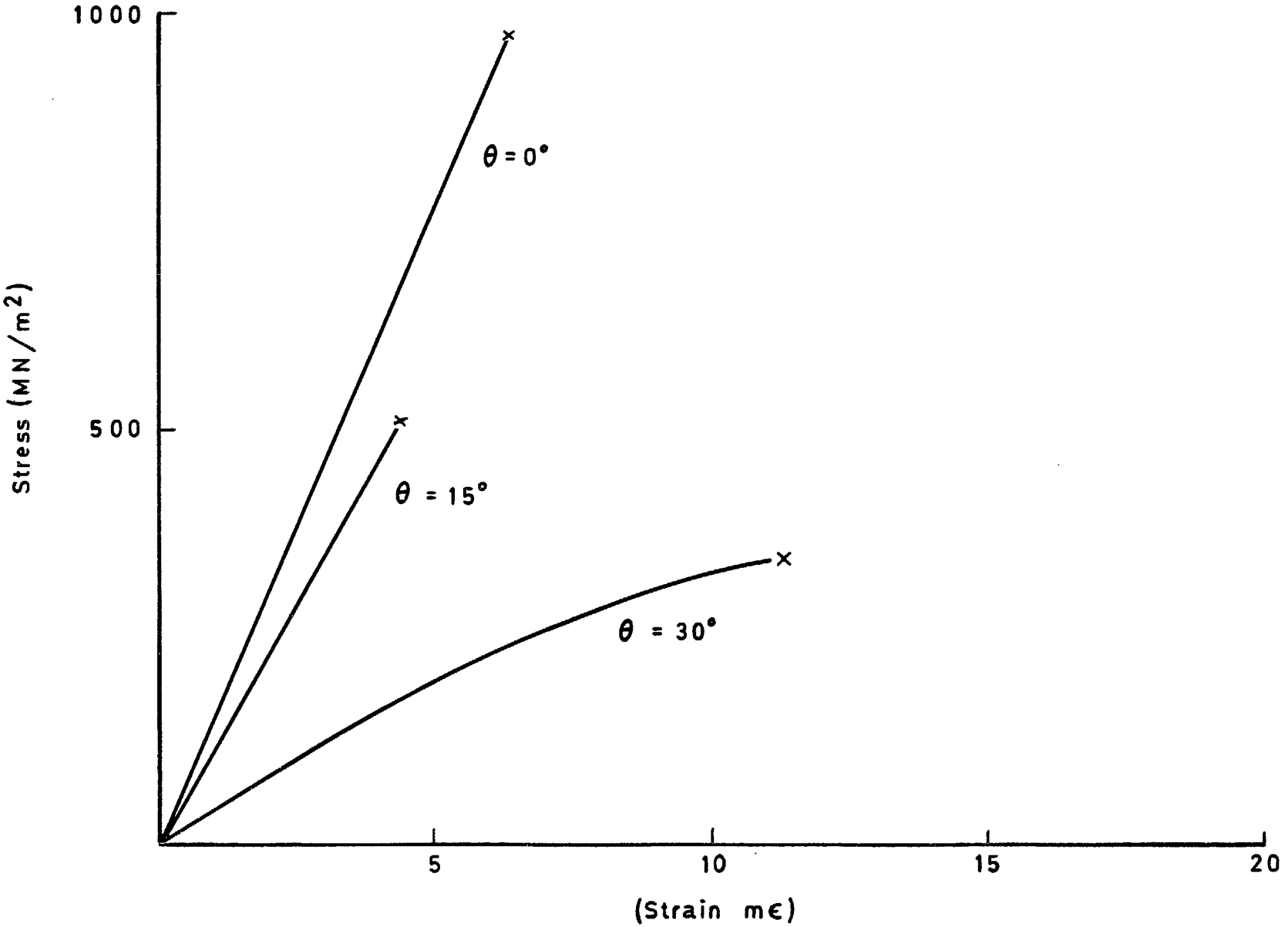

Fig 5 Typical HT-S angle-ply stress/strain curves for various ply orientations

Fig 5

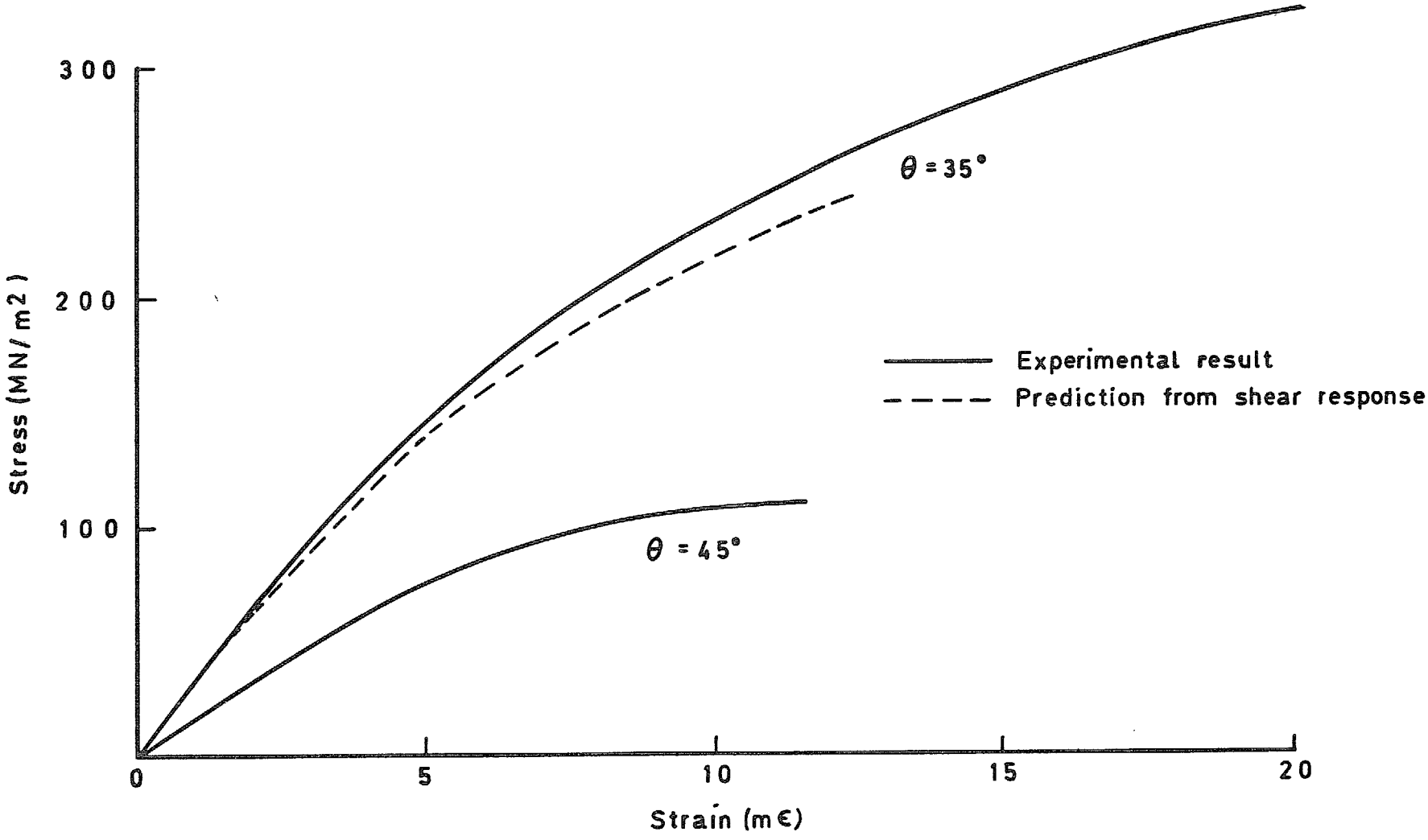

Fig 6 Typical HT-S angle-ply stress/strain curves, for $\theta = 35^{\circ}$ and $\theta = 45^{\circ}$

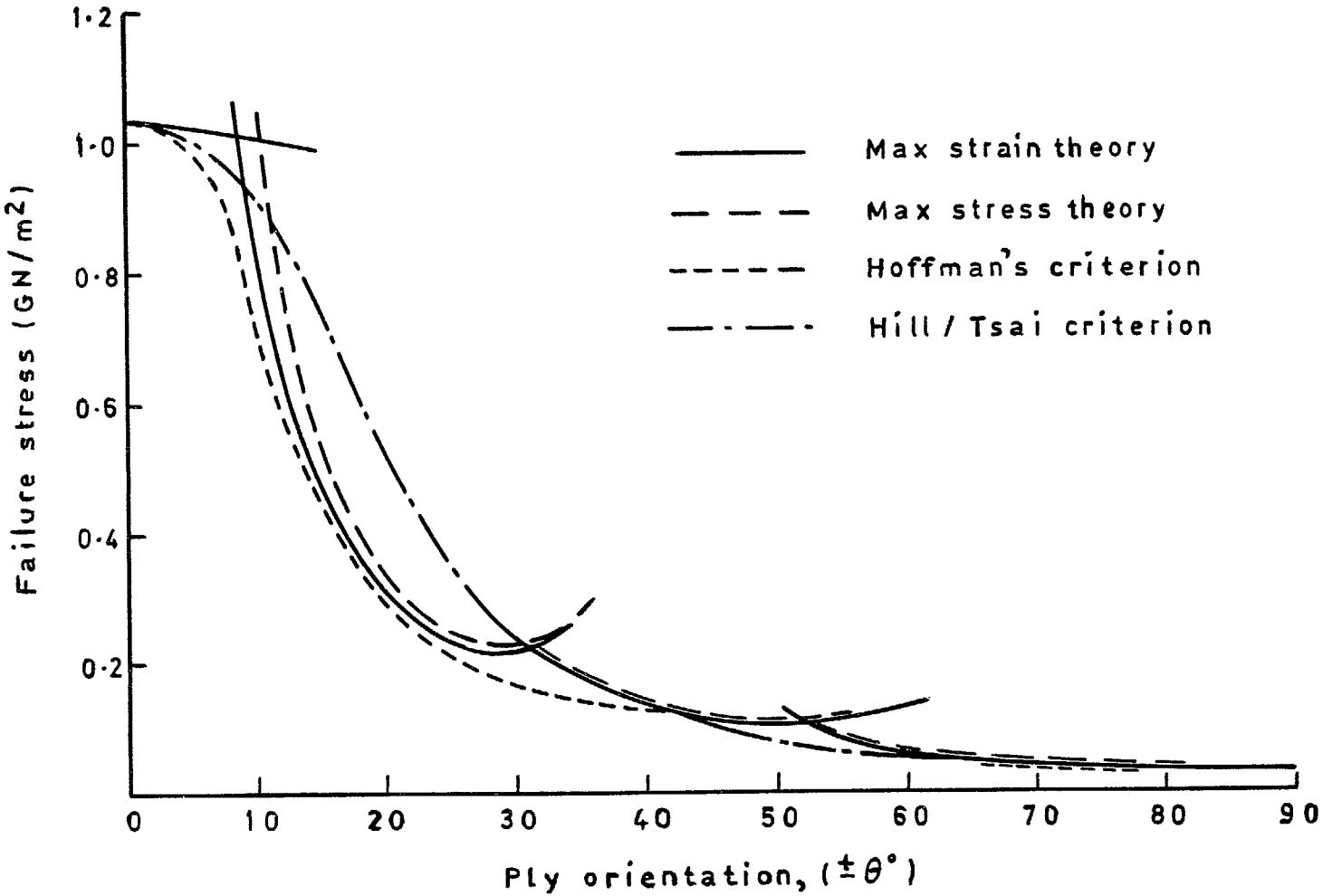

Fig 7 Comparison between theories of failure for tensile strength of angle-ply HT-S laminates

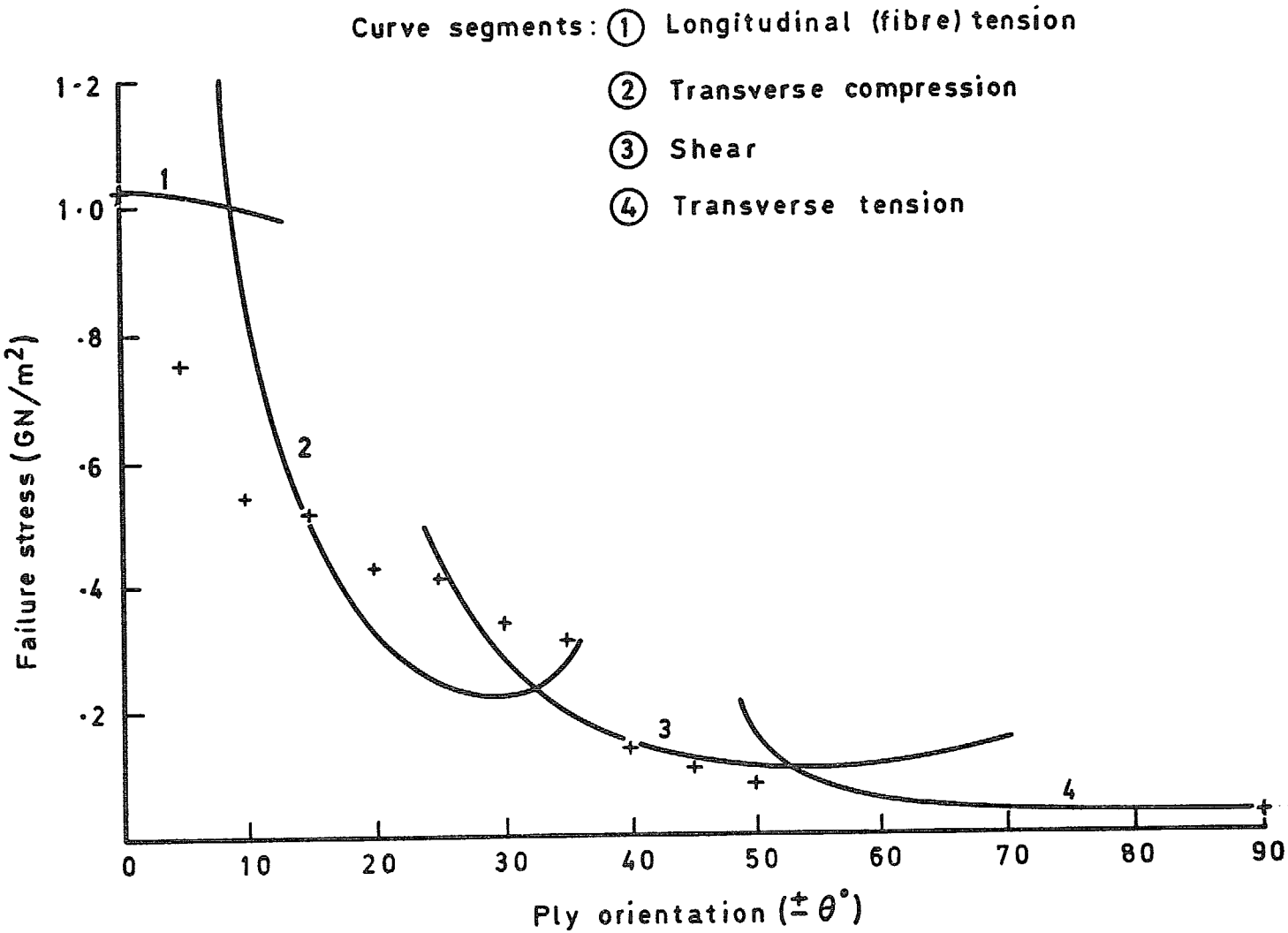

Fig 8 Comparison between experiment and theory of max strain for tensile strength of HT-S angle-ply laminates

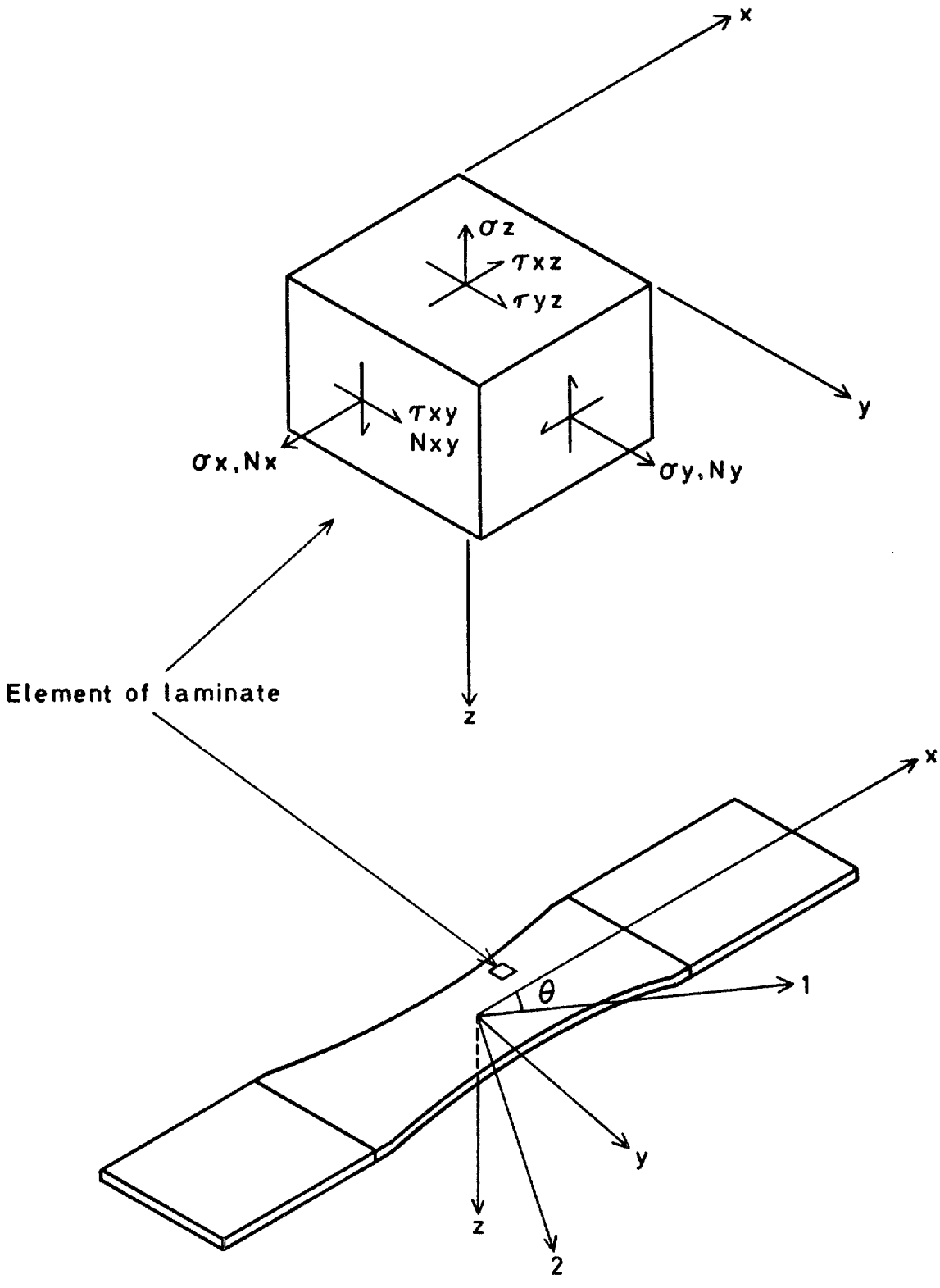

Fig 9 Notation and sign convention for laminate stresses

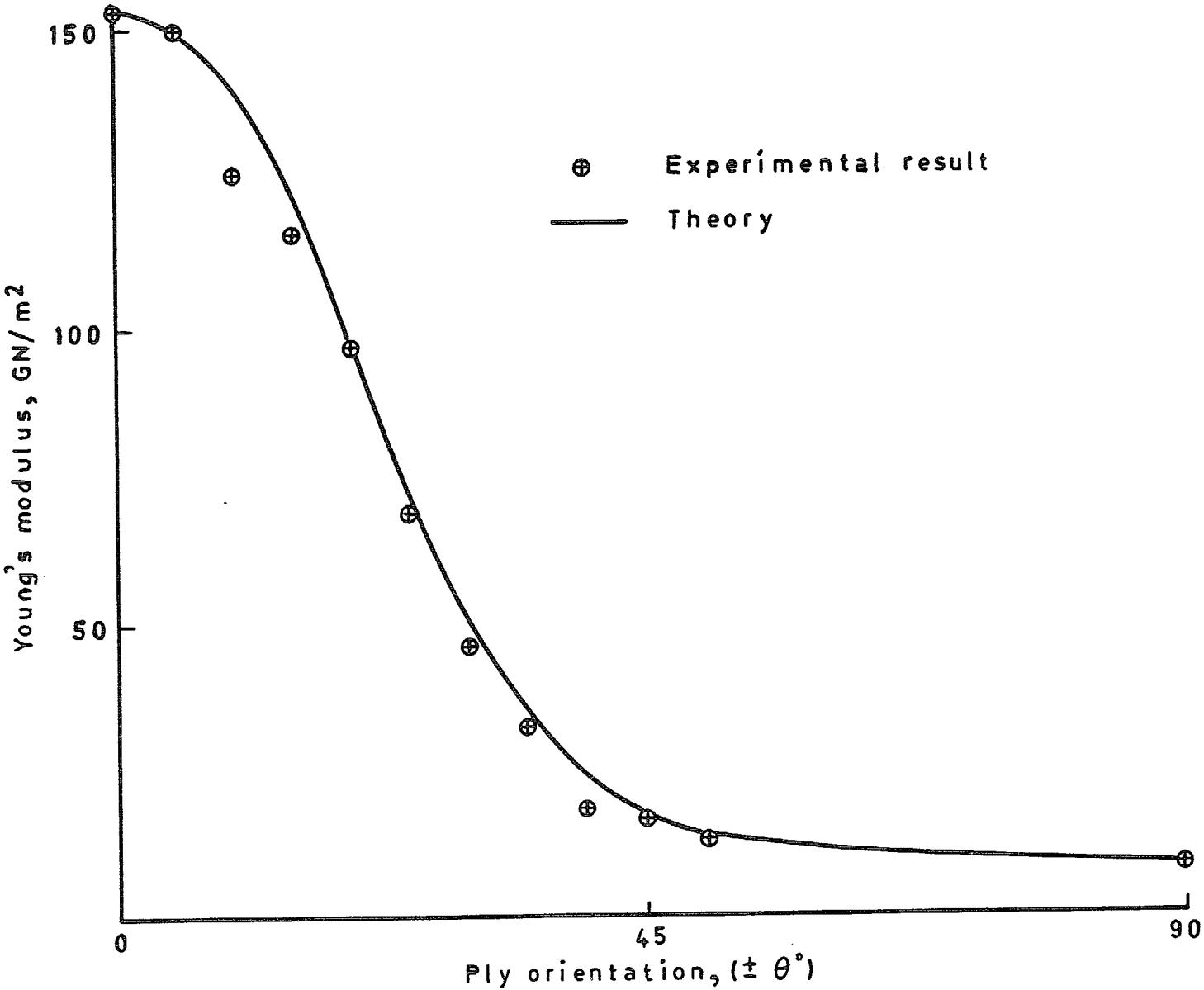

Fig 10 Variation of Young's modulus with orientation for high strength HT-S angle-ply laminates

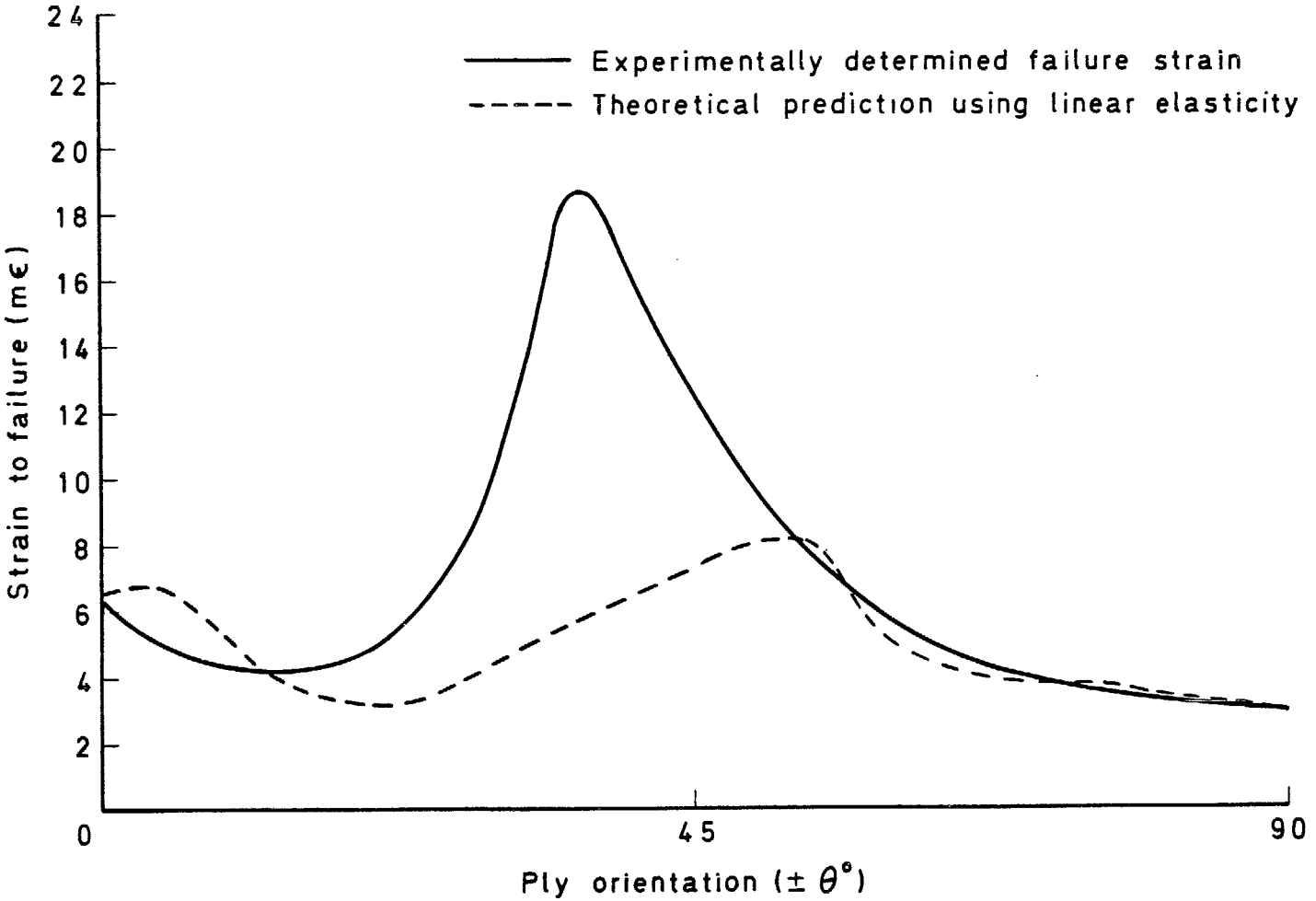

Fig 11 Comparison between experimental and theoretical failure strain for high strength (HT-S) angle-ply laminates

Fig 12

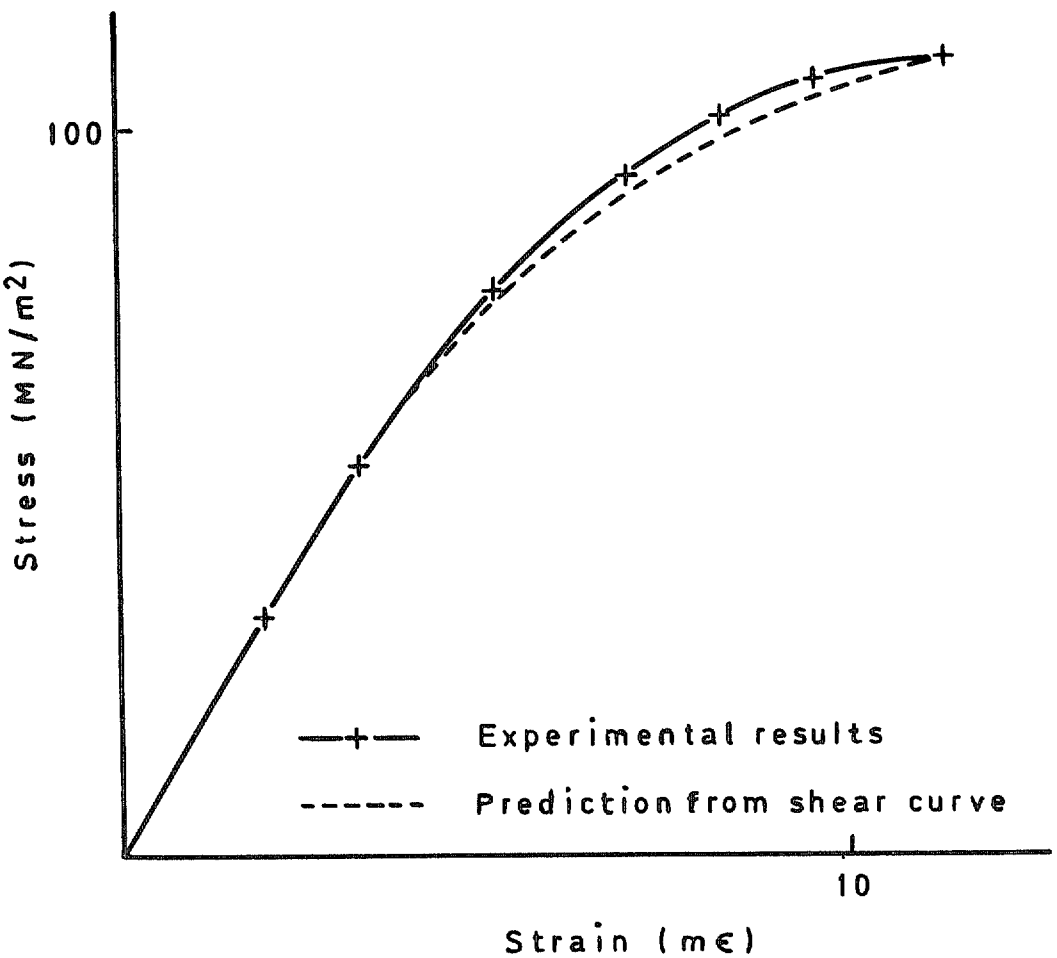

Fig 12 Tensile response of a ±45° high strength (HT-S) angle-ply laminate: experimental results and prediction from unidirectional shear stress/strain curve

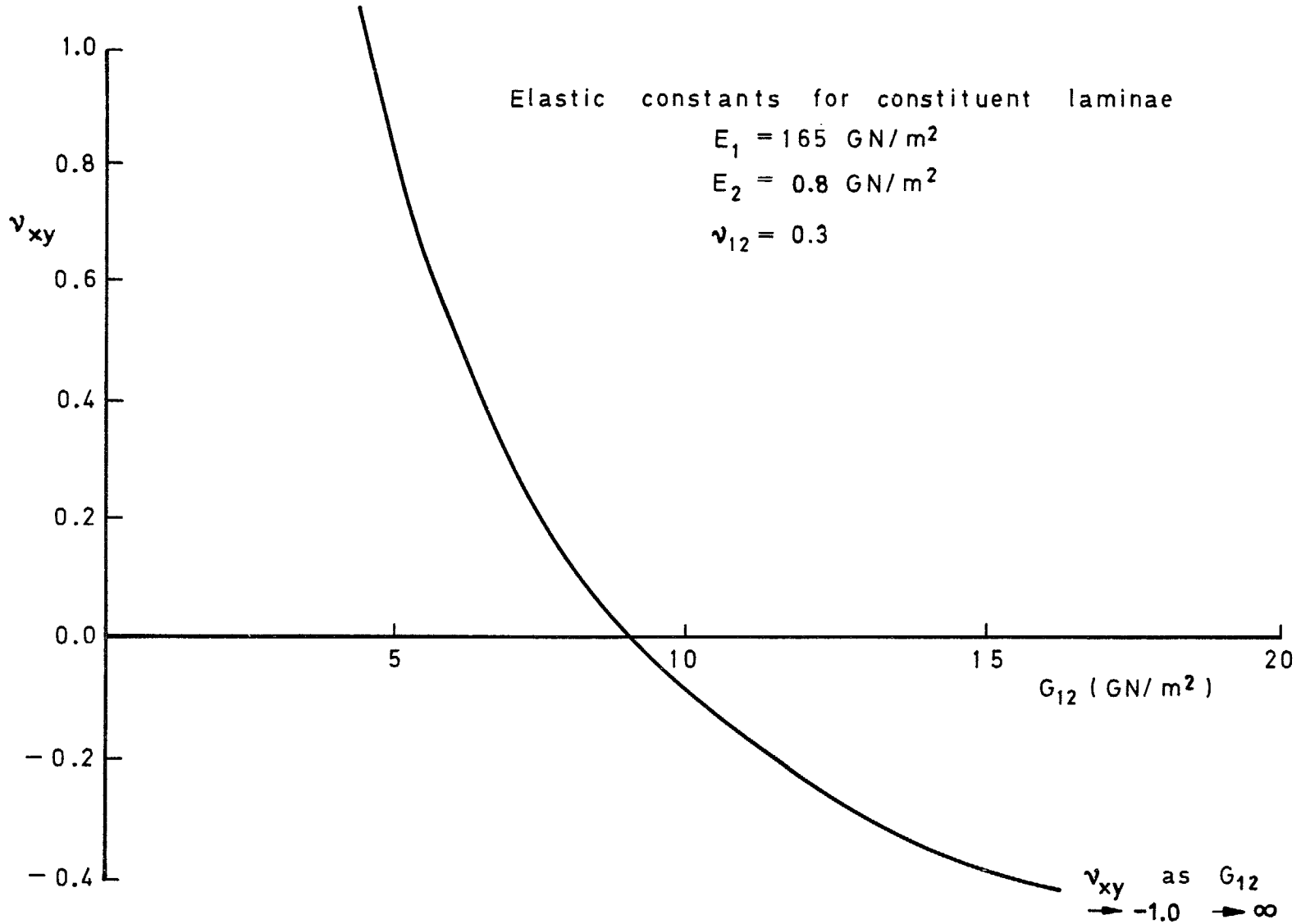

Fig 13 Variation of ν_{xy} with G_{12} for a ±45° high strength (HT-S) angle-ply laminate (Equation (31))

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