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Landing Transition Paths which Optimise Fuel, Time or Distance for Jet-Lift VTOL Transport Aircraft in Steep Approaches

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Summary

Optimal landing transition manoeuvres are studied for jet-lift VTOL aircraft in which the range of engine-tilt is sufficient to allow approach angles up to 20 degrees. The optimal use of incidence, thrust vector angle and thrust to control the transition is studied using simple physical arguments. All the natural constraints such as incidence, engine tilt, maximum permissible decelerations, minimum permissible lift engine thrust, etc., can thus be taken into account and simple rules formulated for the optimal strategy. It is shown that provided control programs are formulated as functions of aircraft speed, flight path shape and the control program can be dealt with as separate problems.

It is known that for minimum distance transitions the optimum path shape involves selecting a let-down speed V^* at which the deceleration function for horizontal flight is a maximum. The optimal manoeuvre is then to decelerate horizontally to V^*, to lose height at this speed and to conclude the transition with a further horizontal deceleration to the hover point (i.e., the 'stepped manoeuvre'). However, for the minimum fuel or time transition the ideal is to use the same stepped manoeuvre but to select the let-down speed as high as is compatible with considerations of the danger of high rates of descent near the ground and the need to avoid obstacles near the airfield.

* Replaces A.R.C. 33 388.

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1. Introduction

In a previous report[1] the problem of minimizing landing transition distance for a jet-lift VTOL aircraft was studied, the aim being to ease the pilot's problem of judging distance to the hover point, perhaps in conditions of poor visibility. A second aspect of this study concerned the desirability of accuracy in achieving the hover in order to reduce to a minimum any final corrections which are known to be very expensive in fuel. The theoretical approach adopted was an unsophisticated one making no use of modern optimal control theory unlike other contributors to this subject, (Refs. 2–5 for example). Instead, following an approach instigated by Lean,[6] the optimal profiles were obtained using simple physical arguments. This meant that full account could be taken of the various, very important, constraints on the state and control vectors, normally difficult to include in numerical optimization algorithms, thus obtaining a clearer idea of the role they play.

In this report the study is taken further to include fuel and time optimisation. The calculated results presented are based on data reasonably representative of a VTOL transport aircraft of 100 000 lbs AUW, with high lift devices in the landing configuration giving a $C_L = 0{\cdot}5$ at zero incidence. Since this configuration is sufficiently different from the smaller aircraft studied in Ref. 1, results for minimum distance transitions are presented but with little discussion. The emphasis is on transition paths to minimize fuel used. It is assumed that the aircraft has to lose both height and speed and perhaps has to circumvent obstacles; mean flight path angles up to -15 degrees are covered.

Besides the effects of the obvious constraints imposed on incidence (by the stall) on thrust vector angles (by built-in engine tilt) and on minimum lift engine thrust setting (by engine response) some discussion is also included on the significance of the restriction on longitudinal acceleration imposed by the passenger. A final section concerns the effect of lift-loss on the topics discussed.

2. Method of Analysis

2.1. Neglecting Lift-Loss

Consider the general jet-lift aircraft configuration having both propulsive and lifting engines. The propulsive engines are assumed fixed relative to the aircraft and their idling thrust T_p is assumed sufficiently small for its effect on normal acceleration to be neglected. Referring to Fig. 1, T denotes the thrust of the lifting engines and ϕ represents the angle of rotation of the lift-engine thrust vector relative to the normal to the aircraft datum such that positive angle gives a decelerating component of thrust. Flight path angle γ is positive when the flight path is inclined upwards.

The pitching degree of freedom is ignored, it being assumed that instantaneous rotation of the aircraft to any desired pitch altitude is possible. The equations of motion are then

$$m\dot{V} = -W\sin\gamma - T\sin(\alpha + \phi) - D + T_p\cos\alpha \tag{1}$$

and

$$mV\dot{\gamma} = -W\cos\gamma + T\cos(\alpha + \phi) + L \tag{2}$$

Assume that the aircraft is controlled on a straight flight path (so that $\dot{\gamma} = 0$) by the use of engine thrust, whilst incidence and thrust vector angle are held constant. Then

$$T = \frac{W\cos\gamma - L}{\cos(\alpha + \phi)} \tag{3}$$

For the aerodynamic forces assume that

$$L = \tfrac{1}{2}\rho V^2 S C_L(\alpha)$$

and

$$D = \tfrac{1}{2}\rho V^2 S C_D(\alpha) + m_e V,$$

where m_e is the engine mass flow linearly related to lift engine thrust so that

$$m_e = m_0 + kT,$$

Equation (1) gives the acceleration of the aircraft along the flight path and a deceleration function $f(V, \gamma)$ may then be obtained in the form

$$f(V, \gamma) = A + BV + CV^2 + DV^3 \quad \text{in g units,} \tag{4}$$

where

$$A = [\tan(\alpha + \phi)\cos\gamma + \sin\gamma - T_p/W],$$

$$B = m_0/W + k\cos\gamma\sec(\alpha + \phi),$$

$$C = [C_D(\alpha) - C_L(\alpha)\tan(\alpha + \phi)]\rho S/2W$$

and

$$D = -kC_L(\alpha)\sec(\alpha + \phi)\rho S/2W.$$

Using the constants listed in Table 1 the deceleration function could be evaluated for a range of incidences, flight path angles and thrust vector angles.

Knowing $f(V, \gamma)$, the transition distance s, measured along the flight path, for a change in speed from V_1 to V_2, could be determined by numerical integration using

$$s = \int_{V_2}^{V_1} \frac{V}{gf(V, \gamma)} dV. \tag{5}$$

The fuel F used in the same manoeuvre is given by

$$F = \frac{1}{3600} \int_{V_1}^{V_2} \frac{(\text{s.f.c.})T}{gf(V, \gamma)} dV,$$

where s.f.c. is the specific fuel consumption in lb/lb thrust/hour.

If s.f.c. is assumed independent of thrust, then a fuel parameter F' can be defined by

$$F' \triangleq \frac{36}{(\text{s.f.c.})} \frac{F}{W} = \int_{V_1}^{V_2} \frac{T/W}{gf(V, \gamma)} dV, \tag{6}$$

where F/W is now the fuel used expressed as a percentage of aircraft weight.

[Data shown in Fig. 13 are assumed typical for the type of lift powerplant under discussion. The assumption of linear variation of fuel with thrust used above is seen to be reasonable for this analysis, provided we include the constraint that T/W should be greater than 0·3 when the lift engines are alight].

Finally, the time taken in the manoeuvre is given by

$$t = \int_{V_2}^{V_1} \frac{1}{gf(V, \gamma)} dV. \tag{7}$$

2.2. Including Lift-Loss

Available data* on lift loss were found to collapse into a curve which could be approximated by the parabola:

$$y = -[0{\cdot}485 - 3{\cdot}359 \times 10^4(x - 0{\cdot}0038)^2]$$

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* This data was kindly provided by Mr. A. A. Woodfield, of R.A.E., Bedford.

in which

$$y \triangleq \Delta L/T \quad \text{and} \quad x \triangleq \rho V^2/T,$$

$$\left(\text{or } \Delta L/W = y(T/W) \text{ and } x = \frac{\rho V^2}{W}\Big/\left(\frac{T}{W}\right)\right).$$

Including lift-loss the equations of motion become

$$m\dot{V} \;=\; -W\sin\gamma - T\sin(\alpha + \phi) - D + T_p\cos\alpha \tag{1}$$

and

$$mV\dot{\gamma} = -W\cos\gamma + T\cos(\alpha + \phi) + L + \Delta L. \tag{8}$$

Previously with $\Delta L \equiv 0$, we could solve equation (8) with $\dot{\gamma} = 0$ and substitute for T in equation (1). Now with $\Delta L \neq 0$, we have a non-linear equation in T/W to solve, namely

$$(T/W)\left\{(\cos(\alpha + \phi) - 0{\cdot}485) + 3{\cdot}359 \times 10^4\left(\frac{\rho V^2}{W}\Big/\frac{T}{W} - 0{\cdot}0038\right)^2\right\} - (\cos\gamma - L/W) = 0.$$

This could be solved without difficulty for most speeds by the Newton–Raphson method. Knowing T/W, the deceleration function was then given by

$$f(V, \gamma) = \sin\gamma + \frac{T}{W}\sin(\alpha + \phi) - \frac{T_p}{W}\cos\alpha + \left[\frac{\rho S}{2W}V^2 C_D + \left(\frac{m_0}{W} + k\frac{T}{W}\right)V\right].$$

Transition distance, fuel and time for straight line transitions could then be determined as before using equations (5), (6) and (7).

2.3. Accelerations Felt by Passengers

During the earlier study[1] it became apparent that with the normal ranges of α and ϕ available, quite large decelerations could be achieved leading naturally to very short transition distances. Some attempt was made to make the solution more realistic, taking account of the opposition by pilots and passengers to excessive deceleration, by imposing an arbitrary limit on $f(V, \gamma)$ and discussing suitable control programs under that assumption. However, the treatment left much to be desired.

The problem concerns the identification of just what it is that the passenger appreciates when the vehicle is subjected to a longitudinal deceleration when in one of several possible attitudes. It is necessary to analyse how these two parameters combine in order to be able to specify what deceleration the passenger will accept. The passenger feels through his seat cushion and seat belt a total force which may be resolved into components perpendicular to and parallel to the aircraft datum respectively. If the aircraft decelerates with deceleration $gf(V, \gamma)$, the passenger is then subjected to the acceleration $gf(V, \gamma)$ along the flight path and the vertical acceleration g. We may assume that he is not particularly concerned about the magnitude of the normal reaction of the seat (provided flight path curvature is small) but is sensitive to fore and aft accelerations which tend to throw him against his seat belt.

The combination of the two external forces then give, along the axis of the aircraft, the acceleration

$$g\sin(\Gamma - \alpha) + f(V, \gamma)g\cos\alpha$$

or

$$g\cos\alpha\,(f(V, \gamma) + \sin\Gamma) - g\sin\alpha\cos\Gamma,$$

(denoting $-\gamma$ by Γ, for convenience).

If as usual we approximate $f(V, \gamma)$ by $f(V, 0) + \sin\gamma$, then

$$a/g = f(V, 0)\cos\alpha - \cos\gamma\sin\alpha.$$

Thus the longitudinal acceleration as felt by the passenger is very weakly dependent upon the flight path angle; on the other hand a large value of α significantly reduces the acceleration experienced.

As we shall see when we discuss the results, there are aerodynamic reasons why α is restricted to be small at certain speeds. One way of relieving the constraint in this case would be by tilting the seat backwards through an angle ε. The corresponding value of the fore and aft acceleration is then

$$\frac{a}{g} = f(V, 0)\cos(\alpha + \varepsilon) - \cos\gamma\sin(\alpha + \varepsilon).$$

Whilst it is conceivable that the seat tilt could be engineered for the passengers it is however, unlikely that it would be a feasible proposition for the pilot.

3. Numerical Results for Aircraft Without Lift-Loss

Using the data of Table 1 and equations (3) and (4) the thrust/weight ratio and deceleration function in various straight-line transitions were determined. Fig. 2 shows the results for the horizontal transition for four incidences and five thrust-vector angles. As illustrated in Ref. 1, the T/W functions for other flight path angles differ only marginally from those shown and the deceleration functions for inclined flight paths are well approximated by

$$f(V, \gamma) = f(V, 0) + \sin\gamma.$$

For the higher incidences the deceleration functions are seen to intersect at a point. Since this implies that $f(V, 0)$ is independent of ϕ it does of course correspond to the speed at which $T/W = 0$.

If we are to choose a control program of α and/or ϕ against V, it is necessary to specify a maximum acceptable value of $f(V, 0)$. Taking for illustration $a/g = 0{\cdot}25$ as the maximum value of forward deceleration that the passenger is prepared to tolerate, we have

$$f(V, 0)_{max} = \frac{0{\cdot}25 + \sin\alpha}{\cos\alpha}.$$

This gives values of 0·25 at $\alpha = 0$ and 0·458 at $\alpha = 12$ degrees. These are shown in Fig. 2 labelled $\varepsilon = 0$.

If one assumed that seat tilt of amount $\varepsilon = 10$ degrees were feasible, then

$$f(V, 0)_{max} = \frac{0{\cdot}25 + \sin(\alpha + \varepsilon)}{\cos(\alpha + \varepsilon)}$$

This gives the values in Fig. 2 labelled $\varepsilon = 10$ degrees.

Clearly if there is no problem in building into the design any desired amount of engine tilt then this passenger toleration constraint applies and we see that there is substantial benefit in choosing a large value of α, other things being equal, since the pilot can then make use of higher deceleration values. Even allowing for that however, there seems to be little case for thrust vector angles greater than, say, 20 degrees; $\phi = 15$ degrees is probably sufficient in most circumstances.

Suppose now that α and ϕ are kept constant at specified values and equations (5), (6) and (7) used to obtain transition distance, time and fuel. The results for various glide path angles shown in Fig. 3 are for transitions from $V_1 = 275$ fps (163 knots) down to zero speed. In the top figure are shown points at which the manoeuvre must be started in order that the aircraft should arrive precisely at the hover point with zero speed. For a given control program these points are seen to lie on a straight line called the starting line. From the programs chosen it is apparent that flight path angles up to -15 degrees are feasible with $\phi = 15$ degrees, provided the value of $f(V, 0)_{max}$ chosen is that applying to $\varepsilon = 10$ degrees.

Corresponding results for time and fuel usage are shown in Figs. 3(b) and 3(c). The interesting point is that in each case these points also lie on a straight line as glide path angle γ is varied. For want of a better term, and by analogy with the distance result, we shall call these the 'time starting lines' and 'fuel starting lines' respectively. These figures provide a very clear picture of the effect of losing height and speed simultaneously on the time and fuel used in straight line transitions with constant values of α and ϕ. The fuel used is typically of the order of 0·5 per cent of the aircraft AUW assuming a specific fuel consumption of 0·6.

Fig. 4 shows similar figures in which the effect of incidence is explored keeping ϕ constant at 15 degrees. As far as distance and time are concerned, there is little to choose between $\alpha = 4$ degrees or 8 degrees, but with regard to fuel it is clearly beneficial to use the higher incidence. (More use is made of aerodynamic lift and less of thrust and so the program is more economical in fuel.)

The constant $\alpha = 8$ degrees program is however not feasible since it requires zero thrust at $V = 275$ fps. Even when fully throttled back the lift engines will have a residual thrust of around 10 per cent AUW and in practice it has been found that the smallest amount of thrust desirable is nearer to 30 per cent AUW for reasons of engine behaviour and response (Fig. 13). If this constraint is to be incorporated into the calculations for $\alpha = 8$ degrees it means that in the initial phase of the transition, incidence must be allowed to vary. We therefore define an incidence program $\alpha_{app} = 8$ degrees in which incidence increases from 4 degrees to 8 degrees progressively as speed falls from 275 fps to 225 fps and remains constant at 8 degrees thereafter.

Referring again to Fig. 4, the lines passing through crosses are the new starting lines. It is apparent that distance and time of transition are each reduced by this modification but that fuel consumption is very slightly increased. Even so, it is still more economical in fuel to use the higher incidence program.

In the earlier paper[1] it was shown that if $f(V, 0)$ is chosen to be sensibly constant then the transition distance is independent of the transition path. This is clearly desirable since it makes for accuracy of the transition. For example, consider the two control programs shown in Fig. 5, both of which provide an approximately constant value of 0·4 g for the deceleration function. In Program 1 incidence is kept constant and thrust vector angle progressively increased; in Program 2, ϕ is kept constant and incidence is increased. The corresponding deceleration functions and thrust/weight functions are illustrated.

It is found that with regard to the results for transition distance, time and fuel there is virtually nothing to choose between these two quite different control programs. This clearly demonstrates the cardinal role played by the level flight deceleration function $f(V, 0)$.

4. Transitions with Variable Flight Path Angle

Having established results for transitions with flight path angle held constant we may next consider the question whether significant performance benefits may be achieved by suitably varying the flight path angle as a function of speed. It is known that for minimum distance transitions the optimal shape of path is the so-called 'stepped approach' in which the aircraft decelerates to an optimal let-down speed V^*; it loses all its height at this speed and then completes the deceleration horizontally. The optimal speed V^* is that at which $f(V, 0)$ has a maximum value. We now seek to extend this analysis to include fuel and time optimisation.

4.1. Formulation as a Line Integral

4.1.1. *Fuel usage*

The rate of fuel utilisation is given by

$$\frac{dF}{dt} = \left(\frac{\text{s.f.c.}}{3600}\right) T$$

where F is in lbs mass. Consequently along an elemental arc, length ds, of the flight path we have, writing F' for $36(F/W)/(\text{s.f.c.})$ per cent,

$$dF' = \frac{T/W}{V} ds.$$

Since we can closely approximate the deceleration function by

$$f(V, \gamma) = f(V, 0) + \sin \gamma,$$

it follows that

$$-\frac{VdV}{ds} = gf(V, 0) + g\frac{dh}{ds}$$

and

$$ds = -\left(\frac{V\,dV}{gf(V,0)} + \frac{dh}{f(V,0)}\right).$$

Now, in the (V, h) plane join the initial point (V_1, h_1) to the final point, the origin, by an arc C. Then

$$F' = -\int_C \left[\frac{T/W\,dV}{gf(V,0)} + \frac{T/W}{Vf(V,0)}dh\right],$$

where we assume T/W is independent of γ. To evaluate the integral we must prescribe the function $h(V)$ defining C and passing through the correct end points. It follows that

$$F' = \int_{V=0}^{V_1} \frac{T/W\,dV}{gf(V,0)} + \int_{V=0}^{V_1} \frac{T/W}{Vf(V,0)}\frac{dh}{dV}dV,$$

and F' is the sum of two terms which may be denoted by F_1 and F_2. F_1 is independent of the choice of $h(V)$ and represents the fuel used in a horizontal transition. In order to minimize F' therefore we have to choose $h(V)$ in order to minimize the integral

$$F_2 = \int_{V=0}^{V_1} \frac{(T/W)}{Vf(V,0)}\frac{dh}{dV}dV.$$

Introducing normalised co-ordinates $\xi = V/V_1$ and $\eta = h/h_1$, in the (ξ, η) plane, let C now denote the arc joining the point (1, 1) to the origin.
Then

$$F' = F_1 + F_2,$$

where

$$F_1 = \frac{V_1}{g}\int_{\xi=0}^{1} \frac{T(\xi)/W}{f(\xi,0)}d\xi = \frac{V_1}{g}b_1$$

and

$$F_2 = \frac{h_1 g}{V_1}\int_{\xi=0}^{1} \frac{T(\xi)/W}{\xi gf(\xi,0)}\frac{d\eta}{d\xi}d\xi = \frac{h_1 g}{V_1}b_2(C);$$

$b_2(C)$ is clearly a function of $d\eta/d\xi$, specifying C. Hence,

$$h_1 = \frac{V_1}{gb_2(C)}\left(F' - \frac{V_1}{g}b_1\right)$$

or

$$h = \frac{V_2}{gb_2(C)}F' - \frac{V^2}{g}\frac{b_1}{b_2(C)}, \tag{9}$$

dropping the subscripts on h and V.

Equation (9) implies that if we fix the deceleration function $f(V,0)$ by choice of control programs for α and ϕ and if we fix the arc C in the (ξ, η) plane, then there is a simple linear relationship between the fuel used and the height lost. This explains the straight 'fuel starting lines' discussed in Section 3.

4.1.2. *Transition time*

For the evaluation of transition time we have

$$-\frac{dV}{dt} = gf(V, \gamma) = gf(V, 0) + g\sin\gamma$$

$$= gf(V, 0) + \frac{g}{V}\frac{dh}{dt}.$$

Hence

$$dt = -\frac{dV}{gf(V, 0)} - \frac{dh}{Vf(V, 0)}$$

and

$$t_f = \int_{V=0}^{V_1} \frac{dV}{gf(V, 0)} + \int_{V=0}^{V_1} \frac{1}{Vf(V, 0)}\frac{dh}{dV}dV$$

$$= t_{f,1} + t_{f,2}.$$

$t_{f,1}$ is independent of the choice of $h(V)$ and represents the time taken in a horizontal transition. We have to choose $h(V)$ to minimize

$$t_{f,2} = \int_{V=0}^{V_1} \frac{1}{Vf(V, 0)}\frac{dh}{dV}dV.$$

Again, introducing normalised co-ordinates (ξ, η) we may conclude that

$$t_f = t_{f,1} + t_{f,2}$$

where

$$t_{f,1} = \frac{V_1}{g}\int_{\xi=0}^{1} \frac{1}{f(\xi, 0)}d\xi = \frac{V_1}{g}c_1$$

and

$$t_{f,2} = \frac{h_1 g}{V_1}\int_{\xi=0}^{1} \frac{1}{\xi g f(\xi, 0)}\frac{d\eta}{d\xi}d\xi = \frac{h_1 g}{V_1}c_2(C).$$

Here c_2 is a function of $d\eta/d\xi$, defining C.

Consequently, solving for h_1,

$$h_1 = \frac{V_1}{gc_2(C)}\left(t_f - \frac{V_1}{g}c_1\right)$$

or

$$h = \frac{V}{gc_2(C)}t_f - \frac{V^2}{g}\frac{c_1}{c_2(C)},$$

upon dropping the subscripts on h and V.

Thus exactly the same conclusion can be made for minimum time optimisation as was made for distance and fuel, namely that for a specific deceleration function and specific flight path shape there is a simple linear relationship between the time taken and the height lost.

4.2. Optimal Shape of Transition Path

Having established in Section 4.1.1 expressions for the fuel-used for any specified shape of path we can now easily determine the path having the smallest fuel cost. In Figs. 6(a) and 6(b) typical thrust (T/W) and deceleration $(f(\xi, 0))$ programs are shown for illustration. These are for the cases $\phi = 15$ degrees combined with incidences of 4 degrees and 8 degrees. Concentrating attention for the present on the $\alpha = 4$ degrees results, in Fig. 6(c) is shown $(T/W)/gf(\xi, 0)$. This function when multiplied by $1/\xi$ (Fig. 6(d)) gives finally the function $(T/W)/\xi gf(\xi, 0)$ shown in Fig. 6(e).

Now to evaluate $b_2(C)$ this must be multiplied by $d\eta/d\xi$, some prescribed function of ξ, (Fig. 6(f) and (g)), and the result integrated over the range (0, 1). In other words, the integral is a weighted average of $(T/W)/\xi gf(\xi, 0)$, the weighting factor being $d\eta/d\xi$.

If one took as a typical choice of C the arc shown in Fig. 6(f), then $d\eta/d\xi$ is a triangular shape having a mean value of 1·0 (as it must). The value obtained for $b_2(C)$ is then relatively large, being around 0·18, because of the large value of the integrand in the range $0 \leqslant \xi \leqslant 0{\cdot}5$. Now if instead one seeks the smallest possible value of $b_2(C)$ this is achieved by taking $d\eta/d\xi$ to be a delta function at $\xi = 1$ (i.e.) $V = V_1$. The integral is then

$$b_2(C) = \left.\frac{T/W}{\xi gf(\xi, 0)}\right|_{\xi=1}$$

The delta function in $d\eta/d\xi$ corresponds to a step in the function η against ξ and hence $d\eta/d\xi = \delta(\xi - 1)$ means that the aircraft loses all its height at constant speed V_1 and thereafter decelerates at constant (zero) height. Now although this is the mathematical optimum it is not realistic in that it involves a steep descent at high speed immediately after lighting the lift engines. What is more feasible is the 'stepped approach', in which the aircraft decelerates horizontally to a satisfactory intermediate speed, V^* say, at which speed the aircraft loses height, and the manoeuvre is completed by a level decelerating transition from $V = V^*$ to zero.

Thus the 'practical optimum' scheme mathematically implies a delta function in $d\eta/d\xi$ at $\xi = \xi^*$ (or a step in the η v. ξ profile at $\xi = \xi^*$). Now delaying the let-down inevitably involves a fuel penalty as may be inferred from Fig. 6(e). However, this figure also defines the trade-off between the fuel penalty and the increased safety as measured by the smaller value of ξ^*. Supposing for illustration that $\xi^* = 0{\cdot}5$ is accepted as a safe let-down speed, then since

$$b_2(C) = \left.\frac{T/W}{\xi gf(\xi, 0)}\right|_{\xi=\xi^*}$$

one obtains the value $b_2(C) = 0{\cdot}10$ for $\xi^* = 0{\cdot}5$ as compared with the value $b_2(C) = 0{\cdot}02$ for $\xi^* = 1{\cdot}0$.

Consider now the alternative control program $\alpha = 8$ degrees. Taking into account the minimum thrust constraint, we see from Fig. 6(a) that $V = 225$ fps is the highest speed at which $T/W \geqslant 0{\cdot}3$ so that 225 fps is the maximum let-down speed for this case. Thus $\xi^* = 0{\cdot}82$ and $b_2(C)$ is calculated to be 0·030. However, a practical scheme with $\xi^* = 0{\cdot}5$ as before gives a slightly lower value of $b_2(C)$ than is given by the $\alpha = 4$ degree program.

It is apparent upon inspection that since T/W is very weakly dependent upon both α and ϕ in the region of $\xi^* = 0{\cdot}5$, the smallest value of $b_2(C)$ at this speed is obtained by selecting a control program which gives a maximum value of $f(\xi, 0)$ at ξ^*. In that case, bearing in mind the overall limitation on deceleration with regard to passenger comfort, it appears that for this aircraft a constant incidence of 8 degrees would be very suitable.

4.3. Fuel Costs

For the various shapes of transition path considered and the data of Fig. 6(a) and (b), the resulting fuel costs were evaluated and are shown in Fig. 7. As was shown in Section 4.1.1 all the curves are linear in h. Considering the results in Fig. 7(a) for the 4 degree incidence case it is apparent that the true optimum implies only a small increase in fuel usage with height lost. The other cases, namely the constant-γ, the 'practical optimum' and the 'typical C', all show significant increases in fuel costs with height. The 'practical optimum' is better than the other two but the constant-γ approach is not so very much worse. In all cases a fuel cost of less than 1 per cent of AUW is incurred for a transition from 2000 ft and a speed of 163 knots.

The results for 8 degree incidence, shown in Fig. 7(b), are in all cases superior to those for 4 degree incidence. The optimum in this case corresponds to zero lift-engine thrust during a let-down at a speed just less than V_1. The only other point of interest is that the constant-γ approach in this case is superior to the stepped transition labelled 'practical optimum'. However, there is clearly some other value of $\xi^* > 0{\cdot}5$ for which a stepped transition would again be superior.

The overall conclusion from this analysis of minimum fuel transitions is that amongst the best feasible shapes of path the fuel usage is not over-sensitive to path shape. On the other hand, by judicious selection of path shape, worthwhile reductions in fuel costs can be achieved without compromising safety and subjecting the passengers to unduly high decelerations. Clearly there is no unique optimum because it all depends upon the weight one attaches to the conflicting requirements. However, the approach formulated here does allow the designer to understand more clearly the implications of his decisions.

It would appear that the use of high incidence is beneficial, both from the point of view of fuel minimisation and passenger comfort. In that case there may be little to choose between a stepped transition with $\xi^* = 0{\cdot}5$ and a constant-γ approach.

Recently there has been a great deal of discussion regarding the noise associated with the operation of VTOL aircraft into urban sites. In an attempt to alleviate the nuisance, a proposal has been made that aircraft could be required to perform the transition horizontally at altitudes between 1000 ft and 2000 ft and then to perform a long vertical descent. The fuel costs for such a manoeuvre from a height of 1000 ft are presented in Fig. 8(a). The two parameters involved in the vertical phase are the acceleration/deceleration parameter Δng and the maximum rate of descent. For the sake of comparison, take $V^* = 137$ fps in a stepped manoeuvre so that the maximum rate of descent is of the order of 40 fps. If one compares the fuel used in this manoeuvre with the fuel used in a vertical descent at 40 fps (with $\Delta n = 0{\cdot}2$ g), then it appears that the latter is of the order of 60 per cent more expensive in fuel. If a descent from 2000 ft were contemplated then the long vertical descent would appear to be a particularly uneconomic proposition.

4.4. Transition Time

There is not very much which needs to be written regarding the results for time minimization apart from the fact that, as in the minimum fuel problem, the let down speed V^* should be as high as possible compatible with safety requirements. Minimum time problems have been studied in the past by Hacker,[2] Brüning[3] and others but the main motive has really been the conservation of fuel and that has been studied explicitly here.

However, it is perhaps worth drawing attention to the simplicity of the present approach which contrasts rather markedly with the complexity of the analysis using a straight application of the Maximum Principle.

5. The Effect of Lift-Loss

For all the results presented so far it was assumed that lift-loss due to interference between the lifting-jet and the free stream could be ignored. The magnitude of the lift-loss can vary considerably with aircraft configuration. To assess its significance in the present study the lift-loss function marked 'basic curve' in Fig. 9 was assumed. This shows the lift-loss parameter $\Delta L/T$ as a parabolic function of $\rho V^2/T$. The shape of the curve for $\rho V^2/T > 0{\cdot}0076$ was unknown so it was arbitrarily assumed that $\Delta L = 0$ in that region.

The resulting deceleration and thrust functions are shown in Fig. 10 for $\phi = 15$ degrees and several incidences. Results for the aircraft with no lift-loss are included for comparison.

The first noteworthy observation is the magnitude of the effect. At a given speed the aerodynamic lift is reduced, requiring more thrust to compensate and leading to increased deceleration.

The second point to note concerns the discontinuities in all the functions (except those for $\alpha = 0$ degrees). These were carefully investigated and shown to be real—assuming the lift-loss data to be correct. The explanation of the effect is given in the Appendix. It could have important implications from the handling point of view and if the form and magnitude of the lift-loss function were substantiated this would require further investigation.

It is possible, however, that the lift-loss expression used gives too large a value for the lift-loss for some configurations. The calculations were therefore repeated with the lift-loss function unchanged in shape but, with the ordinate $\Delta L/T$ halved in magnitude (the 'revised curve' in Fig. 10). The resulting thrust and deceleration functions still showed discontinuities but they were very much reduced in size and it is probably safe to conclude that a lift-loss function of this revised magnitude would probably not provide a significant handling problem.

Whatever the situation is on that score, from the performance point of view, with which we are concerned here, the inclusion of lift-loss makes no qualitative difference to the results. This may be verified by referring to Fig. 11 for there it may be seen that transition distances, time and fuel used for constant-γ approaches all follow the pattern with which we are now familiar. Since for a given setting of the control (α and ϕ) the deceleration function is increased by lift-loss there is a reduction in distance, time and fuel. However, the main point is that whatever the details of the assumed lift-loss expression, the form of the results must remain unchanged and probably most of the qualitative judgements will still apply.

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LIST OF SYMBOLS

a	ft/sec^2	Longitudinal acceleration felt by passengers
C_L		Lift coefficient
C_D		Drag coefficient
D	lb	Drag (including momentum drag)
F	lb	Fuel used
F'	sec	$\frac{36}{\text{s.f.c.}}\left(\frac{F}{W}\right)$
$f(V, \gamma)$	g units	Deceleration function
h	ft	Height
h_1		Initial height
k	(ft/sec)$^{-1}$	Mass flow constant
m	slug	Aircraft mass
m_e	lb/ft/sec	Total mass flow
m_0	lb/ft/sec	Mass flow at zero thrust
S	ft^2	Wing area
s	ft	Distance along flight path
s.f.c.	(lb/lb thrust/hour)	Specific fuel consumption
T	lb	Lift-engine thrust
T_p	lb	Propulsive thrust
t	sec	Time
t_f	sec	Final time of transition
V	ft/sec	Speed
V_1	ft/sec	Initial speed
V^*	ft/sec	Optimal let-down speed
W	lb	Aircraft weight
x	ft^{-2}	$\left.\begin{array}{l}\rho V^2/T\\ \Delta L/T\end{array}\right\}$ parameters in lift loss function
y		
α		Incidence
γ		Flight path angle, positive in climb
Γ		$-\gamma$
ε		Seat tilt angle
ϕ		Thrust vector angle, relative to normal to datum, positive in decelerating sense
ρ	slugs/ft^2	Air density
ξ		V/V_1
η		h/h_1

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APPENDIX

Detailed Analysis of Lift-Loss

When computing the deceleration and thrust functions shown in Fig. 10 for aircraft with lift-loss, difficulty was at first experienced for certain values of speed, in getting the Newton–Raphson process to converge. Further, it appeared in some cases there were two possible solutions for a given speed.

Upon reflection it became apparent that if one imagined progressively decreasing the speed, until $\rho V^2/T$ was just reached (point A in Fig. 9), then any further decrement in speed would lead to an equilibrium point not near A but much nearer to the minimum point B.

For a clearer appreciation of this it is better to replace the normalised (basic) lift-loss plot of Fig. 9 by the set of curves, shown in Fig. 12(a), of $\Delta L/W$ against $\rho V^2/W$ for several T/W ratios. Then for any given speed, $\Delta L/W$ against T/W is obtained as in Fig. 12(b).

Consider the aircraft trimmed at a given speed with no lift-loss (NLL) so that

$$0 = \left(\frac{T}{W}\right)_{NLL} \cos(\alpha + \phi) + \frac{L}{W} - 1.$$

Now suppose that the lift-loss arises as if imposed by some external agency and adjust T/W to retrim. Then

$$\frac{\Delta L}{W} = \left[\left(\frac{T}{W}\right)_{NLL} + \frac{\Delta T}{W}\right] \cos(\alpha + \phi) + \frac{L}{W} - 1,$$

so that

$$\frac{\Delta L}{W} = \frac{\Delta T}{W} \cos(\alpha + \phi).$$

We know $\Delta L/W$ as a function of T/W, shown graphically in Fig. 12(b), which we may denote by $l(T/W)$. So we have to solve the equation

$$l(T/W) = \Delta T/W \cos(\alpha + \phi)$$

for $\Delta T/W$.

In Fig. 12(b) we draw the line

$$\frac{\Delta L}{W} = \left[\frac{T}{W} - \left(\frac{T}{W}\right)_{NLL}\right] \cos(\alpha + \phi)$$

through the point $(T/W)_{NLL}$. This line will intersect the curve at the point D, at which point the lift-loss is compensated for by the increase in thrust. Hence the point D is the trim condition with lift-loss. Now because of the curvature of the function $l(T/W)$, the point C is unstable in the sense that a slight reduction in speed leading to an increase in $(T/W)_{NLL}$ means that the trim condition jumps from the region of C to the region of D.

If the lift-loss formulation is correct in this region of parameter space, there are clearly some conditions of $(T/W)_{NLL}$ which could lead to two solutions of the problem and the discontinuities in thrust. It is entirely possible of course that the formulation of lift-loss is not a good approximation in this region in which case the effects discussed here could be spurious. Also if account is taken of the additional constraint that minimum usable thrust/weight ratio is c. 0·30, then it appears that the constraint operates at just the sort of speed at which this lift-loss feature appears. So, the lighting of the jet-lift engines and the setting up of a trim condition must be combined with the compensation for the lift-loss function and the latter will not then appear as a separate problem to the pilot.

Lastly, the fact that this feature occurs at such low thrust levels implies that there is a paucity of data in this region and one can have correspondingly less confidence in the form of the lift-loss function in this area. However, wind-tunnel research into the magnitude of such lift interference effects is being done within Industry and the Research Establishments and re-examination of the problem with improved data will be feasible at a later date.

TABLE 1

Aircraft Constants

mass	m slugs	3110
weight	W lb	100 000
wing area	S ft^2	1000
wing loading	W/S lb/ft	100
propulsive thrust	T_p lb	1000
engine mass flow constants	$\begin{cases} m_0 \text{ slugs/sec} \\ k \text{ slugs/sec/lb} \end{cases}$	74·60 10^{-3}

Aerodynamic coefficients

$$C_L = 0{\cdot}5 + 4{\cdot}5\,\alpha^c$$

$$C_D = 0{\cdot}075 + 0{\cdot}0763\,C_L^2.$$

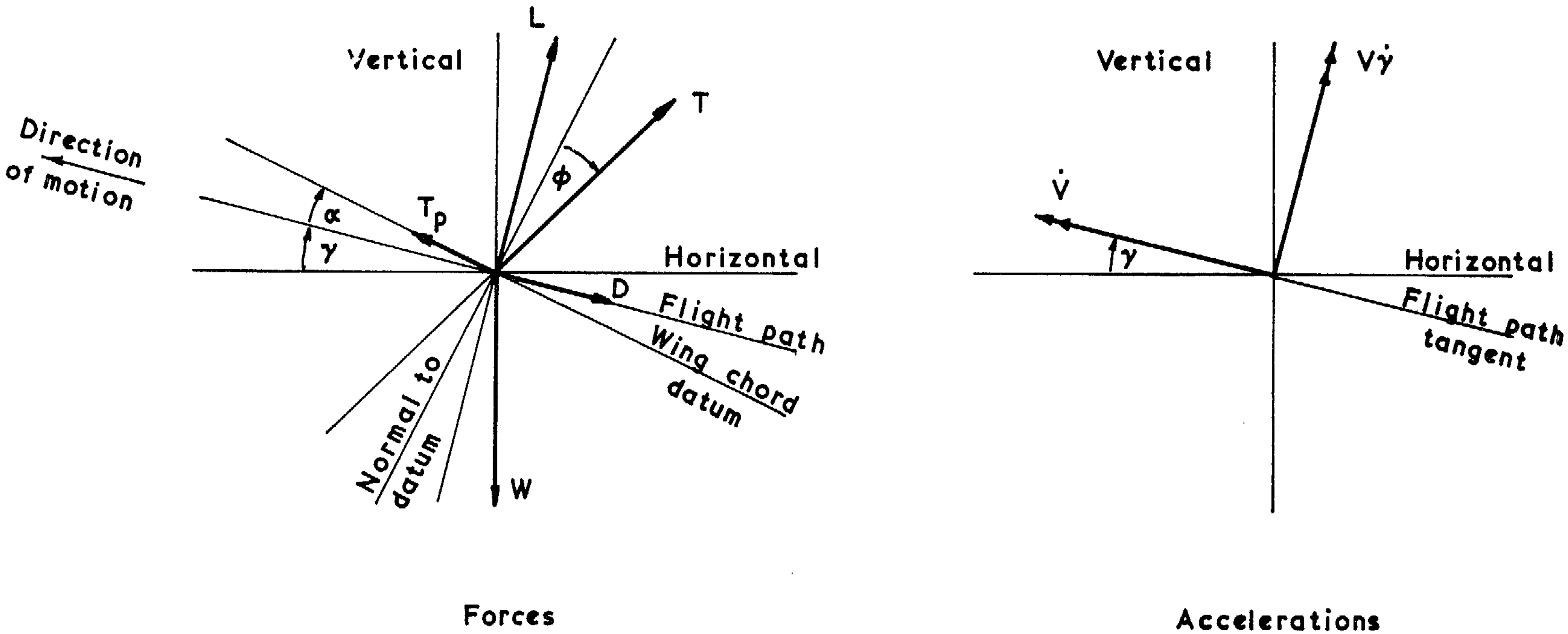


FIG. 1. Force-acceleration diagrams.

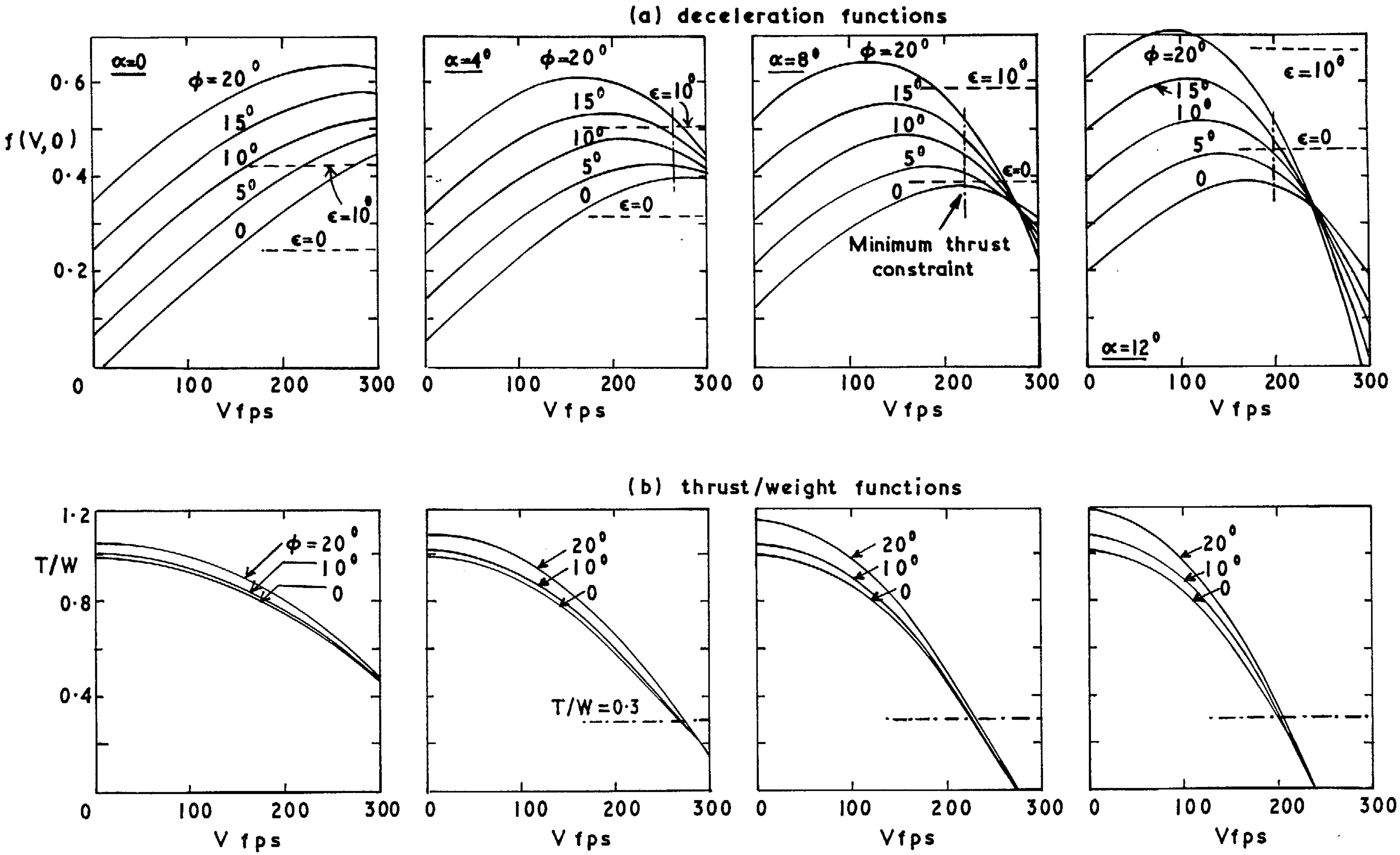
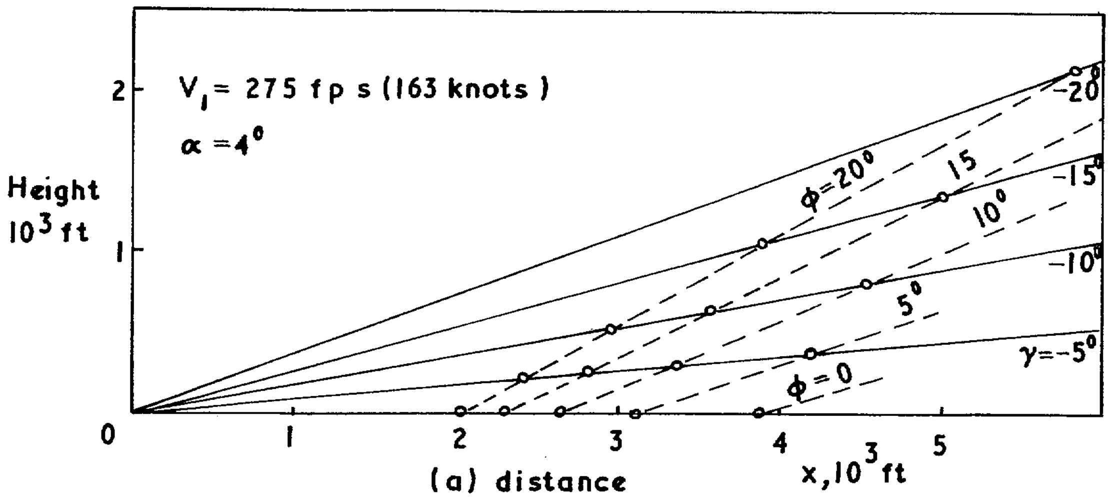
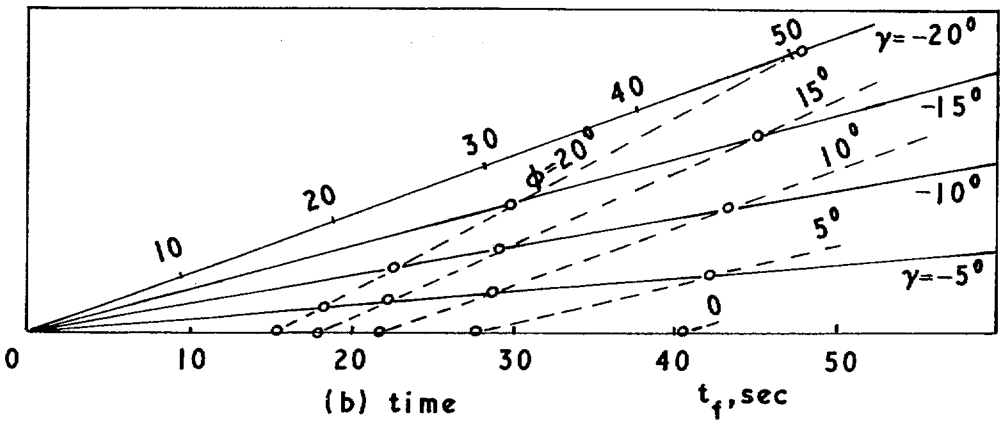


FIG. 2. Deceleration and thrust/weight functions for level flight; no lift-loss.





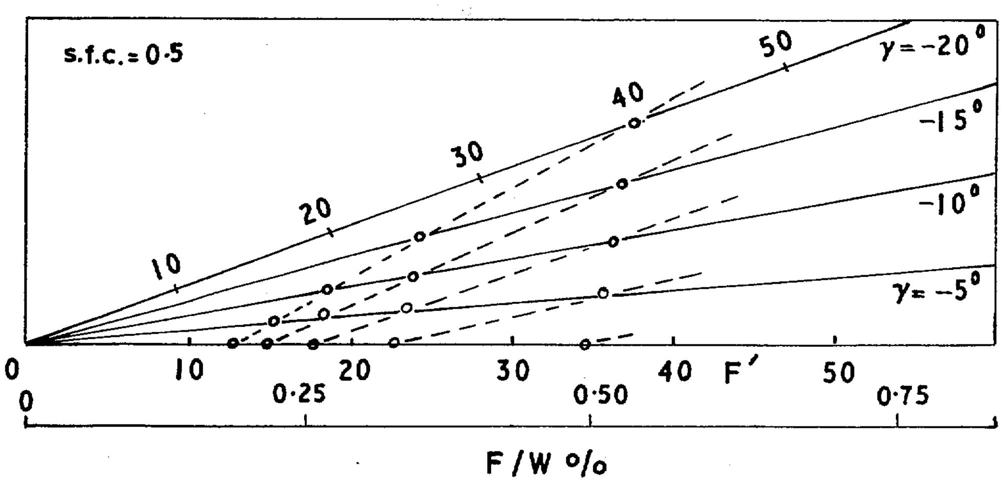


FIG. 3. Transition distance, time and fuel for constant-γ approaches, various thrust-vector angles. (No passenger acceleration constraint.)

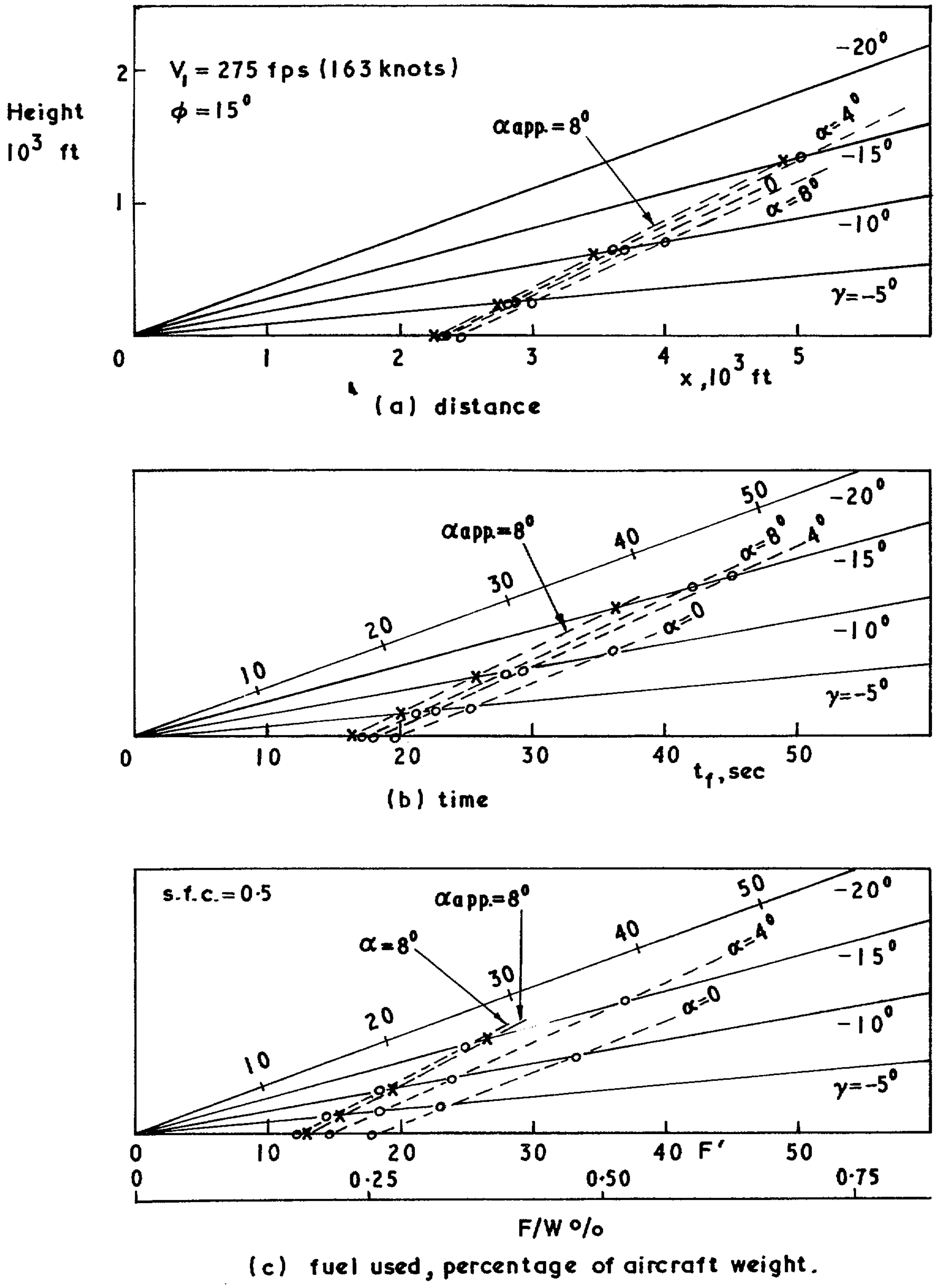


FIG. 4. Transition distance, time and fuel for constant-γ approaches, various aircraft incidences (no passenger acceleration constraint).

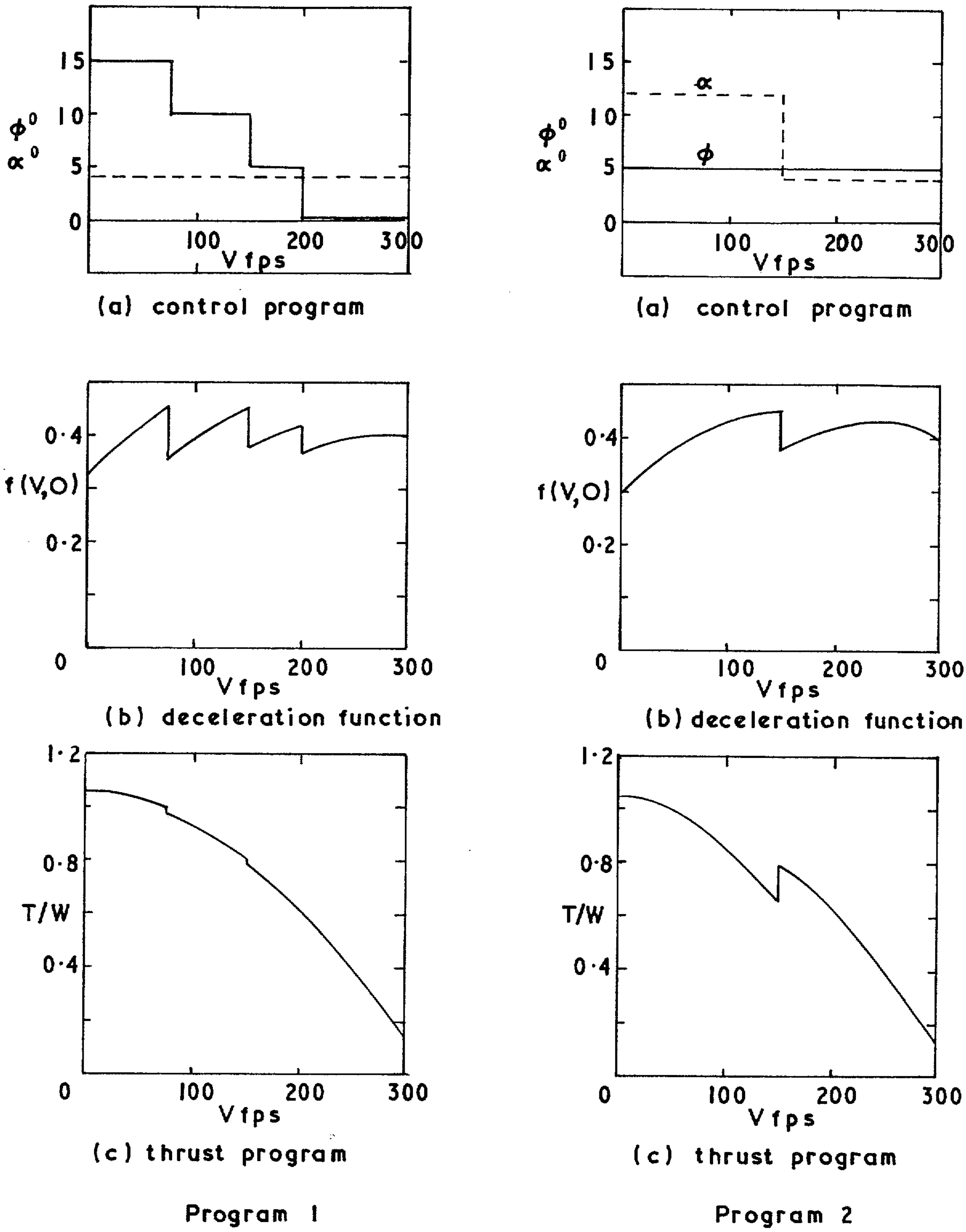


FIG. 5. Feasible control programs for $f(V,0) \doteqdot 0{\cdot}4$.

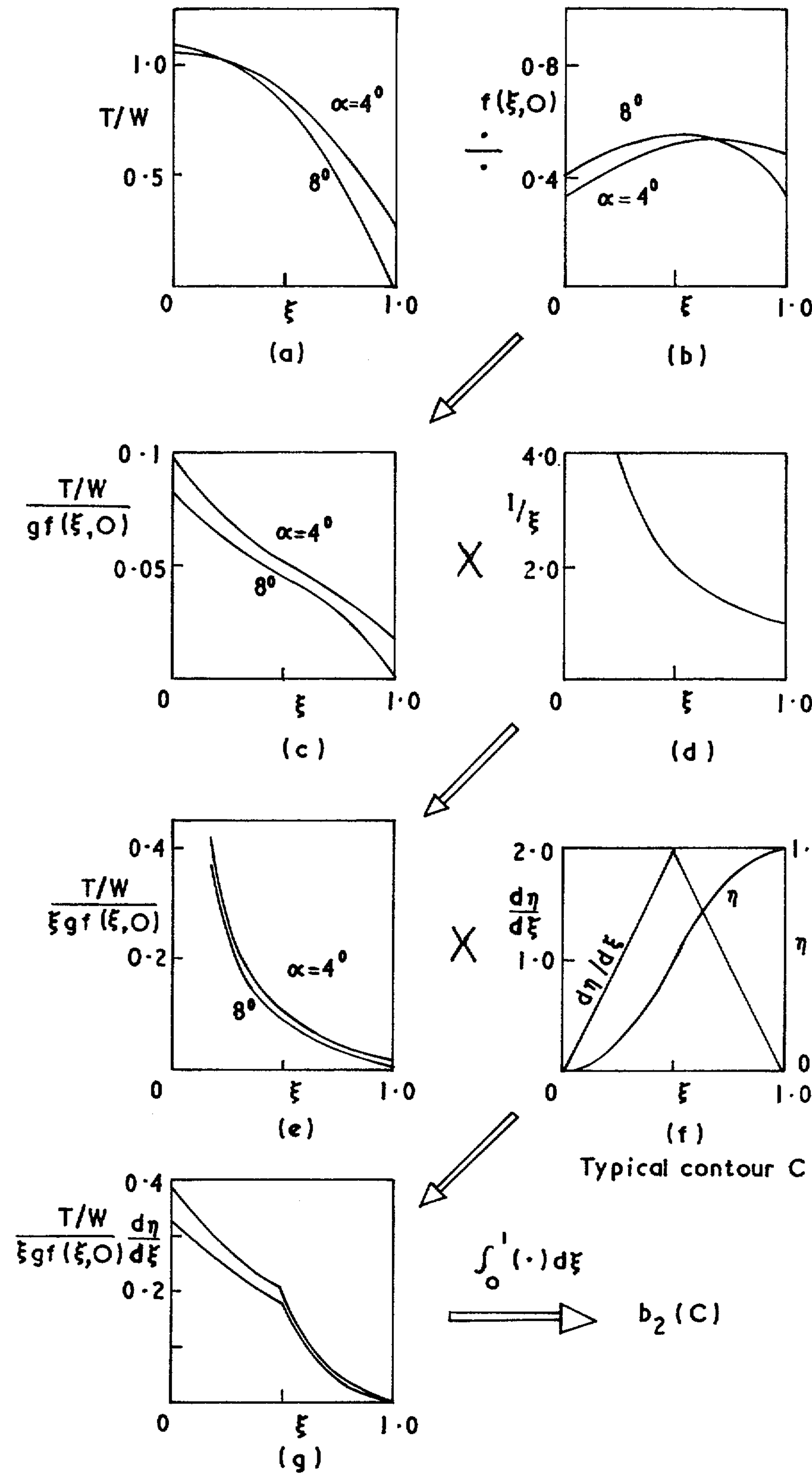


FIG. 6. Schema for the evaluation of $b_2(C)$ for minimum fuel transition paths.

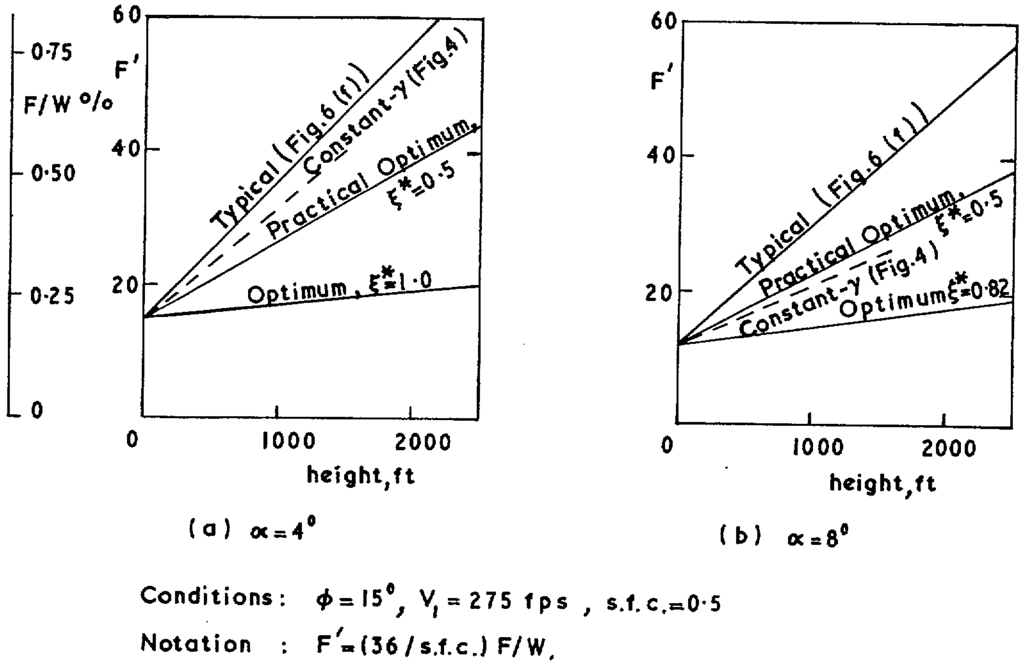


FIG. 7. Fuel costs as a function of height lost, for various shapes of transition path.

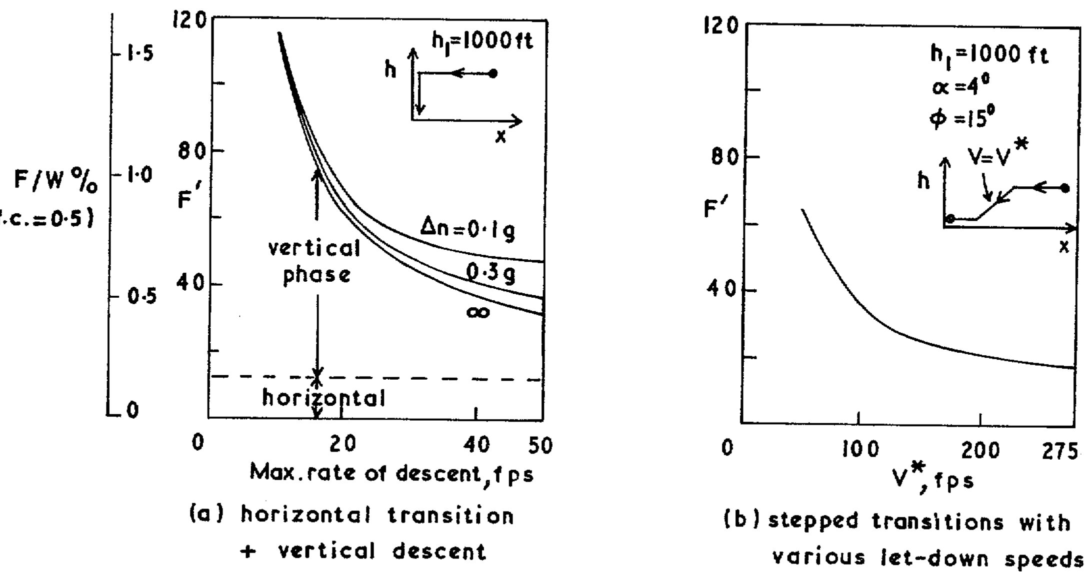


FIG. 8. Comparative fuel costs for stepped transition paths and long vertical descents.

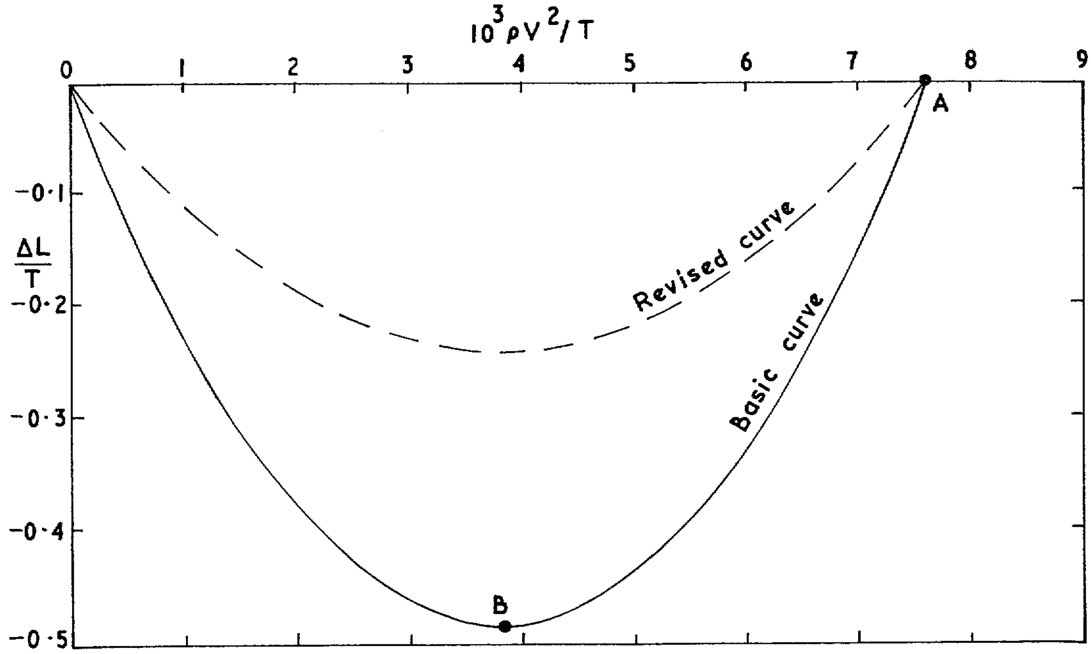


FIG. 9. The lift-loss as a function of thrust and speed.

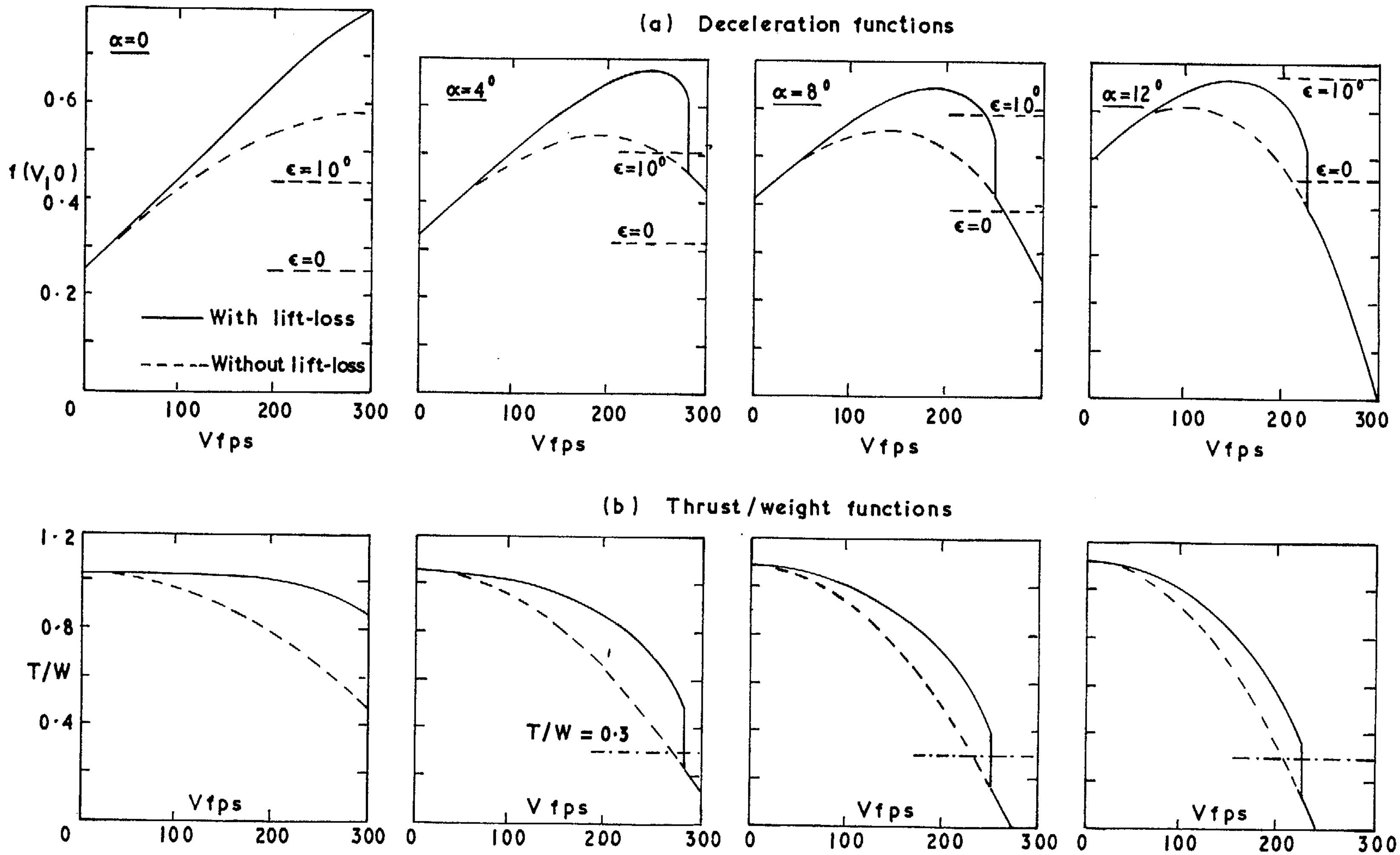


FIG. 10. The effect of lift-loss on deceleration and thrust/weight functions for level flight; $\phi = 15$ degrees

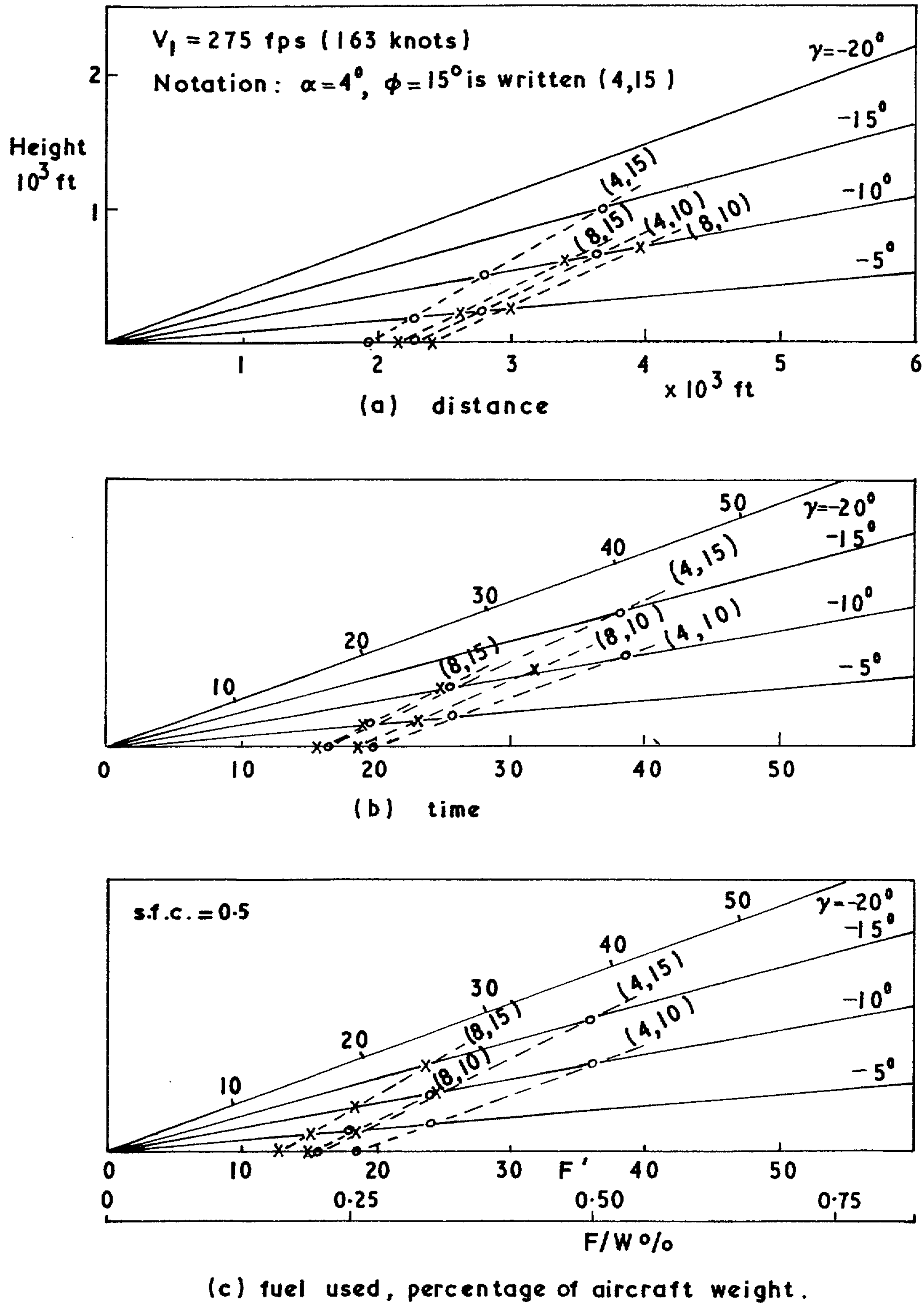


FIG. 11. Transition distance, time and fuel for constant-γ approaches, including the effect of lift-loss.

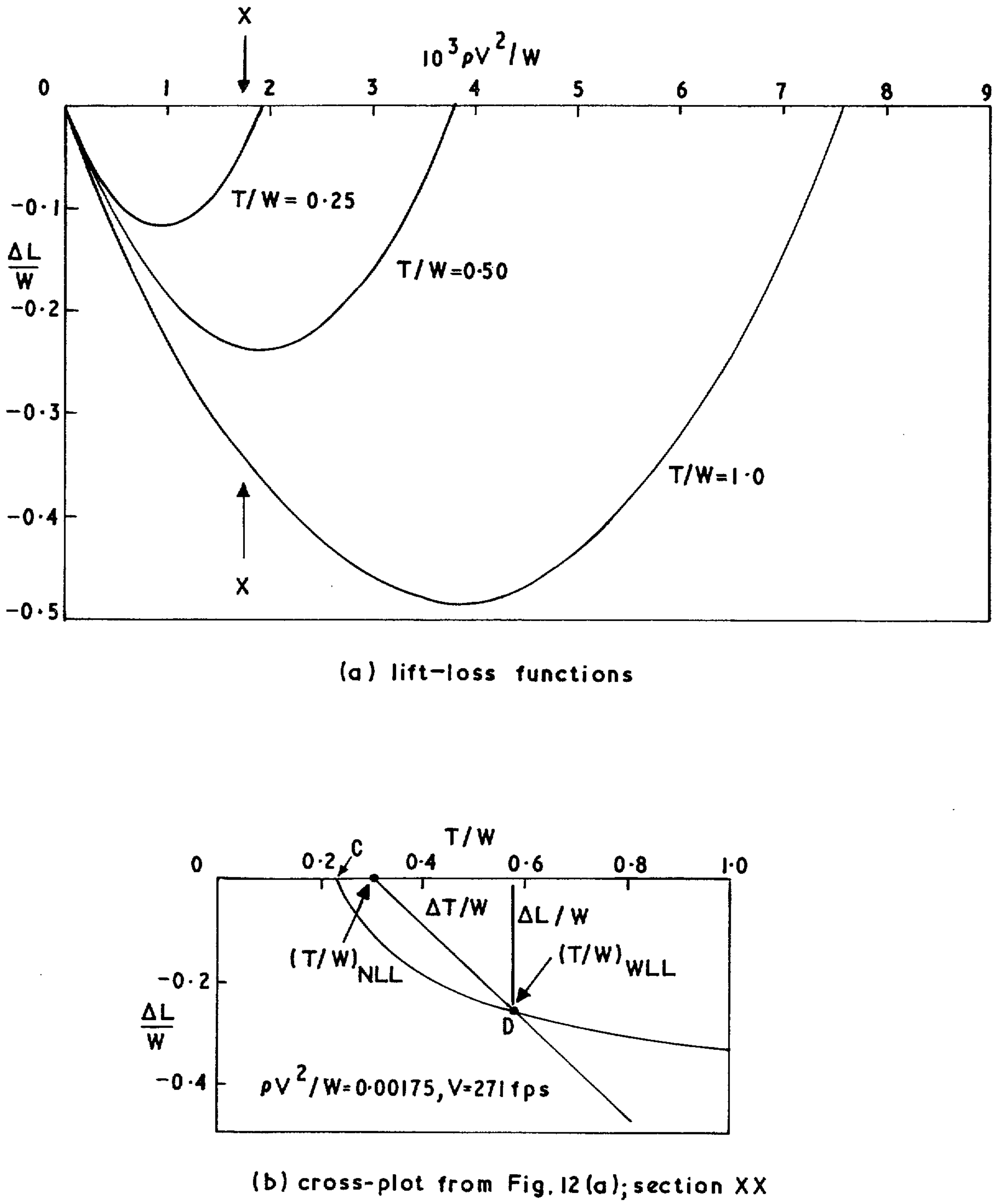


FIG. 12. Analysis of step behaviour in T/W at high speed due to lift-loss.

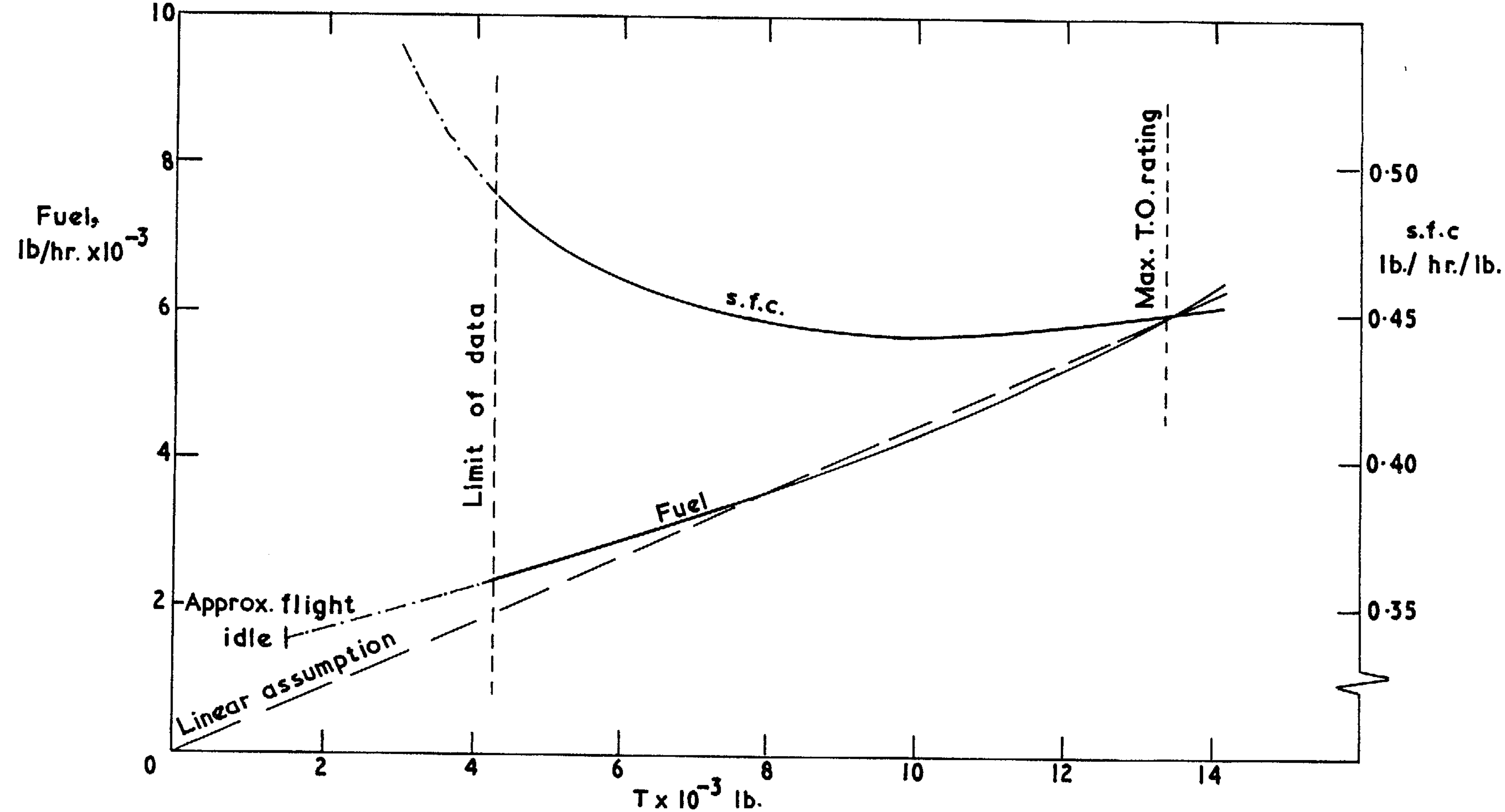


Fig. 13. Data for a typical lifting engine.

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