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A Method of Correlating the Ground Effects on the Longitudinal Characteristics of Slender Wings

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Summary.

This Report describes a method of calculating the ground effect on the longitudinal characteristics of a slender wing when the span/height ratio is small. The results obtained are compared with those obtained from Gersten's method, valid when the span/height ratio is large. From these two sets of results is derived a method of correlating the measured ground effects on the normal force and pitching moment on various slender wings, and hence a means of estimating the ground effect on any slender wing at any height above the ground.

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1. *Introduction.*

Several years ago, during research associated with the development of slender-wing aircraft, wind-tunnel tests indicated that the characteristics of such aircraft close to the ground would be significantly different from their characteristics in free air (*see* Fig. 1); and in particular that the effect of ground proximity could increase their lift by more than 50 per cent[1] at touchdown. The size of this ground effect and of the associated effects on the aircraft's stability emphasized the need for more information about the dependence of the ground effect on the height, shape and incidence of the aircraft.

A considerable amount of experimental work has been done to discover the ground effect on different slender wings and wing-body combinations. Notable contributions have been made by Kirby and Kettle[2], de Sievers[3], Gersten and van der Decken[4], Kirkpatrick[5] and Smith[6] using a variety of models and test techniques. During the same period studies of the different methods used to simulate ground proximity in wind tunnels have shown that for low-aspect-ratio wings the method used had no appreciable influence on the measured results[3,7] and that wind-tunnel results were in close agreement with those obtained from aircraft flying close to the ground at constant height and incidence[8]. These experimental results yielded considerable information about the ground effect phenomena but, in the absence of a ground-effect theory, the dependence of these phenomena on the height, planform and incidence of the wings tested could not satisfactorily be correlated, though some empirical correlations of the lift increment induced by ground proximity have been attempted[1,3].

An endeavour to formulate a ground-effect theory and provide a basis for the analysis of wind-tunnel results was presented in 1965 by Gersten and van der Decken[4]. They considered the case of a plane slender wing at small incidence very close to the ground so that the flow field round the wing and its image in the ground plane could be assumed to be two-dimensional (*see* Fig. 2a), and solved the two-dimensional potential equation in a plane perpendicular to the free stream so that the solution satisfied the appropriate boundary conditions at the spanwise cross-sections of the wing and its image. They then calculated the ground effect on the local normal force coefficient as a function of the local span/height ratio and hence predicted the ground effect on the normal force and pitching moment of slender wings. But this theory is only applicable to wings so close to the ground that the local span/height ratio is everywhere large and at such small incidences that the effect of the leading-edge vortices is negligible. Consequently it is not surprising that at the incidences typical of the approach and landing of a slender aircraft there are considerable discrepancies between the ground effects predicted by their theory and those obtained by experiment.

This present Report presents in Section 2 a theory which can be used to calculate the ground effect on a slender wing at small incidence not very close to the ground so that the span/height ratio is small. In this case the ground effect on the flow field round a slender wing was assumed to be similar to the effect of doublets distributed along the axis of the wing's image from its apex rearwards to infinity (*see* Fig. 2b); these doublets were directed upwards perpendicular to the plane of the image and their strength and distribution were determined from the aerodynamic loading distribution on the wing. Some of the results of this method of calculation are presented and discussed.

It is then demonstrated in Section 3 that, although Gersten's theory[4] and the present theory are valid only when the span/height ratio is very large and very small respectively, the results of these two theories may be used to define the limits of a curve correlating the measured ground effects on the normal force on various slender wings at low incidence and at various heights. The curve may be used to correlate, with reasonable accuracy, the measured ground effects on the pitching-moment characteristics of these wings. This successful correlation at low incidence suggests a method of correlating the ground effects on the longitudinal characteristics of slender wings at incidences appropriate to the take-off and landing of slender aircraft. The correlating curves may be used to estimate the ground effect on any slender wing at any incidence and any height above the ground.

2. *Calculation of the Ground Effect on a Slender Wing.*

The flow field round a slender wing flying close to the ground at subsonic speed and at high incidence, e.g. during take-off or landing, is extremely complex and difficult to calculate. In 1966 Smith[9] presented a method of calculating the conical flow field round a plane delta wing but there is little hope of an early extension of this method to the case of subsonic non-conical flow and still less of the further extension needed to calculate the effects of ground proximity.

It is nevertheless useful to develop methods of calculating the ground effect on slender wings in certain special cases where simplifying assumptions can be made, e.g. the ground effect on wings at small incidence very close to the ground has been calculated by Gersten[4]. The present Report describes a method of calculating the ground effects on slender wings at small incidence *not* very close to the ground. In this method it is assumed that the incidence of the wing is so small that the effects of the leading-edge vortices and of the chordwise variation of its height may be neglected, and that its height is so large that the span/height ratio is small. It should be noted that these assumptions place no restrictions on the chord/height ratio. It is further assumed that the velocity increment induced at the wing by the proximity of the ground is approximately vertical and is similar to that induced by a distribution of doublets along the axis of the wing's image in the ground plane, from its apex rearwards to infinity (*see* Fig. 2b). These doublets are directed upwards and their strength μ/unit length is obtained from the aerodynamic loading distribution on the wing using the equation

$$\mu(X) = 2\int_0^s y\,\gamma(X, Y)\,dY\,,$$

where γ is the x-component of vorticity in the wing surface or the trailing vortex sheet and s is the local semi-span of the wing. In accordance with the slender-wing theory of R. T. Jones[10], the vorticity distribution is assumed to be the same as that on a flat plate of breadth $2s$ normal to a two-dimensional flow having the velocity $U\alpha$, i.e.

$$\gamma(X, Y) = 2U\,\alpha\,Y/\sqrt{s^2 - Y^2}$$

therefore

$$\mu(X) = \pi\,U\,\alpha\,s^2 = U\,N(X)/2q\,,$$

where $N(X)$ is the normal force on the part of the wing forward of station X. The upwash induced at a point (X, Y) on the surface of a slender wing at height H above the ground by doublets of strength $\mu(X')$ distributed along the axis of the wing's image at a distance H below the ground is calculated from the expression

$$w(X, Y) = \int_0^\infty \frac{\mu(X')}{4\pi}\,\frac{8H^2-(X-X')^2-Y^2}{[(X-X')^2+Y^2+4H^2]^{5/2}}\,dX'\,,$$

which is derived from the standard expression for the velocity potential of a doublet. Since the span/height ratio is small the spanwise variation $\frac{\partial w}{\partial Y}$ of the upwash is negligible and the local increase of incidence $\Delta\alpha(X)$ due to ground effect is

$$\Delta\alpha(X) = \int_0^\infty \frac{N(X)}{8\pi\,q}\,\frac{8H^2-(X-X')^2}{[(X-X')^2+4H^2]^{5/2}}\,dX'$$

$$= \int_0^\infty \frac{N(x)}{8\pi\,q\,c^2}\,\frac{8h^2-(x-x')^2}{[(x-x')^2+4h^2]^{5/2}}\,dx'$$

where $h = H/c$, $x = X/c$ and $x' = X'/c$. Because the wing is not very close to the ground and the ground effect on it is therefore small, the aerodynamic loading distribution on the wing and its image may be assumed to be negligibly different from the loading on the wing in free air. Hence

$$\Delta\alpha(x) = \frac{N(1)}{8\pi q c^2} \int_0^\infty \left[\frac{N(x')}{N(1)}\right]_\infty \frac{8h^2-(x-x')^2}{[(x-x')^2+4h^2]^{5/2}}\, dx'$$

$$= \frac{2C_N}{\pi A} \frac{1}{16}\left(\frac{b}{c}\right)^2 \int_0^\infty \left[\frac{N(x')}{N(1)}\right]_\infty \frac{8h^2-(x-x')^2}{[(x-x')^2+4h^2]^{5/2}}\, dx'$$

where the subscript ∞ denotes values taken infinitely far from the ground. Assuming that the normal force on a slender wing in cambered flow is proportional to the incidence at the trailing edge, as predicted by slender-body theory, then the normal force increment induced by the presence of the ground is

$$\Delta C_N = \frac{dC_N}{d\alpha}\Delta\alpha(1)$$

therefore

$$\frac{\Delta C_N}{C_N} = \frac{dC_N/d\alpha}{\pi A/2} \frac{1}{16}\left(\frac{b}{c}\right)^2 \int_0^\infty \left[\frac{N(x')}{N(1)}\right]_\infty \frac{8h^2-(1-x')^2}{[(1-x')^2+4h^2]^{5/2}}\, dx' .$$

If $(x'-1)/2h = \tan\theta$, then to first order in $\Delta\alpha$

$$\frac{\Delta C_N}{C_{N_\infty}} = \frac{[dC_N/d\alpha]_\infty}{\pi A/2} \frac{1}{16}\left(\frac{b}{c}\right)^2 \int_0^\infty \frac{N(x')}{N(1)} \frac{8h^2-(1-x')^2}{[(1-x)^2+4h^2]^{5/2}}\, dx'$$

$$= \frac{[dC_N/d\alpha]_\infty}{\pi A/2} \frac{1}{64}\left(\frac{b}{H}\right)^2 \left\{ 1 + \int_{\theta_1}^{0} \frac{N(\theta)}{N(0)} (2-\tan^2\theta)\cos^3\theta\, d\theta \right\}$$

where $\theta_1 = \tan^{-1}\left(\frac{-1}{2h}\right)$ Hence, denoting the value of $\frac{2}{\pi A}\left[\frac{dC_N}{d\alpha}\right]_\infty$ by F and the function in the curly brackets as $I(\theta_1)$,

$$\frac{\Delta C_N}{C_{N_\infty}} = \frac{F}{64}\left(\frac{b}{H}\right)^2 I(\theta_1) .$$

The variation of h of the value of $I(\theta_1)$ was calculated for two wings whose aerodynamic loading distributions in free air corresponded with those predicted by slender-body theory for a delta and a gothic wing, i.e. $\frac{N(x')}{N(1)} = x'^2$ and $\frac{N(x')}{N(1)} = (2x'-x'^2)^2$ respectively. The calculated variations were plotted in Fig. 3a which shows that the value of $I(\theta_1)$ is not critically dependent on the form of the loading distribution and justifies retrospectively the earlier assumption that $\frac{N(x')}{N(1)} = \left[\frac{N(x')}{N(1)}\right]_\infty$. Since most slender wings have loading distributions similar to those quoted above and since ground effects on slender wings are insignificant when $1/h < 4$, i.e. when $H > c/4$, it may be assumed that $I(\theta_1) = 2$, irrespective of the loading distribution of the wing considered, and Fig. 3b shows that only negligible errors are introduced by this assumption. Hence

$$\frac{\Delta C_N}{C_{N_\infty}} = \frac{\Delta C_L}{C_{L_\infty}} = \frac{F}{32}\left(\frac{b}{H}\right)^2$$

3. *Correlation of the Ground Effect on Slender Wings.*

3.1. *At. Small Incidence.*

It is interesting to compare the results of the theory presented in the preceding Section with those of Gersten's theory[4]. His theory assumes the validity of slender-body theory, i.e. that $\left[\frac{dC_N}{d\alpha}\right]_\infty = \frac{\pi A}{2}$ so $F = 1$, and predicts that for large values of the span/height ratio the local aerodynamic loading on a slender wing at small incidence is

$$dN(x) = 2q\, s\alpha\left[\, 5{\cdot}2 + 2{\cdot}16\frac{s}{H}\right]\frac{ds}{dx}\, dx$$

so the normal-force coefficient near the ground is

$$C_N = \frac{A\alpha}{2}\int_0^1\left(5{\cdot}2\,\eta + 1{\cdot}08\frac{b}{H}\eta^2\right)d\eta\,.$$

Consequently the change in the normal-force coefficient due to the ground effect is

$$\Delta C_N = \frac{A\alpha}{2}\int_0^1\left(5{\cdot}2\,\eta + 1{\cdot}08\frac{b}{H}\eta^2\right)d\eta - \frac{\pi A\alpha}{2}$$

$$\frac{\Delta C_N}{F\,C_{N_\infty}} = 0{\cdot}115\frac{b}{H} - 0{\cdot}173\,,$$

irrespective of the wing's slenderness or loading distribution. The theory in Section 2 predicts that, for small values of the span/height ratio, the ground effect on a slender wing is virtually independent of its planform and equals

$$\frac{\Delta C_N}{F\,C_{N_\infty}} = \frac{1}{32}\left(\frac{b}{H}\right)^2.$$

Thus each theory predicts that the ground effect is primarily dependent on the span/height ratio, b/H.

The variations with b/H of the ground effects predicted by these two theories were plotted in Fig. 4 and compared with experimental values obtained from wind-tunnel tests on various slender wings (*see* Appendix). It was found that at small values of b/H the experimental results were close to the curve calculated from the theory presented in Section 2, but that as b/H increased the experimental results diverged from this curve and tended to approach the curve calculated from Gersten's theory. To correlate the experimental results a curve, referred to as the interpolated curve, was drawn through the experimental results so that it corresponded with Gersten's curve at large values of b/H and with the curve obtained from the present theory at small values of b/H. Fig. 5 shows that all the experimental results lie very close to this interpolated curve which, for $0 < b/H < 6$, may be approximated by the expression

$$\frac{\Delta C_N}{C_{N_\infty}} = 0{\cdot}045\, F\left(\frac{b}{H}\right)^{1{\cdot}42}.$$

The interpolated curve may be used to correlate the measured ground effect on the position of the centres of pressure of various slender wings at small incidence. For this correlation it was assumed that the ground effect on the normal force $N(x)$ on the part of a slender wing forward of station x is dependent on the span/height ratio at that station, i.e. that

$$\frac{\Delta C_{N(x)}}{C_{N(x)_\infty}} = 0{\cdot}045\, F(x)\left(\frac{2s(x)}{H(x)}\right)^{1{\cdot}42} = G(x)\,.$$

This assumption is similar to that underlying slender-body theory. The chordwise position of the centre of pressure is given by the expression

$$x_{cp} = 1 - \int_0^1 \frac{N(x)}{N(1)}\, dx$$

so the change in its position due to ground effect is, for $0 < b/H < 6$,

$$\Delta x_{cp} = \int_0^1 \left[\frac{N(x)}{N(1)}\right]_\infty dx - \int_0^1 \frac{N(x)}{N(1)}\, dx$$

$$= \int_0^1 \left[\frac{N(x)}{N(1)}\right]_\infty \left\{1 - \frac{1+G(x)}{1+G(1)} \quad dx\right\}$$

$$= \frac{G(1)}{1+G(1)} \int_0^1 \left[\frac{N(x)}{N(1)}\right]_\infty \left\{1 - \frac{G(x)}{G(1)}\right\} dx$$

$$= \frac{G(1)}{1+G(1)} \int_0^1 \frac{F(x)}{F(1)}\left(\frac{s(x)}{b/2}\right)^2 \left\{1 - \frac{F(x)}{F(1)}\left(\frac{s(x)}{b/2}\right)^{1{\cdot}42}\right\} dx\,,$$

assuming that the incidence is so small that $\frac{H(x)}{H(1)} = 1$. This expression shows that the ground effect on the centre of pressure, unlike that on the normal force, is dependent on the planform and loading distribution of the wing considered as well as on its span/height ratio. Although the loading parameter $\frac{F(x)}{F(1)}$ is generally not known accurately, the ground effect on the centre of pressure of a particular wing may be estimated using an approximate expression for $\frac{F(x)}{F(1)}$. Experimental measurements of the stability of slender wings[13], and of the distribution of aerodynamic loading on them[14,15] have indicated that for a gothic wing $\frac{F(x)}{F(1)} \simeq 1$ and that for a delta wing $\frac{dN(x)}{dx}$ is proportional to $(1-x^n)$ so that $\frac{F(x)}{F(1)} = \frac{n+2}{n} - \frac{2x^n}{n}$, where $F(1) = \frac{2}{\pi A}\left[\frac{dC_N}{d\alpha}\right]_\infty = \frac{n}{n+2}$. Using these approximations for $\frac{F(x)}{F(1)}$, some values of Δx_{cp} for gothic and delta wings were calculated and plotted in Fig. 6 which shows that the calculated values are close to the experimental results.

3.2. *At Large Incidence.*

Having thus established a successful method of correlating the effect of ground proximity on the longitudinal characteristics of a slender wing at very small incidence, it is appropriate to investigate the possibility of using this method as a basis for correlating the measured ground effects on slender wings at incidences typical of the take-off and landing of a slender aircraft.

It has been demonstrated that at small incidence

$$\frac{\Delta C_L}{C_{L_\infty}} = \frac{\Delta C_N}{C_{N_\infty}} = 0{\cdot}045\, F\left(\frac{b}{H}\right)^{1.42}.$$

If, following the example of Spence[1], the position of a slender wing relative to the ground is defined by the angle of incidence and the height of the mean-quarter-chord point, then the ground effect may be written as

$$\frac{\Delta C_N}{C_{N_\infty}} = F f\left(\alpha, \frac{b}{H_{\frac{1}{4}}}\right),$$

where f is as yet an unknown function of α and $b/H_{\frac{1}{4}}$ and $F = \frac{2}{\pi A}\left[\frac{dC_N}{d\alpha}\right]_\infty$. Experimental values of $\frac{\Delta C_N}{F\,C_{N_\infty}}$ (*see* Appendix) were plotted against $b/H_{\frac{1}{4}}$ in Fig. 7 which shows that $\frac{\Delta C_N}{F\,C_{N_\infty}}$ is independent of wing incidence, planform and aspect ratio and that

$$\frac{\Delta C_N}{F\,C_{N_\infty}} = 0{\cdot}045\left(\frac{b}{H_{\frac{1}{4}}}\right)^{1{\cdot}42}.$$

Consequently the ground effect on the lift characteristics of any slender wing at any height and incidence may be estimated using the measured free air characteristic and the span/height ratio.

Since $\frac{\Delta C_N}{F\,C_{N_\infty}}$ is independent of incidence, the ground effect on the position of the centre of pressure of a wing at moderate incidence may be estimated by following the same procedure as for small incidence in Section 3.1 and assuming

$$\frac{\Delta C_{N(x)}}{C_{N(x)_\infty}} = 0{\cdot}045\, F(x)\left(\frac{2s(x)}{H_{\frac{1}{4}}(x)}\right)^{1{\cdot}42}$$

where $H_{\frac{1}{4}}(x)$ is the height of the mean-quarter-chord point of the part of the wing forward of station x. Hence, for $0 < b/H < 6$,

$$\Delta x_{cp} = \frac{G(1)}{1+G(1)}\int_0^1 \frac{F(x)}{F(1)}\left(\frac{s(x)}{b/2}\right)^2 \left\{1 - \frac{F(x)}{F(1)}\left(\frac{s(x)}{b/2}\right)^{1{\cdot}42}\left(\frac{H_{\frac{1}{4}}(1)}{H_{\frac{1}{4}}(x)}\right)^{1{\cdot}42}\right\} dx\,.$$

Using the same expression for $\frac{F(x)}{F(1)}$ as in Section 3.1, some values of Δx_{cp} for gothic and delta wings at an incidences of 15 degrees were calculated and compared in Fig. 8 with experimental results. Considering the complexity of the flow field round a slender wing at large incidence near the ground, the agreement between the calculated values and the experimental results is reasonably good.

4. *Concluding Remarks.*

This Report presents a method of calculating the ground effect on the normal force on a slender wing at small incidence not very close to the ground.

The Report also describes how the results obtained from this method and those obtained from an earlier method[4] may be used to define the limits of a simple curve correlating the measured ground effects on the normal force on various slender wings at low incidence and at various heights. The curve may be used to correlate, with reasonable accuracy, the measured ground effects on the centres of pressure of these wings. This successful correlation at low incidence suggests a method of correlating the ground effects on the longitudinal characteristics of slender wings at incidences appropriate to the take-off and landing of slender aircraft. The correlating curves, coupled with the empirical expressions for the small thickness effects discussed in the Appendix, provide a method of estimating the ground effect on any slender wing at any incidence and any height above the ground.

APPENDIX

Effects of Wing Thickness.

Both of the ground-effect theories discussed in this Report are strictly valid only for slender wings of zero thickness. All the wind-tunnel tests to investigate ground effects have been made using wings of small but finite thickness; these tests have shown that the ground effect on a slender wing is affected by its thickness and that this thickness effect yields at zero incidence a negative lift increment and a positive pitching moment increment. It was therefore necessary to calculate from the experimental data the values of parameters which were almost independent of wing thickness and could be compared with the theoretical predictions. The parameters chosen and calculated were

$$\frac{\Delta C_N}{C_{N_\infty}} = \frac{dC_N/d\alpha}{(dC_N/d\alpha)_\infty} - 1$$

$$\Delta x_{cp} = \frac{dC_m}{dC_N} - \left(\frac{dC_m}{dC_N}\right)_\infty$$

at zero incidence, and

$$\frac{\Delta C_N}{C_{N_\infty}} = \frac{C_L - (C_L)_{\alpha=0}}{C_{L_\infty}} - 1$$

$$\Delta x_{cp} = \frac{C_m - (C_m)_{\alpha=0}}{C_N - (C_N)_{\alpha=0}} - \left(\frac{C_m}{C_N}\right)_\infty$$

at large incidence. Since the thickness effects were small and their variation with incidence may be presumed small, it was assumed that these parameters were virtually independent of wing thickness and could be compared with results from ground effect theories (Figs. 5 to 8).

The longitudinal characteristics of a thick slender wing near the ground may be estimated by using the values of $\dfrac{\Delta C_N}{C_{N_\infty}}$ and Δx_{cp}, calculated from the interpolated curve in Fig. 5 by the method described in Section 3, in conjunction with the values of the thickness effect obtained from the approximate empirical expressions.

$$(C_N)_{\alpha=0} = -0{\cdot}025\left(\frac{dC_N}{d\alpha}\right)_\infty \frac{t}{H}$$

$$(C_m)_{\alpha=0} = +0{\cdot}009\left(\frac{dC_N}{d\alpha}\right)_\infty \frac{t}{H}$$

derived from the experimental results plotted in Fig. 9.

LIST OF SYMBOLS

A	Aspect ratio
b	Span
c	Chord
C_L	Lift coefficient
C_m	Pitching-moment coefficient
C_N	Normal-force coefficient
F	$\dfrac{2}{\pi A}\left(\dfrac{dC_N}{d\alpha}\right)_\infty$
G	$\dfrac{\Delta C_N}{C_{N\infty}}$
h	H/c
H	Height
$H_{\frac{1}{4}}$	Height of the mean-quarter-chord point
$I(\theta)$	Ground-effect parameter
$N(x)$	Normal force on section of wing forward of station x
n	Aerodynamic loading distribution parameter
q	Dynamic pressure
s	Local semispan
t	Thickness
u	Free-stream velocity
w	Upwash
X	Chordwise distance measured aft from apex
x	X/c

x_{cp}	Distance from apex to centre of pressure
Y	Spanwise distance measured from centreline
α	Incidence
γ	Vorticity
η	$2s/b$
μ	Doublet strength

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TABLE 1

Measured Ground Effects on Slender Wings at Zero Incidence.

Wing	A	Ref.	b/H	$\frac{dC_N}{d\alpha}$	F	$\frac{\Delta C_N}{F\, C_{N_\infty}}$	$\frac{dC_m}{dC_N}$	Δx_{cp}
Delta	1·62	5	0	1·72	0·68	0	0·606	0
			2·04	1·90		0·154	0·611	0·005
			2·68	1·97		0·214	0·613	0·007
			3·04	2·02		0·250	0·615	0·009
			3·98	2·14		0·356	0·617	0·011
			5·02	2·28		0·480	0·621	0·015
			6·08	2·41		0·589		
Delta	1·0	2	0	1·20	0·765	0	0·648	0
			2·02	1·35		0·162		
			4·44	1·55		0·382	0·664	0·016
Gothic	1·0	11	0	1·35	0·86	0	0·494	0
			3	1·67		0·268	0·511	0·017
			6	2·12		0·663	0·526	0·032
Gothic	0·75	12	0	1·22	0·90	0		0
			3	1·45		0·21		0·014
			4·4	1·68		0·41		0·023
			9	2·22		0·90		0·032
Mild Gothic	1·36	2	0	1·66	0·78	0	0·573	0
			4·2	2·12		0·36	0·594	0·021
			5·73	2·35		0·54	0·602	0·029

TABLE 2

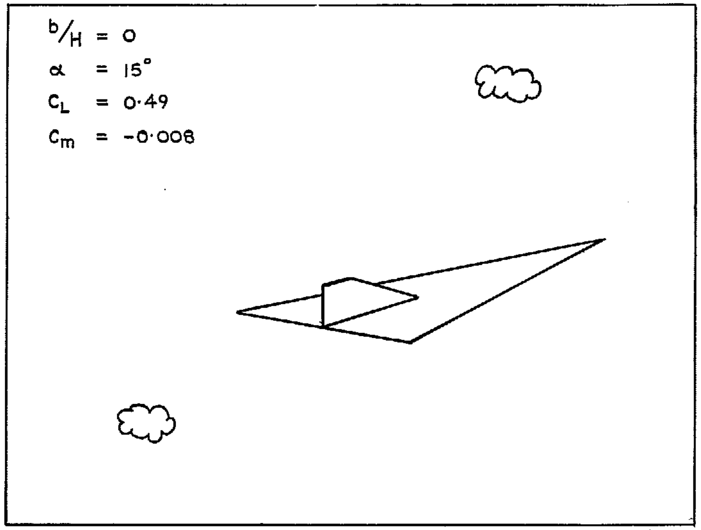
Measured Ground Effects on Slender Wings at Large Incidence.

Wing	A	Ref.	α	$b/H_{\frac{1}{4}}$	$\dfrac{\Delta C_N}{C_N}$	F	$\dfrac{\Delta C_N}{F\,C_N}$	Δx_{cp}
Delta	1·62	5	10	1·99	0·13	0·95	0·14	
				2·91	0·23		0·24	
				3·75	0·31		0·33	
				4·66	0·43		0·46	
				5·55	0·56		0·59	
			15	1·96	0·14	1·06	0·13	0·004
				2·85	0·23		0·22	0·010
				3·64	0·32		0·30	0·017
				4·49	0·44		0·42	0·025
Delta	1·0	2	10	1·85	0·17	1·40	0·12	
				2·92	0·35		0·25	
				3·52	0·47		0·34	
			15	1·71	0·19	1·67	0·12	
				2·7	0·39		0·23	0·024
				3·21	0·55		0·33	0·043
Gothic	1·0	11	10	2·18	0·22	1·74	0·13	
				3·42	0·46		0·27	
			15	1·9	0·21	2·05	0·10	0·019
				2·82	0·42		0·20	0·036
Gothic	0·75	12	10	2·05	0·23	1·82	0·13	
				2·71	0·37		0·20	
				4	0·75		0·41	
			15	1·8	0·25	2·34	0·11	
				2·3	0·35		0·16	0·031
				3·2	0·72		0·31	0·060
Mild Gothic	1·36	2	10	3·35	0·35	1·10	0·32	
				4·32	0·43		0·39	
				5·2	0·54		0·49	
				6·0	0·69		0·63	
			15	3·45	0·37	1·29	0·29	
				4·15	0·47		0·37	
				4·95	0·61		0·47	

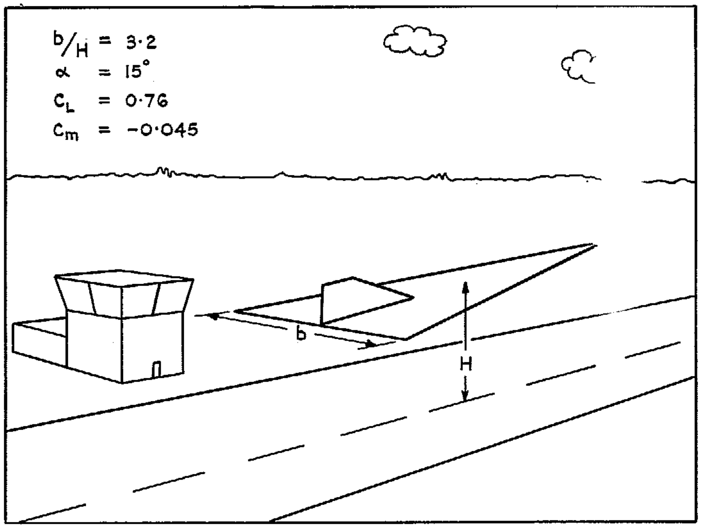
TABLE 3

Effect of Thickness on Ground Effect.

Wing	A	Ref.	t/H	$\left(\frac{dC_N}{d\alpha}\right)_\infty$	$(C_N)_{\alpha=0}$	$\frac{(C_N)_{\alpha=0}}{dC_N/d\alpha}$	$(C_m)_{\alpha=0}$	$\frac{(C_m)_{\alpha=0}}{dC_N/d\alpha}$
Delta	1·62	5	0 0·24 0·30 0·36	1·72	0 −0·003 −0·011 −0·015	0 −0·002 −0·006 −0·009	0 0·0030 0·0050 0·0065	0 0·002 0·003 0·004
Delta	1·0	2	0 0·24 0·53	1·20	0 0 −0·01	0 0 −0·008	0 0 0·003	0 0 0·003
Gothic	1·0	11	0 0·54 1·08	1·35	0 −0·015 −0·045	0 −0·011 −0·033	0 0·004 0·011	0 0·003 0·008
Gothic	0·75	12	0 0·49 0·72 1·48	1·22	0 −0·015 −0·020 −0·040	0 −0·012 −0·016 −0·033	0 0·004 0·006 0·011	0 0·003 0·005 0·009
Mild Gothic	1·36	2	0 0·32 0·37	1·66	0 0 −0·01	0 0 −0·006	0 0·004 0·0055	0 0·002 0·003

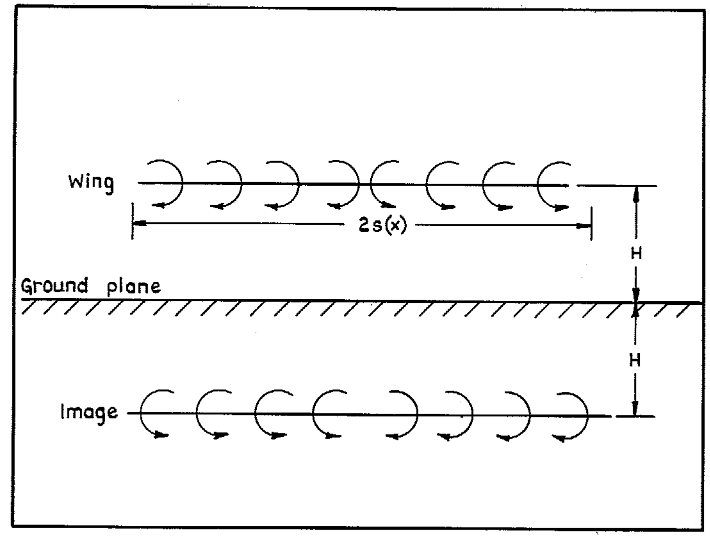


(a) In free air

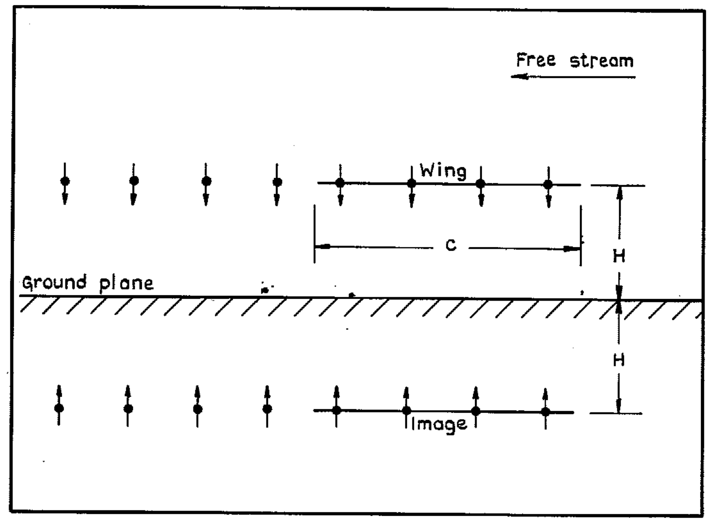


(b) Near the ground

FIG. 1 a & b. Ground effect on a delta wing ($A = 1$).



(a) Gersten's model, plane perpendicular to free stream



(b) Present model

FIG. 2 a & b. Mathematical models of the flow field.

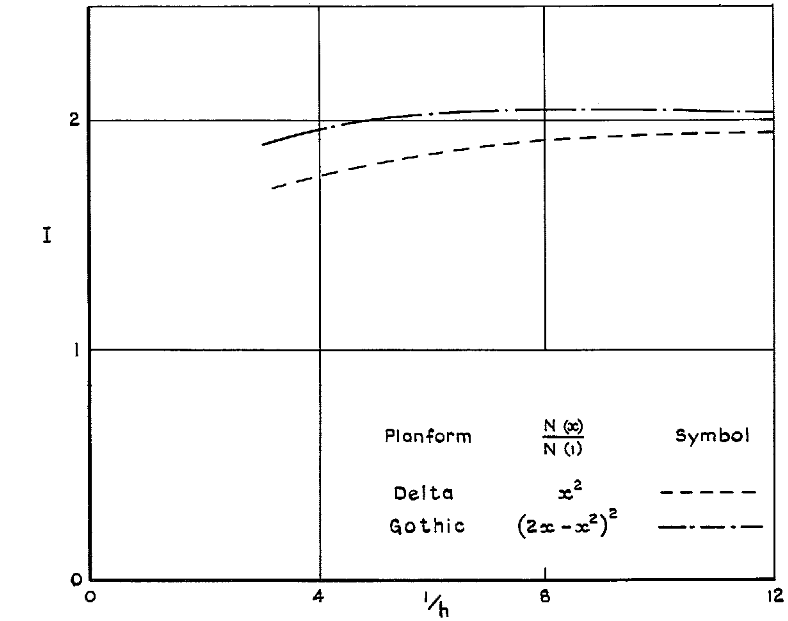


FIG. 3a. Variation of I with height and loading distribution.

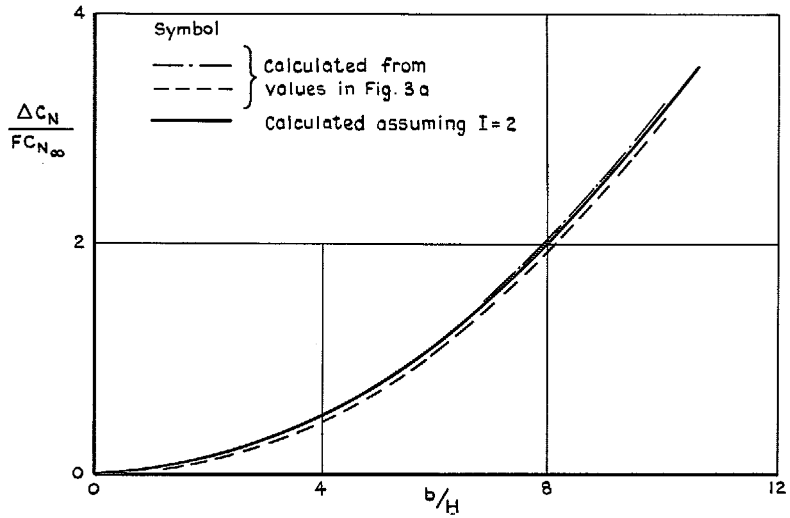


FIG. 3b. Effect of assuming $I = 2$.

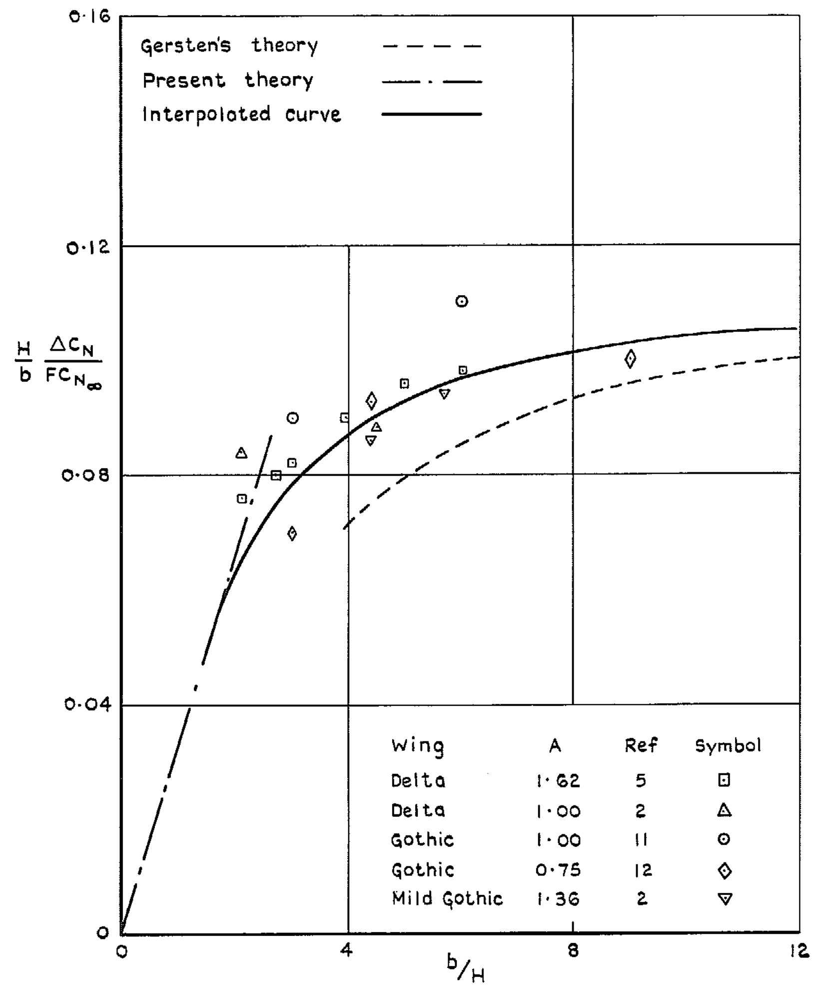


FIG. 4. Comparison of experimental results and theoretical predictions of the ground effect on the normal force on slender wings.

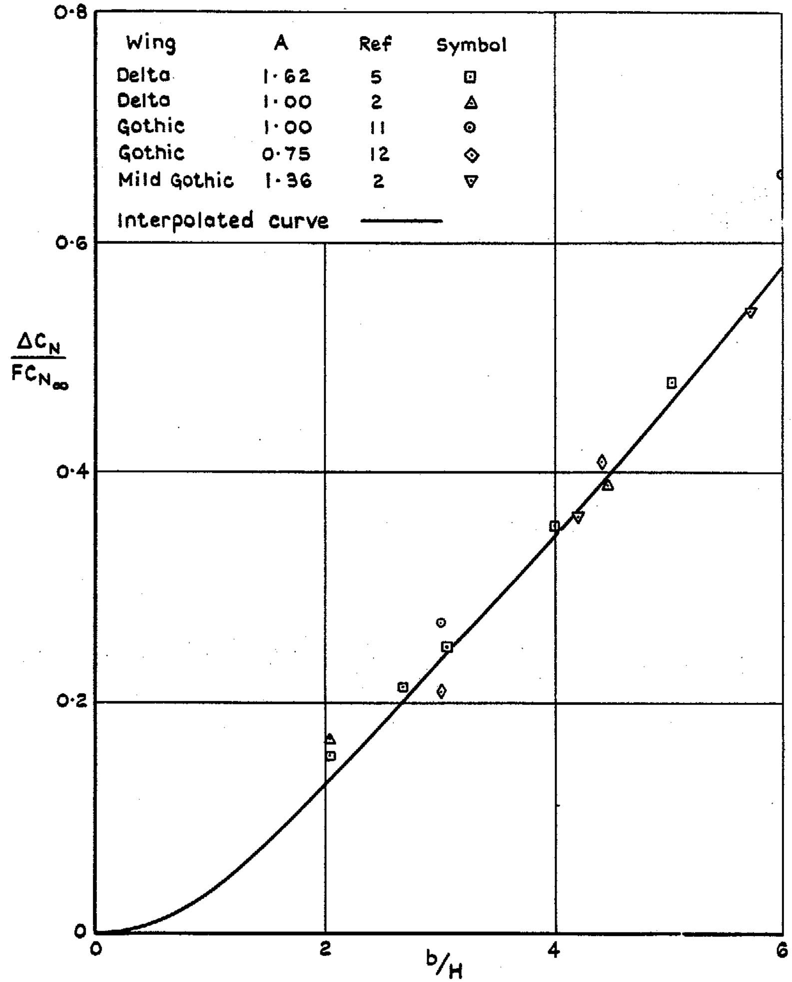


FIG. 5. Comparison of experimental results and the interpolated curve.

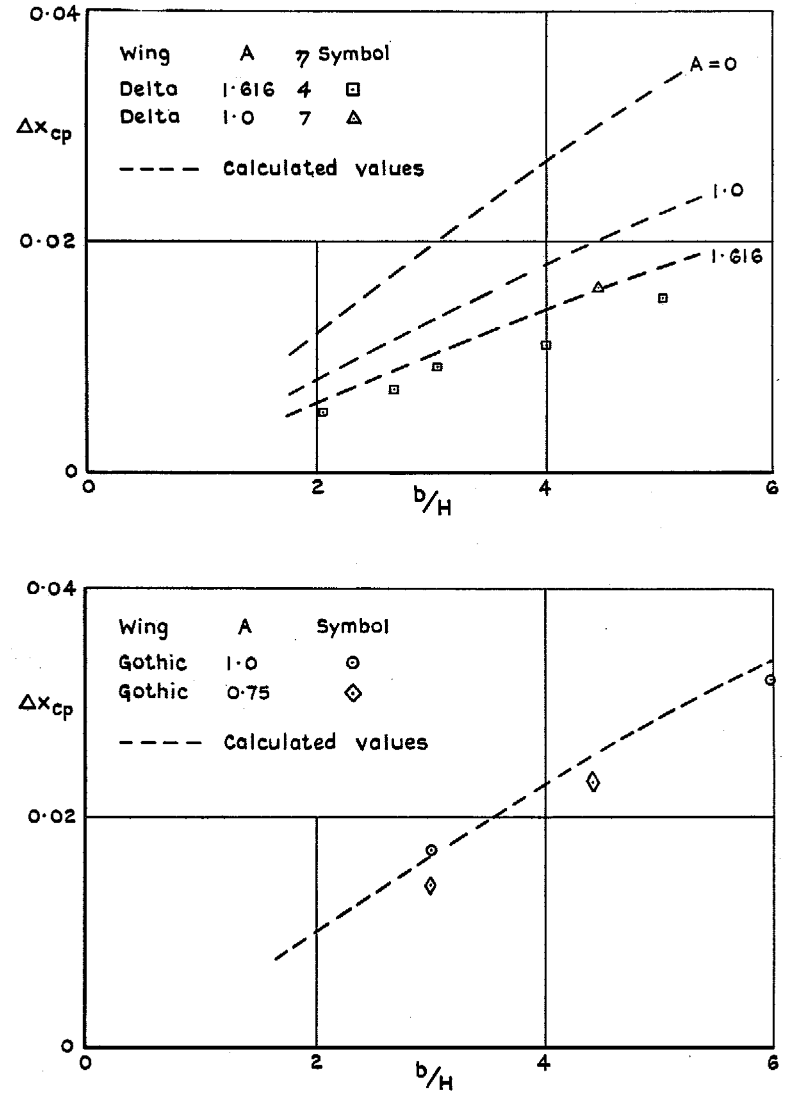


FIG. 6. Comparison of the calculated ground effect on the centre of pressure at zero incidence with experimental results.

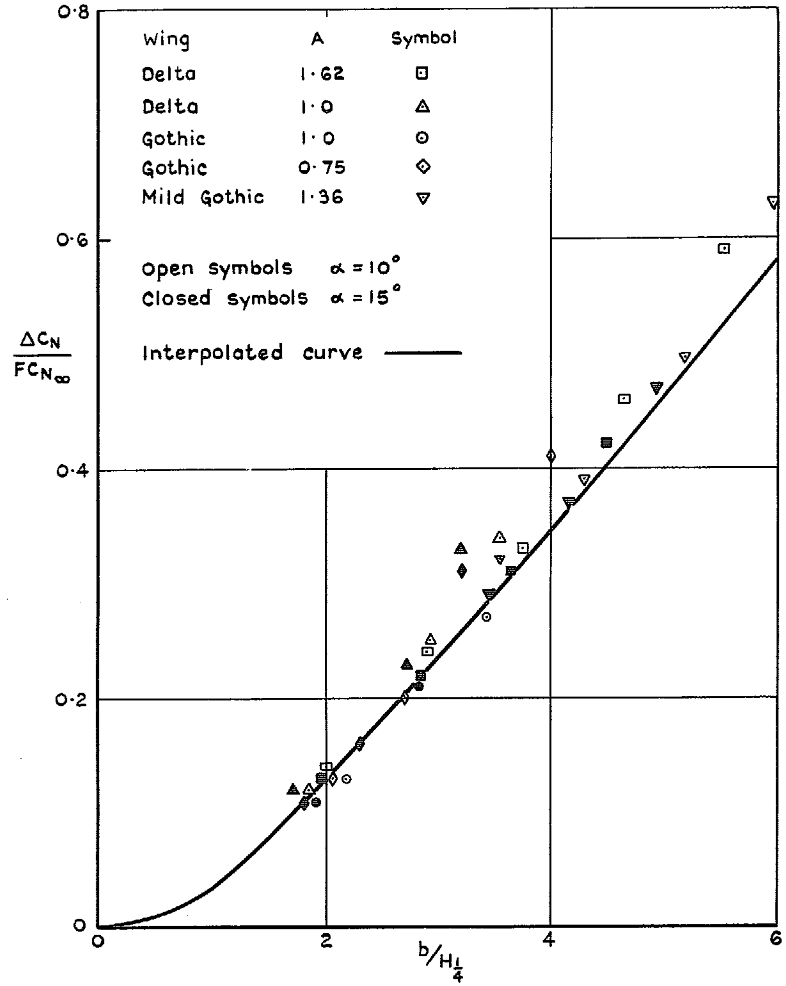


FIG. 7. Correlation of the ground effect on the normal force on slender wings.

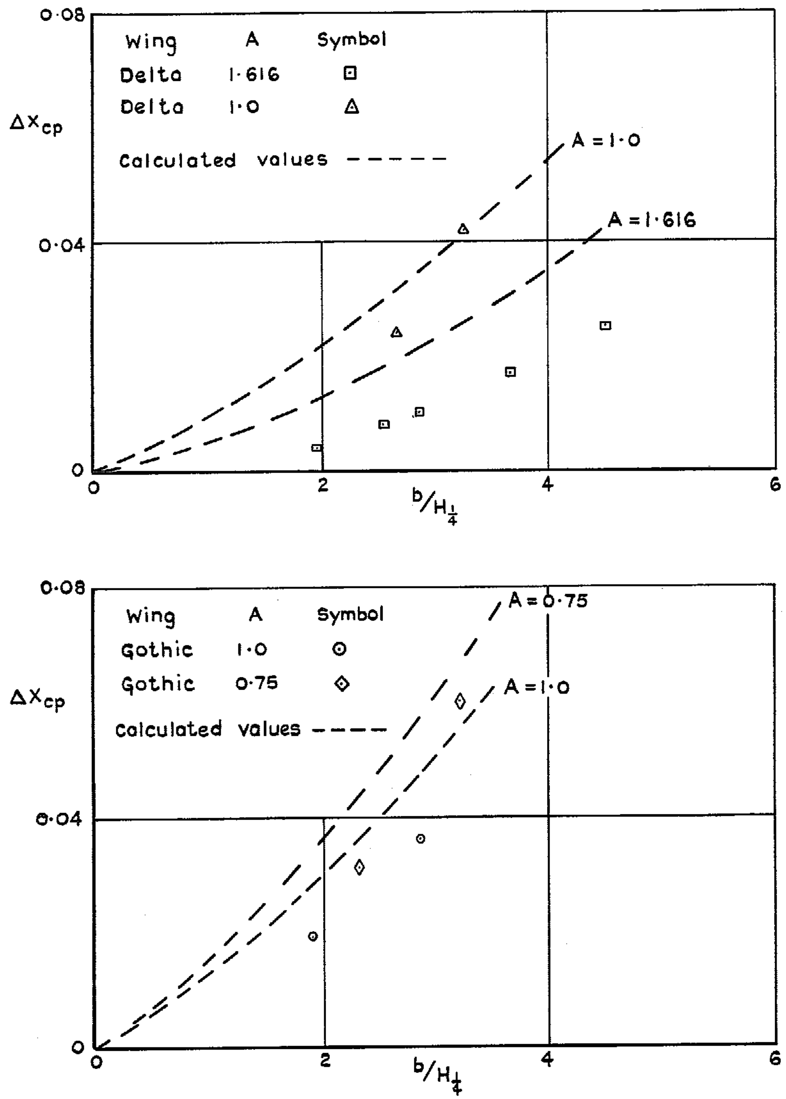


FIG. 8. Comparison of the calculated ground effect on the centre of pressure at an incidence of 15° with experimental results.

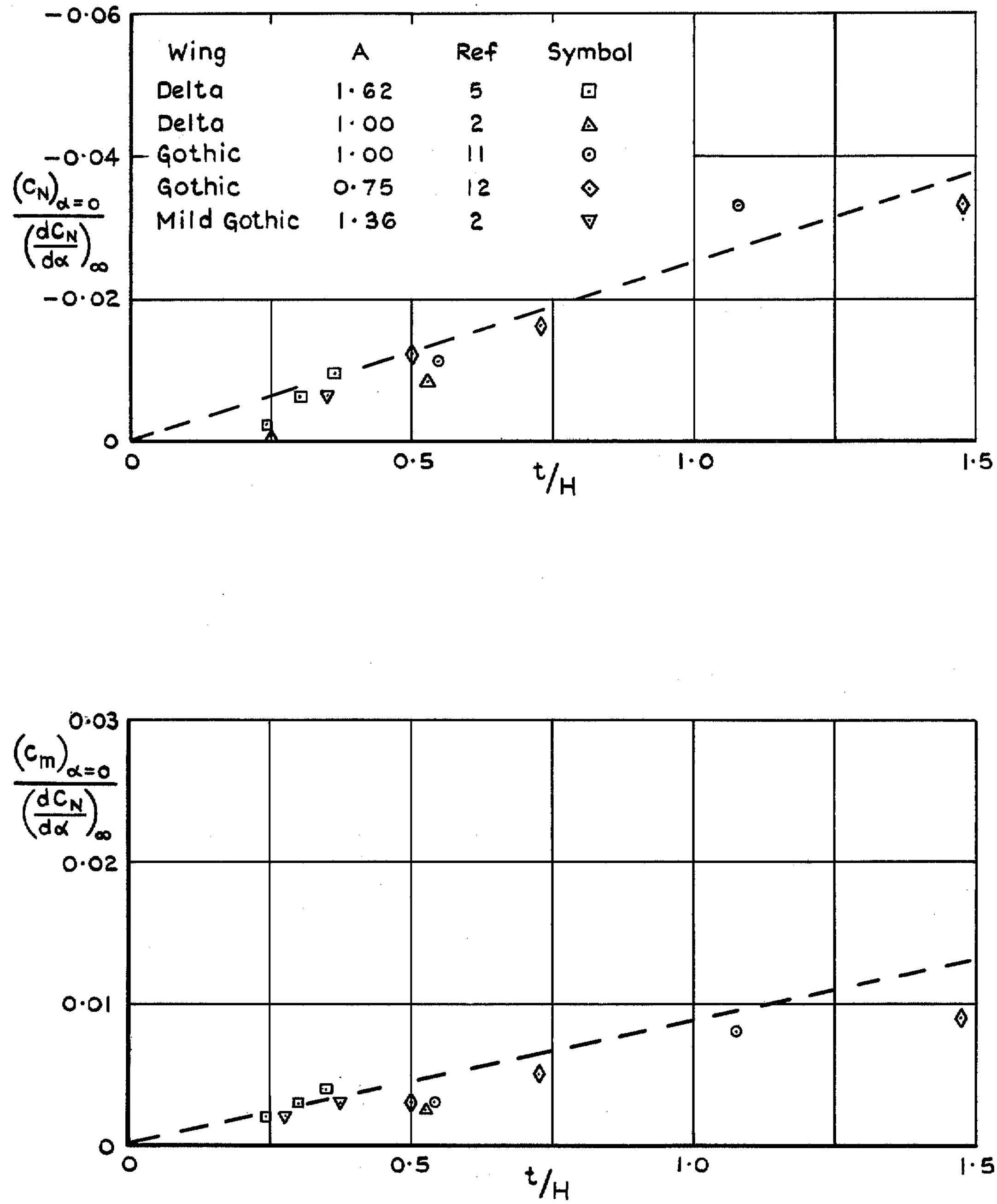


FIG. 9. Effect of wing thickness.

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