R. & M. No. 3644

R. & M. No. 3644

MINISTRY OF TECHNOLOGY

AERONAUTICAL RESEARCH COUNCIL

REPORTS AND MEMORANDA

Wind-Tunnel Investigations of Instability in a Cable-Towed Body System

By A. R. METTAM

Aerodynamics Dept., R.A.E., Farnborough

LONDON: HER MAJESTY'S STATIONERY OFFICE

1970

PRICE 16s. 0d. [80p] NET

Wind-Tunnel Investigations of Instability in a Cable-Towed Body System

By A. R. METTAM

Aerodynamics Dept., R.A.E., Farnborough

Reports and Memoranda No. 3644*

February, 1969

Summary.

A brief investigation was made in R.A.E. wind tunnels into instability of a body towed by a helicopter. This was undertaken after an incident in which a divergent oscillation led to cable failure in flight.

Stability boundaries in terms of cable length and forward speed were determined for the original cable/body system. The effects of various body and suspension modifications were then studied.

Existing theory was compared with the experimental results and both were used in forecasting the behaviour of a new cable/body system. Subsequent experimental work elsewhere confirmed the validity of these estimates.

Enhanced stability of the system was observed at short cable lengths and a possible explanation is suggested.

LIST OF CONTENTS

Section

*Replaces R.A.E. Technical Report 69 022—A.R.C. 31 372.

---

1. *Introduction.*

A Whirlwind helicopter towing a torpedo-shaped body was approaching a hover at low altitude, to enable a ground crew to detach the body from the cable at the completion of tests. As speed was reduced, with the cable being winched-in simultaneously, a lateral oscillation of the body and cable commenced, the mode being that of a simple pendulum, swinging about the aircraft's suspension point. The oscillation diverged so rapidly that after three swings the body was level with the helicopter. At this point, the cable slackened and the subsequent snatch load caused cable failure.

The possibility of this oscillation being due to downwash from the helicopter rotor was readily refuted and it was suggested that the body should be tested in the R.A.E. 24ft tunnel, in an attempt to understand the phenomenon and prevent its recurrence.

These tests were made early in 1967, when it was found that similar oscillations occurred spontaneously at certain combinations of tunnel speed and cable length. A boundary in terms of these parameters was established which agreed well with predictions based on a simple linear theory[1]. At the longer cable lengths (e.g. 30 feet) where the divergence rate was low, the tunnel stream was accelerated as rapidly as possible, to see if an upper speed boundary existed beyond which the oscillation again damped. With the body configuration supplied this proved unattainable, but a speed was reached beyond which no further divergence took place and it was thought that only the tunnel speed limitation prevented an upper boundary being found.

A systematic programme to improve stability of the system was not possible due to time limitations. Accordingly, the body was modified to what was thought to be the optimum configuration, consistent with the operational requirements of the equipment, and re-tested in the 24ft tunnel. Further minor modifications were also briefly investigated. One other test made in the 24ft tunnel was to check the effect of rate of change of cable length during 'winching-in'. This was found to be negligible over the helicopter speed range used during this operation.

To assist in optimising the body shape, a few tests were made of a 1/14 scale model supported on opposed needles in the open jet of a 10in, instrument-calibration wind tunnel. These determined centre of pressure location for the various configurations and enabled the efficiency of the cruciform tail unit in the 0°/90° and 45° positions to be compared.

Following the main tunnel tests, a search of the literature on towed bodies revealed that Glauert[2] had covered the whole stability problem comprehensively but, because of the emphasis on relating the dynamics of the body to the drag characteristics of the cable, it was difficult to interpret and to apply his equations. A simplified extract of Glauert's theory as applied to the present problem was therefore prepared and is presented as an Appendix to this Report.

Information on the contribution of cylindrical bodies (as opposed to their tail units) to the damping derivatives n_r, m_q was found to be very scarce. Accordingly, an analysis was made of the results of several old wind-tunnel tests on model airships, the results being summarised here.

Having gained the foregoing experience of towed-body stability problems, the author was asked to predict stability boundaries for a new body to be smaller, to have a drum-tail and to be supported on a cable of slightly larger diameter. Upper and lower stability boundaries were calculated by Glauert's method, upper boundary points being factored in the light of the tunnel test findings. Subsequent French tunnel tests[3] showed good agreement with the predicted boundaries.

Complementary theoretical work by Hopkin[4] has shown that the upper stability boundary is almost certain to be ill defined experimentally as theory shows the damping gradient to be extremely low in the neighbourhood of this. Conversely, the gradient is large in the vicinity of the lower boundary, which might therefore be expected to be well defined by experiment.

It was felt that a significant change in the upper stability boundary, might result from changes in the mechanical stiffness (EI) of the suspension cable. Accordingly, a simple model of one of the original body configurations, to 1/5 scale, was tested in the R.A.E. 5ft tunnel using three different cables. Two of these were identical aerodynamically but varied in stiffness by a factor of 6·5, whilst the third was the finest single-strand wire suitable for the load. Significant differences were measured between the upper stability boundaries for the three cables but the lower stability boundaries were almost coincident.

These 5ft tunnel tests also demonstrated enhanced stability at very short cable lengths, as did the French tunnel tests[3]. A simple hypothesis is presented to explain this fact.

2. *Wind-Tunnel Tests of Original System.*

2.1. *Description of Towed Body.*

The shape of the original body is shown as Configuration A in Fig. 1. Repairs to the body following the flight cable failure had restored its shape within limits of error which could be ignored.

The body shell was of glass-filled resin with a wooden spar assembly supporting the fins. Non-magnetic construction throughout was required by the operational use of the body, which normally houses two distinct packs of electronic equipment. These have to be as far apart as possible, yielding a high inertia body which was provided for the tunnel tests by mounting lead ballast at the nose and tail ends of the rubber-mounted, central, equipment-tube.

The suspension cable was formed from co-axial phosphor-bronze conductors, sheathed in paper insulation and an outer PVC coating to give an outside diameter of 0·375 inch. At the body end, suspension loads were reacted by a large clamp-block pivoted to the body's dorsal fin, the cable being trapped in a letter *S* pattern by suitably profiled grooves in the clamp block halves. A 40 foot length of cable was supplied for the tunnel tests, one end of this being attached to the hook of a 10 cwt hoist in the 24ft tunnel superstructure. A 2 inch thick plywood door was inserted in the tunnel roof with a central 1 inch diameter hole acting as the aircraft suspension fairlead, about which the pendulum oscillations took place.

2.2. *Test Procedure.*

Cable length, measured between the above suspension point and the body centreline was set at the maximum usable value of 34 feet, which allowed 16 feet amplitude of oscillation to occur within the confines of the tunnel jet.

The tunnel was run to its lowest controllable speed of 20 ft/s, the body being then displaced through some 10° yaw angle by pushing against the fins with a light dural probe. This initiated a predominantly lateral pendulum mode oscillation which damped out completely after five cycles. The experiment was repeated but with the disturbance applied at the nose in case of any momentum effects; the resultant oscillation and damping was, however, identical. Increasing the disturbance to give an initial yaw of some 15° merely resulted in the oscillation persisting for some nine cycles. The experiment was then repeated at an airspeed of 25 ft/s, at which the oscillation diverged to some 12 feet peak-to-peak amplitude; at this point tunnel power was cut and the model restrained by observers stationed at either side of the working section. The boundary between damping and divergent oscillations thus lay between 20 and 25 ft/s; further tests determined 23 ft/s as the speed for which oscillations resulting from a small disturbance damped slowly whilst a larger disturbance produced slow divergence. Thus 23 ft/s was deemed to be the threshold of divergence for the 34 feet pendulum length, and similar points were later obtained at shorter pendulum lengths to define the low-speed stability boundary.

With the 34 feet pendulum length, the tunnel speed was slowly increased, when the lateral pendulum mode oscillation occurred spontaneously at a speed of some 50 ft/s and diverged rapidly, reaching dangerous proportions by the time tunnel speed had increased to 70 ft/s. Emergency-stop braking of the tunnel fan was applied at this point and the model restrained manually. Tunnel speed was then accelerated as rapidly as possible to see if an upper speed limit to the divergent oscillation could be found. The spontaneous divergence again occurred but beyond 80 ft/s the oscillation appeared to be diverging more slowly and it was felt that an upper boundary did exist. However, it was unsafe to permit a further possible growth of the oscillation, which had by this time reached 14 feet amplitude, and the tunnel was shut down at a speed of 110 ft/s.

The whole of the foregoing procedure was repeated for pendulum lengths of 30, 26, 22, 18, 14, 10 and 6 feet, the oscillation period being noted in each case. Only at pendulum lengths greater than 18 feet, however, was there a suggestion of an upper stability boundary being approached. At these lengths, beyond 80 to 90 ft/s, some damping would appear for a few cycles, usually preceded by a 'nodding' oscillation in pitch of a few degrees amplitude. The basic lateral pendulum oscillation would then diverge again slowly to its original amplitude and this would be maintained for up to 2 minutes before the process was repeated. At shorter cable lengths however, as for the odd intermediate value of 26 feet, it was never possible to exceed 75 ft/s airspeed without a dangerously rapid divergence taking place.

A theoretical curve derived by Lipscombe[1] in a preliminary analysis of the test body's stability enabled the tunnel tests to be restricted to a small band of airspeeds at each cable length, thereby saving considerable effort.

Initiation of yaw at shorter cable lengths, where the model was inaccessible from the tunnel floor, was accomplished by pulling a fine thread attached to the lower port fin of the body near its trailing-edge. Checks were first made to ensure that the drag of this thread did not significantly affect the oscillatory behaviour of the system. This thread enabled a closely controlled disturbance to be applied and could also be used to check large oscillations after skill had been acquired in 'playing' the body.

3. *Wind-Tunnel Tests of Modified Body.*

The flight incident, which posed an urgent development problem for the cable-body system occurred at a time when the 24ft tunnel was heavily booked for other work. A systematic investigation of individual parameter effects was thus precluded and it was therefore decided to modify the body, in the light of previous flight experience with towed bodies, to what it was hoped would be the optimum configuration.

The dorsal fin (Fig. 1) seemed entirely redundant and had the additional failing of raising the effective cable/body junction even higher than dictated by the rigid clamp block. All previous experience with towed bodies had shown the need for the suspension point to coincide with the centre of gravity of the body, so this was first effected by forming a stirrup cable end-fitting from streamline section wire and attaching this with pivot pins on either side of the body centreline (Fig. 1, section AA).

Undoubtedly, one large contribution to instability was the high moment of inertia of the body about its centre of gravity. This could not be reduced below a certain value due to the operational requirement for maximum spacing of equipment (Section 2.1). It was, however, thought desirable to compare the effectiveness of inertia reduction to this minimum value with the effectiveness of the aerodynamic changes made. It had to be borne in mind that any changes ultimately adopted had to be capable of manufacture from non-magnetic materials and that large metal surfaces, susceptible to eddy current formation, were unacceptable.

3.1. *Configurations Tested.*

The test configurations are detailed in Table 1 and illustrated in Fig. 1. It was thought that the body should have a higher fineness ratio, combined with a much higher fin/tail volume.

Rearward growth of the body was limited by stowage space on a particular type of helicopter and by a need to restrict all up weight.

A tubular extension was therefore inserted between the body shell and its tail cone, formed from two nesting cylinders which permitted ready rotation of the tail cone for tests with fins in the 0°/90° or 45° configurations. The suspension point was made variable by fitting perforated side plates to the body and was enabled to move forward slightly by the addition of extra ballast in the nose of the body. As shown in Fig. 1, an increase of nearly 50 per cent in the aerodynamic moment arm about the centre of gravity suspension point was achieved by these means.

For tests of inertia reduction effects, (Configuration D Fig. 1) removal of the nose ballast gave a worthwhile reduction of 15 per cent in moment of inertia, although this was accompanied by a slight reduction in aerodynamic moment arm owing to the rearward centre of gravity shift of 3·38 inches.

During tests in Configuration B, a fore-and-aft pendulum-mode oscillation occurred at medium pendulum lengths. Previous flight experience suggested added body drag as a cure; accordingly angle strips were secured to the vertical fin trailing edges as shown in Configuration E, Fig. 1. These drag plates not only cured the fore-and-aft oscillation but had such a marked effect on the lateral pendulum-mode oscillation that this modification was investigated as fully as the other 4 body configurations.

3.2. 10*in Tunnel Tests.*

A scale model of the body was turned up from 5/8 inch dowel rod and fitted with fins carved to correct profile from 3/32 inch thick sheet balsa wood. For the first tests a dorsal fin was similarly attached, this having a row of holes 0·02 inch diameter × 0·15 inch deep drilled into its upper edge at 0·07 inch pitch. A similar row of holes was drilled into the underside of the body, opposite those in the fin, and the model was mounted in the jet of the 10in tunnel by means of upper and lower vertical struts bearing sewing-needles at their tips. These needles entered the model holes referred to above sufficiently to maintain the model in a horizontal flying position and permit free 'weathercocking' about the vertical needle axis. In this way the body-weight moment was reacted by side forces on the needles and had no effect on the aerodynamic moments.

On running the tunnel at 200 ft/s (giving a Reynolds number equivalent to 14 ft/s full scale) the body weathercocked to bring the tail up or downstream of the suspension axis depending on the centre of pressure position. A few rapid tests with the needles in various opposed-hole mountings soon located the centre of pressure to an accuracy of one hole pitch, i.e. 0·07 inch or 1·14 per cent of body length. In practice, the relative torque required for a given out-of-wind displacement permitted accuracy of centre of pressure estimation much higher than this (say 0·25 per cent of body length).

The tests were repeated with the dorsal fin removed and its mounting holes reproduced in the upper surface of the body. Drilling two further rows of holes displaced 45° about the body centreline permitted centre of pressure determination with the fins vertical and horizontal (0°/90° as in Configurations B, D and E).

Similar tests were made with the body lengthened by a further piece of dowel representing the body extension. This could be inserted aft of the elliptical nose section, the three body segments being aligned and secured by an axial steel dowel. The only configuration not tested in the foregoing manner was that with drag plates (Configuration E), since the use of these was not contemplated at the time.

3.3. 24*ft Tunnel Tests* (*Fixed Cable Length*).

The modified body was tested in conjunction with the original cable in each of the configurations B, C, D and E of Table 1 and Fig. 1. The methods used and pendulum lengths were those of Section 2.2, and again the period of the lateral pendulum oscillation was noted at each airspeed, to assist theoretical study. During modification, a turnbuckle eye had been fitted at the nose of the body, enabling it to be suspended, tail down, from a hardened steel-wire pivot. Timing the period of 100 small oscillations about this pivot and location of the body's centre of gravity by balancing on knife edges enabled the moment of inertia about the centre of gravity/suspension point to be calculated accurately for each test configuration.

With the modified body the stability was in all cases sufficiently improved for the existence of the upper stability boundary to be verified experimentally. The scatter in the experimental points was, however, high compared with those locating the lower stability boundary. This is not surprising, since theory shows damping to vary very slowly[4] in the region of the upper boundary.

3.4. *'Winching-in' Experiment.*

Lipscombe's theory[1] had shown that during shortening of the cable one of the oscillation damping terms became negative with increase in rate of change of cable length. R.A.E. was therefore asked for an assurance that this would not in itself lead to a catastrophic divergence during winching-in of the body.

Tunnel availability precluded fitting of a winch but a crude simulation was devised using two pulleys mounted from steel-angle frames attached to the underside of a balance car which was itself suspended from the tunnel roof (Fig. 2a). The suspension cable was passed over the central pulley and then run horizontally to the pulley outside the tunnel jet, where counterbalance weights equal to the weight of the body were attached. A rope attached to these weights enabled them to be hauled down manually, thus raising the body through the tunnel stream, and on its centreline, from an initial pendulum length of 22 feet to zero, when the body contacted a dummy crutch. Plain-journal pulley bearings and a total 'running weight' of 130 lb yielded sufficient frictional damping to give a steady rate of ascent despite manual operation.

Tests were made using this rig at 60 ft/s airspeed and 'winch-in' speeds of 1, 2 and 3 ft/s respectively. In each case one run was made with the body steady initially and one with an initial oscillation of some 5 feet amplitude in the lateral-pendulum mode. No such oscillations were induced by the changing pendulum length and no change of lateral displacement in an existing pendulum oscillation could be detected, although, of course, the angular amplitude increased with cable shortening. Similarly, oscillation frequency appeared constant despite the varying cable length for the short time of each test run. During an attempted test at 4 ft/s the cable parted and time did not permit further experiments.

4. *Results of Tests.*

Stability boundaries as determined by the 24ft tunnel tests are presented in Figs. 3 and 4, for the original body and modified bodies respectively. Similarly, Figs. 5 and 6 present the measured values of oscillation period over the airspeed range for the original and modified bodies respectively.

Fig. 3 shows good agreement between the measured low-speed stability boundary and the theoretical predictions for the original body by Lipscombe[1]. Three points are shown as suggesting an upper boundary to the unstable region, but the accuracy of these in terms of airspeed is probably no better than 30 ft/s. At the low-speed end, however, it was found that the boundary could be located fairly precisely, the scatter about a faired curve through the experimental points being 3 ft/s.

It should be noted that throughout this Report the lower stability boundary is defined as the speed beyond which a definite increase in oscillation amplitude always occurs, on increasing speed. In the case of the upper boundary, however, the definition is that speed beyond which an oscillation definitely damps out, though the rate of damping may be extremely slow.

Turning to the lengthened body results of Fig. 4, comparison with the original body results shows a reduction of some 8 ft/s in the low-speed boundary for Configurations B, C and D, whilst the addition of drag plates (Configuration E) makes a further reduction of 10 ft/s here.

From an operational point of view however, the low-speed boundary is of little significance, since forward flight at cruise or full speed of the towing helicopter was required with the body deployed. Thus the desirable modification was that which moved the upper stability boundary downwards and to the left, leaving the largest field in terms of cable length and forward speed clear of all stability limitations. Of all the modifications tested, Configurations E was clearly the best in this respect, with body inertia reduction (Configuration D) also highly effective.

As a result of the above measurements, R.A.E. recommended that deployment, flying and retraction of the body should all take place above a flight speed of 120 ft/s (70 knots). Assuming winching-in takes place at this speed the possibility of a divergent oscillation occurring need only be considered for cable lengths shorter than 10 feet. At this length the period of oscillation at 120 ft/s airspeed is 3·5 seconds. Hence, if winching-in is carried out at 4 ft/s rate, there is only time for the first swing of an oscillation to occur before the body is safely crutched; this is considered an acceptable risk. It will be shown later in Section 8 that, with a 0°/90° cruciform tail in particular, there is an additional stabilising factor at such small cable lengths which may well result in the upper boundary being reflected to meet the speed axis at a finite value within the flight speed range of the helicopter. This would imply that above a certain speed the cable/body system would be stable at all cable lengths.

As stated earlier, the main reason for recording the period of oscillation throughout the stability tests was to permit its use in possible future theoretical studies. However, some points of interest emerge from Figs. 5 and 6. At any given cable length, it is seen from the lengthened-body results of Fig. 6 that the period corresponds to that measured in still air over the whole speed range within which the system is unstable. Below the low-speed stability boundary there is a marked increase in period and a less well defined increase is also seen at speeds beyond the high-speed stability boundary.

The particular body configuration had little effect on the period at a given value of speed and pendulum length, though a slight increase is seen for the reduced inertia case of Configuration D. This result would be expected in view of the consistently higher values of period seen in Fig. 5, for the original Configuration A, for which the moment of inertia is appreciably lower (Table 1).

5. *Theoretical Studies.*

The first theoretical work considered in these experiments was that of Lipscombe[1], a member of the Establishment conducting the helicopter flight trials. His simple, linearised approach was aimed at finding whether the system could be made completely stable and determining the dominant parameters. His theoretical stability boundary, (Fig. 3) greatly reduced the time taken to map the experimental boundary. It was also as a result of his work that the 'winching-in' experiment of Section 3.4 was devised.

Following the initial stability tests the work of Etkin and Mackworth[5] became available. This confirmed the existence of an upper stability boundary and gave a lucid explanation of the effects of cable drag and inertia during displacement of the system laterally. On the latter point it was suggested that a quasi-rigid assumption for the cable is justified if the cable mass is less than some 10 per cent of the mass of the towed body; coupling of the cable and body motion should then be unimportant.

Following the R.A.E. tunnel programme, the classical theory of Glauert[2] was examined in detail and found to be extremely comprehensive, dealing first with the three oscillations of the cable system in the absence of aerodynamic restoring and damping forces on the body, and then progressing to both longitudinal and lateral stability of the whole cable-plus-body system. To those not familar with dynamic stability analysis, it is difficult to apply his general theory to a particular special case. Therefore, the author has presented in an Appendix a summary of the appropriate portions of Glauert's theory, combined with his own numerical evaluation of the equations leading to the stability boundaries shown in Fig. 8 for the systems of the present tests. Data appertaining to the stability calculations is summarised in Table 2.

Excellent agreement between the theoretical curve and the experimental values for the low-speed stability boundary is seen in Fig. 8, but, for a given cable length, the unmodified Glauert theory gave upper stability boundary values 100 per cent greater than those measured in the tunnel. It is seen that a 40 per cent speed reduction of the upper boundary was shown by the theory to result from addition of drag plates (Configuration E) which gave a threefold increase of body drag. To achieve agreement between theory and experiment, an equally large speed reduction in the theoretical upper boundary was required. The similarity of the two pairs of curves in Fig. 8 suggested that a common factor, which was itself a function of body drag, might, when applied to the theoretical boundaries, achieve the required correlation for both configurations investigated. Such a factor was found to be $k_D^{0 \cdot 438}$ and Fig. 8 shows the excellent fit which its use gave with the measured boundary values.

The foregoing theoretical study by the author required the evaluation of various derivatives appropriate to the body Configurations B and E, the summary paper by Hills[6] being the source of the equations used. Doubt existed on the contribution of the body to the damping derivative $n_r(m_q)$, and the author, therefore made an analysis of results from old tunnel tests[7 to 14] of airship hulls and bombs. The results are presented in Fig. 9, where the approximate values of the derivative as a function of body fineness are given for three basic body forms. Most of the tunnel tests from which this information came were made on models of very similar shape and size to the present test body and at similar airspeeds. Full scale data has also been used, but only where the effects of Reynolds number could be allowed for. The method used in the analysis was to determine the fin/tail contribution to damping moment by the method of Ref. 6 and by subtracting this from the total moment measured in the tunnel test, to obtain the body moment. This quantity was then nondimensionalised, again by the method of Ref. 6.

Further comprehensive studies of the stability of cable-towed body systems have since been made by Hopkin[4] who presents generalised formulae and stability diagrams. In addition, he examines the degree of instability in the region contained by the two boundaries and shows that the maximum instability is obtained quite close to the lower boundary. It is well known that, in an experiment, non-linearities often enforce constant amplitude oscillations and we might expect maximum amplitudes where linear theory predicts maximum instability. From this point of view, the present tunnel tests and those of Ref. 3 are consistent with the linearised theory. Theory also predicts the difficulty in establishing the upper boundary, since the rate of change of the degree of instability is very low over a wide area including this boundary. Maximum amplitude of the oscillation is also expected to be close to the lower stability boundary where damping changes rapidly.

6. *Stability Prediction for a New Cable/Body System.*

Following the previous work, the author was asked to predict stability boundaries for a new body to be carried on a cable of slightly greater diameter. The new body, shown in Fig. 10, was to be smaller than that of the 24ft tunnel tests and was to be equipped with a drum-tail enveloping the rear portion of the four swept fins.

The stability boundaries were first calculated using the Glauert theory and, as described in Section 5, the necessary derivatives were found using the methods of Ref. 6. Advice from previous work on bomb tails was to neglect the contribution to stability of cruciform fins supporting a drum tail. In this case the author treated the empennage as two separate units, a cruciform tail whose fin areas were taken up to the drum leading edge and a separate drum unit in which the internal portion of the fins was neglected.

In view of the close similarity in overall shape of the body and since its inertia had been estimated to give the same ratio of radius of gyration/body length as the body tested in the 24ft tunnel, it was decided to factor the upper stability boundary in the same way as for the old body. Fig. 11 shows the boundaries derived by the Glauert method and the revised upper boundary obtained by applying the factor $k_D^{0 \cdot 438}$ to the Glauert values.

The new body was subsequently tested in the wind tunnels of Avions L. Breguet at Velizy and Fig. 11 also shows that the experimental points agreed well with the above predictions. The new body has since been flown successfully beneath an Alouette III helicopter.

7. *Effects of Cable Mechanical Stiffness.*

Following Hopkin's demonstration[4] that minor changes in damping corresponded to gross changes in the position of the upper stability boundary, it was felt that the damping contribution of the fairly high mechanical stiffness of the support cable in bending should be determined experimentally.

A 1/5 scale model of the old body in Configuration B was therefore tested in the R.A.E. 5ft tunnel using three alternative support cables, none of which were to scale in diameter. Two of these were made from PVC tubing of 2·5 mm outside diameter, one being a loose sliding fit over an inner core of stranded steel cable, whilst the other was filled with lead shot, separated by nylon tube spacers, to give the same density as the former cable but a much lower stiffness. The third cable was a 36 swg steel wire, the finest wire thought to be capable of taking the wind and inertia loads of the body.

To determine mechanical stiffness, a short length of each cable was supported on two knife edges, spaced 4 inches apart. The test specimen was centrally loaded using a fine-wire tipped lever resting on its upper surface and the resultant deflection was measured directly with a scale alongside the specimen. Stiffness, as defined by the product of Young's modulus and moment of inertia, $EI = \frac{WL^3}{48\delta}$ (where W = central load, L = beam span and δ = deflection) was then calculated.

Stiffness of the full scale cable used in the 24ft tunnel tests was similarly determined using a 10 inch beam-span.

Sufficient tests in the 5ft tunnel were made, using each cable, to establish upper and lower stability boundaries, model behaviour being similar to that in the full scale 24ft tunnel tests. The results are presented in Fig. 12 in terms of the corresponding full scale values. Correct scaling of all parameters concerned in towed-body stability is not possible due to conflicting requirements of certain groupings, therefore, since gravity is highly significant and only low airspeeds are involved, correct Froude number scaling is achieved at the expense of correct Mach number and Reynolds number. The appropriate values of scale factor for the 1/5 scale model are as follows:—

Parameter	*Scale Factor*
Linear dimensions	$1/5$
Areas	$1/5^2$
Mass	$1/5^3$
Stiffness EI	$1/5^5$
Linear velocities	$1/5^{\frac{1}{2}}$
Forces	$1/5^3$
Frequencies of oscillations	$5^{\frac{1}{2}}$
Reynolds number	$1/5^{3/2}$
Mach number	$1/5^{\frac{1}{2}}$
Froude number $V/(lg)^{\frac{1}{2}}$	1

Fig. 12 shows a substantially common low-speed stability boundary to exist with all three cables, this agreeing well with the full scale experimental and theoretical values, as might be expected from the insensitive nature of this boundary (*see* Section 5).

In the case of the high-speed stability boundary, Fig. 12 shows a reduction of instability with increasing mechanical stiffness of the cable. Interpolation of a boundary for cable stiffness equivalent to full scale would however suggest greater instability than measured in the 24ft tunnel tests. This is thought to be largely due to the higher ratio of cable to body drag resulting from the oversize 'cables' of the 1/5 scale model. Reynolds number based on cable diameter is only reduced by a factor of 8·5 from full scale as a result of this whilst that based on body area is reduced by a factor of 11·2. These Reynolds number reductions are not likely to be significant since all values based on cable diameter are sub-critical in the present tests.

The high-speed stability boundary for the 36 swg wire suspension is displaced to higher speeds than is suggested merely by its mechanical stiffness, relative to that of the plastic tube suspensions. The fact that its cable drag relative to body drag is only one eleventh of that for the plastic tube suspensions should be a stabilising influence. However, its cable density is 4 times as great as that of the full scale cable, whilst that of the plastic tube suspensions is double the full scale value. Since, for dynamic similarity, density should be constant, this variation probably explains the higher boundary of the 36 swg wire suspension case.

8. *Enhanced Stability at Short Cable Lengths.*

The 5ft tunnel test results of Fig. 12 all show a point of inflexion in the high-speed stability boundary as cable length is reduced; indeed a finite upper speed limit to the unstable region is shown for the 'softer' cables. Preliminary tests of Configuration A in the 24ft tunnel also suggested this trend, but the high-speed boundary for that configuration was so tenuous that it was originally attributed to experimental scatter. However, French experimenters[3] working at 1/10 and 1/5 scale also commented on this deviation from the theoretical curve. The fact that in both R.A.E. tunnels, a tendency appeared for the body to roll about its cable attachment point at these short cable lengths, supports the following hypothesis.

Fig. 13 shows the body performing an oscillation on a cable so short that the vertical fin span, l is an appreciable fraction of the pendulum length b. Thus, as the body moves sideways, the lower vertical fin has a substantially higher lateral velocity v_l, and hence moment about the suspension point, P, than the upper vertical fin. This causes the body to roll as shown and, of course, the roll direction is reversed when the pendulum oscillation reverses the lateral motion. Thus an out-of-phase rolling oscillation ensues, which causes the body mass to act as a pendulum damper of the basic lateral oscillation of the system. The magnitude of the rolling oscillation and hence the stabilising effect should be greater at reduced cable stiffness, since local flexure at M (Fig. 13) opposes the rolling oscillation.

This is in agreement with Fig. 12 where the greatest stability increment at very short pendulum lengths occurred for the cable of lowest stiffness, as shown by the slope reversal of the stability boundary.

9. *Conclusions.*

(1) It was found possible to reproduce in the 24ft tunnel, lateral pendulum-mode oscillations of the test cable/body system which caused a flight cable-failure incident.

(2) Stability boundaries in terms of airspeed and pendulum length were determined experimentally for the cable/body system supplied.

(3) 4 arbitrary modifications were made to the body and stability boundaries were determined for each of these. The most effective in reducing instability was an increase of body drag, but reduction of body inertia is also very effective. In all cases the low speed boundary of oscillation onset is little affected.

(4) Following modification of the body in accordance with the above findings, it was recommended that a minimum-speed limitation of 120 ft/s (70 knots) be imposed for any operational use of the test cable/body system (i.e. for deployment, trailing and retraction of the body).

(5) No changes in oscillatory behaviour of the test system consequent upon rate of change of pendulum length during 'winching-in' were observed up to a 3 ft/s rate of retraction.

(6) Stability boundaries were predicted for a second cable/body system, using theory modified in the light of the present test results. Good agreement was obtained with subsequent French tunnel test results for the new system.

(7) 1/5 scale model tests of the original system showed significant changes in the high-speed stability boundary to result from changes in the mechanical stiffness of the towing cable.

(8) Enhanced stability of cable/body systems at very short pendulum lengths was demonstrated in the present tests. Experimental evidence supports a hypothesis presented here in explanation of the fact.

REFERENCES

No.	Author(s)	Title, etc.
1	J. Lipscombe	Preliminary investigation into the instability of a small helicopter-towed body. A.U.W.E. Technical Note 222/66 (1966).
2	H. Glauert	The stability of a body towed by a light wire. A.R.C. R. & M. 1312 (1932).
3	Staff of Breguet Aviation ..	Determination des coefficients aerodynamiques. C.R.2 (Essais 905) DT/Aé, JCz/JB/JZ, 254/67 (1967).
4	H. R. Hopkin	An approximate treatment of the stability of a towed, unbanked object in a condition of zero lift. R.A.E. Technical Report 69 076. A.R.C. 31 359 (1969).
5	B. Etkin and Jean C. Mackworth	Aerodynamic instability of non-lifting bodies towed beneath an aircraft. UTIA Technical Note 65 (1963).
6	R. Hills	The aerodynamics of armaments, turrets and external stores—Part 1: An analysis of wind tunnel data on the stability of bombs and rockets. R.A.E. Technical Note Aero. 2255 (1953).
7	L. Bairstow, E. F. Relf and J. L. Nayler ..	The stability of airships. Mathematical theory and illustrations of its use. A.R.C. R. & M. 257 (1916).
8	J. R. Pannell and R. Jones ..	Experiments on a model of the German rigid airship L33. A.R.C. R. & M. 361 (1917).
9	J. R. Pannell and R. Jones ..	Experiments on a model of an airship of the 23 class. A.R.C. R. & M. 456 (1918).
10	J. R. Pannell, R. Jones and G. N. Pell ..	Stability and resistance experiments on a model of Vickers rigid airship R.80. A.R.C. R. & M. 541 (1918).
11	R. Jones, D. H. Williams and A. H. Bell ..	Experiments on a model of rigid airship R.29. A.R.C. R. & M. 714 (1920).
12	R. Jones	The motion of airships under certain imposed movements of the rudders. A.R.C. R. & M. 781 (1921).
13	R. Jones and A. H. Bell ..	Experiments on a model of the airship R.101. A.R.C. R. & M. 1168 (1926).
14	J. Trouncer	Wind tunnel tests on two 4500 lb C.P. bombs. R.A.E. Technical Note Aero 1450 (1944).

APPENDIX

Theoretical Derivation of Stability Boundaries.

As discussed in Section 5, the classical work of Glauert[2] considered comprehensively the stability of bodies towed by light wires. Three possible oscillations of the cable system were first considered in the absence of aerodynamic restoring and damping forces on the body; it was shown that the periods of all three increase with cable length and decrease with increasing angle between the cable and the vertical, ϕ (Fig. 7).

Criteria were then derived for the longitudinal and lateral stability of the whole cable-plus-body system, introducing two new periods, corresponding to pitching and yawing oscillations of the body respectively.

Provided the body is symmetrical about its longitudinal axis, the lateral and longitudinal stability derivatives are interchangeable, thus:

$$y_v = z_w \; ; \qquad y_r = -z_q$$

$$n_v = -m_w \; ; \qquad n_r = m_q$$

Glauert nondimensionalised effective pendulum length of the system b (Fig. 7) in terms of the 'fundamental' cable length c, the latter being defined as the length of cable whose normal drag is equal to the tension in it.

The cable angle from the vertical, ϕ (Fig. 7) never exceeded 3·7° at the highest tunnel speed of 160 ft/s in the present tests, this value being used for these calculations. A cable drag coefficient (based on frontal area) of $C_D = 1{\cdot}122$ was assumed, since the cable surface was smooth. Air density used in the calculations was that of sea level, standard atmosphere, i.e. $\rho = 0{\cdot}002378$ slug/ft^3. Thus for the 0·375 inch diameter cable, drag per foot run $= C_D \frac{1}{2} \rho S\, V^2 = \dfrac{1{\cdot}122 \times 0{\cdot}002378 \times 0{\cdot}03125\, V^2}{2} = \dfrac{4{\cdot}1691\, V^2}{10^5}$ lb/ft. Now cable tension $T = W \sec \phi$ (Fig. 7)

therefore
$$c = \frac{W \sec \phi \times 10^5}{4{\cdot}1691\, V^2} = \frac{1{\cdot}55 \times 10^6}{V^2}. \qquad (1)$$

The ratio c/b (where b is the effective pendulum length) was defined by Glauert as $\gamma = c/b$. He then defined a 'typical length' of the system l, as a cable length, related to the nondimensional time parameter τ by,

$$l\tau = \frac{tV}{\mu},$$

where $\mu = \dfrac{m}{\rho l^3}$ and $m =$ mass of body.

Defining τ by $\dfrac{t}{\tau} = \sqrt{\dfrac{m}{R}}$, where $R =$ drag per unit length of cable, gave

$$l^2 = \frac{m}{\rho V}\sqrt{\frac{R}{m}} = \frac{m}{\rho V}\sqrt{\frac{\frac{1}{2}\rho V^2 d\, C_D}{m}},$$

where $d =$ cable diameter.

Squaring both sides of this equation yielded

$$l^4 = \frac{m\,d\,C_D}{2\rho} \qquad \text{or} \qquad l = 4\sqrt{\frac{m\,d\,C_D}{2\rho}}. \tag{2}$$

The lateral stability equation was then developed, that is, the equation determining λ where the equations of motion are assumed to have solutions of the form $e^{\lambda t}$. The equation in the determinant form:

$$\begin{vmatrix} \lambda^2 + y_v\lambda + \gamma & y_v - k_D \\ -\mu\, n_v \lambda & \lambda^2 + n_r\lambda - \mu\, n_v \end{vmatrix} = 0 \qquad \text{(Equation 60 of Ref. 2)}$$

became, in terms of corresponding longitudinal derivatives:

$$\begin{vmatrix} \lambda^2 + z_w\,\lambda + \gamma & z_w - k_D \\ \mu\, m_w \lambda & \lambda^2 + m_q\lambda + \mu\, m_w \end{vmatrix} = 0. \tag{3}$$

The non-dimensional coefficients used here for forces and moments were:

$$k_D = \frac{D}{l^2\,\rho\,V^2}; \qquad z_w = -\frac{Z_w}{l^2\,\rho\,V}$$

$$m_w = -\frac{m\,M_w}{B\,l\,\rho\,V}; \qquad m_q = -\frac{m\,M_q}{B\,l^2\,\rho\,V} \tag{4}$$

where D = drag of towed body

B = body's moment of inertia about its centre of gravity.

The coefficient z_q entered into the stability equation only in the form $(\mu - z_q)$, and since z_q is generally only a small fraction of μ, Glauert neglected it.

On expansion, equation (3) became

$$\lambda^4 + (z_w + m_q)\,\lambda^3 + (\gamma + \mu\,m_w + z_w\,m_q)\,\lambda^2 + (\gamma\,m_q + \mu\,m_w\,k_D)\,\lambda + \gamma\,\mu\,m_w = 0,$$

with Routh's discriminant given by

$$\Delta = (z_w + m_q)(\gamma + \mu\,m_w + z_w\,m_q)(\gamma\,m_q + \mu\,m_w\,k_D) - (\gamma\,m_q + \mu\,m_w\,k_D)^2 - \gamma\,\mu\,m_w\,(z_w + m_q)^2. \tag{5}$$

This expression is a quadratic in $(\mu\,m_w)$; thus

$$\Delta = A(\mu\,m_w)^2 - B(\mu\,m_w) + C$$

and will be negative for a range of positive values of $\mu\,m_w$, if $B^2 - 4AC > 0$.

By deriving these coefficients from equation (5) this inequality was reduced to the form

$$\gamma^2\,(z_w - k_D)^2 - 2\gamma\,k_D\,z_w\,m_q\,(z_w + 2m_q - k_D) + k_D^2\,z_w^2\,m_q^2 > 0.$$

This is satisfied if γ lies outside the range defined by the two critical values:

$$\gamma = \frac{k_D z_w m_q}{(z_w - k_D)^2}\left\{\sqrt{z_w - k_D + m_q} \pm \sqrt{m_q}\right\}^2. \tag{6}$$

Equation (6) gives, for a specific speed, a range of cable lengths within which the lateral motion of the body is stable for all values of μm_w. Outside these limits, however, there is always a range of values of μm_w for which the motion becomes unstable.

Substituting values appropriate to Configuration B of the present test body, the critical values of γ were found to be 0·3836 and 0·004417. Hence instability is only possible where $1/\gamma = b/c$ is less than 2·6067 or greater than 227. From equation (1), $c = 200$ feet when $V = 88$ ft/s, at which speed instability may thus arise for values of b below (200 × 2·6067) or above (200 × 227). This upper limit of $b = 45\,400$ feet is physically absurd, hence the dangerous range of cable lengths is, in practice, between zero and 521 feet.

In Ref. 2, Glauert next listed a number of values of b/c below the lower critical value, appropriate to the aircraft and operating conditions of the time. For each value he substituted the appropriate figure for γ in turn, in the discriminant (equation (5)). By equating the latter to zero, solution yielded two values of μm_w for each value of b/c, these being the boundary values within which oscillations are possible.

For a given configuration of body and cable, however, the value of μm_w is fixed. Hence, the author reversed Glauert's procedure by inserting this value in equation (5), equating to zero, and solving to give boundaries in terms of γ (and hence, pendulum length b) for the region of instability of the system.

The results, plotted against airspeed V are presented in Fig. 8, for Configurations B and E of the original test body (*see* Section 5).

Similar calculations were later made for the new cable/body system of Section 6, the results being plotted in Fig. 11.

Values of physical constants of the three systems for which calculations were made and of the appropriate aerodynamic coefficients are summarised in Table 2.

TABLE 1

Test Configurations of Cable/Body System Investigated in 24ft Wind Tunnel.

Configuration identification letter	Description of body configuration	Body weight (lb)	Moment of inertia, about cg suspension point (slug ft^2)	Moment arm of aerodynamic forces about suspension point in plan (ft)
A	As supplied, with dorsal fin and cable clamp block	65·2	6·36	1·10
B	Increased length body; fins rotated to 0°/90° position; extra ballast in nose to bring cg forward	64·5	9·55	1·86
C	As B, but with fins in original 45° position	64·5	9·55	1·66
D	As B, with fins in 0°/90° positions; extra nose ballast removed and suspension point moved 3·38 inches aft.	57·7	8·10	1·58
E	As B, with drag plates on vertical fin (Fig. 1); external trim weight on nose	65·2	9·79	?

TABLE 2

Summary of Stability Data using Glauert Notation.

Body-cable system data		Original		New body
		Config. B	Config. E	
Body weight (lb)		64·5	65·2	44·1
Body mass (slug)		2·006	2·025	1·371
Body drag (lb) at $V = 60$ ft/s		0·5725	1·7615	0·367
therefore drag coefficient C_D (frontal area)		0·186	0·572	0·210
k_D		0·0173	0·0533	0·0129
Body moment of inertia about cg B(slug ft^2)		9·55	9·79	4·085
M_w/V		+0·0277	+0·0277	+0·0135
Z_w/V		−0·0118	−0·0118	−0·0061
M_q/V		−0·1028	−0·1028	−0·0505
m_w		+0·6325	+0·6325	+0·5691
z_w		+1·283	+1·283	+0·7638
m_q		+2·350	+2·350	+2·1366
System typical length, l(ft)		1·968	1·968	1·826
$cV^2/10^5$		15·55	15·55	9·635
μ		110·66	110·66	94·64
Boundary values of γ for any $\mu\, m_w$ (equation (6))	lower	0·3836	—	0·3716
	upper	0·00442	—	0·00210
Solution of γ for body value of $\mu\, m_w$	lower	99·0	97·7	66·48
	upper	1·07	3·25	1·026
Therefore boundary values of $bV^2/10^4$	lower	1·5707	1·5915	1·4495
	upper	145·320	48·141	93·930

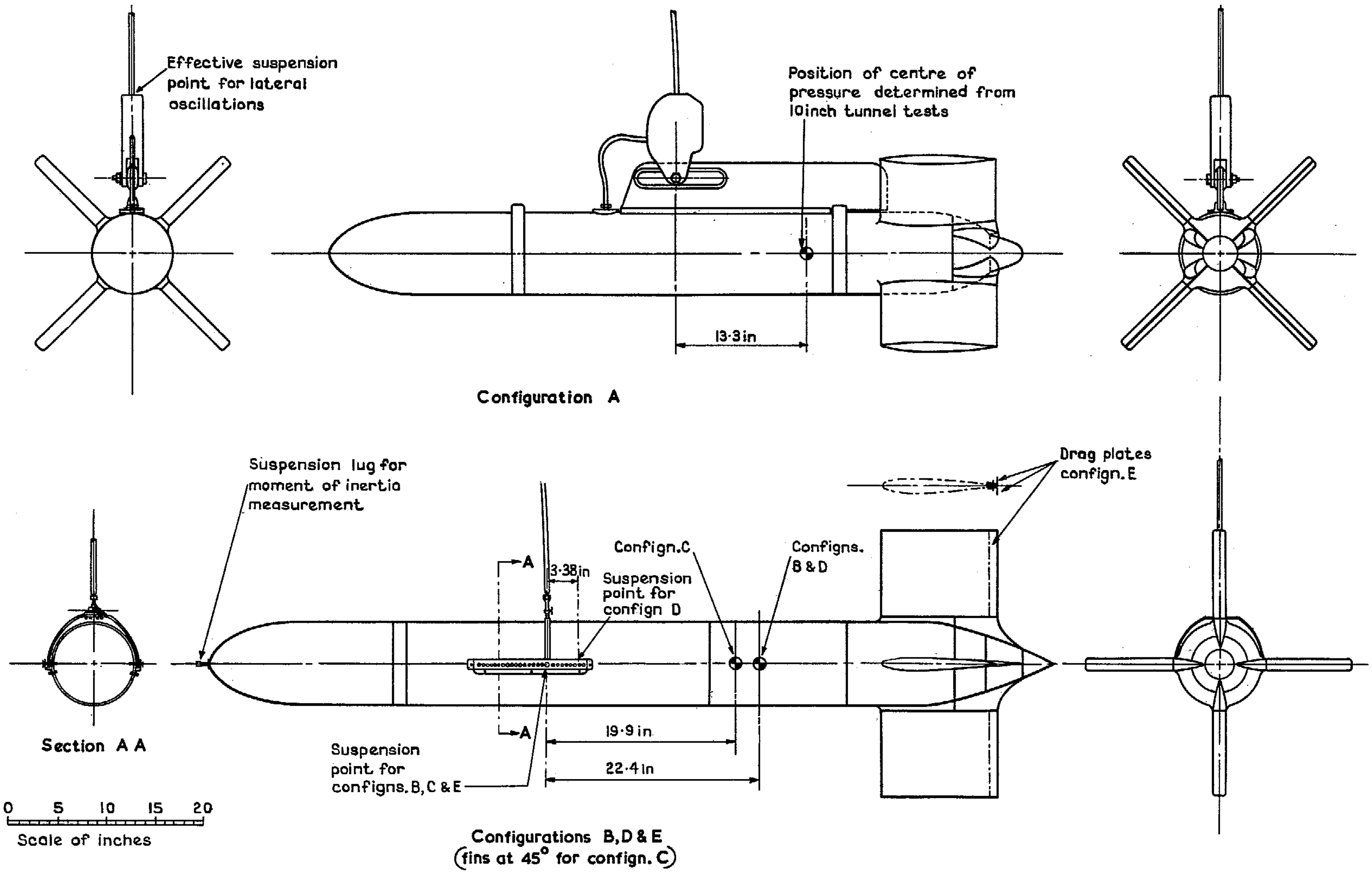
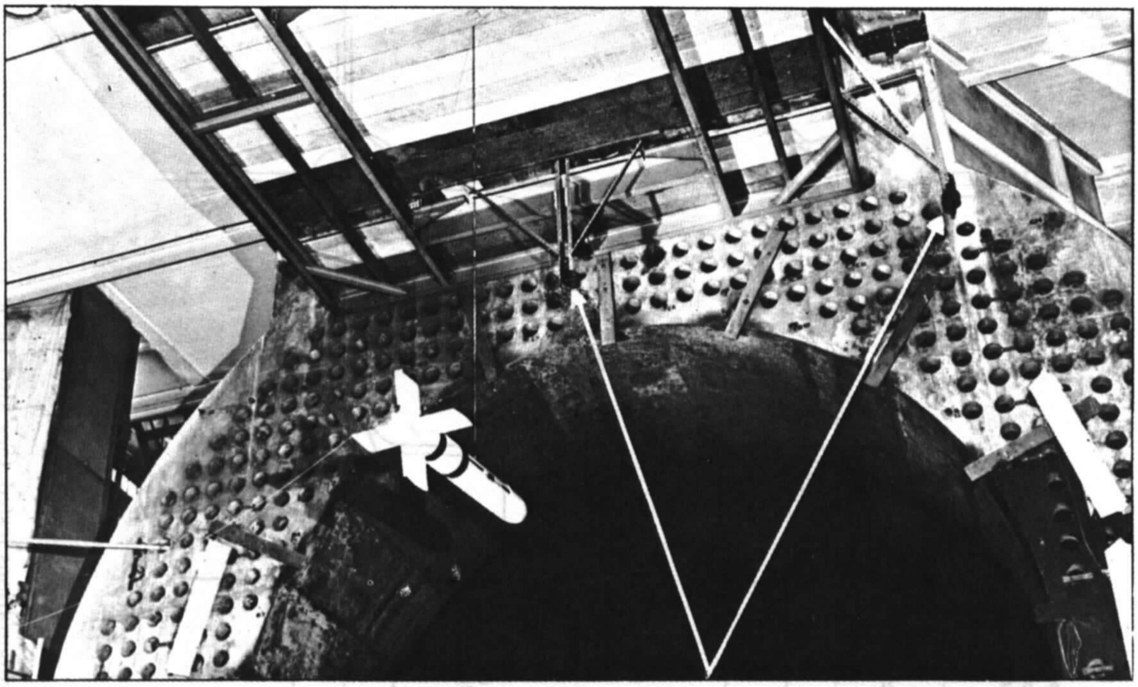


FIG. 1. General arrangement of body, showing modified configurations tested. (*See* also Table 1).



a. Body suspended below balance car in 24ft tunnel

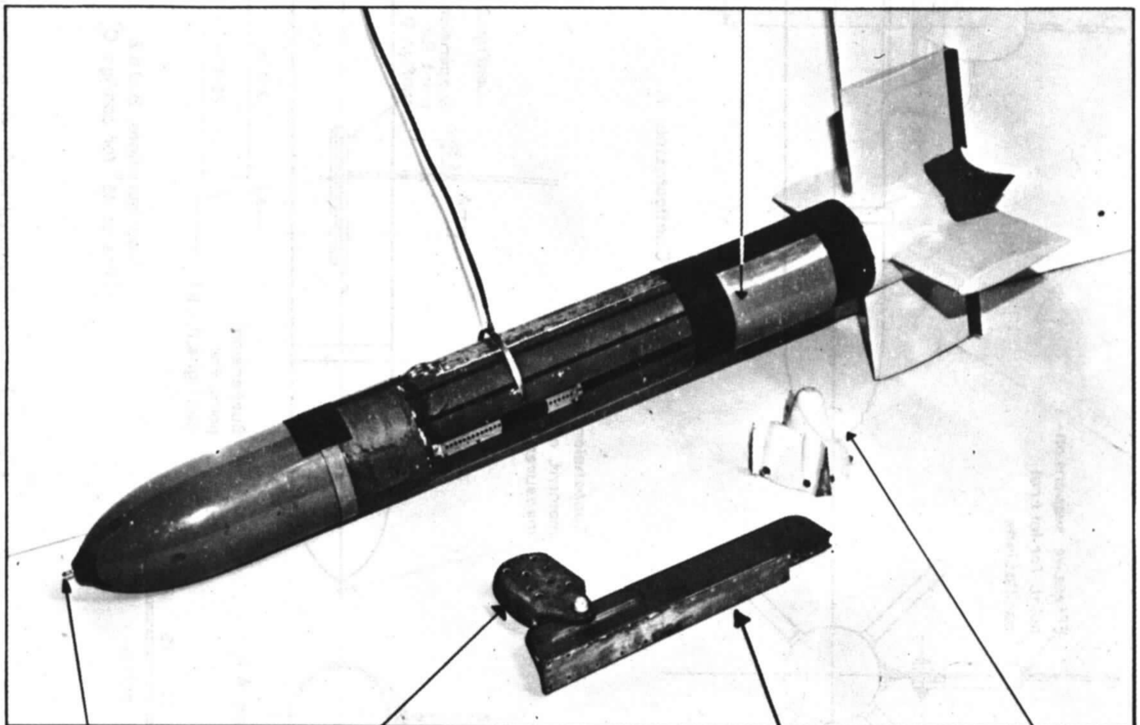


FIG. 2. Modified towed body.

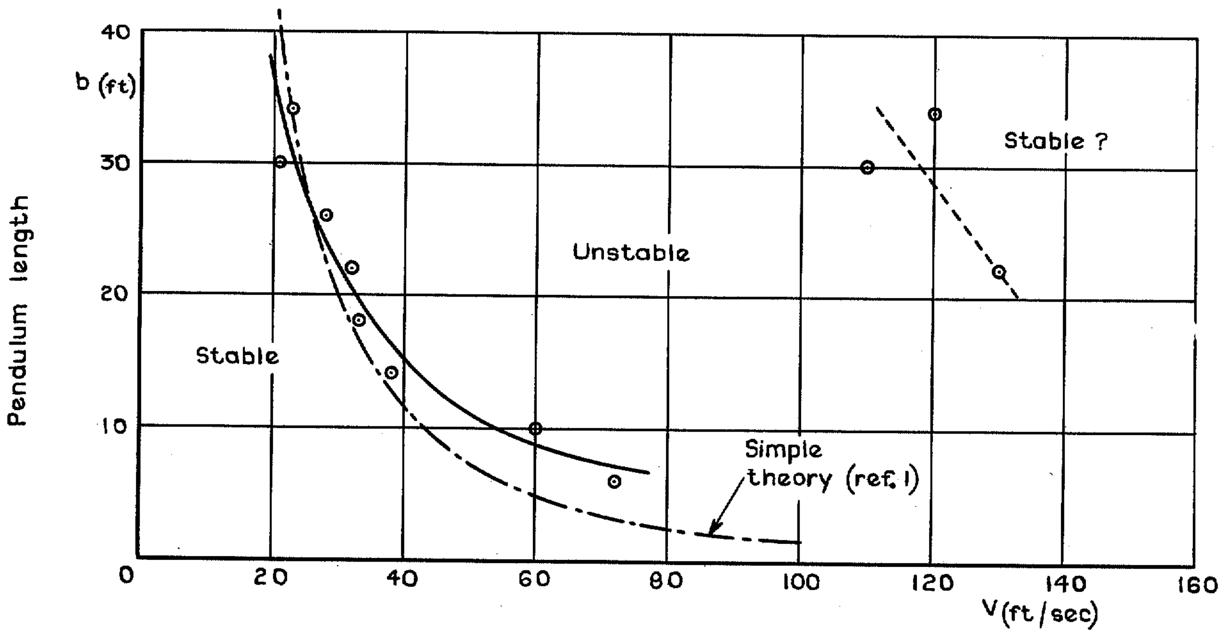


FIG. 3. Stability boundaries of body as supplied—Configuration A.

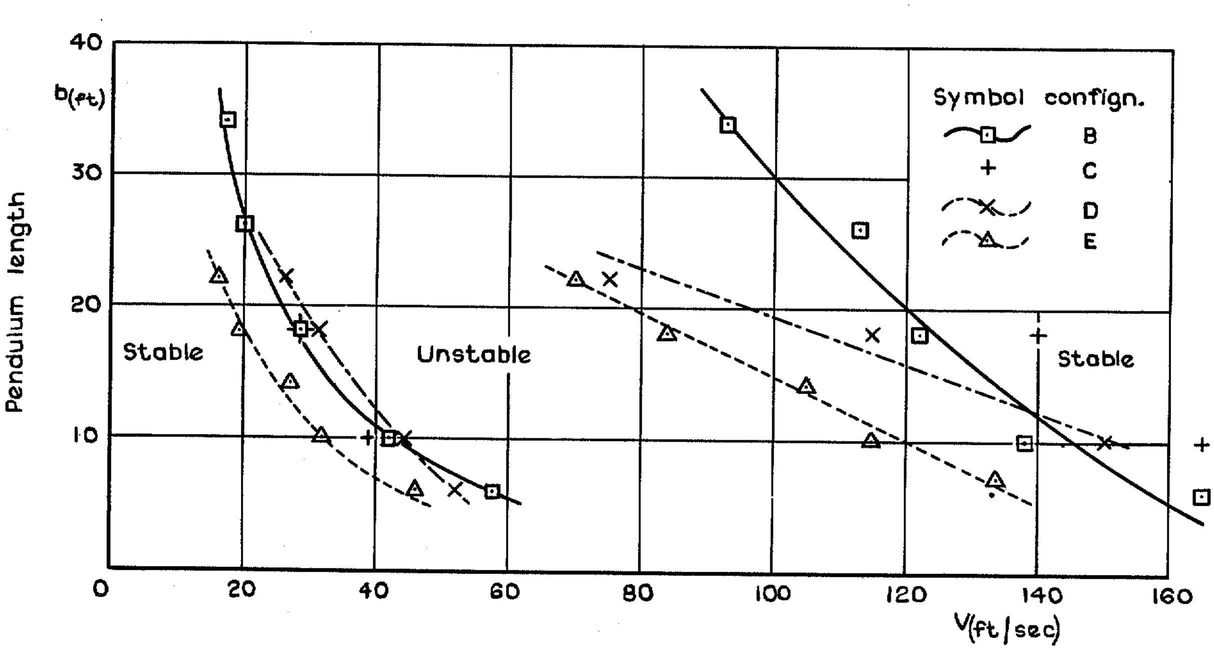


FIG. 4. Stability boundaries of four configurations with lengthened body.

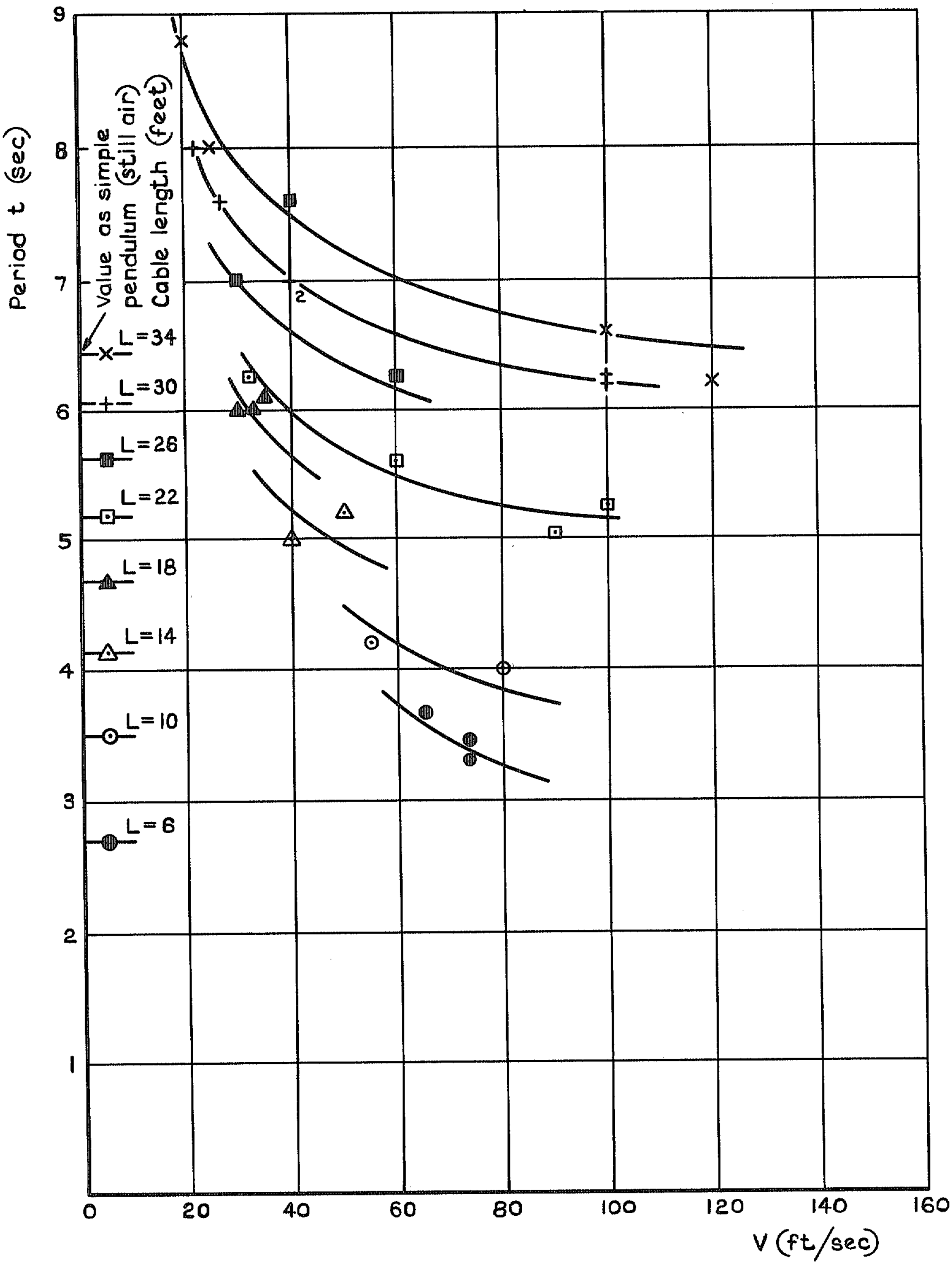


FIG. 5. Period of oscillation—Configuration A.

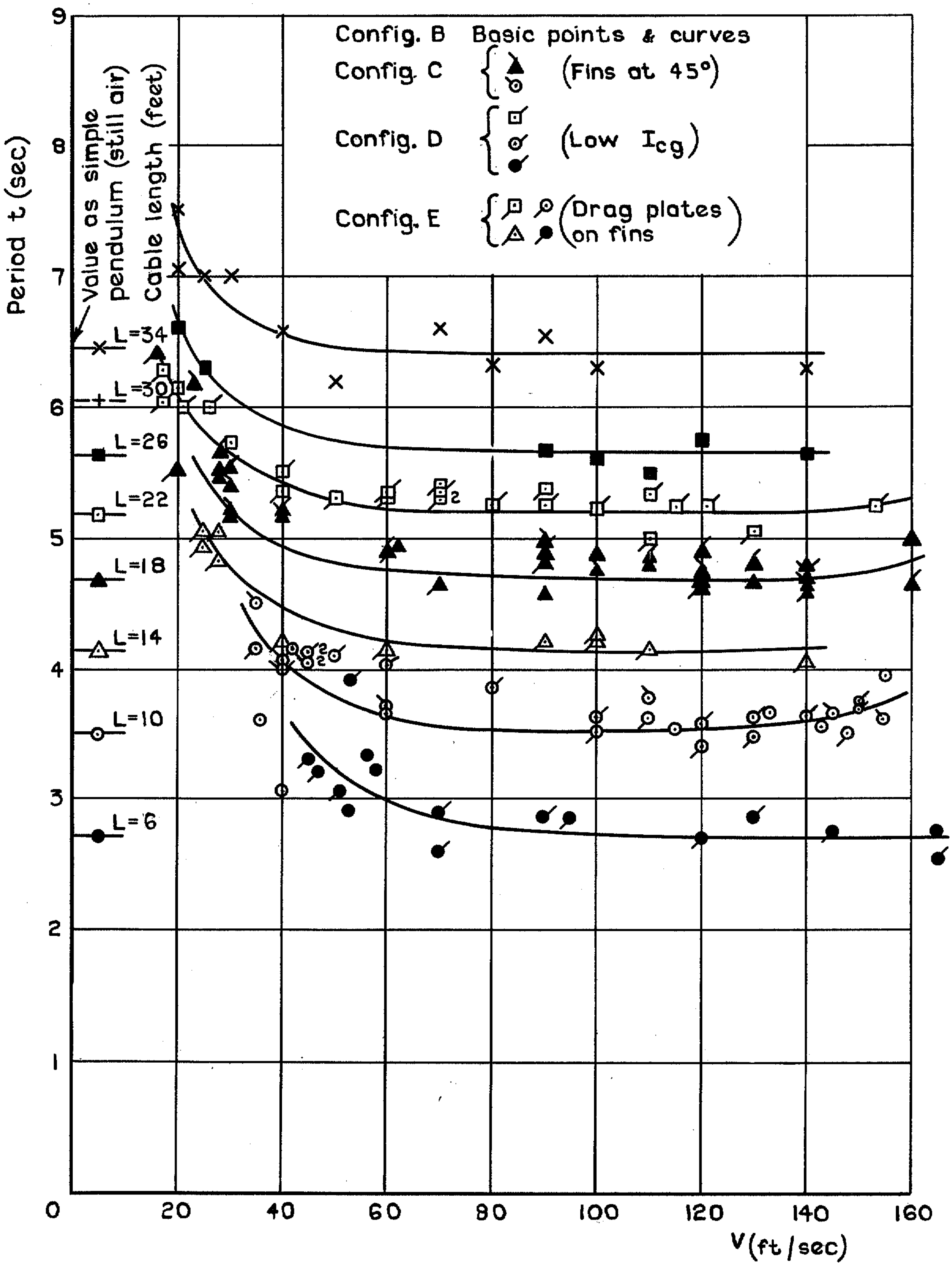


FIG. 6. Period of oscillation with lengthened body—Configurations B, C, D and E.

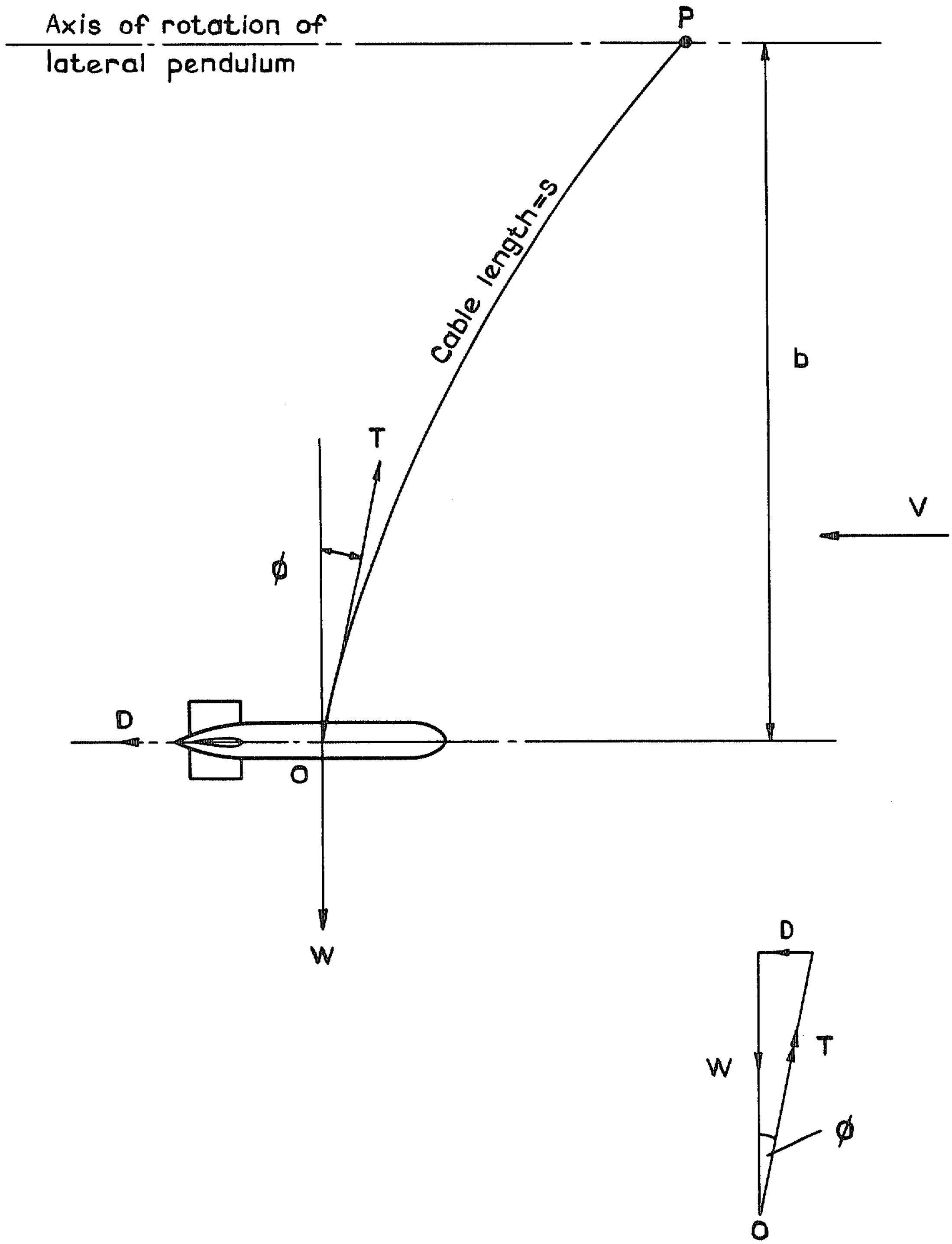


FIG. 7. Stability notation.

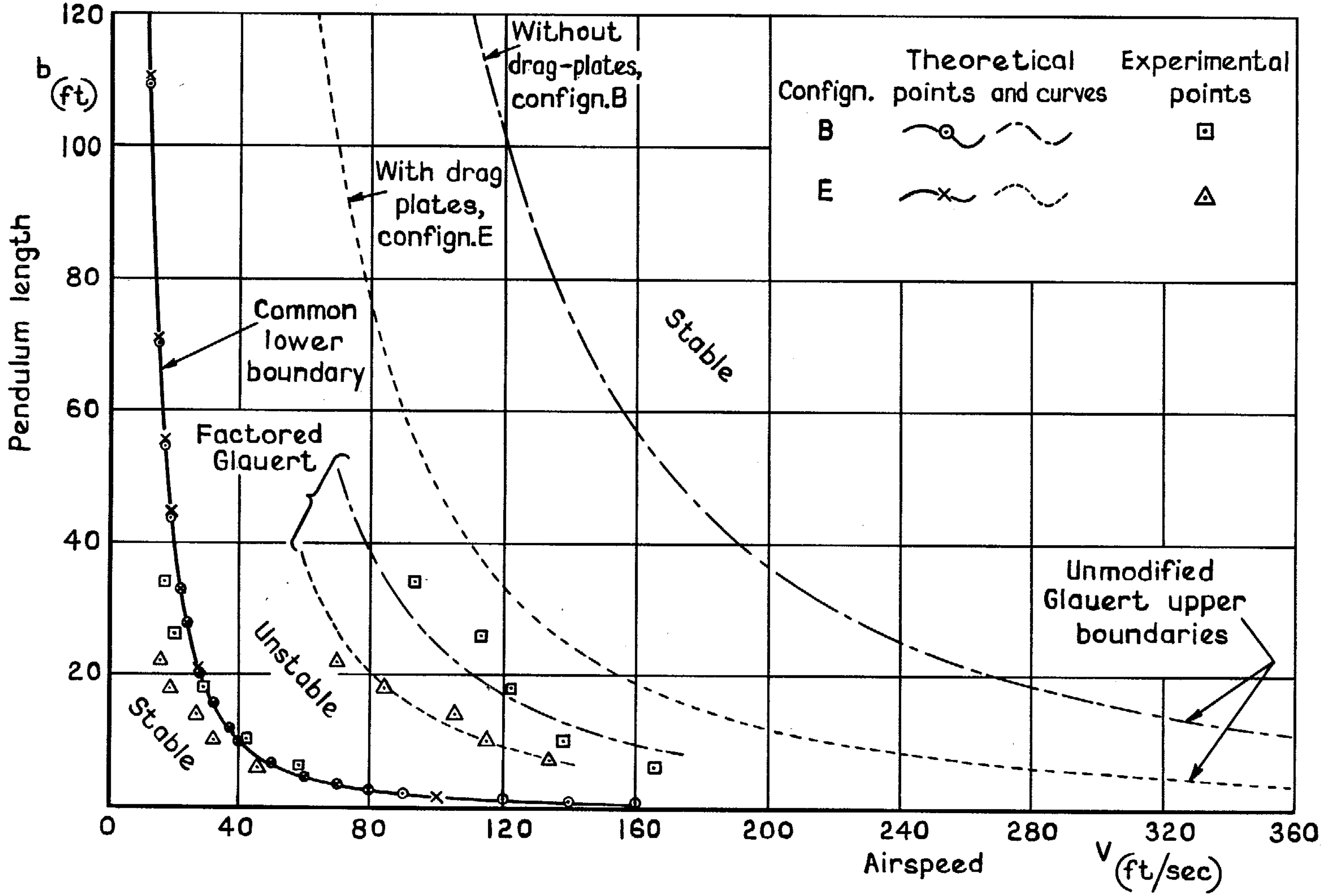


FIG. 8. Comparison of theoretical and experimental stability boundaries for original body extended (Configuration B) and with drag-plates (Configuration E). Note: Glauert theory (Ref. 2) with drag factor (Appendix).

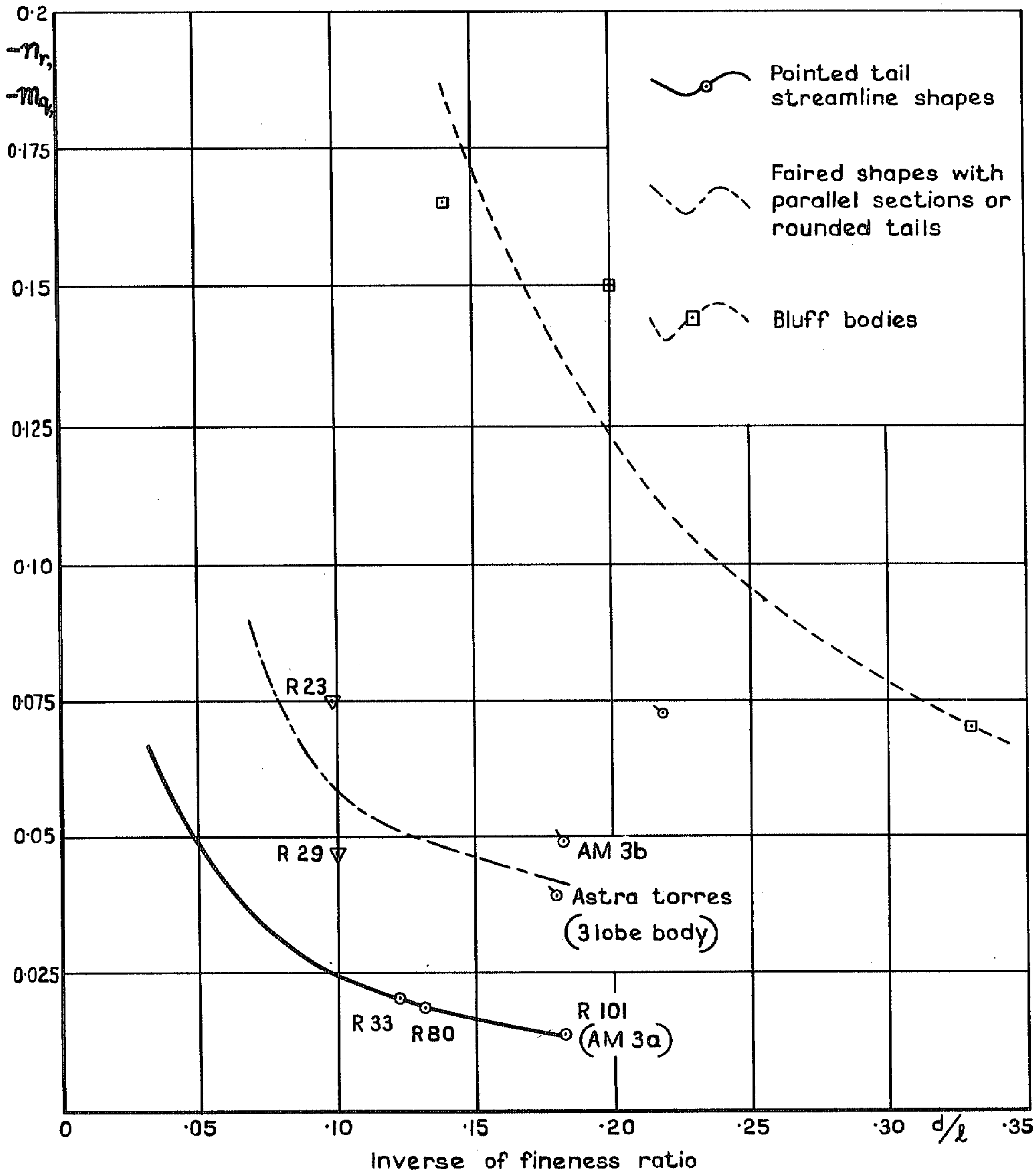


FIG. 9. Contribution of body to damping derivative.

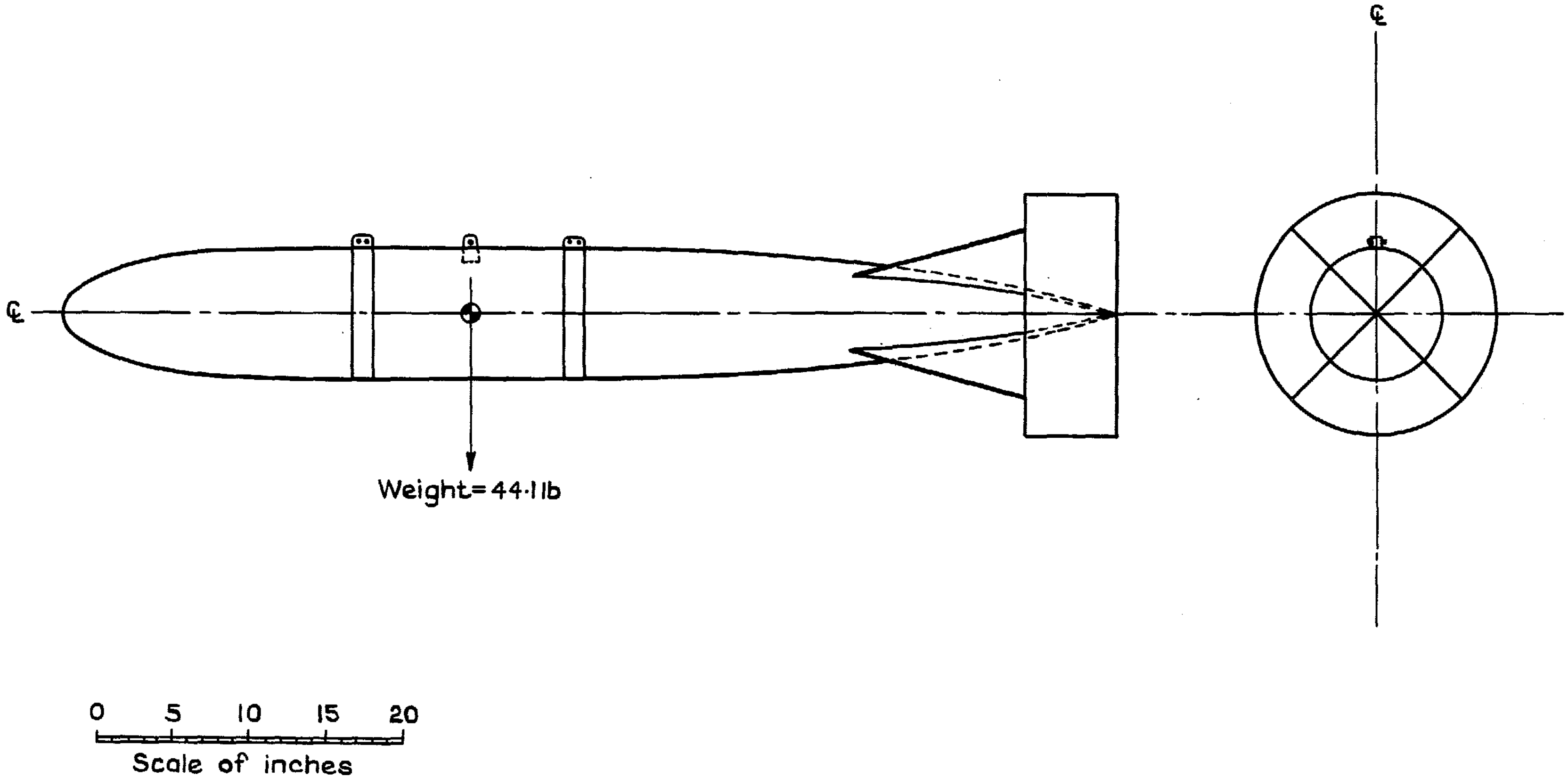


FIG. 10. G.A. of new body.

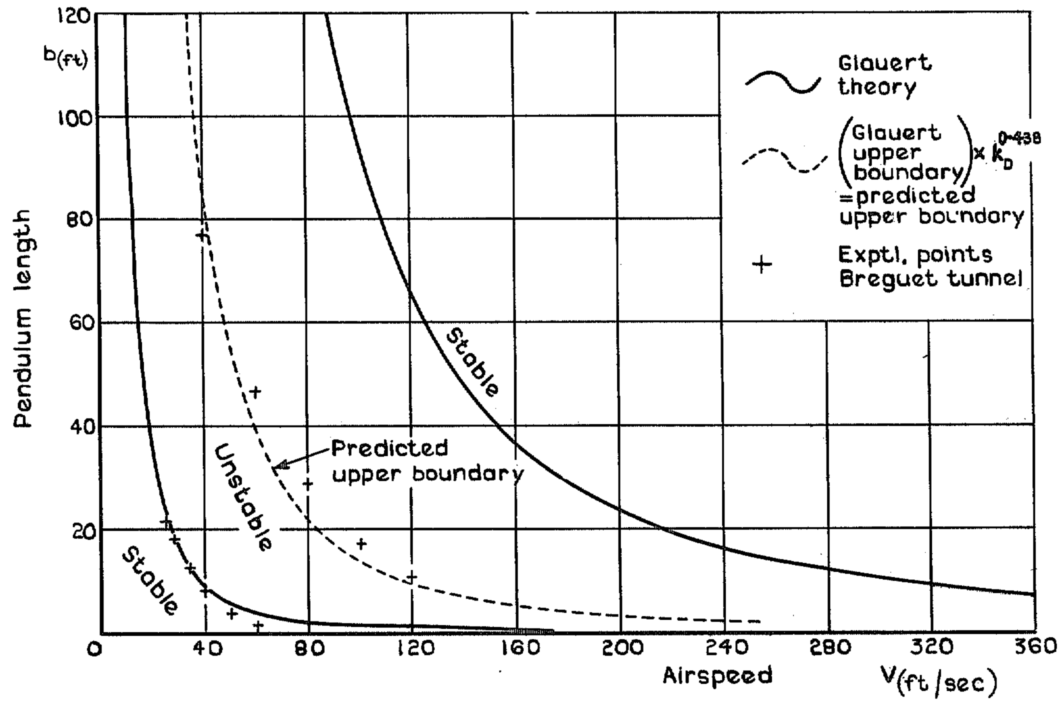


FIG. 11. Predicted stability boundaries for new body, compared with subsequent French tunnel tests.

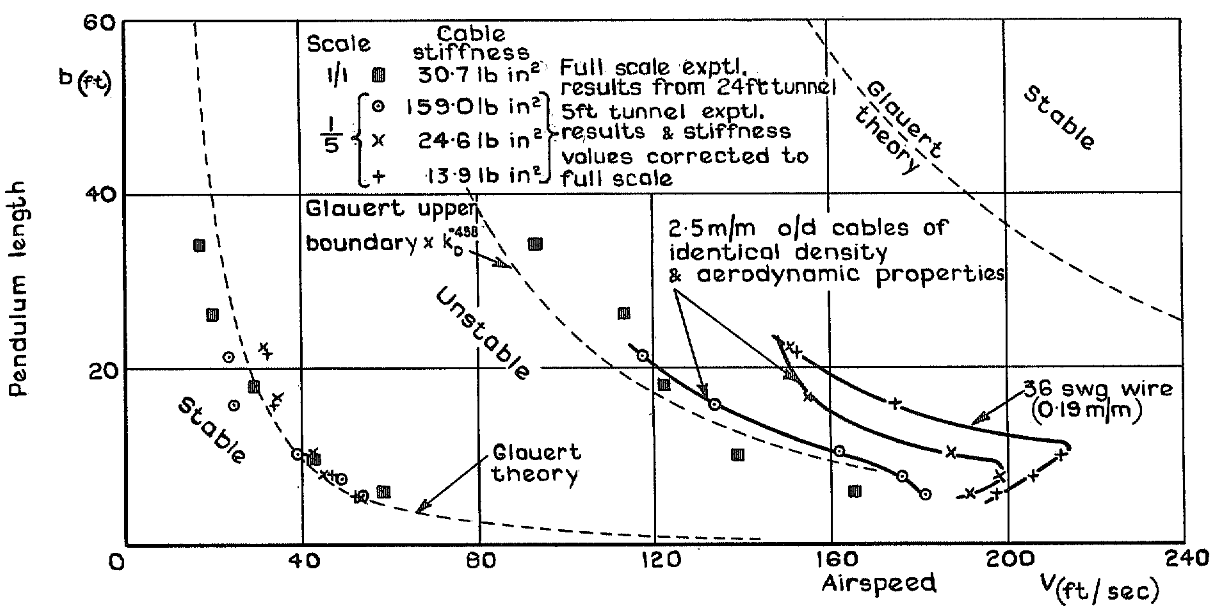


FIG. 12. Configuration B—effect of cable stiffness on stability boundaries.

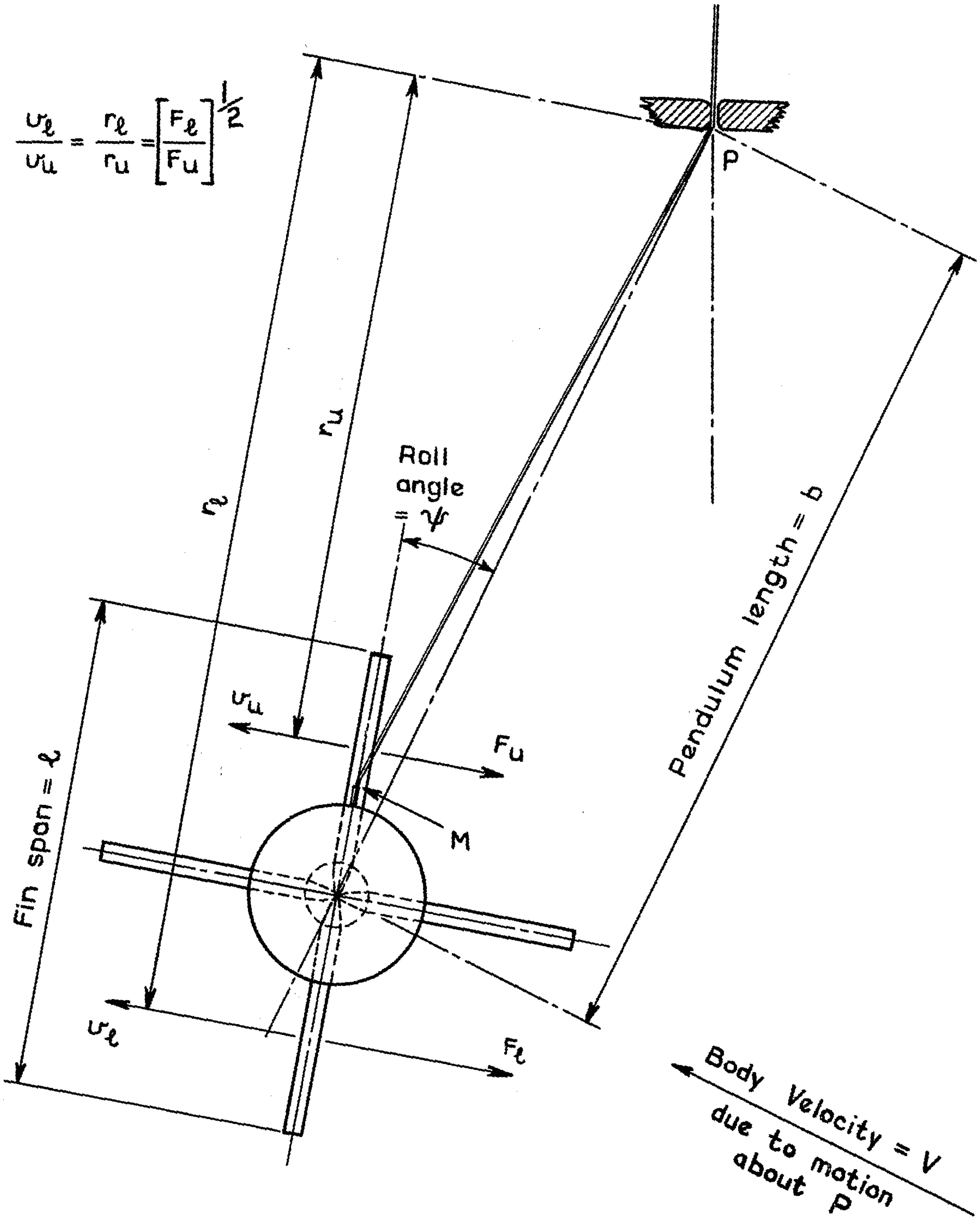


FIG. 13. Body oscillating on short cable (condition generating out-of-phase rolling oscillation).

Printed in Wales for Her Majesty's Stationery Office by Allens Printers (Wales) Limited
Dd. 501372 K.5

Published by
HER MAJESTY'S STATIONERY OFFICE

To be purchased from
49 High Holborn, London WC1
13a Castle Street, Edinburgh EH2 3AR
109 St Mary Street, Cardiff CF1 1JW
Brazennose Street, Manchester M60 8AS
50 Fairfax Street, Bristol BS1 3DE
258 Broad Street, Birmingham 1
7 Linenhall Street, Belfast BT2 8AY
or through any bookseller

R. & M. No. 3644
SBN 11 470364 7