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Conditions for the Existence of an Inertial Subrange in Turbulent Flow

by P. BRADSHAW

Aerodynamics Division, N.P.L.

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CORRECTION

Paragraph 2, line 8, read: Re_{λ}^{2}

Paragraph 4, line 3, read:

. . . " the Kolmogorov length scale $l \equiv (\nu^3/\varepsilon)^{\frac{1}{4}}$ ":

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Summary.

It is found experimentally that the value of K^1 in the spectrum formula $\phi(k_1) = K^1 \varepsilon^{2/3} k_1^{-5/3}$ is constant down to a microscale Reynolds number Re_λ of about 100, both in homogeneous turbulence and in shear flow: this value of Re_λ is much less than that usually predicted. The result can be used to derive dissipation from medium-frequency spectra at moderate laboratory Reynolds numbers.

The inertial subrange is the part of the wave-number spectrum where both the supply and the dissipation of turbulent energy per octave are small compared with the rate of energy transfer through the spectrum, the latter being equal to the total viscous dissipation at high wave numbers, ε. Dimensional arguments[1] show that, for instance, the longitudinal wave-number spectral density of the u-component fluctuations in this range is $\phi_{11}(k_1) = K^1 \varepsilon^{2/3} k_1^{-5/3}$ where K^1 is a universal constant and ϕ_{ij} is the spectral density of $\overline{u_i u_j}$: the v- and w-component spectra are 4/3 times this. It is usually stated that an inertial subrange occurs only at large values of the Reynolds number $Re_\lambda \equiv \sqrt{\overline{u^2}}\, \lambda/\nu$ (the microscale λ will be defined for present purposes by $\varepsilon = 15\, \nu \overline{u^2}/\lambda^2$), and cannot be observed in ordinary laboratory experiments: previous theoretical estimates for the required Reynolds number are discussed in Ref. 2, of which this note is a revision and condensation.

In Fig. 1 are plotted the apparent value of K^1 obtained by drawing a $-5/3$ power law tangent to experimental spectra in grid turbulence, channel flow, jets and the inner and outer parts of boundary layers[3-13]: where possible, direct dissipation measurements have been used, but the tagged symbols were obtained by using the emperical formula for dissipation in a boundary layer, $\varepsilon = (\tau/\rho)^{3/2}/L$, where L/δ is to a first approximation a universal function of y/δ (an assumption justified *a posteriori* in Ref. 14). Although the scatter is rather large it appears that K^1 is constant for all flows down to $Re_\lambda \simeq 100$, compared with theoretical values between 250 (Corrsin[15]) and 1730 (Stewart and Townsend[3]): note that the usual Reynolds number based on mean flow scales is proportional to Re_λ^2. In a boundary layer, Re_λ equals 100 when $u_\tau y/\nu$ equals 250 (say, $y = 0{\cdot}1$ in. at a free stream velocity of 100 ft/sec in air). Once K^1 deviates from its constant value the behaviour may depend on the type of flow.

The practical implication of Fig. 1 is that dissipation can be obtained very easily from medium-frequency spectrum measurements in the laboratory, a much easier task than measuring spectra in the dissipation range itself, which requires about ten times the frequency response and also very short wires. If K^1 is taken as $0{\cdot}5 \pm 0{\cdot}05$ the uncertainty of dissipation measurement is ± 15 per cent which is certainly no more than the uncertainty of direct measurements in laboratory conditions.

The ostensible *physical* implication of Fig. 1 is that an inertial subrange occurs if the energy-containing length scale of the flow (conservatively taken as the dissipation length parameter L which is about $0{\cdot}1\delta$ in the outer part of a boundary layer) is 100 times the Kolmogorov length scale $l = (\nu^3/\varepsilon)^{\frac{1}{4}}$: this figure follows from the definitions of l and L if $Re_\lambda = 100$ and $\tau = 0{\cdot}4\, \overline{u^2}$. It is certain that *all* the attributes of an inertial subrange will not occur at such moderate Reynolds numbers, at least in a shear flow. The

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*Replaces N.P.L. Aero Report 1220—A.R.C. 28 664.

shear correlation coefficient $\phi_{12}(k)/[\phi_{11}(k)\,\phi_{22}(k)]^{\frac{1}{2}}$ may be of the order of 0·2 in the 'inertial' range and several sets of measurements show that the v-component spectral density, and possibly that of the w-component, may be less than $4/3\,\phi_{11}$: anisotropy of intensity is inseparable from non-zero shear stress as the distinction between the two depends on the axes, and this vitiates some of the remarks in Ref. 2. The simplest way of interpreting the results is to suppose that the reduction in spectral density below the subrange value at low Reynolds numbers is opposed by the production of turbulent energy, which first appears in the u-component. The truth is probably more complicated and, despite the further speculations in Ref. 2, all that we can be sure of at present is that Fig. 1 demonstrates a very useful empirical fact.

LIST OF SYMBOLS

K^1	Defined by $\phi_{11}(k_1) = K^1 \varepsilon^{2/3} k_1^{-5/3}$
k	Wave number magnitude
k_1	Longitudinal wave number
L	Dissipation length parameter, $(\tau/\rho)^{3/2}/\varepsilon$
l	Typical length scale of energy-containing eddies
Re_λ	Turbulence Reynolds number, $\sqrt{\overline{u^2}}\,\lambda/\nu \simeq 10[(\tau/\rho)^{\frac{1}{2}} L/\nu]^{\frac{1}{2}}$
u, v, w	Velocity fluctuations in x, y, z directions
u_τ	$\sqrt{\tau/\rho}$
v	Velocity scale of dissipating eddies, $(\nu\varepsilon)^{1/4}$
ε	Dissipation rate
λ	Turbulence microscale, defined by $\varepsilon = 15\nu\,\overline{u^2}/\lambda^2$ and equal to $[\overline{u^2}/\overline{(\partial u/\partial x)^2}]^{\frac{1}{2}}$ if the dissipating eddies are isotropic
ν	Kinematic viscosity
τ	Turbulent shear stress, $-\rho\,\overline{uv}$
$\phi_{ij}(k)$	Spectral density of $\overline{u_i u_j}$
Suffixes	
i, j	1, 2, 3 for x, y, z directions

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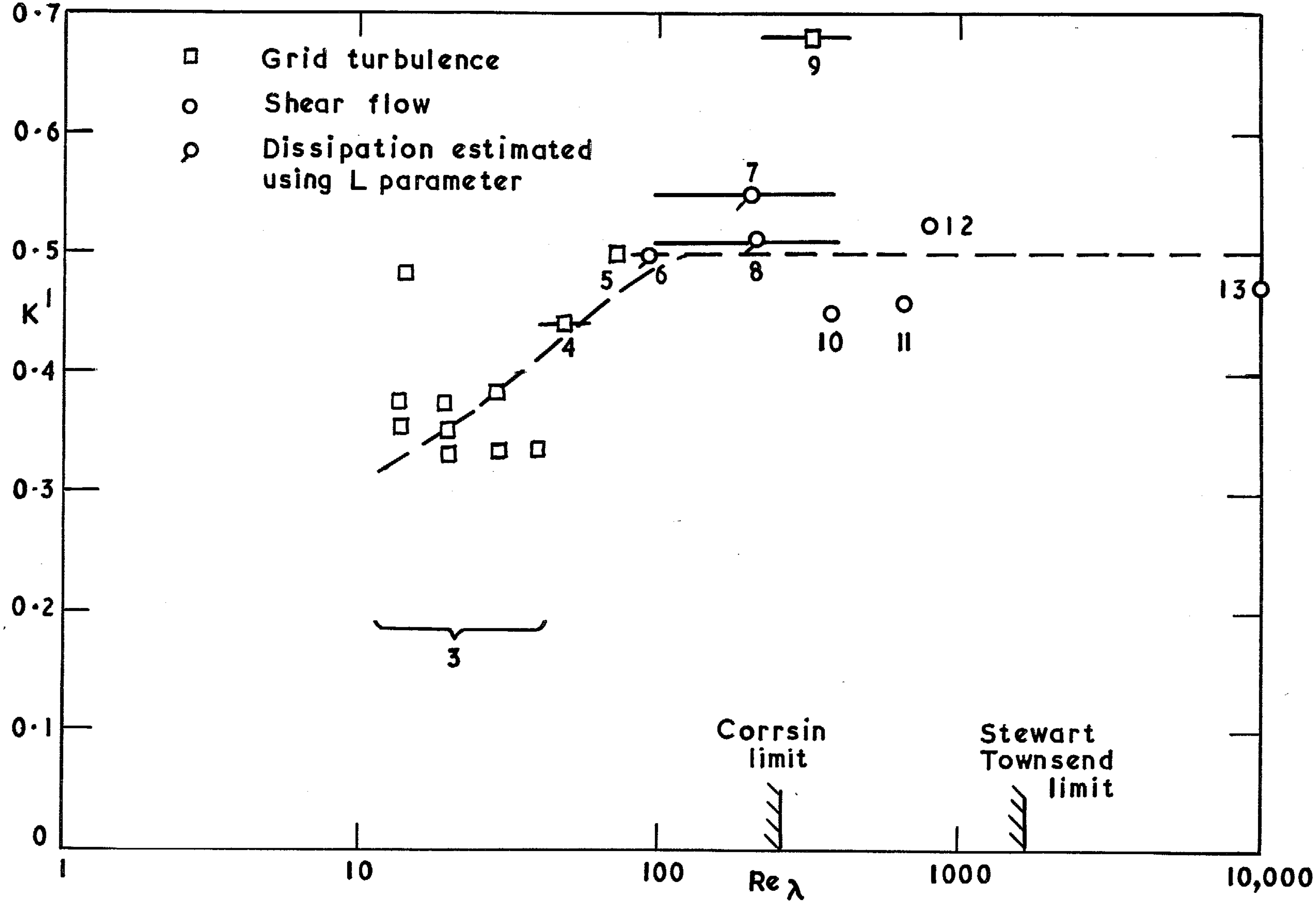

FIG. 1. Variation of subrange factor K^1 with turbulence Reynolds number (numbers indicate References).

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