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Calculations Showing the Influence of Aerodynamic Damping on Binary Wing Flutter

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Summary.

The influence of damping is examined by comparing the flutter critical speeds and frequencies obtained using the complete equations of motion and those obtained when all the damping terms are omitted. The calculations are made for binary flexure-torsion flutter of a wing in incompressible flow and the comparisons extend over ranges of values of the principal parameters.

When the ratio of air density to wing density is high, the damping terms are found to raise the flutter speed by a large amount. But when the ratio of air density to wing density is low, a condition often occurring in practice, it appears that, over a wide range of parameter variation, the flutter speed obtained in the absence of damping is reasonably close to that obtained from the complete equations. Further experience is obviously needed before any general conclusions can be reached, but the present results offer some hope of reducing either the extent of, or the accuracy needed in, the aerodynamic information used for wing flutter calculations.

It is pointed out that with constant aerodynamic coefficients the flutter speed for an undamped system is independent of the wing density.

LIST OF CONTENTS

*Replaces N.P.L. Aero Report 1236—A.R.C. 29264.

1. *Introduction.*

The influence of damping on coupled flutter has remained to a large extent obscure and sometimes conclusions regarding its effects have seemed paradoxical. For instance, although flutter can be prevented if sufficient damping is applied, the application of some lesser amount may be far from beneficial. As early as 1939, Frazer[1] demonstrated both theoretically and experimentally that, in some circumstances, the application of additional damping can change a system from a condition of stability to one of divergent flutter. The recent analytical papers of Nissim[2], Done[3], Niblett[4] and Nemat-Nasser and Herrmann[5] have again drawn attention to the influence of damping and do much to provide an advance in understanding the matter.

The effects of damping on flutter have long been of practical interest in relation to the use of dampers to prevent control surface flutter. Another less obvious but important aspect of the subject concerns the sensitivity of the flutter condition to changes in the values of the aerodynamic damping terms in the equations of motion. Such an interest arises, for instance, when attempting to specify the extent and accuracy of the aerodynamic information needed for acceptable flutter prediction. Also, the matter is of interest to wind-tunnel flutter testing because aerodynamic dampings can be subject to large wall-interference effects[6]. It is therefore of value to make a comparison between the results of flutter calculations when the aerodynamic damping terms are completely ignored and those obtained when all the terms are included.

Equations of motion without damping terms were used by Williams[7] when discussing the dynamic divergence problem of an 'aero-isoclinic' wing, and Lambourne and Chinneck[8] found that calculations based on this method gave excellent agreement with the flutter critical speeds measured for a wind tunnel model. The experiments were designed primarily to elucidate the behaviour of an 'aero-isoclinic' system and no general conclusions regarding the effect of the damping terms could be drawn from these results because of the unconventional features of the system and the unrepresentative mass of the model.

Based on this work, the need for an examination of the effects of ignoring the damping terms for a more typical wing system was realised and led to the present calculations, many of which were made over 12 years ago but have lain unissued because of certain features which, at the time, seemed anomalous. With the appearance of recent analytical papers which provided clarification, and with the growth of interest in vehicles for which the ratio of wing density to air density is high, the original calculations have now been extended.

Over the years various, and apparently independent, applications of the equations for an undamped system have been made in discussions of coupled flutter. Molyneux[9], recognizing that, for an aircraft at high altitude, there may be some justification for neglecting aerodynamic damping, developed a simplified flutter criterion on this basis. Rocard[10] describes the use of 'undamped' equations of motion and discusses the effects of a subsequent introduction of damping terms (*see* Chapter 3 of Ref. 10). Pines[11] has used the method to arrive at a closed solution of the flutter equations, from which the qualitative effects of certain parameters can readily be deduced; a discussion of this particular application is given by Bisplinghoff and Ashley[12].

The present Report gives the results of calculations to show the effect of neglecting all the aerodynamic damping terms for an example of flexure-torsion flutter of a wing in incompressible flow. The wing system, chosen as being most appropriate at the time when the work was started, is that used previously by Duncan and Lyon[13] who calculated by the conventional method the flutter characteristics for a comprehensive range of parameter variation. The calculations of the present Report have been based on the 'undamped' equations of motion using, for the remaining aerodynamic terms, values the same as those used in the calculations of Duncan and Lyon. The calculated critical speeds and frequencies are then compared graphically with those published in Ref. 13. It may be noted that, as early as 1937, Pugsley[14] had proposed a simplified method of flutter calculation which ignored the cross damping terms whilst retaining the direct dampings; he showed, again for the wing of Ref. 13, that good agreement could be obtained with the results from the full equations.

2. *The Equations of Motion and their Solution.*

The full equations of motion for a wing system having two degrees of freedom, ϕ and θ, have the following well known form:

$$\left.\begin{aligned} A_{11}\ddot{\phi}+B_{11}\dot{\phi}+C_{11}\phi+A_{12}\ddot{\theta}+B_{12}\dot{\theta}+C_{12}\theta = 0 \\ A_{21}\ddot{\phi}+B_{21}\dot{\phi}+C_{21}\phi+A_{22}\ddot{\theta}+B_{22}\dot{\theta}+C_{22}\theta = 0 \end{aligned}\right\} . \tag{1}$$

When all the damping terms are omitted we have

$$\left.\begin{aligned} A_{11}\dot{\phi}+C_{11}\phi+A_{12}\ddot{\theta}+C_{12}\theta = 0 \\ A_{21}\dot{\phi}+C_{21}\phi+A_{22}\ddot{\theta}+C_{22}\theta = 0 \end{aligned}\right\} . \tag{2}$$

A general solution of equations (2) is

$$\left.\begin{aligned} \phi = \phi_0\, e^{\lambda t} \\ \theta = \theta_0\, e^{\lambda t} \end{aligned}\right\} , \tag{3}$$

where λ is given by the auxiliary equation

$$A\lambda^4+B\lambda^2+C = 0 \tag{4}$$

which yields

$$\lambda^2 = \frac{-B \pm\sqrt{B^2 -4AC}}{2A}, \tag{5}$$

where

$$\begin{aligned} A &= A_{11}A_{22}-A_{12}A_{21} \\ B &= A_{11}C_{22}+A_{22}C_{11}-A_{21}C_{12}-A_{12}C_{21} \\ C &= C_{11}C_{22}-C_{12}C_{21} \end{aligned} . \tag{6}$$

In a practical system, A is always positive and the following discussion will be restricted to cases for which C is also positive. Following Ref. 8, we can classify the possible types of solution according to the signs of B and (B^2-4AC) as shown in the following table, where α, β, μ and ω represent only positive quantities.

B	B^2-4AC	λ^2	λ	Type of solution
>0	>0	$-\alpha_1$ $-\alpha_1'$	$\pm i\omega_1$ $\pm i\omega_1'$	*1st Type* Two maintained oscillations
>0	<0	$-\alpha_2 \pm i\beta_2$	$\mu_2 \pm i\omega_2$ $-\mu_2 \pm i\omega_2$	*2nd Type* A growing oscillation and a decaying oscillation
<0	<0	$\alpha'_2 \pm i\beta'_2$	$\mu'_2 \pm i\omega'_2$ $-\mu'_2 \pm i\omega'_2$	
<0	>0	$+\alpha_3$ $+\alpha'_3$	$\pm\mu_3$ $\pm\mu'_3$	*3rd Type* Two divergences and two subsidences

The particular condition

$$B^2-4AC = 0, B>0 \tag{7}$$

is of particular interest in the present discussion, since it corresponds to the boundary between neutrally stable oscillations of the 1st type of solution and the existence of a growing oscillation of the 2nd type. This condition represents a stability boundary of the system and will be regarded as a flutter critical condition. It may be compared with the flutter critical condition that is yielded by the complete equations of motion and which corresponds to the boundary between a region of decaying oscillation and a region of growing oscillation.

The stiffness terms C_{11}, C_{12} etc. depend, in general, on air velocity and have the form $E+\rho V^2 F$ where E and $\rho V^2 F$ are respectively the elastic and aerodynamic contributions. The condition $B^2-4AC = 0$ thus provides an equation for the value of ρV^2 corresponding to the critical flutter speed.

When, as is usual, the aerodynamic virtual inertia contributions are neglected, the terms A_{11}, A_{22} and $A_{12} = A_{21}$ refer only to structural inertias and each is proportional to the mass of the wing, or the wing density. Thus, the mass of the wing appears explicitly only as a factor in the expression (B^2-4AC). Then, if the quantities E and F are independent of oscillation frequency (i.e. constant aerodynamic derivatives), the condition $B^2-4AC = 0$ and thus the flutter critical velocity are independent of the mass of the wing. No such conclusion can be reached in general when the damping terms are included.

Another important difference between an undamped and a damped system becomes apparent. In the case of the flutter of a straight semi-rigid wing for which the co-ordinate ϕ and θ refer to flexure and torsion respectively, it is possible by choice of reference axis to make C_{21}, which corresponds to an elastic cross-stiffness, vanish. If, in addition, the mass distribution is such as to make the product of inertia vanish (i.e. $A_{12} = A_{21} = 0$), equations (2) become

$$\left.\begin{aligned} A_{11}\ddot{\phi}+C_{11}\phi+C_{12}\theta &= 0 \\ A_{22}\ddot{\theta}+C_{22}\theta &= 0 \end{aligned}\right\} . \tag{8}$$

It is seen that the equation of torsional moment is independent of flexure and thus no coupled instability involving growing oscillations in both motions can occur.* On the other hand, provided $B_{21} \neq 0$, the motions remain coupled by the inclusion of damping in the complete equations, and the possibility of instability is thus admitted. This indicates one way in which the addition of damping to an initially undamped system might lead to instability.

3. *Application to a Particular Example.*

Calculations based on undamped equations have been made for the cantilever wing treated in Ref. 13. The main features of the wing and the nomenclature, which as far as possible is identical with the original, are as follows:

Planform geometry: *see* Fig. 1.
Flexural axis at 0·3 local chord.
Radius of gyration of each chordwise section at 0·296 local chords aft of flexural axis.
Inertia axis at j local chords aft of flexural axis.
Semi-rigid flexural and torsional modes as specified in Ref. 13.
Reference section at 0·7 span.

s = Semi-span = $2{\cdot}22\, c_0$

d = $0{\cdot}9s$

l = $0{\cdot}7s$

c_0 = Root chord

c_m = Mean chord = $0{\cdot}762\, c_0$

ϕ = Flexural co-ordinate

θ = Torsional co-ordinate

l_ϕ = Flexural stiffness

m_θ = Torsional stiffness

σ_w = Wing density = $\dfrac{\textit{mass of wing}}{\textit{(plan area) (mean chord)}}$

ρ = Air density

ε = ρ/σ_w, (inverse of wing density ratio)

V = Velocity

n = Frequency (cycle/sec.)

U = Non-dimensional velocity coefficient = $\dfrac{V\sqrt{\rho}}{\sqrt{m_\theta\, d{c_m}^2}}$

$$m = \text{frequency coefficient} = \frac{n\, c_m \sqrt{\sigma_w}}{\sqrt{m_\theta/d{c_m}^2}}$$

$$r = \text{stiffness ratio} = \frac{l_\phi}{m_\theta}\left(\frac{c_m}{d}\right)^2$$

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*It may be noted that for the special condition of equal frequencies, when $C_{22}/A_{22} = C_{11}/A_{11}$, the flexural motion alone will grow with time for any non-zero torsional amplitude.

The coefficients of equations (2) are obtained directly from Ref. 13 and are as follows:

Inertia terms

$$A_{11} \qquad = \frac{0{\cdot}405\rho l^3 c_0{}^2}{\varepsilon}$$

$$A_{12} = A_{21} = \frac{0{\cdot}247j\rho l^2 c_0{}^3}{\varepsilon}$$

$$A_{22} \qquad = \frac{0{\cdot}0141\rho l c_0{}^4}{\varepsilon}$$

Stiffness terms

$$C_{11} = l_\phi$$

$$C_{12} = 1{\cdot}039\rho V^2 l^2 c_0$$

$$C_{21} = 0$$

$$C_{22} = m_\theta - 0{\cdot}0317\rho V^2 l c_0^2$$

Then by substitution we have from equations (6)

$$A = \frac{\rho^2 l^4 c_0{}^6}{\varepsilon^2}(a - bj^2) \tag{9}$$

$$B = \frac{m_\theta \rho l^3 c_0{}^2}{\varepsilon}\{e + fr - (g + hj)U^2\} \tag{10}$$

$$C = \frac{m_\theta{}^2 r l^2}{c_0{}^2}(k - pU^2) \tag{11}$$

where $a = 0{\cdot}0057105$ $g = 0{\cdot}017197$

$b = 0{\cdot}061009$ $h = 0{\cdot}34373$

$e = 0{\cdot}40500$ $k = 2{\cdot}8466$

$f = 0{\cdot}040137$ $p = 0{\cdot}12087$

It may be noted from equation (11) that the restriction already made for C to be positive is justified provided the speed is below the static divergence speed corresponding to $C_{22} = 0$; that is provided $U^2 < 23{\cdot}55$.

At this stage, it is of interest to examine the manner in which the roots of equation (4) change with increase of U. As an example, values proportional to the real and imaginary components of the root λ are shown plotted against U in Fig. 2 for the particular condition $j = 0{\cdot}1$ and $r = 5$. It will be seen that with an increase of U from zero, μ remains zero whilst the frequencies of the two oscillations corresponding to the 1st type of solution approach one another and coincide at the flutter critical speed* $U = U_c$. For $U_c < U < U_2$, the solution is of the second type and consists of two oscillations of the same frequency ω, one growing and the other decaying with the same numerical value of the exponent μ. For $U > U_2$, the solution is of the 3rd type with subsidences and divergences.

*Hence the expression 'frequency coincidence flutter' which is sometimes used.

3.1. *Flutter Critical Speed.*

By substitution from equations (9) to (11), the condition $B^2-4AC = 0$ leads to the following biquadratic equation for the flutter critical speed coefficient U_c,

$$(g+hj)^2\, U_c^{\,4}+[4r(a-bj^2)p-2(g+hj)\,(e+fr)]\, U_c^{\,2}+(e+fr)^2-4rk(a-bj^2) = 0\,. \qquad (12)$$

This equation can be used to calculate U_c for chosen values of j and r. It will be seen that U_c is independent of the density ratio ε. Thus as already noted in Section 2, the equivalent air speed for flutter is independent of wing density σ_w.

Values of U_c were calculated for $j = 0{\cdot}05, 0{\cdot}1$ and $0{\cdot}15$ over a range of values of r from 0 to 10; the results together with those for the complete equations obtained from Ref. 13* are shown in Figs. 3 to 5. In the latter case, U_c is dependent on the density ratio and curves are shown for values $\varepsilon = 0, 0{\cdot}0956, 0{\cdot}1275$ and $0{\cdot}1912$.

3.2. *Flutter Frequency at Critical Speed.*

The frequency of the flutter oscillation is obtained from equation (5) and, since $B^2-4AC = 0$ at the flutter critical condition we have

$$2\pi n_c = \omega = (B/2A)^{1/2} = (C/A)^{1/4}. \qquad (13)$$

Values of the critical frequency coefficient

$$m_c = \frac{n_c\, c_m \sqrt{\sigma_w}}{\sqrt{m_\theta/dc_m^{\,2}}}$$

have been calculated and are compared in Figs. 6 to 8 with the values for the complete system with damping terms. Now, although n_c is dependent on the density ratio, m_c is independent of this parameter for the undamped system. Strictly, the value of m_c for the complete system is dependent on density ratio, but the variation is small and thus only a single curve for the damped system is shown in the diagrams.

3.3. *Frequency Parameter.*

The frequency parameter $(\omega c_m/V)$ at the flutter critical condition is related to the coefficients m_c and U_c and the density ratio ε as follows:

$$\left(\frac{\omega c_m}{V}\right) = \frac{2\pi n_c\, c_m}{V_c} = 2\pi\left(\frac{m_c}{U_c}\right)\sqrt{\varepsilon}$$

That is, for fixed ε, the frequency parameter is proportional to (m_c/U_c). Comparisons between the values of (m_c/U_c) for the damped and undamped systems are shown in Figs. 9 to 11; for the system with damping a separate curve is necessary for each value of ε, whereas for the undamped system, the value of (m_c/U_c) is independent of ε. For all the cases that have been examined, removal of the damping terms lowers the flutter frequencies at the critical condition. It will be noted that, for the undamped system, the critical flutter frequency will tend to vanish as $r \to 0$. On the contrary, the frequency remains finite for $r = 0$ when calculated from the complete equations.

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*The data corresponding to the system with damping have been obtained from the rather small graphs in the published document. In a few cases the values were checked by completely new calculations.

4. *Discussion.*

The equations without damping terms that are used in the present Report lead to a flutter condition, specified by the critical velocity coefficient U_c, which is independent of the density ratio ε. That is, the critical value of the equivalent air speed, $(V\sqrt{\rho/\rho_0})_c$, is independent of wing density. When damping terms are included, wing density can, under some circumstances, be an important parameter. However, it is important to note that all the present calculations are based on constant (i.e. frequency independent) aerodynamic derivatives. It could not be concluded that $(V\sqrt{\rho/\rho_0})_c$ is invariant with changes of wing density if the derivatives are frequency dependent.

Inspection of the (U_c, r) diagrams shows that for *large values of* ε *(a light wing or a dense medium)*, the effect of removing all the damping terms is to cause a decrease in the critical-speed coefficient and, it will be noted, this decrease can be large. For *small values of* ε *(a heavy wing or a low density medium)*, the critical-speed coefficients calculated in the absence of damping show the same general trends with variation of the ratio of flexural and torsional stiffness, provided this is not too small, as those calculated using the complete equations; for $\varepsilon = 0$, the removal of the damping terms leads to an increase in the critical speed which, over quite a wide range of stiffness ratio, is no more than about 5 per cent. For some intermediate value of ε, the presence of the damping terms has no influence on the critical speed.

On a physical basis, we might argue that if the wing density is made infinitely large, the frequencies will become vanishingly small and so will the velocities of the wing motions $\dot{\phi}$ and $\dot{\theta}$ for a given amplitude. Under these circumstances, the damping terms (e.g. $B_{12}\dot{\theta}$) would become negligible in comparison with the stiffness and inertia terms. Thus with a gradual increase of wing density it might be expected that the critical speeds calculated from the complete equations would tend asymptotically to the value deduced from the equations without damping. However, the results of the present calculations show that this is not so. Starting from a low value of the wing density (ε large) we find that the critical speed deduced from the full equations falls with increasing density from a value well above that for the undamped system to a value for $\varepsilon = 0$ which is below that for the undamped system. A similar discontinuity between the behaviour of a system with a complete absence of damping and the same system with damping tending to zero has been noted by Nemat-Nasser and Herrmann[5].

The relation between the respective influences of density ratio and of aerodynamic damping are clarified when the graphical method of Niblett[4] is used to represent the solution of the flutter equations. This method depends on plotting a particular conic which varies with density ratio, and a straight line—the damping line—which is independent of density ratio. For the particular example, $j = 0{\cdot}1$, $r = 4$, the Niblett plots in the U^2, m^2) plane are shown in Fig. 12 for $\varepsilon = 0$ and $0{\cdot}1275$; each conic is a hyperbola of which only the left-hand branch is of interest in the present discussion. The flutter solution for a damped system is represented by an intersection of the damping line with the conic (i.e. points A & B), whilst point C, at the nose of the conic for $\varepsilon = 0$, corresponds to the flutter condition for the undamped system. Now, for the condition $\varepsilon = 0$, the conic is independent of aerodynamic damping terms, the effects of which can only be felt by virtue of their influence on the damping line. Thus, since it can be shown that any damping line must intersect the hyperbola, it is clear from the diagram that the critical speed for the undamped system provides an upper bound to the critical speed for the system when $\varepsilon = 0$ with any values of the damping terms. Done[3] has shown analytically that the effect on the critical flutter of the addition of damping terms can be considered as made up of two parts: a 'damping' term which normally represents an increase in critical flutter speed, and a 'frequency parameter' term which can only lower the critical speed and which is dependent on the difference between the values of the critical-frequency parameters for the complete and the undamped systems. It follows that if additional damping leaves the critical-frequency parameter unaltered, then the critical speed will be increased by the damping. This is borne out by the present calculations, for it can be seen from the diagrams that whenever (m_c/U_c) is the same for the complete and the undamped system, the complete system has the higher critical speed. The same conclusion, namely that constancy of frequency parameter would ensure that damping raises the critical speed, would follow immediately from the hypothesis, supported by the results of the calculations, that damping always raises the critical flutter frequency.

The small effect of aerodynamic damping on critical speed found in the present example when the density ratio (ε) is low suggests the need for a more thorough examination of the question as to whether calculations based on the undamped equations could provide acceptable flutter prediction in certain practical cases, for example wings at high altitude or the heavy structures of civil engineering. It is relevant to note that Gaukroger[15], in considering the effect of density ratio on flutter, remarks that density ratios (ε) for aeroplane wings are sufficiently low for the critical equivalent air speed to be independent of wing density. In other words, the critical speeds calculated by the complete equations are, for modern wings, approaching the asymptotic value for infinite-wing density ratio and thus may be close to the speeds obtained from the undamped equations. A cautionary note must be sounded here; there is a class of fluttering system in which the coupling between the freedoms depends solely on the aerodynamic cross dampings (an example is provided by the calculation using quasi-steady aerodynamic derivatives in Ref. 16). In such a case, it is obvious that no flutter can be predicted in the absence of the damping terms.

The attraction in using undamped equations comes not only from a possible simplification in the calculations themselves, but also from a reduction in the number of aerodynamic derivatives required. Even if a complete neglect of aerodynamic damping is not acceptable, it would seem that, in some circumstances, the accuracy required in the aerodynamic terms may be quite low. If this point could be established even for certain cases, it would allow a relaxation in the demands on theoretical estimation and a corresponding lowering in the demands for experimental information.

Finally, a remark may be made with regard to flutter testing in wind tunnels. Since it is known[6] that wall interference effects are more serious for the aerodynamic dampings than for the stiffnesses, a tentative conclusion is that the measured critical speeds of heavy wings are likely to be less subject to wall interference effects than the measurements for light structures.

5. *Acknowledgements.*

Most of the calculations of this report were made by Mrs. I. Manly and Mr. E. Brown.

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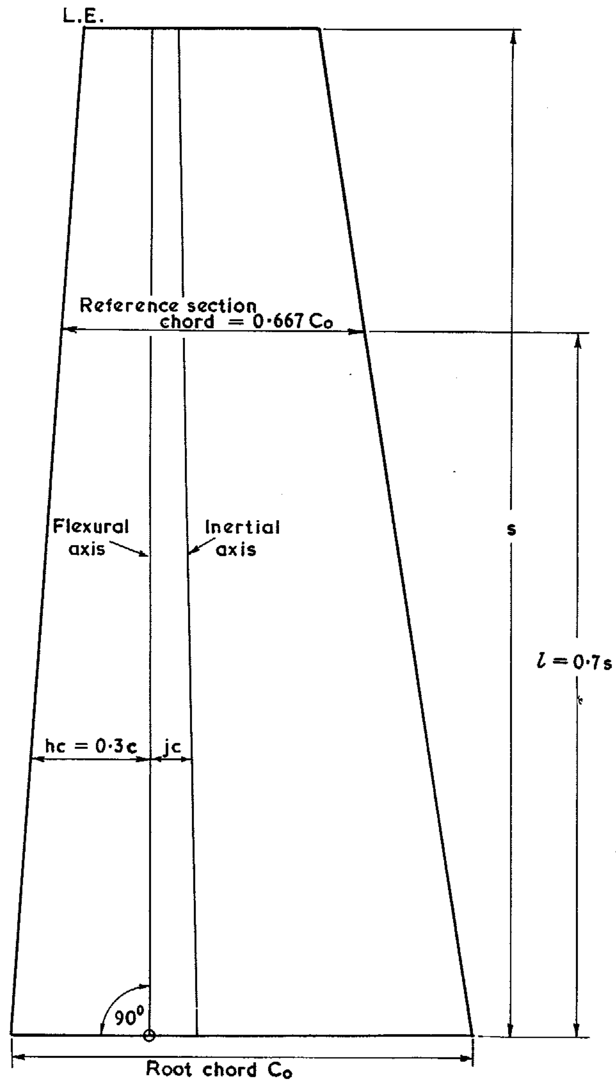

FIG. 1. Diagram of wing.

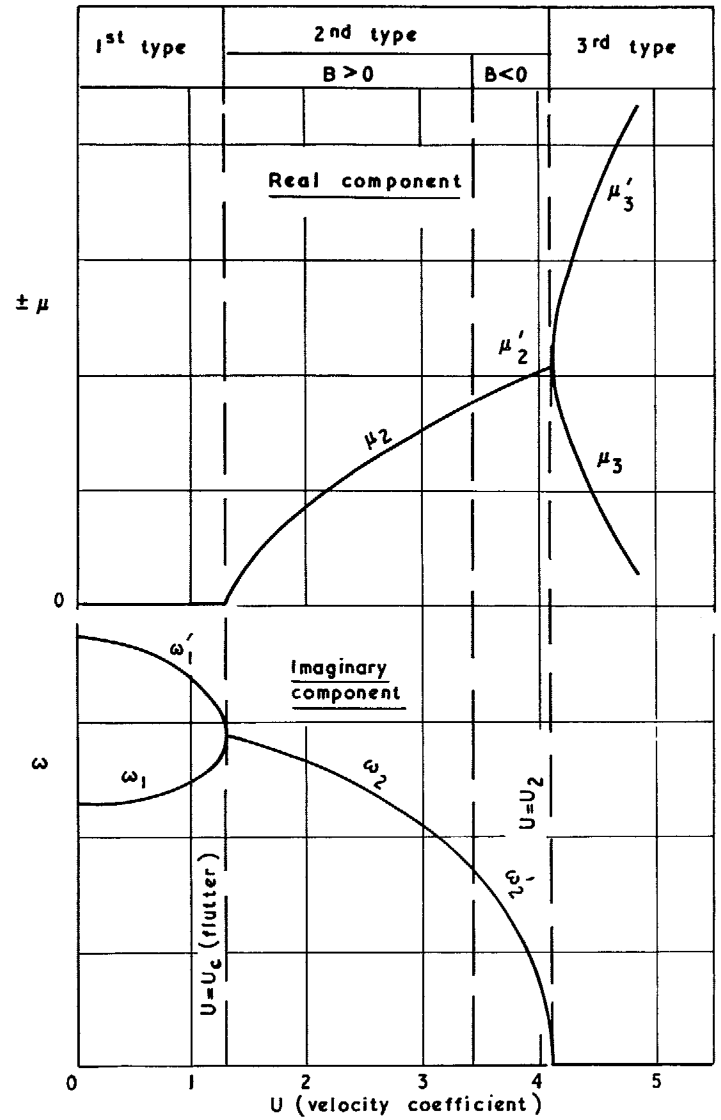

Fig. 2. Variation of root $\lambda = \mu \pm i\omega$ with variation of air velocity; ($j = 0{\cdot}1, r = 5$).

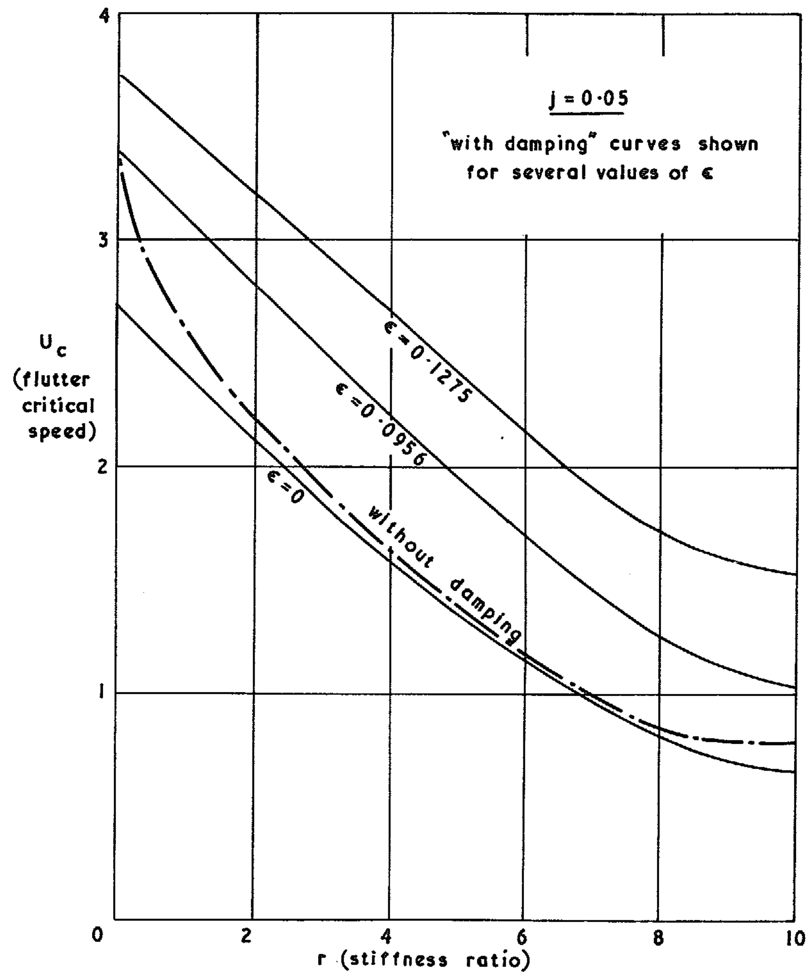

FIG. 3. Variation of flutter critical speed with stifness ratio; $j = 0{\cdot}05$.

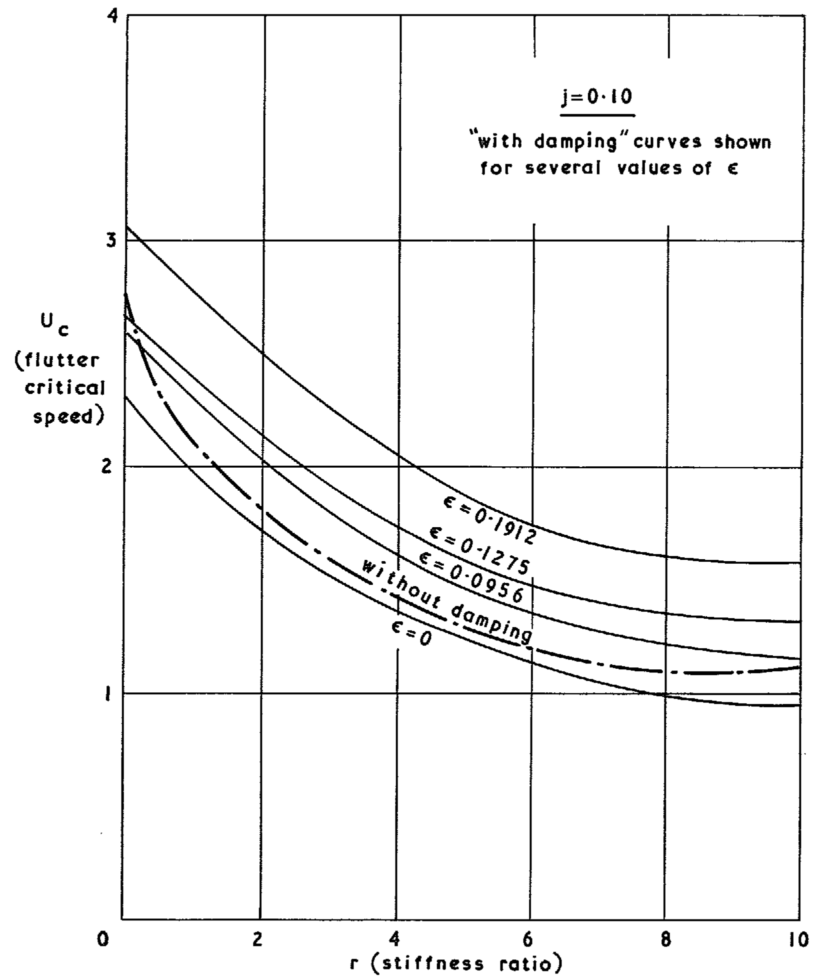

FIG. 4. Variation of flutter critical speed with stiffness ratio; $j = 0{\cdot}10$.

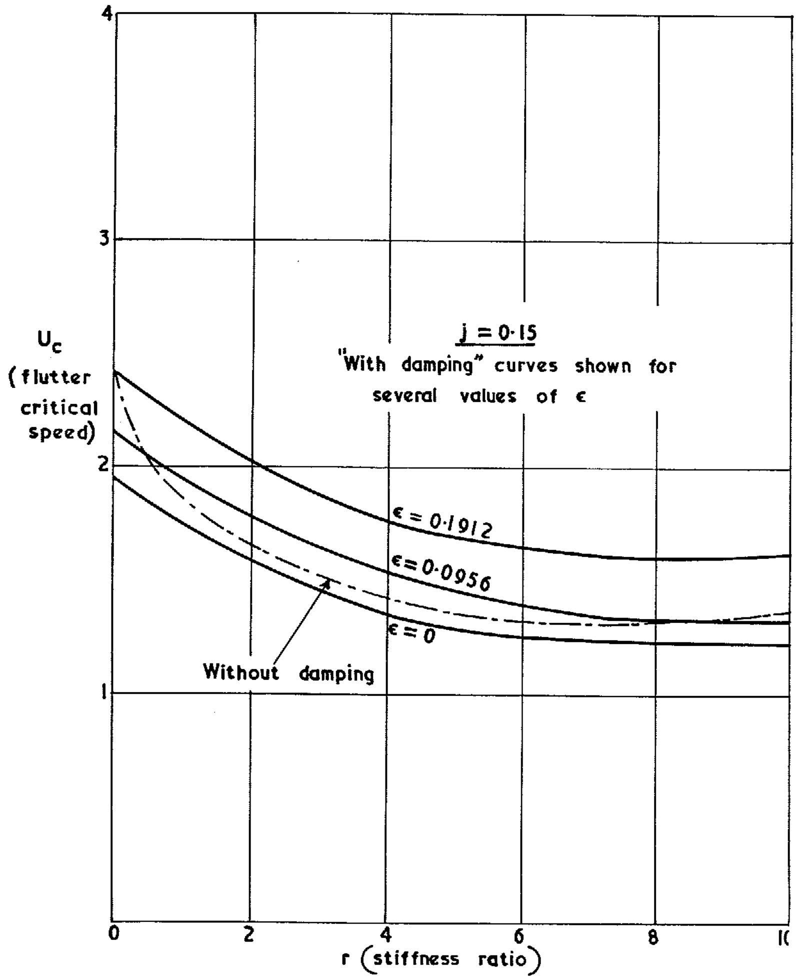

FIG. 5. Variation of flutter critical speed with stiffness ratio; $j = 0{\cdot}15$.

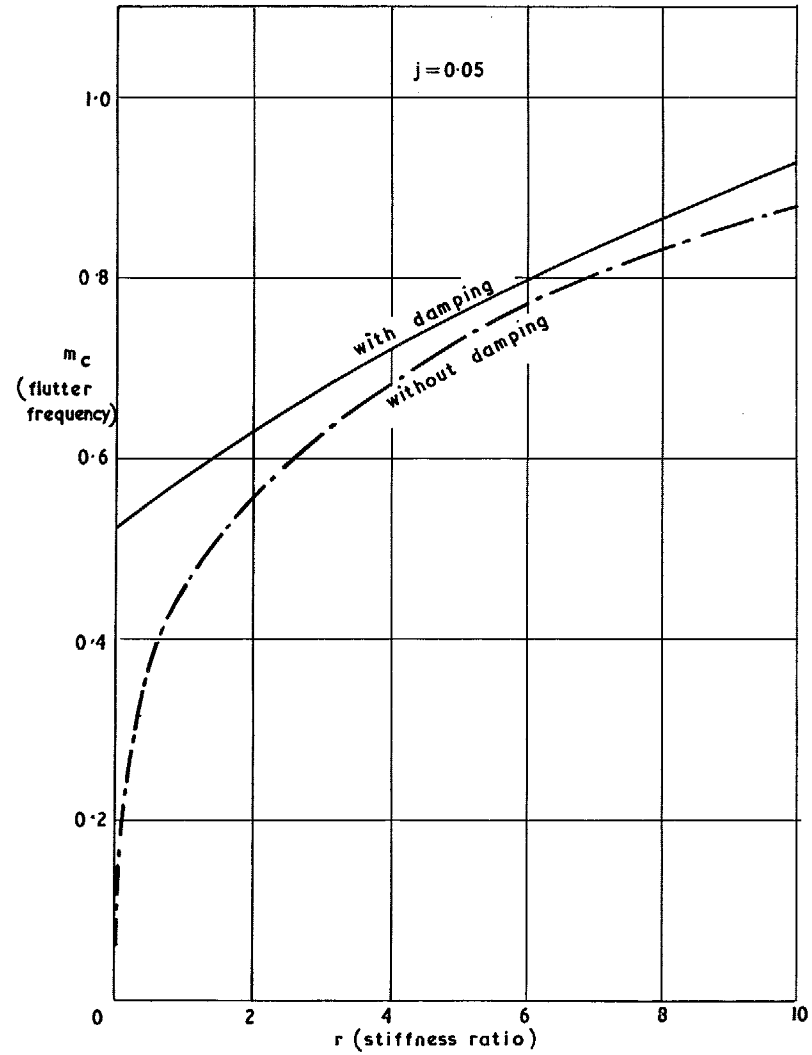

FIG. 6. Variation of flutter frequency with stiffness ratio; $j = 0{\cdot}05$.

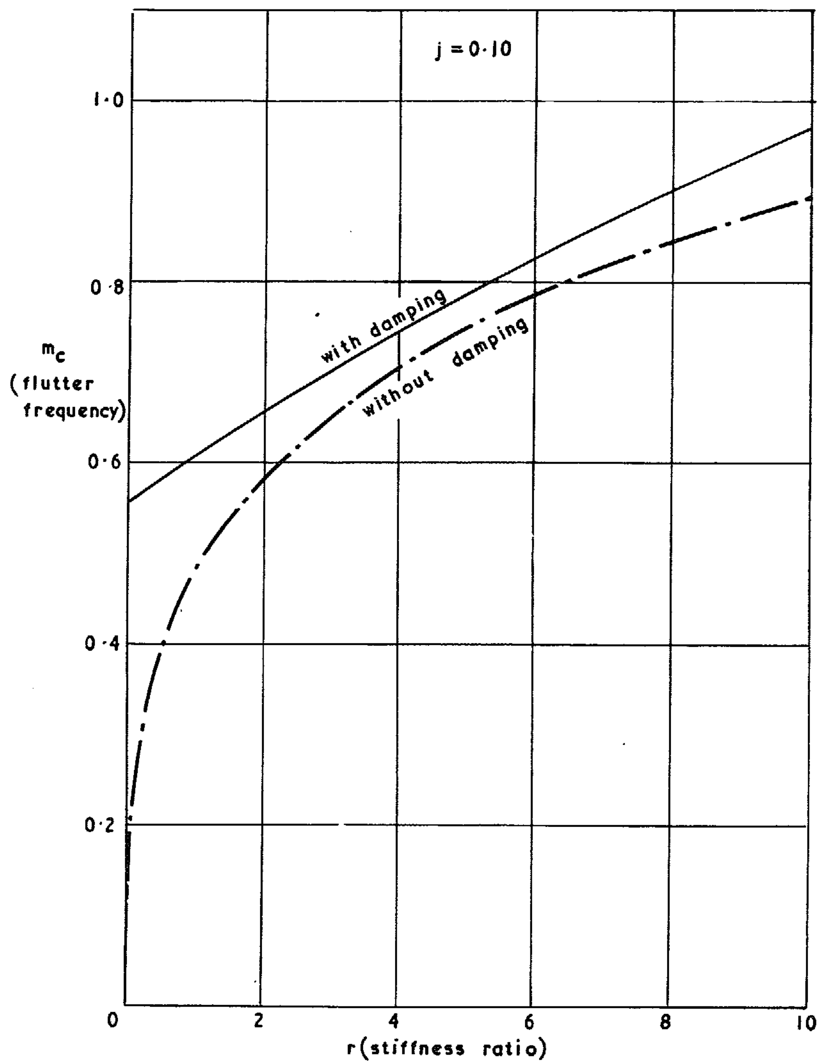

FIG. 7. Variation of flutter frequency with stiffness ratio; $j = 0{\cdot}10$.

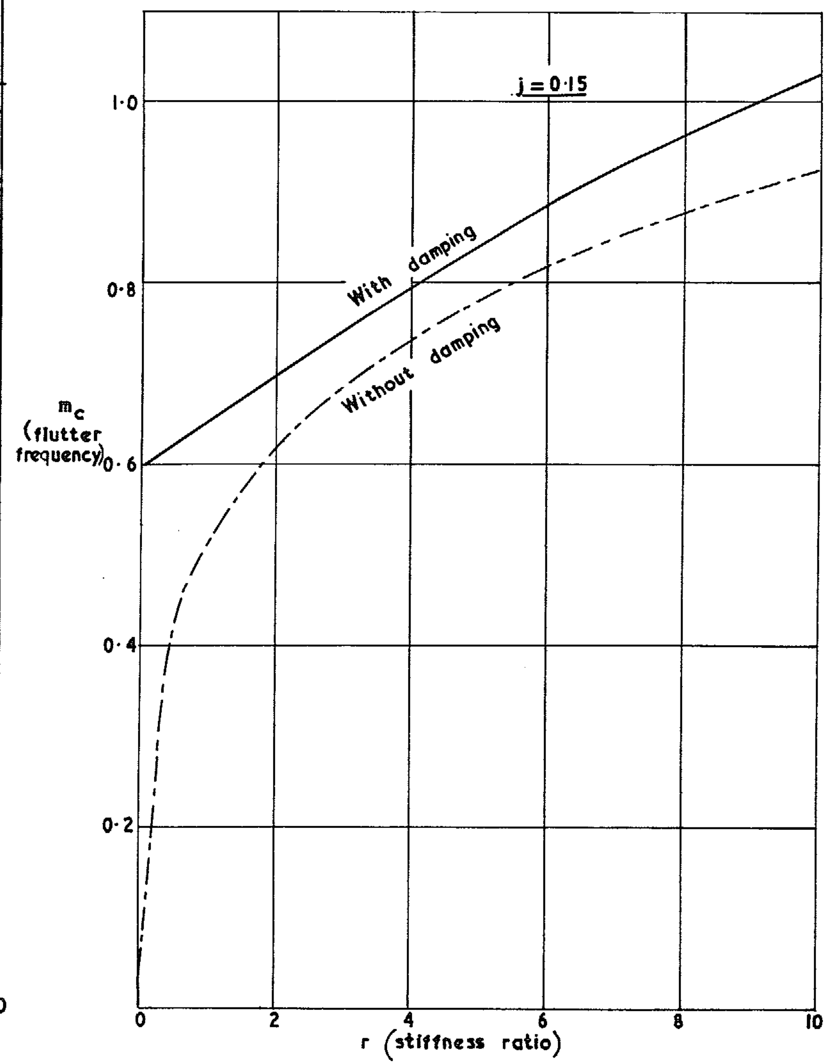

FIG. 8. Variation of flutter frequency with stiffness ratio; $j = 0{\cdot}15$.

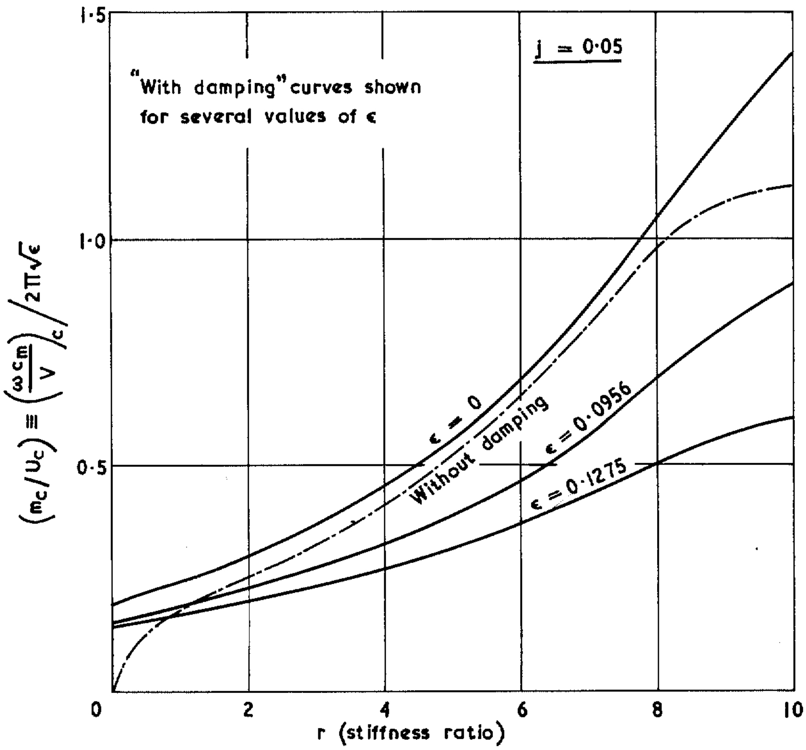

FIG. 9. Variation of frequency parameter with stiffness ratio; $j = 0{\cdot}05$.

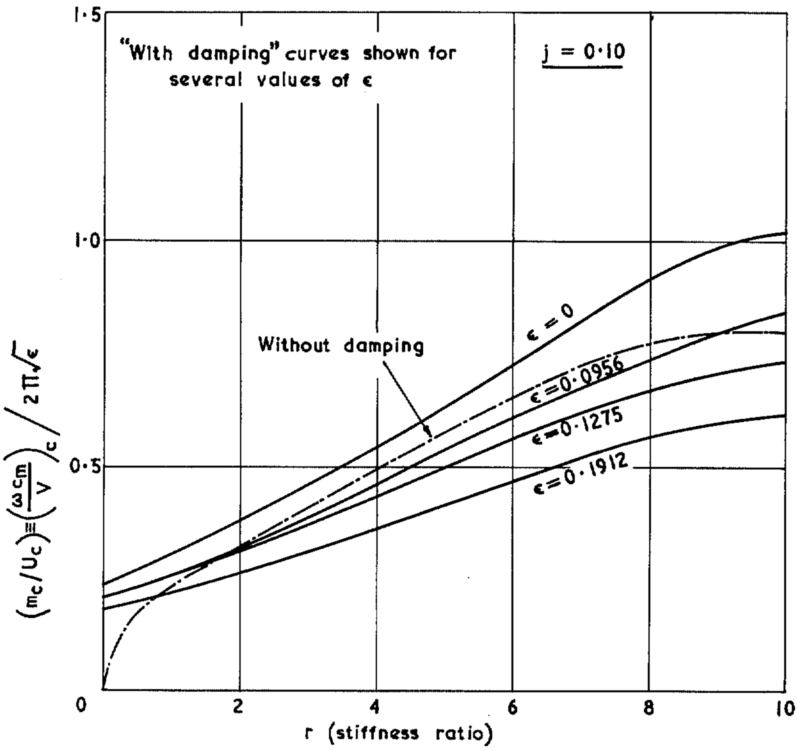

FIG. 10. Variation of frequency parameter with stiffness ratio; $j = 0{\cdot}10$.

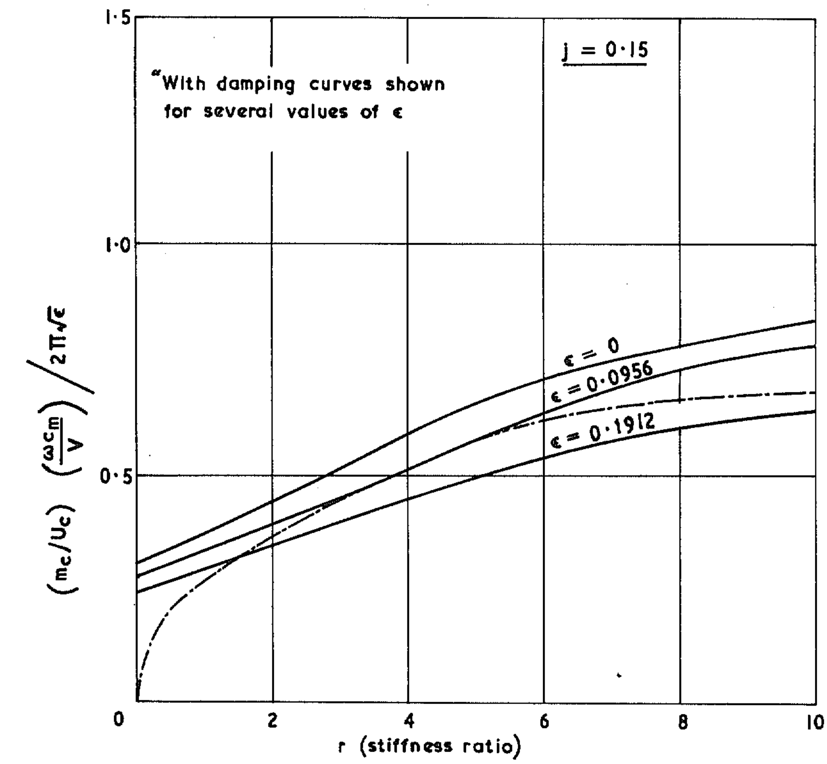

FIG. 11. Variation of frequency parameter with stiffness ratio; $j = 0{\cdot}15$.

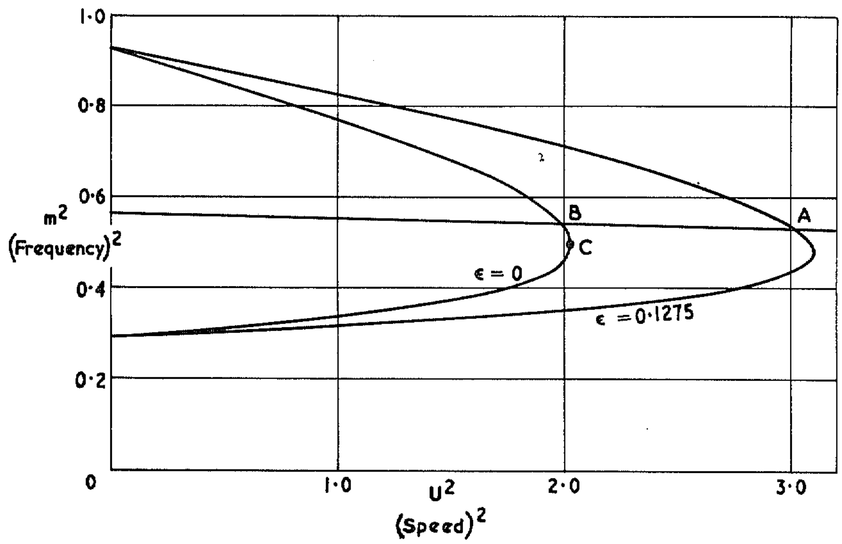

FIG. 12. Graphical solution of flutter equations by Niblett's method for case $j = 0{\cdot}1$, $r = 4$. (*See* Ref. 4 for details of method).

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