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An Investigation of the Feasibility of Reshaping Stick Commands to Improve Hovering Control of Unstabilized Jet-Lift Aircraft

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An Investigation of the Feasibility of Reshaping Stick Commands to Improve Hovering Control of Unstabilized Jet-Lift Aircraft

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Summary.

The Report examines the feasibility of improving hovering control of unstabilized jet-lift aircraft by reshaping stick commands. From basic theoretical considerations it is shown that the open-loop properties of any linear dynamic system operating in an environment without external disturbance can be altered to those of a lower order system by means of a suitable shaping function. Thus there may be numerous shaping laws which, in principle, could simplify control of hovering aircraft. However, it is felt that practical applications of the concept would depend upon the performance of the mechanical system, which must operate under the limitations of control power available and which must satisfy certain safety requirements. A hydraulic integrating mechanism designed to meet such conditions was chosen for detailed study. Simulated hovering flight studies were conducted to observe the effect of variations of mechanism parameters on pilot performance and a discussion on optimum controls is included. A practical hovering control circuit for jet-borne vehicles was thence proposed on the basis of a compromise between mechanism characteristics desirable for normal operation and those that would be necessary to provide adequate control in the event of hydraulic failure.

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*Replaces A.R.C. 28 532.

Introduction.

At very low speeds or in hovering flight, aerodynamic damping of jet-borne aircraft may be practically negligible. Following a small angular displacement in pitch or roll the uncontrolled aircraft is neutrally stable in attitude but unstable in position. Stick movements required to produce simple changes in the attitude or position of the aircraft demand considerable anticipation from the pilot and the work load is severe. In fact, it is easy for the pilot to introduce a larger disturbance than the one he is trying to correct. On the one hand he is, therefore, battling mainly with the consequences of his own mistakes, yet, on the other hand, he cannot desist from active control for more than one or two seconds for otherwise the velocity of translational motion would quickly increase to an unacceptable magnitude.

A feature of hovering flight in jet-lift aircraft is the comparative unimportance of external disturbance due to gusts. In the first place, the vehicle with little or no forward speed is remarkably insensitive to natural winds. Secondly, the pilot is in an environment where a certain amount of vibration and unsteadiness due to turbulence of jets and recirculation effects is always present, which tends to mask the effects of gusts. But the main reason lies in the fact that the pilot is fully occupied in correcting the effects of his own central movements, and these effects are generally larger and more frequent than those due to atmospheric turbulence.

The fact that the absence of damping is the basic reason for the difficulty of hovering jet-borne aircraft naturally suggests the introduction of artificial damping; and indeed this is quite feasible. However, the combination of circumstances outlined above, namely the need for continuous control action, together with the effective absence of external disturbance, has prompted previous investigators [1,2,3] to suggest that improvements in handling properties might be obtainable by simpler means.

An alternative to artificial damping is to reshape the stick commands so that the actions which the pilot actually takes are easier to perform and the consequences of errors in his judgment less severe. The advantage of reshaping is that it can be accomplished with mechanical elements, thus eliminating the need for gyroscopes and electrical systems in the control link. For many purposes, particularly for a cheap expendable military vehicle, the degree of elaboration of triplexed autostabilizers would not seem justified for a manoeuvre which is performed only for a few seconds at the beginning and end of a flight.

Holden[1], has derived the mathematical equation of shaping function which alters the open loop properties of any linear system to those of a velocity control. On this basis he has proposed that the control moment-stick deflection relationship of the attitude controls of unstabilized jet-lift aircraft should be

$$M = \alpha S + \beta \frac{dS}{dt} \qquad (1)$$

where M is the control moment applied to the vehicle, S is the stick position and α and β are constants. The results of simulator tests described in Ref. 2 have shown that the addition of the control moment term proportional to stick velocity could be beneficial to the pilot. With suitable choice of the constants, accuracy of hovering was as good as that which could be achieved with rate damping.

Morris, McCormick and Sinacori[3] take it for granted that reshaping can be useful. Accordingly, their objective was to determine 'What form of shaping best fits the pilot's needs?'. In answer to this question they chose a general equation of the form:

$$\frac{d^2M}{dt^2}+2\zeta\omega_n\frac{dM}{dt}+\omega_n{}^2M = A\left(\frac{d^2S}{dt^2}+z\frac{dS}{dt}\right) \tag{2}$$

where M and S are defined as above, and ζ, ω_n, z and A are constants. By systematic testing in a piloted flight simulator they too established that reshaping stick commands could, with optimum parameters, improve the pilot's control capability to a level obtainable with rate damping.

The method of reshaping stick commands proposed in the present Report differs, for reasons which will be discussed, from the forms described by equations (1) or (2). It is considered that the potential value of the concept is more likely to be realised by devising a simple mechanism which can satisfy the control power and safety requirements of a practical aircraft control system, than by seeking the best possible formula to simplify a pilot's job without regard for practical considerations.

1. *Theoretical Analysis.*

Let us suppose that a quantity ϕ is controlled by a pilot's command S, as shown in Fig. 1, and that the system transfer function is:

$$\frac{\phi}{S} = K\,G(p) \tag{3}$$

where p is the Laplace operator, K is the stick gain, and $G(p)$ is linear and time-invariant. The quantity ϕ, for example, may be the bank angle of a hovering aircraft, S the pilot's stick position and $G(p)$ the transfer function of the rolling motion. In order to simplify the dynamics of a system which is unsatisfactory from the point of view of pilot handling, there are two possible strategies apart from altering the quantity $G(p)$ itself. The first method is the direct application of feedback, as shown in fig. 2. The motion of the controlled output is sensed and a function, $H(p)\phi$, is computed in a feedback path. This time-varying function is subtracted from the pilot's input and the difference is fed as the command to operate the control surface actuator. In this way the overall system transfer function is altered to the form:

$$\frac{\phi}{S} = \frac{K\,G(p)}{1+G(p)\,H(p)} \tag{4}$$

By a suitable choice of the feedback transfer function $H(p)$ the control may possess any desired characteristics.

A second method of altering the system dynamics is illustrated in Fig. 3. A device which is included in series with the control input directly shapes the stick commands independently of the motion of the controlled output, and the overall system transfer function becomes:

$$\frac{\phi}{S} = K\,Y(p)\,G(p) \tag{5}$$

This system too may possess any desired open loop characteristics by a suitable choice of the series transfer function $Y(p)$.

In Ref. 1 it has been suggested that a desirable control law for good handling is a velocity control. A linear system will be altered to this form by appropriate shaping of stick commands when:

$$K\,Y(p)\,G(p) = \frac{\beta}{p} \tag{6}$$

where β is the desired gain of the velocity control. The transfer function of the shaping device is thus:

$$Y(p) = \frac{\beta}{KpG(p)} \tag{7}$$

In practical terms it may not always be possible to accomplish this in a real situation. The output of the shaping device is the movement of the mechanical linkage which operates the control surface actuators. Adequate forces must be available to overcome control linkage friction and inertia and the reshaped stick commands must not exceed the amount of control movement at hand.

When the above theory is applied to the attitude controls of jet-lift aircraft in hovering flight we may assume that $G(p)$ represents a pure acceleration system, i.e.

$$G(p) = \frac{1}{p^2} \tag{8}$$

Hence the transfer law of the shaping device to alter this acceleration control to a velocity control is given, from equation (7), by:

$$Y(p) = \frac{\beta}{Kp.\frac{1}{p^2}} = \frac{\beta}{K}p \tag{9}$$

The relationship between the output of the mechanism, η, and the input stick command is hence:

$$\eta = \frac{\beta}{K}\frac{dS}{dt} \tag{10}$$

The relationship between the control moment and the stick input command is:

$$M = \beta\frac{dS}{dt} \tag{11}$$

This would admit the formidable, although not impossible, requirement of devising a mechanism to perform the function of mechanical differentiation of stick commands. But it would also present the need for having control nozzles capable of instantly producing large control moments to accompany rapid stick movements. Other shaping laws which might, for example, demand infinite control deflections for a short time, or control movements at an infinite rate, although mathematically sound, would not, however, be easily adaptable for practical use.

In order to determine shaping laws which are physically realisable we are guided by the fact that, from the point of view of open loop response to stick commands, an autostabilizer, too, is primarily a device which reshapes stick commands. Hence what has been accomplished practically in jet-lift aircraft control systems by way of feedback systems can equally well be accomplished by a series device which shapes stick commands into the same pattern of control moments.

By equating the right hand sides of equations (4) and (5) we see that when:

$$KY(p)\,G(p) = \frac{KG(p)}{1+G(p)\,H(p)} \tag{12}$$

or,

$$Y(p) = \frac{1}{1+G(p)\,H(p)} \tag{13}$$

the two processes of reshaping and feedback are identical with respect to open loop response to stick movements. If the output control moments are realisable through a feedback system then they must also be realisable through a series device.

For example, in rate stabilization of the attitude controls of the hovering aircraft,

$$H(p) = \tau p \tag{14}$$

where τ is the damping parameter in $\sec^{-1}$. The transfer function of a shaping mechanism which makes the unstabilized aircraft respond to stick movements in exactly the same manner as if rate damping were provided is, from equation (13), given by:

$$Y(p) = \frac{1}{1+\tau p.\dfrac{1}{p^2}} = \frac{p}{\tau + p} \tag{15}$$

Or, in the differential form, the relationship between input movement and output movement of the mechanism is:

$$\frac{d\eta}{dt} + \tau\eta = \frac{dS}{dt} \tag{16}$$

Similarly, we can derive the transfer function of a shaping mechanism to make the aircraft respond to stick movements as if attitude stabilization were present. In this case the feedback function is:

$$H(p) = \tau p + \lambda \tag{17}$$

where λ is the stiffness parameter in $\sec^{-2}$. Hence,

$$Y(p) = \frac{1}{1+\dfrac{\tau p+\lambda}{p^2}} = \frac{p^2}{p^2+\tau p+\lambda} \tag{18}$$

And, in the differential form, the relationship between input and output movement of the mechanism may be expressed as:

$$\frac{d^2\eta}{dt^2} + \tau\frac{d\eta}{dt} + \lambda\eta = \frac{d^2S}{dt^2} \tag{19}$$

It is pointed out that equation (19) is basically that of the mechanism proposed in Ref. 3 for improving hovering control of jet-lift aircraft (c.f. equation (2) of the present Report) except that an output term proportional to stick velocity has been added to it. Thus it is ensured that a pilot would not use up all the available stick travel in any one direction in making adjustments to the aircraft's attitude if it were impulsively displaced from time to time by external disturbances, without using up all the available stick travel in any one direction. However, as it has been remarked in Ref. 2 a control moment term proportional to stick position (or possibly integrals of stick position) is essential if the pilot is to be able to trim an external disturbance sustained for any length of time with the stick. For this reason equation (1) was proposed in Ref. 2 as a more viable suggestion than the basic velocity control of equation (11). In the absence of such a term on the right hand side of equation (2) it is evident that a steady stick deflection will, after a transient period, command zero control moment.

2. *A Practical Form of Shaping for Jet-Lift Aircraft Control Systems.*

The present proposal for reshaping stick commands was derived from equation (6). A mechanism obeying this law, in the same manner as automatic rate damping, causes a step movement of the stick to command a pulse control movement which decays exponentially to zero in a brief transient period. To the pilot the undamped system feels as though damping were present. But there are two obvious drawbacks to using this system as it stands for controlling unstabilized aircraft. Unlike automatic rate damping, direct shaping of stick commands in this way does not allow the pilot to hold on a steady control moment for trimming a sustained disturbance. And, secondly, the pilot is deprived from making more than fleeting use of the maximum control power built into the aeroplane. Both of these objections may be overcome if equation (16) is modified to:

$$\frac{d\eta}{dt}+\frac{\eta}{T} = \frac{dS}{dt}+\frac{S}{\alpha T} \tag{20}$$

where η is the mechanism output movement, T is the time constant of the exponential decay, in seconds, analogous to the inverse of the damping parameter, τ, of the rate damped system, and α is a non-dimensional constant greater than unity.

The control moment-stick deflection relationship may be expressed as:

$$\frac{dM}{dt}+\frac{M}{T} = K_c\left(\frac{dS}{dt}+\frac{S}{\alpha T}\right) \tag{21}$$

where M is in rad/sec^2, and K_c is a gearing constant in rad/sec^2/in.

The physical significance of the constants may be seen by considering the step response of equation (21). Following a step displacement, S, of the stick, with the initial conditions $S = 0$, $M = 0$, we have:

$$M = \frac{K_cS}{\alpha}\left\{1+\alpha-1)e^{-\frac{t}{T}}\right\} \quad \text{for } t \geqslant 0 \tag{22}$$

This is illustrated in Fig. 4. M initially increases to the value K_cS rad/sec^2, and subsequently decays to the steady value $\frac{K_cS}{\alpha}$ rad/sec^2. Thus the parameter α is ratio of the initial control power to the final steady control power for step inputs.

The inclusion of the control moment term proportional to stick position makes it possible for the pilot to apply full control power continuously when the stick is deflected to its maximum position, if control link is provided with a lost motion device. Thus a step movement of the stick, S_1, which is less than S_{max}/α, would momentarily open the control valve to give a control moment M_1, after which it would close exponentially to provide the steady state moment M_1/α. Any larger step movement of the stick, S_2, which is greater than S_{max}/α but less than S_{max}, would open the valve fully to produce the maximum control moment M_{max}, and, after a finite time, this would again close to give the steady control moment S_2M_{max}/S_{max}. A step movement of the stick to its maximum position would open the valve fully and maintain M_{max}. These three conditions are illustrated in Fig. 5. It will be appreciated that the benefits of reshaping small stick movements are thus combined with the ability to sustain full control power for manoeuvring or trimming asymmetrical disturbances.

The idea of saturating control moments in this manner is not entirely novel. Patierno and Isca[4] conducted simulated hovering studies with a non-linear rate stabilized control in which the control power was saturated before the stick reached its maximum deflection. They concluded that the scheme was a useful method for minimizing control power or for obtaining maximum efficiency from a given amount of control power since it allowed the use of a greater stick sensitivity over a range of small control movements which are most frequently used for hovering. Although there are no serious engineering difficulties associated with the introduction of saturating valves, there is, nevertheless, a limit to which the gearing of control systems may be increased. This limitation arises from the need for a satisfactory control link if failure of the aiding device should occur.

3. *Mechanical Realisation.*

The shaping law defined by equation (20) can be mechanically realised by integration of the output, and does not involve differentiation in the link. Fig. 6 illustrates the principle of one type of mechanism which has been produced in the Aeronautical Engineering Department of the Queen's University of Belfast.

It is composed of a four-bar linkage *AB*, *CD*, *EF*, *BD*, shown in the neutral position in Fig. 6a. Stick inputs introduced at *A* cause the link *AB* to rotate about the point *P*. In Fig. 6b the mechanism is shown in a position that it would occupy immediately after a step input command. An output *CC'*, proportional to the input *AA'*, is initially produced. The point *F'* subsequently decays back to the neutral point *F* the output link will remain in the position *D'FC''* as shown in Fig. 6c, leaving on a steady output *CC''*.

The ratio of the steady state output to the initial output commanded by the stick is related to the geometry of the linkage. Thus assuming that $AE = EB = CF = FD$, then $CC'' = DD' = BB'$, and $AA' = CC'$ then:

$$\frac{CC''}{CC'} = \frac{BB'}{AA'} = \frac{BP}{PA}$$

Therefore, the ratio of lengths *PA*/*BP* is the magnitude of the parameter α in the control equation (20).

A hydraulic integrating device to perform the exponential decay, which was originated by Professor A. V. Stephens, is illustrated in Fig. 7. The input is connected to a piston inside a cylinder and the output is connected to the outer case of the cylinder. These are enclosed in an oil bath. Oil under pressure is pumped to the piston and cylinder arrangement which is shown sectioned in Fig. 8. When the piston is suddenly displaced by a stick command, say to the right in Fig. 8, the cylinder will initially move the same distance in the same direction since the oil is virtually incompressible. Oil will then flow immediately from the fixed pipe into the left hand part of the cylinder through the port P_1 at a rate proportional to the port opening. At the same time oil can escape from the right hand part of the cylinder through the port P_2. If the ports consist of narrow slits or rows of small holes the cylinder moves back to its initial position at a rate proportional to its displacement from it.

The springs shown in Fig. 8 are preloaded and their function is to provide a direct link in the control system if an oil pipe fractures allowing the oil to flow freely out of the cylinder. The preload on the springs must be greater than the force required to operate the control valves. In normal operation these springs do not contribute to the feel or to the dynamic response of the mechanism. They are compressed by oil pressure as the piston moves relative to the cylinder, but their presence will not be noticed by the pilot.

The dimensions of the mechanism will depend upon the magnitude of the output forces and the magnitude of the oil pressure which is provided. For a typical fighter type aircraft, with oil pressure of the order of 1,000 p.s.i., the overall length of the cylinder need not be more than a few inches.

An engineered version of this mechanism supplied with oil at a pressure of 100 p.s.i. has been tested on a piloted rig which was mounted on an air-bearing and controlled by jets of low pressure air. The measured parameters of the mechanism were in good agreement with those predicted from geometrical reasoning.

It is pointed out that the device is partly, but not entirely, a passive mechanism. The power required to move the control actuators initially is provided by the pilot. However, the power that is needed to remove the control when the stick is held in a constant position comes from the source which is maintaining the oil pressure, while the pilot's hand reacts to the forces produced as the controls close, without doing any work. This form of mechanism possesses an inherent advantage over one containing only passive components such as springs and dash-pots. In the latter type where there is no external source of power, the pilot must provide not only the energy needed to move the control actuators initially, but must also provide energy, which is temporarily stored within the mechanism, to remove the control again.

The forces which the pilot is required to react from a hydraulic shaping mechanism when maintaining a steady stick deflection need not necessarily be large. If the output friction is relatively small, the pilot's reaction may, with suitable leverage, be made quite unnoticeable in comparison with normal spring feel that would in any case be provided. If the output friction is very large then the system is no worse than manual control by a direct link, and it is likely that the aircraft would possess power assisted controls. In this case the mechanism would merely serve to operate a control valve for another hydraulic system.

4. *The Choice of Parameters for a Practical Mechanism.*

The choice of parameters for a practical shaping mechanism is one of compromise between optimum values for normal operation and those which are most likely to promote safety in event of failure of the mechanism. An important pre-requisite was, therefore, to conduct a programme of simulator tests which would give some measure of the relationship of controllability with the three system parameters, α, K_c and T in idealised circumstances.

4.1. *Description of the Simulation and Experimental Tests.*

For the present purpose it is considered adequate to restrict attention to the case of controlling about a single axis. The two degrees of freedom lateral hovering of a jet-lift aircraft may be represented by the simplified dynamical equations:

$$\frac{d^2\phi}{dt^2} = M \tag{23}$$

and,

$$\frac{d^2y}{dt^2} = g \sin\phi = g\phi \tag{24}$$

where ϕ is the angle of bank in radians, M is the ratio of rolling moment to moment of inertia in rad/sec^2, y is the sideways displacement in feet, and g is the acceleration of gravity in ft/sec^2.

The relationship between control moment and stick position for a direct manual (i.e. acceleration) control is:

$$M = K_A S \tag{25}$$

where S is the stick deflection in inches and K_A is the acceleration gearing constant in rad/sec^2/in.

For a direct control with synthetic rate damping,

$$M = K_R S - \tau \frac{d\phi}{dt} \tag{26}$$

where τ is the damping parameter in sec^{-1} and K_R is the rate stabilized gearing constant in rad/sec^2/in.

And, for the manual control with reshaping according to equation (21),

$$M = K_c S - \frac{1}{T}\int \left(M - \frac{K_c S}{\alpha}\right) dt \tag{27}$$

where α, T and K_c are the mechanism constants already defined.

These equations were simulated in real time as voltages in an electronic analogue computer. Control inputs were fed to the computer from potentiometer pick-offs attached to a centrally located pilot control stick. Angle of bank was displayed as an 'outside-in' artificial horizon line generated on a $4\frac{1}{2}$ in. diameter oscilloscope screen, and aircraft position relative to the ground was represented by a spot of light on a second oscilloscope situated immediately below the artificial horizon. This second screen displayed

± 25 feet of ground relative to a reference point. A diagram of the arrangement of display instruments is shown in Fig. 9. Only the two oscilloscopes were used to display information during this series of tests. The pilot was enclosed in a cockpit which was dimly illuminated by side lighting.

A programme of tests was conducted to assess pilot performance in a hovering steadiness task with a wide coverage of mechanism parameters, and to compare these results with his performance when using both an unaided control and a fully rate damped control. A single objective task was used throughout, one in which the pilot was required to maintain steady hovering at a fixed reference point. No disturbances other than those due to control action by the pilot himself were introduced. Each individual run consisted of two minutes and forty seconds of actual controlling. A rotating cam, with a time of revolution of 80 seconds, alternately switched a scoring circuit in and out. The pilot began controlling when this circuit was switched out, which allowed an amount of preliminary practice each time. Then the scoring circuit switched in for 80 seconds, during which time the absolute displacement of the aircraft from the reference point was integrated in the computer. This quantity was read on a voltmeter and recorded, and the mean absolute displacement, $\bar{y}$, in feet, for each run was subsequently calculated.

A hundred repetitions of the task were made with an optimum geared acceleration control which was found from preliminary tests to be 0·33 rad/sec^2/in. in this particular experimental environment. In a similar manner a hundred performance measurements were made during repeated runs using a rate damped control with 4 sec^{-1} damping and a stick gearing constant $K_R = 0{\cdot}33$ rad/sec^2/in. A systematic coverage of sets of mechanism parameters was made in which ten repetitions of the task were scored using all combinations of:

$$\alpha = 2, 4 \text{ and } 8$$

$$T = 0{\cdot}15, 0{\cdot}25, 0{\cdot}35, 0{\cdot}50 \text{ and } 0{\cdot}75 \text{ sec.}$$

$$K_c = 0{\cdot}167, 0{\cdot}333, 0{\cdot}500, 0{\cdot}667, 0{\cdot}833 \text{ and } 1{\cdot}00 \text{ rad/sec}^2\text{/in.}$$

Precautions were taken in the manner of conducting tests to keep the results as free as possible from bias due to task learning and fatigue, since only one pilot was used throughout. A discussion of these effects and of other random sources of variation, and the methods used in dealing with them is contained in Ref. 5.

4.2. *Results of Tests.*

The mean performance scores and standard deviations of a single score for all the combinations of parameters in the shaping equation that were tested are listed in Table 1. In Table 2 these scores are grouped together for constant values of the parameter α, and are compared with the mean scores obtained with acceleration and rate damped controls. A graphical comparison of the three types of control systems is shown in Fig. 10. It is seen here that the mean level of performance which can be achieved with reshaping approaches that obtainable with rate damping as the magnitude of the parameter α increases. When $\alpha = 8$ the difference between performance with reshaping and performance with rate damping is not discernible. It should be mentioned that this rate damped control was not an optimum for the simulator environment; with the amount of damping present a greater stick sensitivity would have been preferred. However, the value of 0·33 rad/sec^2/in. was chosen because it represents a practical limit to the gearing of rate stabilized controls if the control power is limited to about 1 rad/sec^2. Moreover, performance scores with optimum stick gearing would differ only but little from those values recorded.

In order to illustrate how performance depended upon all three parameters, α, K_c and T, of the shaping mechanism, contours of constant performance score on the T-K_c planes are shown in Fig. 11. These curves were drawn by eye from the figures in Table 1, smoothing out inconsistencies in the process. Shaded regions are those in which the performance scores may be expected to be as good as, or better than, the upper confidence limit of the rate damping scores, i.e. less than 1·4 ft. It is significant that hovering aided by reshaping stick commands may be expected to be as good as that obtained with the rate damped system when the mechanism parameter α is as low as 4 and the parameters T and K_c chosen within a fairly wide range of suitable values. Hovering aided by reshaping with $\alpha = 2$ did not improve accuracy of control to the level obtained with the rate damped system but, nevertheless, led to a significant improvement over the accuracy that could be achieved with a pure acceleration control.

5. *Discussion of Factors Influencing Controllability.*

The view is put forward that in the choice of suitable controls for hovering, it is more relevant to consider the steady state sinoidal response of the systems than the step responses. If sinoidal stick input were fed to a stabilized feedback control system, or to a system with a series shaping device, then the control moments would also be sinoidal at the same frequency, but the whole pattern of the latter would be advanced in phase and time ahead of the stick movement pattern. In practice, pilot inputs are not sinoidal, nor do they occur with an exactly constant frequency, but, because of the rapid pulsing technique which is characteristic of the way in which pilots use the stick to control most types of hovering vehicles, their resemblance to this form is sufficiently close for the sinoidal response to give a working model of what is actually taking place.

5.1. *Significance of Phase Lead.*

It is conjectured that the time or phase lead provided by aided control systems, which reduces the amount of anticipation that the pilot must provide himself, is the important factor which makes some controls more easy for hovering than others. Some support for this view is lent by the results described above.

Let us first consider how the magnitude of the phase lead compares with improvements in hovering performance which have been observed to accompany increasing magnitudes of the parameter α in the shaping equation. If the stick input to the system described by equation (21) were the sinoidal function:

$$S = \sin \omega t \tag{28}$$

where ω is in rad/sec, then the phase lead of control moments is given by:

$$\varepsilon = \tan^{-1} \alpha\omega T - \tan^{-1} \omega T \tag{29}$$

The phase advance angle, ε, is plotted in Fig. 12 as a function of ωT for the values $\alpha = 1$ (which may be regarded as a pure acceleration system), $\alpha = 2$, $\alpha = 4$, $\alpha = 8$ and $\alpha = \infty$ (which may be regarded as a pure rate damped system). It is evident that the magnitude of the phase lead possesses a maximum value which increases with increasing value of α. Insufficient evidence was found to suggest that the pilot, since he is relatively free to choose his own frequency of stick movements, will adjust this rate in keeping with the magnitude of the time constant so that he always operates with maximum phase lead. This clearly would lead to an absurdity where rate damped controls are concerned, since the maximum phase lead is 90 degrees at zero frequency. Instead it seems, on the basis of the simulator results, that there is a preferred frequency band of stick movements in the neighbourhood of one per second which varies little with the control characteristics. Suitable time constants in the shaping equation, which have been observed to be in the range of 0·1 to 0·5 seconds, are those which would allow operation close to the maximum phase lead angle within the preferred frequency band.

The predominant value of ωT observed in the simulated hovering studies was $\omega T = 1$ rad, being the product of a frequency of one stick movement, or π radians per second and the median value of the optimum time constant range, 0·3 seconds. Fig. 12 shows that at this value of ωT the phase lead angle increases from zero at $\alpha = 1$ to 45 degrees at $\alpha =$ infinity. It is also apparent that there is little to be gained in terms of phase advance by making the value of α much greater than 4: the phase lead angle is already two-thirds of that for the corresponding rate damped control, and this, in practice, could prove to be quite acceptable.

We could interpret this in another way. In response to deflections of the stick which are made and held constant for a short time, such as the reaction time of the pilot, the control nozzles open and begin to close exponentially. If the pilot moves the stick quickly to a new position before all the transient control power decays, then it matters little whether all the control power, or only three quarters of it, would have come off had the stick been held in the constant position for a longer period.

5.2. *Significance of Stick Gearing.*

The results of the tests also showed that the pilot's accuracy of control depended upon the sensitivity of the gearing of the systems. The tests were made by holding two of the parameters, α, and T, constant while the third, K_c, was varied. In most cases very sensitive or very sluggish controls were accompanied by poorer performance scores, and somewhere between the extremes an optimum gearing was indicated. The results of Table 1 are plotted in Fig. 13 as functions of K_c to show where these optima occurred. It is evident that the optimum gearing was not a unique value, but depended in some manner upon the other parameters of the mechanism. These optimum controls are plotted as curves on the T-K_c plane in Fig. 14. An explanation of this dependence based on a semi-empirical theory will be given.

As well as causing the control nozzle positions to lead the stick positions, the inclusion of a reshaping mechanism in the control link also modifies the effective mean amplitudes of the nozzle positions in a manner which depends upon the frequency of stick movements. The quantity:

$$A_R = \left\{ \frac{\omega^2 + \frac{1}{\alpha^2 T^2}}{\omega^2 + \frac{1}{T^2}} \right\}^{\frac{1}{2}} \tag{30}$$

which is plotted in Fig. 15 is the ratio of frequency dependent gearing for sinoidal inputs to direct gearing. When the magnitude of the parameter α is greater than unity the value of A_R is always less than unity, hence the mean amplitude of the control nozzle positions will be decreased by the inclusion of the device. It seems plausible that the frequency dependent gearing of an aided system should be correspondingly increased to the optimum value for the unaided system if the control again is to be an optimum one. Thus if the optimum gearing for the unaided system is, say, K_{opt}, then we may argue that the optimum gearing for the aided system is that value of K_c which makes:

$$K_c \left\{ \frac{\omega_p^2 + \frac{1}{\alpha^2 T^2}}{\omega_p^2 + \frac{1}{T^2}} \right\}^{\frac{1}{2}} = K_{opt} \tag{31}$$

where ω_p corresponds to the frequency of stick movements which the pilot is predominantly making. Now, the optimum stick gearing for the unaided acceleration control was found to be 0·33 rad/sec^2/in. It is mentioned in passing that this value is not to be regarded as a universal optimum gearing for all acceleration controls. Optimum controls depend significantly upon the environment and the way in which information is directed to the pilot, the nature of the task and the technique of the individual pilot. However, assuming that these things are equal in this case the optimum stick gearing for the shaping mechanism is given by:

$$(K_c)_{opt} = 0{\cdot}33 \left\{ \frac{\pi^2 + \frac{1}{T^2}}{\pi^2 + \frac{1}{\alpha^2 T^2}} \right\}^{\frac{1}{2}} \tag{32}$$

In accordance with a rate of stick movements of about one per second on the average, the value of ω_p has been taken to be π rad/sec.

The expression (32) is plotted in Fig. 16 as a function of α and T. In comparing these curves with those of Fig. 14 it is seen that there is sufficient agreement to suggest that the pilot does, in fact, control best when the frequency dependent gearing of the aided control system is increased to the value of the optimum unaided system.

6. *The Consequences of Failure of the Mechanism.*

It has been shown that greater accuracy of hovering control in ideal conditions is obtainable with greater values of the parameter α in the shaping mechanism. However, since the condition:

$$M_{max} = \frac{K_c S_{max}}{\alpha} \tag{33}$$

must be satisfied if the pilot is to be able to hold on full control with full stick deflection then the greater the value of α, the greater the control gearing required to maintain full control. But excessively high gearing might make the aeroplane uncontrollable in event of failure of the aiding device; it also restricts the range of stick movements for which the benefits of phase advance are obtainable. A reasonable compromise is to take $\alpha = 4$, for this combined with a stick travel of 6 in. in either direction would enable a maximum control moment of 1·0 rad/sec^2 to be sustained if the stick gearing were 0·667 rad/sec^2/in. From Fig. 14 it is apparent that the best value of T would then be 0·25 seconds.

In the event of oil pressure failing the preloaded springs immediately provide a direct link, but the maximum control moment would now be produced with stick deflections of 1·5 inches or greater. Such a control would be unduly sensitive. To provide a more acceptable manual control in the event of oil pressure failure, an oil operated variable gear mechanism could be included in the control circuit. Such a mechanism is shown diagrammatically in Fig. 17. In normal operation the piston is maintained by oil pressure in the position shown and the gear ratio is $a:b$. If oil and the gear ratio is reduced to $c:d$. The gear ratio could be conveniently halved in this way. After failure of oil pressure the pilot would be left with the stick geared at 0·33 rad/sec^2/in. and full control of 1 rad/sec^2 would thus require a deflection of 3 inches. A control circuit that would seem practical for either the lateral or longitudinal hovering control of a jet-lift aircraft is illustrated in Fig. 18.

7. *Conclusions.*

As a result of the present investigations the following conclusions have been made

(i) When the hovering position of a jet-lift aircraft is maintained by indirect control of attitude the main disturbance tending to upset equilibrium is that introduced by the pilot himself through his control actions.

(ii) Reshaping of stick commands, although it does not alleviate the effects of external forces or moments acting on the vehicle, nevertheless can perform the same function as real or synthetic damping in simplifying the control actions that a pilot must make and thus reduce the amount of disturbance injected. It therefore follows that considerable improvement of handling properties while hovering unstabilized aircraft could be obtained with suitable shaping of the attitude stick commands.

(iii) A practical method of reshaping stick commands using a simple hydraulic integrating mechanism has been described.

(iv) Simulator tests were conducted and have led to a definition of optimum mechanism parameters for hovering in idealised conditions.

(v) Predictions based on the concept of continuous sinoidal stick movements and responses have given values for optimum parameters in good agreement with those derived from measurements of pilot performance.

(vi) A working compromise between optimum parameters for good handling in normal operation and those which could meet the control power and safety requirements of an aircraft control system has been suggested for the shaping mechanism.

(vii) At the present stage of autostabilizer design, it is necessary for a full authority rate damped system to be installed in triplex. Since reshaping mechanisms are basically mechanical they should be much less liable to malfunction than electrical systems, so multiplexing would not be necessary. Therefore, where simplicity and economy are overriding factors, reshaping mechanisms are worth considering as an alternative to autostabilization.

Acknowledgement.

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TABLE 1

Pilot Performance Scores during Simulated Hovering with Reshaping.

no. of tests	T (sec)	K_c (rad/s^2/in)	$\alpha = 2$		$\alpha = 4$		$\alpha = 8$	
			$\lvert\bar{y}\rvert$ (ft)	std. devn. (ft)	$\lvert\bar{y}\rvert$ (ft)	std. devn. (ft)	$\lvert\bar{y}\rvert$ (ft)	std. devn. (ft)
10	0·15	0·167	2·45	0·46	2·00	0·33	2·89	0·28
		0·333	2·05	0·36	1·86	0·46	1·33	0·13
		0·500	1·81	0·27	1·62	0·28	1·06	0·15
		0·667	1·51	0·27	1·68	0·37	1·04	0·19
		0·833	2·36	0·45	1·23	0·12	0·99	0·13
		1·000	2·25	0·88	1·51	0·45	0·99	0·14
10	0·25	0·167	1·88	0·36	1·90	0·37	1·28	0·21
		0·333	1·69	0·51	1·35	0·28	1·11	0·18
		0·500	1·56	0·41	1·36	0·15	1·00	0·11
		0·667	2·16	0·77	1·33	0·24	1·04	0·18
		0·833	2·71	0·48	1·33	0·23	1·14	0·18
		1·000	2·72	0·83	2·03	0·47	1·14	0·21
10	0·35	0·167	2·23	0·51	1·45	0·37	1·35	0·22
		0·333	1·92	0·51	1·35	0·16	1·20	0·10
		0·500	2·07	0·33	1·28	0·21	1·03	0·16
		0·667	1·91	0·42	1·53	0·39	1·20	0·21
		0·833	2·11	0·39	1·54	0·43	1·31	0·21
		1·000	2·49	0·35	1·87	0·41	1·34	0·24
10	0·50	0·167	2·02	0·35	1·66	0·49	1·35	0·19
		0·333	1·81	0·41	1·41	0·30	1·05	0·22
		0·500	1·83	0·46	1·62	0·50	1·13	0·15
		0·667	2·33	0·53	1·89	0·48	1·21	0·12
		0·833	2·72	0·70	2·04	0·53	1·15	0·11
		1·000	3·54	0·97	2·48	0·44	1·30	0·23
10	0·75	0·167	2·09	0·41	1·40	0·32	1·67	0·35
		0·333	1·41	0·24	1·31	0·22	1·37	0·26
		0·500	1·81	0·36	1·53	0·38	1·43	0·27
		0·667	2·21	0·67	1·69	0·30	1·67	0·39
		0·833	3·07	0·43	2·14	0·52	1·40	0·35
		1·000	2·79	0·41	2·40	0·55	1·73	0·20

TABLE 2

Comparison of Different Controls.

control system	no. of tests	$\lvert\bar{y}\rvert$ (ft)	standard deviation (ft)	99·9 % normal confidence limits (ft)
rate damped $\tau = 4\ \text{sec}^{-1}$; $K_R = 0{\cdot}33\ r/s^2/\text{in}$	100	1·35	0·29	0·10
acceleration $K_A = 0{\cdot}33\ r/s^2/in$	100	3·19	1·11	0·36
reshaping $\alpha = 2$	300	2·18	0·67	0·13
reshaping $\alpha = 4$	300	1·63	0·52	0·10
reshaping $\alpha = 8$	300	1·30	0·41	0·08

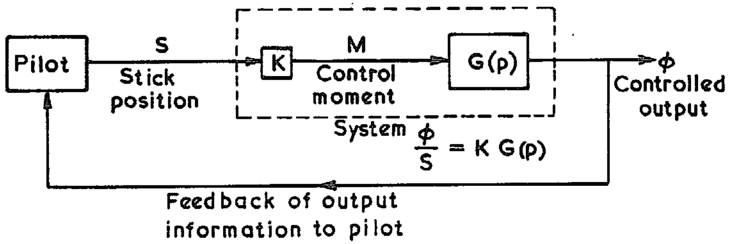


FIG. 1. Basic pilot-aircraft control loop.

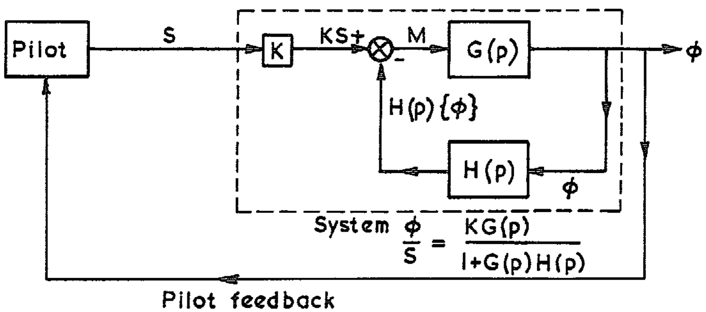


FIG. 2. System modified by feedback.

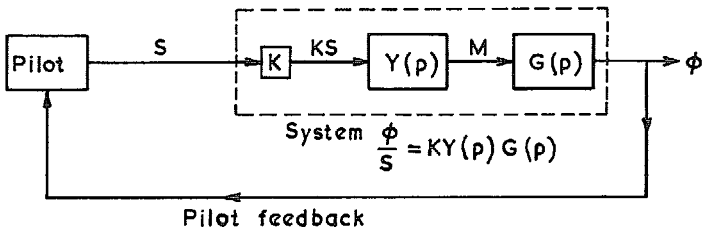


FIG. 3. System modified by reshaping.

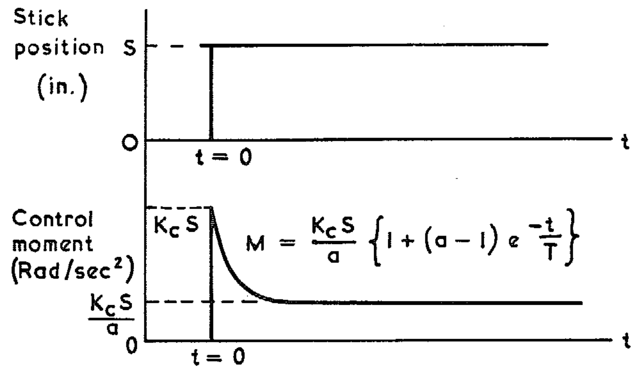


FIG. 4. Step response of shaping mechanism.

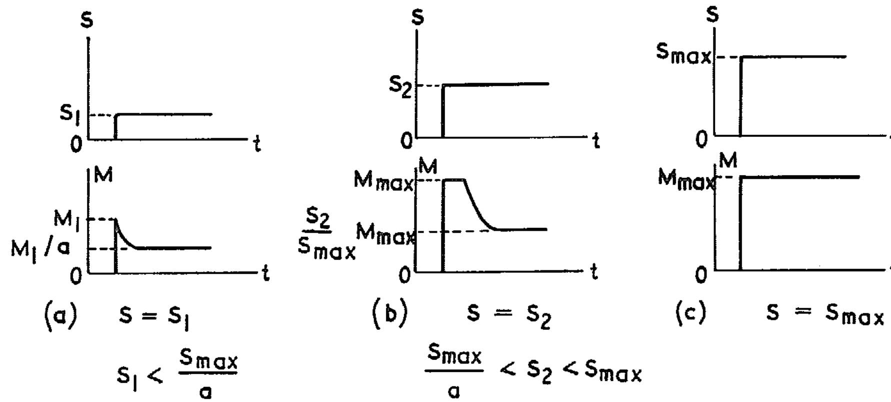


FIG. 5. Step responses of shaping mechanism with non-linear saturation.

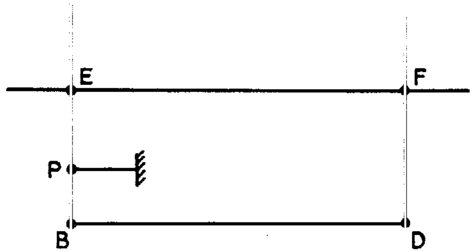


FIG. 6a. Linkage in the neutral position.

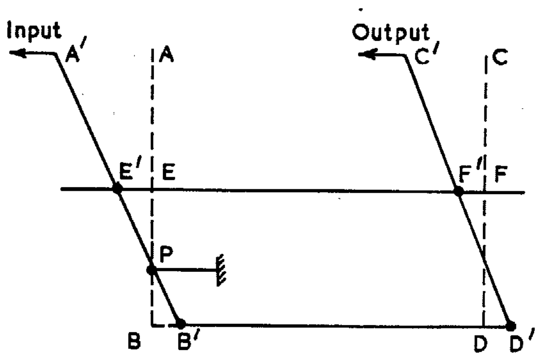


FIG. 6b. Instantaneous position following a step input.

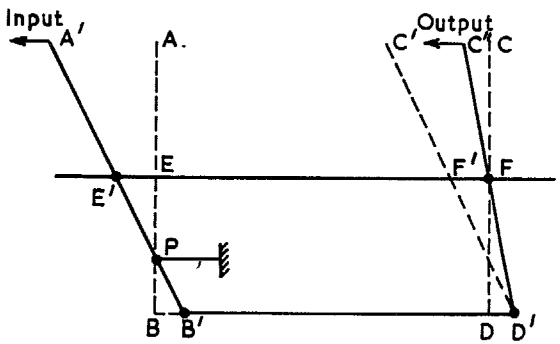


FIG. 6c. Final steady position when the input is constant.

FIG. 6. Principle of the four-bar linkage.

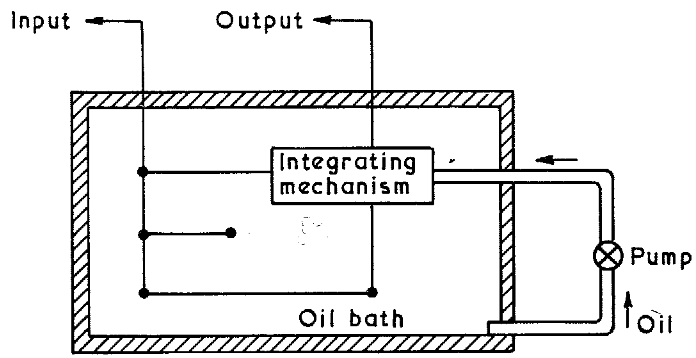


FIG. 7. Arrangement of shaping mechanism.

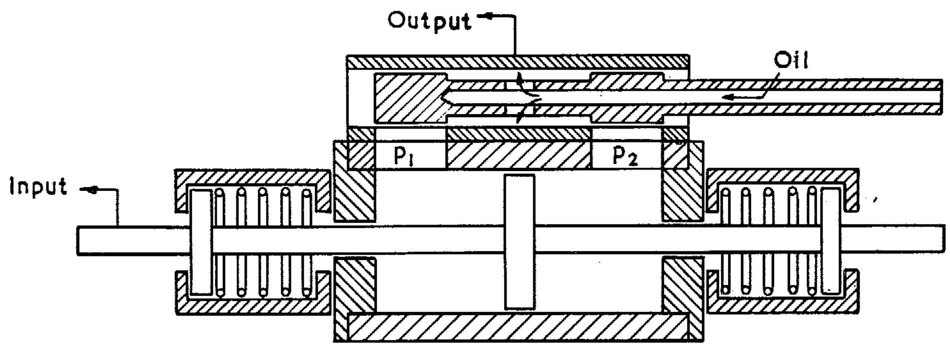
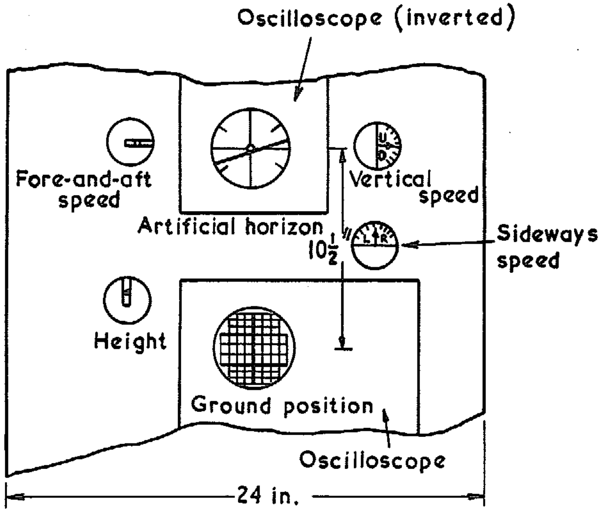
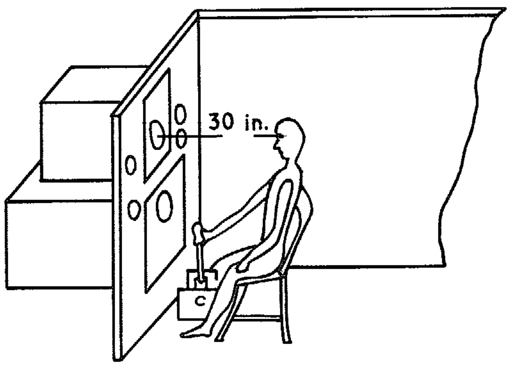


FIG. 8. Hydraulic integrating device.



Display panel.



Interior of cockpit.

FIG. 9. Arrangement of display instruments.

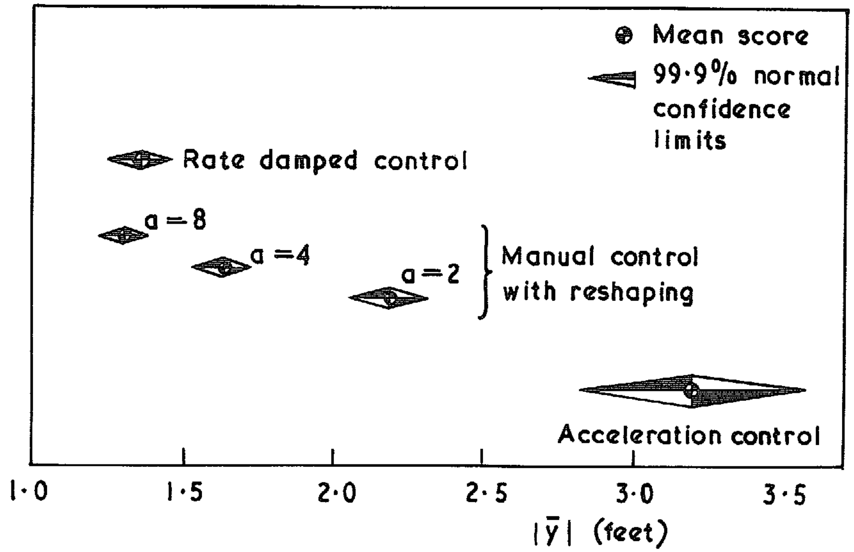


FIG. 10. Comparison of different control systems.

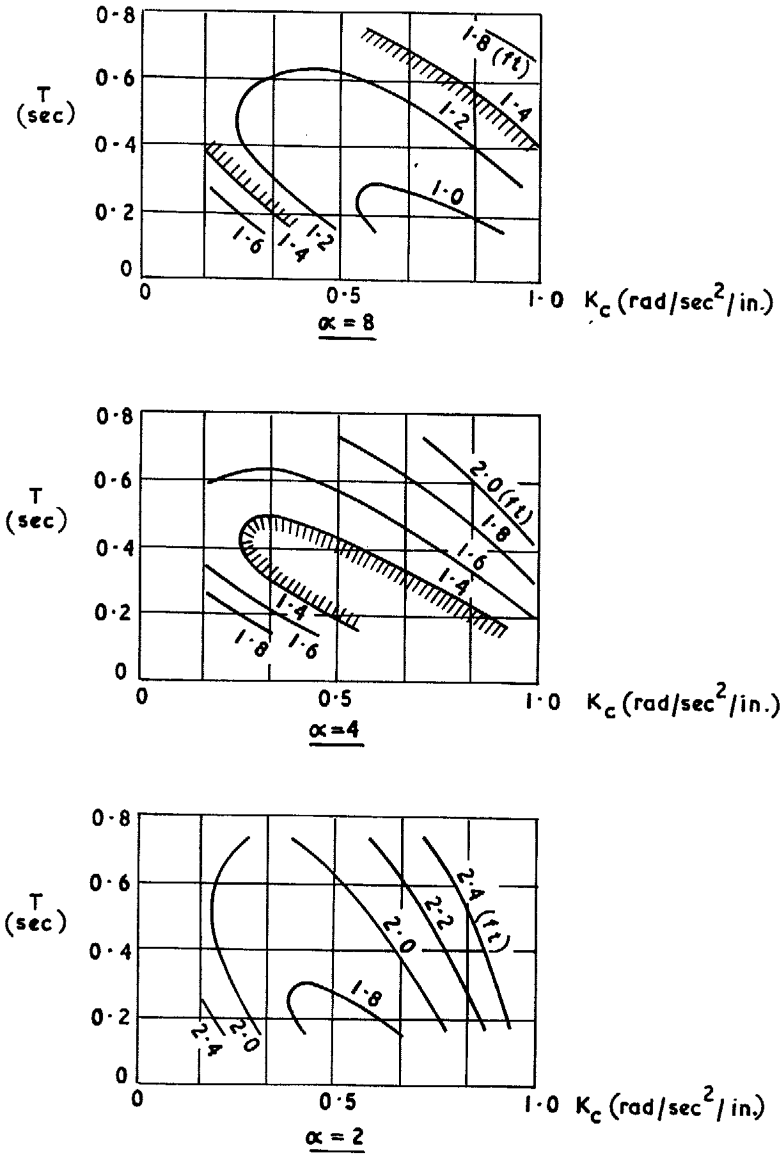


FIG. 11. Contours of constant performance scores in hovering tests.

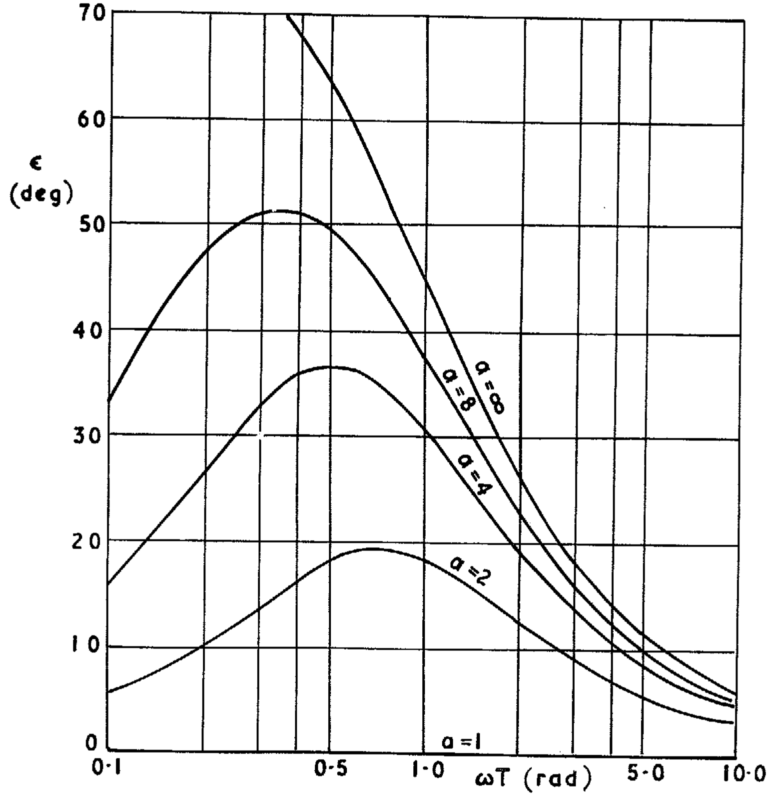


FIG. 12. Phase angles of: $\dfrac{j\omega + \dfrac{1}{\alpha T}}{j\omega + \dfrac{1}{T}}$.

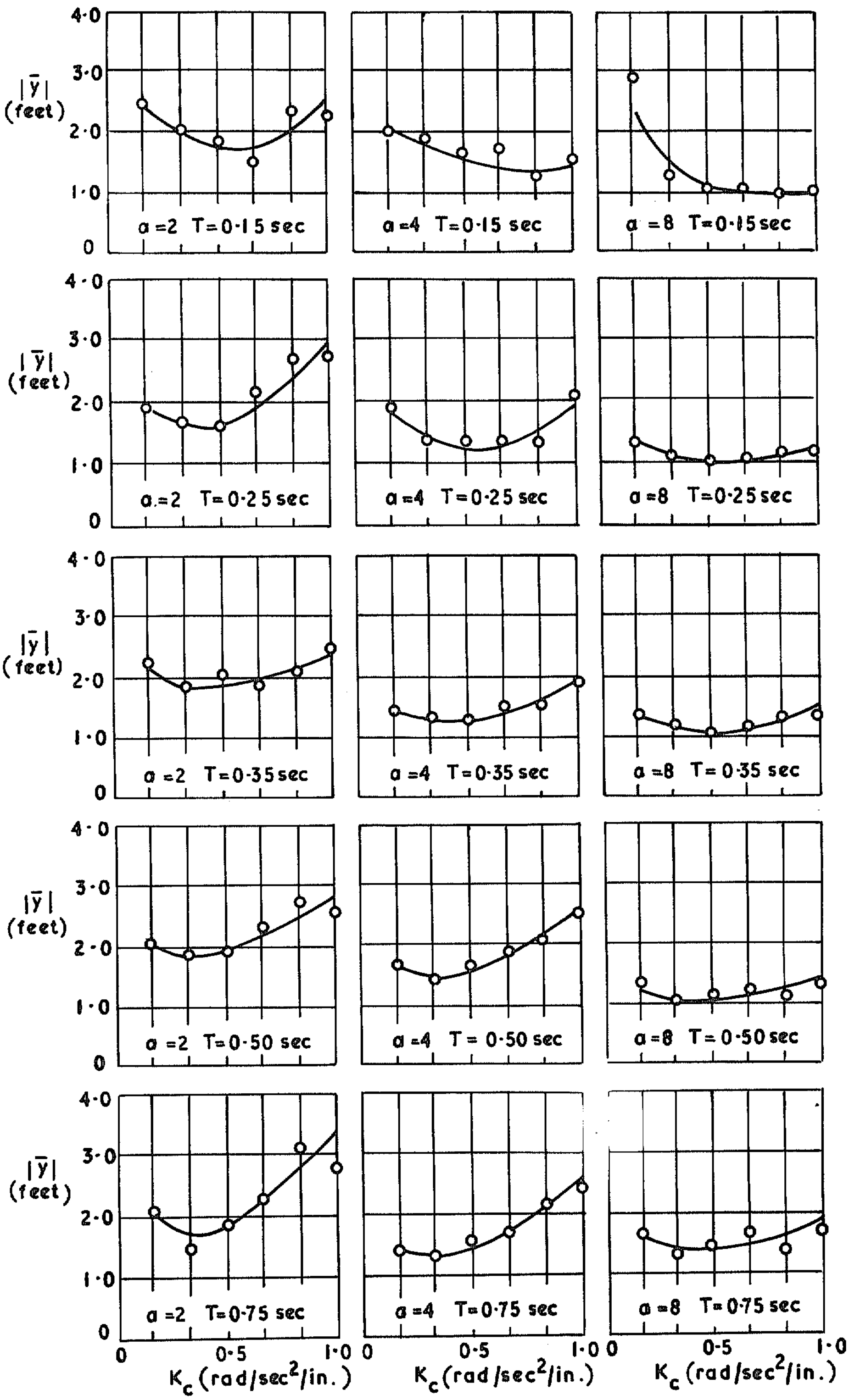


FIG. 13. Performance scores as function of K_c.

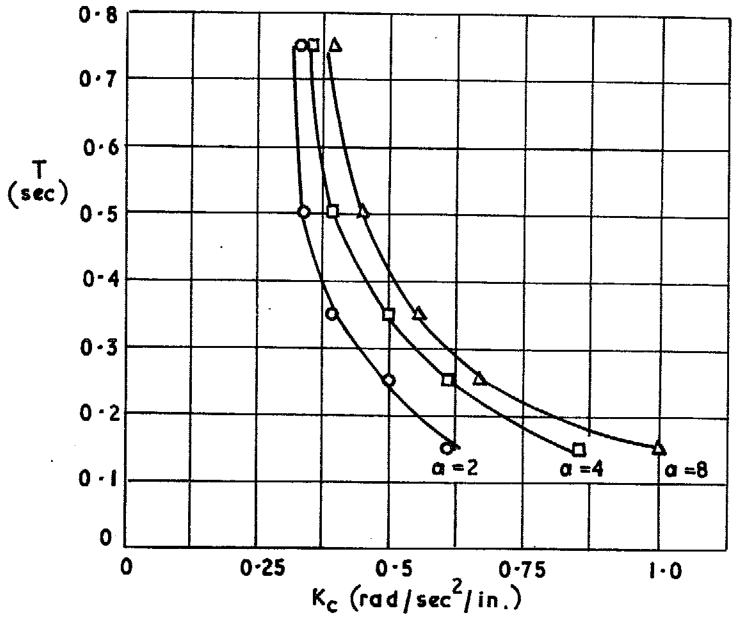


FIG. 14. Experimental optimum controls.

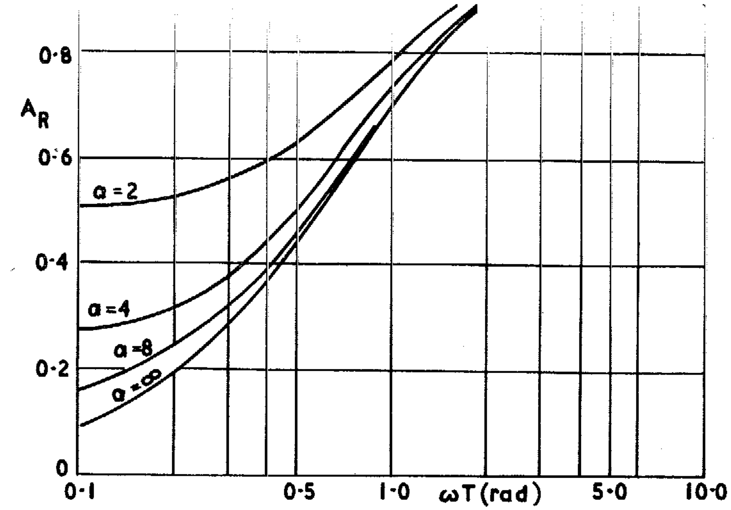


FIG. 15. Modulus of : $\dfrac{j\omega+\dfrac{1}{\alpha T}}{j\omega+\dfrac{1}{T}}$

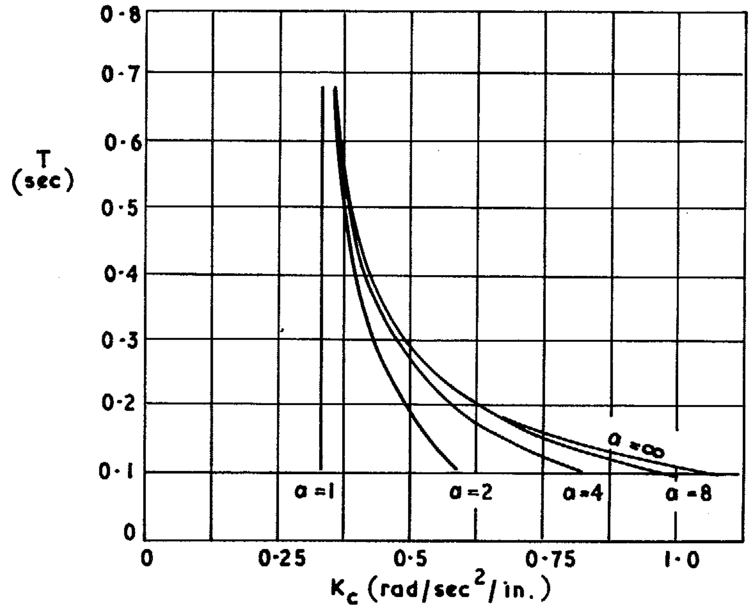


FIG. 16. Theoretical optimum controls.

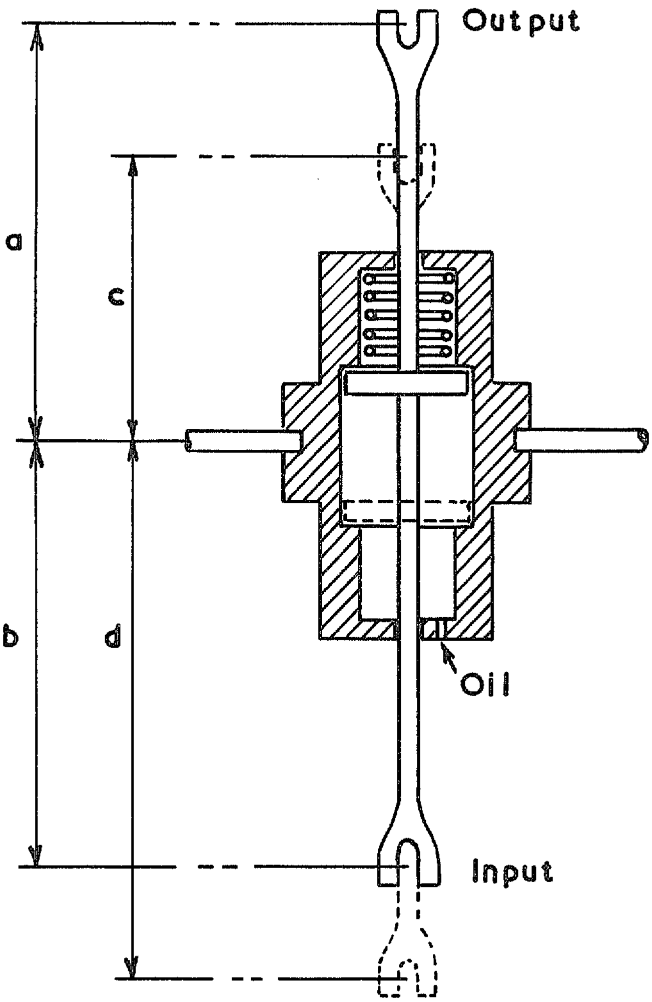


FIG. 17. Oil operated gear changing mechanism.

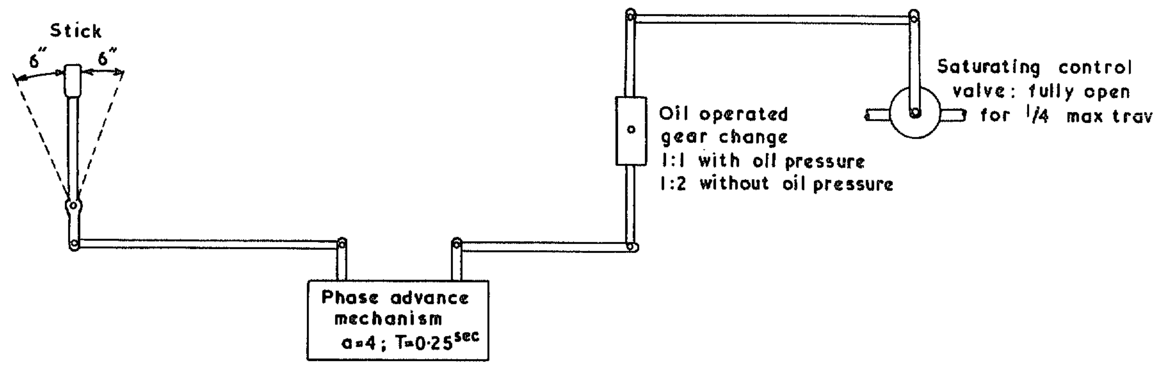


FIG. 18. Suggested hovering control circuit for unstabilized aircraft.

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