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Variations in the Strength of Aluminium Alloy Sheet

By

D. T. Jones, M.A., B.Sc.

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Variations in the strength of aluminium alloy sheet

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SUMMARY

In the design of aircraft structures allowance is made for variation in the material strength of sheet but the thickness of the sheet is usually assumed to be constant at the nominal value for the gauge used.

Thickness variation may appreciably affect the tensile strength of nominally identical pieces of sheet, especially when the tolerance range in thickness is wide. A method is given in this note for calculating the distribution of strength when the mean and scatter of material strength and the mean and scatter of thickness of pieces are known. Expressions are given for the mean and scatter in strength of pieces and, for those cases where the distribution of the strength can be taken as normal, design values can readily be deduced from the expressions.

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ILLUSTRATION

1 Introduction

This note is concerned with two sources of variation in tensile strength of nominally identical pieces of aluminium alloy sheet. The strength of such pieces depends on the strength of the material and the area of the section through which load is applied; in pieces of a given width of section the strength is proportional to the product of material strength and thickness. As both the material strength and the thickness of pieces which have been cut from sheet of a given gauge made of a specified material vary among themselves there are two sources of variation in the strength.

The purpose of the note is to show how the total effect of the two sources may be estimated. Although in design it is usual to take account of variations in material strength only, the thickness being assumed constant at the nominal value[1], there may be occasions when the effect of thickness variation should be included. For instance, if the average thickness is appreciably above nominal value, and there is little scatter in thickness, structures will be stronger and heavier than calculations based on the assumption that the thickness of all sheets is at nominal would indicate. Or if the average thickness is near the nominal but there is a wide scatter in thickness there will be too many weak structures.

It will be shown that when the mean and standard deviation of both material strength and thickness are known, the total effect can be easily calculated if the material strength and thickness of pieces are normally distributed and independent of one another, that is there is no tendency for the material in thick pieces to be stronger or weaker than in thin pieces. In these circumstances the mean and standard deviation of the values of the product can be found and, as in most cases the distribution of the product values is found to be normal, design values* for the pieces of sheet can readily be deduced.

2 Estimation of the form of a distribution from a sample of measurements

Let us consider, to begin with, the way in which a measured variable, say the thickness** of pieces of sheet cut from a bulk supply of sheet of given nominal thickness, are distributed. If the thicknesses of a large sample of pieces taken at random are measured it is found that a considerable number are at and near the mean value and that the numbers with thicknesses on either side of the mean become less as the departure from the mean becomes greater. The numbers can be presented as a histogram in which the height of a column stands for the number of specimens in the sample having thicknesses within the range defined by the width of the column:-

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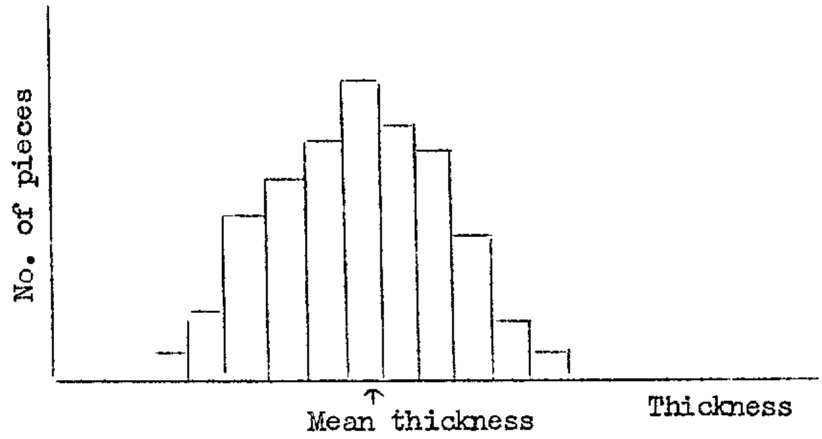
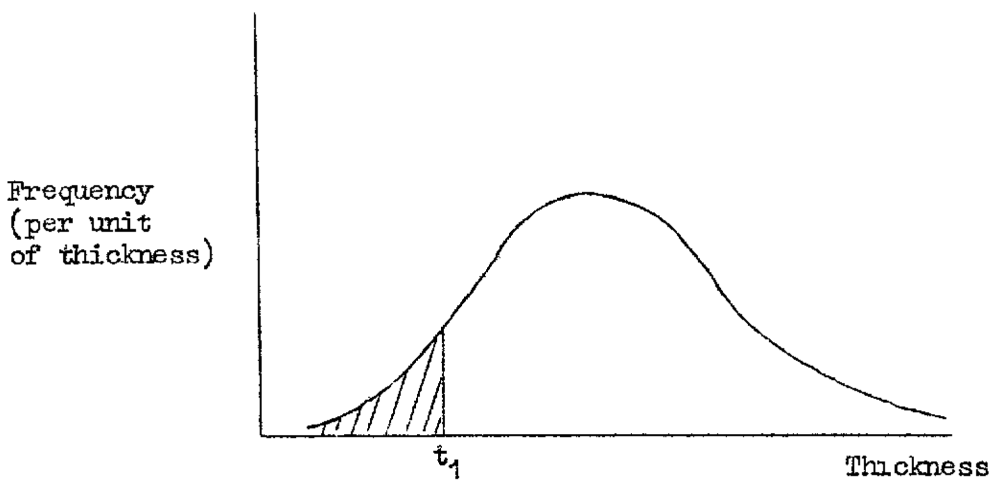
* These design values are for tensile strength. Variations in strength to resist failure by instability are not dealt with.

** Here, and throughout this note, the thickness of a piece of sheet is taken to be the mean thickness of the piece.



A frequency distribution, such as this, not only gives the proportions of sample pieces which fall within given ranges of thickness but also estimates of the proportions in the bulk supply which fall within the given ranges. The larger the sample the closer is this estimate likely to be. If the sample is large enough and it is desired to use it to estimate the proportions of the bulk which fall above or below any proposed value of thickness, a continuous curve may be fitted to the histogram or, alternatively, the mathematical form of the curve may be found. If the curve turns out to be as shown below the proportion of pieces having a thickness less than t_1 is given by the ratio of the shaded area to the whole area under the curve.



The form of curve may be found by calculating first of all the mean of the sample values and the second, third and fourth moments about the mean. If x represents the thickness of a member of the sample and there are n members in the sample the mean is defined as

$$\mu_1' = \frac{\Sigma x}{n} = \bar{x}$$

in which Σx stands for the sum of all the values of x. The moments (which are quantities analogous to the moments used in mechanics) are defined as:

$$\mu_2 = \frac{\Sigma (x - \bar{x})^2}{n}$$

$$\mu_3 = \frac{\Sigma (x - \bar{x})^3}{r}$$

$$\mu_4 = \frac{\Sigma (x - \bar{x})^4}{n}$$

Thus, the second, third and fourth moments about the mean are the means of the sums of squares, cubes and fourth powers respectively of the deviation of sample members from the mean of the distribution. The first moment about the mean, μ_1, is of course always zero. The second moment about the mean, μ_2, is known also as the variance and its square root is the well-known standard deviation, a quantity commonly used as a measure of scatter.

Details of the method by which the moments are used to identify the form of the curve are given in a standard work by Elderton[2]. When the moments have been computed the next step is to evaluate two parameters

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

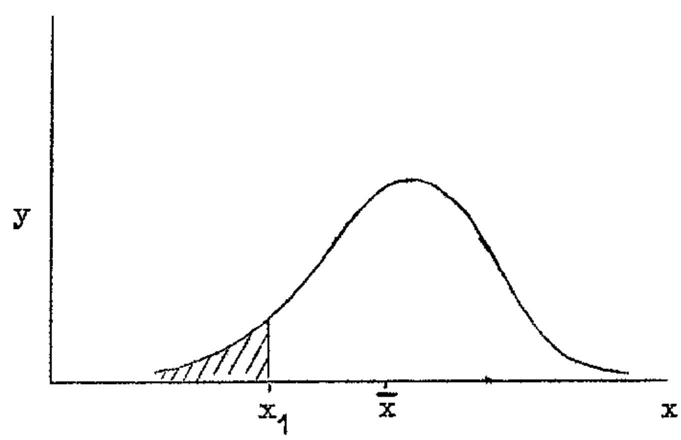
and

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

The values assumed by these identify the curve as one of a wide range of frequency curves due to Pearson. For example, if β_1 is zero and β_2 is 3 the curve is the normal curve of Gauss, the formula for which is

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\bar{x})^2}{2\sigma^2}}$$

where x is the variate, σ is the standard deviation of the distribution, and y is the frequency ordinate. This curve is symmetrical and bell-shaped:-



The area under the curve is unity and the proportion of values below x_1 is $\int_{-\infty}^{x_1} y.dx$. For prescribed values of $\bar{x}$ and σ the value of this expression can be found from tables of the normal probability integral[3].

The normal curve is one of the Pearson range. If the curve does not appear to be normal, the quantities

$$C_1 = 2\beta_2 - 3\beta_1 - 6$$

$$C_2 = \frac{\beta_1 (\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)}$$

provide criteria for selecting the best fitting curve from the other possible curves in the range. The formulae for curves corresponding to given values of the criteria are given by Elderton.

3 Derivation of distribution of strength of pieces of sheet

Now suppose that the thickness x' of pieces of sheet of given gauge is normally distributed with mean m_1 and standard deviation s_1 and the material strength x'' is normally distributed with mean m_2 and standard deviation s_2. All the pieces are of the same width of section so that the strength of each piece is proportional to the product of material strength and thickness. To find the distribution of strength of pieces we therefore have to find the distribution of this product.

There is no general expression for the form of the product distribution but formulae giving its mean and moments in terms of the means and standard deviations of the contributory distributions may be deduced* from a note by Irwin[4]. The mean of the product distribution is given by

$$\mu_1' = m_1 m_2$$

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* Irwin gives, with a slight change of notation, the rth cumulant of two normally distributed independent variates as

$$K_r = (r-1)! \ \tfrac{1}{2}\left\{(s_1^2 m_2^2 + s_2^2 m_1^2) + s_1^2 s_2^2\right\} s_1^{r-2} \ s_2^{r-2}$$

(where r is even)
and

$$K_r = r! \ m_1 m_2 s_1^{r-1} \ s_2^{r-1} \qquad \text{(where } r \text{ is odd)}$$

The cumulants are closely related to moments about the mean and the values of the moments given in the text can be deduced from Irwin's expressions.

and the second, third and fourth moments (the moments we need to determine its form) are

$$\mu_2 = s_1^2 m_2^2 + s_2^2 m_1^2 + s_1^2 s_2^2$$

$$\mu_3 = 6 m_1 m_2 s_1^2 s_2^2$$

$$\mu_4 = 6 s_1^2 s_2^2 (2 s_1^2 m_2^2 + 2 s_1^2 m_1^2 + s_1^2 s_2^2) + 3 \mu_2^2$$

The mean strength of pieces is proportional to the product of the means of thickness and material strength. From the formula for μ_2 we have that the standard deviation of the product value is

$$\sqrt{s_1^2 m_2^2 + s_2^2 m_1^2 + s_1^2 s_2^2} .$$

When the moments have been evaluated the form of the distribution can be found by calculating β_1 and β_2 as described in para 2. Although the strict requirements for normality i.e. $\beta_1 = 0$ and $\beta_2 = 3$ cannot be met except in unreal cases as for example when s_1 or s_2 are zero, it will generally be found that the normal distribution is a sufficiently close approximation for practical purposes. An example shows the method of calculation.

4 Calculated Form of Distribution

Suppose that we have to calculate the strength distribution of pieces of sheet 36 in. wide when it is known that the material strength is normally distributed with a mean m_1 of 30 tons per sq.in. and a standard deviation s_1 of 1 ton per sq.in. The mean thickness is known to be 0.086 in. with a standard deviation of 0.002 in.

We have for the distribution of the product

$$\mu_1' = 2.58$$

$$\mu_2 = 0.011$$

$$\mu_3 = 0.0000619$$

$$\mu_4 - 3\mu_2^2 = 0.000000361$$

and

$$\beta_1 = 0.00038$$

$$\beta_2 = 3.00036$$

As β_1 is very nearly zero and β_2 is very nearly 3 the distribution can be taken to be normal.

The mean strength of pieces 36 in. wide is 36×2.58 tons $= 92.88$ tons and the standard deviation of strength is $36 \times \sqrt{0.011} = 3.78$ tons. The form of the distribution is sketched in Fig.1(a).

Estimating a design value

For design purposes it may be required to find the strength below which a very small proportion, say one in a thousand, may be expected to fall. Tables of the normal probability integral show that the value of strength below which the area under the curve is one thousandth of the whole area is 3.1 times the standard deviation less than the mean. Hence the required value is $92.88 - 3.1 \times 3.78 = 81.16$ tons.

Comparison with design values estimated for no scatter in thickness

It is interesting to compare the above design value with the one which would be obtained on the assumption that there is no scatter in thickness but that all pieces have the average value of thickness and that only the material strength varies. The form of the distribution is shown in Fig.1(b). It has a mean of 92.88 tons and a standard deviation of $0.086 \times 36 = 3.10$ tons. This single variate distribution is clearly not so widely scattered as the product distribution and the design value (corresponding to the requirement that only one in a thousand falls below it) is now

$$92.88 - 3.1 \times 3.10 = 83.27 \text{ tons} .$$

We can also compare the product distribution design value with the one which would be obtained on the assumption that there is no scatter in thickness but all pieces have the nominal value of thickness (the assumption usually made in design). The form of the distribution is shown in Fig.1(c). It has a mean of 86.4 tons and a standard deviation of $0.080 \times 36 = 2.88$ tons. The design value (corresponding to the same requirements as before) is now 77.47 tons. Thus, there is a difference between the design value derived from the product distribution and the one derived from a distribution in which the thickness is assumed constant at nominal of 5.8 tons (or about 7 per cent).

The examples are given only for illustration and are intended to show the order of difference that might be obtained in practice. The standard deviation of thickness influences the difference and the value of 0.002 in. is probably rather higher than would be obtained in many cases. It is very much higher of course than would be obtained in specially made close tolerance sheet used in the construction of some aircraft.

5 Conclusions

The combined effect of variations in material strength and thickness on the tensile strength of pieces of sheet can be calculated if the material strength and thickness are both normally distributed and the mean value and standard deviation of both are known.

The calculation leads to design values which may differ significantly from values derived on the assumption that the material strength is variable but the thickness is constant at the nominal value or the average value for the sheet.

The method of calculation described may be used to find the distribution of the product of any two variables which are independent of one another and are normally distributed.

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2	Sir W.P. Elderton	Frequency Curves and Correlation Cambridge University Press
3	Edited by Karl Pearson	Tables for Statisticians and Biometricians Biometrika Office, University College, London
4	T.O. Irwin	On the Characteristic Function of the Distribution of the Product of Two Normal Variates Proc. Camb. Phil. Soc. Vol.42, Pt.1 1945

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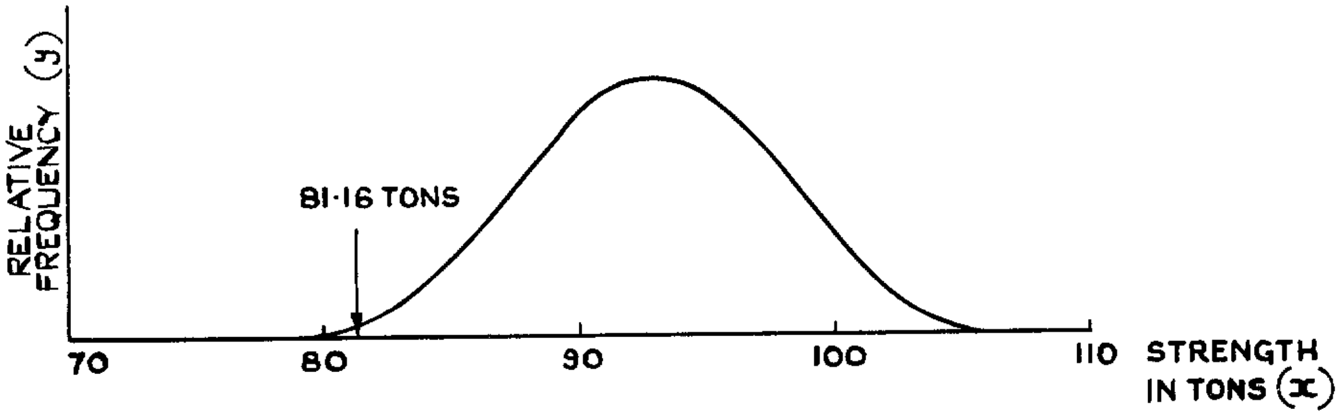
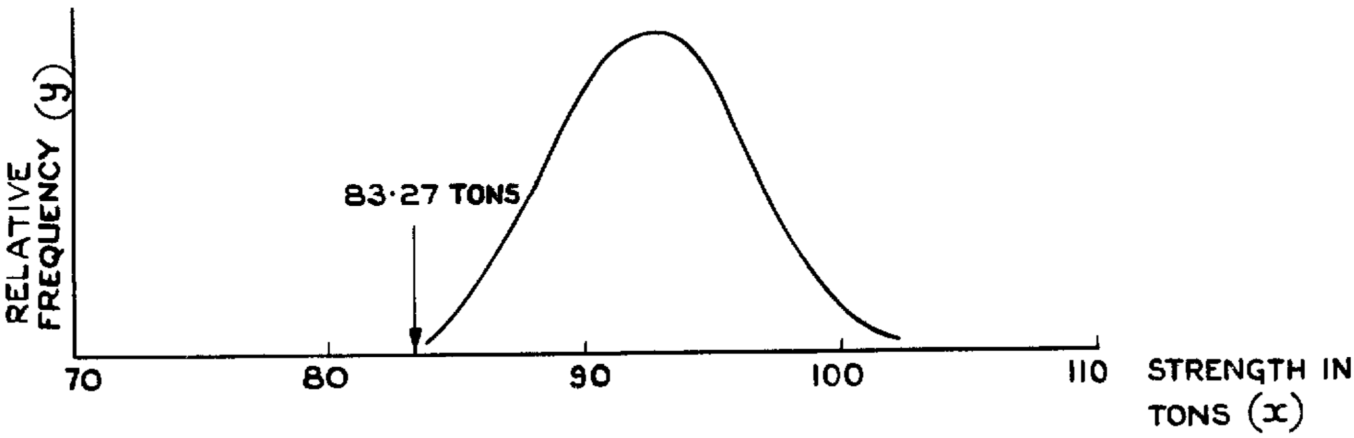
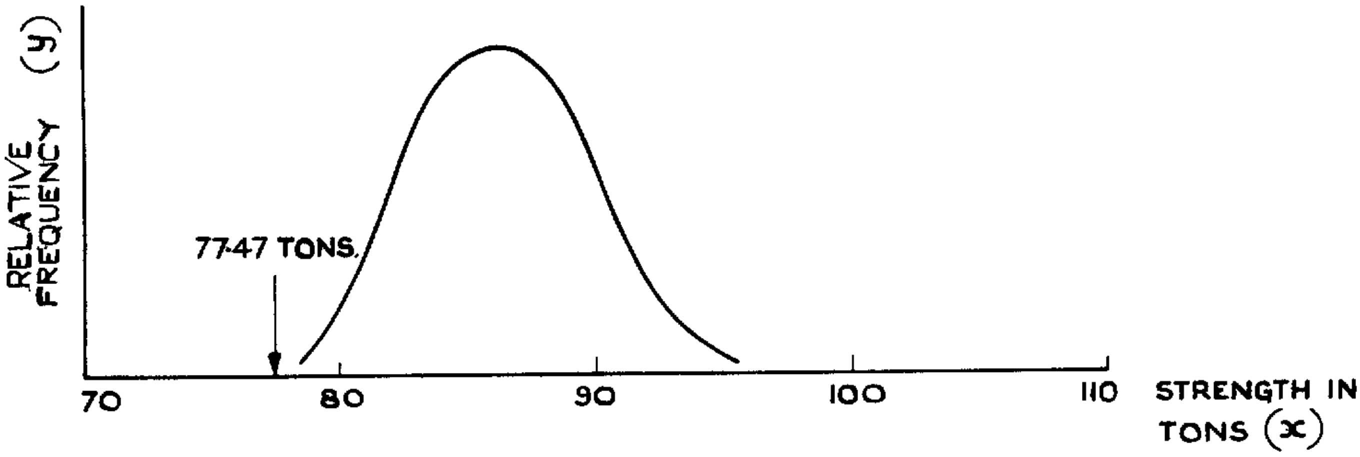
FIG. 1 (a,b&c)



(a) MEASURED VARIATIONS IN MATERIAL STRENGTH AND THICKNESS BOTH TAKEN INTO ACCOUNT.



(b) VARIATION IN MATERIAL STRENGTH TAKEN INTO ACCOUNT. (THICKNESS ASSUMED CONSTANT AT MEASURED MEAN VALUE.)



(c) VARIATION IN MATERIAL STRENGTH TAKEN INTO ACCOUNT. (THICKNESS ASSUMED CONSTANT AT NOMINAL VALUE.)

NOTE:– CURVES ILLUSTRATE WORKED EXAMPLES. ARROWS INDICATE VALUES OF STRENGTH BELOW WHICH ONE SPECIMEN IN A THOUSAND MAY FALL

FIG. 1 (a,b&c.) EXAMPLES OF EFFECT OF THICKNESS VARIATION ON DISTRIBUTION OF STRENGTH.

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