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MINISTRY OF SUPPLY

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CURRENT PAPERS

Investigation of High Length/Beam Ratio Seaplane Hulls with High Beam Loadings

Hydrodynamic Stability Part 2

The Effect of Changes in the Mass, Moment of Inertia and Radius of Gyration on Longitudinal Stability Limits

By

J. K. Friswell, B.Sc.,
A. Kurn, Grad.R.Ae.S., and
D. M. Ridland, G.I Mech.E.

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THE EFFECT OF CHANGES IN THE MASS, MOMENT OF INERTIA AND RADIUS OF GYRATION ON LONGITUDINAL STABILITY LIMITS

by

J. K. Friswell, B.Sc.
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SUMMARY

Tests have been performed to ascertain the effects of varying load, moment of inertia, and radius of gyration on the stability limits of a high length-to-beam-ratio dynamic model. The tests were carried out at high beam loadings, with C_{Δ_0} in the range 2.00 - 3.00. A theoretical analysis has been made of the relation between the effects of the various parameters, and the results of the analysis compared with experimental results. The effect on the limits of a change from a velocity to a draught base has also been considered.

It has been found that the load is the most critical factor, and that provided the load is kept constant increasing the moment of inertia has little effect on the limits. Good agreement has been found between the theoretical treatment and experiment.

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1. INTRODUCTION

Such evidence as was available when this investigation[1,2] was planned indicated that changes in the pitching moment of inertia of a flying boat model did not in themselves, when unaccompanied by changes in mass, have any appreciable effect on the longitudinal hydrodynamic stability limits. For this reason, no particular moment of inertia was aimed at in the construction of models in the series (Table I) nor was any attempt made to vary the moment of inertia according to any particular rule while bringing the mass of each model to the various values at which it was considered desirable to test stability. Extra weights were merely fixed to a bar through the centre of gravity, thus keeping the moment of inertia effectively constant.

Since previous investigations of this matter did not cover the same ranges of values of the various parameters involved as are used in this programme, however, it was felt advisable to carry out limited tests on one model of the series to verify that no particular attention needed to be paid to the value of the moment of inertia. Model B was used as it permitted a more adequate range of values to be covered than other models available.

For completeness three separate tests were performed, in each of which one of the three parameters, mass (m), moment of inertia (I), and radius of gyration (k), was held constant at some appropriate value, and the other two parameters varied over a fairly large range, longitudinal hydrodynamic stability limits being obtained for each combination of values. Mass changes were, however, only considered to show their interaction with changes in the other parameters, moment of inertia and radius of gyration being the factors of direct interest.

Since I, m and k are related by $I = mk^2$, the effects on the limits of changes in them are not independent. They can be related analytically by considering critical trim (i.e. the trim at which longitudinal instability sets in) as a function I, m, k and velocity and taking into account the implicit relations between the parameters. Details of this treatment are given and comparisons made of analytical and experimental results.

It has been suggested that limits plotted on a draught base would show smaller sensitivity to mass and inertia changes than those on a velocity base. Graphs showing the effects of this change of base are therefore included, and the theoretical analysis has been extended to indicate the relation between the two sets of limits.

The centre of gravity has been taken to be fixed throughout the theoretical treatment to correspond with the conditions of the model tests.[3]

In addition to the limits themselves, figures have been included showing the amplitudes of porpoising in the unstable regions. These enable the violence (or otherwise) of the instability to be judged, and comparison of them shows the effect of changes in mass etc. on behaviour in these regions.

2. PREVIOUS INVESTIGATIONS

The mass of a dynamic model is determined directly by the value of C_{Δ_o} at which it is to be tested. The behaviour of the model at different values of C_{Δ_o} is governed by a number of design factors, not all of which are being investigated at present. All models in the programme will be tested at at least two values of C_{Δ_o}, and reference will be made at a later stage to previous work relevant to the behaviour of the models at various loads in the appropriate test conditions. Direct consideration will therefore only be given here to previous investigations into the effects of varying the pitching moment of inertia and radius of gyration, though it should be noted that a change of mass will automatically imply a change either in moment of inertia or radius of gyration.

The effect of moment of inertia on the stability of a seaplane was first considered theoretically by Perring and Glauert[4], who by treating the planing surfaces as flat plates showed that in the single step case too small a moment of inertia would produce instability at an otherwise stable point while in the two-step case too large a moment of inertia would have this effect. Their general conclusion was that in model tests the ratio mass/moment of inertia was the most critical factor, i.e. that the radius of gyration should be given its correct scale value, and that if the model was then stable an increase in the radius of gyration from this value would produce instability in the two-step case while a decrease would produce instability in the one-step case. No specific consideration was, however, given to which, if any, of I, m and k were to be kept constant during the changes mentioned for the conclusions to be valid.

Richards and Hutchinson[5] also considered radius of gyration to be the factor which would have most effect on stability, and mentioned that changes in mass while retaining the radius of gyration at its scale value (by altering the moment of inertia) still resulted in a movement of the stability limits. The latter point was investigated by means of the Routh discriminant, and led to the conclusion that both mass and radius of gyration should be given correct scale values in model tests. The size of the effect referred to in this report was illustrated in Reference 6 for one particular model, the mass being increased by 15% and the moment of inertia by 100%; the movement of the stability limit here was very slight, being approximately one-fifth of the change produced by a 30% change of mass at constant moment of inertia.

In Reference 7, the results of fairly extensive tests on the effects of radius of gyration and moment of inertia changes were given both on critical trim and amplitudes of porpoising; the planing surface used represented the forebody only of a flying boat hull, so that the treatment was concerned with the lower limit. The tests covered a range of values of C_{Δ_0} from 0 to 2, of C_V from 3 to 7 and of radius of gyration from 0.5 to 1.3 beams. An increase in radius of gyration at constant load was found to lower the critical trim, while an increase in load at constant radius of gyration raised it. Both these effects were fairly large, being of the order of 2 degrees for 100% change in the former case and 1 degree for a change from $C_{\Delta_0} = 0.27$ to 0.40 in the latter. Porpoising amplitudes were found to increase markedly with decrease in radius of gyration at constant load. Further tests with a dynamic model showed that these amplitudes also increased with moment of inertia at constant radius of gyration. An analysis in this report of conventional flying boats showed them to have radii of gyration of at most 1.55 beams, associated with a C_{Δ_0} of the order of 1.

Further limited data on the subject were given by Olson and Land[8]. Little significant change was found to result from increasing the moment of inertia of a dynamic model by 25% at constant load ($C_{\Delta_0} = 0.72$). Similar results were quoted by Davidson for 100% change in moment of inertia at constant C_{Δ_0} of 0.89 in Reference 9.

The general conclusions of the various reports mentioned are substantiated in other sources but no quantitative data are given.

It will be seen that the experimental data mentioned all relates to fairly low values of C_{Δ_0}. However, the general theoretical and experimental conclusions may be expected to extend to higher values of C_{Δ_0}.

/ 3. DESCRIPTION

3. DESCRIPTION OF EXPERIMENTS

As already stated, the tests were carried out on Model B of the series described in Part I of this account[3]. Full details are given there of the considerations affecting the design of the models, but it may be mentioned here that Model B has a length to beam ratio of 11 (the forebody being 6 beams in length and the afterbody 5 beams), 4° forebody warp per beam, an afterbody to forebody keel angle of 6° and a straight transverse step with a step depth of 0.15 beams. Figure 1 gives the hull lines of the model and Figure 2 photographs of it.

Longitudinal stability tests were made by towing the model from the wing tips on the lateral axis through the centre of gravity, the model being free in pitch and heave. Values of C_{Δ_0}, moment of inertia, radius of gyration and elevator setting were selected before each run, and the model towed at constant speed. The angle of trim was noted in the steady condition, and if the model proved stable at the speed selected it was given nose down disturbances to determine whether instability could be induced, the amount of disturbance necessary to cause instability being in the range $0 - 10^{\circ}$. Stability limits were built up by these methods, the disturbed limits evidently representing the worst possible case. When steady porpoising did occur, either with or without disturbance, the amplitude was noted, amplitude for this purpose being defined as the difference between the maximum and minimum trims attained in the oscillation. Full details of the techniques used are given in Reference 3.

The minimum value of C_{Δ_0} which could be achieved was 2.00, and the minimum moment of inertia 21.3 lb . ft^2. A range of values of C_{Δ_0} was covered at this minimum moment of inertia by adding lead weights to a bar through the centre of gravity (Figure 3(a)). The addition of these weights produced a change in the moment of inertia of less than 1%, so that it can fairly be said that the moment of inertia remained constant. A second series of tests was performed at constant radius of gyration with C_{Δ_0} varying between 2.00 and 3.00, this constant value being chosen as 1.26 ft since this was the only value which could be obtained at all the values of C_{Δ_0} required. Finally, with C_{Δ_0} fixed at 2.50, the centre of the range, the moment of inertia was increased by 40%, almost the maximum increase obtainable at this C_{Δ_0} and one which is likely to exceed any natural increase which arises in the manufacture of the models; moreover, the range covered was much wider than would be likely full-scale. In these last two cases the chosen moment of inertia was obtained by sliding lead weights along a light bar running fore and aft inside the model; as shown in Figure 3(b).

The stability limits obtained in these tests are shown in Figures 4-9, and the porpoising amplitudes in Figures 10-17, the limits also being reproduced in these latter figures for convenience.

4. EXPERIMENTAL RESULTS

The results of individual tests are given in Figures 10-17, and the stability limits are compared in Figures 4-9.

With the mass held constant and the moment of inertia and radius of gyration varied (Figures 6 and 7) almost no change in the undisturbed limits results; what difference there is can be attributed to experimental error. The disturbed limits are rather more widely separated, but the amount is still not significant. The fact that the limits are not in order here tends to confirm this view.

Finally, Figures 8 and 9 show that with radius of gyration held constant the variation of the limits with load is of the same order as in the case of constant moment of inertia, though here there are no cases of curves being positioned out of order. The variation here can also of course be considered as a moment of inertia effect.

It is interesting to note that in all cases the separations of the undisturbed lower limits are of the same order as the changes in hump trims from load to load and that at the higher speeds instability occurs at about the same elevator settings in all cases. Figures showing trim curves have not been included in this report since it is with the limits themselves that it is concerned, but these figures will be given in the data report on Model B.

Considering the three sets of limits as a whole, it seems that over the ranges of values considered the value of C_{Δ_0} is the most critical factor, and that neither changes in the radius of gyration nor in the moment of inertia will have any significant effect unless accompanied by changes in C_{Δ_0}.

The effects of the various changes on the amplitudes of porpoising (Figures 10-17) are in general less marked, though in all cases there is a large difference between the amplitudes at corresponding points in the undisturbed and disturbed cases. With moment of inertia constant, an increase in load and decrease in radius of gyration produces a small change in the amplitudes in the disturbed case and no discernible change in the undisturbed case. At constant load there is a small increase with increasing radius of gyration and moment of inertia in the disturbed case, and a most marked increase in the undisturbed case. In the remaining case, with radius of gyration constant, there is no evidence of change in either direction.

It is interesting to compare these results with those quoted in Section 2 as relevant to lower values of C_{Δ_0}. While the general, qualitative, conclusions of those references are confirmed, the radius of gyration has not been found to have the importance it possessed at lower loads; as already mentioned, C_{Δ_0} seems the only critical factor. Of course, if, as is common in model tests, the moment of inertia is held appreciably constant while the load is increased, than a change in C_{Δ_0} is accompanied by a change in radius of gyration, so that in this sense the value of the radius of gyration can be said to be critical. However, the results quoted in Section 2 referred to limit changes resulting from changes in radius of gyration at constant load; this effect is not noticeable in the present case, though it is possible, but unlikely, that it exists at other values of C_{Δ_0} in the range 2.00 - 3.00. It may be noted that the value of radius of gyration in the present tests ranges between 2.17 and 2.82 beams, somewhat higher values than those relevant to Reference 7; since the radius of gyration of a full-scale version of the design now tested would be about 2.2 beams, however, this range of values is a realistic one.

/ 5. THEORETICAL

5. THEORETICAL ANALYSIS

Let V denote velocity
α_k keel attitude
d draught
m mass
I moment of inertia
k radius of gyration
and C critical trim (i.e. the trim at which longitudinal instability sets in for any particular velocity or draught).

Then the stability limits plotted against V and α_k (as in Figures 4 - 9; C_V is merely a constant multiple of V) can be regarded as graphs of C as a function of V and two of I, m, and k: in Figures 4 and 5 C is represented as a function of V, m and I, or V, k and I: in Figures 6 and 7 of V, I and m, or V, k and m; and in Figures 8 and 9 of V, I and k, or V, m and k.

Because of the implicit relationship $I = mk^2$ the separations of the critical trim lines on these various graphs are not all independent. These separations can be represented analytically by partial derivatives of the type $\left(\frac{\partial C}{\partial I}\right)_{V,k}$, where the suffices indicate the variables taken as the independent variables other than the one with respect to which differentiation is being effected. For convenience this derivative will be written $C_I^{V,k}$, and others written similarly. The complete set of these derivatives in the (α_k, V) plane is

$$C_V^{m,I}\ ,\quad C_m^{V,I}\ ,\quad C_I^{V,m}\ ,$$

$$C_V^{m,k}\ ,\quad C_m^{V,k}\ ,\quad C_k^{V,m}\ ,$$

$$C_V^{I,k}\ ,\quad C_I^{V,k}\ ,\quad C_k^{V,I}\ .$$

For relations between them we proceed as follows;

let $C = f(V,m,I)$

then $dC = \frac{\partial f}{\partial V}\ dV + \frac{\partial f}{\partial m}\ dm + \frac{\partial f}{\partial I}\ dI$

and since $I = mk^2 = \phi\ (m,k)$ say,

$$dI = \frac{\partial \phi}{\partial m} dm + \frac{\partial \phi}{\partial k}\ dk$$
$$= k^2\ dm + 2mkdk$$

To find $C_h^{i,j}$ where h, i and j are the three variables chosen as independent variables, dC must first be expressed in terms of dh, di and dj only. $C_h^{i,j}$ is then the coefficient of dh in this expression.

e.g. $C_V^{m,I} = \frac{\partial f}{\partial V}$, $C_I^{V,m} = \frac{\partial f}{\partial I}$, $C_m^{V,I} = \frac{\partial f}{\partial m}$, and since

$$dC = \frac{\partial f}{\partial V}\ dV + \frac{\partial f}{\partial m}\ dm + \frac{\partial f}{\partial I}\left(k^2\ dm + 2mkdk\right),$$

$C_m^{V,k} = \frac{\partial f}{\partial m} + k^2\frac{\partial f}{\partial I} = C_m^{V,I} + k^2\ C_I^{V,m}$ etc. Other relations are obtained by eliminating dm instead of dI.

/ The

The set of relations of this kind is

$$C_V^{m,I} = C_V^{m,k} = C_V^{I,k} \qquad \ldots\ldots\ldots (1)$$

$$C_m^{V,k} = C_m^{V,I} + k^2 \; C_I^{V,m} \qquad \ldots\ldots\ldots (2)$$

$$C_k^{V,m} = 2mk \; C_I^{V,m} \qquad \ldots\ldots\ldots (3)$$

$$C_I^{k,V} = C_I^{V,m} + \frac{1}{k^2} \; C_m^{V,I} \qquad \ldots\ldots\ldots (4)$$

$$C_k^{I,V} = - \frac{2m}{k} \; C_m^{V,I} \qquad \ldots\ldots\ldots (5)$$

A similar set of relations holds with the draught d replacing V throughout, viz,

$$C_d^{m,I} = C_d^{m,k} = C_d^{I,k} \qquad \ldots\ldots\ldots (6)$$

$$C_m^{d,k} = C_m^{d,I} + k^2 \; C_I^{d,m} \qquad \ldots\ldots\ldots (7)$$

$$C_k^{d,m} = 2mk C_I^{d,m} \qquad \ldots\ldots\ldots (8)$$

$$C_I^{k,d} = C_I^{d,m} + \frac{1}{k^2} \; C_m^{d,I} \qquad \ldots\ldots\ldots (9)$$

$$C_k^{I,d} = - \frac{2m}{k} \; C_m^{d,I} \qquad \ldots\ldots\ldots (10)$$

The two sets can be linked as follows :-

In general $d = d\ (V,m,\eta)$ and the trim curves give $\eta = \eta(\alpha_k, V, m)$ so that $d = d\ (V,m,\alpha_k)$. In the transformation of stability limits from a velocity to a draught base however all the points considered are points on critical trim lines so that $\alpha_k = C$ and $d = d\ (V,m,C)$. Since C is already known as $f\ (V,m,I)$ this can be reduced to $d = d\ (V,m,I)$. Then we have

$$\left.\begin{aligned} C &= f\ (V,m,I) \\ I &= mk^2 \\ d &= \psi\ (V,m,I) \end{aligned}\right\}$$

and a similar treatment to that already employed gives relations linking the various derivatives. We obtain

$$C_m^{d,I} = C_m^{V,I} - \frac{d_m^{I,V}}{d_V^{m,I}} \; C_V^{m,I} \qquad \ldots\ldots\ldots (11)$$

$$C_I^{m,d} = C_I^{m,V} - \frac{d_I^{m,V}}{d_V^{m,I}} \; C_V^{m,I} \qquad \ldots\ldots\ldots (12)$$

and $$C_d^{I,m} = \frac{C_V^{m,I}}{d_V^{m,I}} \qquad \ldots\ldots\ldots (13)$$

/ which

which with the other two sets of relations are sufficient to determine all other possible relations.

6. RELATION OF THEORY TO EXPERIMENT

As already noted, the separations of the various limits plotted for comparison purposes in Figures 4 - 9 can be related to the partial derivatives enumerated in the previous section, as can the slopes of these limits. This is equally true of both disturbed and undisturbed limits, but consideration will only be given here to the latter.

For example, consider Figure 4. The slopes of the curves are given by $C_V^{I,m}$, and their separations normal to the velocity axis by $C_m^{V,I}$ (it is immaterial that the non-dimensional parameters C_V and $C_{\wedge o}$ have been used in annotating the figure itself rather than V and m - the effect is merely to change the units of measurement). That the slopes and separations are different in different sections of the diagram merely indicates that the derivatives are not constants but are themselves functions of V, I, m and k.

In a similar manner $C_V^{I,m}$ and $C_I^{V,m}$ give slopes and separations on Figure 6 and $C_V^{k,m}$ and $C_m^{V,k}$ on Figure 8. It should perhaps be noted that in all the cases so far mentioned there is an alternative choice of independent variables; e.g. $C_V^{I,m}$ (Figure 4) could equally well have been $C_V^{I,k}$, and $C_m^{V,I}$ have been $C_k^{V,I}$. The fact that the existence of this choice does not affect the slopes of the limits is expressed by Equation (1) of section 5; this equation also takes account of the fact that the various sets of limits consist in part of the same limits collected together in different combinations.

Equations (2) to (5) give the theoretical relations between the vertical separations of the limits in Figures 4, 6 and 8. If it is assumed that the movement of the limits in Figure 6 is negligible, being only of the order of possible experimental error, then we have $C_I^{V,m} = 0$ and $C_k^{V,m} = 0$ (this is self-consistent: see Equation (3)). Equations (2) and (4) then reduce to

$$C_m^{V,k} = C_m^{V,I}$$

and $$k^2 \; C_I^{k,V} = C_m^{V,I}$$

respectively. The first of these equations is in direct accord with the evidence of Figures 4 and 8, the vertical separation of the limits for $C_{\wedge o} = 2.00$ and for $C_{\wedge o} = 3.00$ being the same in both cases, within about 10%: all of this discrepancy could be attributed to experimental error. Verification of the second relation is not directly possible without expressing the various derivatives as functions of I, m, k and V, but a brief calculation readily shows it to give results of the correct order of magnitude. Equation (5) is self-evident.

remember that d in these equations denotes draught at points on critical trim lines only). No experimental readings of draught were obtained during the tests, but the requisite information can be obtained by a direct comparison of Figures 4, 6 and 8 with 18 - 20; this only involves the assumption that the difference between the draught of the forebody and that of the corresponding wedge shape is small. These figures show that $d_m^{V,I}$ is positive and $d_V^{m,I}$ negative, and both will be less than 1 in the units chosen. Equation (13) gives the relation between the slopes of the limits on the two bases, and because of the facts just mentioned shows that they will be of opposite sign and that the slope on the draught base will be greater than that on the velocity base. This is confirmed by all of Figures 18 - 20. (Note that in these figures draught increases from right to left so that slopes are reversed).

Equation (11) connects Figures 4 and 18. For the lower limits $C_V^{m,I}$ is negative, so that $C_m^{d,I}$ is predicted to be less than $C_m^{V,I}$, which is in fact the case. It is interesting to note that this tendency for the limits to collapse in changing to a draught base is only possible when the slope of the limits is negative; if it were positive the collapse would occur in transferring from a draught to a velocity base.

Equation (12) cannot be checked accurately with the figures available; it was observed previously that the scatter of the lines in Figure 6 could all be attributed to experimental error, and if the curves were corrected before transposing to a draught base, then there would be a complete collapse on both bases. $d_I^{m,V}$ would be zero under these circumstances, which since $C_I^{m,V}$ is zero would imply that $C_I^{m,d}$ is also zero; this would be completely self-consistent.

An analysis can be made of Equations (6) to (10) in exactly the same manner as was done with (1) to (5), and gives very similar results. Details will not therefore be given here.

It will be seen that all the analytical predictions have been verified, and that therefore in general it would not be necessary to cover a complete range of all the parameters in order to ascertain the effect of varying them; this could be done by a limited series of tests together with the results of section 5. In a similar manner the effects of any change of base could be predicted without actually carrying out the work.

7. CONCLUSIONS

The experimental evidence obtained in this series of tests indicates that within the range of values of parameters covered, only the load has an appreciable effect on stability limits. When the load is held constant, moment of inertia increases of up to 40% have no appreciable effect on the limits.

Increase of the radius of gyration at constant mass has the effect of increasing the amplitude of porpoising particularly in the undisturbed case, while the amplitudes are not noticeably affected by changes of mass.

All the general predictions of the theoretical analysis have been verified; this indicates that to obtain complete information on the behaviour of a model under variations of the various parameters involved, it is unnecessary to perform a large number of tests, since all the results can be forecast from a limited number of experiments. In the same way the effect of a change of base on stability limits can be accurately predicted analytically.

/ LIST OF SYMBOLS

LIST OF SYMBOLS

b	beam
C	critical trim
C_V	velocity coefficient = $V/\sqrt{gb}$
C_Δ	load coefficient $= \Delta/wb^3$
C_{Δ_o}	load coefficient at $V = 0$
$C_I^{V,k}$	etc. see section 5
d	draught
I	pitching moment of inertia
k	pitching radius of gyration
m	mass
w	water density
α_k	keel attitude
Δ	load on water
η	elevator setting

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LIST OF REFERENCES

A. Reports directly related to M.A.E.E. Hull Design Investigation (Ref. 2).

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1	A. G. Smith J. E. Allen	Water and air performance of seaplane hulls as affected by fairing and fineness ratio. R. & M. 2896. August, 1950.
2	A. G. Smith J. A. Hamilton	Notes on a detailed research programme on aero and hydrodynamics of hulls with high fineness ratio and full step fairings. F/Res/221. 1951. A.R.C. 13,877. March, 1951.
3	D. M. Ridland J. K. Friswell A. Kurn	Investigation of high length/beam ratio seaplane hulls with high beam loadings. Hydrodynamic stability Part I. Techniques and presentations of results. Current Paper No. . September, 1953.

B. Other Reports

No.	Author(s)	Title
4	W. G. A. Perring H. Glauert	Stability on the water of a seaplane in the planing condition. R. & M. 1493. 1932.
5	G. J. Richards J. L. Hutchinson	Some notes on the mathematical investigation of porpoising. F/Res/72, 1934. A.R.C. 1259.
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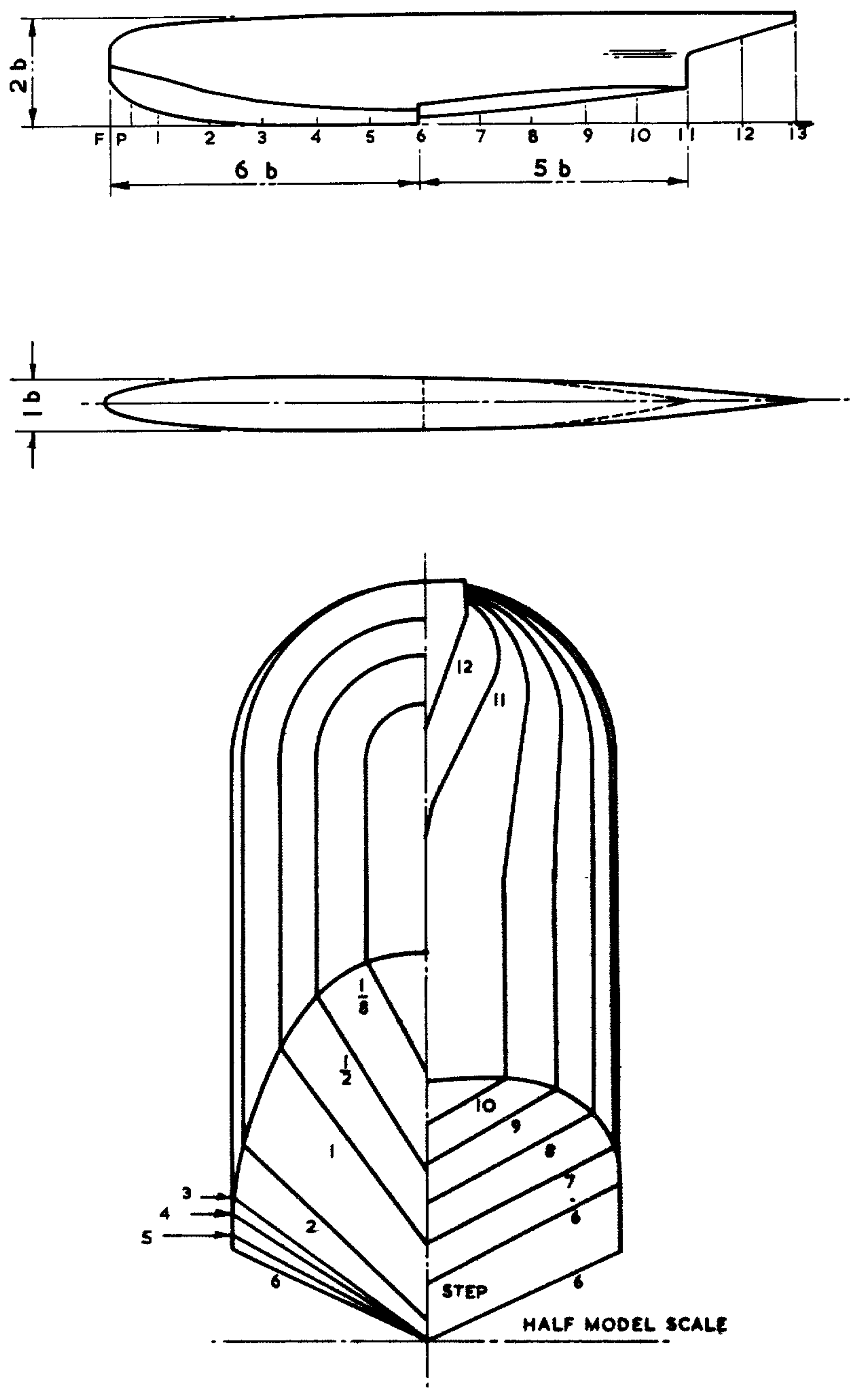
/TABLE I

TABLE I

Models for hydrodynamic stability tests

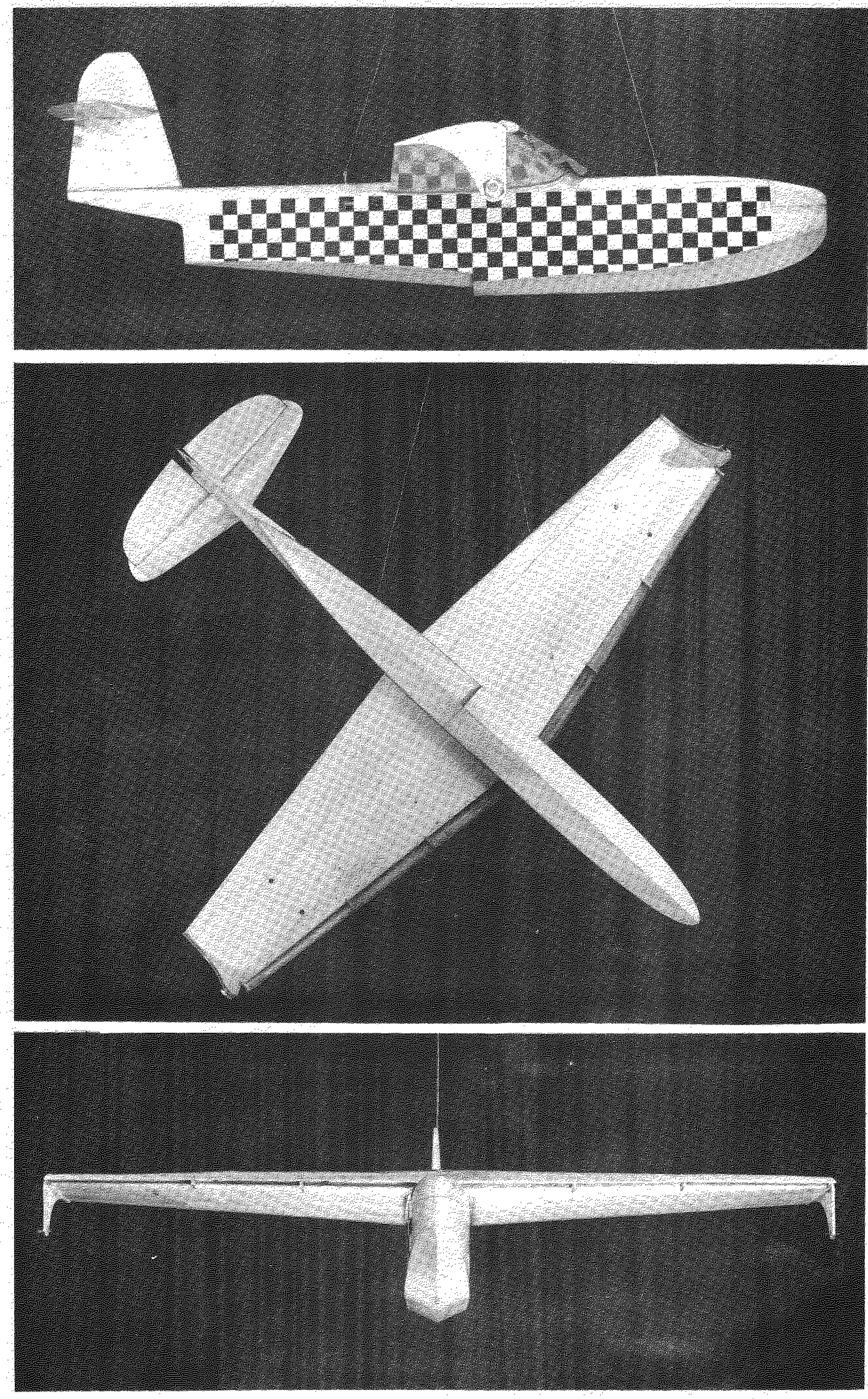
Model	Forebody warp	Afterbody length	Afterbody-forebody keel angle	Step form	To determine effect of
	degrees per beam	beams	degrees		
A	0	5	6	Unfaired transverse Step depth 0.15 beam.	Forebody warp
B	4	5	6		
C	8	5	6		
D	0	4	6		Afterbody length
A	0	5	6		
E	0	7	6		
F	0	9	6		
G	0	5	4		Afterbody angle
A	0	5	6		
H	0	5	8		



MODEL B.

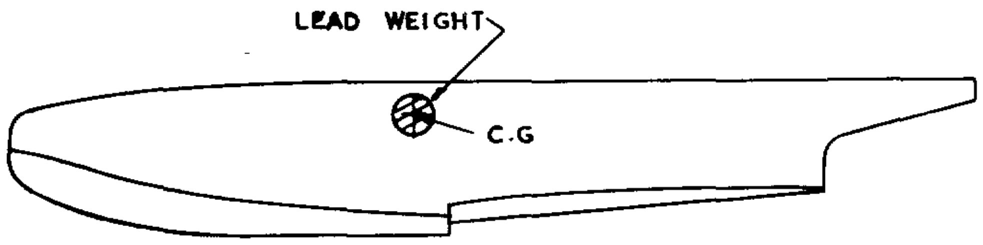
HULL LINES.

FIG. 2.

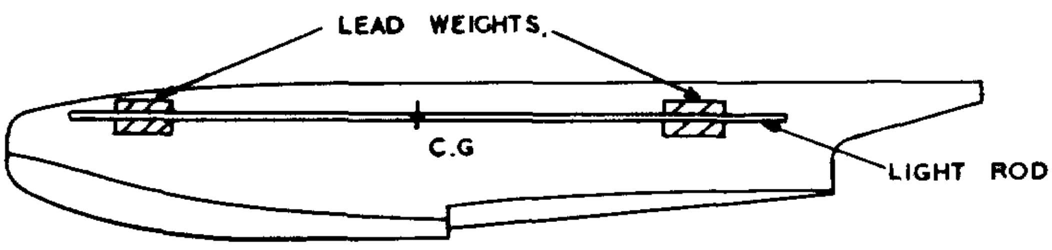


PHOTOGRAPHS OF MODEL B

FIG. 3.

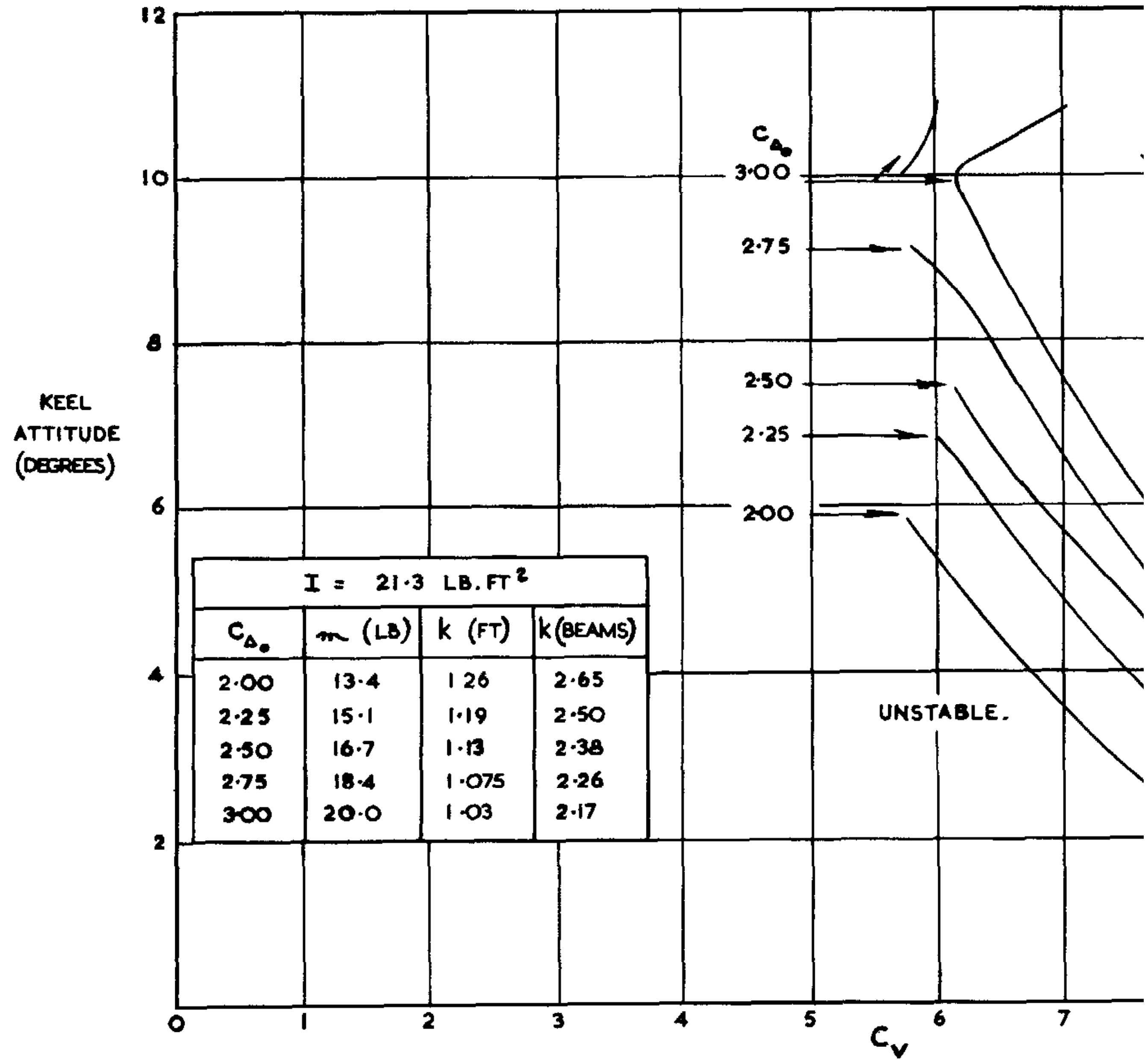


(A) MASS



(B) MOMENT OF INERTIA.

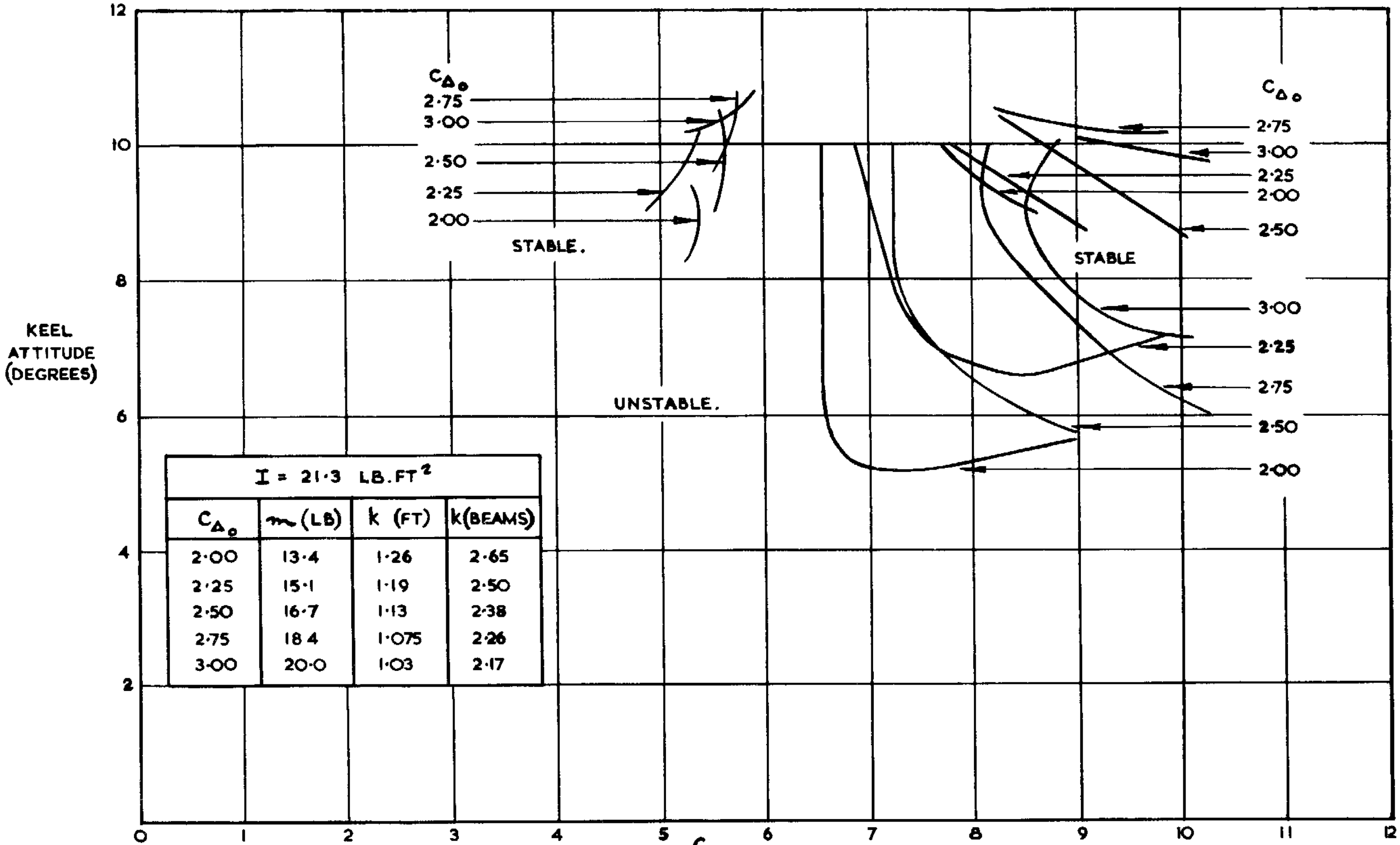
SCHEMATIC DIAGRAM OF ARRANGEMENTS
FOR VARYING MASS AND MOMENT OF INERTIA.



I = 21·3 LB. FT^2

C_{Δ_0}	m (LB)	k (FT)	k (BEAMS)
2·00	13·4	1·26	2·65
2·25	15·1	1·19	2·50
2·50	16·7	1·13	2·38
2·75	18·4	1·075	2·26
3·00	20·0	1·03	2·17

MODEL B.
COMPARISON OF UNDISTURBED STABILITY LIMITS AT C



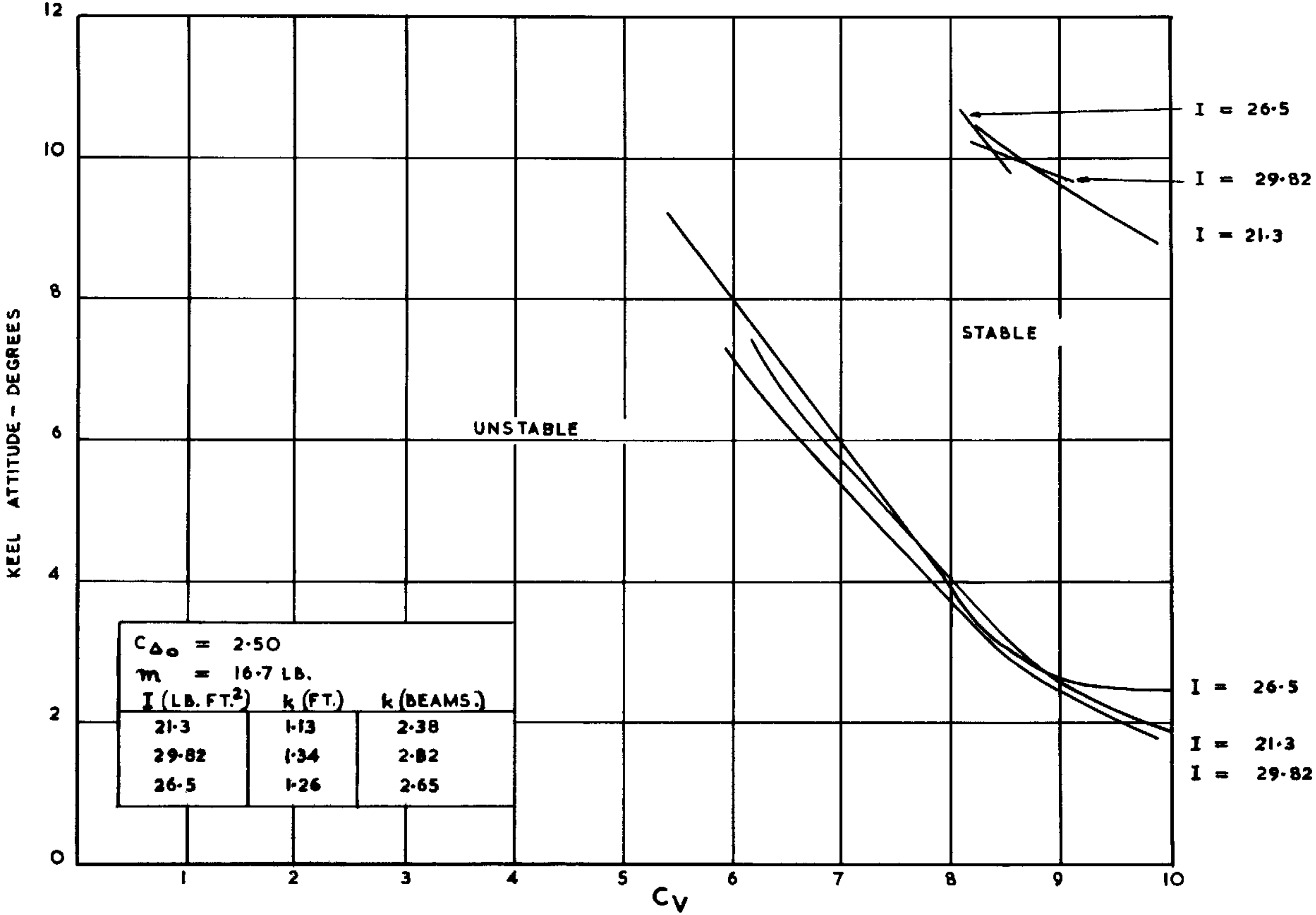
I = 21·3 LB.FT²

C_{Δ_0}	m (LB)	k (FT)	k(BEAMS)
2·00	13·4	1·26	2·65
2·25	15·1	1·19	2·50
2·50	16·7	1·13	2·38
2·75	18 4	1·075	2·26
3·00	20·0	1·03	2·17

MODEL B.

COMPARISON OF DISTURBED STABILITY LIMITS AT CONSTANT MOMENT OF INERTIA.

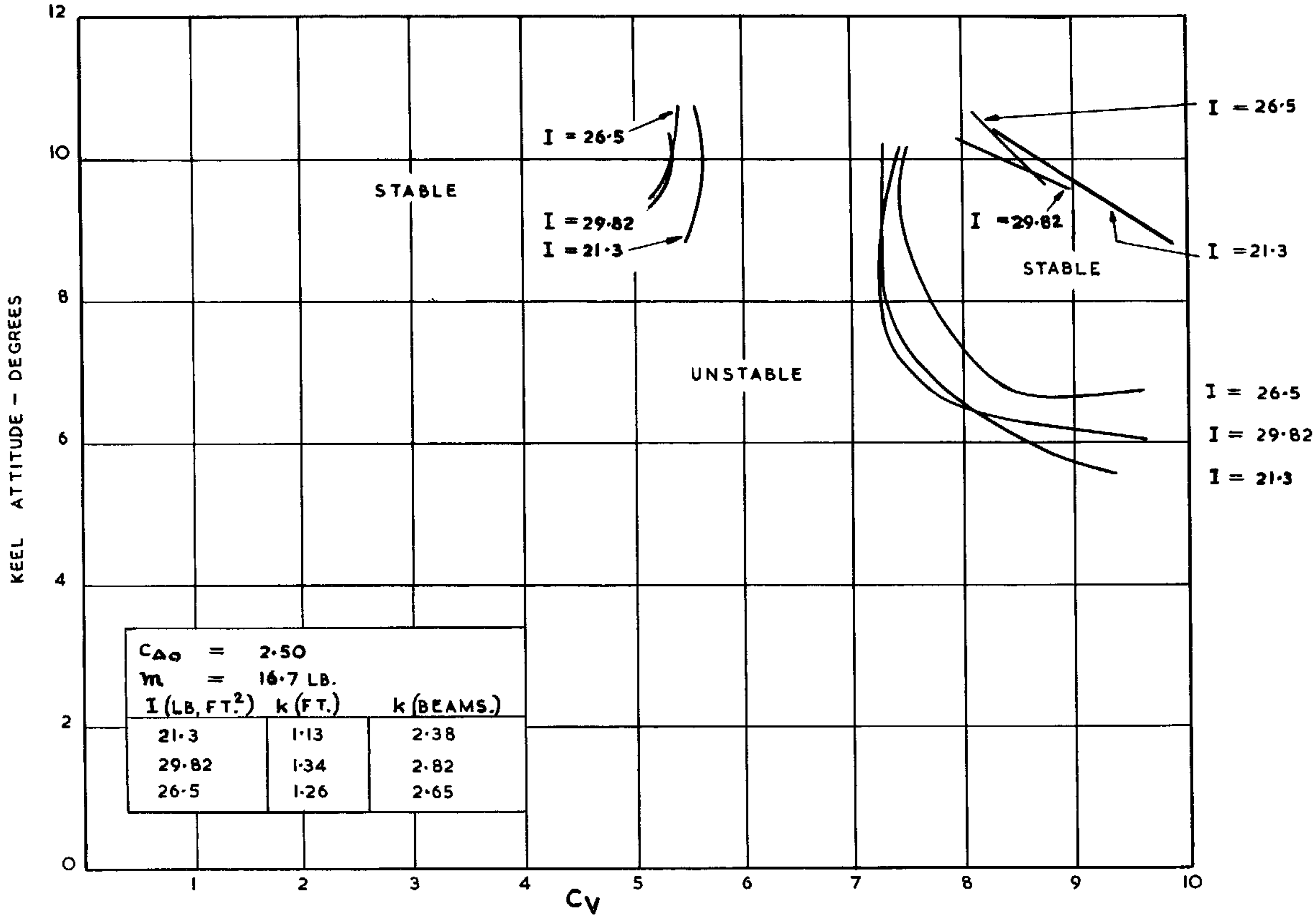
FIG. 5.



MODEL B. COMPARISON OF UNDISTURBED STABILITY LIMITS AT CONSTANT MASS.

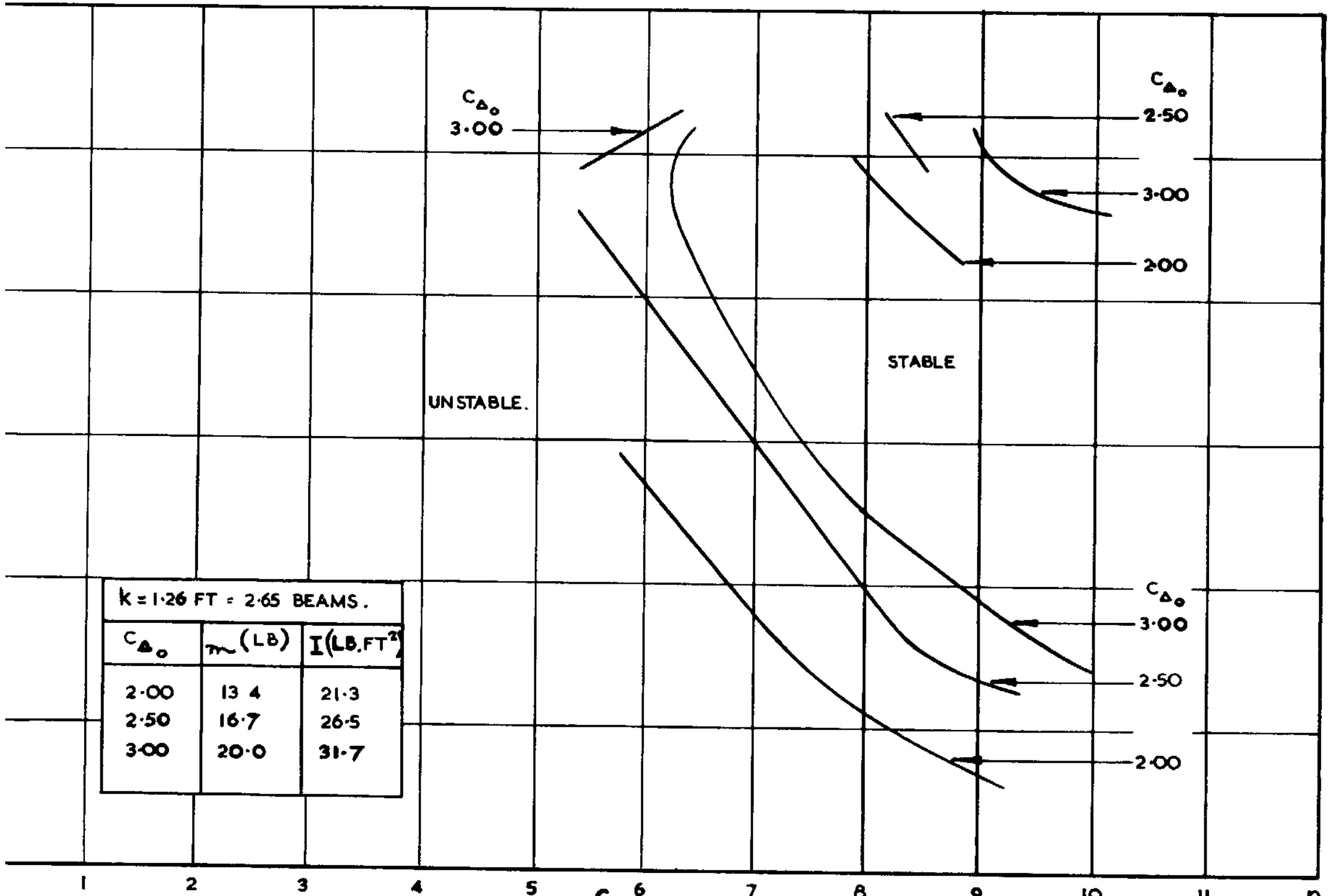
FIG.6

FIG.7

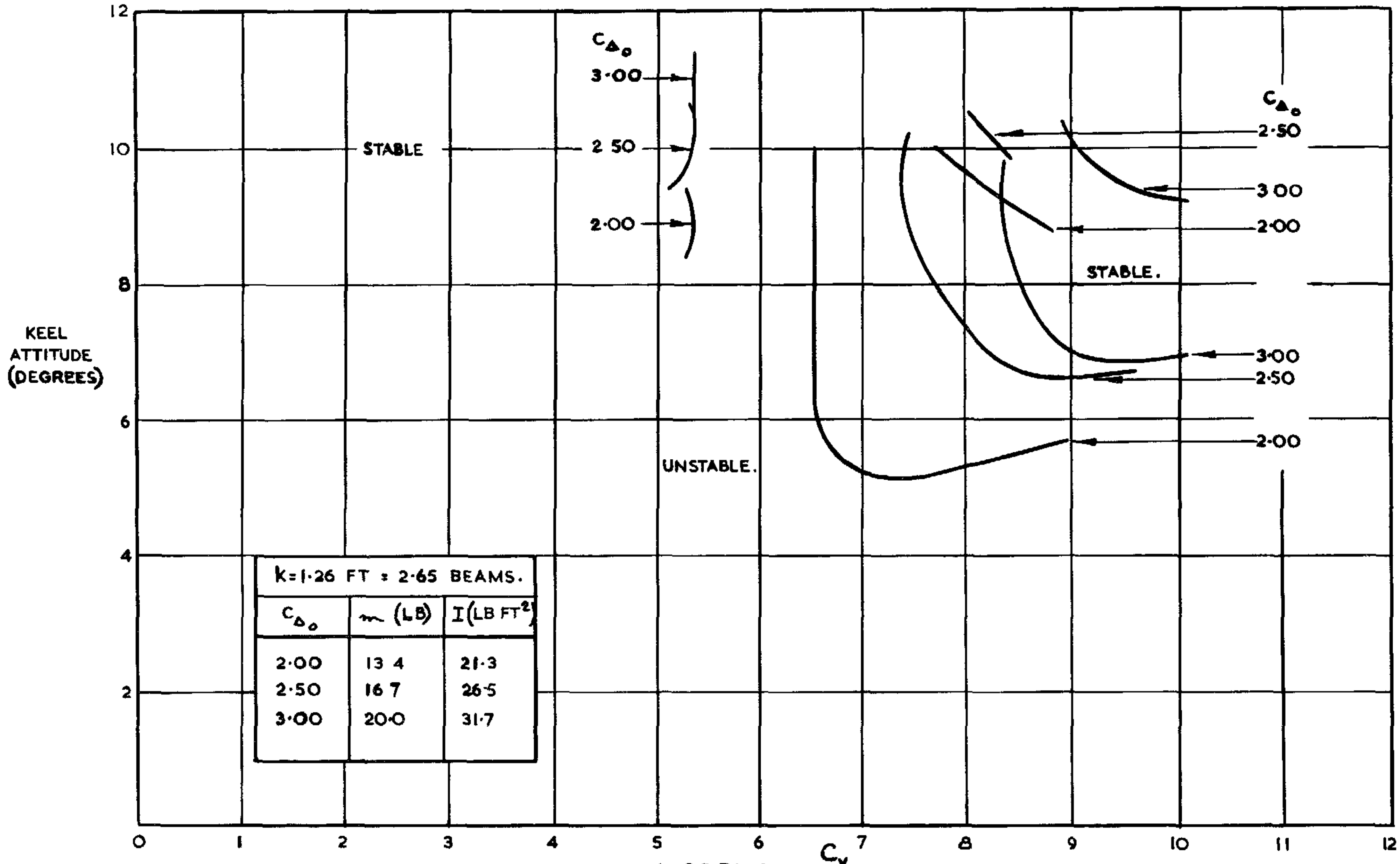


MODEL B . COMPARISON OF DISTURBED STABILITY LIMITS AT CONSTANT MASS.

FIG. 8.



MODEL B.
COMPARISON OF UNDISTURBED STABILITY LIMITS AT CONSTANT RADIUS OF GYRATION.

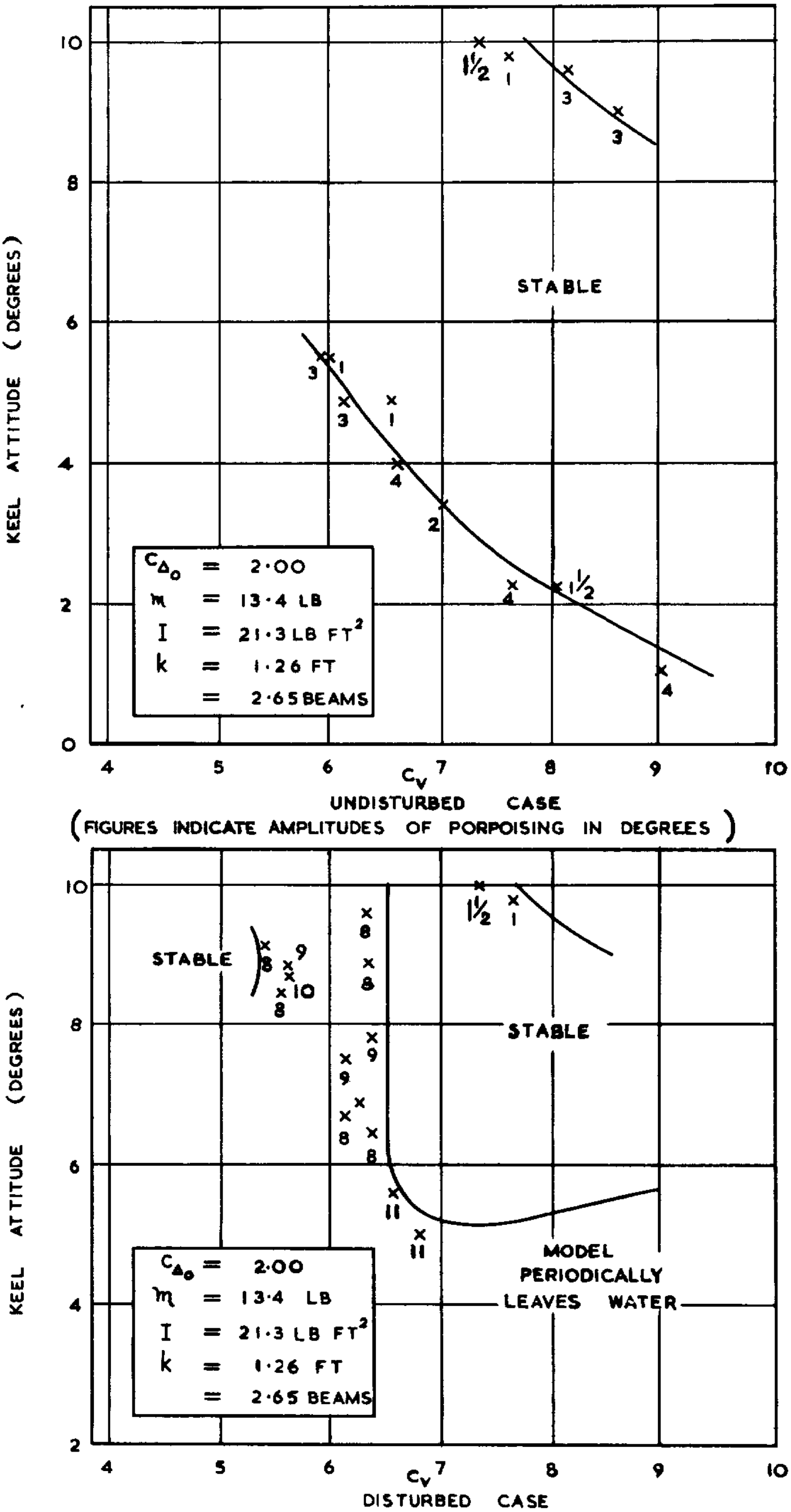


MODEL B.

COMPARISON OF DISTURBED STABILITY LIMITS AT CONSTANT RADIUS OF GYRATION.

FIG. 9.

FIG. 10

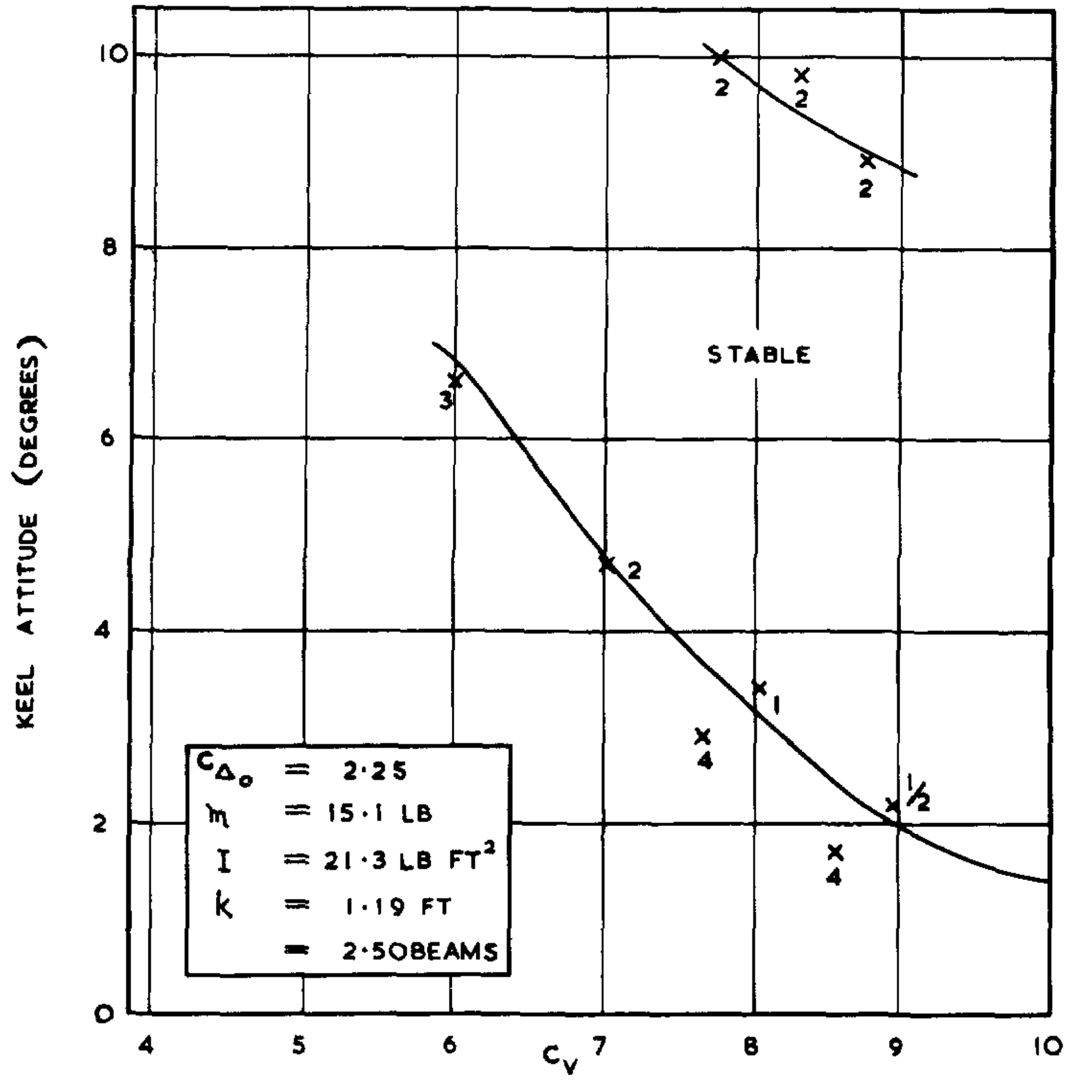


MODEL B

PORPOISING AMPLITUDES

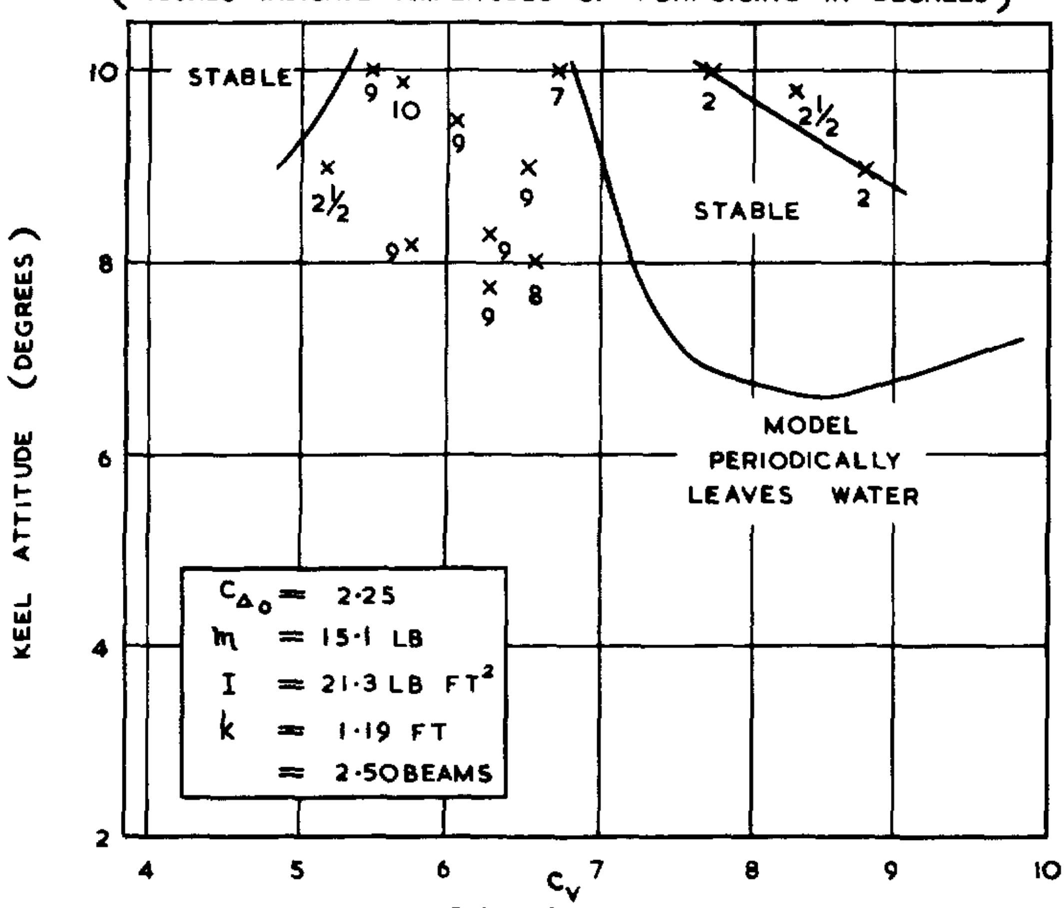
AND STABILITY LIMITS (1)

FIG. 11



UNDISTURBED CASE

(FIGURES INDICATE AMPLITUDES OF PORPOISING IN DEGREES)



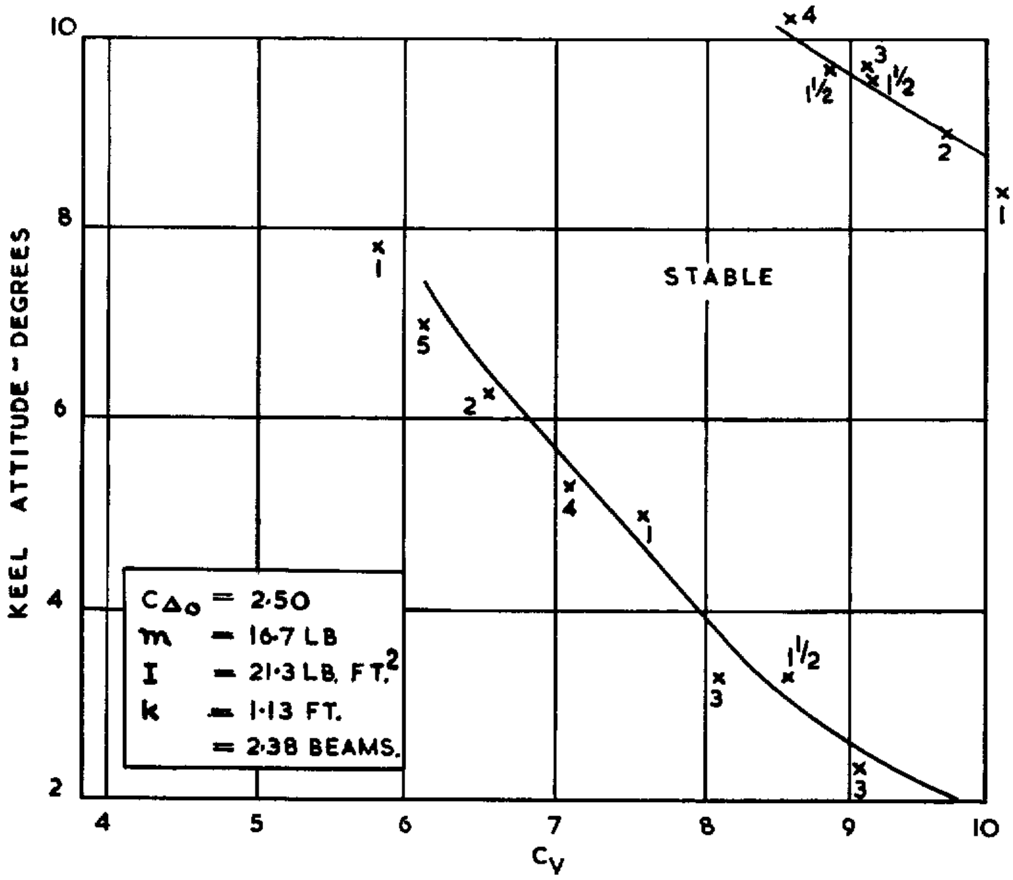
DISTURBED CASE

MODEL B

PORPOISING AMPLITUDES

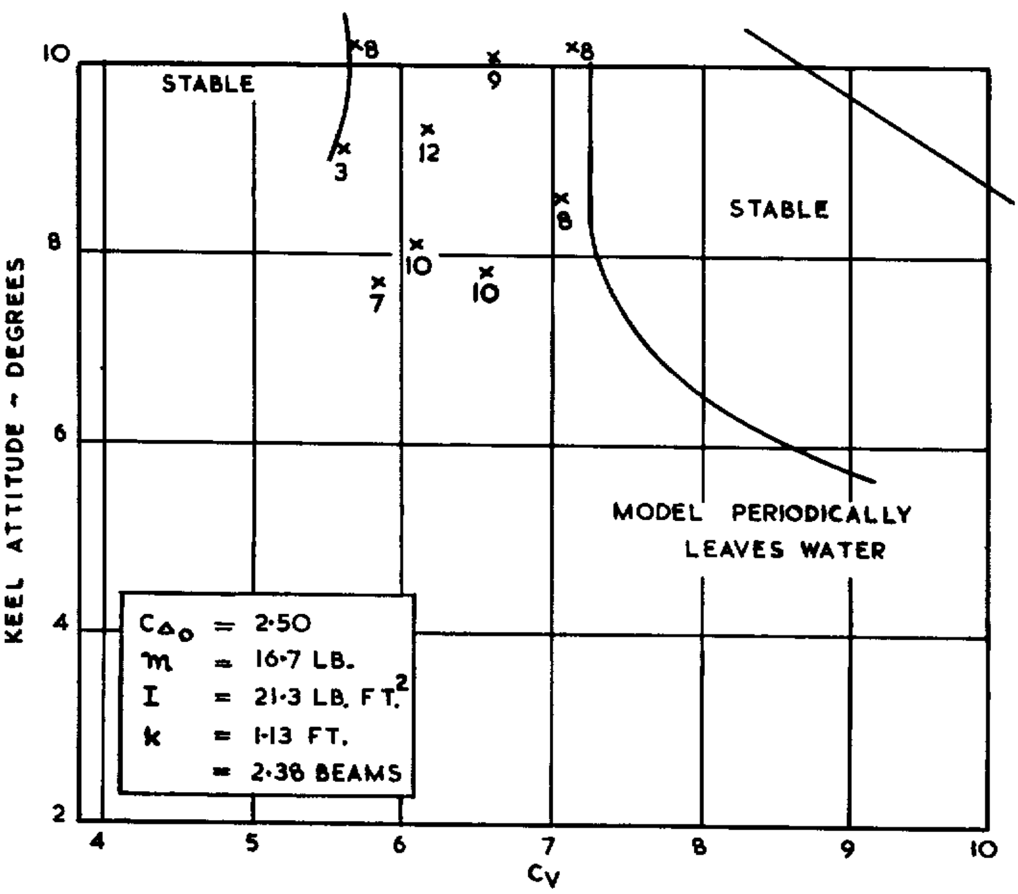
AND STABILITY LIMITS (2)

FIG.12



UNDISTURBED CASE

(FIGURES INDICATE AMPLITUDES OF PORPOISING IN DEGREES)

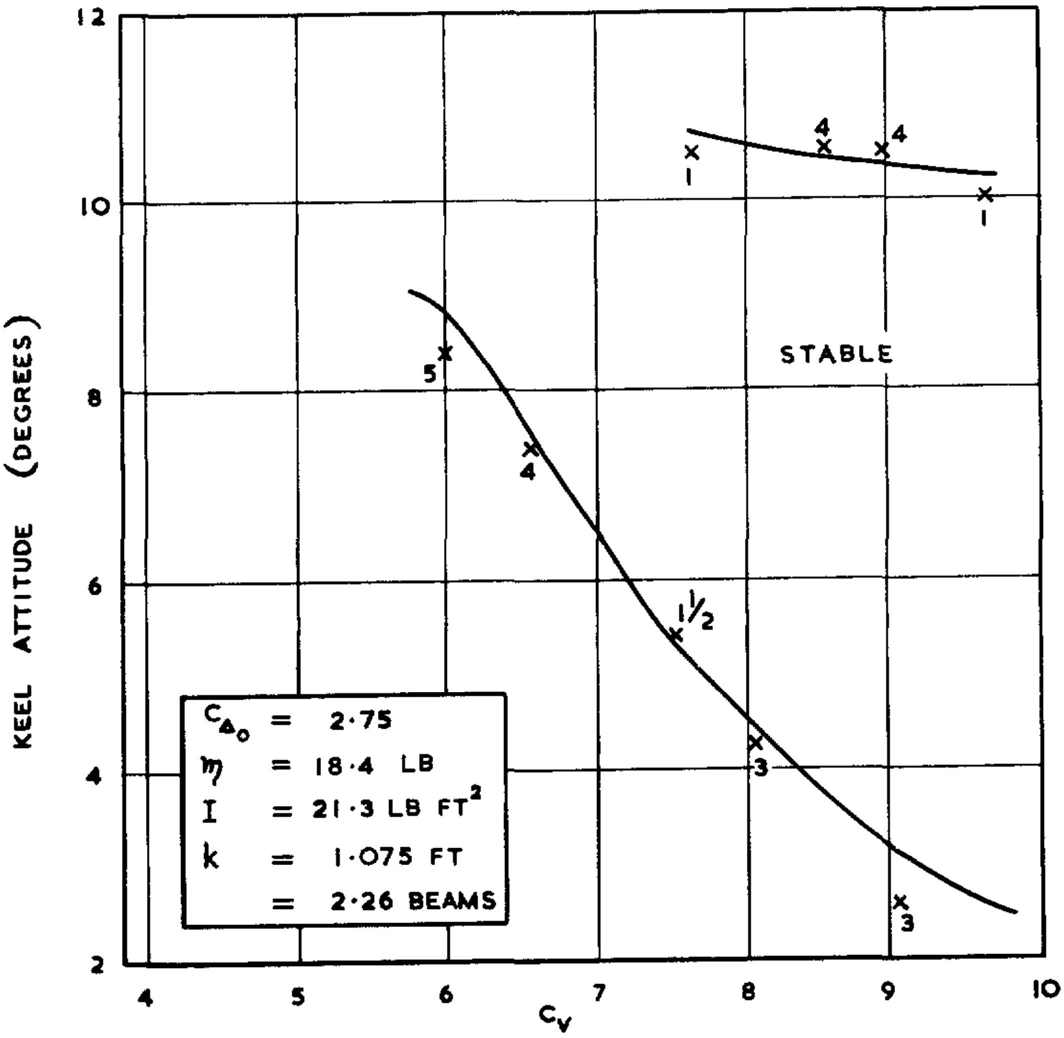


DISTURBED CASE

MODEL B.

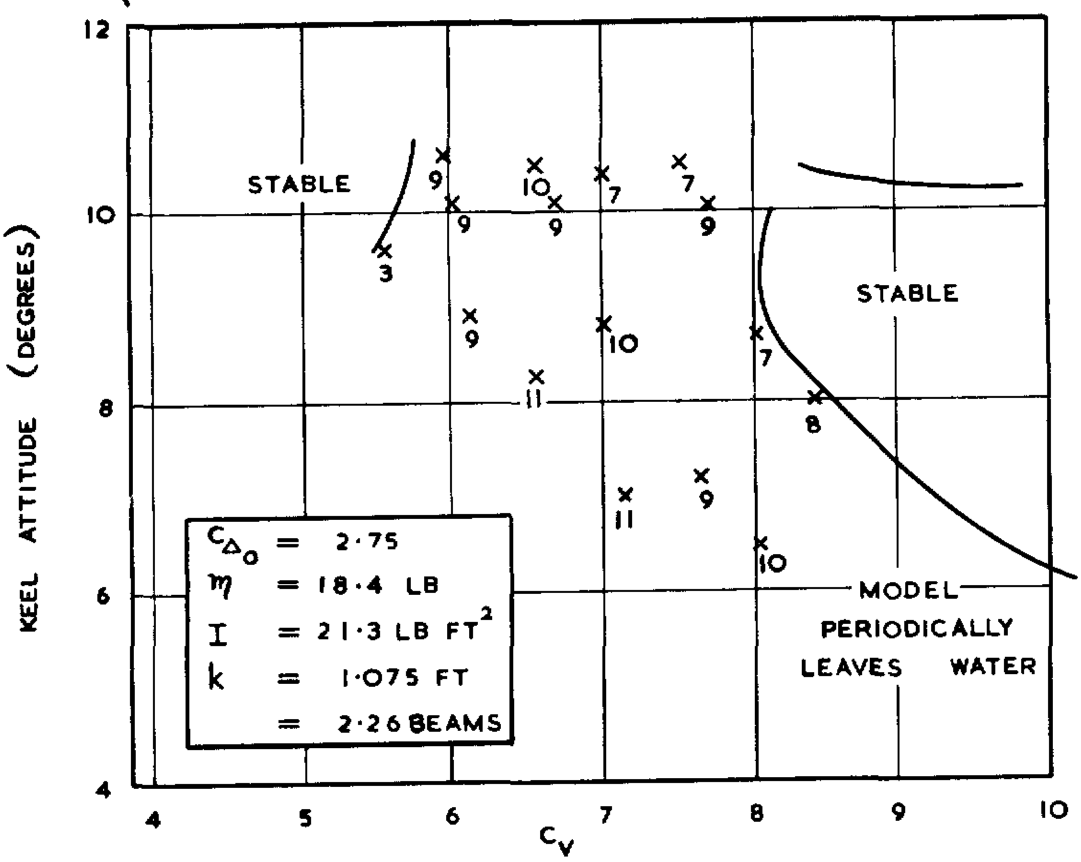
PORPOISING AMPLITUDES AND STABILITY LIMITS (3)

FIG. 13



UNDISTURBED CASE

(FIGURES INDICATE AMPLITUDES OF PORPOISING IN DEGREES)



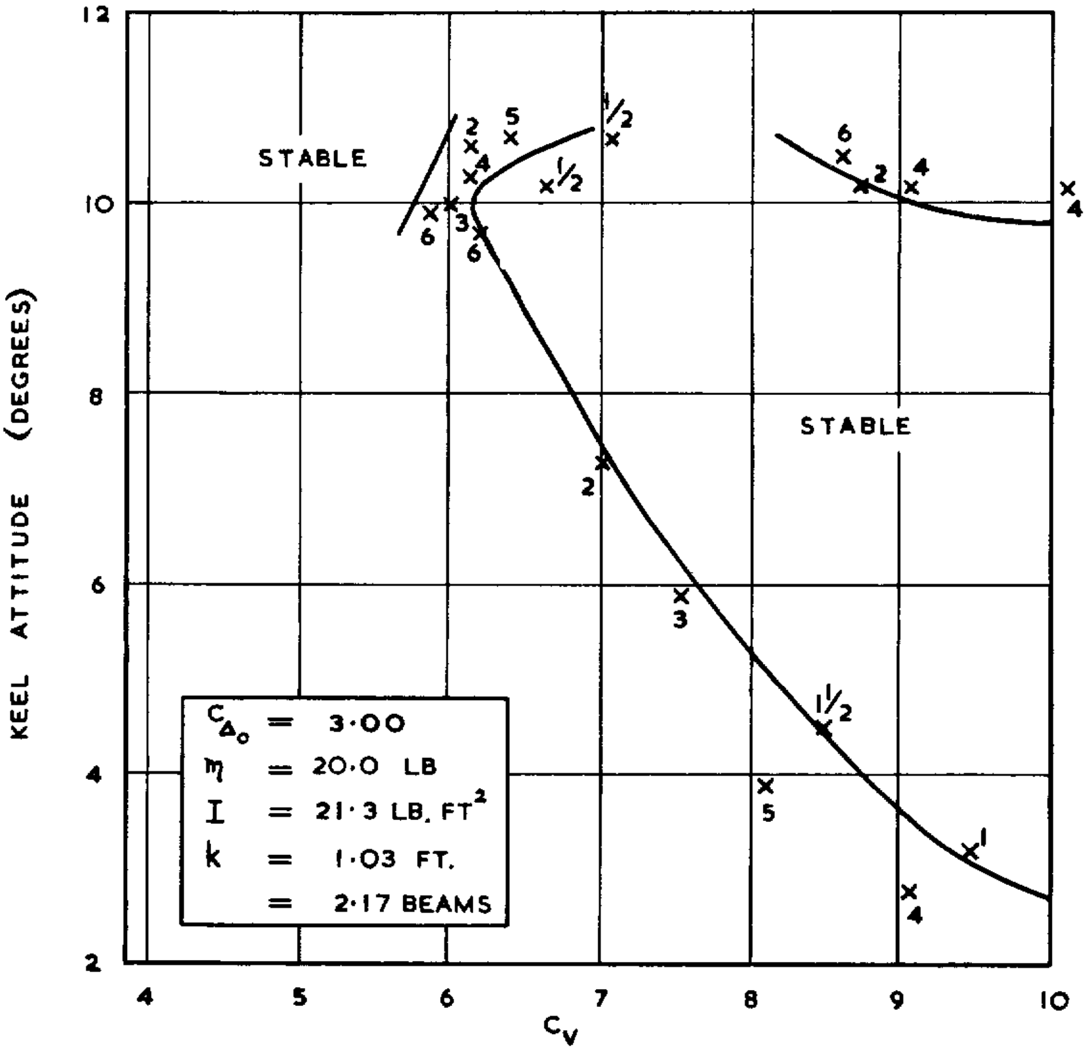
DISTURBED CASE

MODEL B.

PORPOISING AMPLITUDES

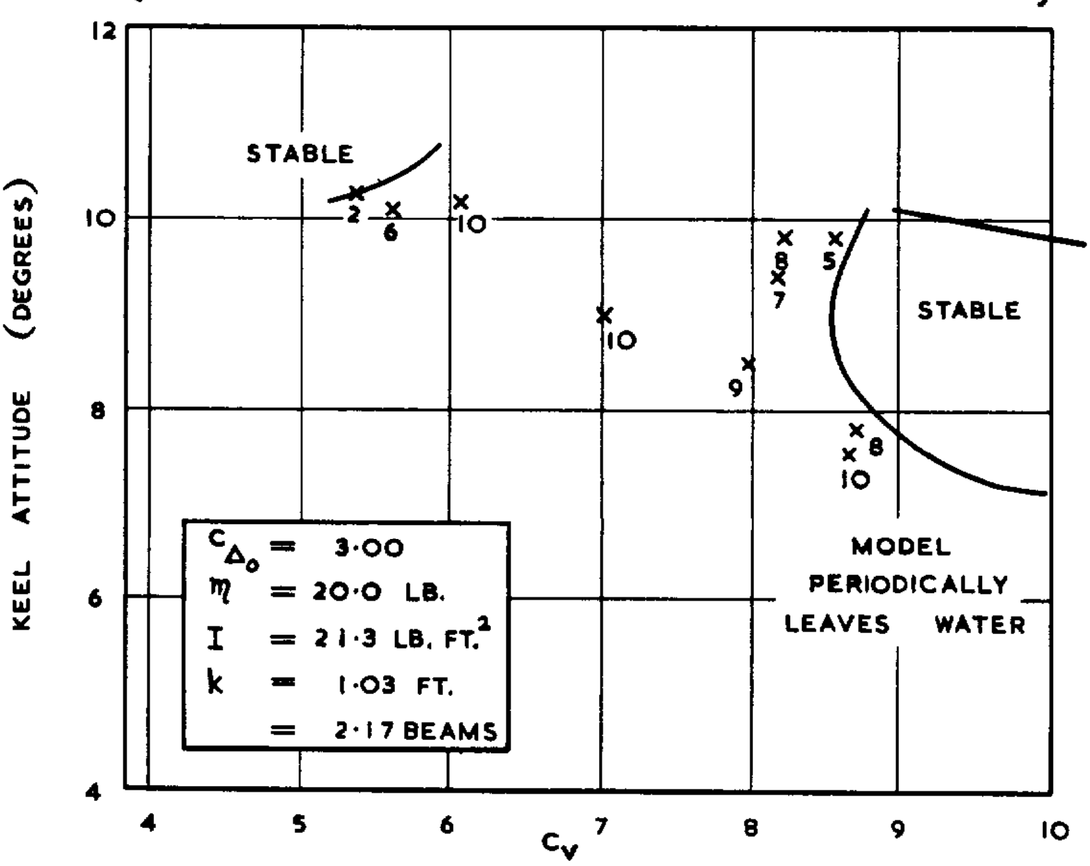
AND STABILITY LIMITS (4)

FIG. 14



UNDISTURBED CASE

(FIGURES INDICATE AMPLITUDES OF PORPOISING IN DEGREES)

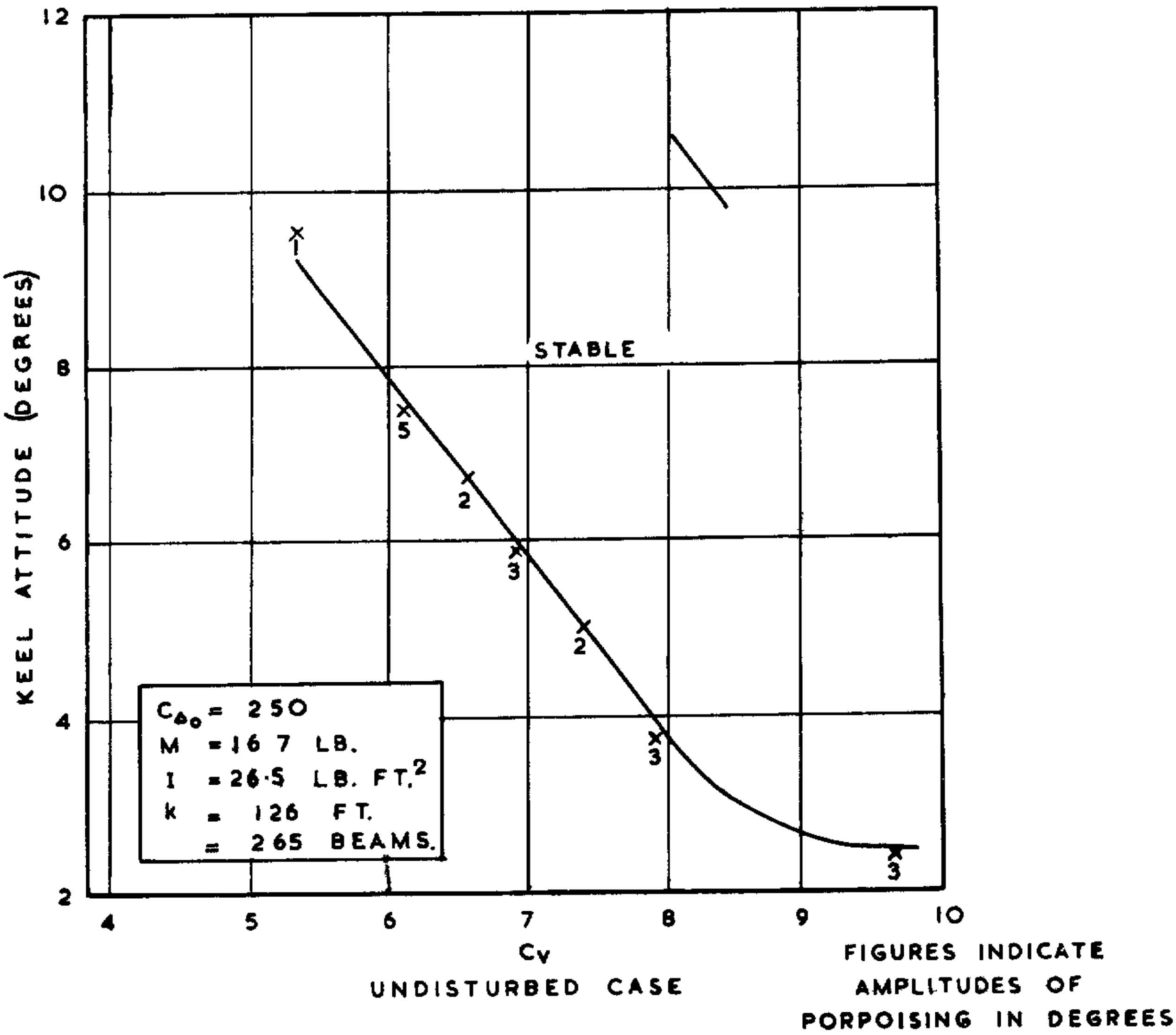


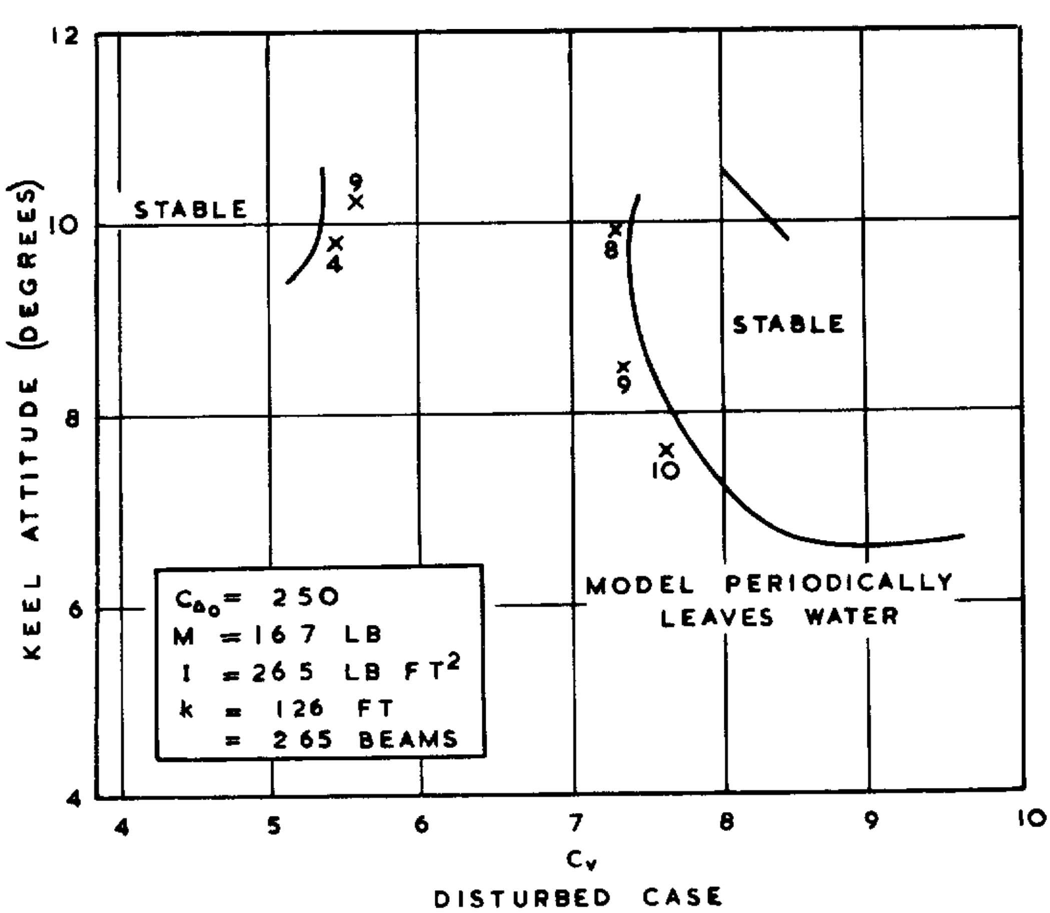
DISTURBED CASE

MODEL B.

PORPOISING AMPLITUDES

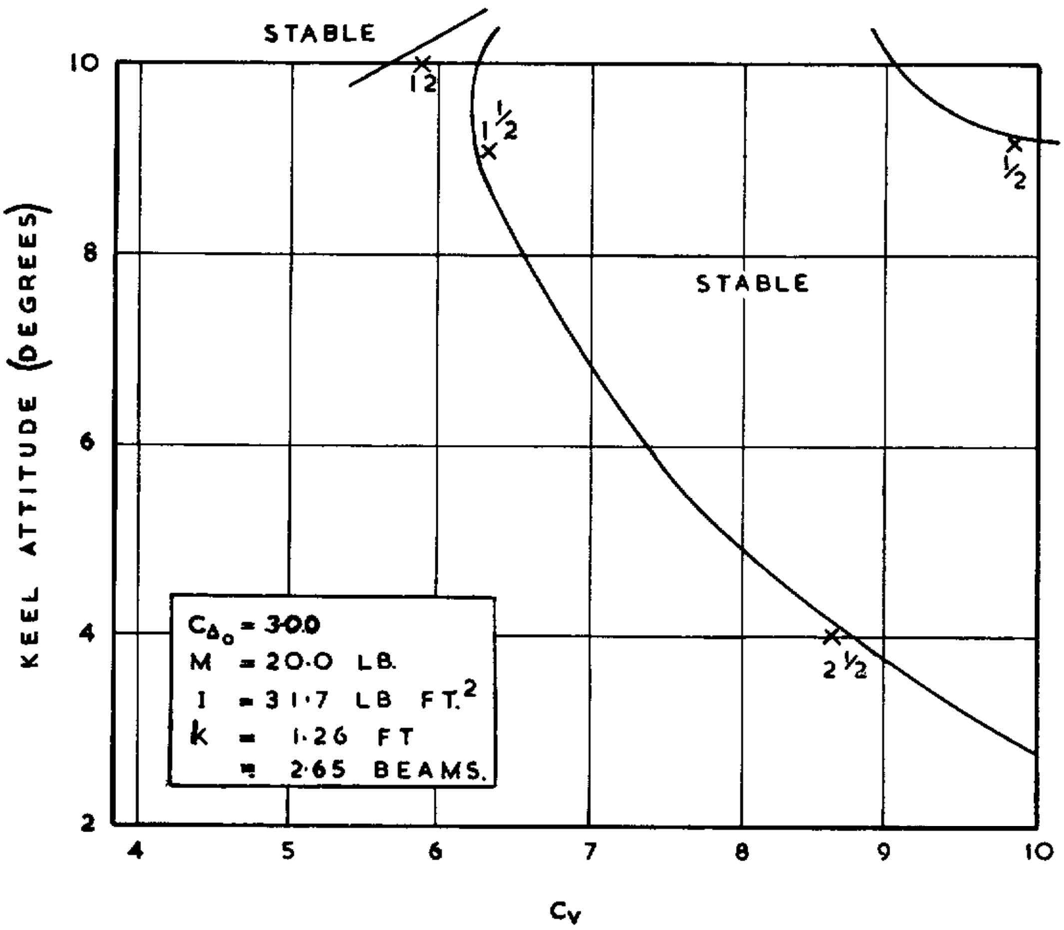
AND STABILITY LIMITS (5)





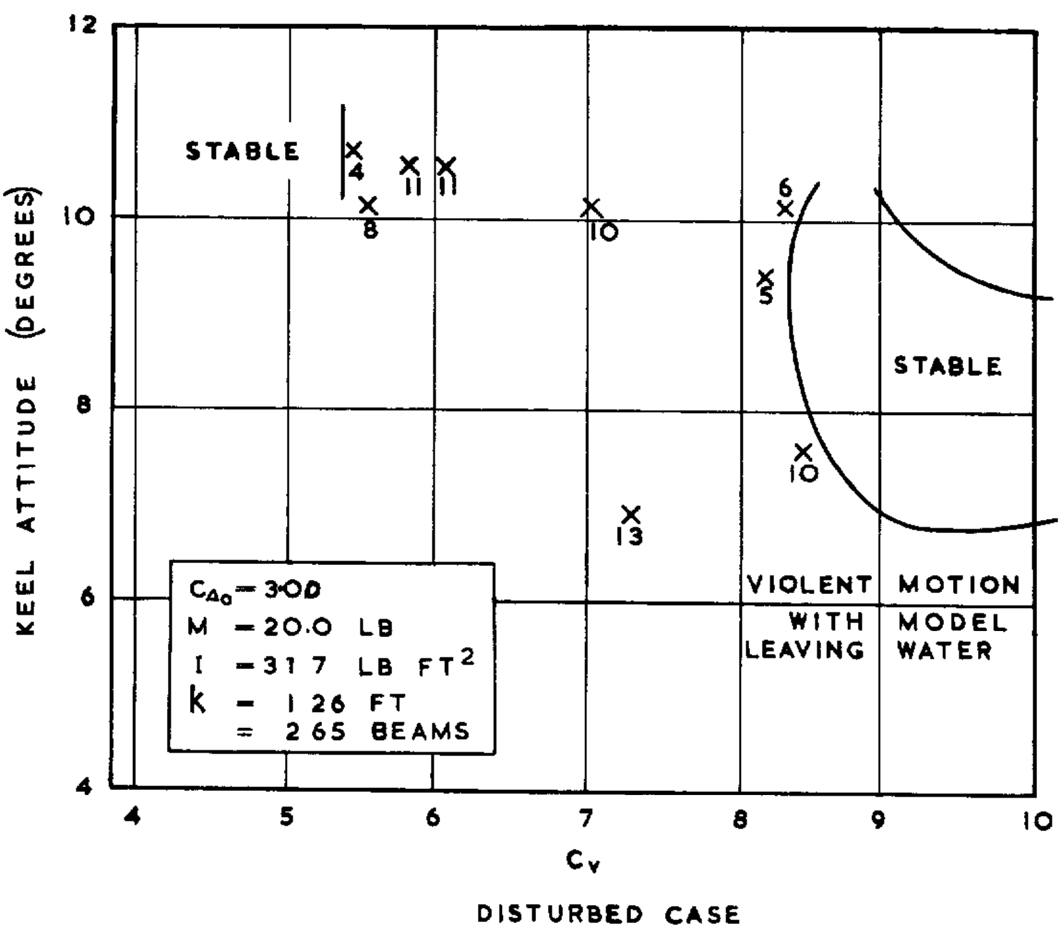
MODEL B.
PORPOISING AMPLITUDES
AND STABILITY LIMITS (6)

FIG. 16



UNDISTURBED CASE

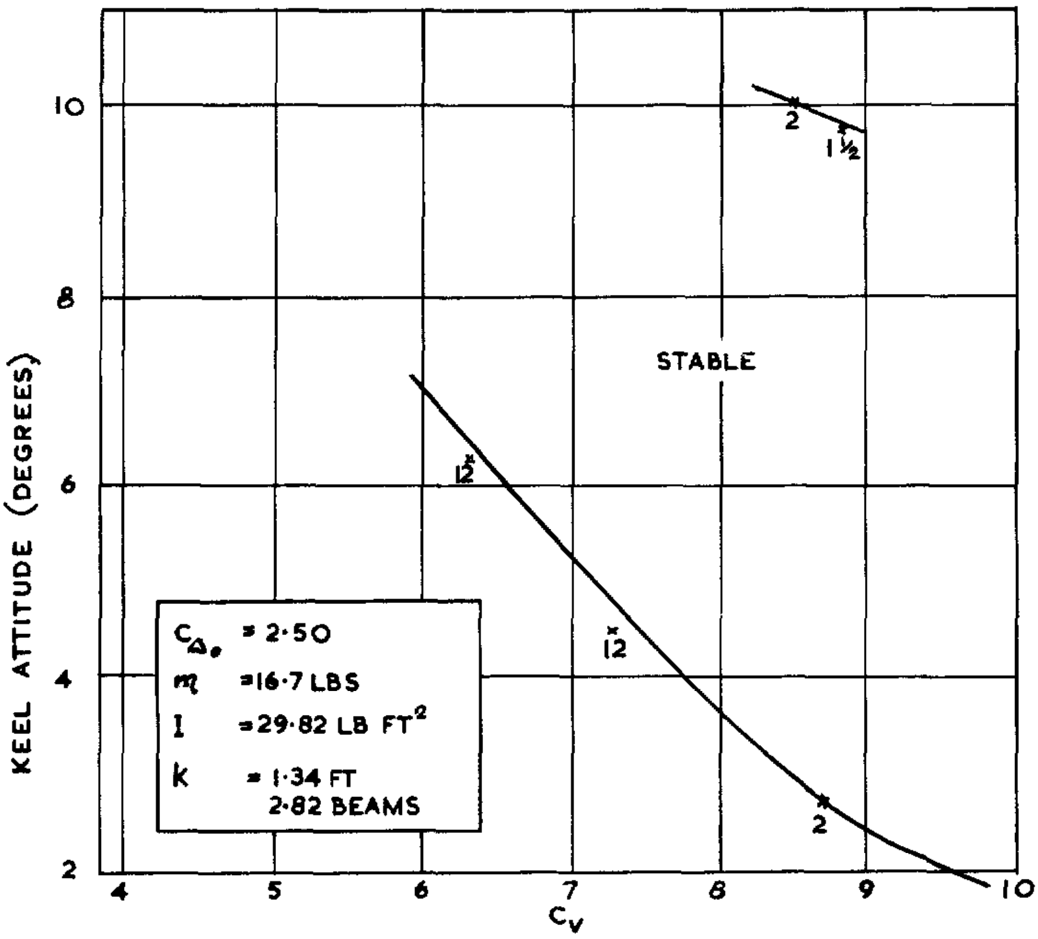
FIGURES INDICATE AMPLITUDES OF PORPOISING IN DEGREES



DISTURBED CASE

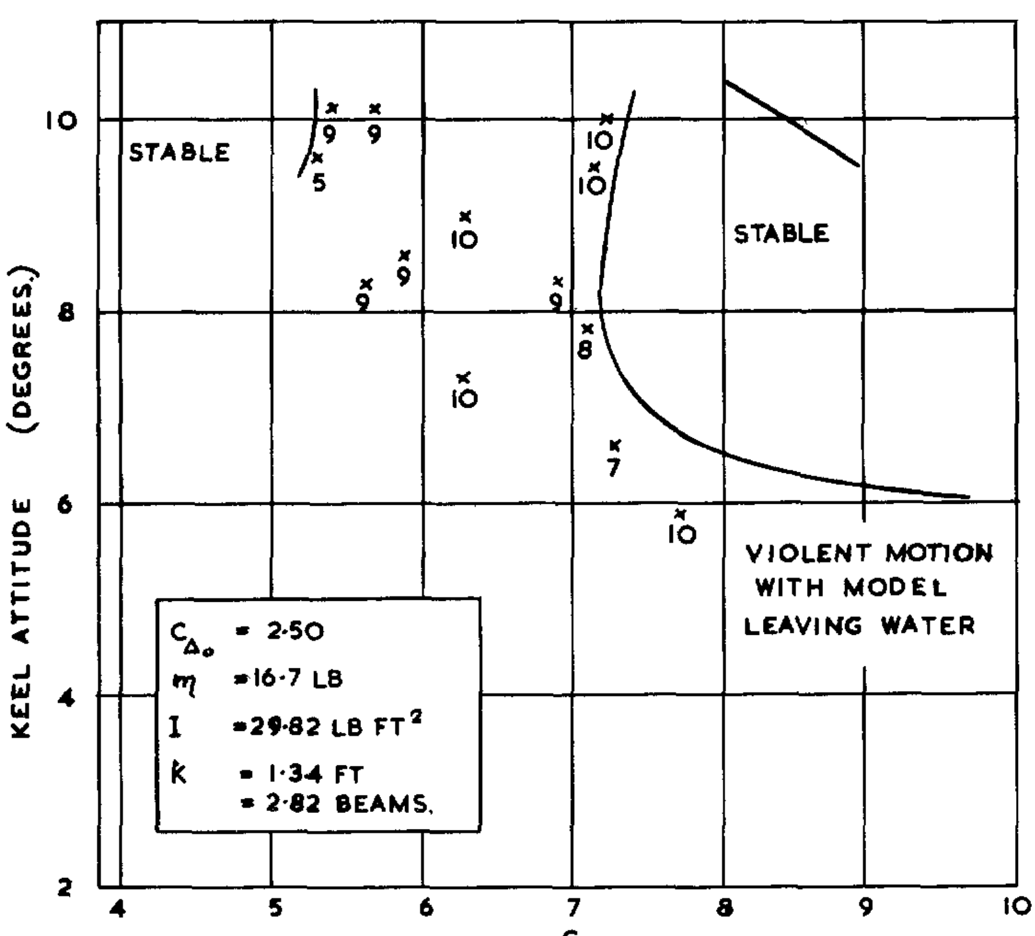
MODEL B.
PORPOISING AMPLITUDES AND STABILITY LIMITS (7)

FIG. 17.



UNDISTURBED CASE.

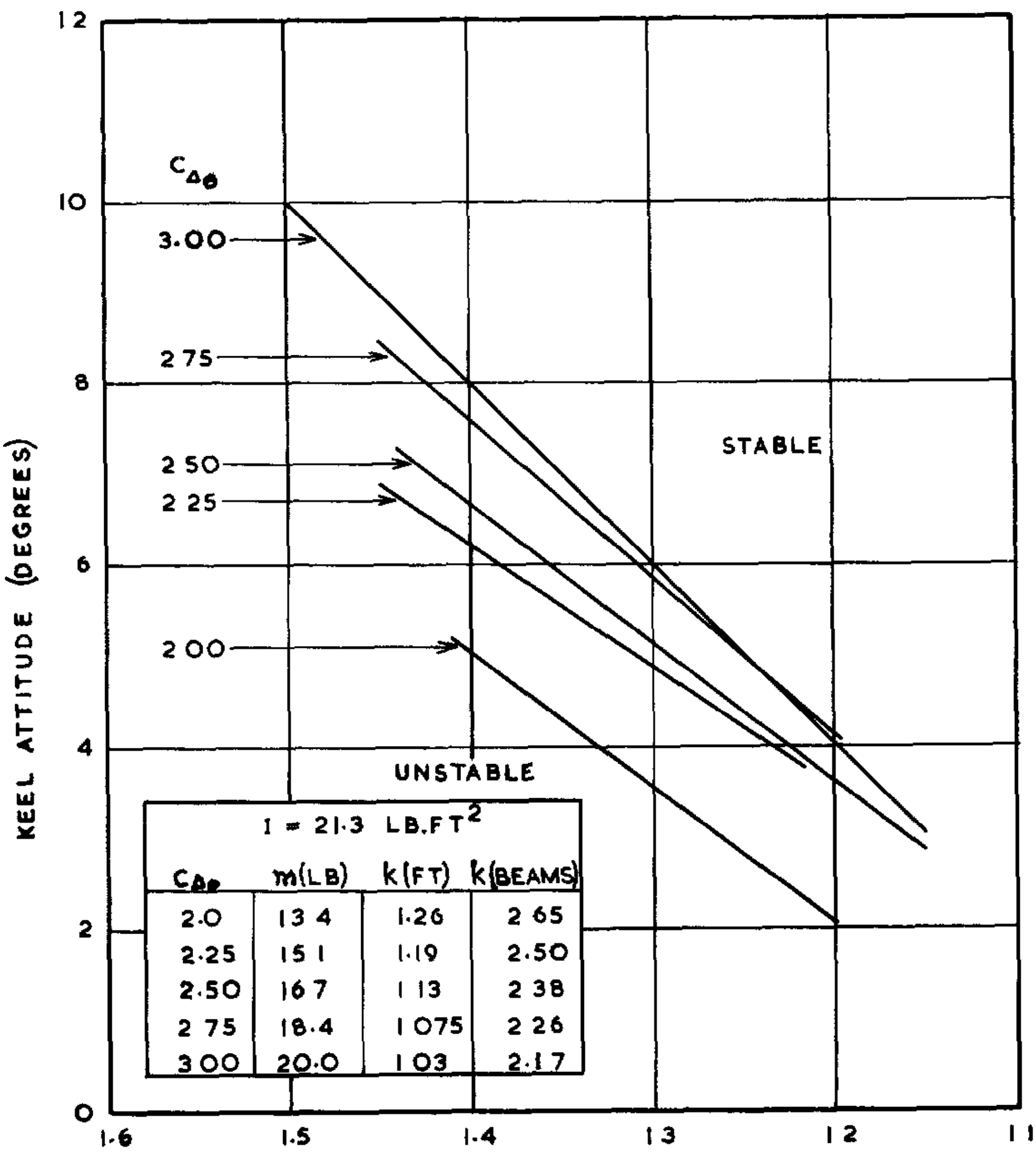
(FIGURES INDICATE AMPLITUDES OF PORPOISING IN DEGREES)



DISTURBED CASE

MODEL B.
PORPOISING AMPLITUDES
AND STABILITY LIMITS (8)

FIG. 18



I = 21.3 LB.FT^2

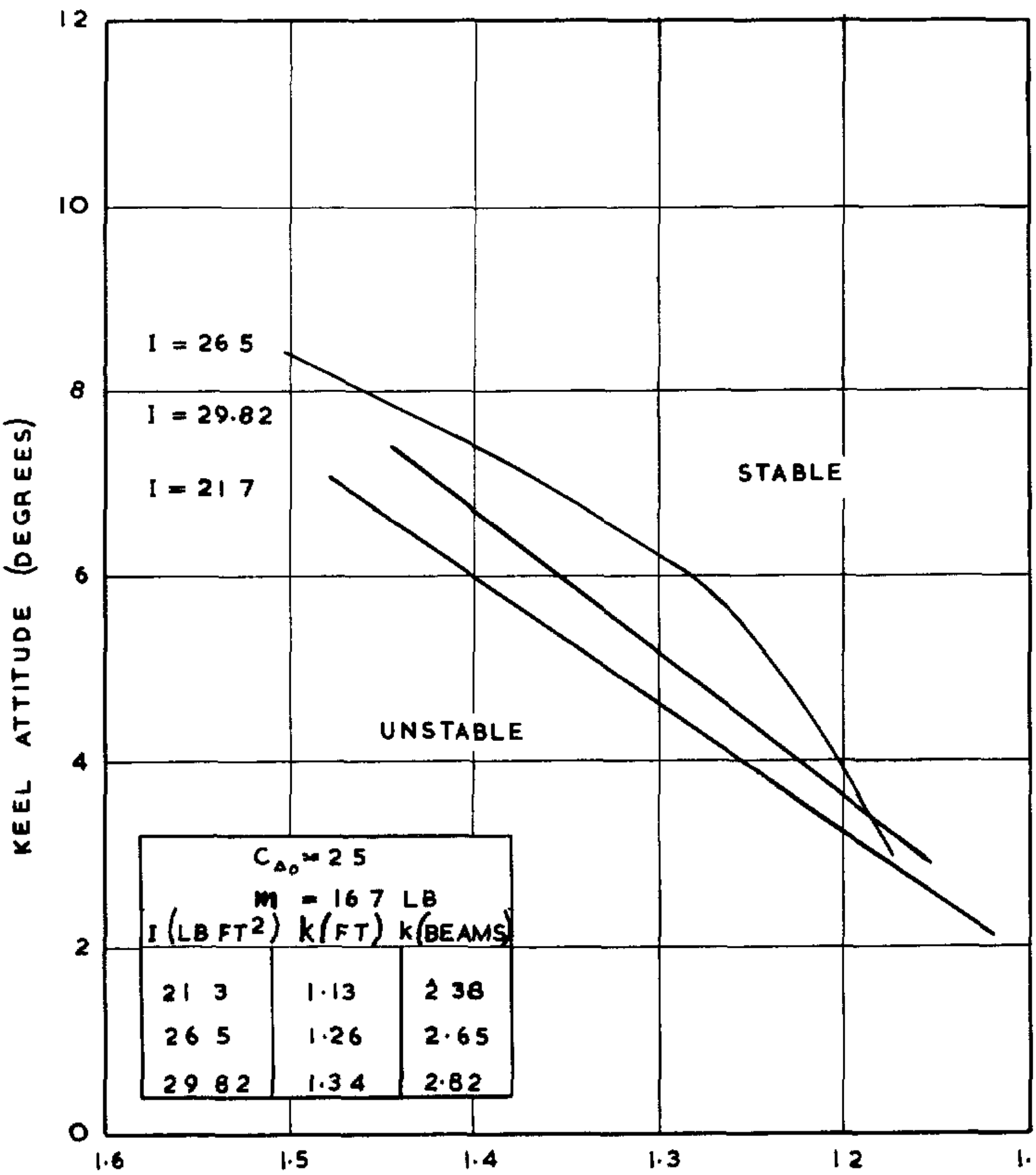
C_{Δ_0}	m(LB)	k(FT)	k(BEAMS)
2.0	13 4	1.26	2 65
2.25	15 1	1.19	2.50
2.50	16 7	1 13	2 38
2 75	18.4	1 075	2 26
3 00	20.0	1 03	2.17

DRAUGHT $d = \dfrac{4\,06}{\alpha_K^{0.265}} \cdot \dfrac{\sqrt{C_\Delta}}{C_V}$ (INS)

MODEL B.

COMPARISON OF UNDISTURBED LOWER LONGITUDINAL STABILITY LIMITS ON A DRAUGHT BASE.

(1) CONSTANT MOMENT OF INERTIA.

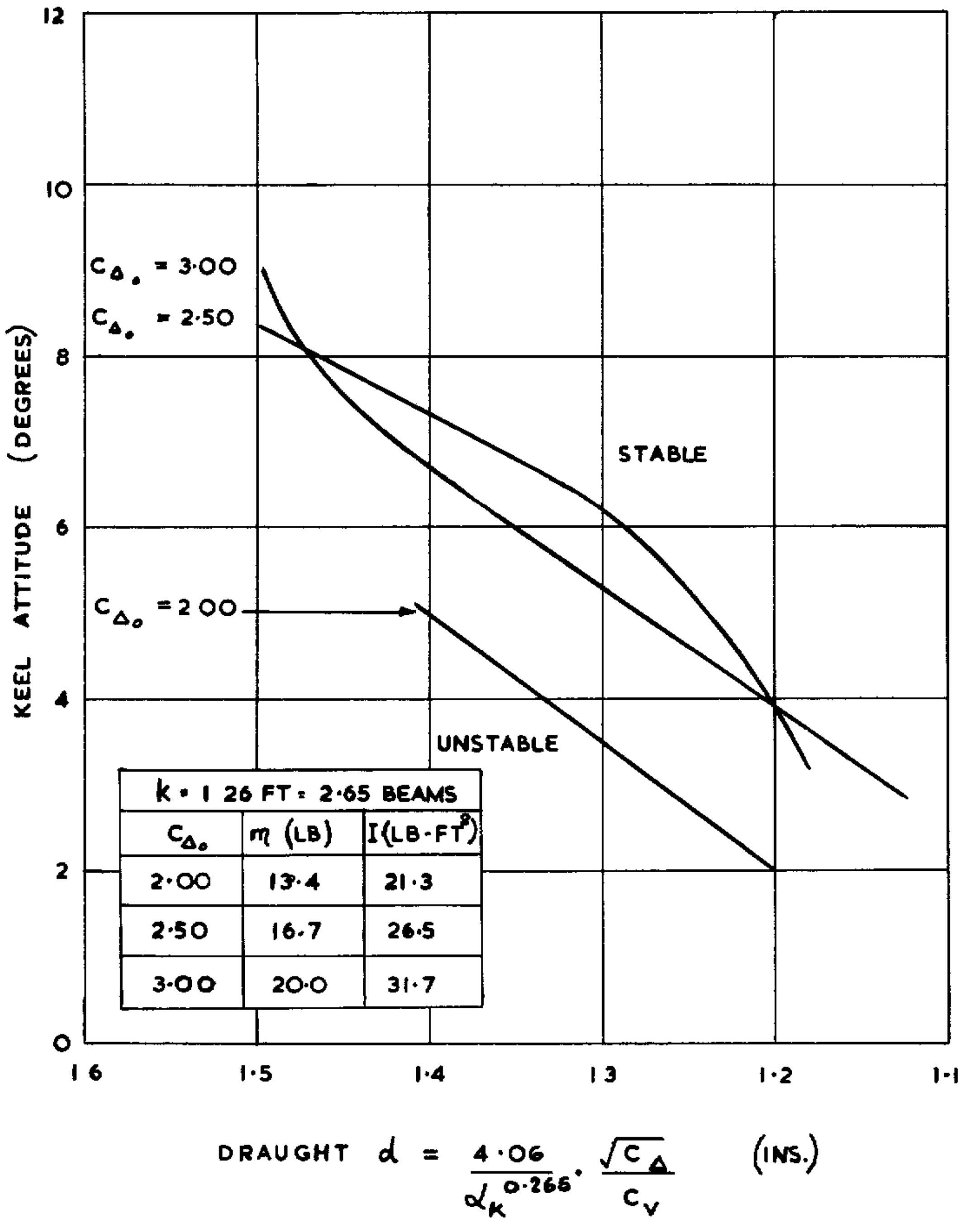


$$\text{DRAUGHT } d = \frac{4{\cdot}06}{\alpha_K^{0\,265}} \cdot \frac{\sqrt{C_\Delta}}{C_V} \quad \text{(INS)}$$

MODEL B.

COMPARISON OF UNDISTURBED LOWER LONGITUDINAL STABILITY LIMITS ON A DRAUGHT BASE.

(2) CONSTANT MASS.



k = 1 26 FT = 2·65 BEAMS		
C_{Δ_0}	m (LB)	I (LB·FT²)
2·00	13·4	21·3
2·50	16·7	26·5
3·00	20·0	31·7

MODEL B

COMPARISON OF UNDISTURBED LOWER LONGITUDINAL STABILITY LIMITS ON A DRAUGHT BASE

(3) CONSTANT RADIUS OF GYRATION

C.P. No. 202

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