C.P. No. 158
(15,889)
A.R.C. Technical Report

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The Thermodynamics of Frictional Resisted Adiabatic Flow of Gases Through Ducts of Constant and Varying Cross Section

By

W. R. Thomson

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1954

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ERRATA

Page 6 - Equation 19 should read:-

$$\frac{dV}{V} = \frac{2}{2 + (\gamma - 1) M^2} \frac{dM}{M} \qquad \ldots(19)$$

Page 7 - Equations 21, 23 and 24 should read:-

$$\frac{dP}{P} = - \frac{2 \{1 + (\gamma - 1) M^2\}}{2 + (\gamma - 1) M^2} \frac{dM}{M} - \frac{dA}{A} \qquad \ldots(21)$$

$$\frac{gvdP}{V^2} = - \frac{1}{\gamma M^2} \frac{dA}{A} - \frac{2 \{1 + (\gamma - 1) M^2\}}{2 + (\gamma - 1) M^2} \frac{dM}{\gamma M^3} \qquad \ldots(23)$$

$$\frac{fdL}{2h} - \frac{dA}{\gamma M^2 A} - \frac{2 (1 - M^2) dM}{\gamma M^3 \{2 + (\gamma - 1) M^2\}} = 0 \qquad \ldots(24)$$

Fig. 11. In the upper series of curves, on the extreme left, the space between $M = 0.35$ and $M = 0.4$ (both correctly positioned), has been divided into eight parts instead of 10.

C.P. No. 158.

Report No. R. 119

September, 1952

NATIONAL GAS TURBINE ESTABLISHMENT

The Thermodynamics of Frictional Resisted Adiabatic Flow of Gases through Ducts of Constant and Varying Cross Section

- by -

W. R. Thomson

SUMMARY

The report presents an analytical study dealing with the adiabatic flow of gases with frictional losses through ducts of constant and varying cross section. The thermodynamic treatment is along lines published by other workers such as Bailey and Fabri and is essentially one-dimensional in character in so far that frictional effects are assumed to be uniformly distributed over the total cross sectional area of flow. With this simplifying assumption, relationships are deduced connecting the pressure, temperature, velocity and flow area of the gas at any one plane with those at any other plane in a duct.

The main relationships are unusable for quantitative estimation except through graphs and the main value of the report lies in the presentation of these graphs, the use of which should facilitate the solution of duct flow problems.

APPENDICES

ILLUSTRATIONS

ILLUSTRATIONS (cont'd)

1 0 Introduction

The subject of the flow of gases in ducts forms an important application for the science of gas dynamics and its analytical treatment is of obvious importance in those branches of engineering involving flow machinery such as turbine engines. The part of the subject of duct flow dealt with in this report comprises cases where it may be assumed that the flow is adiabatic i.e. no heat is transmitted to or from external sources. Such cases have application to flow in diffusers and propelling nozzles of gas turbine and ram jet engines.

The treatment given here is for ducts of varying cross sectional area, includes for the effects of friction, and makes the usual simplifying assumption that the flow is one-dimensional i.e. the effects of friction are distributed uniformly over the cross-sectional area of flow instead of being confined to the boundary layers as they are in practice. Nothing original is claimed for the analysis; it is considered that the main value of the work lies in the resulting generalised curves forming part of the report which are, as far as is known, presented for the first time to a large enough scale and in sufficient detail to facilitate the solution of duct flow problems.

Work by Neil P. Bailey (reference 1) and Jean Fabri (reference 2) has been freely used by the Author in this treatment and acknowledgement is made of the help their original work has afforded.

2.0 The basic equations

At any plane in a duct the flow equation is

$$\frac{Q\sqrt{T_t}}{AP_t} = \sqrt{\frac{g\gamma}{R}} \; \frac{M}{\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \quad \ldots\ldots\ldots\ldots(1)$$

Q is the fluid mass flow, constant over the length of the duct. T_t is the total head temperature which from the principle of the conservation of energy is constant over the length of the duct i.e. the flow is adiabatic.

P_t is the total head pressure at the plane considered. This in the presence of friction, will fall over the duct length. A is the area of cross section at the plane considered and M the Mach numner of the flow at that plane.

R is the gas constant and γ the ratio of specific heats K_p/K_v.

Eqn. (1) is plotted in Fig. 1, for subsonic flow only, in the form of three parameters: $\frac{Q\sqrt{T_t}}{AP_t}$ against M with curves of γ.

Also at any plane the relationship between total head and static pressures is given by

$$\frac{P_t}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

This is plotted in Fig. 2, for subsonic flow only, in the form of three parameters: P_t/P against M with curves of γ.

Finally the relationship between total head and static temperatures, in the form for most accurate calculation, is given by

$$\frac{\delta T}{T_t} = \frac{1}{1 + \frac{2}{(\gamma - 1)\, M^2}} \qquad \text{.............................(3)}$$

Here δT is the difference between the total head and static temperatures i.e. the temperature equivalent of the velocity. This eqn. is plotted in Fig. 3 up to M = 2.6 in the form of three parameters: $\delta T/T_t$ against M with curves of γ.

3.0 Evaluation of Mach number

In the classical proof of the equations for maximum mass flow under insentropic expansion in a nozzle, the ratios of the throat or critical values of the static temperature and pressure to the total head values are given by

$$\frac{T_c}{T_t} = \frac{2}{\gamma + 1} \qquad \text{......................................(4)}$$

$$\frac{P_c}{P_t} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}} \qquad \text{..................................(5)}$$

while the critical velocity is given by

$$V_c^{\,2} = g\gamma R T_c \qquad \text{......................................(6)}$$

It is to be noted that in this classical proof γ is defined as the mean value between T_t and T_c i.e. over the range of the expansion.

In turn the Mach number of any velocity of flow V is defined by

$$M = \frac{V}{\sqrt{g\gamma RT}} \qquad \text{...................................(7)}$$

where T is the static temperature corresponding to velocity V and γ is the mean value between T_t and T.

It will be found (see Appendix VI) that the treatment here developed of flow in a nozzle with friction yields M = 1 in the throat when the mass flow is a maximum.

4.0 Main analysis

The differential equation for resisted flow may be written

$$VdV + gvdP + \frac{fV^2 dL}{2h} = 0 \qquad \text{.............................(8)}$$

where v is the specific volume $({}^{1}/_{\rho})$

dL is an elemental length of the flow path

f is the friction coefficient

and h is the hydraulic mean depth.

This will be combined with the equations

$$M^2 = \frac{V^2}{g\gamma RT} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$P = \rho RT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

$$Q = \rho AV \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$T_t = T + \frac{V^2}{2gJKp} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

and with their respective differential equations

$$\frac{dM}{M} + \frac{dT}{2T} - \frac{dV}{V} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$\frac{dP}{P} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \quad \text{(Q being constant)} \ldots\ldots\ldots\ldots (15)$$

$$dT + \frac{VdV}{gJK_p} = 0 \quad (T_t \text{ being constant}) \ldots\ldots\ldots (16)$$

to obtain differential equations for any change in terms of M as the independent variable.

4.1 Derived differential equations

In all the above equations consistent values of K_p and γ, i.e. mean values over the temperature range T_t to T, enable combination of various equations to be effected.

Thus from equations

(9) and (16), $$\frac{dV}{V} = - \frac{1}{(\gamma - 1)\, M^2} \frac{dT}{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

(13) and (17), $$\frac{dT}{T} = - \frac{2\,(\gamma - 1)\, M^2}{2 + (\gamma - 1)\, M^2} \frac{dM}{M} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

(13) and (18), $$\frac{dV}{V} = - \frac{2}{2 + (\gamma - 1)\, M^2} \frac{dM}{M} \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

(14) and (15), $$\frac{dP}{P} = \frac{dT}{T} - \frac{dV}{V} - \frac{dA}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

(18), (19), and (20), $$\frac{dP}{P} = \frac{2\left\{1 + (\gamma - 1) M^2\right\}}{2 + (\gamma - 1) M^2} \frac{dM}{M} - \frac{dA}{A} \qquad (21)$$

The basic eqn. (8) can be written

$$\frac{fdL}{2h} + \frac{dV}{V} + \frac{gvdP}{V^2} = 0 \qquad (22)$$

Now $$\frac{gvdP}{V^2} = \frac{gRT}{V^2} \frac{dP}{P} \text{ using (10),}$$

$$= \frac{1}{\gamma M^2} \frac{dP}{P} \text{ using (9),}$$

$$= \frac{1}{\gamma M^2} \frac{dA}{A} - \frac{2\left\{1 + (\gamma - 1) M^2\right\}}{2 + (\gamma - 1) M^2} \frac{dM}{\gamma M^2} \qquad (23)$$

by using (21).

Then combining eqns. (22) and (19) and (23)

$$\frac{fdL}{2h} - \frac{dA}{\gamma M^2 A} - \frac{2\,(1 - M^2)\,dM}{\gamma M^2 \left\{2 + (\gamma - 1) M^2\right\}} = 0 \qquad (24)$$

The required differential equations are then (21) for the pressure change and (24) for the friction-length effect.

At this point the further analysis may conveniently be divided into two parts - one for constant area, and the other for variable area ducting.

5.0 The equations for constant area ducting

Here dA = 0 and eqn. (21) becomes

$$\frac{dP}{P} = -\frac{2\left\{1 + (\gamma - 1) M^2\right\} dM}{M\left\{2 + (\gamma - 1) M^2\right\}} \qquad (25)$$

This equation is integrated to give

$$\log_e P = -\log_e M - \frac{1}{2}\log_e \left\{2 + (\gamma - 1) M^2\right\} + \text{constant} \qquad (26)$$

or between planes 1 and 2 in the flow path

$$\frac{P_2}{P_1} = \frac{M_1}{M_2} \sqrt{\frac{2 + (\gamma - 1) M_1^2}{2 + (\gamma - 1) M_2^2}} \qquad (27)$$

Also, eqn. (24) becomes

$$\frac{\gamma f dL}{2h} = \frac{2\,(1 - M^2)\,dM}{M^3\left\{2 + (\gamma - 1) M^2\right\}} \qquad (28)$$

This equation is integrated to give

$$\frac{\gamma f L}{2h} = \frac{M_2^2 - M_1^2}{2M_2^2 M_1^2} - \frac{\gamma + 1}{4} \log_e \frac{M_2^2}{M_1^2} \frac{2 + (\gamma - 1) M_1^2}{2 + (\gamma - 1) M_2^2} \quad \text{.........(29)}$$

where L is the pipe length between the two planes considered. $\gamma fL/2h$ is conveniently shortened to ε and may be called the "pipe function".

5.1 The graphs for constant area ducting

The treatment is simplified by replacing the second plane referred to in para. 5.0 above, by that plane, actual or hypothetical, where M = 1. This critical plane is then referred to under suffixed symbols P_c, ε_c, etc. At the same time the numbering of the primary plane may be omitted and that plane referred to by symbols without suffices as P, M, etc.

Then eqns. (27) and (29) become respectively

$$\frac{P}{P_c} = \frac{1}{M} \sqrt{\frac{\gamma + 1}{2 + (\gamma - 1) M^2}} \quad \text{..............................(30)}$$

$$\varepsilon_c = \frac{\gamma f L_c}{2h} = \frac{1 - M^2}{2M^2} - \frac{\gamma + 1}{4} \log_e \frac{1}{M^2} \frac{2 + (\gamma - 1) M^2}{\gamma + 1} \quad \text{.....(31)}$$

where L_c is the pipe length between the section under consideration (normally either entrance or exit) and that plane, actual or hypothetical, where M = 1.

In the plotting here given, only subsonic flow is covered. Three graphs are included, Fig. 5 covering the range M = 1 to 0.1, but, for the sake of accuracy, being actually used for the lower portion only of that range viz. from M = 0.16 to 0.1.

Two enlargements of the upper portion are then given viz. Fig. 6 covering the range M = 0.35 to 0.16, and Fig. 7 for the remaining range, M = 1.0 to 0.35.

In these graphs pressure ratio, $p_c = P/P_c$, is plotted against pipe function, $\varepsilon_c = \gamma f L_c/2h$ with intersecting curves of M and γ.

To illustrate the use of these graphs an example is included in Appendix III. The usual problem of finding the total head pressure drop in a length of ducting is set and the method of accurately estimating this may be followed in the example.

6.0 The equations for convergent and divergent ducting

Eqn. (21) is integrated directly to give

$$\log_e P = -\frac{1}{2} \log_e \left\{2 + (\gamma - 1) M^2\right\} - \log_e M - \log_e A + \text{constant} \quad \text{..(32)}$$

or between planes 1 and 2 in the flow path

$$\log_e \frac{P_2}{P_1} = \frac{1}{2} \log_e \frac{2 + (\gamma - 1) M_1^2}{2 + (\gamma - 1) M_2^2} + \log_e \frac{M_1}{M_2} + \log_e \frac{A_1}{A_2} \quad \text{..........(33)}$$

This simplifies to the equation

$$\frac{P_2}{P_1} = \frac{A_1 M_1}{A_2 M_2} \sqrt{\frac{2 + (\gamma - 1) M_1^2}{2 + (\gamma - 1) M_2^2}} \quad \text{.............................(34)}$$

Equation (24) is rewritten

$$\frac{\gamma f dL}{2h} - \frac{dA}{M^2 A} = \frac{2 (1 - M^2) dM}{M^3 \left\{2 + (\gamma - 1) M^2\right\}}$$

$$\left(\frac{\gamma f}{2} \frac{A}{h} \frac{dL}{dA} - \frac{1}{M^2}\right) \frac{dA}{A} = \frac{2 (1 - M^2) dM}{M^3 \left\{2 + (\gamma - 1) M^2\right\}}$$

Now $A/h = S$, the perimeter of the cross-section, hence the eqn. becomes

$$\left(\frac{\gamma f}{2} \frac{S\ dL}{dA} - \frac{1}{M^2}\right) \frac{dA}{A} = \frac{2 (1 - M^2) dM}{M^3 \left\{2 + (\gamma - 1) M^2\right\}} \quad \text{.....................(35)}$$

If the first term within the bracket were constant, integration would be possible following separation of the variables. As the $\gamma f/2$ is constant for purposes of the integration, it remains only to examine the remaining term $S\ dL/dA$. It is found that, for certain simple tapering ducts formed by conic and pyramidal frusta, this term does indeed remain constant.

This being so it is convenient to rewrite eqn. (35) in the form

$$\left(\alpha - \frac{1}{M^2}\right) \frac{dA}{A} = \frac{2 (1 - M^2) dM}{M^3 \left\{2 + (\gamma - 1) M^2\right\}} \quad \text{...........................(36)}$$

where $$\alpha = \frac{\gamma f}{2} \frac{S\ dL}{dA} \quad \text{................................(37)}$$

Certain cases are cited in Appendix IV in which the quantity $S\ dL/dA$ is derived in terms of the duct geometry.

Thus for the special cases of circular or square cross-section frusta

$$\alpha = \frac{\gamma f}{2 \operatorname{Tan}\beta} \quad \text{....................................(38)}$$

where β is the half-angle of the cone or pyramid.

Further, for the special cases of elliptical or rectangular cross-section frusta

$$\alpha = \frac{\gamma f\ (a + b)}{4\ b\ \operatorname{Tan}\beta} \quad \text{.....................................(39)}$$

where β is the larger of the two half-angles of the cone or pyramid.

Finally for the general case of a break-down of the duct length into a number of short lengths

$$\alpha = \frac{\gamma f}{2} \frac{S_m \delta L}{\delta A} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(40)$$

where S_m is the mean perimeter of the short length δL, δA being the change in cross-sectional area in that length.

Eqn. (36) is integrated to give

$$\log_e A = -\log_e M - \frac{1-\alpha}{\gamma - 1 + 2\alpha} \log_e (1 - \alpha M^2) + \frac{\gamma + 1}{2(\gamma - 1 + 2\alpha)} \log_e \left\{2 + (\gamma - 1)M^2\right\} + \text{constant} \qquad \ldots\ldots\ldots(41)$$

Then between planes 1 and 2 in the flow path

$$\log_e \frac{A_2}{A_1} = \log_e \frac{M_1}{M_2} + \frac{1-\alpha}{\gamma - 1 + 2\alpha} \log_e \frac{1 - \alpha M_1^2}{1 - \alpha M_2^2} + \frac{\gamma + 1}{2(\gamma - 1 + 2\alpha)} \log_e \frac{2 + (\gamma - 1) M_2^2}{2 + (\gamma - 1) M_1^2} \qquad \ldots..(42)$$

which simplifies to

$$\frac{A_2}{A_1} = \frac{M_1}{M_2} \left(\frac{1 - \alpha M_1^2}{1 - \alpha M_2^2}\right)^{\frac{1-\alpha}{\gamma-1+2\alpha}} \left\{\frac{2 + (\gamma - 1) M_2^2}{2 + (\gamma - 1) M_1^2}\right\}^{\frac{\gamma+1}{2(\gamma-1+2\alpha)}} \qquad \ldots\ldots\ldots\ldots(43)$$

α being as defined above in eqns. (37) to (40) and being negative for convergent ducts and positive for divergent ducts.

6.1 The graphs for convergent and divergent ducting

The treatment is simplified by replacing the first plane, referred to in Section 6.0 above, by that plane, actual or hypothetical, where $M = 1$. This critical plane is then referred to under the suffixed symbols P_c, A_c, etc. (M_c being 1). At the same time the numbering of the second plane may be omitted and that plane referred to under symbols without suffixes P, A, M, etc.

Then eqns. (34) for the pressure ratio and (43) for the area ratio become respectively

$$\frac{P}{P_c} = \frac{A_c}{AM} \sqrt{\frac{\gamma + 1}{2 + (\gamma - 1) M^2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

$$\frac{A}{A_c} = \frac{1}{M}\left(\frac{1-\alpha}{1-\alpha M^2}\right)^{\frac{1-\alpha}{\gamma-1+2\alpha}}\left\{\frac{2+(\gamma-1)M^2}{\gamma+1}\right\}^{\frac{\gamma+1}{2(\gamma-1+2\alpha)}} \qquad \ldots\ldots\ldots\ldots(45)$$

In the graphs, Figs. 8 to 19, P/P_c (for subsonic flow) or P_c/P (for supersonic flow), is plotted against A/A_c with intersecting curves of M and α. Three sets of curves for $\gamma = 1.3$, 1.35, and 1.4 admit of interpolation for any normal value of γ. Subsonic and supersonic flows are covered separately. Subsonic flow is taken down to about $M = 0.1$ and supersonic flow up to about $M = 2$. As regards friction a range of α of from -0.1 to 0 for convergent ducts and from 0 to +0.1 for divergent ducts enables cones of very small apex angles to be included.

To illustrate the use of these graphs an example is given in Appendix V. The case chosen is that of expansion in the convergent-divergent nozzle of a jet engine.

7.0 The Mach number of flow in the throat for maximum mass flow

Appendix VI gives the proof that in the simple one-dimensional treatment of flow with friction used in this work, the Mach number of the flow in the throat of a duct under maximum mass flow conditions is unity. This is regarded as a most satisfactory feature of the treatment in so far that it agrees with the simple qualitative result based on the fact that pressure effects in a fluid can only be transmitted with sonic velocity.

8.0 The temperature-entropy diagram for duct flow

Appendix VII gives the equation for entropy change during an expansion or compression. By analysing the change in the form $dT/d\phi$, the general shapes of the expansion and compression temperature-entropy curves may be inferred in explanation of the friction process accompanying the change. These are illustrated in Fig. 20.

9.0 Conclusion

It is considered that the plotting of the equations (unusable directly, except through a graph) resulting from the simple treatment of flow with friction enables ducting problems to be solved with a high degree of consistency of result. The scope of the supersonic graphs to cover higher Mach numbers of flow can readily be extended by additional graphs as the requirement arises.

REFERENCES

No.	Author	Title
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2	Jean Fabri	"Methode Rapide de Determination des Caracteristiques d'un Ecoulement Gazeux a Grande Vitesse". O.N.E.R.A., 1947
3	D. Fielding and J. E. C. Topps	"Thermodynamic Properties of Air and Combustion Products of Hydrocarbon Fuels: Part I". N.G.T.E. Report R. 74. 1950. A.R.C. 13,672. July, 1950.

APPENDIX I

10.0 List of symbols

A = Area

α = Variable area ducting friction index

β = Half-angles of conic and pyramidal frusta

γ = K_p/K_v

D = Diameter

ε = Pipe Function

f = Friction Coefficient

h = Hydraulic mean depth

J = Mechanical equivalent of heat

K_p = Specific heat at constant pressure

K_v = Specific heat at constant volume

L = Length of ducting

M = Mach number

P = Pressure

p = Pressure ratio

Q = Mass flow

q = Fuel-air ratio

R = Gas constant

r = Radius

ρ = Density

S = Perimeter

T = Temperature

V = Velocity

v = Specific volume

ϕ = Entropy

10.1 List of subscripts

A = Aircraft

c = Critical

m = Mean

List of subscripts (cont'd)

t = Total head

1 = Initial plane of reference

2 = Final plane of reference

APPENDIX II

11.0 The value of γ

γ is obtained fundamentally from K_p using the fact that R is constant at 96 ft. lb. p. lb. - °C., i.e. $K_p - K_v$ is constant at 0.0686 C.H.U. p. lb. -°C. for air and combustion products of hydrocarbon fuels. In turn K_p is dependent upon T. However, frequently the range of an expansion or compression is indicated primarily by the pressure ratio. Making use of data given in Reference 3 dealing with the thermodynamic properties of air and combustion products Fig. 4 has been prepared to read directly the temperature range corresponding to any pressure ratio (and efficiency of process) so that, knowing the initial temperature of the change, the final temperature can be read with sufficient accuracy to make a satisfactory estimate of the mean K_p for the change.

Further, instead of obtaining the mean K_p over the temperature range so given, it is suggested that the true K_p at the arithemetical mean temperature be used. Not only is this a much easier operation than that of obtaining the mean K_p but for low pressure ratios (less than 2) it gives a more accurate answer since the random error to which the mean K_p method is subject exceeds the systematic error present in the suggested method.

Thus Fig. 4 covers five parameters, p (pressure ratio), η_∞ (polytropic efficiency), q (fuel-air ratio), T_1 and T_2 (initial and final temperatures) and is used to obtain $T_m = \frac{1}{2}(T_1 + T_2)$, T_1 being known. K_p is then read from the curves of Reference 3.

The presence of η_∞ is unnecessary for the particular application to flow in ducts where an assumption of 100 per cent will introduce very little error into the preliminary calculation for mean temperature but has the advantage of rendering the graph of general application to compression and expansion in compressors and turbines of gas turbine plant.

For calculations involving the relationship between total head and static conditions (as in Section 2.0 of the text) the true value of γ at $T_m = \frac{1}{2}(T_t + T)$ would be used.

For other calculations the true value of γ at $T_m = \frac{1}{2}(T_t + T)$ would be used, T being the static temperature at the end of the expansion or beginning of the compression.

APPENDIX III

12.0 Example of a calculation for constant section ducting

12.1 Problem

Find the total head pressure drop in 10 ft. length of ducting of 15 in. x 9 in. rectangular internal section with $Q = 40$ lb. p.sec., $q = 0.0185$, $T_t = 850°K.$, $P_{t1} = 34.1$ p.s.i.a., $f = 0.005$.

12.2 Solution

$$h = \frac{15 \times 9}{48 \times 12} = 0.2345 \text{ ft.}$$

$$\frac{Q\sqrt{Tt}}{AP_t} \text{ at entry} = \frac{40\sqrt{850}}{15 \times 9 \times 34.1} = 0.2532$$

Guess $\gamma = 1.35$. Fig. 1 gives $M_1 = 0.443$.

Fig. 3 gives $\delta T/T_t = 0.03365 \therefore \delta T = 28.6$; $T_m = 836$.

$K_p = 0.2718$; $K_v = 0.2032$; $\gamma = 1.337$

Fig. 1 gives $M_1 = 0.445$ (no need to repeat calculations for γ)

Fig. 2 gives $P_{t1}/P_1 = 1.1385 \therefore P_1 = 29.95$

Guess $\gamma = 1.337$ for the whole expansion through the pipe.

$$\varepsilon_{c1} - \varepsilon_{c2} = \frac{0.005 \times 1.337 \times 10}{2 \times 0.2345} = 0.1425$$

Enter Fig. 7 at $M_1 = 0.445$, $\gamma = 1.337$ and read $\varepsilon_{c1} = 1.152$ and

$P_1/P_c = 2.390$. Then $\varepsilon_{c2} = 1.152 - 0.1425 = 1.0095$

Enter Fig. 7 at $\varepsilon_{c2} = 1.0095$ and $\gamma = 1.337$ and read $M_2 = 0.462$ and $P_2/P_c = 2.295$.

(N.B. γ could now be recalculated from a new δT_2 using Fig. 3 with $M_2 = 0.462$ and $\gamma = 1.337$. However in this case this refinement is unnecessary owing to the small degree of extra expansion in the pipe.)

Then $P_1/P_2 = 2.390/2.295 = 1.0414$

Hence $P_2 = 29.95/1.0414 = 28.75$

Fig. 2 for $M_2 = 0.462$ and $\gamma = 1.337$ gives $P_{t2}/P_2 = 1.1495$

Hence $P_{t2} = 28.75 \times 1.1495 = 33.02$

and $\delta P_t = P_{t1} - P_{t2} = 34.10 - 33.02 = 1.08$ p.s.i.

APPENDIX IV

13.0 The value of α

$$\alpha = \frac{\gamma f}{2} \frac{S\,dL}{dA} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

is repeated for reference.

13.1 Cases follow for which α remains constant over the whole length of the duct.

13.1.1 Right circular cone of radius r

Here $S = 2\pi r$, $A = \pi r^2$, $dA = 2\pi r dr$.

Then $S\,dL/dA = dL/dr$ which is constant and equal to $1/\text{Tan}\beta$ where β is the half-angle at the cone apex Thus as quoted in the text, for this case (and for that below in 13.1.2)

$$\alpha = \frac{\gamma f}{2\text{Tan}\beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

13.1.2 Right square pyramid of side 2a

Here $S = 8a$, $A = 4a^2$, $dA = 8a\,da$.

Then $S\,dL/dA = dL/da$ which is constant and equal to $1/\text{Tan}\beta$ where β is the half-angle at the pyramid apex i.e. $\text{Tan}\beta = \frac{a_2 - a_1}{L}$ where $2a_2$ and $2a_1$, are the frusta sides of the larger and smaller ends respectively and L the length normal to the end planes. α is then given in eqn. (38)

13.1.3 Right elliptical cone of semi-diameters a and b

Here b/a = constant, say c; $S = \pi(a + b) = \pi(1 + c)a$; $A = \pi ab = \pi ca^2$; $dA = 2\pi ca\,da$.

Then $S\,dL/dA = \frac{1 + c}{2c}\frac{dL}{da} = \frac{a + b}{2b}\frac{dL}{da}$ which is constant.

$dL/da = 1/\text{Tan}\beta$ where β is the half-angle at the apex in the plane of the major semi-diameter.

Thus as quoted in the text for this case (and for that below in 13.1.4)

$$\alpha = \frac{\gamma f\,(a + b)}{4b\ \text{Tan}\beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

13.1.4 Right rectangular pyramid of sides 2a and 2b

Here b/a = constant, say c; $S = 4(a + b) = 4(1 + c)a$; $A = 4ab = 4ca^2$; $dA = 8\,ca\,da$

Then $S\,dL/dA = \frac{1 + c}{2c}\frac{dL}{da} = \frac{a + b}{2b}\frac{dL}{da}$ which is constant.

$dL/da = 1/\text{Tan}\beta$ where β is the half-angle at the apex to the bisector of the lesser side. α is given in eqn. (39).

13.2 For cases other than the foregoing it is unlikely that α will remain constant over the whole length of the duct. Then the duct length must be broken down into short lengths for each of which it must then be assumed that α will remain constant.

Then for any one of these sections

$$\alpha = \frac{\gamma f S_m \ \delta L}{2\delta A} \qquad \text{.................. ....(40)}$$

where S_m is the mean perimeter of the section

δL is the length of the short section

δA is the change in cross-sectional area over the short section considered.

13.3 In all cases δA or dA, hence β and $\tan\beta$, hence α, will be negative for convergent ducts and positive for divergent ducts.

APPENDIX V

14.0 Example of a calculation for convergent-divergent ducting

14.1 Problem

Find the thrust given by a jet engine at its design point when fitted with a convergent-divergent nozzle: altitude 36,000 ft., flight Mach number 1.4. At entry to the nozzle the flow conditions are T_t = 1087.5°K. q = 0.02092; P_t = 23.075; M_1 = 0.45. Assume for the nozzle design conical ducting for both convergent and divergent portions of the nozzle, $\beta = -7^1/_2°$ for the former and $+7^1/_2°$ for the latter. f = 0.005. Atmospheric pressure, Pa = 3.283. Aircraft Velocity, V_A = 1355.6. Q = 77.65.

14.2 Solution

Using γ roughly as 1.35 with M_1 = 0.45, Fig. 3 gives for entry conditions $\delta T_{t1}/T_t$ = 0.034 i.e. δT = 37. T_m = 1069. K_p = 0.2852. K_v = 0.2166 γ = 1.317.

Using γ = 1.317 and M_1 = 0.45, Fig. 1 gives $\frac{Q\sqrt{T_t}}{AP_{t1}} = 0.2665$ whence A_1 = 2.892. Fig. 2 gives P_{t1}/P_1 = 1.1411 whence P_1 = 20.22

A value of γ is now required to cover the whole expansion from total head inlet conditions to static conditions at exit from the divergent portion where P_a = 3.283. The pressure ratio over the whole expansion is thus 23.075/3.283 = 7.03. Using η_∞ = 100 per cent and T_t = 1087.5 Fig. 4 gives T_2 = 665 approx.

(N.B. If desired an efficiency can be applied to this calculation without much effect on the value of γ obtained).

Then $T_m = \frac{1}{2}(1087.5 + 665) = 876$. K_p = 0.2752. K_v = 0.2066 γ = 1.333.

For the convergent portion $\alpha = -\frac{1.333 \times 0.005}{2 \times 0.1317} = -0.0253$

Use of a pair of subsonic graphs (for γ = 1.30 and 1.35) gives the following table using M_1 = 0.45 and α = - 0.0253:

Fig.	5	6	Diff.		Diff.
γ	1.30	1.35	0.05	1.333	0.033
A_1/A_c	1.454	1.448	- 0.006	1.450	- 0.004
P_1/P_c	1.614	1.634	0.020	1.627	0.013

Then A_c = 2.892/1.450 = 1.995

P_c = 20.22/1.627 = 12.43

14.2.2 Design of the divergent portion

The pressure ratio over this portion is $P_c/P_2 = 12.43/3.283 = 3.785$. $\alpha = +0.0253$ with γ remaining at 1.333 and again use a pair of supersonic graphs (for $\gamma = 1.30$ and 1.35) gives the following Table using P_c/P_2 and α:

Fig.	12	13	Diff.		Diff.
γ	1.30	1.35	0.05	1.333	0.033
M_2	1.918	1.918	0	1.918	0
A_2/A_c	1.697	1.668	- 0.029	1.678	- 0.019

Then $M_2 = 1.918$ and $A_2 = 1.995 \times 1.678 = 3.345$

Fig. 3 gives $\delta T_2/T_t = 0.379$ i.e. $\delta T_2 = 412$

$V_2^2 = 2gJ \times 0.2752 \times 412 = 10.23 \times 10^6 \therefore V_2 = 3200$

14.2.3 Thrust of the nozzle

$$\text{Net thrust} = \frac{77.65}{g}(3200 - 1355.6) = 4450 \text{ lb.}$$

APPENDIX VI

15.0 The Mach number of flow in the throat for maximum mass flow

If the known conditions of the flow at entry are suffixed "1", and those at any subsequent section carry no suffix, the changes in condition are covered by the following equations:

Combination of eqns. (34) and (43) gives

$$\frac{P}{P_1} = \left(\frac{1 - aM^2}{1 - aM_1^2}\right)^{\frac{1-a}{\gamma-1+2a}} \left\{\frac{2 + (\gamma - 1) M_1^2}{2 + (\gamma - 1) M^2}\right\}^{\frac{\gamma+a}{\gamma-1+2a}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(46)$$

For the two static pressures there are two total head pressures to correspond, P_{t1} and P_t respectively, related to P_1 and P by eqn. (2) whence

$$\frac{P_t}{P_{t1}} = \left(\frac{1 - aM^2}{1 - aM_1^2}\right)^{\frac{1-a}{\gamma-1+2a}} \left\{\frac{2 + (\gamma - 1) M^2}{2 + (\gamma - 1) M_1^2}\right\}^{\frac{a(\gamma+1)}{(\gamma-1)(\gamma-1+2a)}} \quad \ldots\ldots\ldots\ldots\ldots(47)$$

The mass flow eqn. (1) then becomes

$$\frac{Q}{AP_{t1}}\sqrt{\frac{RT_t}{g\gamma}} = \frac{P_t}{P_{t1}}\,\frac{Q}{AP_t}\sqrt{\frac{RT_t}{g\gamma}}$$

$$= M\left(\frac{1 - aM^2}{1 - aM_1^2}\right)^{\frac{1-a}{\gamma-1+2a}} \left\{\frac{2 + (\gamma - 1) M^2}{2 + (\gamma - 1) M_1^2}\right\}^{\frac{a(\gamma+1)}{(\gamma-1)(\gamma-1+2a)}} \left\{\frac{2}{2 + (\gamma - 1) M^2}\right\}^{\frac{\gamma+1}{2(\gamma-1)}}$$

or, collecting all the constant quantities with Q/A on the left-hand side

$$(1 - aM_1^2)^{\frac{1-a}{\gamma-1+2a}} \left\{2 + (\gamma - 1) M_1^2\right\}^{\frac{a(\gamma+1)}{(\gamma-1)(\gamma-1+2a)}}\, 2^{-\frac{\gamma+1}{2(\gamma-1)}}\, \frac{Q}{AP_{t1}}\sqrt{\frac{RT_t}{g\gamma}}$$

$$= \frac{M\,(1 - aM^2)^{\frac{1-a}{\gamma-1+2a}}}{\left\{2 + (\gamma - 1) M^2\right\}^{\frac{\gamma+1}{2(\gamma-1+2a)}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(48)$$

Then for a given mass flow Q, the duct area A becomes a minimum when Q/A is a maximum i.e. when the right-hand side of eqn. (45) is a maximum, i.e. when

$$\frac{d}{dM}\,\frac{M\,(1 - aM^2)^{\frac{1-a}{\gamma-1+2a}}}{\left\{2 + (\gamma - 1) M^2\right\}^{\frac{\gamma + 1}{2(\gamma-1+2a)}}} = 0$$

i.e. $$\frac{d}{du}\frac{uv}{w} = 0$$

or in its most convenient form

$$\frac{v}{w}\left(1 + \frac{u}{v}\frac{dv}{du} - \frac{u}{w}\frac{dw}{du}\right) = 0$$

$$\text{or} \quad 1 + \frac{u}{v}\frac{dv}{du} - \frac{u}{w}\frac{dw}{du} = 0$$

$$\text{i.e.} \quad 1 - \frac{2a\,(1 - a)\,M^2}{(1 - aM)(\gamma - 1 + 2a)} - \frac{(\gamma - 1)\,(\gamma + 1)\,M^2}{(\gamma - 1 + 2a)\left\{2 + (\gamma - 1)M\right\}} = 0$$

..................(49)

the solution of which is M = 1.

APPENDIX VII

15.0 The temperature-entropy diagram for duct flow

The basic equation for the friction work is

$$dF = gJTd\phi \text{ in absolute units} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(50)$$

Written in the same units eqn. (8) of section 4.0 becomes

$$dF = \frac{fV^2dL}{2h} = -VdV - gvdP \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(51)$$

15.1 Constant area ducting

Using eqns. (12), (16), (20), and (10), with (50) and (51)

$$d\phi = K_V \frac{dT}{T} - \frac{K_p - K_V}{2} \frac{dT}{T_t - T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(52)$$

which may be rewritten

$$\frac{dT}{d\phi} = \frac{T}{K_V} \frac{1 - T/T_t}{1 - T/T_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(53)$$

Further, integration of eqn. (52) gives

$$\phi_2 - \phi_1 = K_V \left(\log_e \frac{T_2}{T_1} - \frac{\gamma - 1}{2} \log_e \frac{T_t - T_1}{T_t - T_2} \right) \quad \ldots\ldots\ldots\ldots(54)$$

for the entropy change.

It is to be noted that this reaches a maximum (from eqn. (53)) when $T = T_c$.

The interpretation of this result is that with subsonic flow at the entry to a pipe, provided the pipe is of sufficient length, the leaving velocity will have a Mach number of unity and this cannot be exceeded. Also with supersonic flow at the entry to a pipe, again provided the pipe is of sufficient length, the leaving velocity will again have a Mach number of unity and no further diffusion can take place.

Eqn. (53) shows that the $T\phi$ curve representing subsonic expansion in a pipe has negative slope at the start, this slope becoming steeper until at $T = T_c$ it is running vertically. Similarly the $T\phi$ curve representing supersonic diffusion in a pipe has positive slope at the start, this slope becoming steeper until at $T = T_c$ it is running vertically.

15.2 Convergent and divergent ducting

Using eqns. (12), (24), and (36) with (50) and (51)

$$d\phi = -\frac{4K_V a\,(T_t - T)(1 - M^2)\,dM}{M\,(1 - aM^2)\left\{2 + (\gamma - 1)\,M^2\right\}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(55)$$

But $\quad M_2 = \dfrac{2\,(T_t - T)}{(\gamma - 1)\,T}$ and $MdM = -\dfrac{T_t\,dT}{(\gamma - 1)\,T^2}$

whence eqn. (55) becomes

$$d\phi = \frac{K_v a \left\{ (\gamma + 1) T - 2T_t \right\} dT}{T \left\{ (\gamma - 1 + 2a) T - 2aT_t \right\}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

$$\frac{dT}{d\phi} = \frac{T}{K_v a} \frac{(\gamma - 1 + 2a) T - 2aT_t}{(\gamma + 1) T - 2T_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

Three cases may be examined

15.2.1 When $T = T_t$ (a purely hypothetical case) i.e. at the start of an expansion from, or at the end of a diffusion to total head conditions,

$$\frac{dT}{d\phi} = \frac{T_t}{K_v a} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (58)$$

Thus for a subsonic expansion in a convergent duct (a - ve) the slope of the $T\phi$ curve would be - ve, whilst for a subsonic diffusion in a divergent duct (a + ve) the slope would be + ve.

Between this case and the next the slope would become steeper.

15.2.2 When $T = T_c$

$$\frac{dT}{d\phi} = \infty \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (59)$$

i.e. for both expansion and diffusion the $T\phi$ curve would run vertically.

Between this case and the next the slope of the curve would become less steep accompanied of course by increasing entropy.

15.2.3 When $T = 0$ (a purely hypothetical case) i.e. at the end of an expansion to, or at the commencement of a diffusion from limiting conditions,

$$\frac{dT}{d\phi} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (60)$$

i.e. both curves run horizontally.

15.2.4 The entropy change between any two planes of flow 1 and 2 during expansion or diffusion is obtained by integration of eqn. (56) as

$$\phi_2 - \phi_1 = K_v \left\{ \log_e \frac{T_2}{T_1} - \frac{(1 - a)(\gamma - 1)}{\gamma - 1 + 2a} \log_e \frac{(\gamma - 1 + 2a) T_2 - 2aT_t}{(\gamma - 1 + 2a) T_1 - 2aT_t} \right\}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

15.2.5 The general shapes of the change lines on the temperature-entropy diagram may be sketched from the information of eqns. (58), (59), and (60).

These curves are shown in Fig. 20, above for an expansion, and below for a diffusion.

In Fig 20A a shockless expansion from subsonic conditions at 1 to supersonic conditions at 2 is shown on the general curve, the friction energy or heat of the two portions of this expansion, before and after the critical point c, being represented by the two areas $\phi_1 1 C \phi_c$ and $\phi_c C 2 \phi_2$ respectively. The general curve extends from the total head conditions at t, referred to in eqn. (58) and Section 15.2.1 above, to fully expanded conditions at O, referred to in eqn. (60) and Section 15.2.3 above, and passing through the critical point C referred to in eqn. (59) and Section 15.2.2 above

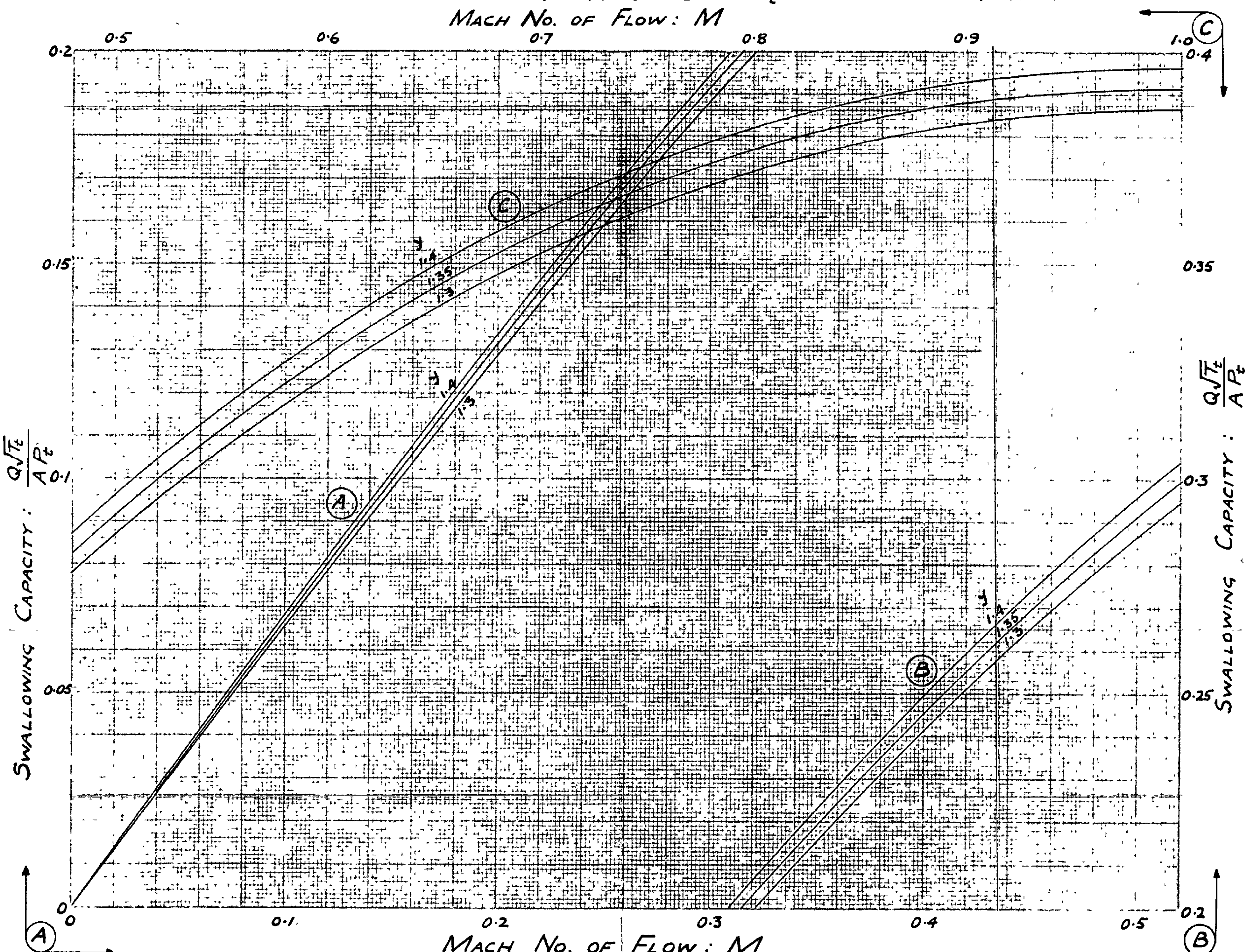
In Fig. 20B a shockless diffusion from supersonic conditions at 3 to subsonic conditions at 4 is shown on the general curve, the friction energy or heat of the two portions of this diffusion, before and after the critical point C, being represented by the two areas $\phi_3 3 C \phi_c$ and $\phi_c C 4 \phi_4$ respectively. The general curve extends from zero pressure conditions at O, referred to in eqn. (60) and Section 15.2.3 above, to fully diffused total head conditions at t, referred to in eqn. (58) and Section 15.2.1 above, and passing through the critical point C, referred to in eqn. (59) and Section 15.2.2 above.

FLOW IN PIPES : SWALLOWING CAPACITY AGAINST MACH NUMBER.

FIG. 1.

FORMULA $$\frac{Q\sqrt{T_t}}{AP_t} = \sqrt{\frac{g\gamma}{R}}\,\frac{M}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

Q = MASS FLOW LB.P.SEC. T_t AND P_t = TOTAL HEAD TEMP AND PRESS.
A = AREA OF CROSS SECTION (AP_t = LB.)
$R = J(c_p - c_v) = 96$ M = MACH NO.
γ = MEAN VALUE BETWEEN T_t AND T TO SIMPLIFY THE FORMULA.

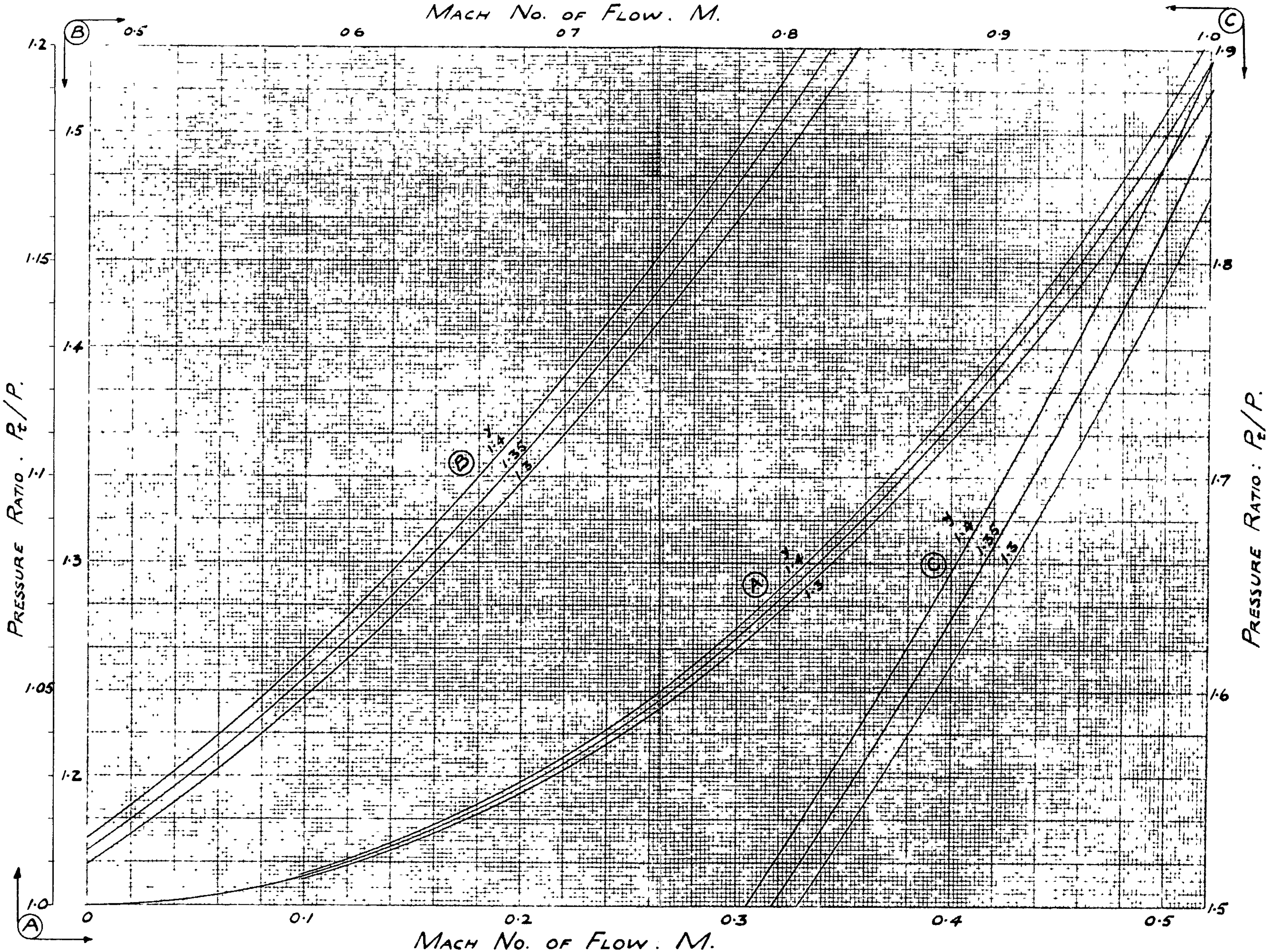


USE EACH SET OF CURVES WITH ITS APPROPRIATE PAIR OF SCALES.

RATIO, TOTAL HEAD TO STATIC PRESSURE AGAINST MACH NUMBER. FIG. 2.

FORMULA: $\frac{P_t}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$

P_t = TOTAL HEAD PRESSURE
P = STATIC PRESSURE
M = MACH NO. OF FLOW
γ = MEAN VALUE BETWEEN T_t AND T TO SIMPLIFY FORMULA



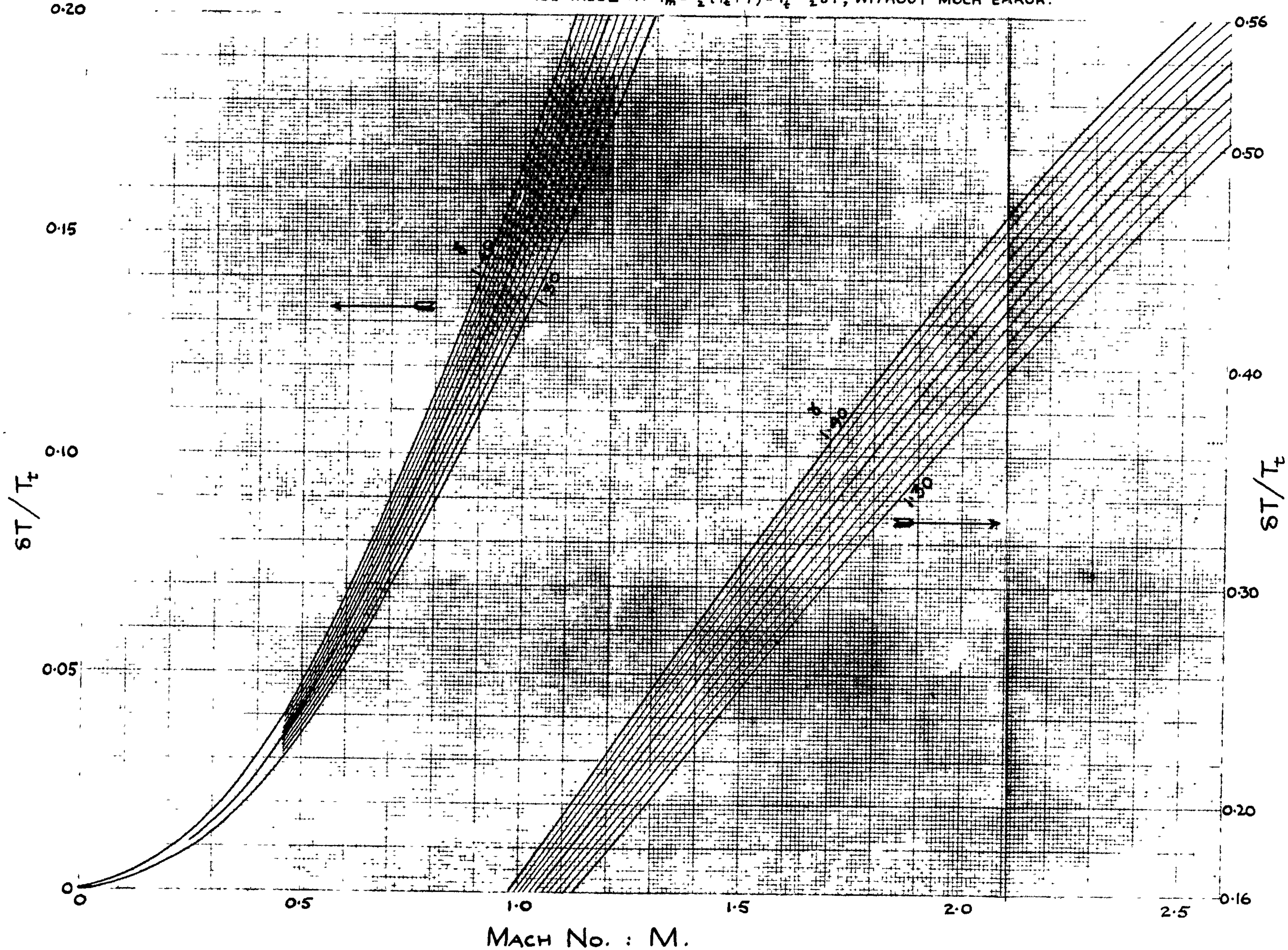
USE EACH SET OF CURVES WITH ITS APPROPRIATE PAIR OF SCALES.

FLOW IN DUCTS : TEMPERATURE DROP AND MACH NUMBER.

FIG.3.

$$\delta T / T_t = 1 \Big/ \left\{ 1 + 2 / (\overline{\gamma - 1}\, M^2) \right\}$$
$$= (\gamma - 1)/(\gamma + 1) \text{ AT } M = 1.$$

δT = TEMPERATURE DROP FROM
T_t = TOTAL HEAD TEMPERATURE TO
T = STATIC TEMPERATURE OF STREAM.
M = MACH NO. OF FLOW = $V/\sqrt{g\gamma RT}$
γ = MEAN VALUE BETWEEN T_t AND T
= TRUE VALUE AT $T_m = \frac{1}{2}(T_t + T) = T_t - \frac{1}{2}\delta T$, WITHOUT MUCH ERROR.



COMPRESSION & EXPANSION PROCESSES.

FIG 4.

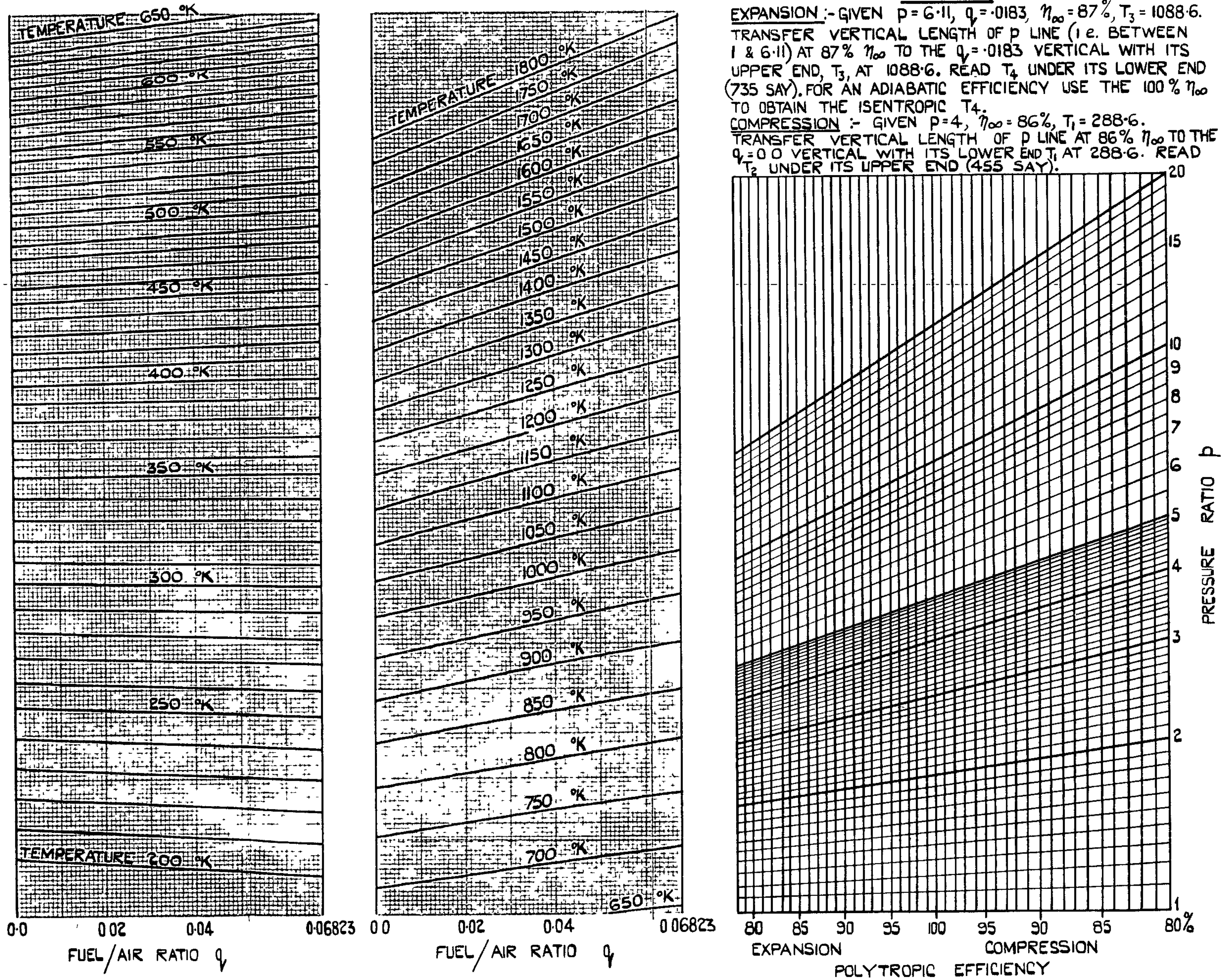
PRESSURE RATIO, FUEL/AIR RATIO, AND TEMPERATURES FOR STANDARD FUEL COMBUSTION PRODUCTS

FOR RAPID ESTIMATION OF APPROXIMATE FINAL TEMPERATURE

EXAMPLES

EXPANSION :- GIVEN $p = 6{\cdot}11$, $q = {\cdot}0183$, $\eta_\infty = 87\%$, $T_3 = 1088{\cdot}6$. TRANSFER VERTICAL LENGTH OF p LINE (i.e. BETWEEN 1 & 6·11) AT 87% η_∞ TO THE $q = {\cdot}0183$ VERTICAL WITH ITS UPPER END, T_3, AT 1088·6. READ T_4 UNDER ITS LOWER END (735 SAY). FOR AN ADIABATIC EFFICIENCY USE THE 100% η_∞ TO OBTAIN THE ISENTROPIC T_4.

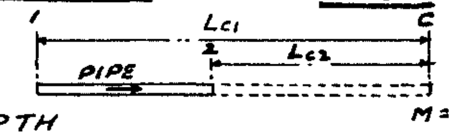
COMPRESSION :- GIVEN $p = 4$, $\eta_\infty = 86\%$, $T_1 = 288{\cdot}6$. TRANSFER VERTICAL LENGTH OF p LINE AT 86% η_∞ TO THE $q = 0{\cdot}0$ VERTICAL WITH ITS LOWER END T_1 AT 288·6. READ T_2 UNDER ITS UPPER END (455 SAY).



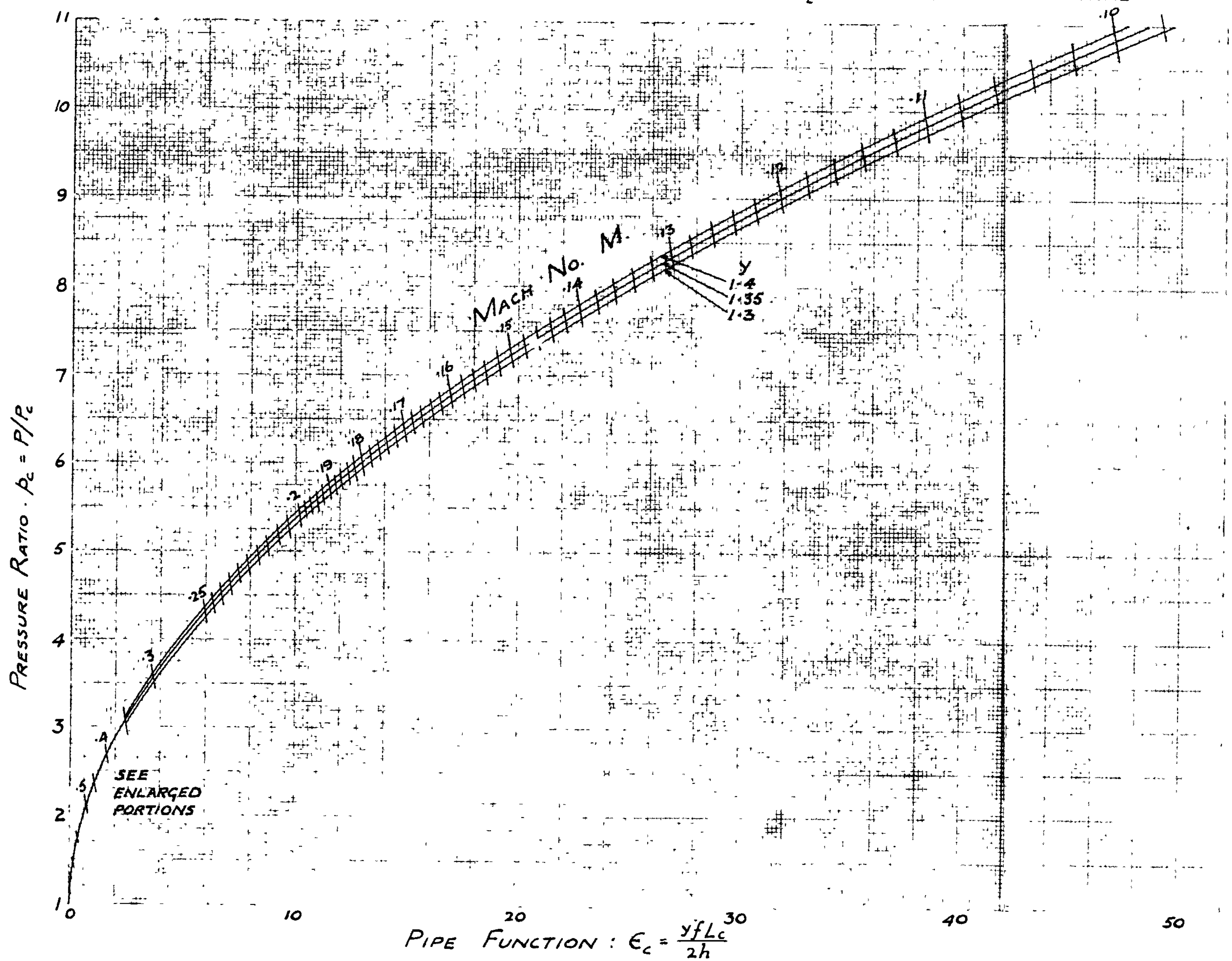
FLOW IN PIPES : LENGTH, MACH NO., AND PRESSURE RATIO.

FIG.5.

FORMULAE: $p_c = \dfrac{P}{P_c} = \dfrac{1}{M}\sqrt{\dfrac{y+1}{2+(y-1)M^2}}$ $\qquad \epsilon_c = \dfrac{yfL_c}{2h} = \dfrac{1-M^2}{2M^2} - \dfrac{y+1}{4}\log_e \dfrac{1}{M^2}\,\dfrac{2+(y-1)M^2}{y+1}$



P = STATIC PRESS. AT SECTION CONSIDERED.
P_c = " " " " WHERE $M = 1$.
M = MACH NO. OF FLOW AT SECTION CONSIDERED
f = FRICTION COEFFICIENT.
L_c = PIPE LENGTH FROM SECTION CONSIDERED TO THAT WHERE $M = 1$
y = MEAN VALUE BETWEEN T_t AND T TO SIMFLIFY THE FORMULAE
h = HYDRAULIC MEAN DEPTH



FLOW IN PIPES : LENGTH, MACH NO., AND PRESSURE RATIO.

FIG 6.

1st. Enlargement. See Formulae on Sk. 2840.

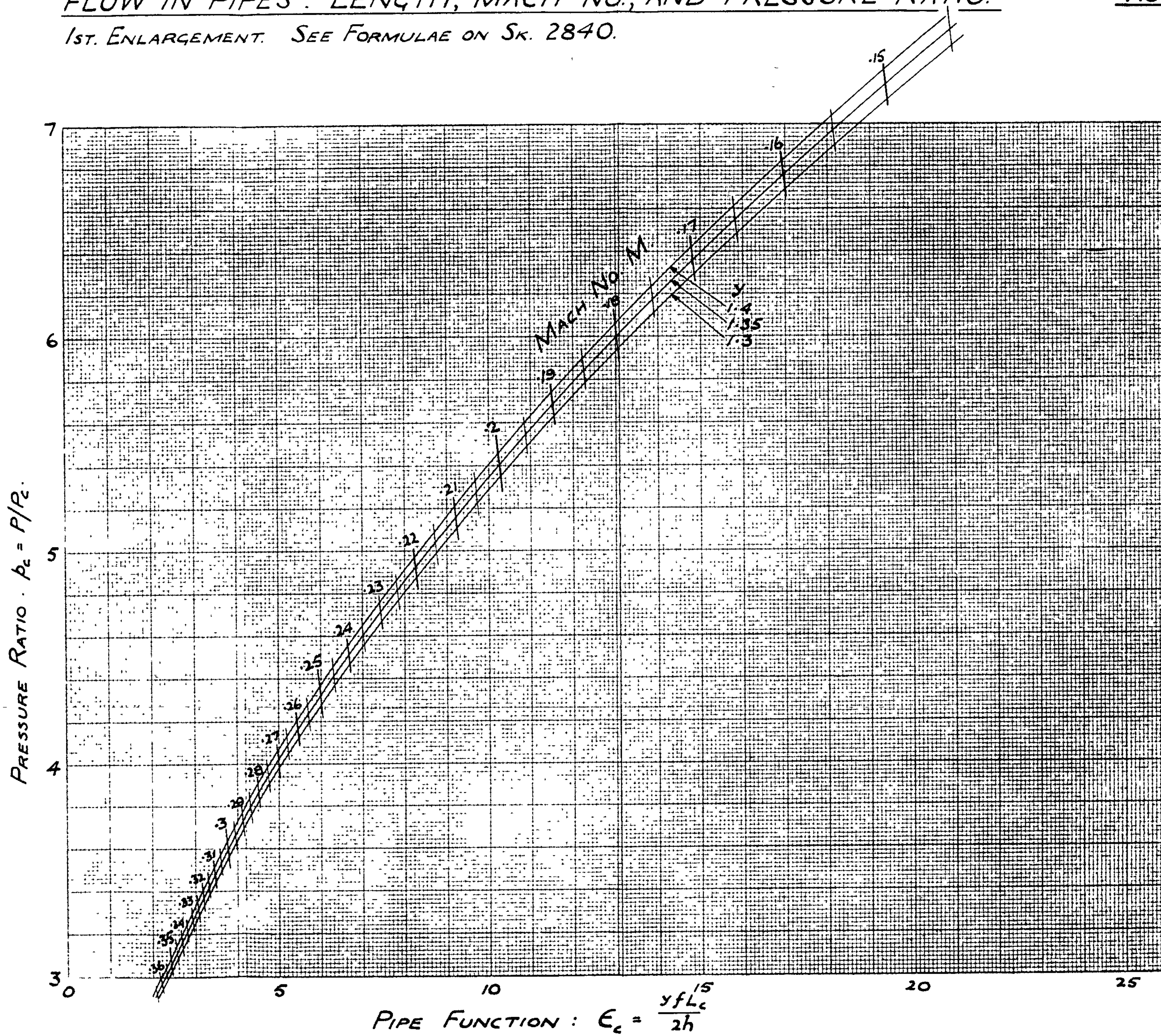
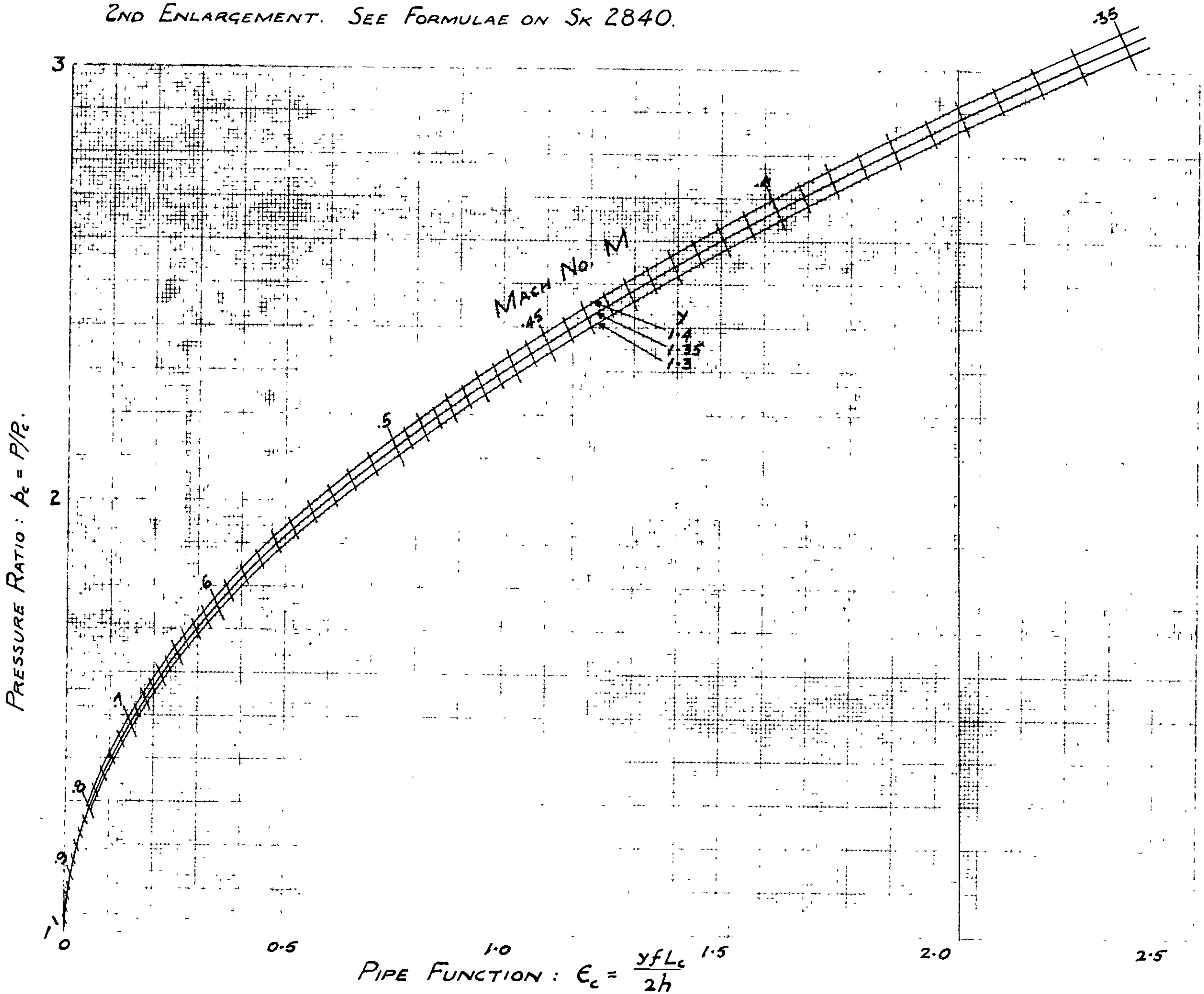


FIG. 7.

FLOW IN PIPES : LENGTH, MACH NO., AND PRESSURE RATIO.

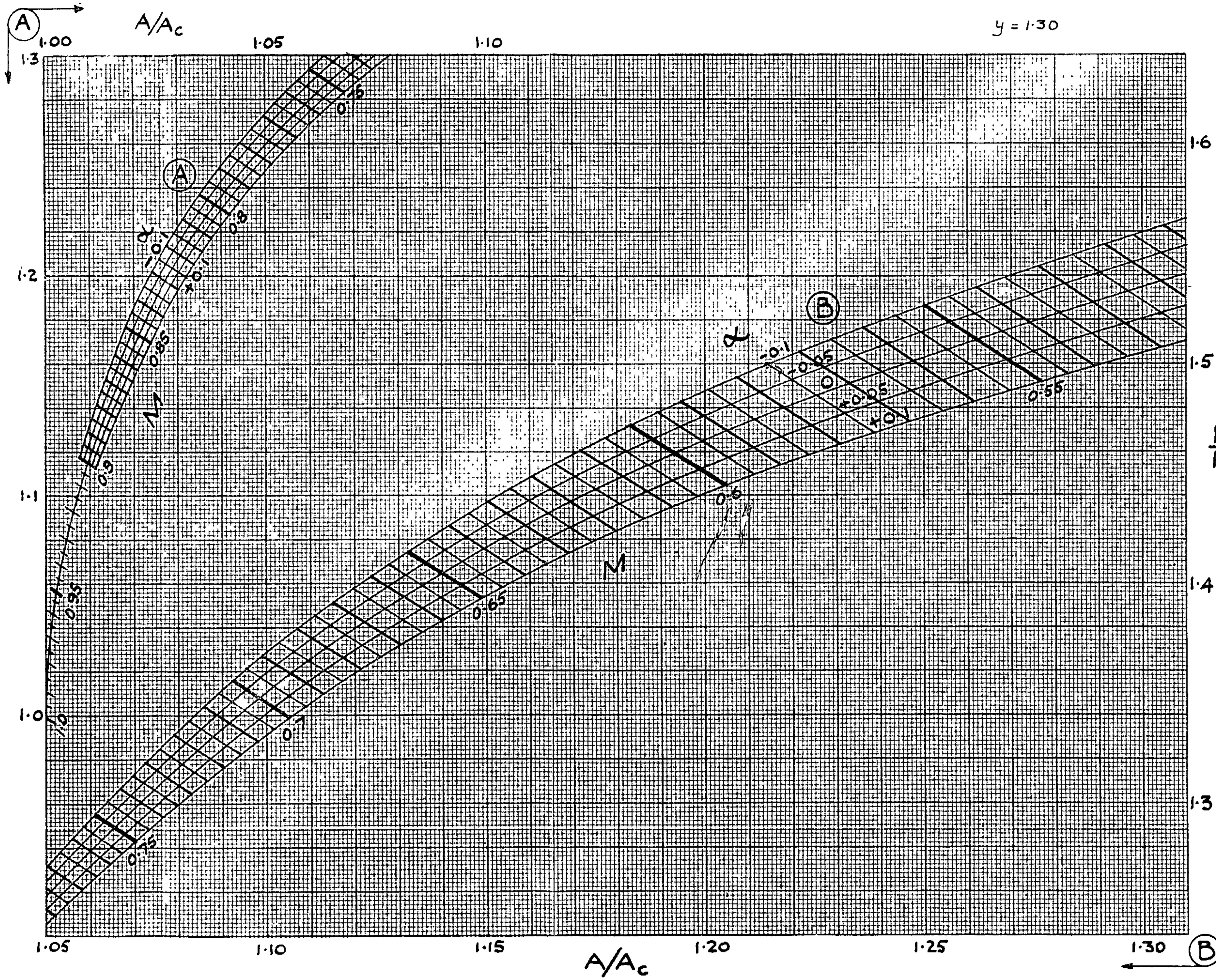
2ND ENLARGEMENT. SEE FORMULAE ON SK 2840.



SUBSONIC FLOW IN CONVERGENT AND DIVERGENT DUCTS WITH FRICTION.

FIG. 8

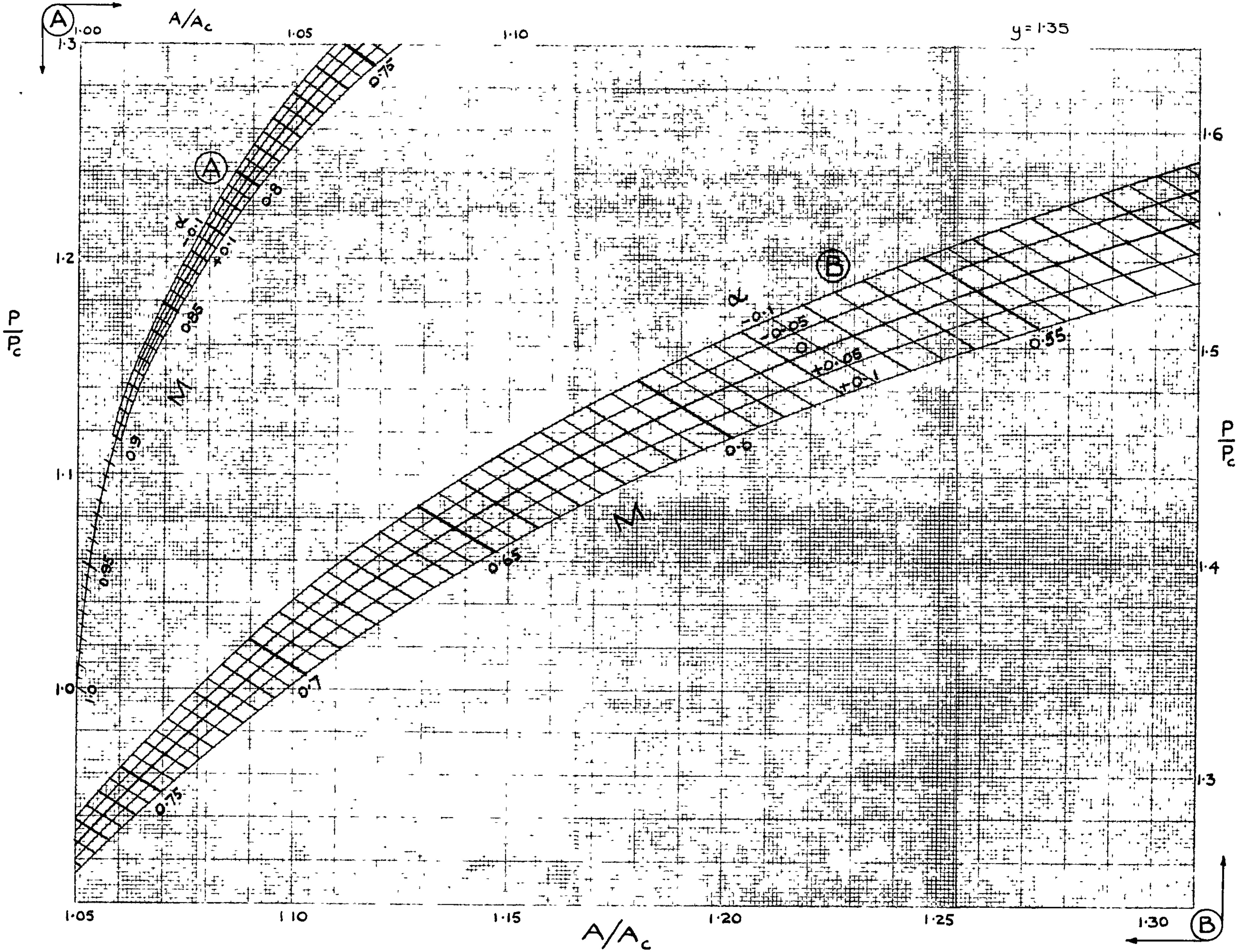
A, P, AND M = AREA, PRESSURE (STATIC), AND MACH NO. AT THE SECTION UNDER CONSIDERATION.
A_c AND P_c = AREA AND PRESSURE (STATIC) AT THE SECTION (HYPOTHETICAL OR ACTUAL) WHERE M = 1.
$\alpha = \gamma f/2 \tan\beta$ WHERE f = FRICTION COEFFICIENT AND $\tan\beta = dA/S\,dL = \delta A/S_m \delta L$ WHERE δA AND δL = CHANGE IN AREA AND LENGTH AND [S_m = MEAN PERIMETER
OR, IN THE SPECIAL CASE OF A DUCT OF CIRCULAR CROSS-SECTION, β = CONE HALF-ANGLE. α IS +VE FOR DIVERGENTS AND −VE FOR CONVERGENTS.



USE EACH SET OF CURVES WITH ITS APPROPRIATE PAIR OF SCALES.

SUBSONIC FLOW IN CONVERGENT AND DIVERGENT DUCTS WITH FRICTION. FIG 9

A, P, AND M = AREA, PRESSURE (STATIC), AND MACH NO. AT THE SECTION UNDER CONSIDERATION.
A_c AND P_c = AREA AND PRESSURE (STATIC) AT THE SECTION (HYPOTHETICAL OR ACTUAL) WHERE M = 1
$\alpha = \gamma f/2 \tan\beta$ WHERE f = FRICTION COEFFICIENT AND $\tan\beta = dA/S\,dL = \delta A/S_m \delta L$ WHERE δA AND δL = CHANGE IN AREA AND LENGTH AND [S_m = MEAN PERIMETER
OR, IN THE SPECIAL CASE OF A DUCT OF CIRCULAR CROSS-SECTION, β = CONE HALF-ANGLE. α IS +VE FOR DIVERGENTS AND −VE FOR CONVERGENTS.

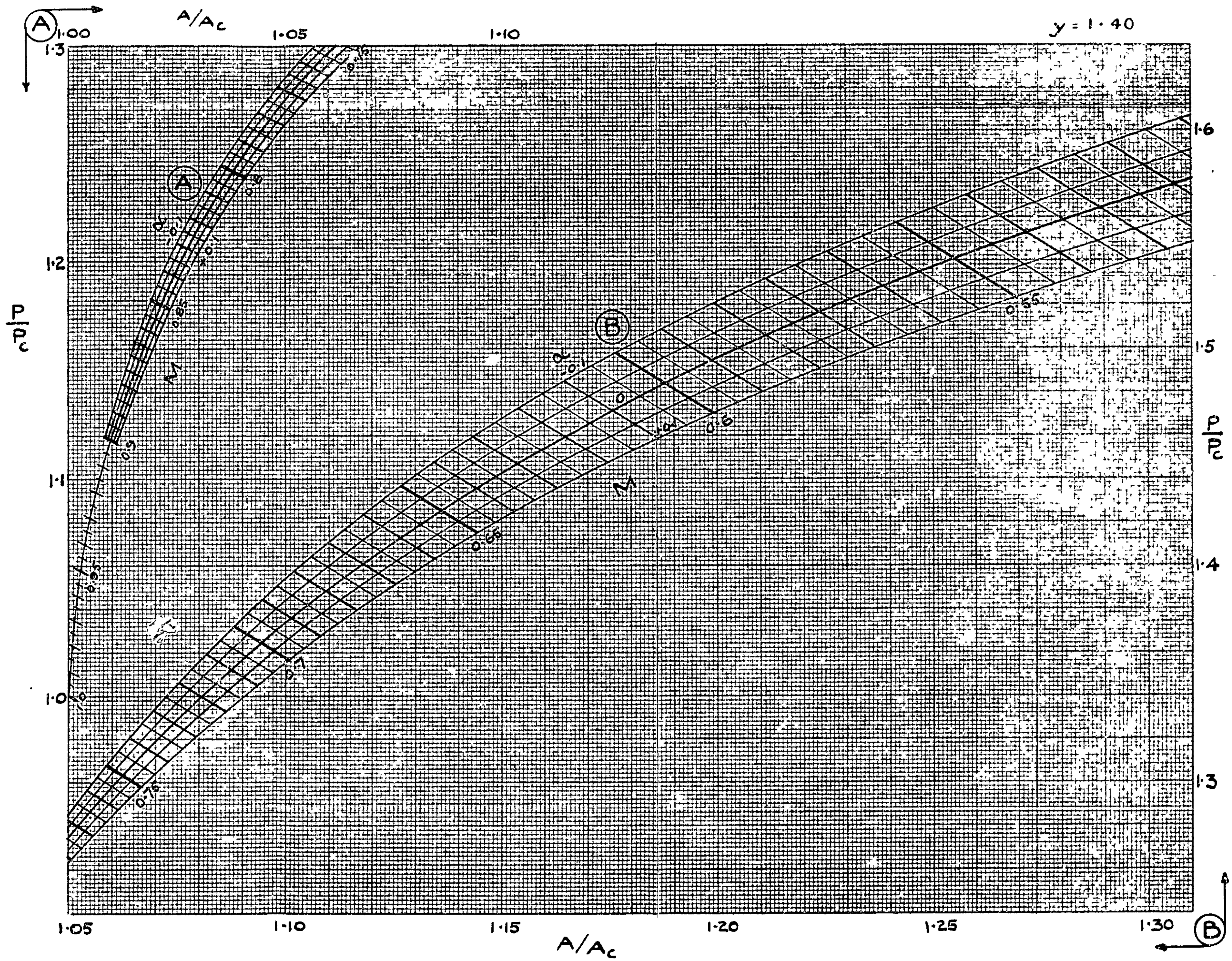


USE EACH SET OF CURVES WITH ITS APPROPRIATE PAIR OF SCALES.

SUBSONIC FLOW IN CONVERGENT AND DIVERGENT DUCTS WITH FRICTION.

FIG. 10

A, P, AND M = AREA, PRESSURE (STATIC), AND MACH NO. AT THE SECTION UNDER CONSIDERATION.
A_c AND P_c = AREA AND PRESSURE (STATIC) AT THE SECTION (HYPOTHETICAL OR ACTUAL) WHERE M = 1. [S_m = MEAN PERIMETER
$\alpha = \gamma f / 2 \tan\beta$ WHERE f = FRICTION COEFFICIENT AND $\tan\beta = dA/S\,dL = \delta A / S_m \delta L$ WHERE δA AND δL = CHANGE IN AREA AND LENGTH AND
OR IN THE SPECIAL CASE OF A DUCT OF CIRCULAR CROSS-SECTION β = CONE HALF-ANGLE α IS +VE FOR DIVERGENTS AND –VE FOR CONVERGENTS.

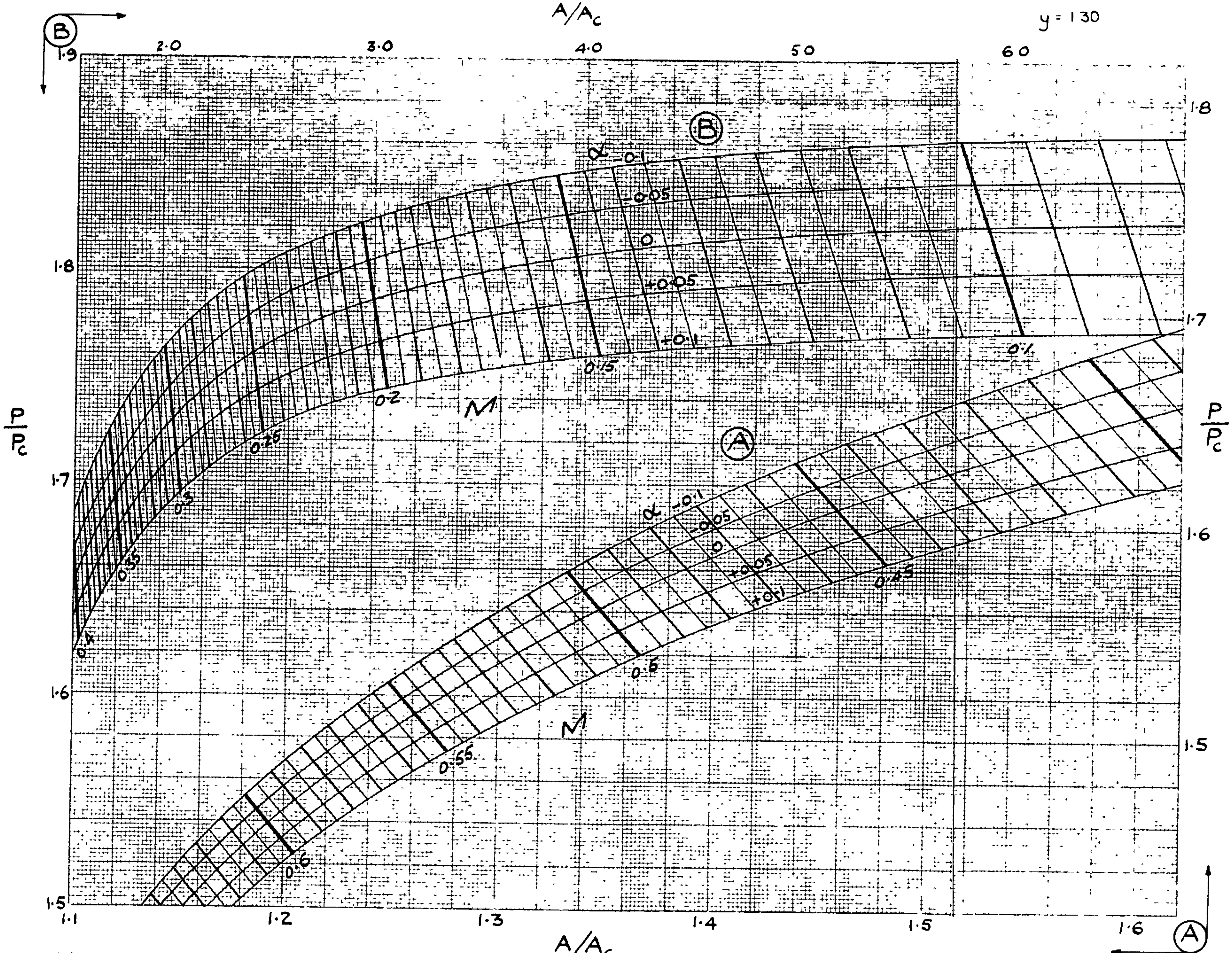


USE EACH SET OF CURVES WITH ITS APPROPRIATE PAIR OF SCALES.

SUBSONIC FLOW IN CONVERGENT AND DIVERGENT DUCTS WITH FRICTION.

FIG. 11.

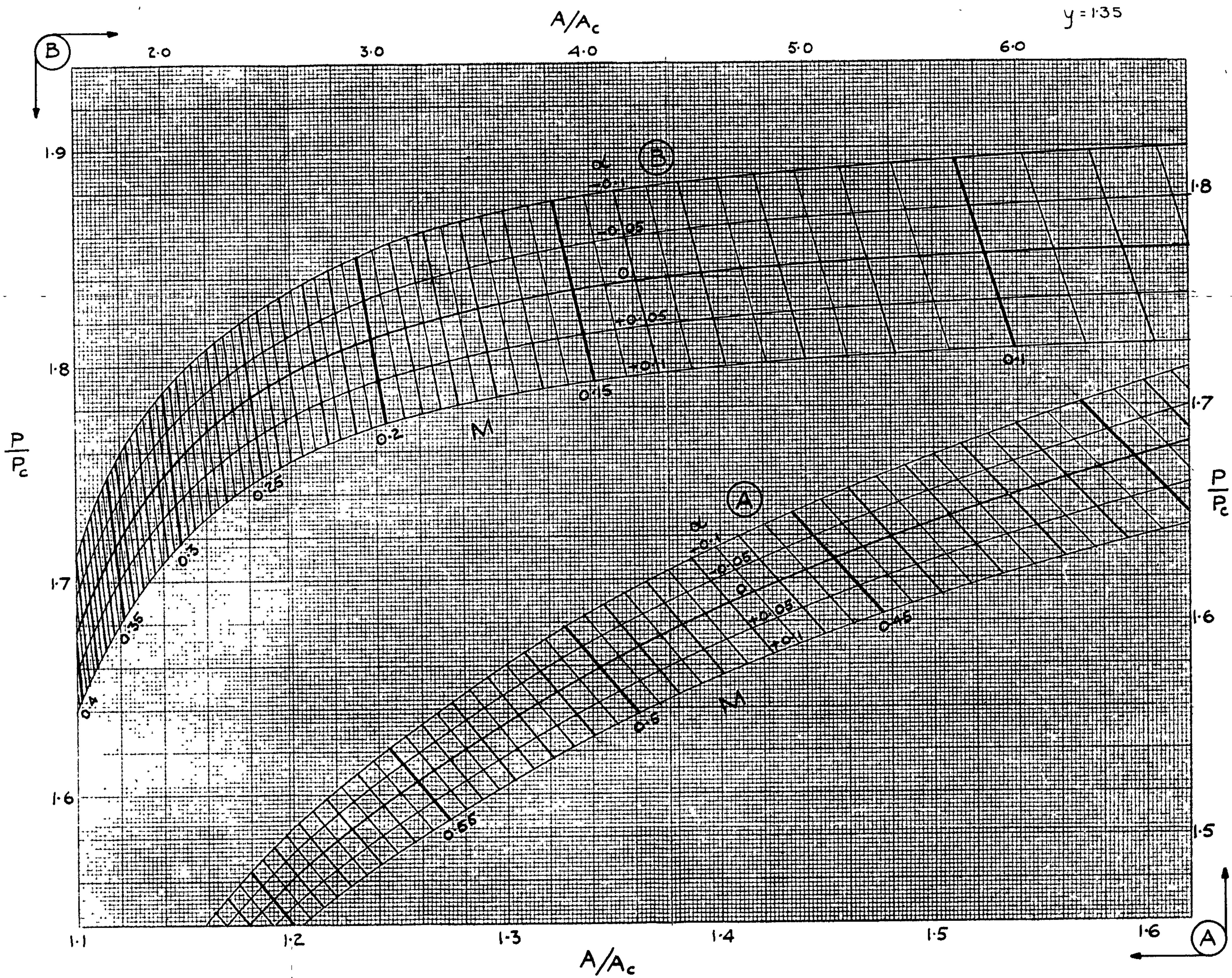
FOR EXPLANATION SEE 1ST. GRAPH. α IS +VE FOR DIVERGENTS AND −VE FOR CONVERGENTS.



USE EACH SET OF CURVES WITH ITS APPROPRIATE PAIR OF SCALES.

SUBSONIC FLOW IN CONVERGENT AND DIVERGENT DUCTS WITH FRICTION. FIG. 12.

FOR EXPLANATION SEE 1ST. GRAPH. α IS +VE FOR DIVERGENTS AND −VE FOR CONVERGENTS

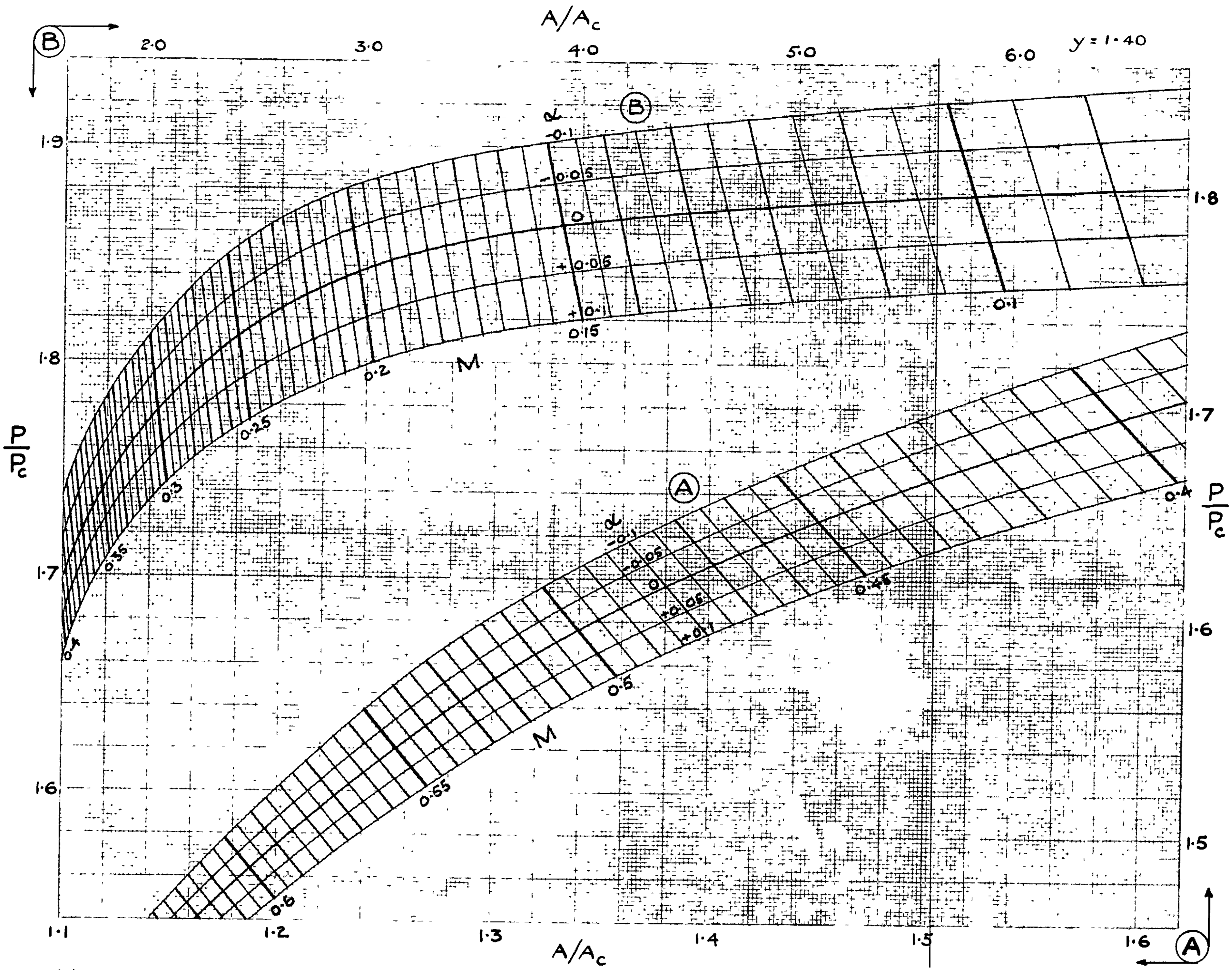


USE EACH SET OF CURVES WITH ITS APPROPRIATE PAIR OF SCALES.

SUBSONIC FLOW IN CONVERGENT AND DIVERGENT DUCTS WITH FRICTION.

FIG. 13

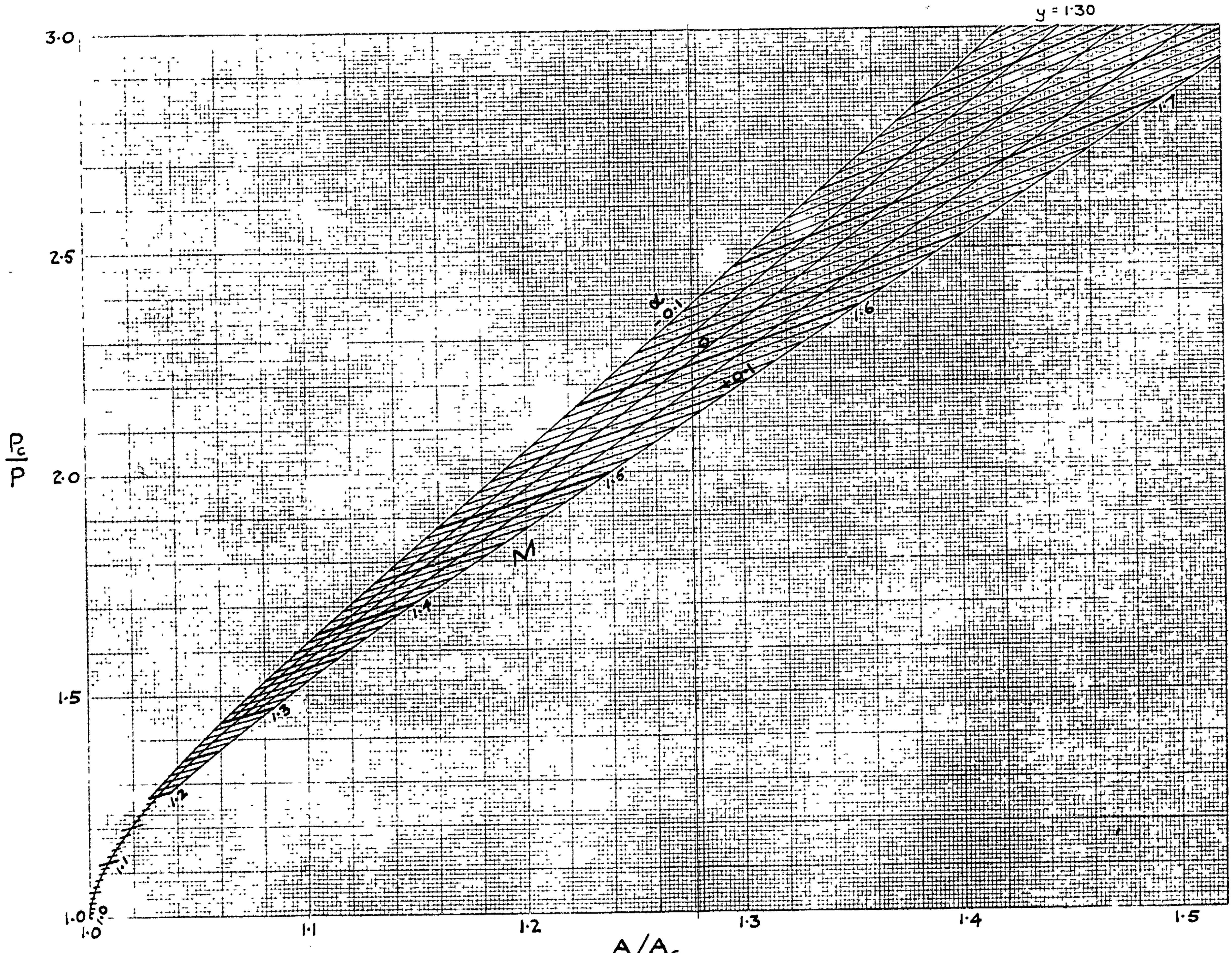
FOR EXPLANATION SEE 1ST GRAPH α IS +VE FOR DIVERGENTS AND −VE FOR CONVERGENTS.



USE EACH SET OF CURVES WITH ITS APPROPRIATE SET OF SCALES.

SUPERSONIC FLOW IN CONVERGENT AND DIVERGENT DUCTS WITH FRICTION. **FIG. 14**

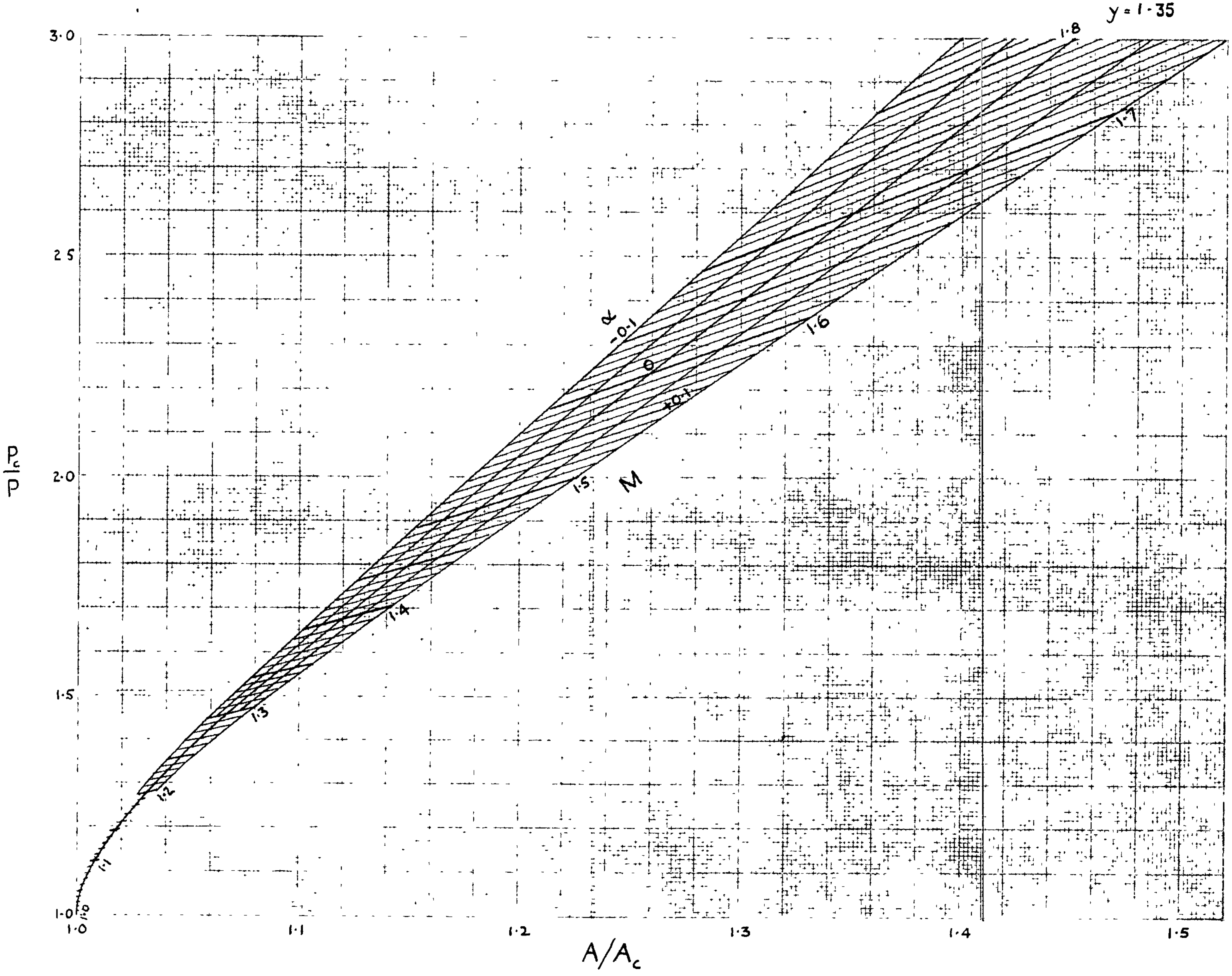
A, P, AND M = AREA, PRESSURE (STATIC), AND MACH NO. AT THE SECTION UNDER CONSIDERATION.
A_c AND P_c = AREA AND PRESSURE (STATIC) AT THE SECTION (HYPOTHETICAL OR ACTUAL) WHERE M = 1. [S_m = MEAN PERIMETER,
$\alpha = \gamma f / 2 \tan\beta$ WHERE f = FRICTION COEFFICIENT AND $\tan\beta = dA/S\,dL = \delta A / S_m \delta L$ WHERE δA AND δL = CHANGE IN AREA AND LENGTH, AND
OR, IN THE SPECIAL CASE OF A DUCT OF CIRCULAR CROSS-SECTION, β = CONE HALF-ANGLE. α IS +VE FOR DIVERGENTS AND −VE FOR CONVERGENTS.



SUPERSONIC FLOW IN CONVERGENT AND DIVERGENT DUCTS WITH FRICTION

FIG. 15

A, P, AND M = AREA, PRESSURE (STATIC), AND MACH NO AT THE SECTION UNDER CONSIDERATION.
A_c AND P_c = AREA AND PRESSURE (STATIC) AT THE SECTION (HYPOTHETICAL OR REAL) WHERE M = 1
$\alpha = \gamma f / 2 \tan\beta$ WHERE f = FRICTION COEFFICIENT AND $\tan\beta = dA/s\,dL = \delta A / s_m \delta L$ WHERE δA AND δL = CHANGE IN AREA AND LENGTH AND S_m = MEAN PERIMETER,
OR IN THE SPECIAL CASE OF A DUCT OF CIRCULAR CROSS-SECTION, β = CONE HALF-ANGLE α IS +VE FOR DIVERGENTS AND −VE FOR CONVERGENTS.



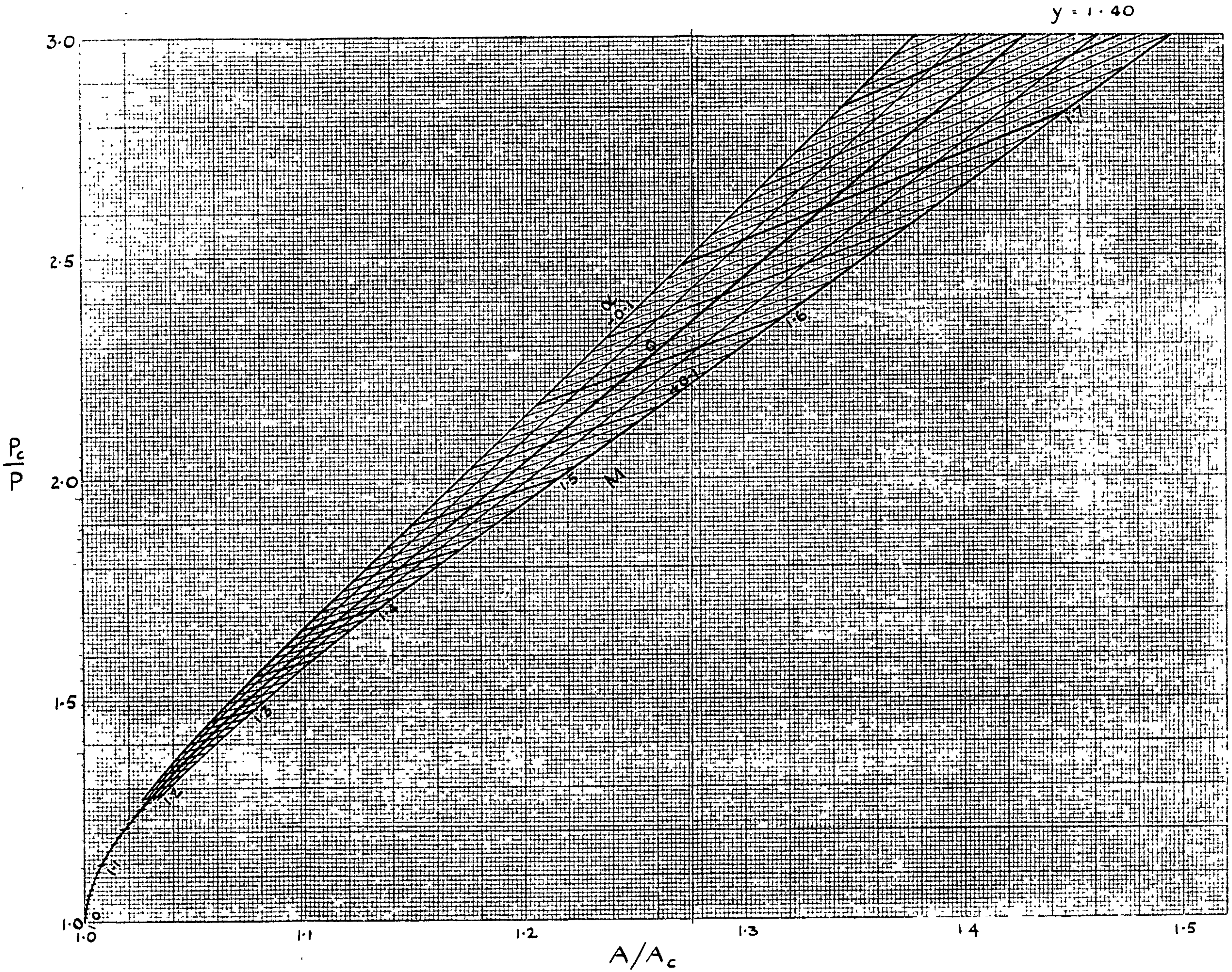
SUPERSONIC FLOW IN CONVERGENT AND DIVERGENT DUCTS WITH FRICTION.

FIG. 16.

A, P, AND M = AREA, PRESSURE (STATIC), AND MACH NO. AT THE SECTION UNDER CONSIDERATION

A_c AND P_c = AREA AND PRESSURE (STATIC) AT THE SECTION (HYPOTHETICAL OR ACTUAL) WHERE M = 1

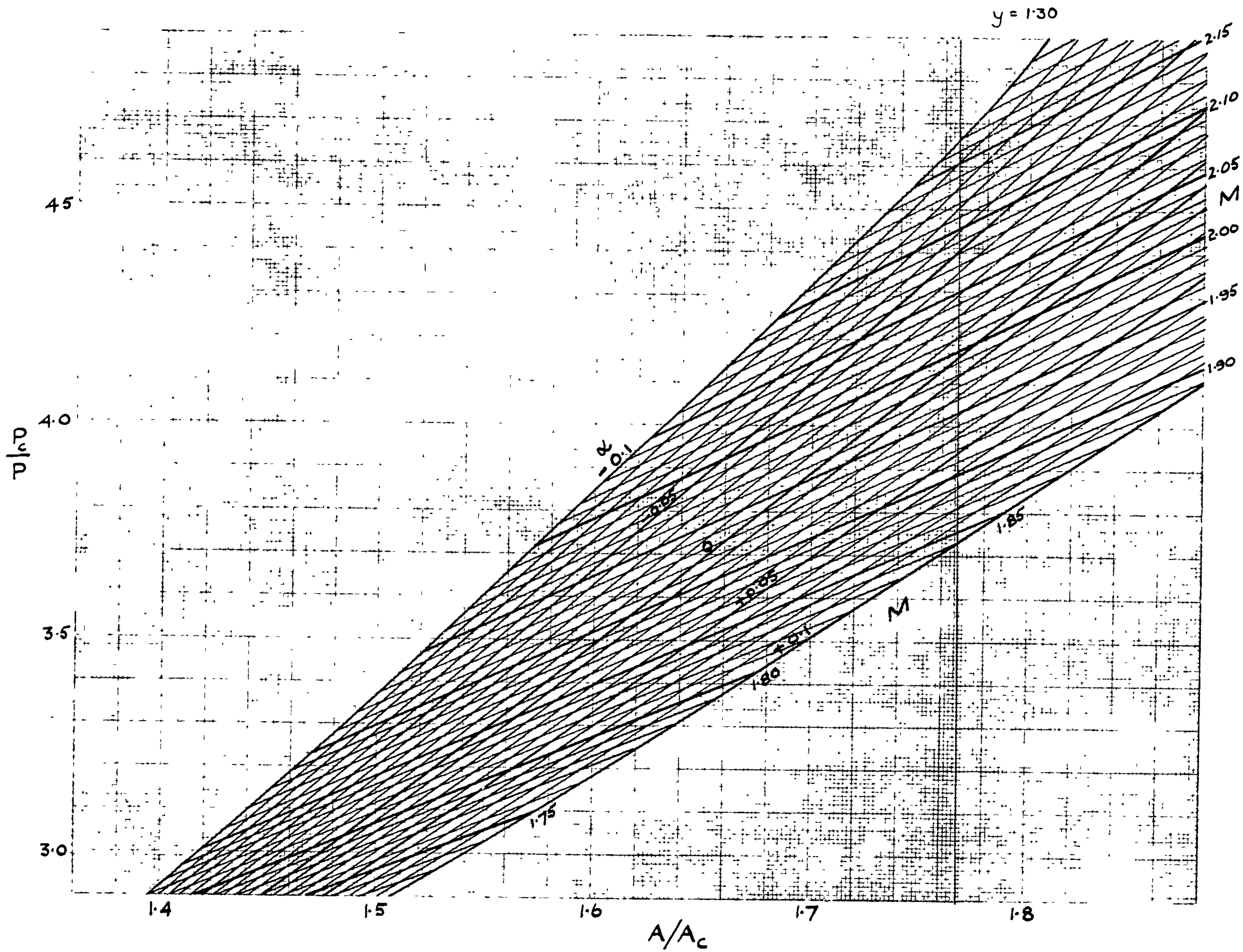
$\alpha = \gamma f / 2 \tan\beta$ WHERE f = FRICTION COEFFICIENT AND $\tan\beta = dA/s\,dL = \delta A/s_m \delta L$ WHERE δA AND δL = CHANGE IN AREA AND LENGTH AND [S_m = MEAN PERIMETER, OR, IN THE SPECIAL CASE OF A DUCT OF CIRCULAR CROSS-SECTION, β = CONE HALF-ANGLE. α IS +VE FOR DIVERGENTS AND -VE FOR CONVERGENTS



SUPERSONIC FLOW IN CONVERGENT AND DIVERGENT DUCTS WITH FRICTION.

FIG. 17.

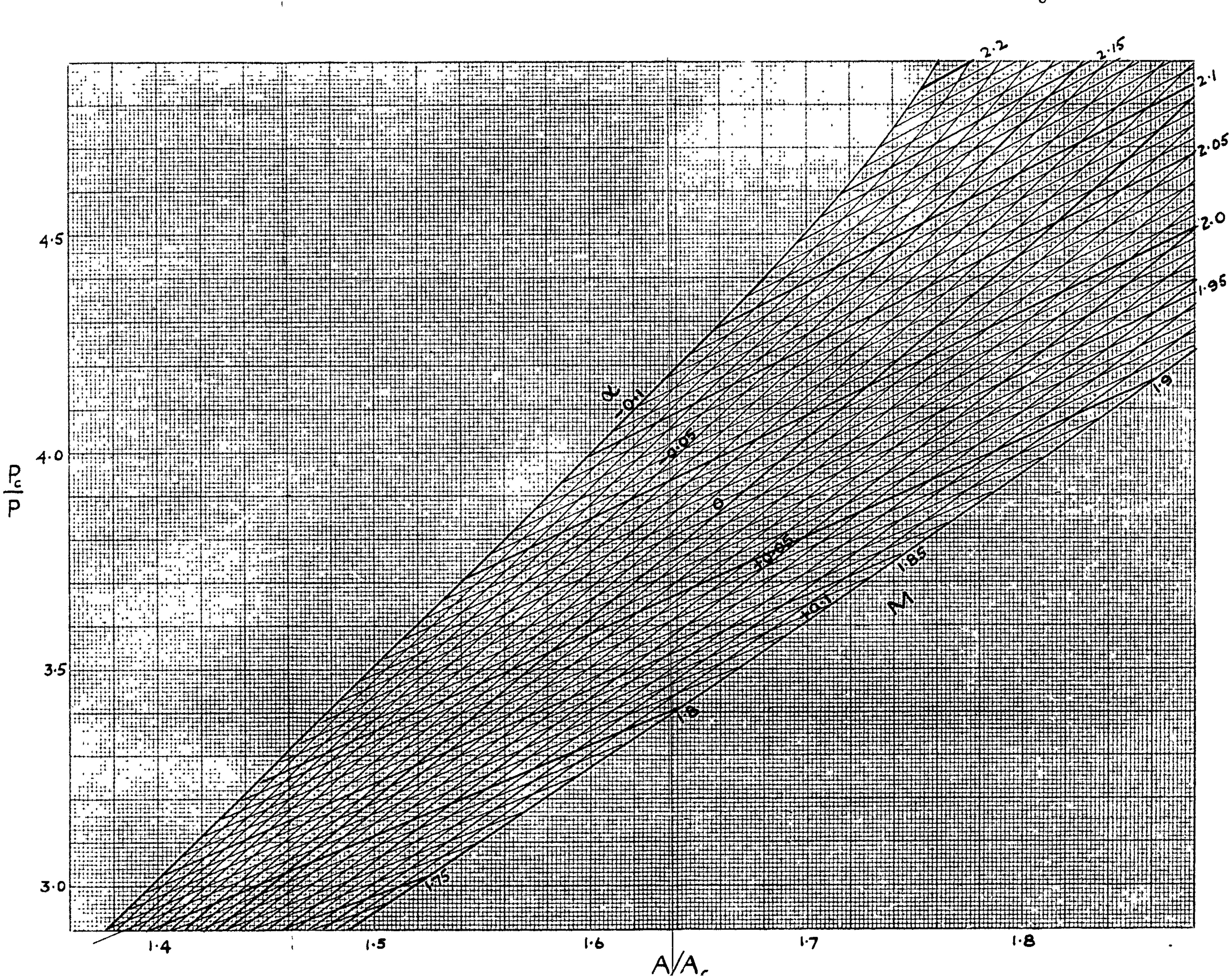
FOR EXPLANATION SEE 1ST. GRAPH. α IS +VE FOR DIVERGENTS AND −VE FOR CONVERGENTS.



SUPERSONIC FLOW IN CONVERGENT AND DIVERGENT DUCTS WITH FRICTION.

FIG. 18

FOR EXPLATION SEE 1ST GRAPH   α IS +VE FOR DIVERGENTS AND −VE FOR CONVERGENTS.



SUPERSONIC FLOW IN CONVERGENT AND DIVERGENT DUCTS WITH FRICTION.

FIG. 19

FOR EXPLANATION SEE 1ST GRAPH. α IS +VE FOR DIVERGENTS AND −VE FOR CONVERGENTS.

$\gamma = 1{\cdot}40$

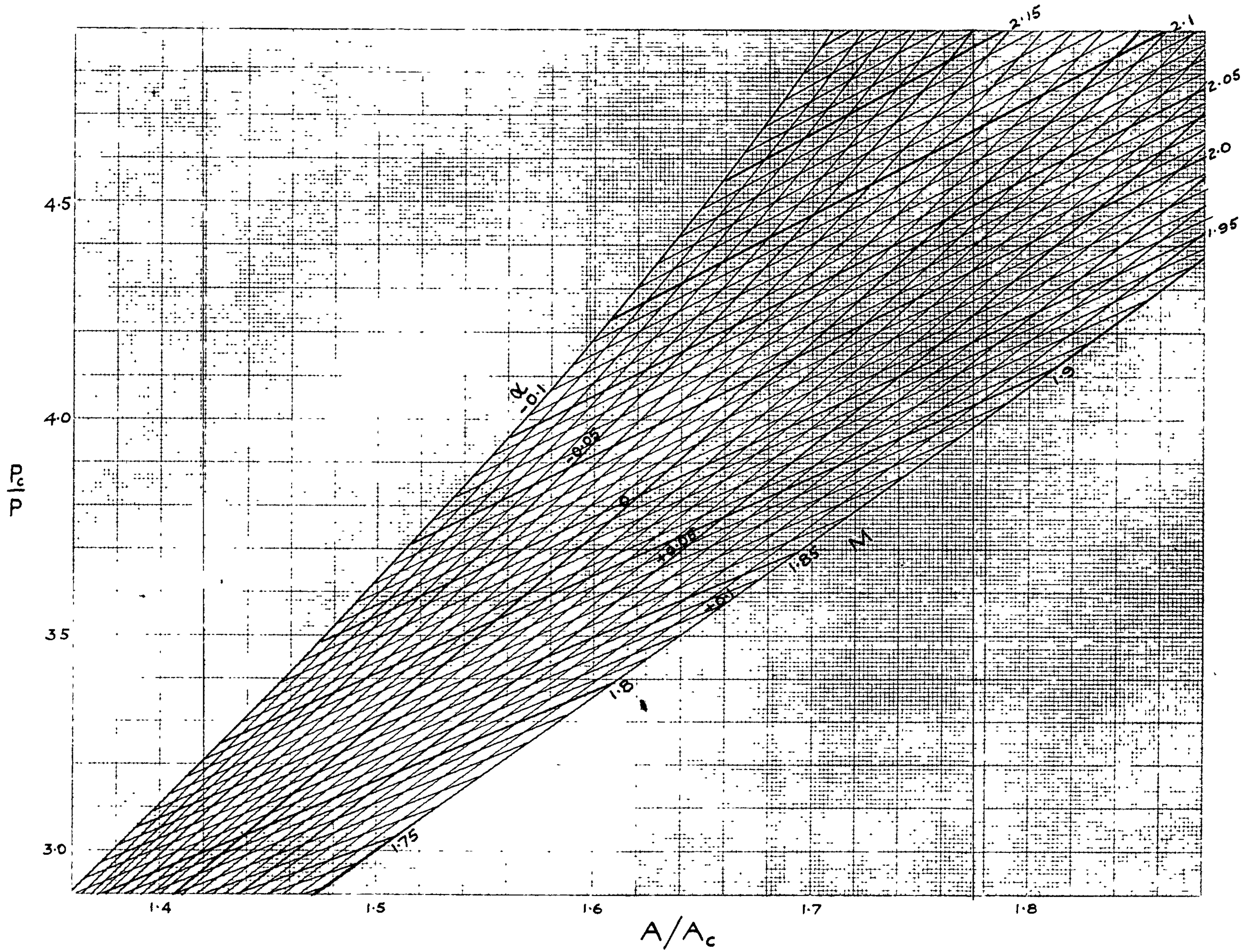


FIG 20.

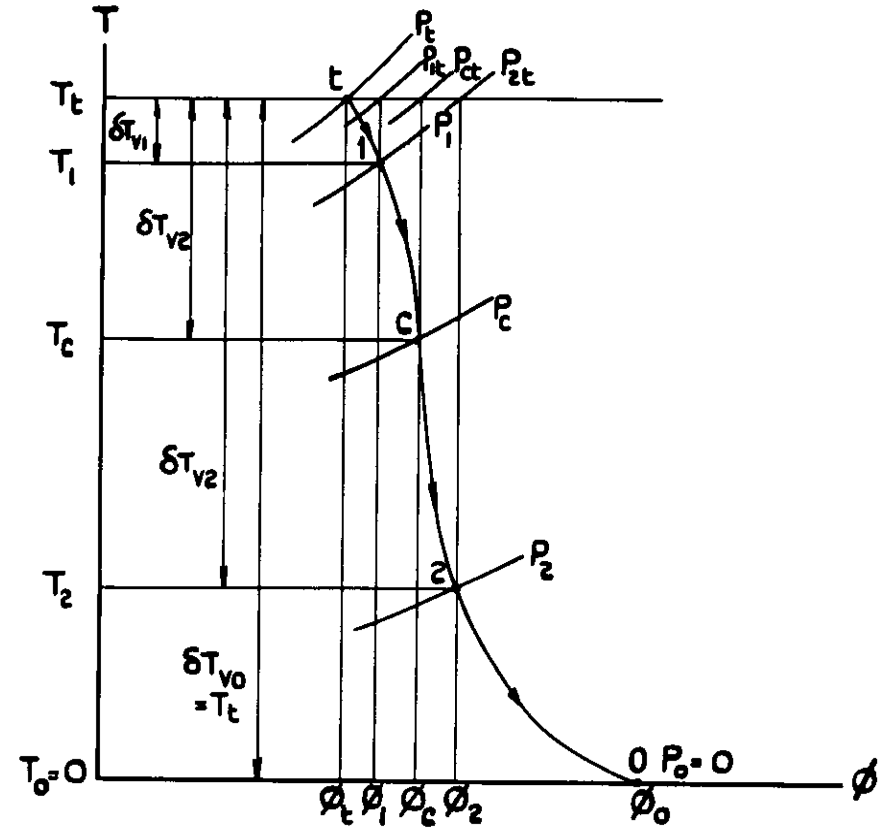


FIG. 20A: $T\phi$ DIAGRAM FOR AN EXPANSION (SHOCKLESS.)

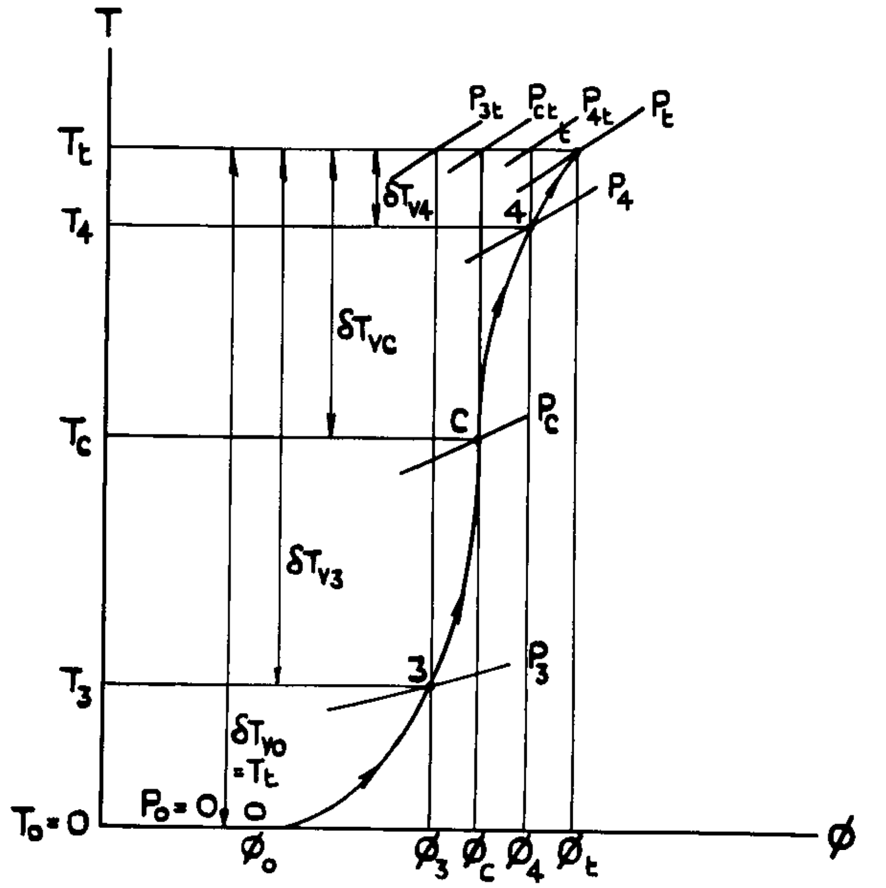


FIG 20B: $T\phi$ DIAGRAM FOR A DIFFUSION (SHOCKLESS.)

$$\delta T_v = \frac{V^2}{2gJC_p}$$

NOT TO SCALE

TEMPERATURE - ENTROPY DIAGRAMS FOR FLOW IN DUCTING WITH FRICTION.

C.P. No. 158
(15,889)
A.R.C. Technical Report

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