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Design of a Right-angled Bend with Constant Velocities at the Walls

By

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Summary

In designing a corner in a two-dimensional duct it is possible, by the insertion of an aerofoil, to maintain the same constant velocity on the outer and inner walls. It is, however, necessary to shape these walls to suit the conditions. The present paper gives a method whereby the aerofoil and walls can be designed. Two examples are given.

List of Symbols

z = $x + iy$, the complex variable in the physical plane.

q, θ = modulus and amplitude of velocity vector.

w = $\phi + i\psi$.

ϕ = stream function.

ψ = velocity potential.

m, n = ϕ-interval from leading edge to trailing edge along the aerofoil, upper and lower surfaces respectively.

Δ = central value of a variable in a diamond, less mean of corner values.

ϵ = side of square in w-plane.

r = distance in physical plane measured perpendicular to a stream line. Hence $\frac{\partial r}{\partial x} = \sin\theta$.

a, b = numerical multipliers.

1.0 Design of a Right-angled Bend with Constant Velocities at the Walls

The rapid changes in velocity at an ordinary right-angled bend in a channel must inevitably produce disturbed conditions. For two-dimensional flow of a perfect fluid, it is possible, by inserting a suitable aerofoil at the corner and re-shaping the walls, to keep the velocities at the walls constant. An infinite number of shapes can be designed; two are given in this paper. The shape of the more promising of the two is shown in Figure 1. Aerofoil and boundaries of the corner are so designed that the following conditions will prevail:-

(a)/

(a) Constant velocity (unity) along the outer and inner walls AED and BCM respectively.

(b) Constant velocity (> 1) along KH.

(c) Constant velocity (< 1) along LJ.

(d) Zero velocity at the singularity G, rising monotonically to the required values at K and L (see Table 5 (i)).

(e) The axis of symmetry, MJHD, will lie at 45° to the straight.

The present paper deals only with the symmetrical case when the leading and trailing halves of the aerofoil are identical.

2.0 The Field of $\log q^{-1}$

When $\log q^{-1}$ in the w-plane is examined, it is seen that the required values and their location are as follows (see Fig. 2.1):-

Location		$\log q^{-1}$
along	AED	Zero
"	BCM	Zero
"	KH	a
"	LJ	b
at singularity G.		$-\infty$

2.1 Addition of Three Fields.- The required $\log q^{-1}$ field indicated in Fig. 2.1 may be obtained by the addtion of multiples of three separate fields. Three fields, A, B and C are indicated on Figs. 2.2, 2.3, and 2.4, respectively; field B is field A inverted. The final field is the sum (a x field A + b x field B + field C). It is obviously permissible to make this addition since each of the fields A, B and C represents a solution of

$$\nabla^2 \log q^{-1} = 0$$

with the same boundaries.

2.2 ϕ-intervals.- It must be noted that, since there is lift on the aerofoil, the number of ϕ-intervals along the top is different from the number along the bottom. It is, however, interesting to note that the Kutta-Joukowsky relation between lift and circulation does not apply since the whole flow is being turned through a definite angle $(\pi/2)$.

2.3 Squaring Fields.- Field A can be "squared" and finally determined; field B immediately follows. Field C, which has a singularity, requires special treatment, described in Appendix I. The $\log q^{-1}$ values used on the nose are given in Table 5 (i). Field C can be determined finally for these chosen boundary conditions.

3.0./

3.0 Solution

In the final solution there are four unknowns, namely a, b, m and n. It is permissible to choose one of these, say n, arbitrarily. The effect of this choice is that the length/width ratio of the lower passage is fixed. For this example n has been chosen as 4. The other three unknowns are not free and must be found from the mathematical conditions of the problem.

3.1 Equations.- Two conditions are obtained by equating the directions of the tangents at D and M to $+\pi/4$. The third condition is that the line MD must lie at $-\pi/4$, or that the traverse AEDMCBA must close.

The first equation, for the outer wall, is

$$\pi/4 \;=\; -0.4443 - b \times 0.1901 - a\left(\frac{m}{2} + 0.1645\right) \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

the numerical values being obtained respectively from field C, field B and field A. In each of the fields A, B and C,

$$\frac{\partial \log q^{-1}}{\partial \psi} = \frac{\partial \theta}{\partial \phi},$$

and $\delta\theta$ can be obtained for each ϕ-interval along the boundary being considered. From field C, (Figure 5.1), by this means, $\Delta\theta$ from end to end is -0.4443. Likewise the angle obtained from field B, (Figure 4.1), is $-b \times 0.1901$. In field A, (Figure 4.1), $\Delta\theta$ from end to end is -2.1645, and, for each internal of $1/4\ \phi$ at the right hand end, $\delta\theta = -0.250$. At $\phi = 2$, the values of $\log q^{-1}$ have become practically independent of ϕ, and the curvature is, with sufficient accuracy, constant. Consequently, after 2 units of ϕ measured from $\phi = 0$, the angle turned through would be -2.0 radians. The correction to be applied to the angle turned in $m/2$ units of ϕ is thus $(-2.1645 + 2.0)$. Consequently the angle turned through by the boundary of field A is $a(-m/2 - 0.1645)$.

Similarly the second equation, for the angle turned through by the inner wall BCM, is

$$+\pi/4 \;=\; +0.4443 + a \times 0.1901 + b\left(\frac{n}{2} + 0.1645\right). \quad \ldots \quad \ldots \quad (2)$$

When the final $\log q^{-1}$ field has been obtained, the distances necessary for the calculation of co-ordinates in the z-plane are easily found. We know q and θ. x, for example, follows from

$$x = \int \cos\theta/q \; d\phi .$$

3.2 Length of Fields.- The fields are taken to be infinitely long to the right and to the left. Obviously after quite a short distance in either direction each field will settle down and all subsequent sections will be sensibly identical. The solution is valid only if the passages are sufficiently long for this to occur. Over the central portion of the bend the assumption is that the flow in each passage is identical with that in a free vortex.

3.3 Numerical Solution.- With $n = 4$, equations (1) and (2) together with the third condition (para. 3.1) give, after a number of trials, final values of the constants as follows:-

$$m = 9.254,$$

$$a = -0.2640, \text{ and}$$

$$b = 0.1808.$$

The line from M to D is found to lie at the required angle $(-\pi/4)$, within the limits of accuracy used. Tables 1 and 2 show the calculations for the shape of the boundary walls AED and BCM.

4.0 Co-ordinates of H and J

The co-ordinates of H and J are calculated using

$$q = \frac{\partial \psi}{\partial r},$$

the values for q being obtained from the final $\log q^{-1}$ field. The distance apart of the stream lines is

$$r = \int_{\psi_1}^{\psi_2} q^{-1} \, d\psi .$$

4.1 Aerofoil Boundaries.- The shape of the aerofoil boundaries HG and JG are worked out, see Tables 3 and 4, using the values of a, b, m, n and the co-ordinates of H and J already calculated.

4.2 Final Shape.- The final shape is shown plotted in Fig. 1. The position of the stagnation point is determined by making a large scale sketch of the closing lines approaching the nose. The integrals along the top and bottom surfaces of the aerofoil do not result in exactly the same position for the stagnation point, but the "closing error" is less than 0.02 of the width of the channel and a mean position is assumed. It is not possible to carry the integrals right up to the singularity, but a consideration of the conditions in this neighbourhood on the lines given in Ref. 2, enables an estimate to be made.

5.0 Conclusion

Once fields A, B and C have been obtained, a variety of solutions can be worked out with different values of n . If it is desired to sharpen the nose of the aerofoil, it will be necessary to "square" another field like C with a different distribution of velocity at the nose. The field A (and B) will apply to any design of corner with the aerofoil on the central stream line.

It will be noted that the final shape is not known until the problem is completed. An earlier solution with a different assumption for the velocity distribution at the nose gave an aerofoil too thick to have any practical application. The results of the calculation, with the assumed velocity distribution, are shown in Fig. 3 and Table 5(ii), respectively.

The/

The problem becomes much more difficult if an attempt be made to incorporate a rounded leading edge and a sharp trailing edge in the aerofoil, as symmetry can no longer be assumed.

It is not at all certain that the design given would produce the desired result in practice. There will be a cross flow in the boundary layer on the end walls, and in any actual channel the velocity distribution in the approaching fluid will not be uniform. The departures from the assumed conditions, together with the growing boundary layer on the walls and aerofoil, and the break away at the trailing edge of the latter, may produce conditions which will prevent the scheme from being effective. Nevertheless some of the difficulties might be overcome by an empirical modification of the design.

Acknowledgement

This problem was suggested to me by my father, Professor A. Thom, and I am indebted to him for guidance throughout the work.

References

No.	Author(s)	Title, etc.
1	A. Thom and Laura Klanfer	Designing a slot for a given wall velocity. O.U.E.L. 49. Current Paper No. 76. December, 1950.
2	A. Thom	Treatment of the stagnation point in arithmetical methods. O.U.E.L. Report No. 53. A.R.C. 14,118. May, 1951. (To be published as R. & M. 2807.)

APPENDIX I/

APPENDIX I

References 1 and 2 are intended to be used for symmetrical fields. In the present problem the field is ultimately asymmetric, so that there will be a different velocity distribution on the top and bottom surfaces of the "nose". In the absence of better information it seems advisable, when carrying out calculations near the stagnation point as described in Ref. 2, to use the mean of the values on the top and bottom surfaces.

Field A

In field A, Sheet 4, Fig. 4.2, at the stagnation point, the value of 0.417 is obtained by taking the mean of $(0.250 + 0.370 + 0.549 + \frac{1}{2} \times 1.000)$. This is in agreement with the convention used in field C, and recommended in Ref. 2.

Field C

The method used in field c is described in References 1 and 2. Referring to Sheet 4, Fig. 5.2, the innermost sheet of the field, the convention used is to put the mean of the four surrounding points for the value at the singularity. Each of the surrounding points is obtained in the same way, except that an appropriate Δ-value is added. For points C and G the Δ-values were obtained from Fig. 5, Ref. 2, where*

$$L_Q = \frac{1}{3} \{(L_B + L_G + L_C) - (L_A + L_H + L_D)\} .$$

Δ-values for points F and E were obtained from Para. 2.4, Ref. 2, and for seven other points near, Δ-values were obtained from Table I, Ref. 2.

It should be noted that the values of Δ are unaffected by the scale or size of the diamonds, and so for sheets 3, 2 and 1, of the field, Δ-values are also applied on lines $\phi = -2\epsilon$, $\psi = 2\epsilon$ and $\psi = 3\epsilon$ as above.

Final Field

In the final log 1/q field obtained by adding the three fields together, the final L_Q value will differ from the value obtained in field C alone. At point O, for example, when L_Q is calculated in this way, the change in Δ is 0.002. This discrepancy is not serious in view of the general difficulty of working near a stagnation point.

/Table 1

*An error in transcription appears in Ref. 2 where, in Fig. 5 and on p.5, the 1/3 was omitted from the formula for L_Q . The correct value is given here.

Table 1

a = -0.2645 m = 9.24
b = 0.1808 Boundary AED. ψ = +1 n = 4.00

ϕ	δθ in Field (-)A	(-)B	(-)C	A x a	B x b	Δθ Final	θ	x = $\int \cos\theta d\phi$	y = $\int \sin\theta d\phi$
$3\frac{1}{4}$	0	0	1	0	0	-1	-1	-.250	-1.000
-3	.1	.3	1	0	0	-1	-2	0.000	-1.000
$2\frac{3}{4}$	.4	.5	2	0	0	-2	-4	.250	-1.001
$2\frac{1}{2}$	1.1	1.1	4	0	0	-4	-8	.500	-1.002
$2\frac{1}{4}$	2.2	2.2	6	1	0	-5	-13	.750	-1.004
-2	4.1	3.8	9	1	-1	-9	-22	1.000	-1.007
$1\frac{3}{4}$	7.2	6.2	13	2	-1	-12	-34	1.250	-1.012
$1\frac{1}{2}$	12.3	9.5	19	3	-2	-18	-52	1.500	-1.021
$1\frac{1}{4}$	20.4	14.0	28	5	-3	-26	-78	1.749	-1.034
-1	33.6	19.5	40	9	-4	-35	-113	1.999	-1.053
$\frac{3}{4}$	54.3	25.1	54	14	-5	-45	-158	2.247	-1.082
$\frac{1}{2}$	84.8	28.8	65	22	-5	-48	-206	2.494	-1.121
$\frac{1}{4}$	124.8	27.8	65	33	-5	-37	-243	2.739	-1.172
0	167.3	21.4	54	44	-4	-14	-257	2.981	-1.232
$\frac{1}{4}$	201.9	13.4	38	54	-2	14	-243	3.223	-1.296
$\frac{1}{2}$	224.2	7.5	22	59	-1	36	-207	3.466	-1.356
$\frac{3}{4}$	237.0	3.9	12	63	-1	50	-157	3.711	-1.407
+1	243.8	2.1	6	64	-0	58	-99	3.957	-1.446
$\frac{1}{4}$	247.0	1.3	3	65	-0	62	-37	4.206	-1.471
$\frac{1}{2}$	248.5	1.0	2	65	"	64	+27	4.456	-1.480
$\frac{3}{4}$	249.5	0.7	1	66	"	65	+92	4.706	-1.473
+2	250	0	0	132	"	132	224	4.955	-1.450
$\frac{1}{2}$	250	0	0	132	"	132	356	5.442	-1.339
+3	"	"	"	132	"	132	488	5.911	-1.165
$\frac{1}{2}$	"	"	"	132	"	132	620	6.353	-0.931
+4	"	"	"	133	"	133	753	6.759	-0.640
$4\frac{1}{2}$	"	"	"	32	"	32	785	7.124	-0.298
m/2 4.62			Radians x 1000					7.209	-0.213

Table 2/

Table 2

a = -0.2645
b = 0.1808

Boundary BCM. $\psi = -1$

m = 9.24
n = 4.00

ϕ	δθ in Field: A	B	C	A x a	B x b	Δθ Final	θ	x = ∫ cos θd	y = ∫ sin θdφ
$3\frac{1}{4}$	0	0	1	0	0	1	1	-.250	1.000
-3	.3	.1	1	0	0	1	2	0.000	1.000
$2\frac{3}{4}$	.5	.4	2	0	0	2	4	.250	1.001
$2\frac{1}{2}$	1.1	1.1	4	0	0	4	8	.500	1.002
$2\frac{1}{4}$	2.2	2.2	6	-1	0	5	13	.750	1.004
-2	3.8	4.1	9	-1	1	9	22	1.000	1.007
$1\frac{3}{4}$	6.2	7.2	13	-2	1	12	34	1.250	1.012
$1\frac{1}{2}$	9.5	12.3	19	-3	2	18	52	1.500	1.021
$1\frac{1}{4}$	14.0	20.4	28	-4	4	28	80	1.750	1.034
-1	19.5	33.6	40	-5	6	41	121	1.999	1.053
$\frac{3}{4}$	25.1	54.3	54	-7	10	57	178	2.247	1.084
$\frac{1}{2}$	28.8	84.8	65	-8	15	72	250	2.493	1.128
$\frac{1}{4}$	27.8	124.8	65	-7	23	81	331	2.735	1.190
0	21.4	167.3	54	-6	30	78	409	2.972	1.271
$\frac{1}{4}$	13.4	201.9	38	-4	36	70	479	3.201	1.370
$\frac{1}{2}$	7.5	224.2	22	-2	41	61	540	3.423	1.486
$\frac{3}{4}$	3.9	237.0	12	-1	43	54	594	3.637	1.614
+1	2.1	243.8	6	-1	44	49	643	3.845	1.754
$\frac{1}{4}$	1.3	247.0	3	0	45	48	691	4.044	1.904
$\frac{1}{2}$	1.0	248.5	2	0	45	47	738	4.237	2.063
$\frac{3}{4}$	.7	249.5	1	0	45	46	784	4.422	2.231
n/2 2	0	250	0	0				4.599	2.408

Radians x 1000

Table 3/

Table 3

a = -0.2640
b = 0.1808

Boundary GH. ψ = 0

m = 9.254
n = 4.000

ϕ	$\delta\theta$ in Field (-) A	B	C	A x a	B x b	$\Delta\theta$ Final	θ	Log 1/q + 0.264	Log $\frac{1}{q}$	$\frac{1}{q}$ cos θ	x = $\int \frac{1}{q}$ cos $\theta d\phi$	$\frac{1}{q}$ sin θ	y = $\int \frac{1}{q}$ sin $\theta d\phi$
0													
1/32							-1253	0.773	0.509		3.417		-0.202
1/16	127	48	166	34	9	209	-1044	0.386	0.122	0.354	3.428	-1.075	-0.236
3/32	96	30	106	25	5	136	-908	0.248	-0.016	0.495	3.443	-0.850	-0.263
1/8	78	21	68	21	4	93	-815	0.180	-0.084	0.565	3.461	-0.725	-0.286
5/32	68	16	47	18	3	68	-747	0.136	-0.128	0.603	3.480	-0.640	-0.306
3/16	61	12	36	16	2	54	-693	0.106	-0.158	0.627	3.500	-0.580	-0.324
7/32	55	10	31	15	2	48	-645	0.085	-0.179	0.644	3.520	-0.534	-0.341
1/4	101	14	49	27	3	79	-566	0.066	-0.198	0.656	3.541	-0.493	-0.356
5/16	91	10	38	24	2	64	-502	0.039	-0.225	0.675	3.583	-0.428	-0.383
3/8	83	7	32	22	1	55	-447	0.017	-0.247	0.685	3.626	-0.376	-0.406
7/16	78	5	26	21	1	48	-399	0.004	-0.260	0.696	3.669	-0.333	-0.427
1/2	147	7	37	39	1	77	-322	0.000	-0.264	0.707	3.713	-0.299	-0.446
5/8	139	6	23	37	1	61	-261	"	"	0.729	3.804	-0.243	-0.476
3/4	266	6	21	70	1	92	-169	"	"	0.742	3.897	-0.198	-0.501
1	257	2	8	68	0	76	-93	"	"	0.758	4.086	-0.129	-0.533
1/4	253	1	5	67	"	72	-21	"	"	0.765	4.277	-0.071	-0.551
1/2	252	1	4	67	"	71	50	"	"	0.768	4.469	-0.016	-0.555
3/4	251	1	1	66	"	67	117	"	"	0.767	4.661	0.038	-0.546
2	250	0	0	66	"	66	183	"	"	0.764	4.852	0.090	-0.524
1/4	"	"	"	66	"	66	249	"	"	0.755	5.041	0.140	-0.489
1/2	"	"	"	66	"	"	315	"	"	0.745	5.227	0.190	-0.442
3/4	"	"	"	66	"	"	381	"	"	0.730	5.409	0.238	-0.383
3	"	"	"	66	"	"	447	"	"	0.713	5.587	0.286	-0.311
1/4	"	"	"	66	"	"	513	"	"	0.693	5.760	0.332	-0.228
1/2	"	"	"	66	"	""	579	"	"	0.669	5.927	0.377	-0.134
3/4	"	"	"	66	"	"	645	"	"	0.640	6.087	0.420	-0.029
4	"	"	"	66	"	"	711	"	"	0.614	6.240	0.461	0.086
1/4	"	"	"	66	"	"	777	"	"	0.583	6.386	0.501	0.211
1/2	32	"	"	8	"	"	785	"	"	0.548	6.523	0.538	0.346
										0.545	6.592	0.545	0.415
4.627				Radians x 1000				"	"				

Table 4

a = −0.2640
b = 0.1808

Boundary GJ $\psi = 0$

m = 9.254
n = 4.000

ϕ	$\delta\theta$ in Field (−)A	B	(−)C	A x a	B x b	$\Delta\theta$ Final	θ	$\text{Log}\,\frac{1}{q}$ −0.181	$\text{Log}\,\frac{1}{q}$	$\frac{1}{q}\cos\theta$	$x = \int \frac{1}{q}\cos\theta\, d\phi$	$\frac{1}{q}\sin\theta$	$y = \int \frac{1}{q}\sin\theta\, d\phi$
0													
1/32							0.995	0.773	0.954		3.514		0.112
1/16	48	127	166	13	23	−130	0.865	0.386	0.567	0.962	3.544	1.480	0.158
3/32	30	96	106	8	17	−81	0.784	0.248	0.429	0.998	3.575	1.170	0.194
1/8	21	78	68	6	14	−48	0.736	0.180	0.361	1.017	3.607	1.015	0.226
5/32	16	68	47	4	12	−31	0.705	0.136	0.317	1.019	3.639	0.922	0.255
3/16	12	61	36	3	11	−22	0.683	0.106	0.287	1.016	3.671	0.863	0.282
7/32	10	55	31	3	10	−18	0.665	0.085	0.266	1.012	3.703	0.824	0.308
1/4	14	101	49	4	18	−27	0.638	0.066	0.247	1.008	3.734	0.790	0.333
5/16	10	91	38	3	16	−19	0.619	0.039	0.220	1.000	3.796	0.742	0.379
3/8	7	83	32	2	15	−15	0.604	0.017	0.198	0.995	3.858	0.707	0.423
7/16	5	78	26	1	14	−11	0.593	0.004	0.185	0.991	3.920	0.682	0.465
1/2	7	147	37	2	27	−8	0.585	0.000	0.181	0.993	3.982	0.670	0.507
5/8	6	139	23	2	25	+4	0.589	"	"	1.000	4.107	0.662	0.590
3/4	6	266	21	2	48	29	0.618	"	"	0.996	4.232	0.667	0.673
1	2	257	8	1	47	40	0.658	"	"	0.976	4.476	0.684	0.844
1/4	1	253	5	0	46	41	0.699	"	"	0.948	4.713	0.733	1.027
1/2	1	252	4	0	46	42	0.741	"	"	0.917	4.942	0.772	1.220
3/4	1	251	1	0	45	44	0.785	"	"	0.884	5.163	0.808	1.422
2								"	"	0.846	5.374	0.846	1.633

Radians x 1000

Table 5/

Table 5

Velocity distribution along $\psi = 0$

(i)		(ii)	
ϕ	$\log \frac{1}{q}$	ϕ	$\log \frac{1}{q}$
0		0	
1/64	1.079	1/16	0.733
1/32	0.733	1/8	0.386
3/64	0.531	3/16	0.245
1/16	0.386	1/4	0.180
3/32	0.248	5/16	0.136
1/8	0.180	3/8	0.106
5/32	0.136	7/16	0.085
3/16	0.106	1/2	0.066
7/32	0.085	5/8	0.035
1/4	0.066	3/4	0.014
5/16	0.039	1	0.000
3/8	0.017		
7/16	0.004		
1/2	0.000		

The values for q in Table 5 (i), from $\phi = 0$ to $\phi = 1/16$ inclusive are calculated from $q = e\phi^{\frac{1}{2}}$; which gives the velocity along the boundaries of a right-angled bend. The values in (ii) from $\phi = 0$ to $\phi = 1/8$ are calculated from

$$q = \frac{e}{\sqrt{2}} \phi^{\frac{1}{2}} .$$

AA

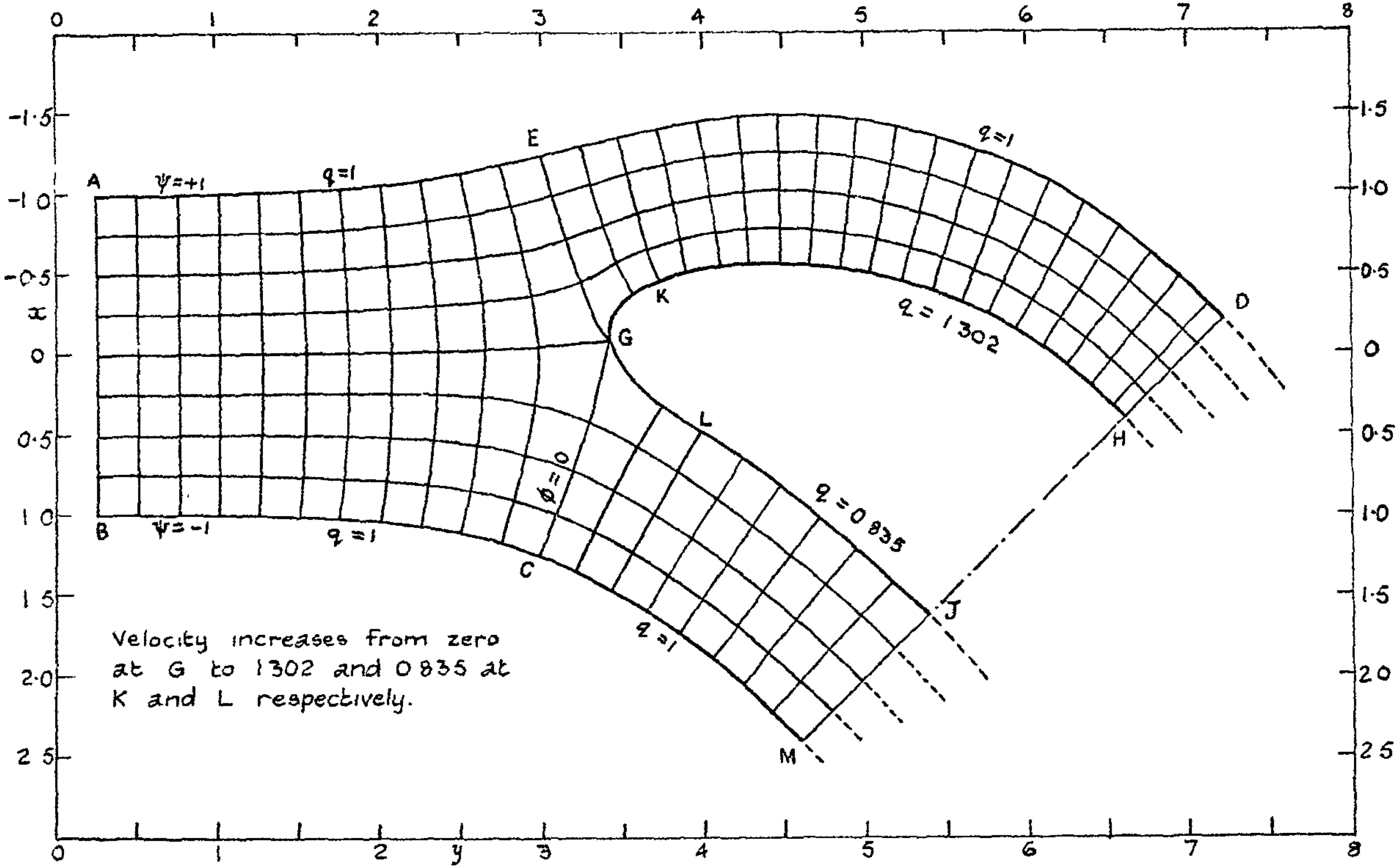
A
B
C
D
E
G
H
J
K
L
M
ψ=+1
ψ=-1
q=1
q=1
q=1
q=1
q = 1302
q = 0 835
φ=0
x
y
Velocity increases from zero at G to 1302 and 0 835 at K and L respectively.

FIG. 1.

FIGS. 2.1.-2.4.

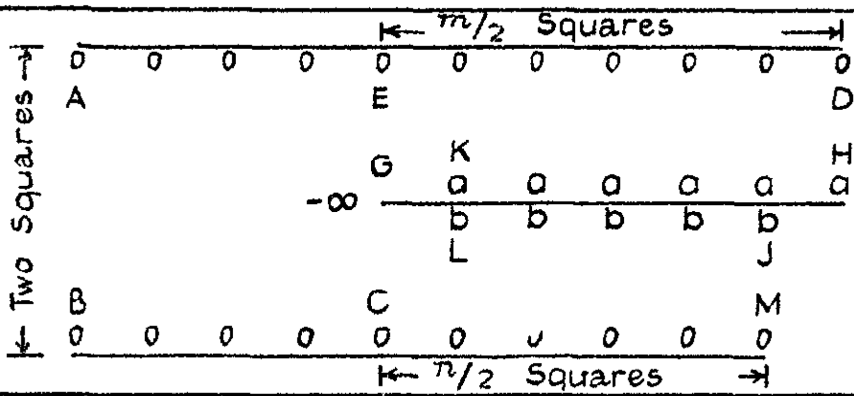

FIG 2.1.

Final Field

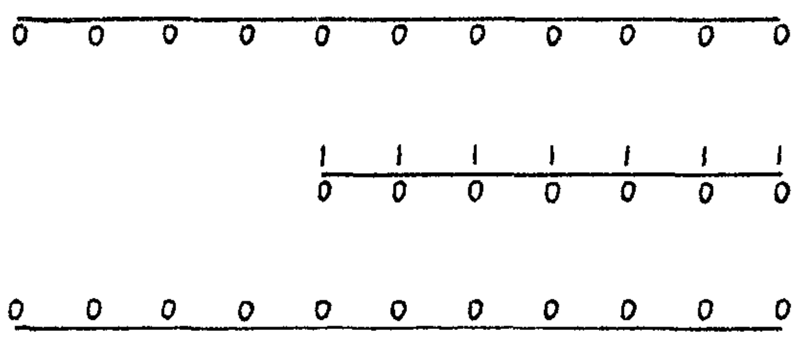

FIG. 2.2

Field A.

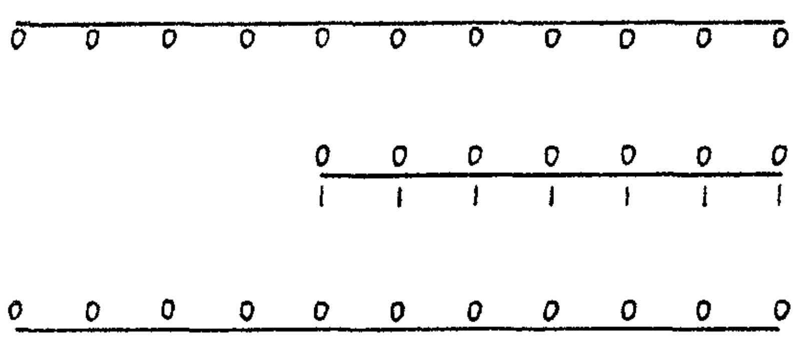

FIG. 2 3

Field B
(Field A inverted)

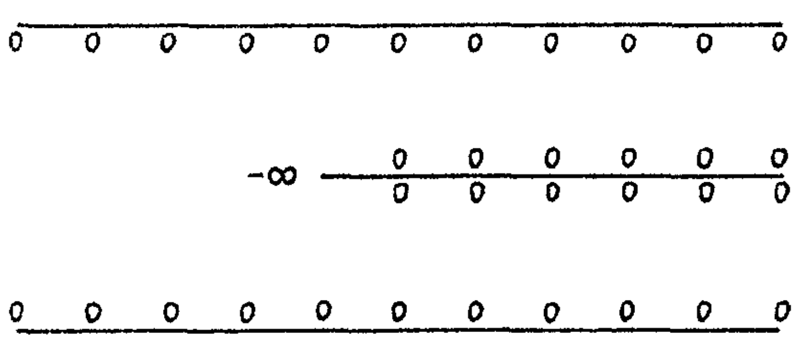

FIG. 2.4

Field C.

Log $\frac{1}{q}$ values on (ϕ, ψ) field

((A×a + B×b + C) gives required final field)

FIG. 3.

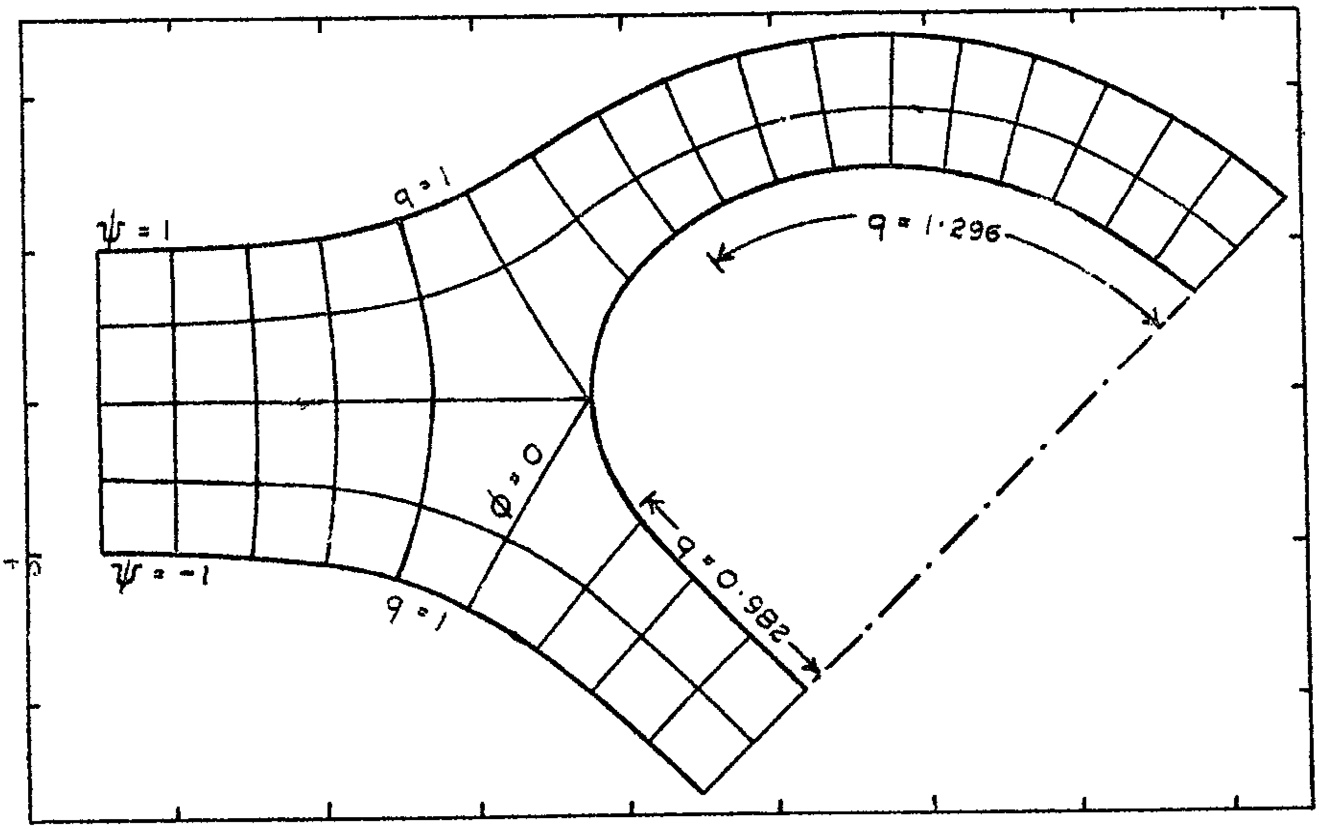

FIG. 4.1.

Sheet № 1. Field A (Major part) $1000 \log \frac{1}{q}$

$(\delta\theta)$: 20·4, 33·6, 54·3, 84·8, 124·8, 167·3, 201·9, 224·2, 237·0, 243·8, 247·0, 248·5, 249·5 $(\Delta\theta = 2164\cdot5)$

ψ \ ϕ =	$-1\frac{1}{4}$	-1	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\frac{3}{4}$	15	25	41	66	102	147	189	217	233	242	246	248	249	250
$\frac{1}{2}$ ($+\psi$)	29	47	76	122	194	296	392	447	475	488	494	497	499	500
$\frac{1}{4}$	37	59	95	153	256	452	637	705	730	741	746	748	749	750
0	38	60	92	141	222			1000 / 0	1000 / 0	1000 / 0	1000 / 0	1000 / 0	1000 / 0	1000 / 0
$-\frac{1}{4}$ ($-\psi$)	32	49	72	100	126	104	39	14	6	3	2	1	1	0
$-\frac{1}{2}$	23	34	47	60	68	57	32	16	8	4	2	1	1	0
$-\frac{3}{4}$	11	17	23	28	31	26	17	10	5	3	2	1	1	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Along $\psi = 0$, between columns $\frac{3}{4}$ to 2 (above the line): 266, 257, 253, 252, 251; (below the line): 6, 2, 1, 1, 1

$(\delta\theta)$: 14·0, 19·5, 25·1, 28·8, 27·8, 21·4, 13·4, 7·5, 3·9, 2·1, 1·3, 1·0, 0·7 $(\Delta\theta = 100\cdot1)$

Figures inside the dotted rectangle come from the next inner Sheet

Sheet No 4   Field A.   1000 Log $\frac{1}{q}$

$\phi =$	$-\frac{1}{8}$		$-\frac{1}{16}$		0		$\frac{1}{16}$		$\frac{1}{8}$		$\frac{3}{16}$		$\frac{1}{4}$	
	353		429		533		646		722		769		798	$\psi = \frac{1}{8}$
		382	427	485	552	625	693	741	777	803	821	834		
	333	368	414	478	565	680	759	808	841	863	877	887	895	$\psi = \frac{1}{16}$
		343	385	446	549	771	856	895	916	929	937	943		
$(\delta\theta)$							127		96		78		68	61	55
	288	310	336	370	417	1000 / 0	1000 / 0	1000 / 0	1000 / 0	1000 / 0	1000 / 0	1000 / 0	1000 / 0	0
$(\delta\theta)$							48	30	21	16	12	10		
		274	282	282	250	110	57	35	24	18	14	11		
	231	237	236	225	192	130	86	59	43	32	26	21	17	$\psi = -\frac{1}{16}$
		207	202	188	164	131	97	73	56	43	35	28		
	184		177		147		98		63		42		29	$\psi = -\frac{1}{8}$

Figures on edge come from next outer Sheet.

FIG 4.2.

Sheet 1 Field C (Major part) $1000 \text{ Log } \frac{1}{q}$

ψ \ ϕ =	$-1\frac{3}{4}$	$-1\frac{1}{2}$	$-1\frac{1}{4}$	-1	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
($\Delta\theta = 444.3$), between columns	12.7	19	28	40	53.5	65	65.5	54.5	38	22	12	6	3	1.5	1	
$\frac{3}{4}$	10	15·	23	33	47	61	71	62 Δ(-2)	47	29	15	7	3	2	1	0
$\psi = \frac{1}{2}$	19	28	42	63	91	127 Δ(+2)	158 Δ(+2)	148 Δ(-8)	101 Δ(+2)	52	25	12	5	2	1	0
$\frac{1}{4}$	25	37	57	85	126	189 Δ(+2)	279 F	305 G	146 E	54	22	10	4	2	1	0
0	27	41	62	93	141	217 Δ(-8)	382 C			0	0	0	0	0	0	0
($\delta\theta$), between columns											21	8	5	4	1	

Figures inside the dotted rectangle come from the next inner Sheet

FIG. 5.1

Sheet 4. Field C. 1000 Log $\frac{1}{q}$

$\phi = \frac{1}{8}$		$\frac{1}{16}$		0		$\frac{1}{16}$		$\frac{1}{8}$		$\frac{3}{16}$		$\frac{1}{4}$	
459		507		494		397		282		197		139	$\psi = \frac{1}{8}$
	544	580	599	583 Δ(-2)	523	439	358	287	230	188	154		
539	600	670 Δ(+2)	730 Δ(+2)	723 Δ(-8)	619 Δ(+2)	475	361	276	216	171	138	113	$\psi = \frac{1}{16}$
	650	761 Δ(+2)	913 F	993 G	745 E	486	335	244	185	144	115		
							166	106	68	47	36	31	($\delta\theta$)
579	672	807 Δ(-8)	1061 C	955	773	386	248	180	136	106	85	66	0

FIG. 5.2

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