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VIBRATIONS OF RAFWIRES.—By R. G. HARRIS, of the ROYAL AIRCRAFT ESTABLISHMENT. Presented by the DIRECTOR OF RESEARCH.

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VIBRATIONS OF RAFWIRES.

By R. G. Harris, of the R.A.E.

Presented by The Director of Research.

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Summary.—This report describes experimental and theoretical investigations of the vibrations of rafwires during "singing." The experimental work was confined to the laboratory. Observations were made on a rafwire in a wind channel at various angles of yaw, at various wind speeds and under various tensions. Subsidiary aerodynamic and other measurements were also made. The principal reason for the experiment was the wish to find an explanation for fractures which have occurred from time to time.

The singing appears to be primarily due to torsional vibrations of the wire in the fundamental mode. A steady amplitude is maintained by energy derived from the air flowing past the wire; this balances the losses of energy due to (*a*) hysteresis, (*b*) the forcing of vibrations in the structure, and (*c*) friction at the attachments. The air moments have been shown to be of the required nature over a certain range of angles of yaw (*i.e.*, N_r is positive). The note depends only upon the dimensions, density, and rigidity of the wire. Amplitudes up to $\pm$ 70° were observed.

Lateral vibrations of very small amplitude (about 0·1 inch or less) were never entirely absent during singing. Lateral resonance with the torsional vibration is possible, through the intermediary of air forces, at a series of tensions. At these tensions the torsional amplitude is much reduced, *e.g.*, by friction at the attachments, but the note is then observed to be loudest.

The stresses in a singing wire have been calculated, and are large, but do not appear to be dangerous when the tension is not excessive. A $2\frac{1}{2}$ hours' fatigue test in the channel, during which the amplitude decayed from 68° to 64°, did not lead to fracture. Fracture appears more probable under excessive tensions, and would then be determined by lateral resonance (when the torsion is a minimum) and is to be expected at the first lateral loop from the end of the wire.

Singing can be prevented by wrapping rubber round the wire at some point, and fixing lead round this.

1. *Introductory*.—It is well known that aeroplane rafwires "sing" under conditions that have not hitherto been determined. Some years ago, the following observations were made on full scale aeroplanes:

(*a*) The pitch of the note of a singing wire was estimated by ear to be about middle C, corresponding to a frequency of about 260 vibrations per second.

(*b*) During singing, a main lift wire was found to be alive to the touch, and singing stopped when the wire was gripped by hand.

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The present series of experiments were undertaken with the aim of studying the problem more conveniently in a wind channel. They consist of :

(A) Main Experiments, in which "singing" is reproduced in the channel under certain conditions (§ 2).

(B) Subsidiary experiments, in which the supply and expenditure of energy is studied with a view to explaining the main "singing" experiments (§ § 12–14).

2. *Singing experiments.*—(Tables 1–5, Figs. 1–5).—A rafwire, ten feet long, of section 0·410″ by 0·094″ (Fig. 2) was supported vertically between two beams so that the central seven foot length was inside a seven foot wind channel (Fig. 1). Standard aeroplane shackles were used (Fig. 3), and provision was made for varying the tension from 15 to 1,015 lbs. by steps of 25 lbs., and the steady wind speed from 61·5 ft/sec. to 132 ft/sec.

The wire sang loudly and spontaneously under certain conditions, as detailed below, the sound being clearly audible, anywhere in the channel, above the noise of the channel. The conditions for clear singing were :

(1) The angle of rigging (yaw) of the wire must be between $10\frac{1}{2}°$ and 28° to the wind,

(2) The tension must exceed a minimum, in the neighbourhood of 300 lbs.

(3) At the lowest angle of rigging ($10\frac{1}{2}°$) the wind speed must exceed a certain minimum, about 90 ft/sec. At other riggings singing commenced at all wind speeds used, down to the lowest (61·5 ft/sec.).

By means of optical arrangements it was observed that the wire executed violent torsional vibrations when singing. The greatest observed amplitude was $\pm 69\frac{1}{2}°$ (at middle of rafwire), at a wind speed of 132 ft/sec. On close examination a lateral vibration was also seen to exist; this was usually of small amplitude, of the order 0″·1. The frequency of the note was observed to be 217 vibrations per second, independent of wind speed, tension, and angle of rigging within the error limits of the experiment, suggesting that the vibration was primarily torsional.

The fundamental frequencies of the wire were calculated, with the following results :

Mode of vibration.	Frequency (vibrations per second).
Lateral (*i.e.*, transverse).	27 for 1,000 lbs. load.
Longitudinal.	421 independent of load.
Torsional.	225 independent of load.

The lateral frequency varies approximately as $\sqrt{\text{load}}$. In these frequencies no correction has been made for the screwed and forked part of the wire, and in the first and third cases it has been assumed that the wire is of elliptical section (*cf.* Fig. 2).

It is concluded that the torsional vibration during singing is in the fundamental mode and has the natural free period approximately. This conclusion is supported also by previous observations of singing (*e.g.*, § 1 (*a*).).

3. *The source of energy.*—(*a*) *A priori indications.*—The torsional vibration involves a continued supply of rotational energy from the air, to balance the dissipation of energy due to :

(1) Torsional hysteresis and forced vibrations in the structure,
(2) Friction at the attachments.

The air moments about the longitudinal axis of the wire are defined, for convenience, as yawing moments. Consider a wire rigged vertically in a wind channel. The yawing moments which can act on the wire at a constant wind speed are :

(1) Yawing moment due to the position of the centre of pressure (in front of the axis),
(2) Yawing moment due to the velocity of rotation ω about the axis of the wire.

The first, the N_v moment, being a function of the orientation of the elements of the wire, acts in the same fashion as the rigidity of the material ; it directly affects the period of vibration (to about 1 per cent., § 15), but cannot supply energy for the maintenance of a *pure* torsional vibration (*i.e.*, in the absence of a lateral vibration in a different phase). The second moment, the N_r moment, has the effect of a positive or negative damping according to its sign, and is therefore indicated as a probable source of energy.

(*b*) *Existence of the N_r supply of energy.*—The magnitude of the N_r couple was measured on a 15 scale model of a rafwire in two different ways (§ 13). It is approximately equivalent to an impulsive couple occurring every time the wind is at 15° to the wire (§ 13, Fig. 7). During one complete torsional cycle of an element of the wire, four impulses occur, provided the amplitude at the element is sufficient to cover both of the $\pm$ 15° positions.

(*c*) *Sufficiency of the energy supply.*—The energy demanded of each air impulse is less than 1 per cent. of the total energy of the torsional vibration. The balance between the supply and the dissipation of energy is worked out below (§ 16). Fig. 11 shows two of the main parts in the balance of energy, viz., the supply due to N_r without the "speed ratio" correction (§ 13) and the hysteresis loss.

(*d*) *The torsional vibration does not resonate with a free external source.*—It has been suggested by the N.P.L. that the singing of rafwires is a vibration forced by the periodic throwing off of eddies in the same manner as a short round wire is caused to vibrate in a wind.* The free eddy frequency is, however, very much higher than the frequency of the note produced under the ordinary conditions of flight. For example, the present ten foot experimental rafwire might resonate with the eddies at about 20 m.p.h.; at 100 m.p.h. a resonating wire would require to be about two feet long, and the higher the speed the shorter the wire.

The torsional vibration in singing has approximately the fundamental free period of the wire, and cannot therefore be regarded as a forced oscillation under the action of a free periodic flow. It is an oscillation whose frequency is determined (to a first approximation) by the dimensions, density, and elastic properties of the wire (§ 15), maintained by energy derived from the periodic flow produced by the oscillation, *e.g.*, as in the motion of a clock, the timing of the supplies of energy is necessarily determined by the motion of the oscillating system.

4. *The effect of tension.*—The part played by the tension is mainly twofold :—

(1) The tension determines the frictional couples between the pins and the wiring plates. These couples enable the wire to vibrate torsionally with fixed ends until a "critical" amplitude is exceeded. Beyond this amplitude the pins begin to slip against these couples, energy is thereby absorbed, and the torsion is less than it would be if the ends were clamped. The critical amplitude is proportional to the tension, and is indicated by the dotted straight line in Fig. 4. This line assumes a coefficient of friction of 0·15.

(2) The tension determines the lateral resonance, which has the double effect of determining the loudness of the note and the point of the wire at which fracture is most likely to occur. The lateral vibrations are produced by energy derived from the air, due to Y_v; the amount of this energy depends on the phase difference between the lateral and torsional vibrations, and this depends on the tension. When the lateral vibration is large enough to cause the pins to lose contact at each side of the wiring plates alternately, frictional damping occurs as described in (1), and the torsion is reduced. On the other hand, when the torsion is large, the lateral vibration has more energy absorbed, due to the more stable values of Y_v beyond 25° yaw (Fig. 10). Thus a large torsional vibration tends to decrease the lateral, and vice versa. It

* (" On the Sound Produced by Circular Wires in an Air Current." *Philosophical Magazine,* Vol. 42, p. 173. July, 1921. By E. F. Relf, A.R.C. Sc.)

is calculated that lateral resonance would occur at the tensions stated in Table 9; in these cases the phase difference is 90°.

In the singing experiments it was observed that the torsional amplitude varied with the tension according to a curve which consists of a series of waves about a mean straight line passing through the origin (Fig. 4). The mean straight line represents the critical amplitude mentioned above. The minima correspond to the maximum lateral oscillations, in agreement with the above theory. The sound is loudest at the torsional minima, which is again consistent with the present theory, because the sound is produced mainly by the lateral vibrations.

The absence of *fundamental* lateral vibrations during singing is to be explained as follows: Assume for the sake of clearness that there is no phase difference. The air supplies energy (due to Y_v, Fig. 10) to a pre-existing lateral vibration between $10\frac{1}{2}°$ and 25° yaw, but absorbs energy between 0° and $10\frac{1}{2}°$ and again between 25° and larger angles, so that the lateral vibration tends to be damped out more and more as the torsional amplitude becomes greater and greater (beyond 25°). To take an experimental example, when the wire was rigged at $10\frac{1}{2}°$, the fundamental lateral vibration was violent at the lower wind speeds and singing was not audible at any tension. When the wind speed was raised to 90 ft/sec. the torsional amplitude at a suitable high tension became large enough for audible singing, and at the same time the lateral vibration became so small that it was difficult to detect; when the wind speed was now lowered the fundamental lateral vibration did not occur again, owing to the magnitude of the torsion, until the wind speed was about 70 ft/sec.

5. *The commencement of singing.*—Before singing begins, the wire must have an angular velocity or its aerodynamical equivalent. This condition is satisfied if a lateral oscillation already exists, because the acceleration in yaw is aerodynamically equivalent to an angular velocity. In agreement with this, the range of angles of rigging at which the experimental wire sang ($10\frac{1}{2}°$ to 28°) corresponds to the range of angles of yaw within which small lateral vibrations tend to arise owing to the lateral force on the wire (Y_v) being unstable (Fig. 10). Outside this range a lateral oscillation arising, say, from a change in the direction or velocity of the wind, tends to be damped out by the wind forces due to the stable Y_v, and before damping it has not enough energy to produce equivalent angles of yaw of 15°. Consequently singing does not arise at angles of rigging outside the above range.

6. *Singing on aeroplanes.*—On aeroplanes in flight singing could begin from the following causes:—

(1) The necessary initial angular velocity can arise through
 (*a*) independent lateral vibrations,
 (*b*) twist due to gusts.

(2) The necessary angle of yaw could be produced by
   (*a*) side slipping,
   (*b*) flying at an angle of incidence greatly different from that for which the wires are rigged, *e.g.*, at very low or very high speeds, and some special manœuvres,
   (*c*) rotation of propeller slipstream,
   (*d*) bad rigging,
   (*e*) lateral vibrations (partly).

(3) The high tension necessary for the maintenance of singing is most likely to be found in lift wires (" flying " wires) during manœuvres which involve high accelerations, such as flattening out at high speeds, spins, loops, &c.

Except in a particularly bad example of (2) (*d*), singing would always be preceded by strong lateral vibrations of the wire, in the fundamental mode, since the wire would have to pass through $10\frac{1}{2}°$ before singing commences.

To an observer on the ground the constancy of the pitch of the note is generally obscured by the Doppler effect. When the aeroplane approaches and recedes from the observer at *ground* speeds of 140 m.p.h., a note of frequency 217 would be heard as notes of frequencies 257 and 177 respectively, an interval of about half an octave. The singing note is therefore generally heard as a continuously varying one, especially when the aeroplane is manœuvring in the neighbourhood of the observer.

7. *Fracture.*—Two cases are to be distinguished from each other, according as the dangerous stresses arise mainly from torsion or tension. These cases are referred to as " torsional fatigue " and " bending under tension," for the sake of distinction, although each involves both torsion and extension.

(A) *Torsional fatigue.*—It is estimated, on the basis of the experiments, that the torsional amplitude would be of the order 80° for the experimental wire at a wind speed of 140 m.p.h., at a suitable tension (about 965 lbs.). The shear stress corresponding to this amplitude is as follows :—

Tension.	Maximum shearing stress.	Shearing planes.
Zero	17·5 tons/sq. in.	90° and 0°
965 lbs.	18·8 tons/sq. in.	79° and − 11°

If the wire were much thinner compared with its length, it is estimated (§ 20) that the shear stress would be somewhat increased at the same wind speed, up to a limit of about one ton per square inch more than the above figures.

If the fatigue limit is about 21 tons/sq. in. these calculations indicate that torsional fatigue fracture is improbable. For example, if the calculated stress is underestimated about 10 per cent. for any reason, fatigue fracture would be possible for thin wires, but only after a comparatively large number of vibrations. This would involve :—

(*a*) prolonged singing,

(*b*) fairly constant high tension, in order to maintain the large amplitude, because according to Fig. 4, the amplitude is sensitive to the tension.

The satisfaction of these two conditions is improbable under ordinary flight conditions; for example, the necessary high tension is available only in exceptional cases, such as diving or flattening out from loops, spins, &c., *i.e.*, manœuvres in which the conditions are not likely to be constant or prolonged.

This type of fracture might be expected to occur anywhere between the end of the wire and the first lateral loop, according to the amount of lateral "resonance"; it would start always at the end of the minor axis of the cross section.

(B) *Bending under tension.*—If lateral resonance occurs in r loops when the mean tensile stress in the wire is T, this gives rise to an additional alternating stress p. The maximum value of p occurs at the loops; this is estimated to be as follows for the experimental wire *when the lateral vibration has an amplitude of* 0″·1.

Direction of vibration.	r	Mean tensile stress (T) (tons/sq. in.).	p (tons/sq. in.)	Total tensile stress (tons/sq. in.).
Parallel to the minor axis.	8	13·4	2·8	16·2
	9	10·4	3·6	14·0
	10	8·3	4·4	12·7
	11	6·7	5·3	12·0
	12	5·4	6·3	11·7
	13	4·4	7·4	11·8
	14	3·5	8·6	12·1
Parallel to the major axis.	7	13·1	9·4	22·5
	8	7·5	12·2	19·7

If the torsional shear stress is q, the maximum value of the resultant shear stress $\sqrt{\frac{1}{4}(p + \mathrm{T})^2 + q^2}$ occurs very close to the first loop from the end of the wire, which is therefore the point at which the wire would be expected to break if the tension became excessive during singing. In contrast with a torsional fatigue fracture, this type could occur at the end of either the minor or the major axis according to the direction of vibration.

The torsion has here little effect beyond being the origin of the lateral vibration; the latter is unimportant compared with the tension, except in so far as it determines the position of the point of fracture. Fracture could be due to a momentarily excessive tension or to tensile fatigue.

(C) *Flaws.*—If a flaw exists in a wire, singing could be the cause of fracture at the flaw. "Torsion" could cause it anywhere except in the middle of the wire, while "Bending under Tension" could cause it anywhere except at the nodes. Tension alone could of course cause it anywhere.

8. *Broken wires.*—Figs. 12 and 13 show photographs of rafwires broken on aeroplanes and submitted to the R.A.E. for metallurgical examination. These show the following features:—

(1) The fractures occur a short distance from the end of the wire. On a ten foot wire the position of the loops are:—

Overtone.	Distance of loop from pin.
7	8″·6
8	7″·5
9	6″·7
10	6″·0
11	5″·5
12	5″·0

(2) The appearance of the fractures is consistent with them starting at points on the minor axis (Fig. 12) and on the major axis (Fig. 13).

(3) The angles at which the fractures commence (viewed from the front) are visible in Fig. 12 only. The shearing planes at the end of the minor axis appear to be at angles of about 40° and 50° to the plane of the cross-section.

Each of these features is consistent with fracture of the "Bending under Tension" type. The fracture at the major axis (Fig. 13) is inconsistent with the "torsional fatigue" type, in the absence of flaws, since this is the point of the perimeter at which the torsional shear stress is a minimum. The shearing planes of Fig. 12 are consistent with "torsional fatigue" fracture if the number of vibrations is relatively great; as noted above, this would involve improbable conditions of flight.

9. *Conclusions as to fracture.*—The general conclusion is that the singing of a rafwire is not dangerous when the tension is moderate, and when no flaws exist. When the tension is excessive, lateral "resonance" provides an additional alternating tensile stress, varying along the wire, which determines the point of fracture. Fracture due to torsional fatigue could occur under improbable conditions of flight (prolonged constant high tension).

APPENDIX.

I.—*Minor Singing Experiments.*

10. *Prevention of singing.*—Experiments indicate that singing can be prevented by attaching to the wire a small body capable of absorbing energy, *e.g.*,

(*a*) Rubber tape, 3 feet long by ½ inch broad, was wrapped round 3 inches of the wire, about a foot from one end, and a piece of lead sheet was clinched on this. A torsional vibration of 1° occurred at 127 ft/sec. and there was no audible singing.

(*b*) At wind speeds below 100 ft/sec. singing was easily prevented by attaching any loose fitting device to the wire, such as a small piece of aluminium or lead sheet.

It remains to be noted that duplicate wires, joined to each other, would probably not sing.

11. *Experiment on fatigue.*—A rough fatigue experiment was performed, in which a wire was allowed to sing for 2½ hours under a load of 965 lbs. at 127 ft/sec. The total number of vibrations was 1,950,000. Although this is a small number for a fatigue experiment, it is to be remembered that a wire usually sings in flight for only a short time, and then rests for a longer time.

In this experiment the amplitude of vibration decayed gradually from 68° to 64° (Table 12). At the end of the experiment the temperature of the lower wiring plate and the screw thread were 9° C. and 4° C. respectively, above that of the surrounding air. This can be wholly accounted for by :—

(*a*) end friction at the pins,

(*b*) hysteresis.

II.—*Subsidiary Experiments.*

12. *Experiments on hysteresis and minor end effects.*—Experiments were made in which the decay of a torsional oscillation was observed when the wind was off (period adjusted to be about one second). The rafwire was used,

(*a*) with the same end fittings as in the singing experiments, and

(*b*) with clamps attached to the outer roof and the floor. In cases (*a*) and (*b*) the effective length of the wire was 119 inches and 101 inches respectively.

The resulting loss of energy, due to hysteresis and the forcing of vibrations in the structure, is shown in Fig. 6. The two curves of Fig. 6 corresponding to cases (*a*) and (*b*) are not strictly comparable, since the twist at an element of the wire is not the same function of the position of the element in both cases. The curves, however, show that the effect of using shackles at the ends of the wire is to increase the dissipation of energy, presumably by

(i) forcing vibrations in the supports,

(ii) minor frictional effects, such as the friction between the screw thread and the forked end.

The hysteresis loss will appear as heat mainly toward the ends of the wire, since the potential energy is greatest there.

13. *Experiments on* N_r.—The following experiments were performed on N_r :—

(A) The usual small oscillation method was used by Messrs. Alford and Younger to determine N_r for a 15 scale model of elliptical section. The results are shown in Fig. 7.

(B) In order to check the applicability of these results to finite oscillations, an oil damping method was used (Fig. 9 and Table 8). The energy supplied by N_r balanced the energy absorbed by the oil, and a finite constant amplitude of rotation was maintained. The results of experiments A and B agree within 10 per cent.

(C) The foregoing experiment (B) was repeated on an accurate 15 scale model of the rafwire, the cross-section of which was similar to that shown in Fig. 2. It appeared from this that the flattening of the front and rear edges of the rafwire reduces the N_r supply of energy by 16 per cent., *i.e.*, it reduces the singing (Tables 6, 7).

(D) The impossibility of perpetual motion proves that the value of $\frac{N_r}{U}$ (usually assumed constant) must begin to decrease when the angular velocity becomes great enough. To distinguish this from the usual scale effect, it will be called a "speed ratio" effect, since it presumably depends on the angles of yaw produced at various elements of the wire, *i.e.*, on the ratio $\frac{\omega a}{U}$ where ω is the angular velocity, U the wind speed, and a the major diameter of the wire. As a simple demonstration of this, a short piece of stout rafwire was supported between centres at its ends ; a small velocity of rotation was given to it by hand, and this increased spontaneously in the wind until it settled down to an apparently steady value. This proves that the torsional amplitude of a singing rafwire must be less than that which would be predicted directly from the value of N_r given by experiment C.

When the value of ωN_r is integrated over an oscillation between 0° and 30° yaw, it is found that the energy absorbed by N_r below 7° yaw is almost exactly equal to that supplied beyond 20° yaw, with the result that not much error is to be expected if it is assumed, for convenience, that all the N_r energy is supplied in the neighbourhood of the peak of Fig. 7 ; *i.e.*, N_r has practically the effect of an impulsive couple.

14. *Experiments on lateral forces and moments.*—(A) The value of the yawing moment due to N_v was obtained from the experiments described in § 13 (B) and (C), by noting the change of zero when the wind was on. The results are given in Table 11 and Fig. 8.

(B) The value of the cross wind force was measured by supporting a two-foot length of the experimental rafwire on a spindle. The results are shown in Table 10 and Fig. 10. Lateral vibrations were observed to occur at angles of yaw between 10°·5 and an upper limit which varied with the wind speed roughly as follows :—

Wind speed (ft /sec.).	Upper limit of yaw.
40	17°
50	17°
60	19°
70	20°
75	25°

On the present theory these vibrations (due to unstable values of Y_v) are the most usual starting point of singing (*cf.* § 5).

III.—*Analysis of Singing.*

15. *Pitch of the note.*—In calculating the pitch of the note for the purpose of the present analysis it is assumed :—

(*a*) that the wire is an elliptic cylinder from pin to pin.

(*b*) that N_v is zero.

This leads to the expression—

$$n = \frac{1}{l\left(F + \frac{1}{F}\right)} \sqrt{\frac{Ng}{\rho}}$$

where n = fundamental torsional frequency,
l = length of wire,
F = fineness ratio of wire,
N = modulus of rigidity of the material,
ρ = volume density of the material.

It will be noted that the frequency depends only on :

(1) The length of the wire.

(2) The shape of the cross-section (but not the dimension of the section),

(3) The material (as defined by N and ρ).

The effect of N_v was calculated in a typical case (rigging 15°, amplitude ± 60°) with the following results :—

Wind speed (ft/sec.).	Frequency.
60	225
100	225
160	224
220	223

The unstabilising movement of the C.P. between ± 12° yaw (Fig. 8) is thus equivalent to a negligibly small weakening of the steel.

16. *Torsional amplitude.*—The torsional amplitude is determined by the balance between the supply and the dissipation of the rotational energy. In calculating the supply it is assumed, on the basis of § 13 (Fig. 7) that every element of the wire is acted upon by an impulsive N_r couple every time it passes through ± 15° yaw, and that N_r is elsewhere zero. The phases at which the supplies are received lead to an integration by summation of an infinite series ; the condition that this series is convergent is also the condition that N_r energy is supplied to an element. The results of the calculation are shown for a typical case (rigging 15°) in Fig. 11.

In calculating the loss of energy due to hysteresis, &c., the upper curve of Fig. 6 is first expressed as a linear function of $\frac{d\theta}{dz}$, where θ is the angle of twist at an element whose distance from the end of the wire is z. This function is taken to apply to each element, and is integrated over the whole wire. The result is shown in Fig. 11. The loss of energy due to the pins slipping against friction is equal to the frictional couple multiplied by the angle (β) through which slipping occurs. This angle is too small to observe, and the loss is treated by calculating the values of β that are necessary to account for the difference between the energy supply and the hysteresis loss.

In these calculations the "speed ratio" effect on N_r is not estimated, hence the calculated values of β will be too great. A minor effect, also ignored, is the frequency correction to hysteresis.

The following table shows the angles (β) through which the pins must rotate in order to preserve the balance of energy. The case treated is that of Fig. 4; the minima in that diagram are not treated for the present (*cf.* § 17).

Load (lbs.).	Amplitude (degrees).	Energy.			β angle through which the pin slips (degrees).
		N_r supply (inch-lbs.).	Hysteresis loss (inch-lbs.).	Remainder (inch-lbs.).	
315	12	0·036	0·003	0·033	0·21
390	16·5	0·050	0·007	0·043	0·22
465	19·5	0·059	0·011	0·048	0·21
540	25·5	0·077	0·022	0·055	0·21
615	28·3	0·086	0·029	0·057	0·19
715	28	0·085	0·028	0·057	0·16
790	33·5	0·125	0·045	0·080	0·21
865	44·5	0·205	0·099	0·106	0·25
915	49	0·24	0·130	0·110	0·25
1015	34·5	0·135	0·050	0·085	0·17

This shows that slipping through ¼° is sufficient to account for the balance of energy; this is well within the range of movement of the pins (± 1°).

As the wind speed increases, the N_r energy is increased in direct proportion, and the amplitude ought to increase in an asymptotic manner, determined by the intersection of curves such as those of Fig. 11. The amplitude does increase with the wind speed (Fig. 5), but not to such an extent as Fig. 11 would suggest. The reason is partly that more energy is lost by slipping against friction (not plotted in Fig. 11) and also, as suggested above, that the N_r supply of energy is over-estimated, owing to the "speed ratio" effect.

In the case of the most energetic torsional vibration (amplitude 69°·5) the energy balance worked out as follows :—

Supply of energy (N_r)	=	0·630 inch lbs. per swing.
Loss (hysteresis, &c.)	=	0·353 ,, ,, ,,
Remainder	=	0·277 ,, ,, ,,

The latter, if due entirely to slipping of the pins, would involve an angle of slipping of 0°·60, enough to heat the wiring plate and forked end through 5° C. per minute. § 11 shows that this is an excessive estimate, again pointing to a large "speed ratio" effect on N_r. In the two cases treated above the upper limit to the "speed ratio" effect is 46 per cent. and 44 per cent. respectively.

As a further check on the foregoing theory, a rough test was performed on a clamped wire. The torsion was thereby increased about 30 per cent. by getting rid of the energy loss due to slipping at the pins. This test gave an upper limit of about 50 per cent. for the "speed ratio" effect on N_r.

17. *Variation of torsional amplitude with tension.*—Table 9 gives the loads necessary to produce the various lateral overtones of the observed frequency 217 for a wire of constant section throughout its length; *i.e.*, it ignores the facts that the ends are screwed and forked, &c. Let it be assumed that the various corrections (*e.g.*, for the screwed ends) amount to a reduction of the value of ml^2n^2 by 5·2 per cent., where l and n are defined in § 15, and m is the mass of the wire per unit length. The following results are obtained for vibrations parallel to the minor axis:

Number of tone.	Calculated load (lbs.).	Observed load at minimum amplitude (lbs.).
8	840	840
9	654	665
10	518	490
11	415	415
12	335	330
13	271	270

It is therefore consistent with the observations to assume that these particular torsional minima occur when there is lateral resonance parallel to the minor axis.

There are other minima at 940 and 565 lbs., an interval of 375 lbs. Table 9 suggests that lateral resonance parallel to the major axis should occur at 870 and 497 lbs. an interval of 373 lbs. In this case the stiffness is about 20 times more important than it is for vibrations parallel to the minor axis, hence the loads are more sensitive to small errors in the assumptions. The minima at 940 and 565 lbs. are therefore regarded as being due to lateral vibrations parallel to the major axis.

During singing lateral vibrations were never quite absent; they were usually too small to measure conveniently, but they became very violent at the best marked minimum (840 lbs.), so violent that the torsional vibration ceased after a few seconds, and then the lateral resonance ceased also. Singing was loudest at 940 and 665 lbs.; this is consistent with the above theory.

IV.—*Fracture.*

18. *Combined tension and shear. Range of stress.*—If p and q are the tensile and the maximum torsion shear stress respectively at a definite point, the greatest range of shear stress is from $+q$ to $-q$, and occurs in transverse and longitudinal planes, whereas the maximum shear stress is $\sqrt{\frac{1}{4}p^2 + q^2}$ and occurs in planes somewhat inclined to these. For example, in the case where the maximum torsion occurred in the channel experiments (torsion 69°·5, load 940 lbs.) the shear stress is :—

$$6{\cdot}7 \sin 2\theta + 15{\cdot}2 \cos 2\theta \sin pt \text{ (tons/sq. in.)}$$

where θ is the inclination of the plane considered. The maximum range was therefore $+15{\cdot}2$ to $-15{\cdot}2$ in planes $\theta = 0°$ and $90°$, whereas the maximum stress occurs in planes $\theta = -12°$ and $78°$, the stress varying from 16·6 to $-11{\cdot}2$, a range $8\frac{1}{2}$ per cent. smaller than the preceding one.

19. *Shearing planes at lateral resonance.*—When the forced lateral vibration is a maximum, it will be assumed for convenience in this calculation that the torsional amplitude is one half of the critical amplitude (*cf.* § 4 and Fig. 4). This leads to the following upper limits to the values of $\frac{q}{p}$:

·075 F. for points at the end of the minor axis,
·075 for points at the end of the major axis,

where F is the fineness ratio. The shearing planes then differ from the 45° diagonal planes by amounts whose upper limits are 4·3 F° and 4·3° according as the fracture begins at the ends of the minor or major axes.

20. *Torsional shear stress, conjectured variation with diameter : length ratio.*—It is desired to estimate what the worst case would be for torsional stresses, hence the tension is supposed to be such as will not produce lateral resonance (*cf.* Fig. 4). The torsional amplitude depends upon :—

(1) that which would result from the balance between the supply of air energy and the "hysteresis" loss (*cf.* Fig. 11).

(2) that at which the pins would commence to slip, *i.e.*, the "critical" amplitude.

If (1) is less than (2), the amplitude is given by (1), but in the usual case (1) is greater than (2), and the amplitude then lies somewhere between the two values.

(1) Assume that the "hysteresis" loss of energy (Fig. 6) is a function of the surface stress rather than of the distortion, *i.e.*, that it depends on $\frac{a\,B}{l}$ rather than $\frac{B}{l}$, where B is the torsional amplitude at the middle of the wire, a is the major diameter of the cross-section, and l is the length of the wire. This leads to the quadratic :—

$$\frac{a\,B}{l}\left(\frac{a\,B}{l} + 0{\cdot}000322\right) = k\mathrm{UP}$$

where U is the wind speed (ft/sec.), P is a factor depending on the phases at which the N_r energy is supplied, and k is a constant covering the value of N_r, the "speed ratio" effect, the slope of the "hysteresis" curve (Fig. 6) and the frequency effect on hysteresis. The experimental results are roughly consistent with this equation, and Fig. 5 gives an empirical value of k, viz. : $k = 3{\cdot}8 \times 10^{-8}$. P varies when the length or diameter of the wire varies ; when the wire is too short or too thick it does not sing at all owing to the small value of P. At large amplitudes P is nearly constant, tending to unity as a limit.

The following table expresses the solution of the above equation for the simplest case (rigging 0°, wind speed 150 ft/sec.).

Torsional amplitude (degrees).	Major diameter, ÷ Length.	Shear stress. (Tons /sq. in.).
15	0	0
16	$4{\cdot}1 \times 10^{-3}$	4·1
17	5·8 ,,	6·1
18	7·0 ,,	7·8
19	7·4 ,,	8·7
20	7·6 ,,	9·4
30	6·7 ,,	13·0
40	5·7 ,,	14·1
50	4·7 ,,	14·8
60	4·0 ,,	15·1
80	3·1 ,,	15·5
100	2·5 ,,	15·7
120	2·1 ,,	15·8
		limit 15·9

In this case the maximum possible steady stress (due to torsion only) is in the neighbourhood of 16 tons/sq. in., occurring for long or narrow wires, and singing does not take place if the major diameter of the wire is greater than 0·008 times its length. The diameter of the experimental wire is 0·0034 times its length.

The first five values in the above table represent unstable conditions, and do not concern us.

(2) When the tensile stress p is low enough to permit of the pins slipping, the shear stress at which this happens (*i.e.*, at the "critical" amplitude) is 0·15 F p (*cf.* § 19). Hence the shear stress at the critical amplitude is 0·67 p when the fineness ratio is $4\frac{1}{2}$. When the stress in the foregoing table exceeds 0·67 p, the actual stress during singing would lie between 0·67 p and that of the table.

The quadratic equation above shows that an increase in the wind speed increases the possible stress in the ratio of $\sqrt{U}$ approximately. The following table gives the maxima possible, on this assumption, and also gives the tensile stresses necessary to prevent slipping at the pins; the stresses in column 3 must exist before those in column 2 could occur :—

Wind speed, m.p.h.	Maximum possible shear stress (torsion). (tons/sq. in.).	Necessary tensile stress (tons/sq. in.).
100	15·7	23·4
120	17·2	25·7
140	18·6	27·8
160	19·9	29·7
180	21·1	31·5
200	22·2	33·2

TABLE 1.

TORSIONAL AMPLITUDES OF A SINGING WIRE.

Variation with load (wind speed 78·5 ft/sec., setting 10°·5).

Load (lbs.).	Amplitude (degrees).	Character of note.	Load (lbs.).	Amplitude (degrees).
1015	34·5	Loud, rather rough	515	24·0
990	31·5	Loud, very clear	490	10·5
965	25·8	Loud, very clear	465	19·5
940	16·5	Loudest	440	19·0
915	49·0	Loud	415	4·9
890	46·0	Loud	390	16·5
865	44·5	Loud	365	15·5
840	10	Interference	340	13·0
815	12·5	Interference	315	12·0
790	33·5	Very rough	290	7·7
765	32·3	Very rough	265	3·4
740	30·5	Very rough	240	4·8
715	28·0	Rough	215	4·8
690	25·0	Louder	190	4·4
665	24·0	Rough	165	4·3
640	26·5	Less rough	140	3·9
615	28·3	Rough	115	3·6
590	26·3	Rough	90	0·3
565	23·5	Rough	—	—
540	25·5	Rough	—	—

TABLE 2.

TORSIONAL AMPLITUDES OF A SINGING WIRE.

Variation with wind speed.

Setting : 10°·5. Load : 915 lbs.		Setting : 18°·6. Load : 940 lbs.	
Wind speed (ft /sec.).	Amplitude (degrees).	Wind speed (ft /sec.).	Amplitude (degrees).
133	67·0	132	69·5
127	66·5	127	67·5
118	65·5	118	65·5 63·5
108	62·5	108	64·0 62·5
98	61·0	98	60·5
88	58·0	88	56·0
78·5	45·0	78·5	48·0
78·5	18	—	—
73·5	5·0	—	—
68·5	4·0	—	—
63·5	3·0	—	—
61·4	0·5	—	—

TABLE 3.

EXPERIMENTS ON A SINGING WIRE.

Observations when the setting was 20°·9.

Load (lbs.).	Wind Speed (ft /sec.).	Amplitude (degrees).	Quality.
1015	78·5	35·5	Very clear and strong. (rows 1–3)
1015	73·5	33·3	
1015	68·5	31·8	
915	63·5	41·3	
815	63·5	36·3	
715	63·5	36·0	
965	61·4	too small	Stronger than at 940 lbs.
915	61·4	37·8	Very clear, rapid beats.
865	61·4	36·0	
840	61·4	damped out	Violent interference, beats.
815	61·4	32·8	
765	61·4	too small	Clear and loud.
715	61·4	31·5	Clear.
690	61·4	too small	Clear and loud.
615	63·5	,,	Rough, some interference.
590	63·5	,,	Clear and loud.
565	63·5	,,	Faint.
540	63·5	,,	Fainter.
515	63·5	,,	Rough, some interference.
490	63·5	,,	Rough, some interference.
465	63·5	,,	Fainter, beats.
440	63·5	,,	Singing stops.

TABLE 4.

EXPERIMENTS ON A SINGING WIRE.

Observations at a setting of 18°·8.

Load (lbs.).	Wind Speed (ft /sec.).	Amplitude (degrees).	Quality, etc..
1015	78·5	33·0	
990	61·4	35·0	
965	61·4	35·8 to 37·0	
940	61·4	36·8	
915	61·4	36·5	
890	61·4	35·3	
865	61·4	32·8 to 34·5	Beats, 5 in 18 seconds.
840	61·4	too small	Violent interference.
815	68·5	35·8	
790	68·5	32·0	
765	68·5	38·3	Loud.
740	68·5	37·8	Not so loud.
715	68·5	34·8	Loud.
690	68·5	too small	Very loud.
615	68·5	,,	Interference.
590	63·5	,,	
515	63·5	,,	Interference.
490	63·5	,,	
465	63·5	,,	Clear.
440	63·5	,,	Rough.
415	63·5	,,	Clear, faint.
390	63·5	,,	Rough.
365	63·5	,,	Clear.
315	63·5	,,	Singing stops.

TABLE 5.

EXPERIMENTS ON A SINGING WIRE.

Variation of the setting. (Loads 715 and 1,015 lbs., various wind speeds).

Setting.	Load (lbs.).	Wind speed.	Amp. (degrees).	Setting.	Load (lbs.).	Wind speed.	Amp. (degrees).
10°·5	1015	78·5	34·5	22°·6	1015	78·5	37·3
	715	78·5	28·0		1015	73·5	35·5
					1015	68·5	33·3
17°·0	715	63·5	34·0		715	63·5	37·3
	715	68·5	35·8		715	68·5	39·0
	715	73·5	37·3		715	73·5	41·0
	715	78·5	38·5				
	1015	78·5	34·2	22°·9	715	61·4	33·8
18°·8	1015	78·5	32·5	24°·2	1015	78·5	33·8
	1015	88	35·5		1015	73·5	32·3
	1015	98	38		1015	68·5	30·3
	715	68·5	35·0		1015	63·5	27·8
					715	61·4	26 to 30
20°·9	715	73·5	36·8		715	63·5	33·0
	715	58·5	31·0		715	68·5	34·8
	715	63·5	35·3				
	1015	78·5	35·5	25°·3	1015	78·5	34·8
	1015	73·5	33·3		1015	73·5	32·3
	1015	68·5	31·8		1015	68·5	30·8
					1015	63·5	28·8
22°·4	1015	78·5	35·8		715	63·5	31·3
	1015	73·5	33·8		715	68·5	34·5
	1015	68·5	32·3				
	1015	63·5	30·5	26°·7	715	61·5	31·0
	715	63·5	29·8				
				27°·9	1015	68·5	34·3
					1015	63·5	32·5
					1015	61·4	30·0

TABLE 6.

MEASUREMENT OF N_r ON 15 SCALE MODEL.

Later model. Oil depth 1·1 cm.

Wind speed (ft./sec.).	Angle of yaw (degrees).	Oil damping coefficient (λ)	Period (secs.).	Initial amplitude necessary for starting (degrees).	Static twist (degrees).	Amplitudes on each side of the swing	
						Pos. yaw (degrees).	Neg. yaw (degrees).
35·8	−5·5	0·173	1·16	—	−2·7	—	—
35·8	−1·3	0·174	1·15	—	−0·8	—	—
35·8	5·0	0·176	1·1	—	2·9	—	—
35·8	8·4	0·177	1·02	7·8	4·3	7·0	9·0
35·8	10·7	0·176	1·00	—	5·2	10·6	15·9
35·8	11·8	0·178	0·98	—	5·4	11·6	16·8
35·8	13·4	0·177	0·96	—	4·8	14·7	19·1
35·8	14·5	0·176	0·92	—	4·0	17·0	19·7
35·8	16·3	0·176	0·90	—	3·3	19·1	19·3
35·8	17·7	0·177	0·88	2·9	3·2	20·1	19·7
35·8	19·6	0·177	0·88	5·9	3·2	21·8	20·8
35·8	21·6	0·177	0·88	8·3	3·3	22·4	21·6
35·8	23·5	0·174	0·86	10·6	3·3	21·9	21·1
35·8	25·7	0·169	0·86	14·0	3·3	20·6	20·1
35·8	27·7	0·171	0·86	—	3·3	—	—
35·8	29·3	0·170	0·86	—	3·3	—	—
35·0	10·6	0·191	0·98	1·1	5·1	9·1	13·9
40·0	12·2	0·190	1·04	—	6·7	11·8	19·2
45·0	13·2	0·189	1·10	—	7·7	14·5	24·2
50·0	14·2	0·188	0·98	—	8·7	38·8	52·8
49·0	14·3	0·186	1·00	—	8·8	34·3	46·6
47·9	14·1	0·185	1·00	—	8·6	33·6	45·8
46·8	14·0	0·184	1·00	—	8·5	26·5	36·3
45·6	13·8	0·183	1·08	—	8·3	18·5	26·2
45·0	13·8	0·182	1·10	—	8·3	17·6	24·7
40·0	13·1	0·181	1·02	—	7·6	15·2	21·3
35·0	12·1	0·180	0·98	—	6·6	11·8	16·8

TABLE 7.

MEASUREMENT OF N_r ON 15 SCALE MODEL.

Later model. Oil depth 1·7 cm. Wind speed 35·8 ft./sec.

Angle of yaw (degrees).	Oil damping coefficient (λ)	Period (secs.).	Initial amplitude necessary for starting (degrees).	Static twist (degrees).	Amplitudes on each side of the swing.	
					Pos. yaw (degrees).	Neg. yaw (degrees).
−5·5	0·239	1·1	—	−2·6	—	—
5·2	0·238	1·1	—	3·0	—	—
10·1	0·238	1·0	2·2	5·0	7·8	11·3
12·3	0·238	0·96	—	5·1	10·6	14·6
14·0	0·238	0·92	—	4·6	13·1	15·7
15·6	0·238	0·90	—	3·8	15·2	15·8
17·4	0·238	0·88	3·1	3·3	16·7	15·9
18·5	0·238	0·86	5·2	3·2	16·5	15·5
20·8	0·236	0·86	8·4	3·2	16·8	15·5
25·6	0·236	0·86	—	3·2	—	—

TABLE 8.

MEASUREMENT OF N_r ON 15 SCALE MODEL.

Old model (elliptical cross-section).

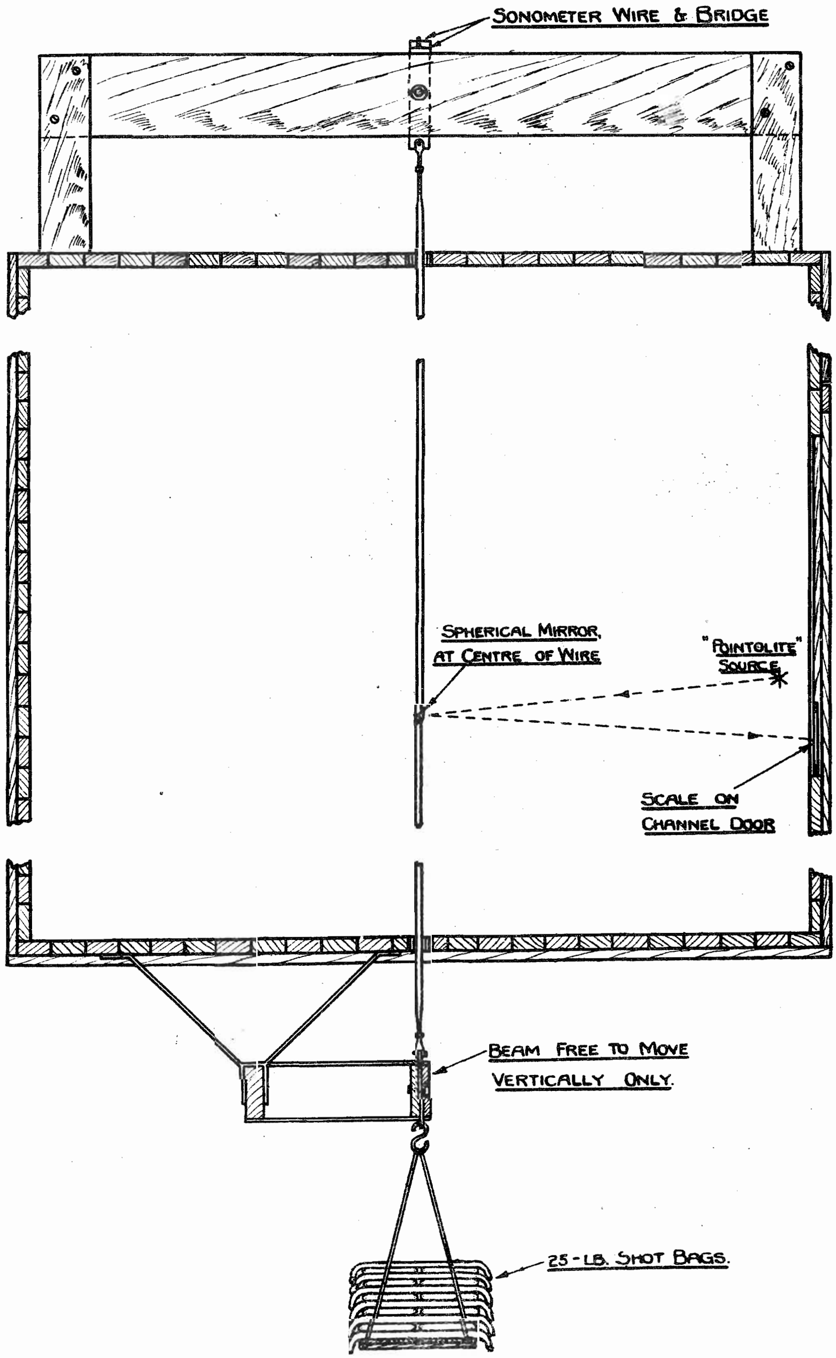
Wind speed (ft./sec.).	Angle of yaw (degrees)	Oil damping coefficient (λ)	Period (secs.).	Initial amplitude necessary for starting (degrees)	Static twist (degrees)	Amplitudes on each side of the swing.	
						Pos. yaw (degrees).	Neg. yaw (degrees).
35·2	−3·8	0·194	0·98	18·6	−1·9	30·5	28·6
35·2	−0·6	0·200	0·98	20·7	−0·3	27·3	27·6
35·2	3·5	0·197	0·99	19·8	1·9	27·1	30·3
36·0	11·3	0·219	1·01	2·6	5·6	10·9	16·3
36·0	13·2	0·219	1·00	—	6·0	12·3	18·6
36·0	14·2	0·220	0·97	—	5·0	15·1	18·8
36·0	15·6	0·221	0·94	—	3·9	19·0	19·5
36·0	17·6	0·220	0·92	2·2	3·4	19·8	19·1
36·0	21·0	0·222	0·90	6·5	3·3	22·1	21·1
36·0	25·3	0·218	0·87	11·6	3·6	22·8	21·8
36·0	28·5	0·221	0·865	16·6	3·6	19·4	18·9
36·0	30·1	0·219	0·87	—	3·6	—	—
36·0	14·0	0·223	0·98	—	5·6	14·4	19·3
39·7	14·4	0·224	1·02	—	6·0	16·3	21·9
43·3	14·8	0·225	1·06	—	6·4	18·7	24·6
46·9	15·2	0·225	1·11	—	6·8	21·2	28·0
50·4	15·6	0·226	1·02	—	7·2	43	54

REPORT Nº 759.

FIG Nº 1

EXPERIMENTS ON A SINGING R.A.F. WIRE.

GENERAL ARRANGEMENT OF APPARATUS.



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Report No. 759.

Fig No. 2

Dimensions of Cross Section of Rafwire

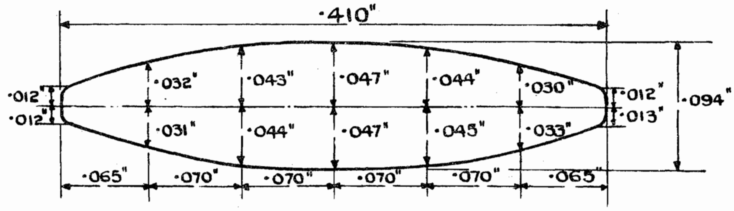
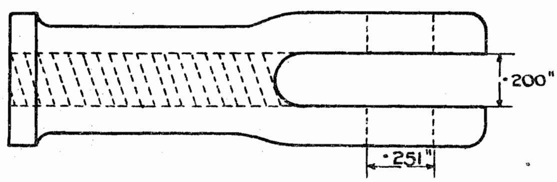


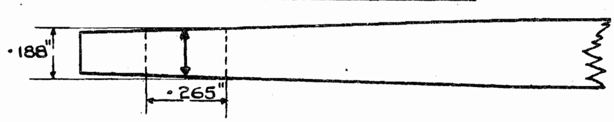
Fig No. 3

Dimensions of End Fittings

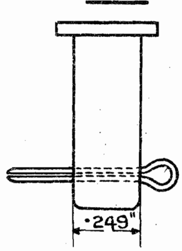
¼" Forked Ends S.W.G. 414.



Wiring Plate



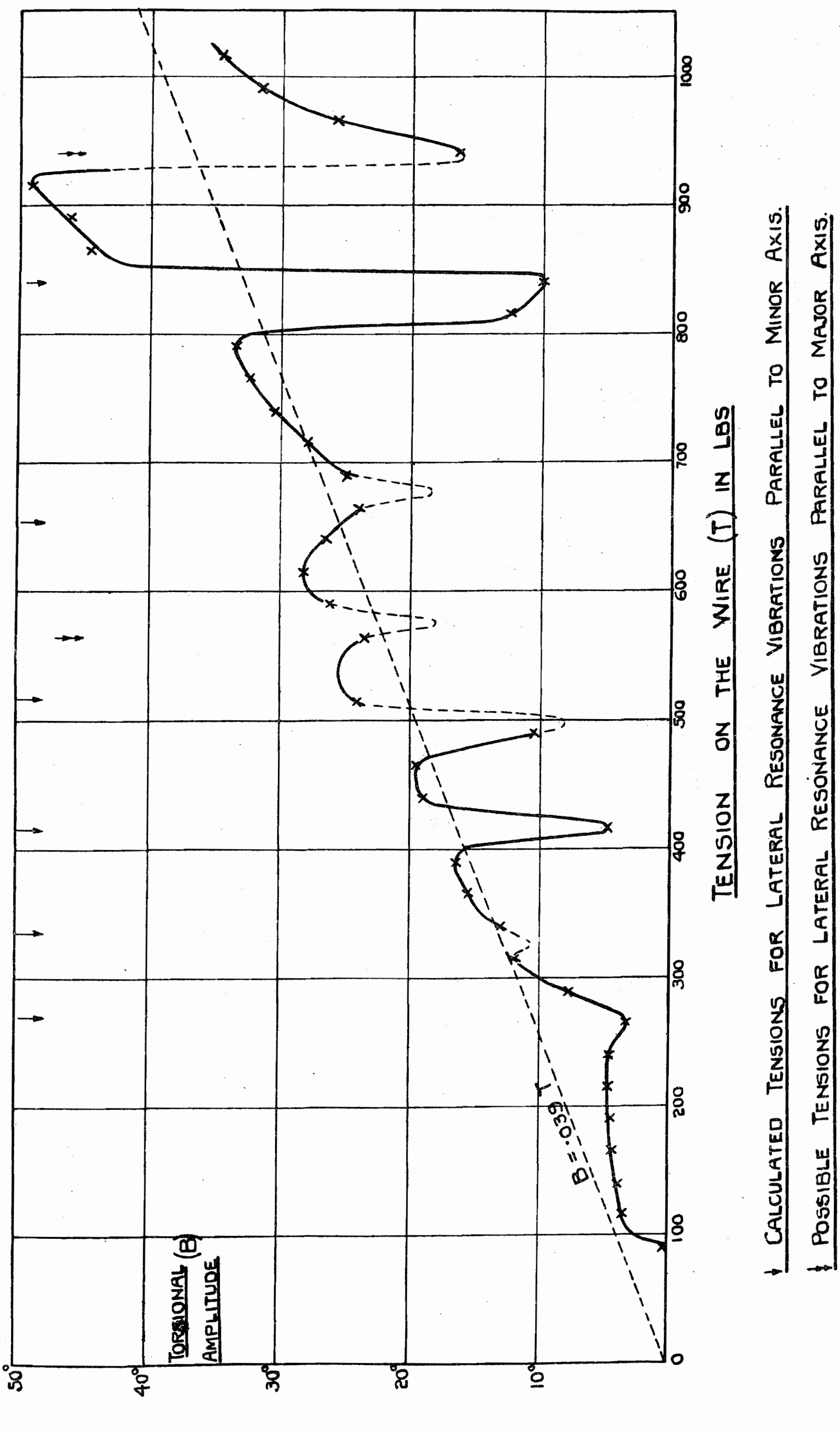
Pin



REPORT Nº 759. FIG Nº 4

VARIATION OF TORSIONAL AMPLITUDE WITH TENSION IN A SINGING R.A.F. WIRE.

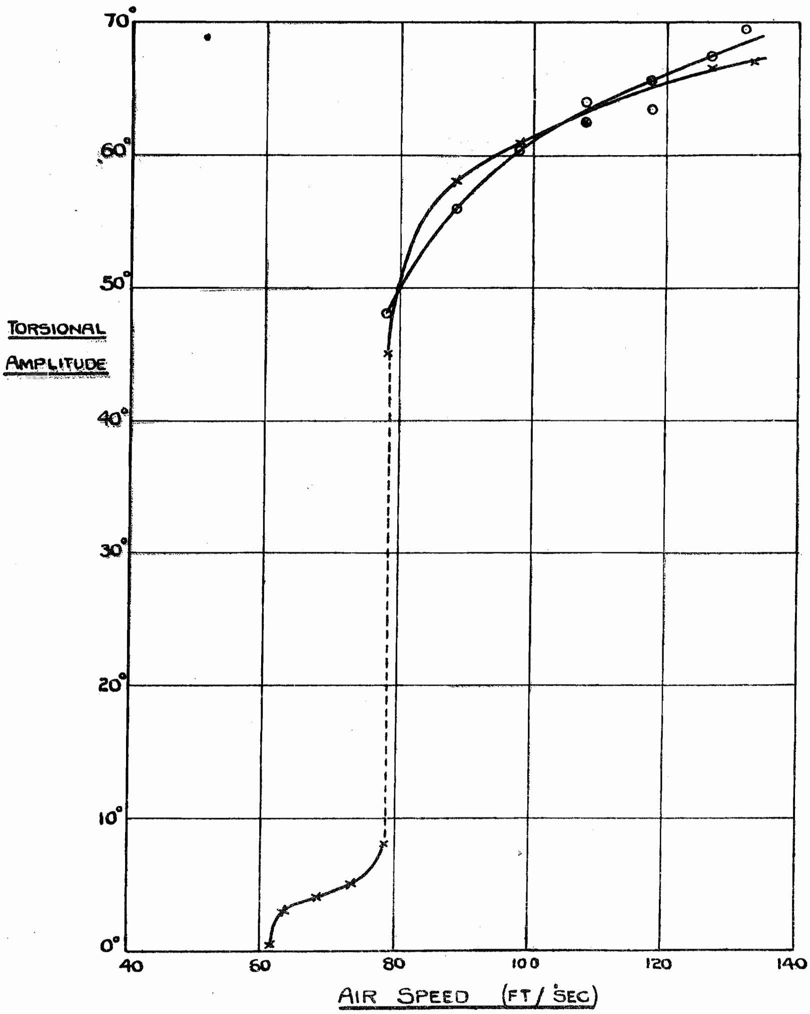
WIRE SET AT 10°5 TO THE WIND. AIR SPEED 78·5 FT/SEC.



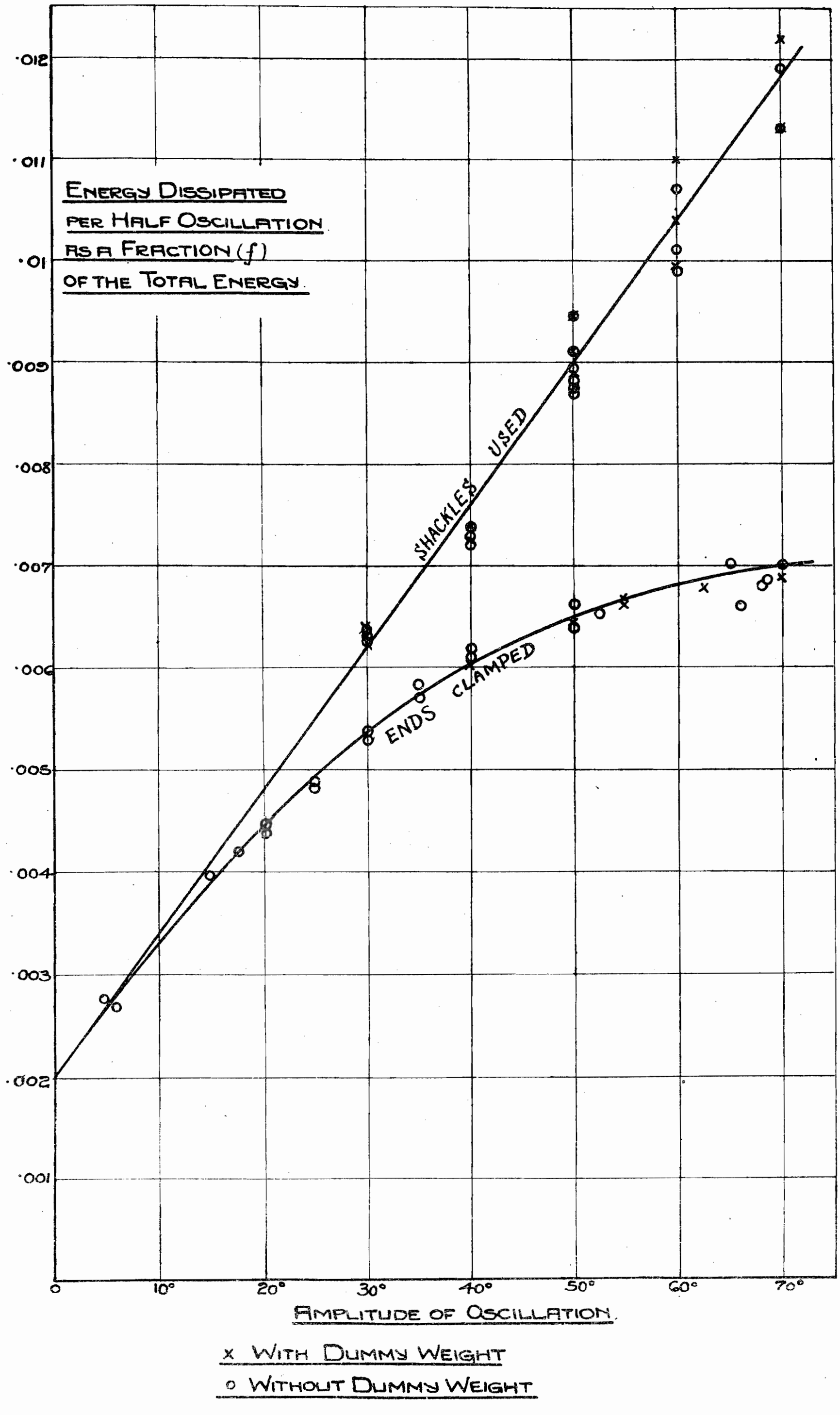
VARIATION OF TORSIONAL AMPLITUDE WITH AIR SPEED, IN A SINGING RAFWIRE.

x WIRE SET AT 10°·5 TO THE WIND, TENSION 915 LBS.

o WIRE SET AT 18°·6 TO THE WIND, TENSION 940 LBS.

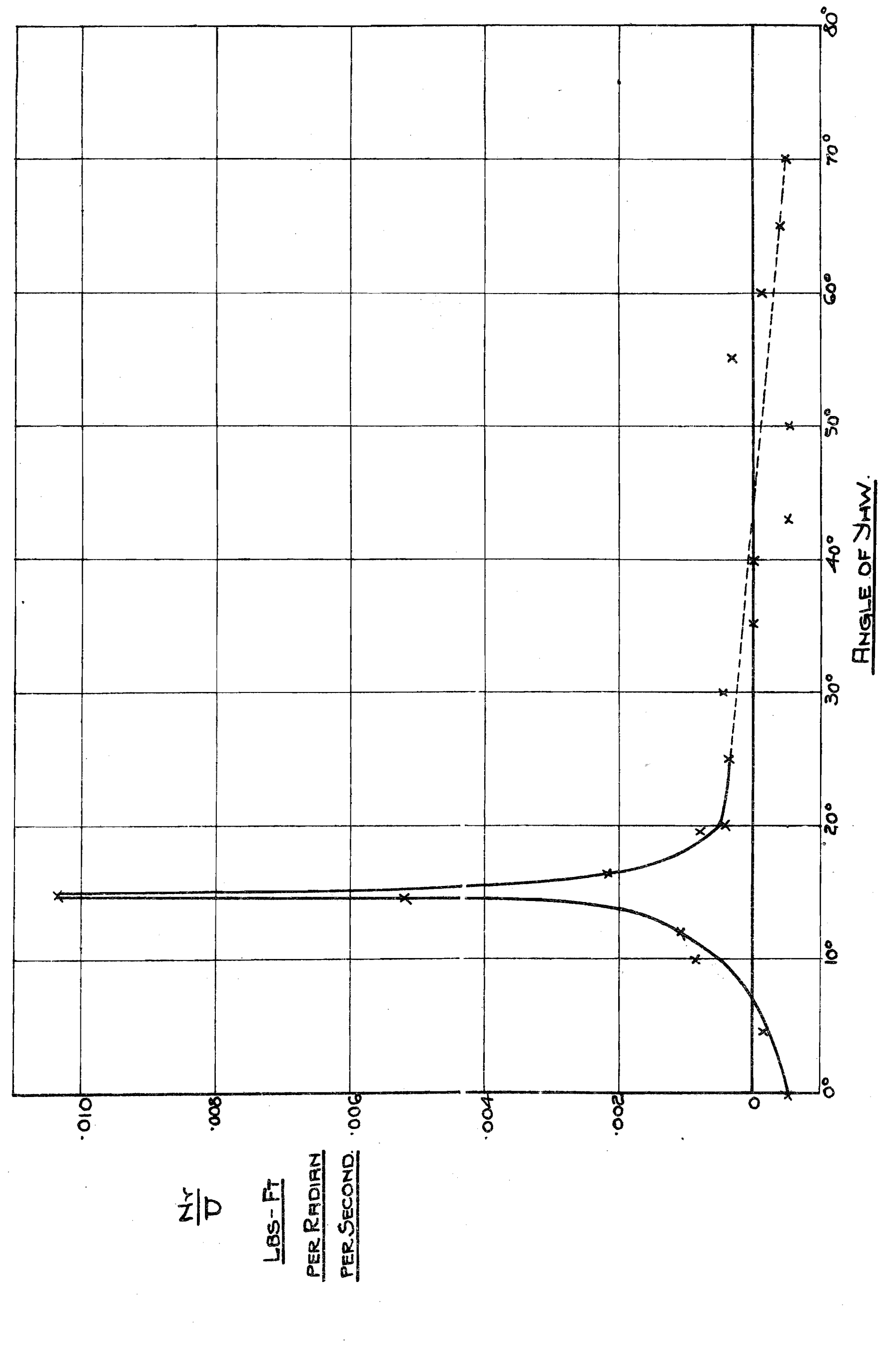


DISSIPATION OF ENERGY, DUE TO HYSTERESIS, ETC., IN A RAFWIRE.

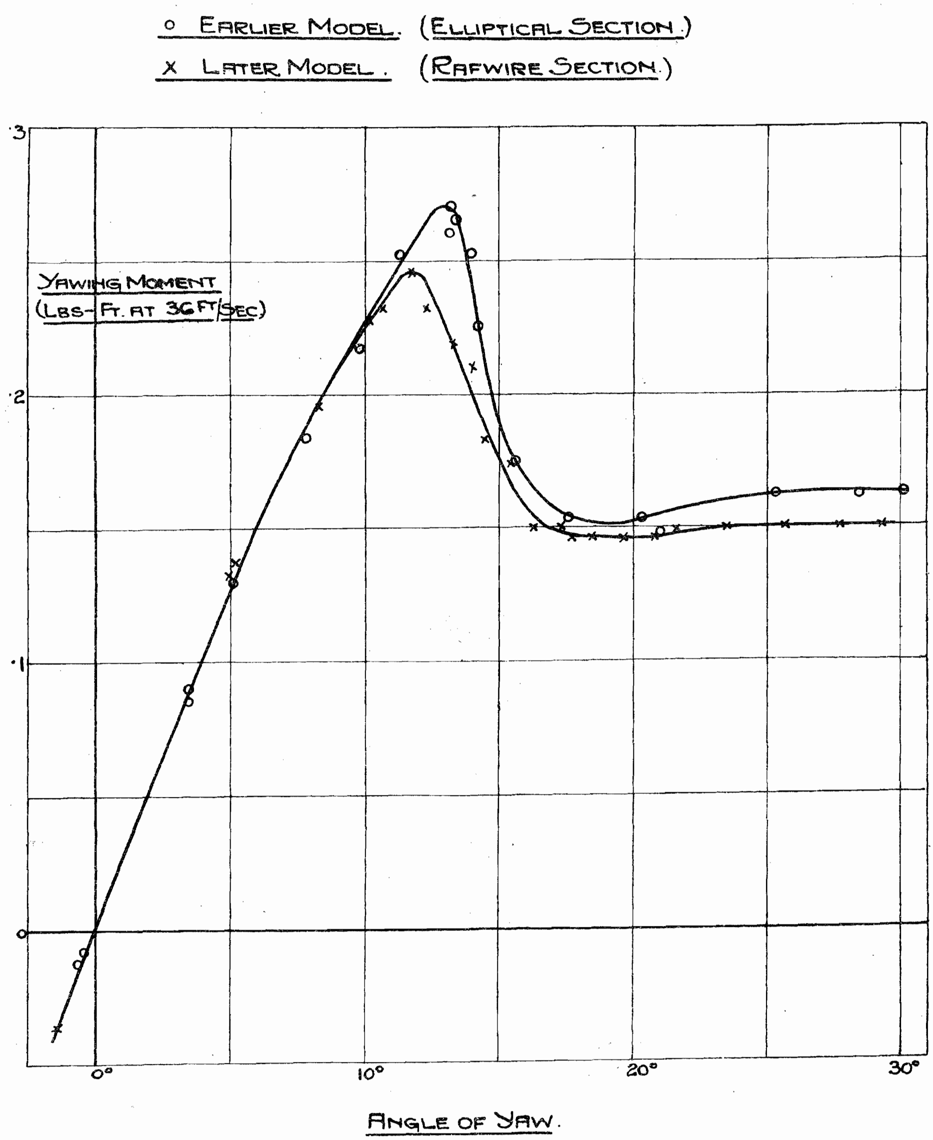


$\frac{N_r}{U}$ FOR 15 SCALE MODEL OF RAFWIRE.

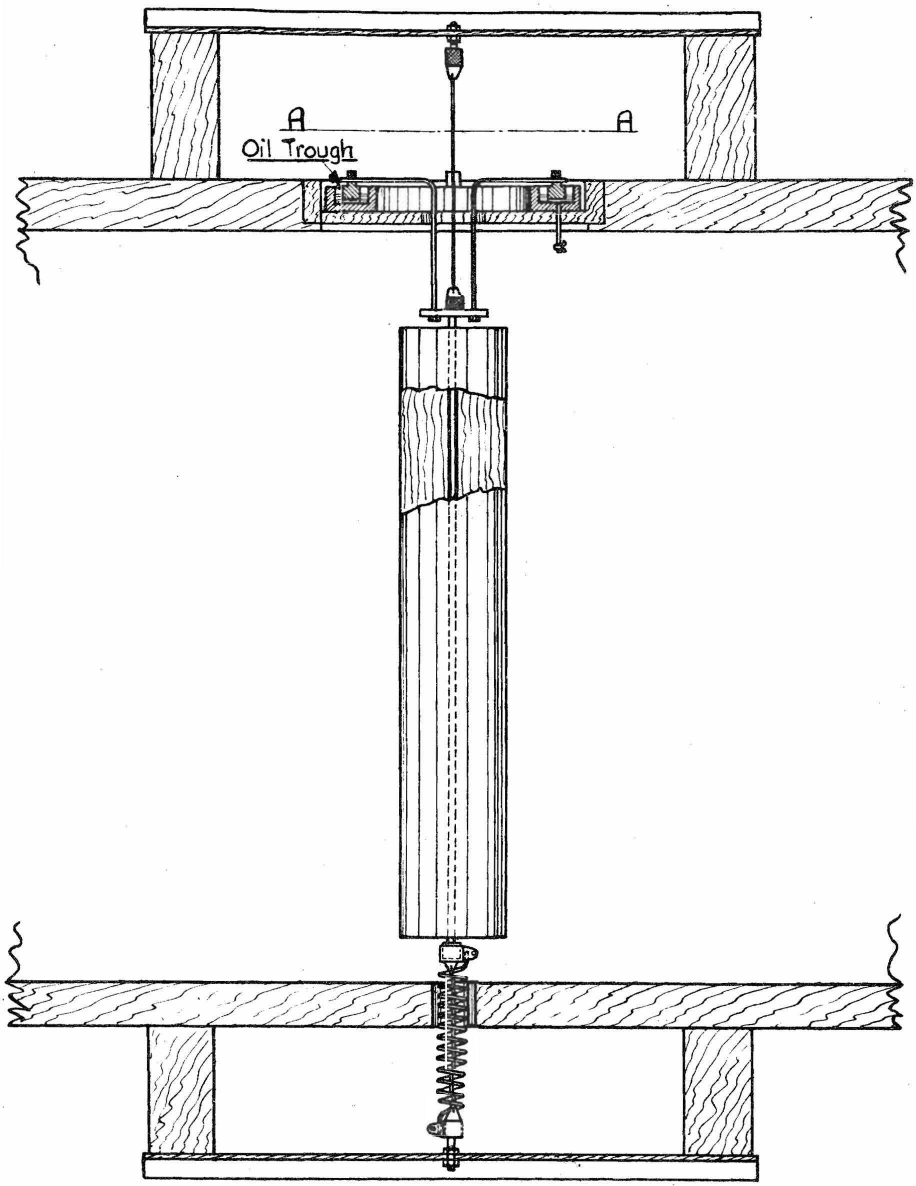
(EARLIER MODEL, ELLIPTICAL SECTION, 6" x 1·33")

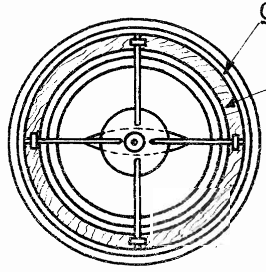


Nv MOMENTS ON 15 SCALE MODELS.



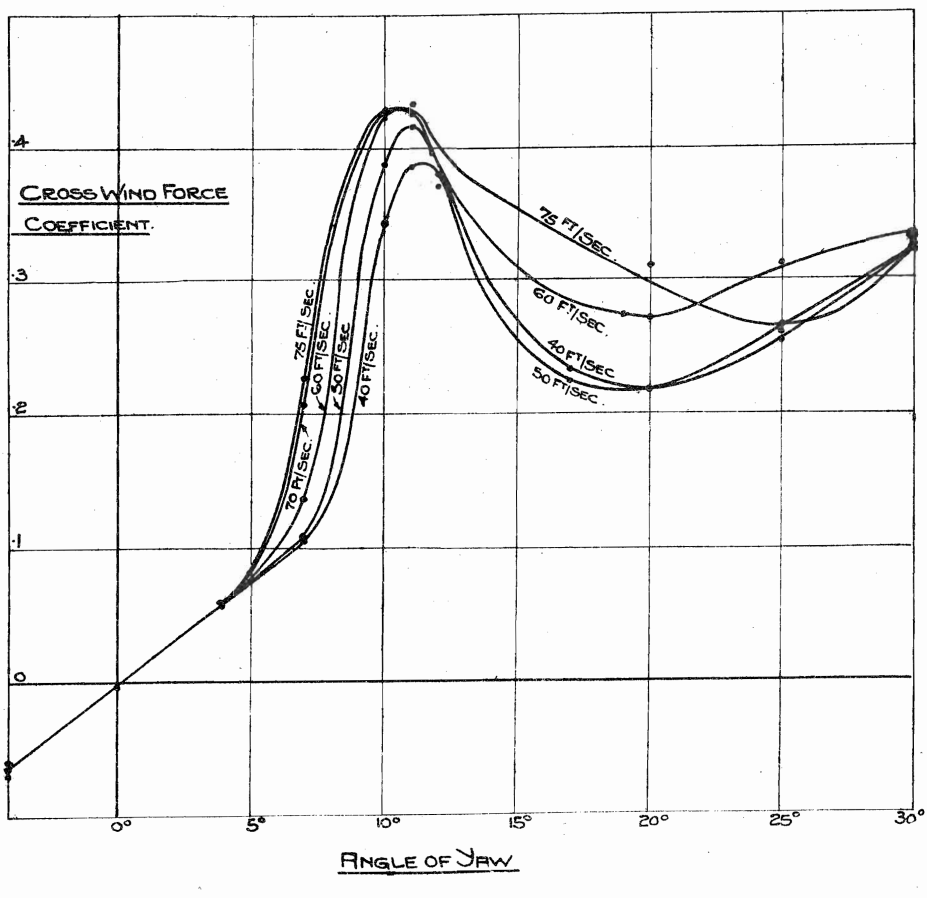
Apparatus for Experiments on Nr.





Section Thro' "AA"

CROSS WIND FORCE ON A RAFWIRE AT VARIOUS AIR SPEEDS.



Fracture at the end of the minor axis of the cross section.

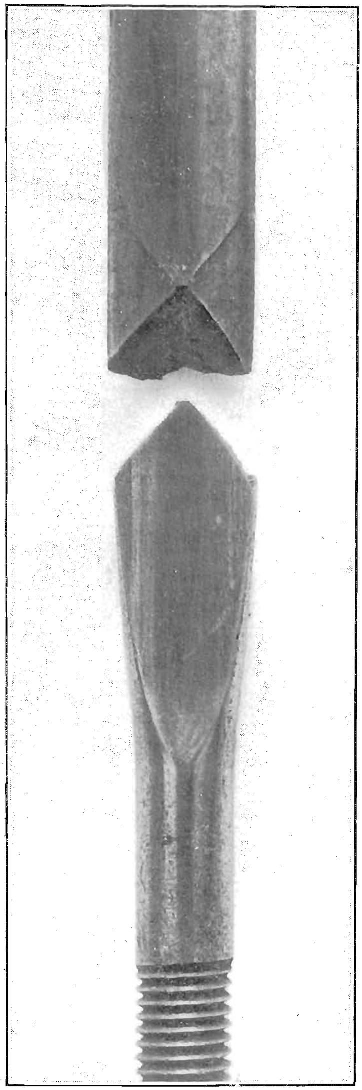


Fig. 12.

Fracture at the end of the major axis of the cross section.

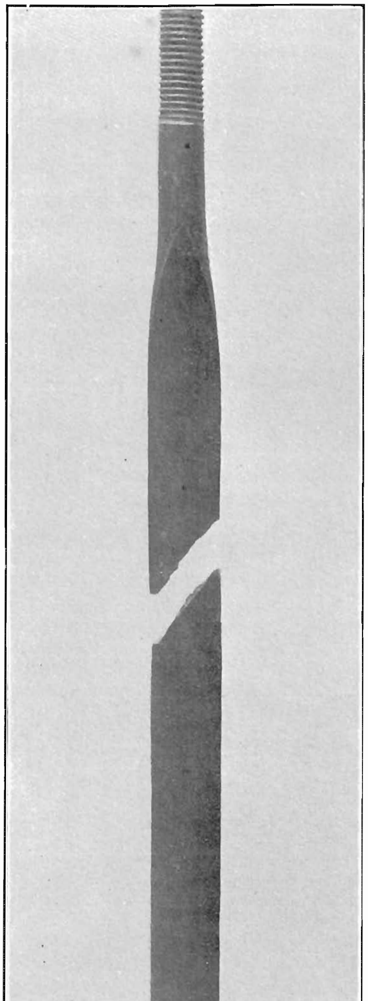


FIG. 13.

TABLE 9.

LATERAL OVERTONES.

Number of Tone (r)	Load necessary for vibrations parallel to	
	Major axis (lbs.).	Minor axis (lbs.).
1	58,100	58,100
2	14,500	14,500
3	6,390	6,450
4	3,530	3,630
5	2,160	2,310
6	1,380	1,600
7	870	1,170
8	497	887
9	197	691
10	−62	548
11	—	440
12	—	356
13	—	289
14	—	232
15	—	184
16	—	143
17	—	106
18	—	73
19	—	43
20	—	14

TABLE 10.

CROSS WIND FORCE ON RAFWIRE.

Angle of yaw.	Cross Wind Force coefficient at				
	40 ft/sec.	50 ft/sec.	60 ft/sec.	70 ft/sec.	75 ft/sec.
−4°	−0·0621	−0·0621	−0·0643	−0·0661	−0·0645
0°	−0·0031	−0·0031	−0·0027	−0·0020	−0·0024
4°	0·0564	0·0567	0·0560	0·0565	0·0592
7°	0·106	0·108	0·138	0·208	0·227
10°	0·346	0·392	0·421	0·427	0·428
11°	0·386	0·416	0·427	0·432	0·428
12°	—	—	0·38	—	0·37
17°	0·232	0·224	—	—	—
19°	—	—	0·272	—	—
20°	0·218	0·218	0·271	0·402	0·31
25°	0·263	0·255	0·312	—	0·26
30°	0·324	0·320	0·33	—	0·323

TABLE 11.

YAWING MOMENT DUE TO N_v.

Earlier Model (elliptical cross-section).		Later Model (true cross-section).	
Angle of yaw (degrees).	Yawing moment lbs./ft. at 36 ft./sec.	Angle of yaw (degrees).	Yawing moment lbs./ft. at 35·8 ft./sec.
−3·9	−0·0855	−4·9	−0·117
−0·6	−0·013	−0·8	−0·036
3·5	0·085	5·6	0·130
5·1	0·130	9·0	0·193
7·8	0·184	10·7	0·225
9·8	0·217	12·4	0·243
11·3	0·252	12·9	0·229
13·2	0·270	14·0	0·216
14·0	0·252	14·6	0·207
14·2	0·225	15·1	0·180
15·6	0·175	16·1	0·171
17·6	0·153	16·9	0·148
20·4	0·153	18·0	0·148
21·0	0·148	18·3	0·144
25·3	0·162	19·1	0·144
28·5	0·162	20·2	0·144
30·1	0·162	21·4	0·144
—	—	22·2	0·146
—	—	24·1	0·148
—	—	26·3	0·148
—	—	28·3	0·148
—	—	29·0	0·148

TABLE 12.

VARIATION OF TORSIONAL AMPLITUDE WITH TIME IN A SINGING RAFWIRE.

Time (minutes).	Torsional amplitude (degrees).
0 to 15	68
20 to 25	67
30 to 45	66
50	65
55	66
60 to 80	65
85 to 150	64

Owing to the method of observation, a reading of the amplitude denotes anything between that reading and one degree more.

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