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THE THEORETICAL RELATIONSHIPS FOR AN AEROFOIL WITH A MULTIPLY HINGED FLAP SYSTEM.

By W. G. A. PERRING, R.N.C., A.M.I.N.A.

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AERODYNAMIC SYMBOLS

1. GENERAL

m	mass
t	time
V	resultant linear velocity
Ω	resultant angular velocity
ρ	density, σ relative density
ν	kinematic coefficient of viscosity
R	Reynolds number, $R = lV/\nu$ (where l is a suitable linear dimension), to be expressed as a numerical coefficient x 10^6

Normal temperature and pressure for aeronautical work are 15° C. and 760 mm.

For air under these conditions $\begin{cases} \rho = 0{\cdot}002378 \text{ slug/cu. ft.} \\ \nu = 1{\cdot}59 \times 10^{-4} \text{ sq. ft./sec.} \end{cases}$

The slug is taken to be 32·2 lb.-mass.

α	angle of incidence
ϵ	angle of downwash
S	area
c	chord
s	semi-span
A	aspect ratio, $A = 4s^2/S$
L	lift, with coefficient $k_L = L/S\rho V^2$
D	drag, with coefficient $k_D = D/S\rho V^2$
γ	gliding angle, $\tan \gamma = D/L$
L	rolling moment, with coefficient $k_l = L/sS\rho V^2$
M	pitching moment, with coefficient $k_m = M/cS\rho V^2$
N	yawing moment, with coefficient $k_n = N/sS\rho V^2$

2. AIRSCREWS

n	revolutions per second
D	diameter
J	V/nD
P	power
T	thrust, with coeffici
Q	torque, with coefficie
η	efficiency, $\eta = TV/P$

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Summary.—Introductory (*Purpose of Investigation*).—Theoretical expressions for the lift and pitching moment of an aerofoil in two dimensional motion, were developed in R. & M. 910. This theory was extended in R. & M. 1095, to include the hinge moment of a flap in the case of a rectangular aerofoil of finite span.

Range of present investigation.—The analysis has now been extended to an aerofoil fitted with a multiply hinged flap system, and theoretical expressions for lift and pitching moment of the aerofoil, and the hinge moment about any hinge position have been deduced in the case of a rectangular aerofoil of finite span. Application of the theoretical expressions to the particular case of an aerofoil fitted with a servo operated flap is considered, and the theory compared with experiment.

Conclusions.—The agreement between theory and experiment is very satisfactory in the case of a flap $0 \cdot 3$ of the aerofoil chord, but ceases to be as good when the flap or servo chord becomes very small.

Introduction.—Theoretical expressions for the lift and pitching moment of an aerofoil, and for the hinge moment of a flap about any hinge position, can be obtained, so long as the lift bears a linear relationship with the angle of incidence, and with the flap angles. Expressions for the lift and pitching moment of an aerofoil were developed in R. & M. 910* and the theory was extended to include hinge moments on a flap in R. & M. 1095.† The present report extends the theory to an aerofoil with any number of hinged flaps, expressions being deduced for the lift and pitching moment of the aerofoil and the hinge moment about any hinge position in the case of a rectangular aerofoil of finite span. It is interesting to

*R. & M. 910—A theory of thin aerofoils. By H. Glauert.

†R. & M. 1095.—Theoretical relationships for an aerofoil with hinged flap. By H. Glauert.

note that the relationships given in the present report may be used to deduce the expressions obtained by Glauert in his " Theory of thin aerofoils," R. & M. 910.

Following the methods of R. & M. 1095 the present investigation relates to the case of an aerofoil of symmetrical section, and the results can be modified to apply to aerofoils of more general shape by the addition of constant terms k_{L_0} k_{m_0} $k_{H_{r0}}$ etc., to the equations for lift, pitching moment, and the hinge moment coefficient respectively.

Consider an aerofoil with p flaps represented in outline by $A\,A_1\,A_2..A_r..A_p\,T$, fig. 1, where $A\,A_1$ is the front portion of the aerofoil and A_1, $A_2..A_r..A_p$ are hinge positions. A T is the line joining the leading and trailing edges of the aerofoil and is sensibly the same length as the aerofoil chord.

Let

c = total chord of the aerofoil (approximately equal to AT)

$E_1c, E_2c..E_rc..E_pc$ = chord of flaps $A_1\,T$, $A_2\,T$ ...... $A_r\,T$......$A_p\,T$ respectively, and these are approximately equal to $A'_1\,T$, $A'_2\,T$...$A'_r\,T$...$A'_p\,T$.

$\gamma_1c, \gamma_2c...\gamma_rc...\gamma_pc$ = height of hinges A_1, A_2...A_r..A_p above base line AT.

α = angle of incidence to base line AT.

α' = angle of incidence of front part $A\,A_1$.

$\xi_1, \xi_2 .. \xi_r .. \xi_p$ = flap angles, that is the acute angles between $A\,A_1$ and $A_1\,A_2$; $A_1\,A_2$ and $A_2\,A_3$; etc.

From these definitions it follows that

$$\frac{\gamma_1}{1 - E_1} = (\alpha - \alpha') \quad ..$$

or in general

$$\frac{\gamma_s - \gamma_{s+1}}{E_s - E_{s+1}} = \sum_{s=1}^{s} \xi_s - (\alpha - \alpha') \quad .. \quad (1)$$

and at hinge P

$$\frac{\gamma_p}{E_p} = \sum_{s=1}^{p} \xi_s - (\alpha - \alpha') \quad ..$$

Considering the case of two dimensional motion for an aerofoil of infinite span then the lift L, may be written

$$L = k_L\, c\, \rho\, V^2$$

and the pitching moment M, about the leading edge, which is positive when it tends to increase the angle of incidence may be written

$$M = k_m c^2 \rho V^2$$

It is convenient to express the coefficients k_L and k_m in the form

$$k_L = a_1 (\alpha' + \sum_{s=1}^{p} \lambda_s \xi_s) \quad \ldots \quad \ldots \quad (2)$$

and

$$k_m = -\frac{1}{4} k_L - \sum_{s=1}^{p} m_s \xi_s \quad \ldots \quad \ldots \quad (3)$$

The hinge moment H_r of the flap $A_r \ldots P_p T$ about the hinge A_r due to the aerodynamic force, regarded positive when it tends to increase the flap angle, can be expressed as a non-dimensional coefficient k_{H_r} defined by

$$H_r = k_{H_r} (E_r c)^2 \rho V^2$$

and this coefficient can also be expressed in the form

$$k_{H_r} = -\beta_r k_L - \sum_{s=1}^{p} b_{rs} \xi_s \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

$$= -a_1 \beta_r \alpha' - \sum_{s=1}^{p} (b_{rs} + a_1 \beta_r \lambda_s) \xi_s$$

2. *Effect of aspect ratio.*—Since only the slope of the lift curve, and not the angle of no lift, is affected by a change of aspect ratio, the lift coefficient equation will remain in the same form as that given by (2) and the coefficients λ_s will be independent of aspect ratio. The value of a_1 will, however depend on the aspect ratio and appropriate values in the case of a rectangular aerofoil of constant section and incidence are given in Table 1.

Glauert has shown *that when the moment coefficient in two dimensional flow is of the form given by equation (3), then the moment coefficient of a finite rectangular aerofoil will be of the same form. By the same argument it follows that if the hinge moment coefficients are of the form given by equation (4), then the hinge moment coefficients will have the same form in the case of a finite rectangular aerofoil. Consequently the coefficients defined by

$$m_s, \ \beta_r \text{ and } b_{rs}$$

in the equations for pitching moment, and hinge moment coefficient (3) and (4), will all be independent of aspect ratio.

* The elements of aerofoil and airscrew theory. By H. Glauert, p. 150.

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3. *General analysis.*—Regarding A, the leading edge of the aerofoil as the origin, and the base line AT as the axis of x, then it is convenient to replace the coordinate x by the angle θ, where

$$x = \frac{1}{2} c (1 - \cos \theta) \quad \ldots \quad \ldots \quad (5)$$

So that in passing from A to T along the aerofoil θ varies from 0 to π. The position of the hinges $A_1 A_2 \ldots A_r \ldots A_p$ may be defined by the angles $\varphi_1, \varphi_2 \ldots \varphi_r \ldots \varphi_p$ and can in general be represented by the equation.

$$\cos \varphi_s = -(1 - 2 E_s) \quad \ldots \quad \ldots \quad (6)$$

Also the shape of the aerofoil is defined by the equations

$$\theta = 0 \text{ to } \varphi_1, \quad \frac{dy}{dx} = \frac{\gamma_1}{1 - E_1} = (\alpha - \alpha')$$

and in general

$$\theta = \varphi_s \text{ to } \varphi_{s+1}, \quad \frac{dy}{dx} = -\frac{\gamma_s - \gamma_{s+1}}{E_s - E_{s+1}} = (\alpha - \alpha') - \sum_{s=1}^{s} \xi_s \quad \ldots \quad (7)$$

and for

$$\theta = \varphi_p \text{ to } \pi, \quad \frac{dy}{dx} = -\frac{\gamma_p}{E_p} = (\alpha - \alpha') - \sum_{s=1}^{p} \xi_s$$

Assuming a distribution of vorticity along the aerofoil in the form of the series

$$k\, dx = cV \left\{ A_0 (1 + \cos \theta) + \sum_{n=1}^{\infty} A_n \sin n \theta \sin \theta \right\} d\theta \quad (8)$$

then it can be shown that

$$\left.\begin{aligned} \alpha - A_0 &= \frac{1}{\pi} \int_0^\pi \frac{dy}{dx}\, d\theta \quad \ldots \quad \ldots \\ A_n &= \frac{2}{\pi} \int_0^\pi \frac{dy}{dx} \cos n \theta\, d\theta \quad \ldots \end{aligned}\right\} \ldots \quad (9)$$

Substituting in equation (9) the values given in (7) and integrating

$$\alpha - A_0 = \frac{1}{\pi} \left[(\alpha - \alpha') \varphi_1 + \sum_{s=1}^{p} \left\{ (\alpha - \alpha') - \sum_{s=1}^{s} \xi_s \right\} (\varphi_{s+1} - \varphi_s) \right]$$

$$A_n = \frac{2}{n\pi} \left[(\alpha - \alpha') \sin n \varphi_1 + \sum_{s=1}^{p} \left\{ (\alpha - \alpha') - \sum_{s=1}^{s} \xi_s \right\} (\sin n \varphi_{s+1} - \sin n \varphi_s) \right]$$

from which, because

$$\varphi_{p+1} = \pi$$

$$A_0 = \alpha' + \sum_{s=1}^{p} \frac{\pi - \varphi_s}{\pi} \xi_s \quad \ldots \quad \ldots$$

and

$$A_n = \frac{2}{n\pi} \sum_{s=1}^{p} \xi_s \sin n\,\varphi_s \quad \ldots \quad \ldots \qquad (10)$$

The lift coefficient has been obtained as

$$k_L = \pi\left(A_0 + \frac{1}{2} A_1\right)$$

$$= \pi\left[\alpha' + \sum_{s=1}^{p}\left(\frac{\pi - \varphi_s}{\pi} + \frac{\sin\varphi_s}{\pi}\right)\xi_s\right]$$

from which it follows, by comparison with equation (2) that

$$a_1 = \pi \quad \ldots \quad \ldots \quad \ldots$$

$$\lambda_s = \frac{\pi - \varphi_s}{\pi} + \frac{\sin\varphi_s}{\pi} \quad \ldots \qquad (11)$$

Also the moment coefficient has been obtained as

$$k_m = -\frac{1}{4} k_L - \frac{\pi}{8}(A_1 - A_2)$$

$$= -\frac{1}{4} k_L - \sum_{s=1}^{p} \frac{1}{4} \sin\varphi_s (1 - \cos\varphi_s)\,\xi_s$$

from which by comparison with equation (3)

$$m_s = \frac{1}{4} \sin\varphi_s \;(1 - \cos\varphi_s) \quad \ldots \quad \ldots \qquad (12)$$

4. *Hinge moments about hinge point A_r.*—To calculate the hinge moment about A_r due to the aerodynamic forces on the flap, since the lift force on an element dx is simply $\rho\, V\, k\, dx$, then the hinge moment H_r is given by

$$H_r = -\int_{A_r}^{T} \rho V k \{x - (1 - E_r)\,c\}\, dx$$

and by substitution from equations (5), (6) and (8),

$$H_r = -\frac{1}{2} c^2 \rho V^2 \int_{\varphi_r}^{\pi} \left\{A_0 (1 + \cos\theta) + \sum_{n=1}^{\infty} A_n \sin n\,\theta \sin\theta\right\} (\cos\varphi_r - \cos\theta)\, d\theta$$

So that

$$H_r = -\frac{1}{2} c^2 \rho V^2 \left\{ A_0 K_r + \sum_{n=1}^{\infty} A_n K_{rn} \right\}$$

where

$$K_r = \sin \varphi_r \left(1 - \frac{1}{2} \cos \varphi_r\right) - (\pi - \varphi_r)\left(\frac{1}{2} - \cos \varphi_r\right)$$

and the general term for K_{rn} when $n > 2$ is given by

$$K_{rn} = \frac{1}{4}\left(\frac{\sin (n-2)\varphi_r}{n-2} - \frac{\sin (n+2)\varphi_r}{n+2}\right) - \frac{\cos \varphi_r}{2}\left(\frac{\sin (n-1)\varphi_r}{(n-1)} - \frac{\sin (n+1)\varphi_r}{n+1}\right)$$

and the particular values when $n = 2$, and $n = 1$ are

$$K_{r_2} = \frac{1}{4}\left\{-(\pi - \varphi_r) - \frac{\sin 4\varphi_r}{4}\right\} - \frac{\operatorname{Cos} \varphi_r}{2}\left\{\sin \varphi_r - \frac{\sin 3\varphi_r}{3}\right\}$$

$$K_{r_1} = \frac{1}{4}\left\{\sin \varphi_r - \frac{\sin 3\varphi_r}{3}\right\} - \frac{\cos \varphi_r}{2}\left\{-(\pi - \varphi_r) - \frac{\sin 2\varphi_r}{2}\right\}$$

Putting the values of A_0 and A_n from equation (10) in the hinge moment equation found above then

$$H_r = -\frac{1}{2} c^2 \rho V^2 \left[K_r \left\{\alpha' + \sum_{s=1}^{p} \frac{\pi - \varphi_s}{\pi} \xi_s\right\} + \sum_{n=1}^{\infty}\left\{\frac{2K_{rn}}{n\pi} \sum_{s=1}^{p} \xi_s \sin n\varphi_s\right\}\right]$$

and comparing it with the equation

$$H_r = -E_r^2 c^2 \rho V^2 \left[a_1 \beta_r \alpha' + \sum_{s=1}^{p} (b_{rs} + a_1 \beta_r \lambda_s)\xi_s\right]$$

we have

$$\beta_r = \frac{K_r}{2\pi E_r^2} \qquad \text{since } a_1 = \pi$$

and if

$$C_{rs} = \sum_{n=1}^{\infty} \frac{2\,K_{rn}}{n\,\pi} \sin n\,\varphi_s$$

we have

$$b_{rs} + a_1\,\beta_r\,\lambda_s = \frac{1}{2\,E_r^2}\left\{C_{rs} + K_r\,\frac{\pi - \varphi_s}{\pi}\right\}$$

or

$$b_{rs} = \frac{1}{2\,E^2_r}\left\{C_{rs} - \frac{K_r \sin\varphi_s}{\pi}\right\}$$

The summation C_{rs} is carried out in Appendix I where it is shown that

$$C_{rs} = \frac{\sin\varphi_s\,\sin\varphi_r}{2\,\pi} + \frac{\pi - \varphi_r}{\pi}\sin\varphi_s\left(\cos\varphi_r - \frac{1}{2}\cos\varphi_s\right) + \frac{(\cos\varphi_s - \cos\varphi_r)^2}{4\,\pi}\log\frac{1 - \cos(\varphi_s + \varphi_r)}{1 - \cos(\varphi_s - \varphi_r)}$$

and the value of the summation when $s = r$ is given by

$$C_{rr} = \frac{\sin^2\varphi_r}{2\pi} + \frac{\pi - \varphi_r}{\pi}\,\frac{\sin 2\,\varphi_r}{4}$$

It should be noted that the summation C_{rs} is not identical with or equal to the summation C_{sr}.

6. *Summary of results.*—The formulae which have been derived for the parameters

$$\lambda_s,\ m_s,\ \beta_r,\ b_{rs}$$

determine completely the lift, pitching moment, and hinge moments, for an aerofoil with any multiply hinged flap system, since they apply equally to two dimensional motion or to a finite rectangular aerofoil. The only other parameter is a_1 and its value under different conditions is given in Table 1.

Summarising the results for the other parameters we have

$$\lambda_s = \frac{\pi - \varphi_s}{\pi} + \frac{\sin\varphi_s}{\pi}$$

$$m_s = \frac{1}{4}\sin\varphi_s\,(1 - \cos\varphi_s)$$

$$\beta_r = \frac{K_r}{2\,\pi\,E_r^2}$$

and

$$b_{rs} = \frac{1}{2E_r^2}\left\{ C_{rs} - \frac{K_r \sin \varphi_s}{\pi} \right\}$$

where

$$K_r = \sin \varphi_r \left(1 - \frac{1}{2} \cos \varphi_r\right) - (\pi - \varphi_r)\left(\frac{1}{2} - \cos \varphi_r\right)$$

and

$$C_{rs} = \frac{\sin \varphi_s \sin \varphi_r}{2\pi} + \frac{\pi - \varphi_r}{\pi} \sin \varphi_s \left(\cos \varphi_r - \frac{1}{2} \cos \varphi_s\right) + \frac{(\cos \varphi_s - \cos \varphi_r)^2}{4\pi} \log \frac{1 - \cos(\varphi_s + \varphi_r)}{1 - \cos(\varphi_s - \varphi_r)}$$

The complete expressions for lift and pitching moment of the aerofoil with p flaps and the hinge moment coefficient about any hinge position A_r may be summarised as

$$k_L = a_1 (\alpha' + \sum_{s=1}^{p} \lambda_s \xi_s)$$

$$k_m = -\frac{1}{4} k_L - \sum_{s=1}^{p} m_s \xi_s$$

$$k_{H_r} = -\beta_r k_L - \sum_{s=1}^{p} b_{rs} \xi_s$$

All the parameters have been evaluated. The results for parameters λ_s, m_s, and β_r, are given in Table 2, and are shown plotted in Figs. 2–4. The values of the parameter b_{rs} (which includes also the parameter b_{rr}) are given in Table 3, and are plotted in Fig. 5. It should be noted that the parameter b_{rs} values in Table 3 and Fig. 5 refer to the equation for hinge moments about A_r.

7. *Application to aerofoil fitted with servo operated flap.*—In this case $p = 2$, and the flap positions are defined by φ_1 and φ_2, and the flap angles by ξ_1, ξ_2.

Then the following relationships hold :—

for lift coefficient

$$k_L = a_1 (\alpha' + \lambda_1 \xi_1 + \lambda_2 \xi_2)$$

for pitching moment

$$k_m = -\frac{1}{4} k_L - m_1 \xi_1 - m_2 \xi_2$$

for aileron hinge moment

$$k_{H_1} = -\beta_1 k_L - b_{1,1} \xi_1 - b_{1,2} \xi_2$$

and for servo hinge moment

$$k_{H_2} = -\beta_2 k_L - b_{2,1}\xi_1 - b_{2,2}\xi_2$$

The parameters in these equations can be readily written down from the results summarised in section 6.

8. *Comparison with experimental results.*—Tests were carried out on a R.A.F. 28 aerofoil fitted with a servo operated flap, and the results are given in report T.2608.* The aerofoil had an aspect ratio of 5, and flaps of 0·3 of the chord, and a servo of 0·09 of the chord. The values of the various parameters deduced from the experimental results are compared with the theoretical values in the following table :—

Comparison of Experimental Results with Theory for Aerofoil R.A.F. 28 with servo operated flap.

$E_1 = 0{\cdot}30$ $E_2 = 0{\cdot}09$ Aspect ratio $= 5$.

Parameter ..	a_1	λ_1	λ_2	m_1	m_2	$-\beta_1$	$b_{1,1}$	$b_{1,2}$
Theory ..	2·16	0·66	0·38	0·32	0·26	0·10	0·28	0·58
Experiment	2·04*	0·63	0·26	0·30	0·19	0·10†	0·22	0·37

* Deduced from R. & M. 1027.
† Assumed.

Experimental data were only available at one angle of incidence, viz., 2·0°, so that to compare the experimental results with the theory the parameter a_1 has been deduced from the lift results of R. & M. 1027† and β_1 has been assumed equal to the theoretical value.

The agreement between theory and experiment for the parameters λ_1, m_1, and $b_{1,1}$, is fairly good, but the agreement for λ_2, m_2 and $b_{1,2}$ is not so good. A further comparison can be made, using the experimental results contained in N.A.C.A. report No. 278.‡ In this report results are given for an aerofoil of symmetrical section, M–1, fitted with a hinged flap. The aerofoil was rectangular in plan form, and of aspect ratio 3, and the flap had a chord of 0·3c. Correcting the results given in the report for tunnel constraint, then the comparison of the theory and experiment becomes :—

Parameter ..	a_1	λ_1	$-\beta_1$	$b_{1,1}$
Theory	1·80,	0·66	0·10	0·28
Experiment ..	1·56,	0·64	0·08	0·27

As in the previous case, the agreement between theory and experiment for the parameters λ_1, and $b_{1,1}$ is fairly good.

* T.2608.—Wind Tunnel Tests on aerofoil R.A.F.28 fitted with servo operated flap. Perring (unpublished).

† R. & M. 1027.—Tests of two aerofoils R.A.F.27 and R.A.F.28. Hartshorn and Davies.

‡ N.A.C.A. Report No. 278.—Lift, drag and elevator hinge moments for Handley Page control surfaces. R. H. Smith.

APPENDIX I.

Summation

$$\sum_{n=1}^{\infty} \frac{2\,K_{rn}}{n\,\pi} \sin n\,\varphi_s$$

When the general term of K_{rn} for $n > 2$ is given by

$$K_{rn} = \frac{1}{4}\left(\frac{\sin(n-2)\,\varphi_r}{(n-2)} - \frac{\sin(n+2)\,\varphi_r)}{(n+2)}\right)$$
$$- \frac{\cos\varphi_r}{2}\left(\frac{\sin(n-1)\,\varphi_r}{(n-1)} - \frac{(\sin(n+1)\,\varphi_r}{(n+1)}\right)$$

and the particular values when $n = 2$ and $n = 1$ are

$$K_{r_2} = \frac{1}{4}\left\{-(\pi - \varphi_r) - \frac{\sin 4\,\varphi_r}{4}\right\}$$
$$- \frac{\cos\varphi_r}{2}\left\{\sin\varphi_r - \frac{\sin 3\,\varphi_r}{3}\right\}$$

$$K_{r_1} = \frac{1}{4}\left\{\sin\varphi_r - \frac{\sin 3\,\varphi_r}{3}\right\}$$
$$- \frac{\cos\varphi_r}{2}\left\{-(\pi - \varphi_r) - \frac{\sin 2\,\varphi_r}{2}\right\}$$

To avoid these particular values it will be convenient to carry out the summation from 3 to ∞, and add the terms involving $K_{r,2}$ and $K_{r,1}$. We can then write—

$$\sum_{n=3}^{\infty} \frac{2\,K_{rn}}{n\,\pi} \sin n\,\varphi_s$$

$$= \frac{2}{\pi}\left[\frac{1}{4}\left(\sum_{n=3}^{\infty}\left\{\frac{\sin n\,\varphi_s \sin(n-2)\,\varphi_r}{n\,(n-2)} - \frac{\sin n\,\varphi_s \sin(n+1)\,\varphi_r}{n\,(n+1)}\right\}\right)\right.$$

$$\left. - \frac{\operatorname{Cos}\varphi_r}{2}\left(\sum_{n=3}^{\infty}\left\{\frac{\sin n\,\varphi_s \sin(n-1)\,\varphi_r}{n\,(n-1)} - \frac{\sin n\,\varphi_s \sin(n+1)\,\varphi_r}{n\,(n+1)}\right\}\right)\right]$$

and if we again write this as a summation from 1 to ∞ then

$$\sum_{n=3}^{\infty} \frac{2\, K_{rn}}{n\, \pi} \sin n\, \varphi_s$$

$$= \frac{2}{\pi} \left[\frac{1}{4} \left\{ \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2} \right) (\sin (n+2)\, \varphi_s \sin n\, \varphi_2 \right.\right.$$

$$\left. - \sin n\, \varphi_s \sin (n+2)\, \varphi_r) \right\}$$

$$+ \frac{1}{4} \left(\frac{\sin \varphi_s \sin 3\, \varphi_r}{1 \cdot 3} + \frac{\sin 2\, \varphi_s \sin 4\, \varphi_r}{2 \cdot 4} \right)$$

$$- \frac{\cos \varphi_r}{2} \left\{ \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right) (\sin (n+1)\, \varphi_s \sin n\, \varphi_r \right.$$

$$\left. - \sin n\, \varphi_s \sin (n+1)\, \varphi_r) \right\}$$

$$- \frac{\cos \varphi_r}{2} \left(\frac{\sin \varphi_s \sin 2\, \varphi_r}{1 \cdot 2} + \frac{\sin 2\, \varphi_s \sin 3\, \varphi_r}{2 \cdot 3} \right.$$

$$\left.\left. - \frac{\sin 2\, \varphi_s \sin 2\, \varphi_r}{1 \cdot 2} \right) \right]$$

This can be still further simplified to

$$\frac{2}{\pi} \left[\left\{ \frac{1}{4} (\cos 2\, \varphi_s - \cos 2\, \varphi_r) \right.\right.$$

$$\left. - \cos \varphi_r (\cos \varphi_s - \cos \varphi_r) \right\} \sum_{n=1}^{\infty} \frac{\sin n\, \varphi_s \sin n\, \varphi_r}{n}$$

$$+ \frac{1}{4} \left(\frac{\sin \varphi_s \sin 3\, \varphi_r}{1 \cdot 3} + \frac{\sin 2\, \varphi_s \sin 4\, \varphi_r}{2 \cdot 4} \right)$$

$$- \frac{\cos \varphi_r}{2} \left(\frac{\sin \varphi_s \sin 2\, \varphi_r}{1 \cdot 2} + \frac{\sin 2\, \varphi_s \sin 3\, \varphi_r}{2 \cdot 3} \right.$$

$$\left.\left. - \frac{\sin 2\, \varphi_s \sin \varphi_r}{1 \cdot 2} \right) \right]$$

We have now to consider the summation

$$\sum_{n=1}^{\infty} \frac{\sin n\, \varphi_s \sin n\, \varphi_r}{n}$$

This may be written

$$\frac{1}{2} \sum_{n=1}^{\infty} \left(\frac{\cos n\, (\varphi_s - \varphi_r)}{n} - \frac{\cos n\, (\varphi_s + \varphi_r)}{n} \right)$$

Both these terms are similar, so that it is sufficient to consider the summation of one only.

Writing

$$\varphi_s - \varphi_r = \Phi$$

Then

$$\sum_{n=1}^{\infty} \frac{\cos n\,(\varphi_s - \varphi_r)}{n}$$

is equal to the real part of

$$\sum_{n=1}^{\infty} \frac{\cos n\,\Phi + i \sin n\,\Phi}{n}$$

or the real part of

$$\sum_{n=1}^{\infty} \frac{e^{\,i\,n\,\Phi}}{n}$$

We have

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{e^{\,i\,n\,\Phi}}{n} &= -\log\,(1 - e^{\,i\,\Phi}) \\ &= -\log 2 \sin \frac{\Phi}{2}\left(\sin \frac{\Phi}{2} - i \cos \frac{\Phi}{2}\right) \\ &= -\log\left(2 \sin \frac{\Phi}{2}\right) + i\left(\frac{\pi - \Phi}{2}\right)\end{aligned}$$

Consequently

$$\sum_{n=1}^{\infty} \frac{\cos n\,(\varphi_s - \varphi_r)}{n} = -\log 2 \sin\left(\frac{\varphi_s - \varphi_r}{2}\right)$$

and

$$\sum_{n=1}^{\infty} \frac{\operatorname{Sin} n\,\varphi_s \sin n\,\varphi_r}{n} = \frac{1}{2} \log \frac{\sin\left(\dfrac{\varphi_s + \varphi_r}{2}\right)}{\sin\left(\dfrac{\varphi_s - \varphi_r}{2}\right)}$$

which can be more conveniently written

$$= \frac{1}{4} \log \frac{1 - \cos\,(\varphi_s + \varphi_r)}{1 - \cos\,(\varphi_s - \varphi_r)}$$

The summation

$$\sum_{n=3}^{\infty} \frac{2\,K_{r\,n}}{n\,\pi} \sin n\,\varphi_s$$

therefore equals

$$\frac{2}{\pi}\Bigg[\frac{1}{8}(\cos\varphi_s-\cos\varphi_r)^2\log\left\{\frac{1-\cos(\varphi_s+\varphi_r)}{1-\cos(\varphi_s-\varphi_r)}\right\}$$

$$+\frac{1}{4}\left(\frac{\sin\varphi_s\sin 3\varphi_r}{1\cdot 3}+\frac{\sin 2\varphi_s\sin 4\varphi_r}{2\cdot 4}\right)$$

$$-\frac{\cos\varphi_r}{2}\left(\frac{\sin\varphi_s\sin 2\varphi_r}{1\cdot 2}+\frac{\sin 2\varphi_s\sin 3\varphi_r}{2\cdot 3}-\frac{\sin 2\varphi_s\sin\varphi_r}{1\cdot 2}\right)\Bigg]$$

and combining this with the terms given by $K_{r,2}$ and $K_{r,1}$,

$$\sum_{n=1}^{\infty}\frac{2K_{rn}}{n\pi}\sin n\varphi_s$$

becomes

$$C_{rs}=\frac{\sin\varphi_s\sin\varphi_r}{2\pi}$$

$$+\frac{\pi-\varphi_r}{\pi}\sin\varphi_s\left(\cos\varphi_r-\frac{1}{2}\cos\varphi_s\right)$$

$$+\frac{(\cos\varphi_s-\cos\varphi_r)^2}{4\pi}\log\frac{1-\cos(\varphi_s+\varphi_r)}{1-\cos(\varphi_s-\varphi_r)}$$

and the particular value of this when $s=r$ can easily be shown to be

$$C_{rr}=\frac{\sin^2\varphi_r}{2\pi}+\frac{\pi-\varphi_r}{\pi}\,\frac{\sin 2\varphi_r}{4}$$

TABLE I.

Aspect Ratio	∞	8	6	4	2
Value of a_1 ..	3·14	2·42	2·27	2·01	1·52

TABLE II.

Value from curves.

Ratio E_s or E_r	λ_s	m_s	β_r	b_{rr}
0	0	0	0	0·425
0·05	0·282	0·207	0·038	0·399
0·10	0·396	0·270	0·055	0·373
0·15	0·480	0·304	0·068	0·348
0·20	0·550	0·320	0·080	0·324
0·30	0·660	0·321	0·100	0·277
0·40	0·746	0·294	0·117	0·230
0·50	0·818	0·250	0·136	0·182
0·60	0·876	0·196	0·155	0·137
0·80	0·960	0·080	0·194	0·057
1·00	1·000	0	0·250	0

TABLE III.

Value from curves.

Ratio E_s	Value of E_r								
	0	0·05	0·10	0·15	0·20	0·30	0·40	0·50	0·60
	Parameter b_{rs}								
0	0·425								
0·02	0	0·673	0·598	0·511	0·452	0·368	0·312	0·270	0·237
0·05	0	0·399	0·627	0·631	0·595	0·515	0·450	0·397	0·350
0·10	0	0·225	0·373	0·540	0·593	0·585	0·540	0·490	0·442
0·15	0	0·164	0·251	0·348	0·471	0·555	0·551	0·519	0·480
0·20	0	0·121	0·185	0·248	0·324	0·475	0·515	0·511	0·486
0·25	0	0·100	0·147	0·193	0·247	0·370	0·455	0·479	0·471
0·30	0	0·083	0·122	0·160	0·195	0·277	0·378	0·430	0·441
0·40	0	0·059	0·085	0·109	0·129	0·172	0·230	0·306	0·348
0·50	0	0·040	0·058	0·073	0·086	0·110	0·143	0·182	0·239
0·60	0	0·028	0·039	0·046	0·055	0·075	0·089	0·109	0·137

Note.—E_r is the ratio defining the particular hinge position about which the moment is being considered.

E_s is the ratio for any hinge position.

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FIG. 1.

AEROFOIL WITH MULTIPLY HINGED FLAP SYSTEM.

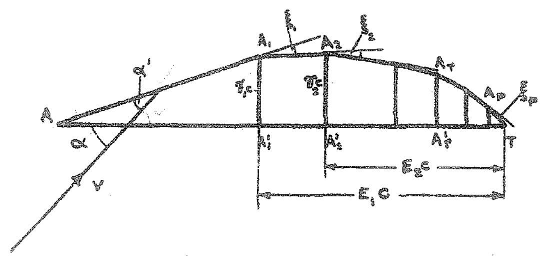

FIG. 2.

PARAMETER FOR LIFT COEFFICIENT EQUATION.
VALUES OF λ_s

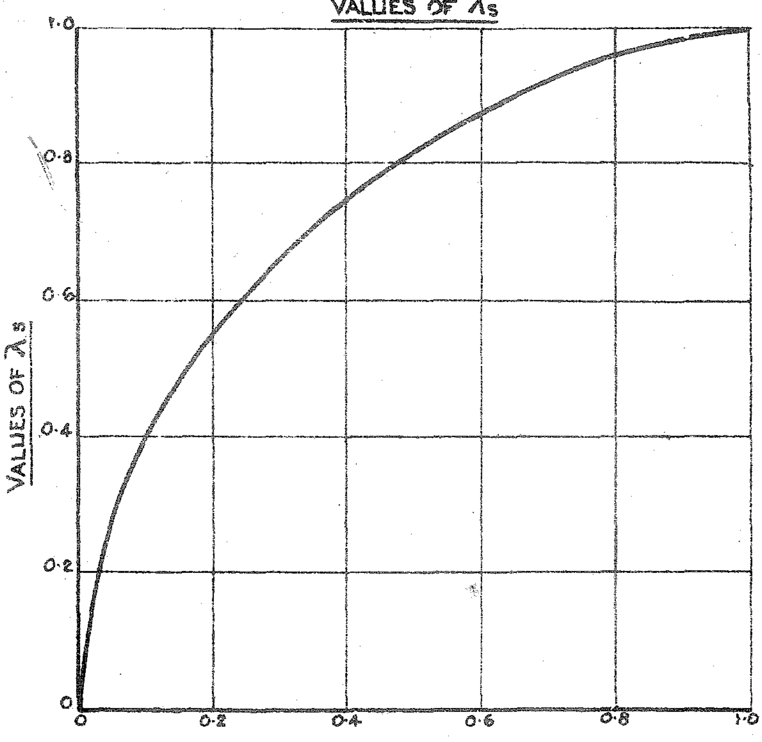

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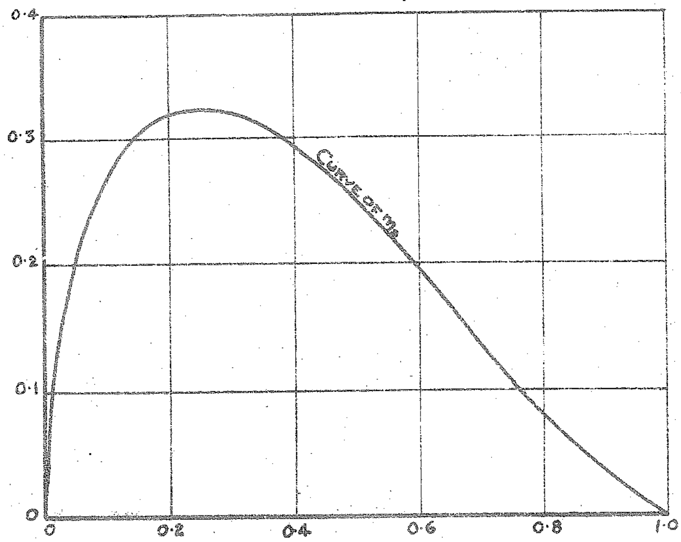
FIG. 3.

AEROFOIL WITH MULTIPLY HINGE FLAP SYSTEM.

PARAMETER OF PITCHING MOMENT EQUATION.

VALUES OF m_s

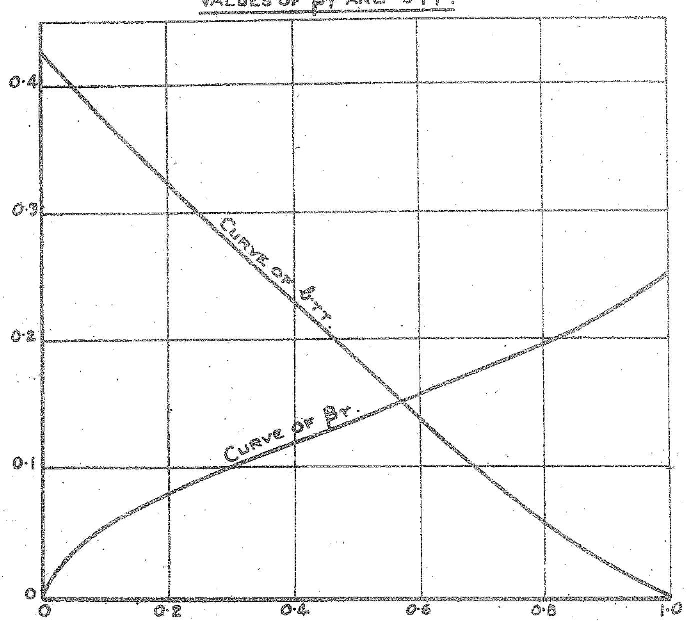


RATIO E_s

FIG. 4.

PARAMETER OF HINGE MOMENT EQUATION.

VALUES OF β_γ AND $b_{\gamma\gamma}$.



HINGE POSITION E_γ.

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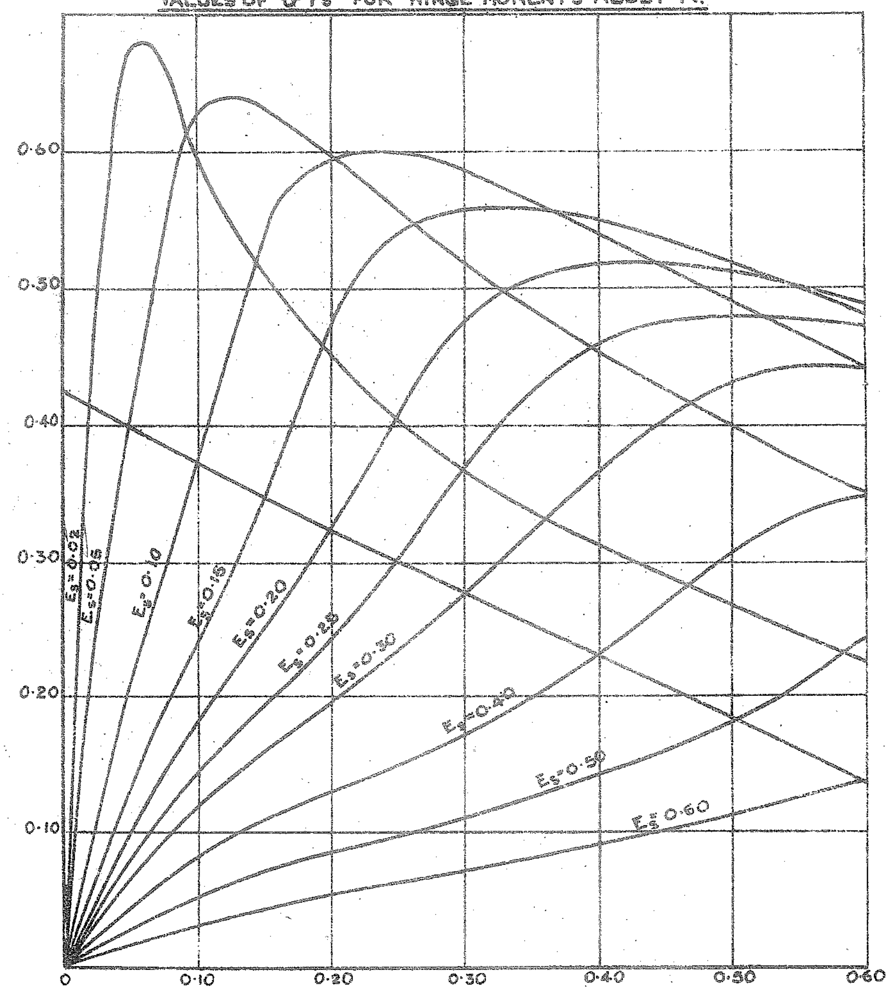
AEROFOIL WITH MULTIPLY HINGED FLAP SYSTEM.

PARAMETERS OF HINGE MOMENT EQUATION.

VALUES OF $\mathcal{E}_{YS}$ FOR HINGE MOMENTS ABOUT R.



SCALE OF E_Y

NOTE:- E_Y IS THE RATIO DEFINING THE PARTICULAR HINGE POSITION ABOUT WHICH THE MOMENT IS BEING CONSIDERED.

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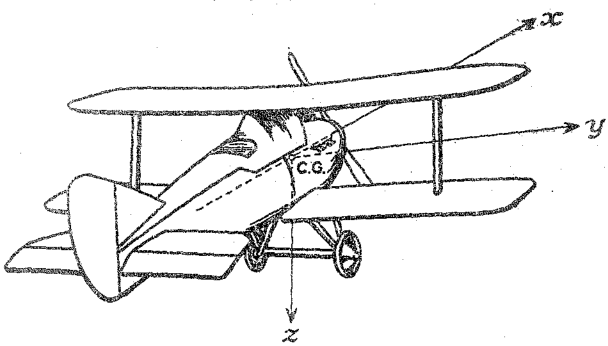
SYSTEM OF AXES.



Axes	Symbol Designation / Positive direction	x longitudinal / forward	y lateral / starboard	z normal / downward
Force	Symbol	X	Y	Z
Moment	Symbol Designation	L rolling	M pitching	N yawing
Angle of Rotation	Symbol	ϕ	θ	ψ
Velocity	Linear Angular	u p	v q	w r
Moment of Inertia		A	B	C

Components of linear velocity and force are positive in the positive direction of the corresponding axis. Components of angular velocity and moment are positive in the cyclic order y to z about the axis of x, z to x about the axis of y, and x to y about the axis of z.

The angular movement of a control surface (elevator or rudder) is governed by the same convention, the elevator angle being positive downwards and the rudder angle positive to port. The aileron angle is positive when the starboard aileron is down and the port aileron is up. A positive control angle normally gives rise to a negative moment about the corresponding axis. The symbols for the control angles are :—

ξ aileron angle
η elevator angle
η_T tail setting angle
ζ rudder angle