The Effect of Blade Twist on the Characteristics of the C.30 Autogiro

By

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SUMMARY.—*Introductory.*—An analysis of the blade motion and force characteristics of the standard Cierva C.30 autogiro rotor is made, taking into account the torsional flexibility of the blades. The results are applied to the steady motion and pitching equilibrium of the whole machine.

Range of investigation.—Using the physical constants of the blades, the analysis has been carried out for the cases of a mean profile drag coefficient over the blade elements equal to 0·014 and 0·012, and a speed range from zero to 130 m.p.h. The most important assumption of the present investigation is that the blades remain straight under all circumstances. The other approximations are not expected to have any great influence.

Conclusions.—The blades are found to twist to the extent of several degrees, in the sense that the mean pitch angle (at any radius) round the circle is decreased and that superimposed on this there is a periodic variation. Both effects increase with the forward speed until at the highest speed the outer portion of the advancing blade is twisted to below the no-lift angle of the section.

As a result of thus taking torsional flexibility into account the axial thrust of the rotor corresponding to the observed rotational speed is reduced to a more nearly constant value, which is dependent upon the mean profile drag coefficient assumed. At the same time the longitudinal force is reduced by 40–60 per cent., but the final lift/drag ratio is not affected very appreciably, being decreased by only some 8 per cent. at its maximum.

Applying the results to the motion of a complete machine much better agreement is now found with the experimental values obtained for incidence and stick position in gliding tests at the R.A.E., Farnborough.[5] In particular, the somewhat anomalous reversal of stick position at the higher speeds is predicted.

Further developments.—An attempt should be made on the more complex problem of the bending of the blades, where the inertia is not negligible as is the torsional moment of inertia of the section.

In addition further consideration may be needed with regard to the questions of tip loss and varying induced flow over the disc.

Wind tunnel measurements of the fuselage drag and rotor downwash on to the tail are very desirable in order to make a more complete comparison of performance.

The question of longitudinal and lateral stability can also be attacked from the theoretical side.

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1. *Introduction.*—A general theory of the autogiro was first formulated by Glauert in R. & M. 1111;[1] this was extended in R. & M. 1127[2] to give a better approximation at high rates of advance, and to the flapping motion. The theory has been further extended by Wheatley[3] to take account of a blade pitch angle which is not constant along the blade radius, the case of a linear variation being considered. In view of the fact that the blades of the standard Cierva C.30 autogiro are known to twist periodically in flight, it seemed worth while to attempt a further extension of the theory to the case where the blades are flexible (in torsion) and their twist is dependent on the resultant of the aerodynamic and other forces and couples acting on them.

The authors wish to acknowledge their indebtedness to some notes on blade twist communicated to the A.R.C. by Señor J. de la Cierva.

The importance of flexibility in bending is also subsequently discussed, but detailed consideration is reserved for a possible later note.

In the present work frequent reference is made to the theory of R. & M. 1127, Part II,[2] and the notation of that paper is adopted (see Appendix I), except that λ is used in place of x as a coefficient of effective normal velocity through the rotor disc, b the number of blades is replaced by N, and the angle of pitch θ is re-defined. Also, to accord with the new convention, the coefficients a and δ now have double their previous values. For the present purpose it was considered sufficiently accurate to retain only first harmonic terms in all expressions depending on the rotation of the rotor, although some estimate is made of the effect of second harmonic terms.

2. *Forces on an element of blade.*—The C.30 autogiro blade consists essentially of a tubular spar to which the canvas covered plywood forming the blade surface is secured by means of equally spaced ribs. By test on a complete blade it has been found that most (about 89 per cent.) of the resistance to twist is given by the spar, and it will be convenient and sufficiently accurate to assume the whole twisting effect to take place about the centre line of this spar.

The calculation for the C.30 blades is considerably simplified by the fact that the aerodynamic forces may be reduced to a lift acting through the centre line of the spar, (0·23 chords from the leading edge) together with a pitching moment which is constant for all fairly small angles of incidence and a drag whose effect on the twist is negligible. This statement is based on tests in the Compressed Air Tunnel at full scale Reynolds number $2{\cdot}08 \times 10^6$, on an aerofoil whose section (Fig. 10 at the end of the report) agrees closely with that of the C.30 autogiro blade. On correction to infinite aspect ratio these gave a lift coefficient slope $a = \frac{dC_L}{d\alpha} = 5{\cdot}72$, a no-lift angle $-2{\cdot}58°$, and a pitching moment coefficient, when the lift acts through the spar centre, $C_M = -0{\cdot}052$ (for incidences between $-4°$ and $+9°$). The measured values are given in Table 5, which is also reserved to the end.

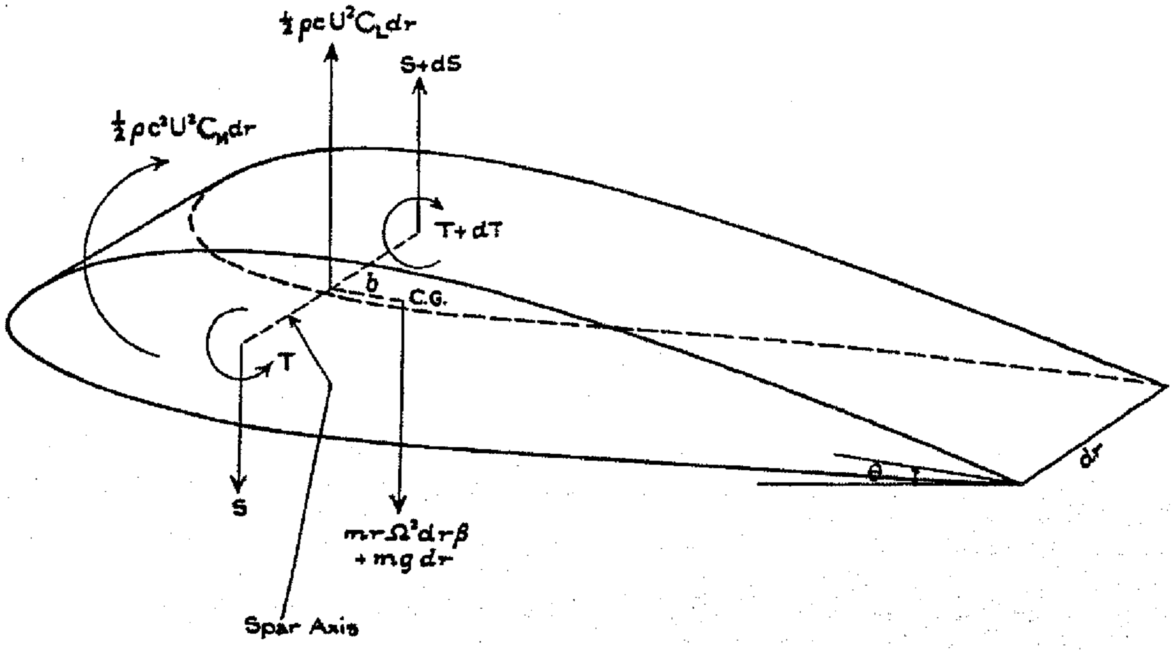

Fig. 1.—Components in the plane perpendicular to the spar axis of forces and couples acting upon an element of blade.

The system of components in the plane perpendicular to the spar axis of forces and couples acting upon an element of length dr is as shown in Fig. 1. S and T represent the shearing force and twisting couple acting across the end of the section, S being chosen to pass through the spar axis and T adjusted accordingly. Centrifugal force introduces a term whose component perpendicular to the spar axis is $mdr\, \Omega^2\, r\beta$, and gravity a term $mg\, dr$, m being the mass per unit length of blade and β being small. These act through the centre of gravity of the blade, which is at a distance b ($= 0{\cdot}065$ chord) behind the spar axis. In accordance with the remark above, all these forces are taken as directly applied to the spar, and the discontinuities in the actual case of spaced ribs are ignored.

The acceleration of the element normal to the spar in the plane containing the rotor axis is given by the equation of motion :

$$m\,dr\;.\;r\ddot{\beta} = dS + \tfrac{1}{2}\varrho c\,dr\;.\;U^2\,C_L - m\,dr\,\Omega^2\,r\beta - mg\,dr. \quad .. \quad (1)$$

The moment of inertia of the section is very small, so that we may assume that for rotation in the plane perpendicular to the spar axis the couples are in equilibrium. Thus, taking moments about the centre of gravity :

$$0 = dT + b\;.\;dS + \tfrac{1}{2}\varrho c^2\,dr\,U^2\,C_M + \tfrac{1}{2}\varrho c\,dr\,U^2\,C_L\,b\,. \quad .. \quad (2)$$

Eliminating dS ;

$$-\,dT = m\,b\,r\,(\ddot{\beta} + \Omega^2\beta)dr + m\,b\,g\,dr + \tfrac{1}{2}\varrho c^2\,U^2\,C_M\,dr. \quad .. \quad (3)$$

Assuming the blades to be always straight, β is independent of radius and can be put equal to $a_0 - a_1 \cos\psi - b_1 \sin\psi$, neglecting terms in 2ψ and higher orders. Also, as in R. & M. 1127 (page 31),

$$U = r\,\Omega + \mu R\,\Omega \sin\psi\,.$$

Hence ;

$$-\,dT/dr = m\,b(a_0\,\Omega^2 r + g) + \tfrac{1}{2}\varrho c^2\,C_M\,\Omega^2\,(r + \mu R\sin\psi)^2\,. \quad .. \quad (4)$$

2.1. *Differential equation of twist.*—The twisting couples on the ends of an element dr of the spar being $-$ T and T $+\,dT$, the twist in the element is

$$d\theta = K\,T\,dr$$

where $\frac{1}{K}$ is the torsional stiffness as determined by statical tests on the blade. Hence we have

$$-\,d^2\theta/dr^2 = \tfrac{1}{2}K\varrho c^2\,C_M\,\Omega^2\,(r + \mu R\sin\psi)^2 + K\,m\,b\,(a_0\,\Omega^2 r + g)\ldots \quad (5)$$

If we define θ as the pitch angle measured from the chord, and θ_{root} as its setting at the root, the end conditions to be fulfilled are

$$\begin{cases} \theta = \theta_{root} \text{ when } r = 0\,, \\ \dfrac{d\theta}{dr} = 0 \quad \text{when } r = R. \end{cases}$$

The integration is readily effected, and leads to the result :

$$\theta = \frac{1}{12}A\,(x^4 - 4x) + \left(\frac{1}{3}A\,\mu\sin\psi - \frac{1}{6}Ba_0\right)(x^3 - 3x)$$
$$+ \left(\frac{1}{2}A\,\mu^2\sin^2\psi - \frac{1}{2}C\right)(x^2 - 2x) + \theta_{root} \quad .. \quad .. \quad (6)$$

where

$$\begin{cases} x = r/R \\ A = -\tfrac{1}{2}K\varrho c^2 C_M R^4 \Omega^2 \\ B = K\, m\, b\, R^3\, \Omega^2 \\ C = K\, m\, b\, R^2 g\,. \end{cases}$$

In Fig. 2 are drawn curves showing the variation of θ along the blade for a particular set of values of Ω, μ and a_0 and four positions of a blade in azimuth.

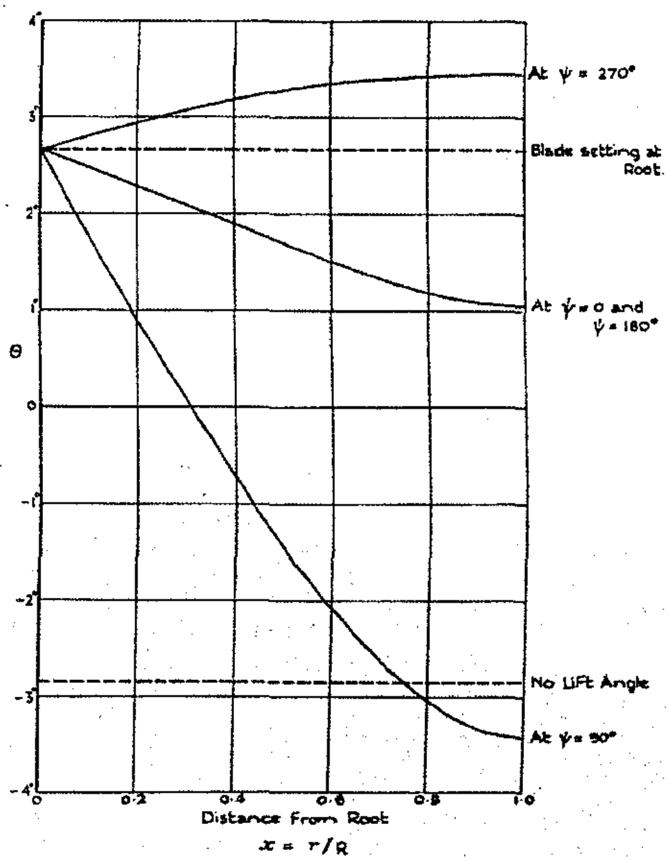

FIG. 2.—Blade Twist in the C.30 Rotor.

For the particular case $\delta = 0{\cdot}014$, $\mu = 0{\cdot}4$, $\Omega = 245$ r.p.m., $a_0 = 5{\cdot}32°$ } Forward speed 129 m.p.h.

It should be remarked that no allowance has here been made for the breakdown of the above theory in the reversed velocity region, due to violation of the conditions for constancy of C_M; but since for $\psi = 270°$ the velocities are low and the twist small there is little loss of accuracy from this cause.

2.2. *Approximate formula for the pitch.*—The expression for the pitch angle (equation (6)) is somewhat complicated and further detailed discussion is relegated to Appendix IV where the general rotor equations are solved for two working conditions only. In the first draft of the report the approximate formula $\theta = \theta_0 + \theta_1 \sin \psi$ was adopted, the values of θ_0 and θ_1 being chosen to make θ coincide as nearly as possible with its value at 0·7R from the root, as given by the exact expression. This position was taken since owing to the higher velocity there the outer portions of the blades are much the most important. In Appendix IV the results of this approximation are compared with the more accurate values and the difference is seen to be negligible except for a small increase in thrust.

On this approximation we have :

$$\left\{\begin{aligned} \theta_0 &= 0{\cdot}293\, a_0 B - (0{\cdot}213 + 0{\cdot}228\mu^2)A + 0{\cdot}455C + \theta_{root} \quad \ldots \quad (7) \\ \theta_1 &= -\, 0{\cdot}586\, \mu A \quad \ldots \quad (8) \end{aligned}\right.$$

The values of the constants for the C.30 machine are given in the following table :—

TABLE 1

Rotor Characteristics

Torsional Stiffness 1/K	17,720* lb.-ft./radian per foot run of blade.
Chord c	0·917* feet.
Tip Radius R	18·5* feet.
Mass of blade per unit length m	0·0615* slug per foot.
Distance b (Fig. 1)	0·06* foot.
Geometrical Pitch at Root θ_{root}	0·0465* radians.
C_M	— 0·052.

Equations (7) and (8) therefore become

$$\left\{\begin{aligned} \theta_0 &= (0{\cdot}1694\, a_0 - 0{\cdot}0320 - 0{\cdot}0342\mu^2)\, \Omega'^2 + 0{\cdot}0475 \quad \ldots \quad (9) \\ \theta_1 &= 0{\cdot}0879\mu\, \Omega'^2 \quad \ldots \quad (10) \end{aligned}\right.$$

where $\Omega' = 0{\cdot}0477\, \Omega$ = ratio of the rotor angular velocity to a standard value of 200 r.p.m.

3. *The general rotor equations.*—The equations of R. & M. 1127, Part II, expressing the thrust, thrust moment, drag and torque of the rotor have to be modified when varying pitch is introduced. It simplifies matters to consider in the first place

* These values are derived from measurements on the full scale blades made at the R.A.E. A couple of 10 lb.-ft. applied near the tip of the blade produced a twist there of 0·535° when a section 16 feet 7 inches away was held fixed.

only first harmonic terms in ψ. Thus we have $\beta = a_0 - a_1 \cos\psi - b_1 \sin\psi$, and the expansion of equation (12) in that report is found to give as *conditions of zero thrust moment*:

$$-\frac{1}{\gamma} a_0 + \frac{1}{3}\lambda + \frac{1}{4}(1+\mu^2)\theta'_0 - \frac{1}{3}\mu\theta_1 - \frac{1}{\gamma}C' = 0 \quad \ldots \quad \ldots \quad (11)$$

(from constant term)

$$-\frac{1}{3}\mu a_0 + \frac{1}{4}\left(1+\frac{1}{2}\mu^2\right) b_1 = 0 \quad \ldots \quad \ldots \quad (12)$$

(from coefficient of $\cos\psi$)

$$\frac{1}{2}\mu\lambda - \frac{1}{4}\left(1-\frac{1}{2}\mu^2\right) a_1 + \frac{2}{3}\mu\theta'_0 - \frac{1}{4}\left(1+\frac{3}{2}\mu^2\right)\theta_1 = 0 \quad \ldots \quad \ldots \quad (13)$$

(from coefficient of $\sin\psi$)

where θ'_0 is θ_0 measured not from the chord but from the no-lift angle.

The equation of *zero torque* corresponding to (15) of R. & M. 1127 is now:

$$\lambda^2 + \mu\lambda a_1 + \frac{2}{3}\lambda\theta'_0 - \frac{1}{2}\mu\lambda\theta_1 + \frac{1}{2}\mu^2 a_0^2 - \frac{2}{3}\mu a_0 b_1 + \frac{1}{4}\left(1+\frac{3}{2}\mu^2\right) a_1^2$$
$$+ \frac{1}{4}\left(1-\frac{1}{2}\mu^2\right) a_1\theta_1 + \frac{1}{4}\left(1+\frac{1}{2}\mu^2\right) b_1^2 - \frac{\delta}{2a}(1+\mu^2) = -\frac{4}{a} q = 0; \quad (14)$$

of *Thrust*:

$$t = \frac{1}{2} a \left\{\frac{1}{2}\lambda + \frac{1}{3}\left(1+\frac{3}{2}\mu^2\right)\theta'_0 - \frac{1}{2}\mu\theta_1\right\}; \quad \ldots \quad \ldots \quad (15)$$

of *Longitudinal Force*:

$$h = \frac{1}{4}\mu\delta + \frac{1}{2} a \left\{\lambda\left(\frac{3}{4} a_1 - \frac{1}{2}\mu\theta'_0 + \frac{1}{4}\theta_1\right)\right.$$
$$+ a_0\left(\frac{1}{4}\mu a_0 - \frac{1}{6} b_1\right)$$
$$\left. + a_1\left(\frac{1}{4}\mu a_1 + \frac{1}{3}\theta'_0 - \frac{1}{4}\mu\theta_1\right)\right\} \quad \ldots \quad \ldots \quad (16)$$

4. *Solution of the equations.*—The method adopted for solving the above equations was to substitute at each μ the known values of Ω' and an assumed value of δ. The value $\delta = 0{\cdot}014$ is adopted here, having been first of all estimated in an unpublished report on the basis of Fage's observations on symmetrical aerofoils.[4] The value of C_{D_0} measured in the Compressed Air Tunnel was $0{\cdot}0106$ for a C_L value of $0{\cdot}520$, so that the assumption $\delta = 0{\cdot}014$ includes a liberal allowance for the effect of stalling of blade sections near the centre, roughness of surface,

tip loss, etc. θ_0 and θ_1 are then known in terms of a_0, and the zero thrust moment and zero torque equations can be solved directly to give λ, a_0, a_1, b_1, θ_0 and θ_1. In practice the easiest method is to assume values for λ, obtain a_0, θ_0, b_1 and a_1 in succession from the conditions of zero thrust moment (equations (11), (12) and (13)), and substitute in equation (14) to find values of q. Interpolation then gives the values consistent with $q = 0$.

t and h are easily determined from these values, and the rotor incidence i follows at once from

$$\tan i = \frac{\lambda}{\mu} + \frac{\sigma}{2\mu^2} t \left(\text{or more exactly, } = \frac{\lambda}{\mu} + \frac{\sigma}{2\mu\sqrt{\mu^2 + \lambda^2}} t\right) \quad \ldots \quad (17)$$

Lift and drag are given by

$$X = T \sin i + H \cos i$$

$$Z = T \cos i - H \sin i.$$

Also, since by energy considerations (R. & M. 1127, page 20) it can be shown that

$$\frac{X}{Z} = \frac{\delta\,(1 + 3\mu^2)}{8\mu t} + \frac{\sigma t}{2\mu^2}, \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

another value may be obtained for h, say,

$$h_E = \frac{\delta\,(1 + 3\mu^2)}{8\mu} - \frac{\lambda}{\mu} t \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

which has been verified to be algebraically identical with (16) and affords a useful check on the calculations.

5. *Numerical results.*—Figures for the rotational speed (and also for incidence and stick position) for a speed range up to 100 m.p.h. are available from the results of gliding tests at the R.A.E., Farnborough.[5] From these and by extrapolation to higher values of μ the actual values of Ω appropriate to μ have been taken and a complete calculation made to obtain the twist and flapping for $\mu = 0, 0{\cdot}1, 0{\cdot}15, 0{\cdot}2, 0{\cdot}3, 0{\cdot}35, 0{\cdot}4$ and $\delta = 0{\cdot}014$ and $0{\cdot}012$. The results are given in Table 2 and plotted in Fig. 3 for the case of $\delta = 0{\cdot}014$.

Deduced values of incidence, thrust and longitudinal force (coefficients and full values) are shown in Table 3 and Figs. 4, 5 and 6. The corresponding figures when blade twist is not taken into account, are also given, together with the experimental incidence curve obtained at Farnborough.[5]

TABLE 2

Flapping and Twisting of C.30 Blades

	μ	Ω (r.p.m.).	λ	Flapping.			Twist.	
				a_0	a_1	b_1	θ_0	θ_1
$\delta = 0{\cdot}014$	0	208	0·0160	8·54°	0	0	2·31°	0
	0·1	203	0·0146	8·37°	1·00°	1·05°	2·27°	0·49°
	0·15	206	0·0131	7·99°	1·42°	1·59°	2·16°	0·80°
	0·2	210	0·0116	7·58°	1·71°	1·99°	2·00°	1·12°
	0·3	227	0·0110	6·49°	1·75°	2·49°	1·53°	1·95°
	0·35	238	0·0132	5·88°	1·44°	2·58°	1·18°	2·50°
	0·4	251	0·0188	5·23°	0·85°	2·59°	0·74°	3·17°
$\delta = 0{\cdot}012$	0	208	0·0141	8·27°	0	0	2·27°	0
	0·1	203	0·0127	8·10°	0·96°	1·08°	2·20°	0·49°
	0·15	206	0·0113	7·72°	1·37°	1·53°	2·11°	0·80°
	0·2	210	0·0099	7·31°	1·67°	1·91°	1·96°	1·12°
	0·3	227	0·0096	6·20°	1·69°	2·41°	1·48°	1·95°
	0·4	251	0·0171	4·96°	0·70°	2·47°	0·67°	3·17°
$\delta = 0{\cdot}014$ and assuming no twist.	0	208	0·0154	8·96°	0	0	2·67° (all rows)	0° (all rows)
	0·1	203	0·0127	8·74°	1·65°	1·16°		
	0·15	206	0·0095	8·53°	2·39°	1·69°		
	0·2	210	0·0053	8·21°	3·14°	2·15°		
	0·3	227	—0·0060	7·49°	4·44°	2·85°		
	0·4	251	—0·0192	6·63°	5·47°	3·27°		

TABLE 3

Incidence and Forces of C.30 Rotor in Steady Flight

	μ	Incidence i	Coefficients.		Thrust T (lb.).	Long. force H (lb.).	X/Z
			Thrust t	Long. force h			
$\delta = 0{\cdot}014$	0	90°	0·1090	0	2,140	0	—
	0·1	21·6°	0·1066	0·00249	1,990	46_5	0·421
	0·15	11·0°	0·1022	0·00351	1,960	67	0·229
	0·2	6·60°	0·0970	0·00411	1,935	82	0·159
	0·3	3·40°	0·0841	0·00429	1,960	100	0·110
	0·35	3·00°	0·0770	0·00401	1,975	99	0·104
	0·4	3·28°	0·0695	0·00321	1,980	91_5	0·104
$\delta = 0{\cdot}012$	0	90°	0·1054	0	2,060	0	—
	0·1	20·3°	0·1029	0·00226	1,920	42	0·394
	0·15	10·1°	0·0986	0·00322	1,890	62	0·211
	0·2	6·02°	0·0940	0·00385	1,875	77	0·146
	0·3	3·08°	0·0812	0·00387	1,890	90	0·101
	0·4	3·02°	0·0661	0·00273	1,885	78	0·095
$\delta = 0{\cdot}014$ and assuming no twist.	0	90°	0·1141	0	2,240	0	—
	0·1	21·3°	0·1112	0·00381	2,070	71	0·430
	0·15	10·0°	0·1087	0·00572	2,090	110	0·230
	0·2	5·05°	0·1050	0·00701	2,095	140	0·159
	0·3	0·31°	0·0960	0·00930	2,230	216	0·102
	0·4	—1·30°	0·0867	0·01059	2,465	301	0·100

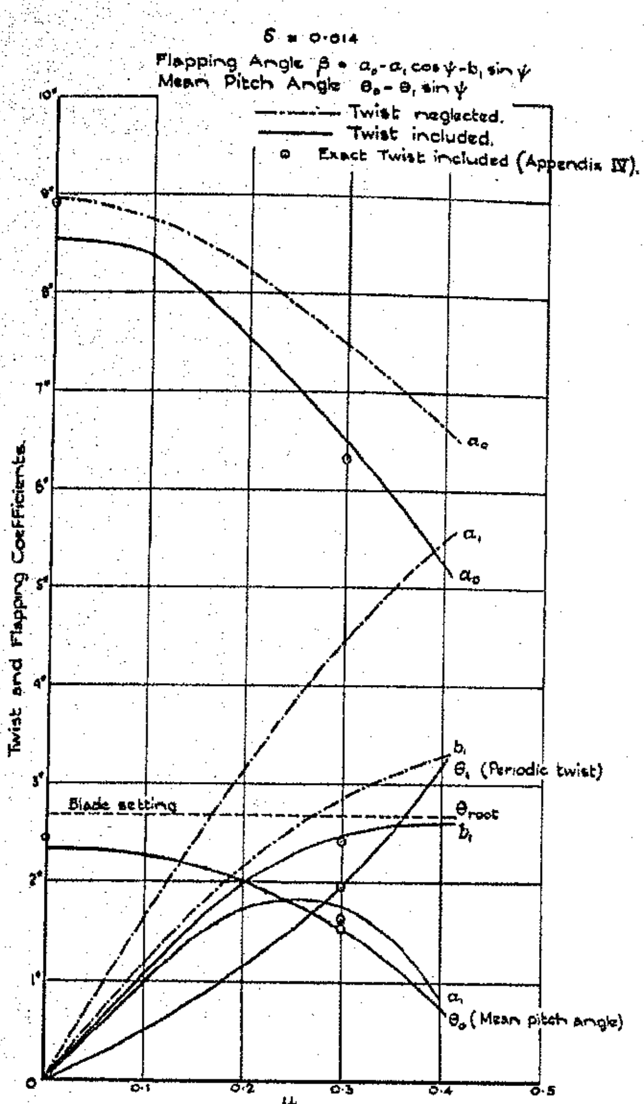

FIG. 3.—Twist and Flapping of C.30 Rotor Blades.

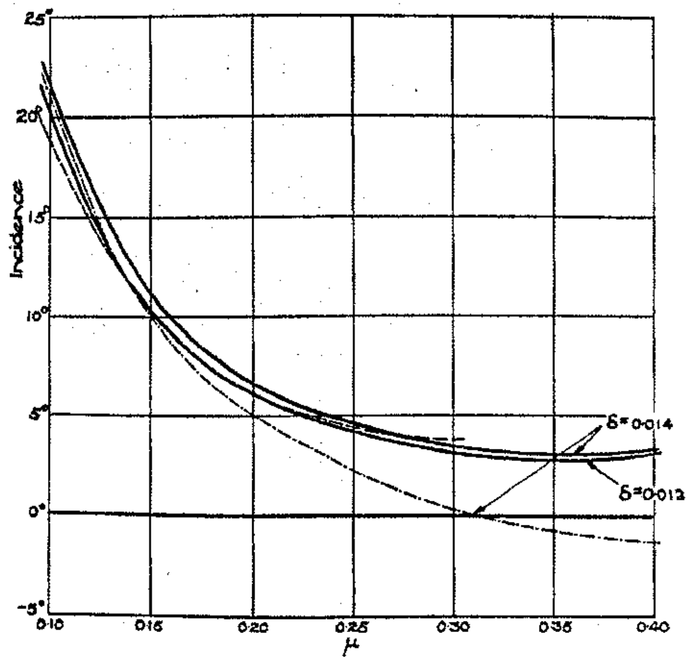

FIG. 4.—Rotor Incidence of C.30 Autogiro.

——— Allowing for twist.

—·—·— Twist neglected.

– – – – Experimental curve.

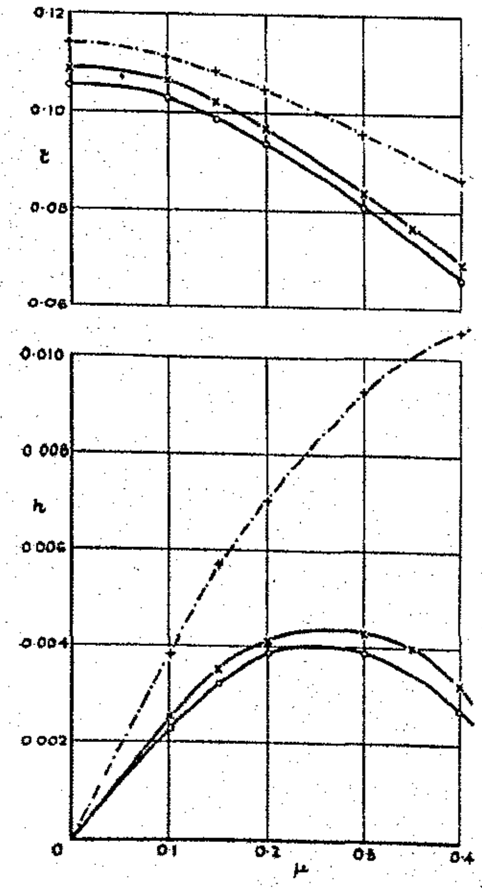

FIG. 5.—C.30 Rotor Thrust and Longitudinal Force Coefficients.

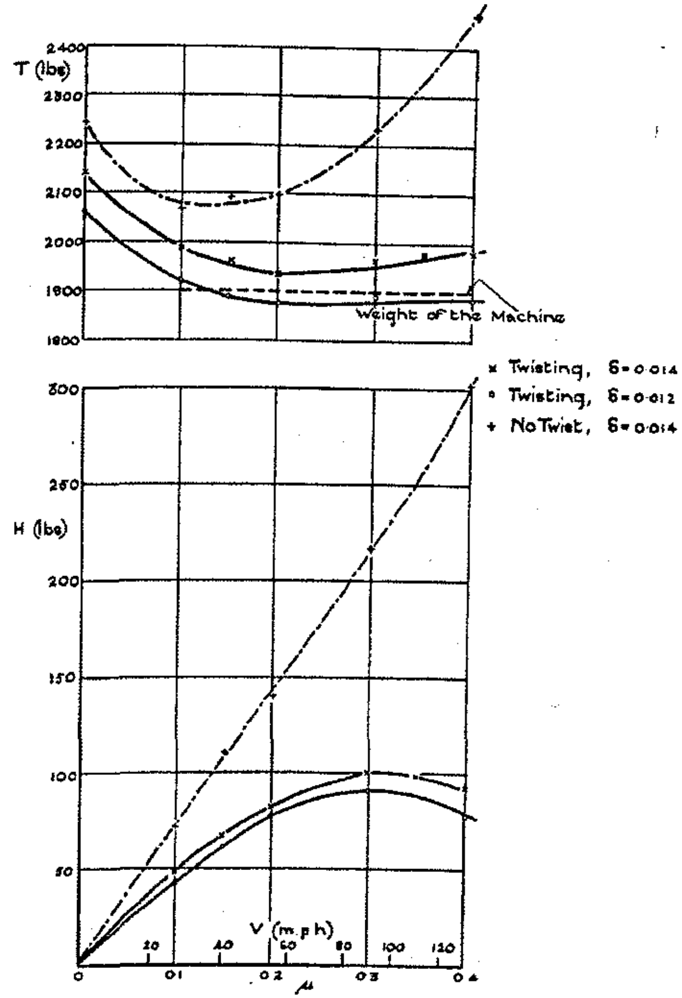

FIG. 6.—C.30 Rotor Thrust and Longitudinal Force.
(Based on rotational speed deduced from R.A.E. gliding tests.)

× Twisting, $\delta = 0{\cdot}014$ O Twisting, $\delta = 0{\cdot}012$ + No twist, $\delta = 0{\cdot}014$

For comparison with similar curves in R. & M. 1127 a further figure (7) shows the effect of blade twist and change of δ upon the drag/lift ratio (X/Z).

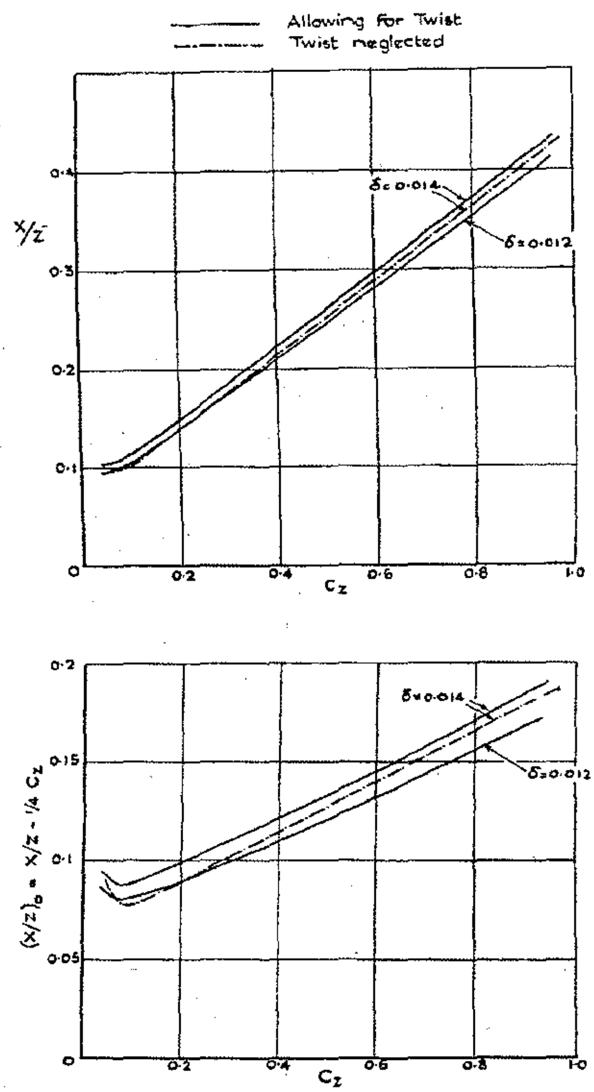

FIG. 7.—Lift and Drag of C.30 Autogiro Rotor.

The important question of pitching equilibrium and stick position is considered separately, in §7 below and Appendix II.

6. *Discussion of results.*—It is at once clear that the occurrence of blade twist has an important influence on certain of the autogiro characteristics, and brings the theoretical results into much closer accord with practice. As has been predicted by Cierva, the effect increases with speed, so that a periodic twist θ_1 of $\pm$ 1·9° at a forward speed of 89 m.p.h. ($\mu = 0\cdot3$) becomes $\pm$ 3·2° at 129 m.p.h. ($\mu = 0\cdot4$)

and at the same time the *mean* pitch angle θ_0 round the circle is decreased (Fig. 3 and table 2). Cierva's conclusion that the twist to some extent takes the place of flapping, is also confirmed, as may be seen from the fact that at $\mu = 0{\cdot}4$, a_1 is $4{\cdot}6°$ less than it would be with no twist. This is in agreement with the theory of R. & M. 1127, Part I, where a non-twisting flapping blade and a non-flapping blade whose pitch angle is varied sinusoidally around the circle are compared.

The twisting of the blades also affects the rotor attitude for steady flight (Fig. 4). As might be expected from R. & M. 1127 (page 16), the increase in incidence, $4{\cdot}58°$ at $\mu = 0{\cdot}4$ (Table 3), for example, is nearly equal to the change in a_1, the primary longitudinal component of flapping, $4{\cdot}62°$ (Table 2). The observed and calculated values of incidence are in agreement to within a degree up to the highest speed measured, and this indicates that the simplifying assumptions made in this paper are justified, at least at this stage.

The most striking effects of twist are found in the rotor force characteristics (Figs. 5 and 6). Apart from the lower end of the speed range, where the rotor angular velocity upon which the calculations are based has had to be somewhat doubtfully extrapolated from the Farnborough curves, the twist results show a fairly constant thrust. Since for steady flight and not too large incidence the thrust should nearly equal the weight of the machine (1,900 lb.), we might deduce a value for δ, except that the neglected factor of tip loss is certainly of importance as regards thrust. In fact a crude application of strip theory to the thrust and thrust moment equations (but not to the torque equation because the drag on a blade element will be little reduced), on a basis of the reduction of chord at the tip, indicates a thrust of about 90 lb. less than that so far calculated, which would bring the values for $\delta = 0{\cdot}014$ into fair agreement with the weight.

Of great interest is the much decreased longitudinal force H when twist is included. This is not only less than half as large as for infinitely stiff blades but actually begins to decrease above about 100 m.p.h. As a result, the pitching moment equation (see Appendix II) is profoundly modified, and so in consequence is the stick position for equilibrium. The effect on drag is of course not so marked, on account of the thrust component, and as may be seen from Fig. 7 the lift/drag ratio is hardly affected at all.

7. *Pitching equilibrium.*—It is known that the C.30 autogiro exhibits a curious reversal effect on the stick position to trim. To maintain steady flight at both high and low speeds the stick has to be held further back (rotor tilted in the sense of greater incidence) than for the intermediate speeds (see the experimental curve in Fig. 8). This may be considered to imply the existence of some form of instability, for since a backward movement of the control column always produces a nose up pitching moment, it follows that if the machine is flying in equilibrium at a fairly high speed and the speed then increases with the stick held fixed, a nose

down pitching moment is produced which will tend to increase the speed still further. The phenomenon is qualitatively predicted on the twist theory, as is shown by Fig. 8, in which the angle γ between the rotor axis and the perpendicular to the body datum is plotted against μ. The remaining discrepancy there shown between the theoretical value and the approximate curve obtained in gliding tests is not considered serious, in view of the critical dependence of γ on the exact fore and aft position of the centre of gravity of the whole machine, and on the aerodynamical characteristics and downwash on the tailplane.

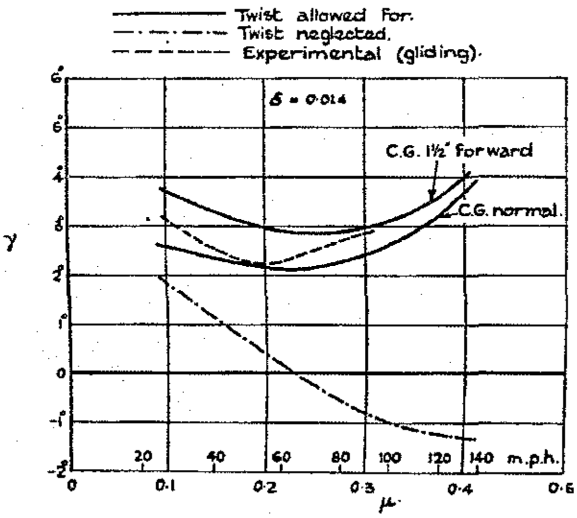

FIG. 8.—Stick Position to Trim on C.30 Autogiro.

Details of a complete calculation of pitching equilibrium are given in Appendix II, and table 4 shows the contributions of the various items to the pitching moment of the whole autogiro.

The more general problem of the pitching stability is under consideration.

8. *General theory of blade twist.*—It will be noticed that the theory given in this paper is applicable only to a rotor whose blades have the special characteristics (as in the C.30 machine) that the aerodynamic reactions can be reduced to a lift and drag and a pitching moment with *constant* coefficient when the lift is chosen to act through the twisting axis, here taken to be the centre line of the spar.

In the general case the pitching moment coefficient C_M is not constant, but is approximately linearly dependent on the incidence of the particular element under consideration. The problem is thus more complicated, but a solution is possible by a method of series, and is developed in a further Appendix III.

9. *Further developments.*—The present application of the theory of twist given above depends upon the following assumptions or approximations.

For the blades—

(1) They are assumed straight and infinitely rigid in bending ;

(2) the whole stiffness in torsion is taken as concentrated in the spar ;

(3) the discontinuities from rib to rib along the spar are neglected ;

(4) the moment of inertia of the blade section about the spar is small ;

(5) an approximate formula giving constant twist along the blade is used in the rotor equations.

For the air velocities—

(6) The component parallel to the axis of the spar of the relative velocity at a section of the blade is neglected ;

(7) end effect at the tip has not been considered ;

(8) the blades do not in fact extend to the rotor axis ;

(9) no allowance has here been made for the changed conditions over those parts of the swept disc area at which reversed velocities and large angles of attack occur ;

(10) λ is assumed constant over the swept disc area.

For the forces—

(11) the aerodynamic forces on a section may be reduced to a lift and drag acting through the spar and a pitching moment with constant coefficient ;

(12) the direct effect of drag on the twist of a section is neglected.

For the rotor in general—

(13) only first harmonic terms in ψ have been retained.

Of these, (2), (3), (11) and (12) are justifiable approximations, (7) and (5) are discussed elsewhere (§6 and Appendix IV), and (6) in R. & M. 1127 (page 3). We consider the others in order.

In regard to bending (1), an estimate has been made of the *statical* deflection which a beam of the stiffness of the autogiro blade would undergo, when subjected to the instantaneous forces given by the present theory of twist. For the particular case of $\mu = 0 \cdot 3$, this bending appears to be of the order 1 ft. upwards at the tip together with a periodic term of amplitude $1 \cdot 1$ ft. Since however the lowest frequency of oscillation of the blade in bending is calculated from the stiffness, neglecting damping, to be $3 \cdot 4$ periods per second, which is about equal to the frequency of rotation of the rotor, little reliance can be placed on this estimate.

Some recent unpublished photographs taken in flights at Farnborough[6] seem to indicate maximum flexural deformations of much smaller magnitude than this. It should be remarked that the effect of a constant curvature has been considered in R. & M. 1127, Part II.

The moment of inertia of a blade section about the spar axis (assumption 4) has been determined as 0·00231 slug ft.^{2} per ft. run of blade and gives for the whole blade a frequency of torsional oscillation of 38 per second, neglecting some reduction due to the unknown internal structural damping. This is very high compared with the fundamental frequency of rotation of the rotor, about 3·5 per second, and hence the forced oscillations at the latter frequency will be little affected.

Although the actual blades do not reach the rotor axis (assumption 8) by some 15 in. and there are heavy friction dampers between, the aerodynamic effects at the centre have been estimated as very small due to the low velocity there, and inertia forces are also unimportant on account of their small moment.

Some consideration might be given, as in Ref. 5, to those regions of the swept disc area in which the resultant velocity on the blades is in the direction from the trailing to the leading edge (assumption 9). It is obvious that the aerofoil sections are here working under quite other conditions than have been assumed above. The regions of transition offer no difficulty, since although there the angle of attack passes through 90° the velocity at the same time becomes very small. Wheatley[3] has made allowance in this respect by splitting his integral for the backward moving blade into two relevant parts and evaluating these separately.

Wheatley[3] and Glauert[1] also consider a varying induced flow of a particular type, and find that the blade motion, though not the net rotor forces, are somewhat affected. Thus it may be worth while examining the case of varying λ, when blade twist is not neglected (assumption 10).

Finally, the retention (13) of only first harmonic terms in ψ is partly justified by the results of R. & M. 1127, Part II, in which the effect of cos 2ψ and sin 2ψ terms is found to be comparatively small for the particular machine considered there. For the C.30 autogiro a very rough estimate on the basis of the R. & M. 1127 formulae (page 34) gives values of a_2 and b_2 of less than 0·5° even for the highest $\mu = 0{\cdot}4$. It therefore seems unnecessary to embark upon the very considerable increase of labour required for the inclusion of these terms.

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APPENDIX I

Notation

Dimensions and constants of blades

N		number of blades.
c		chord of blade element.
r		radius to blade element.
R		tip radius.
x	=	r/R.
σ		the solidity $= Nc/\pi R$.
θ		the pitch angle.
θ_0		mean pitch angle (see text) measured from chord line.
θ'_0		mean pitch angle (see text) measured from zero lift.
θ_1		amplitude of periodic change in pitch angle.
$1/K$		torsional stiffness of blade.
m		mass of blade per foot run.
γ	=	$c\varrho a R^4/I_1$ where I_1 is the moment of inertia of one blade about its hinge.
C′	=	$G_1/I_1\Omega^2$ where G_1 is the moment of gravity for one blade about its hinge.

Motion of blades

ψ		angular position of blade measured from the backward position.
Ω	=	$\dot{\psi}$ the angular velocity of the blade.
Ω'	=	$0{\cdot}0477\Omega$ = ratio of rotor angular velocity to a standard value (200 r.p.m.).
β		the flapping angle of a blade, measured upwards, $= a_0 - a_1 \cos\psi - b_1 \sin\psi \, (- a_2 \cos 2\psi - b_2 \sin 2\psi \ldots\ldots)$

Velocity of the air relative to the centre of the autogiro,

$\lambda R\,\Omega$	parallel to the axis.
$\mu R\,\Omega$	normal to the axis.
U	component in a plane normal to the blade of the resultant relative velocity at an element.
i	angle of incidence between normal to rotor axis and the direction of flight.

Forces on blade elements

S	shearing force, acting through centre of spar.
T	twisting couple in spar.
a	slope of lift curve of blade element.*
δ	mean profile drag coefficient of blade element.*

* a and δ have double their values in R. & M. 1127 in accordance with the usage of modern coefficients of the type C_L and C_D.

Forces on rotor as a whole

T		thrust parallel to the axis.
H		longitudinal force perpendicular to the axis.
Q		torque.
X		drag.
Z		lift, coefficient C_z (based on disc area).
t	=	$T/Nc\varrho R^3 \Omega^2$, thrust coefficient.
h	=	$H/Nc\varrho R^3 \Omega^2$, longitudinal force coefficient.
h_E		longitudinal force coefficient deduced by energy considerations.
q	=	$Q/Nc\varrho R^4 \Omega^2$.

APPENDIX II

The pitching equilibrium of the C.30 autogiro.—We may represent the forces acting on the machine as in Fig. 9, which is drawn for the general case of steady but not necessarily horizontal flight. Here Z_t is the resultant lift on the tailplane, acting

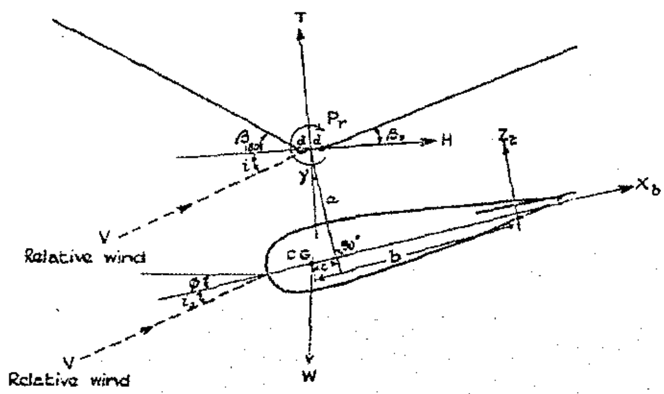

FIG. 9.

through its centre of pressure distant b from the c.g. of the whole machine; X_b the body drag; i_0 the incidence of the machine as given by the angle between the relative wind V and the fore and aft datum line, relative to which the mean angle of no-lift of the tailplane is set at $+ 2°$ ($= + 0{\cdot}035$ radian); ϕ the tilt of the datum line from the horizontal; P_r a correction on the total pitching moment due to the offset d of the blade hinges and also to the direct twisting couple at the root of a blade when in transverse position; and a, b and c are constant dimensions of the machine. As in §7, γ denotes the angle between the rotor axis and the perpendicular to the body datum.

Taking moments about the c.g. and initially assuming ϕ and γ small, the pitching moment on the machine for a given γ, i.e., given stick position is

$$T(a\gamma - c) + Ha + P_r - Z_t b.$$

Z_t may be expressed as $1\cdot5\ \varrho V^2 S_2\ (i - \gamma + 0\cdot035 - i_d)$ where the slope of the normal force curve of the tailplane, area S_2, against tailplane incidence is taken as $1\cdot5$ per radian, and i_d represents an allowance for downwash from the rotor. This latter has been calculated on the assumption that the rotor is equivalent to a monoplane aerofoil of the same lift and span with elliptic distribution of lift; [6] which leads to the result

$$i_d = 0\cdot88\,\frac{\sigma}{\mu^2}\,z$$

where

$$z\ (= Z/Nc\ \varrho R^3\ \Omega^2) = t \cos i - h \sin i.$$

P_r consists of two terms. If for the moment we consider a four-bladed rotor with two blades in the fore and aft position, then the differences between the magnitudes and directions of the forces acting at the hinges, which are offset a distance d from the rotor axis, will produce a couple tending to pitch the machine. The expression for this is found to be

$$\frac{1}{12} c\ \varrho\ a\ \Omega^2 R^3 d \left(1 - \frac{3}{2}\mu^2\right) b_1 + m\Omega^2 R^2 d\ a_1$$

and may be corrected to the actual case of 3 blades by the factor 3/4. The second term in P_r is due to the direct twisting couples transmitted at the roots of the blades, and hence is the average around the circle of $\frac{3}{K}\left(\frac{d\theta}{dr}\right)_{r=0} \sin\ \psi$, using the exact formula for θ. This is found to reduce to $\frac{3}{4}\mu\varrho c^2\ \Omega^2\ R^3 C_M$ and is therefore negative and nearly proportional to μ.

Hence for a given μ the expression for the pitching moment is a linear function of γ. The equilibrium stick position for steady flight is clearly determined by that value of γ for which this moment vanishes. Fig. 8 of §7 gives curves for the variation of γ with speed, making use of values of T and H, etc., drawn from the curves of this paper. The very much better agreement with experiments when twist is taken into account is clearly shown, together with the reversal of slope also then introduced. It should be remarked that an assumed $\delta = 0\cdot012$ gives practically identical curves, but that a different position for the c.g. of the machine modifies them considerably, as may be seen from the curve calculated for a c.g. assumed $1\frac{1}{2}$ inches forward of the actual position in the gliding tests. A comparison between the two cases of twist included and twist neglected shows clearly how the increased equilibrium value of γ at high speed in the former case is produced by a decrease in the contribution of longitudinal rotor force and to a smaller degree by a decrease in P_r.

For the equilibrium values of γ, Table 4 shows the contributions of the various terms to the total (zero) pitching moment on the machine, for the case of $\delta = 0\cdot014$, and also the restoring pitching moment on the machine when the rotor is in its equilibrium condition relative to the wind for the given speed, but the body is tilted so that γ is 1° too great (stick pulled back). It will be noted, as expected, that this is always positive and increases with speed.

TABLE 4

Pitching Moments about the Centre of Gravity

Pitching moment $T(a\gamma - c) + Ha - Z_t b + P_r$

where Z_t = net tailplane lift

$$= 1 \cdot 5\, \varrho V^2 S_2 \left(i - \gamma + 0 \cdot 035 - 0 \cdot 88 \frac{\sigma}{\mu^2} z\right)$$

a = distance between rotor centre and c.g. of machine, measured perpendicularly to the body axis, = 5·78 ft.

c = ditto, measured parallel to the body axis, = 0·42 ft.

b = distance from centre of pressure of tailplane to c.g., parallel to the body axis, = 10·4 ft.

d = offset of blade hinge from rotor centre, = 1·75 in.

μ	0·1	0·15	0·2	0·3	0·35	0·4
V (m.p.h.).	27	41	55·5	90	110	132·5
Including twist $\delta = 0 \cdot 014$						
γ for equilibrium	2·55°	2·35°	2·16°	2·43°	2·87°	3·69°
Pitching moment (lb.ft.) due to :						
Total rotor thrust, $T(a\gamma - c)$ . .	−325	−355	−391	−341	−256	−95
Longitudinal rotor force, Ha	269	387	474	578	572	529
Tailplane :						
Gross $-1 \cdot 5 \varrho V^2 S_2 (i - \gamma + 0 \cdot 035)\, b$	−507	−617	−663	−797	−855	−934
Downwash $1 \cdot 5 \varrho V^2 S_2 \left(0 \cdot 88 \frac{\sigma}{\mu^2} z\right) b$	564	585	590	598	601	606
P_r Offset of blade hinges	21	32	39	48	43	32
P_r Direct pitching moment on blades	−22	−34	−49	−84	−104	−136
Increase of γ by 1°, $0 \cdot 0175\, (Ta + 1 \cdot 5 \varrho V^2 S_2 b)$ . .	224	253	299	466	602	790
Neglecting twist $\delta = 0 \cdot 014$						
γ	1·81°		0·37°	−0·81°		−1·29°
Pitching moment due to :						
Total rotor thrust	−495		−805	−1120		−1357
Longitudinal rotor force	416		839	1250		1740
Tailplane Gross	−517		−689	−841		−1166
Tailplane Downwash	586		636	684		759
P_r Offset of blade hinges	33		67	110		162
P_r Direct pitching moment on blades	−22		−49	−84		−136
Increase of γ by 1°	234		314	495		840

APPENDIX III

More general theory of blade twist.—As a first attempt at removing the restriction to an assumed constant pitching moment (when the lift is chosen to act through the twisting axis), we consider the case of a linear variation with incidence α;

$$C_M = e\alpha + f, \text{ say.}$$

Now the velocity components of the relative wind in the plane perpendicular to the blade at the section are

$$U_x = r\Omega + \mu R\Omega \sin \psi$$

$$U_y = \lambda R\Omega - r\dot{\beta} - \mu R\Omega\beta \cos \psi$$

where U_x is also in the plane perpendicular to the rotor axis and U_y passes through the rotor axis (see R. & M. 1127, page 8).

Hence the incidence

$$\alpha = \theta + \frac{\lambda R\Omega - r\dot{\beta} - \mu R\Omega\beta \cos \psi}{r\Omega + \mu R\Omega \sin \psi} .$$

Now take $y = \frac{r}{R} + \mu \sin \psi$ and make the same approximation as before that $\beta = a_0 - a_1 \cos \psi - b_1 \sin \psi$ only. Then α reduces to

$$\alpha = (\theta - a_1 \sin \psi + b_1 \cos \psi) + \frac{1}{y}(\lambda + \mu a_1 - \mu a_0 \cos \psi) .$$

Also the differential equation of twist becomes

$$-\frac{d^2\theta}{dy^2} = \tfrac{1}{2}K\, \varrho c^2 C_M\, \Omega^2 R^4 y^2 + m\, b\, (a_0\, \Omega^2 R y + g - a_0\, \Omega^2 R\mu \sin \psi) .$$

Thus, substituting for C_M,

$$\frac{d^2\theta}{dy^2} = Ay^2\theta + By^2 + Cy + D$$

where

$$\begin{cases} A = - \quad K\, \varrho c^2 \Omega^2 R^4 e \\ B = - \tfrac{1}{2}K\, \varrho c^2 \Omega^2 R^4 \{(- a_1 \sin \psi + b_1 \cos \psi)e + f\} \\ C = - \tfrac{1}{2}K\, \varrho c^2 \Omega^2 R^4\, (\lambda + \mu a_1 - \mu a_0 \cos \psi)e - mba_0\, \Omega^2 R \\ D = + mb\, (a_0\, \Omega^2 R\mu \sin \psi - g) . \end{cases}$$

We simplify further by putting $y = A^{-1/4}z$, thus obtaining an equation of the form

$$\frac{d^2\theta}{dz^2} = \theta\, z^2 + B'z^2 + C'z + D'$$

where $B' = B/A$, $C' = A^{-3/4}C$, $D' = A^{-1/2}D$.

(If l is not negative, so that A is negative, the development proceeds very similarly if now $y = (-A)^{-1/4}z$.)

Assume a solution in series, say,

$$\theta = A_0 + A_1 z + A_2 z^2 + \ldots\ldots$$

By substitution it is easily found that

$$2.1 \quad A_2 = D'$$

$$3.2 \quad A_3 = C'$$

$$4.3 \quad A_4 = A_0 + B'$$

$$5.4 \quad A_5 = A_1$$

$$6.5 \quad A_6 = A_2$$

$$\ldots\ldots\ldots$$

Hence

$$\theta = A_0\left(1 + \frac{z^4}{4.3} + \frac{z^8}{8.7.4.3} + \frac{z^{12}}{12.11.8.7.4.3} + \ldots\right)$$

$$+ A_1\left(z + \frac{z^5}{5.4} + \frac{z^9}{9.8.5.4} + \ldots\right)$$

$$+ D'\left(\frac{z^2}{2.1} + \frac{z^6}{6.5.2.1} + \frac{z^{10}}{10.9.6.5.2.1} + \ldots\right)$$

$$+ C'\left(\frac{z^3}{3.2} + \frac{z^7}{7.6.3.2} + \ldots\right)$$

$$+ B'\left(\frac{z^4}{4.3} + \frac{z^8}{8.7.4.3} + \ldots\right).$$

For given values of all the quantities in the expressions for A, B, C and D the twist at every point would then be calculable from this equation (in which the series converge very rapidly), but in general the values of λ, a_0, a_1, b_1, for an assumed μ and Ω, are still unknown. The two constants of integration A_0 and A_1 are, of course, settled by the end conditions; in this case, that $\theta = \theta_{\text{root}}$ for $r = 0$, and $\frac{d\theta}{dr} = 0$ for $r = R$.

In order to make practical application of this formula we may adopt the same approximate method as already described in this paper, of taking the mean pitch of the blade at any azimuth ψ to be $\theta_0 - \theta_1 \sin \psi$, where the constants θ_0 and θ_1 are chosen to make the pitch angle as nearly as possible equal to that at $0{\cdot}7$R from the centre given by the exact formula. By use of the equations of thrust moment and zero torque, the solution may then be carried through as before.

Practical arrangement of the calculation.—By a systematic arrangement of the work the solution can be made considerably less laborious than might perhaps be expected; and the flapping, twist and other characteristic quantities for a given steady condition (μ and Ω specified) can be comparatively easily obtained.

The first step is to work out the values of z corresponding to $r = 0$, $0{\cdot}7$R and R and $\psi = 0$, 90°, 180°, 270°. It is to be noted that for a given rotor,

$$z = \text{constant} \times \sqrt{\Omega}\left(\frac{r}{\mathrm{R}} + \mu \sin \psi\right).$$

Then by the use of easily constructed graphs of the series functions $\left(1 + \frac{z^4}{4.3} + \frac{z^8}{8.7.4.3} + \ldots\right)$ etc. and of their derivatives $\frac{d}{dz}\left(1 + \frac{z^4}{4.3} + \ldots\right)$, θ can be worked out from the end conditions and its value at $0{\cdot}7$R for each of $\psi = 0$, 90°, 180°, 270° expressed as a linear function of B′, C′ and D′.

Substituting the corresponding expressions (in terms of λ, a_0, a_1, b_1), for B′, C′ and D′, the average round the circle of the four values of θ at $0{\cdot}7$R gives θ_0 as a linear function of λ, a_0, a_1, b_1 and half the difference between the values at 180° and 0° gives θ_1.

The three conditions of zero thrust moment and the equation of zero torque being unchanged, there are thus six relations between the quantities λ, a_0, a_1, b_1, θ_0, θ_1, five linear and one quadratic. On inspection it will be seen that these can be quickly reduced by direct substitution to one quadratic and two linear equations in λ, a_1, b_1, and these may be solved most easily by the method already proposed of assuming values for λ in the linear equations and interpolating to find that value for which the quadratic is also satisfied.

The general rotor characteristics, i, t, h, etc., are deduced as before.

Non-linear variation of pitching moment.—It is clear that a similar method of solution by series would apply to any other case where the pitching moment as here defined is expressible as a power, or sum of powers, of the incidence. But the calculations would be very laborious, and happily the need of this extension is unlikely to arise; since except when the centre of pressure for ordinary incidences lies near the twisting axis, the pitching moment coefficient is linearly dependent on the lift coefficient, which itself is usually linear with incidence.

APPENDIX IV

Use of the exact formula for twist in the rotor equations.—As explained in §2.2, wherever the rotor equations (of thrust, thrust moment, torque, etc.) involve the pitch angle θ, an approximate expression of the form $\theta_0 - \theta_1 \sin \psi$ has been used, θ_0 and θ_1 being chosen to give the best agreement with the exact expression for the pitch at radius 0·7R from the root. In order to see whether this approximation was justified a calculation with the exact formula was carried out for two cases, viz.: $\mu = 0$ and $\mu = 0 \cdot 3$, $\delta = 0 \cdot 014$.

The pitch angle measured from the no lift angle, equation (6), was rewritten:

$$\theta = P_4 x^4 + (P_3 + Q_3 \sin \psi) x^3 + (P_2 + R_2 \sin^2 \psi) x^2 + (P_1 + Q_1 \sin \psi + R_1 \sin^2 \psi) x + P_0 \quad \text{(6a)}$$

where

$$P_4 = \frac{1}{12} A ;$$

$$P_3 = -\frac{1}{6} B a_0 ; \qquad Q_3 = \frac{1}{3} A\mu ;$$

$$P_2 = -\frac{1}{2} C ; \qquad R_2 = \frac{1}{2} A\mu^2 ;$$

$$P_1 = -\frac{1}{3} A + \frac{1}{2} B a_0 + C ; \quad Q_1 = -A\mu ; \qquad R_1 = -A\mu^2 ;$$

$$P_0 = \theta_{\text{root}} .$$

The rotor equations of §3 then became :—

From *the condition of zero thrust moment,*

$$-\frac{1}{\gamma}(a_0 + C') + \frac{1}{3}\lambda + \left(\frac{1}{8} + \frac{1}{12}\mu^2\right) P_4 + \left(\frac{1}{7} + \frac{1}{10}\mu^2\right) P_3 + \frac{1}{6}\mu Q_3 + \left(\frac{1}{6} + \frac{1}{8}\mu^2\right) P_2 + \left(\frac{1}{12} + \frac{3}{32}\mu^2\right) R_2 + \left(\frac{1}{5} + \frac{1}{6}\mu^2\right) P_1 + \frac{1}{4}\mu Q_1 + \left(\frac{1}{10} + \frac{1}{8}\mu^2\right) R_1 + \left(\frac{1}{4} + \frac{1}{4}\mu^2\right) P_0 = 0 \quad \text{(11a)}$$

$$-\frac{1}{3}\mu a_0 + \frac{1}{4}\left(1 + \frac{1}{2}\mu^2\right) b_1 = 0 \quad \text{(12a)}$$

$$\frac{1}{2}\mu\lambda - \frac{1}{4}\left(1 - \frac{1}{2}\mu^2\right) a_1 + \frac{2}{7}\mu P_4 + \frac{1}{3}\mu P_3 + \left(\frac{1}{7} + \frac{3}{20}\mu^2\right) Q_3 + \frac{2}{5}\mu P_2 + \frac{3}{10}\mu R_2 + \frac{1}{2}\mu P_1 + \left(\frac{1}{5} + \frac{1}{4}\mu^2\right) Q_1 + \frac{3}{8}\mu R_1 + \frac{2}{3}\mu P_0 = 0 \quad \text{(13a)}$$

From *the condition of zero torque,*

$$\lambda^2 + \mu\lambda a_1 + \frac{1}{2}\mu^2 a_0^2 - \frac{2}{3}\mu a_0 b_1 + \frac{1}{4}\left(1 + \frac{3}{2}\mu^2\right) a_1^2 + \frac{1}{4}\left(1 + \frac{1}{2}\mu^2\right) b_1^2$$

$$- \frac{\delta}{2a}(1 + \mu^2) + \frac{1}{7}\lambda P_4 + \frac{1}{6}\lambda P_3 + \left(\frac{1}{10}\mu\lambda - \frac{1}{14}a_1 + \frac{1}{40}\mu^2 a_1\right) Q_3$$

$$+ \frac{1}{5}\lambda P_2 + \left(\frac{1}{10}\lambda - \frac{1}{20}\mu a_1\right) R_2 + \frac{1}{4}\lambda P_1 + \left(\frac{1}{6}\mu\lambda - \frac{1}{10}a_1 + \frac{1}{24}\mu^2 a_1\right) Q_1$$

$$+ \left(\frac{1}{8}\lambda - \frac{1}{16}\mu a_1\right) R_1 + \frac{2}{3}\lambda P_0 \left(= -\frac{4}{a}q\right) = 0 \quad .. \quad .. \quad \text{(14a)}$$

Thrust,

$$\frac{2}{a}t = \frac{1}{2}\lambda + \left(\frac{1}{7} + \frac{1}{10}\mu^2\right) P_4 + \left(\frac{1}{6} + \frac{1}{8}\mu^2\right) P_3 + \frac{1}{5}\mu Q_3 + \left(\frac{1}{5} + \frac{1}{6}\mu^2\right) P_2$$

$$+ \left(\frac{1}{10} + \frac{1}{8}\mu^2\right) R_2 + \left(\frac{1}{4} + \frac{1}{4}\mu^2\right) P_1 + \frac{1}{3}\mu Q_1 + \left(\frac{1}{8} + \frac{3}{16}\mu^2\right) R_1$$

$$+ \frac{1}{3}\left(1 + \frac{3}{2}\mu^2\right) P_0 \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad \text{(15a)}$$

The methods of §4 were used to solve these equations for given μ and Ω and the following results were obtained, the approximate values being repeated for comparison.

μ		λ	a_0	a_1	b_1	θ at 0·7R
0	" Exact "	0·0160	8·92°	0	0	2·44°
	Approximate	0·0160	8·54°	0	0	2·31°
0·3	" Exact "	0·0119	6·31°	1·63°	2·42°	1·52°–1·95° sin ψ
	Approximate	0·0112	6·48°	1·75°	2·49°	1·53°–1·95° sin ψ

μ		i	t	h	T	H
0	" Exact "	(90°)	0·1097	0	2,145 lbs.	0 lbs.
	Approximate	(90°)	0·1090	0	2,130 lbs.	0 lbs.
0·3	" Exact "	3·58°	0·0868	0·00396	2,020 lbs.	92 lbs.
	Approximate	3·40°	0·0841	0·00429	1,960 lbs.	100 lbs.

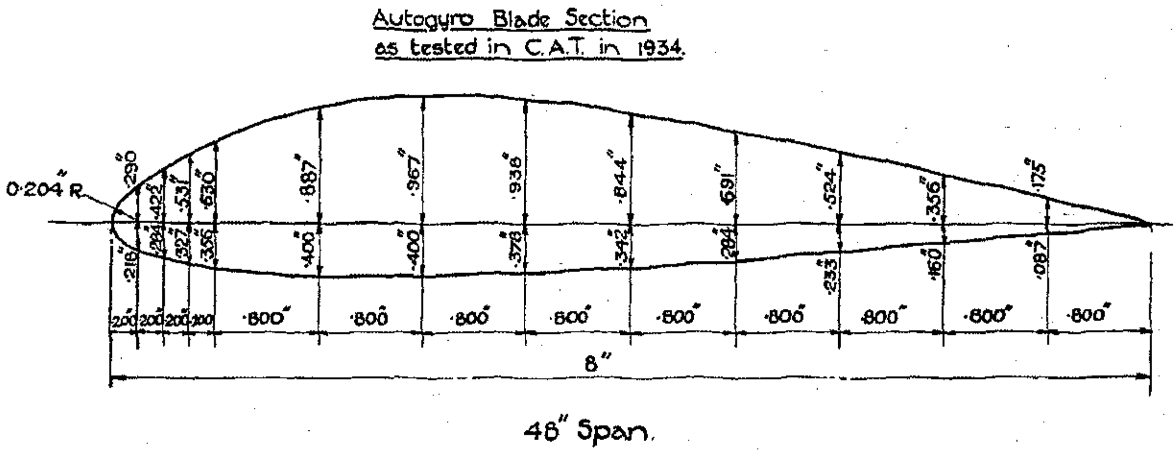

FIG. 10.

TABLE 5

Autogiro Blade Section tested in Compressed Air Tunnel

$P = 1$ atmos.; $V = 75{\cdot}9$ f.s.;

$\frac{1}{2}\rho V^2 = 6{\cdot}71$; $R = 0{\cdot}313 \times 10^6$

α	C_L	C_D	C_M	C_{D_0}
$-1{\cdot}9_5$	0·071	0·0160	—0·0498	0·0158
$+1{\cdot}2$	0·287	0·0188	—0·0446	0·0142
$4{\cdot}3_5$	0·512	0·0288	—0·0416	0·0142
$7{\cdot}3_5$	0·736	0·0486	—0·0488	0·0168
10·4	0·948	0·0714	—0·0412	0·0214
12·5	1·070	0·0888	—0·0376	0·0252
14·5	1·164	0·107	—0·0314	0·0316
15·6	1·148	0·123	—0·0356	0·0484
17·8	1·078	0·153	—0·382	
20·0	1·046	0·184	—0·0448	
23·2	1·024	0·229	—0·0536	
25·6	0·644	0·343	—0·0928	
28·9	0·654	0·402	—0·106	

TABLE 5—*continued*

P = 11·1 atmos. ; V = 46·4 f.s. ;
$\frac{1}{2}\varrho V^2$ = 27·6 ; R = 2·08 × 10^6

α	C_L	C_D	C_M	C_{D_0}
$-4\cdot2_5$	−0·108	0·0102	−0·0502	0·0098
−2·0	+0·050	0·0098	−0·0480	0·0096
$+1\cdot2_5$	0·286	0·0142	−0·0440	0·0098
4·5	0·520	0·0256	−0·0406	0·0106
7·5	0·750	0·0436	−0·0384	0·0124
10·7	0·952	0·0672	−0·0334	0·0168
12·8	1·048	0·0874	−0·0320	0·0264
$13\cdot8_5$	1·062	0·104	−0·0374	0·0416
14·9	1·072	0·119	−0·0402	
$15\cdot9_5$	1·078	0·135	−0·0436	
$18\cdot1_5$	1·059	0·165	−0·0466	
20·3	1·058	0·196	−0·0562	
$23\cdot4_5$	1·052	0·242	−0·0650	
$25\cdot6_5$	1·046	0·273	−0·0720	
$28\cdot9_5$	0·992	0·317	−0·0850	

P = 4·3 atmos. ; V = 56·5 f.s. ;
$\frac{1}{2}\varrho V^2$ = $15\cdot9_5$; R = 0·993 × 10^6

α	C_L	C_D	C_M	C_{D_0}
−4·2	−0·094	0·0114	−0·0514	0·0110
−2·0	+0·063	0·0105	−0·0488	0·0102
$+1\cdot2_5$	0·290	0·0150	−0·0456	0·0104
4·4	0·516	0·0250	−0·0408	0·0102
$7\cdot4_5$	0·742	0·0420	−0·0390	0·0114
10·5	0·958	0·0642	−0·0354	0·0134
12·6	1·056	0·0836	−0·0330	0·0218
13·7	1·062	0·0960	−0·0360	0·0334
14·7	1·066	0·114	−0·0388	
15·8	1·068	0·129	−0·0400	
17·9	1·060	0·159	−0·0466	
20·1	1·046	0·189	−0·0530	
22·3	1·050	0·210	−0·0650	
25·5	1·042	0·269	−0·0710	
28·8	1·002	0·339	−0·0836	

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