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(Ae. 451.)

THE STABILITY OF A BODY TOWED BY A LIGHT WIRE.

By H. Glauert, M.A.

Presented by
The Director of Scientific Research, Air Ministry.

FEBRUARY, 1930.

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AERODYNAMIC SYMBOLS.

1. General

m	mass
t	time
V	resultant linear velocity
Ω	resultant angular velocity
ρ	density, σ relative density
ν	kinematic coefficient of viscosity
R	Reynolds number, $R = lV/\nu$ (where l is a suitable linear dimension), to be expressed as a numerical coefficient x 10^6

Normal temperature and pressure for aeronautical work are 15° C. and 760 mm.

For air under these conditions $\begin{cases} \rho = 0{\cdot}002378 \text{ slug/cu. ft.} \\ \nu = 1{\cdot}59 \times 10^{-4} \text{ sq. ft./sec.} \end{cases}$

The slug is taken to be 32·2 lb.-mass.

α	angle of incidence
ϵ	angle of downwash
S	area
c	chord
s	semi-span
A	aspect ratio, $A = 4s^2/S$
L	lift, with coefficient $k_L = L/S\rho V^2$
D	drag, with coefficient $k_D = D/S\rho V^2$
γ	gliding angle, $\tan \gamma = D/L$
L	rolling moment, with coefficient $k_l = L/sS\rho V^2$
M	pitching moment, with coefficient $k_m = M/cS\rho V^2$
N	yawing moment, with coefficient $k_n = N/sS\rho V^2$

2. Airscrews

n	revolutions per second
D	diameter
J	V/nD
P	power
T	thrust, with coefficier
Q	torque, with coefficien
η	efficiency, $\eta = TV/P =$

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Summary.—Introductory (*Purpose of Investigation.*)—Owing to the practice of towing instruments below an aeroplane, the conditions for the stability of a towed body required investigation.

Range of investigation.—The stability of a body towed by a light inextensible wire has been investigated on certain simplifying assumptions regarding the force experienced by the wire.

Conclusions.—In addition to the pitching and yawing oscillations of the body there are three oscillations of the whole system. The most important oscillation is associated with a bowing of the wire in the plane of symmetry, and, even if the body has satisfactory statical stability, this oscillation may become unstable if the body is too short or if the drag of the body is low compared with that of the wire.

Further developments.—Further investigation is necessary to examine the dynamical effects on the wire which are ignored in the present analysis.

Introduction.—The form assumed by a light wire, which is used to tow a body behind an aeroplane at a constant speed, has been known for many years, and in report R. & M. 554, A. R. McLeod* has derived the corresponding form for a heavy wire. No attempt, however, appears to have been made to determine the stability of the body, and this problem is becoming important owing to the practice of towing aerodynamic instruments below an aeroplane.

In the following pages an attempt is made to determine the conditions for the stability of a body towed by a light inextensible wire on the usual assumption that the aerodynamic force on any element of the wire is normal to its length. The shape of the wire, when the body is displaced from its equilibrium position, will depend on the displacement and on the rate at which the displacement is changing. In order to simplify the analysis, however, it is assumed that the second factor may be ignored and that the characteristics of the wire system during any disturbance may be expressed purely in terms of the instantaneous displacement. This assumption requires further investigation, since it introduces some uncertainty into the validity of the conclusions, but the present analysis does, in fact, appear to represent the actual conditions with reasonable accuracy

* R. & M. 554. "On the action of wind on flexible cables, with application to cables towed below aeroplanes, and balloon cables." (1918).—A. R. McLeod.

The report is divided into three parts. Part I deals with the wire system and determines three fundamental periods of oscillation, two in the plane of symmetry and one normal to it. Parts II and III develop respectively the criteria for the longitudinal and lateral stability of the system, and introduce two more periods of oscillation, corresponding to the pitching and yawing of the body. If the body has a reasonable amount of statical stability, it appears that instability arises only in the normal oscillation in the plane of symmetry which is associated with bowing of the wire, but other forms of instability may arise if the wire is unduly short. The factors which tend to produce instability of the bowing oscillation, when the body has a reasonable amount of statical stability, are a short length of the body and a low drag of the body compared with the drag per unit length of the wire.

PART I.

The Wire System.

1. *Steady motion.*—Consider a light inextensible wire, and let R be the drag per unit length of the wire when at right angles to a stream of velocity V. This drag will be of the form

$$R = k_R\, d\, \varrho\, V^2 \qquad (1)$$

where d is the diameter of the wire and k_R is a non-dimensional drag coefficient. When the wire is inclined at an angle θ to the stream, the force F per unit length of the wire will be assumed to be at right angles to the length of the wire and of magnitude

$$F = R \sin^2 \theta \qquad (2)$$

This assumption is a very close approximation* to the actual experimental results unless the angle θ is extremely small.

Since the aerodynamic force on any element ds of the wire is normal to the element, the tension T of the wire will be constant throughout its length and the shape of the wire will be governed by the equation

$$T\frac{d\varphi}{ds} = F$$

where φ is the angle of inclination to the vertical of the element ds (*see* Fig. 1). Writing now

$$T = R\,c \qquad (3)$$

where c is the length of wire whose normal drag is equal to the tension, the differential equation for the shape of the wire becomes

$$\frac{ds}{d\varphi} = c \sec^2 \varphi \qquad (4)$$

---

* cf. R. & M. 554.

Take as origin O the bottom point where the wire is vertical and would support a body of weight T and of zero drag. Then, on integrating equation (4),

$$s = c \tan \varphi$$

Also the rectangular co-ordinates of any point of the wire, referred to the origin O, can be derived from the differential equations.

$$\frac{dx}{ds} = \sin \varphi$$

$$\frac{dy}{ds} = \cos \varphi$$

and, after integration, the complete solution can be expressed in the form

$$\left.\begin{aligned} \sigma &= \tan \varphi = \sinh \eta \\ \xi &= \cosh \eta - 1 \end{aligned}\right\} \quad \ldots \quad (5)$$

where

$$\sigma = \frac{s}{c},\ \xi = \frac{x}{c},\ \eta = \frac{y}{c} \quad \ldots \quad (6)$$

At any point B of the wire the tension T has horizontal and vertical components which would suffice to support a body of weight W and drag D, provided

$$\left.\begin{aligned} \mathrm{W} &= \mathrm{R}\, c \cos \varphi \\ \mathrm{D} &= \mathrm{R}\, c \sin \varphi \end{aligned}\right\} \quad \ldots \quad (7)$$

Now consider a wire BA of length s as shown in Fig. 2, supporting a body of weight W and drag D. The shape assumed by the wire can be determined from the equations (5) by imagining the wire to be extended to the point O at which it would be vertical. Then, denoting values corresponding to the points B and A by the suffices (1) and (2) respectively, the appropriate system of equations is

$$\sigma_1 = \sinh \eta_1 = \tan \varphi = \frac{\mathrm{D}}{\mathrm{W}}$$

$$\sigma_2 = \sinh \eta_2$$

$$\xi_1 = \cosh \eta_1 - 1 = \sec \varphi - 1$$

$$\xi_2 = \cosh \eta_2 - 1$$

and

$$\xi_2 - \xi_1 = \alpha = \frac{a}{c}$$

$$\eta_2 - \eta_1 = \beta = \frac{b}{c}$$

$$\sigma_2 - \sigma_1 = \sigma = \frac{s}{c}$$

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The shape of the wire is obtained by eliminating the coordinates (ξ, η, σ), and $(\xi, \eta, \sigma)_2$ from these equations. Thus

$$\sigma + \tan\varphi = \sigma_2 = \sinh(\beta + \eta_1)$$
$$= \sinh\beta\sec\varphi + \cosh\beta\tan\varphi$$

and

$$\alpha + \sec\varphi = 1 + \xi_2 = \cosh(\beta + \eta_1)$$
$$= \cosh\beta\sec\varphi + \sinh\beta\tan\varphi$$

and hence the two equations which determine the shape of the wire are

$$\left.\begin{aligned}\sigma &= \sinh\beta\sec\varphi + (\cosh\beta - 1)\tan\varphi\\ \alpha &= \sinh\beta\tan\varphi + (\cosh\beta - 1)\sec\varphi\end{aligned}\right\} \quad \cdots \quad \cdots \quad (8)$$

Alternative forms of these equations which are useful in the subsequent analysis are

$$\left.\begin{aligned}\sigma - \alpha\sin\varphi &= \sinh\beta\cos\varphi\\ \alpha - \sigma\sin\varphi &= (\cosh\beta - 1)\cos\varphi\end{aligned}\right\} \quad \cdots \quad \cdots \quad \cdots \quad (9)$$

and

$$\left.\begin{aligned}1 + \alpha\cos\varphi &= \cosh\beta + \sinh\beta\sin\varphi\\ \sin\varphi + \sigma\cos\varphi &= \sinh\beta + \cosh\beta\sin\varphi\end{aligned}\right\} \quad \cdots \quad \cdots \quad (10)$$

Finally, on eliminating the hyperbolic functions from the equations (9),

$$(\alpha - \sigma\sin\varphi + \cos\varphi)^2 - (\sigma - \alpha\sin\varphi)^2 = \cos^2\varphi$$

or

$$(\sigma^2 - \alpha^2)\cos\varphi = 2(\alpha - \sigma\sin\varphi) \quad \cdots \quad \cdots \quad \cdots \quad (11)$$

2. *Disturbed Motion.*—During the disturbance of the steady motion the behaviour of the wire system will depend on the displacement of the body B and on the rate at which the displacement is increasing or decreasing. In order to simplify the analysis, however, the effects of the velocity of displacement will be ignored, and the modified form of the wire will be calculated on the assumption that the body B is at rest, relative to the point A, in its displaced position. On this assumption, a lateral displacement of the body B corresponds to a rotation of the whole system about a horizontal axis through the point A and parallel to the velocity V, and hence the motion is essentially that of a pendulum of length b. Ignoring any aerodynamic restoring or damping forces on the body, the system would perform an oscillation of period

$$P_L = 2\pi\sqrt{\frac{b}{g}} \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (12)$$

Since b is obtained for any wire system as a multiple of the fundamental length c, it is convenient to calculate

$$p_L = 2\pi\sqrt{\frac{b}{c}} \qquad \ldots \quad (13)$$

and the corresponding period is then derived by multiplying p_L by $\sqrt{c/g}$.

The distortion of the wire due to a longitudinal or normal displacement of the body in the plane of symmetry is more complex in form. Consider a displacement of the body B relative to the point of attachment A, choose the equilibrium position B_0 of the body as origin of co-ordinates, and denote the horizontal and vertical displacements of the body by x and y respectively. Owing to this displacement there will be changes in the horizontal and vertical components of the wire tension acting on the body, and in general these forces will tend to restore the body to its equilibrium position. The horizontal and vertical components of the wire tension in the displaced position may therefore be expressed conveniently as $(T \sin \varphi - T_x)$ and $(T \cos \varphi - T_y)$. In general these forces will depend not only on the displacement (x, y) but also on the velocity of the body relative to the point A. Any effects due to this velocity will be ignored in the present analysis, and the shape of the wire in the displaced position of the body can therefore be derived from the equations (7) and (8) by considering the effect of small increments of the weight W and drag D of the body. Equations (7) determine the corresponding increments of c and φ, and equations (8) then determine the corresponding increments of a and b. During this calculation R and s remain constant. Note also that

$$\delta a = -x, \; \delta b = -y \qquad \ldots \quad (14)$$

and

$$\delta D = -T_x, \; \delta W = -T_y \qquad \ldots \quad (15)$$

From equations (7)

$$\delta D = R(\sin\varphi \, \delta c + c\cos\varphi \, \delta\varphi)$$
$$\delta W = R(\cos\varphi \, \delta c - c\sin\varphi \, \delta\varphi)$$

and when the increments of c and φ are expressed in terms of the increments of a and b, these equations assume the form

$$\delta D = R(a_1 \, \delta a + a_2 \, \delta b)$$
$$\delta W = R(b_1 \, \delta a + b_2 \, \delta b)$$

or

$$\left.\begin{aligned} T_x &= R(a_1 x + a_2 y) \\ T_y &= R(b_1 x + b_2 y) \end{aligned}\right\} \qquad \ldots \quad (16)$$

where (a_1, b_1, a_2, b_2) are four non-dimensional coefficients depending only on the initial shape of the wire and determined by the equations

$$\left.\begin{aligned} a_1 \, \delta a + a_2 \, \delta b &= \sin\varphi \, \delta c + c\cos\varphi \, \delta\varphi \\ b_1 \, \delta a + b_2 \, \delta b &= \cos\varphi \, \delta c - c\sin\varphi \, \delta\varphi \end{aligned}\right\} \qquad \ldots \quad (17)$$

The first equation (8) is

$$\frac{s}{c}\cos\varphi = \sinh\frac{b}{c} + \sin\varphi\,(\cosh\frac{b}{c} - 1)$$

and hence

$$\cos\varphi\;\delta\left(\frac{s}{c}\right) - \frac{s}{c}\sin\varphi\;\delta\varphi$$

$$= \left(\cosh\frac{b}{c} + \sin\varphi\sinh\frac{b}{c}\right)\;\delta\left(\frac{b}{c}\right) + \cos\varphi\left(\cosh\frac{b}{c} - 1\right)\;\delta\varphi$$

or by virtue of equations (9) and (10)

$$(1 + a\cos\varphi)\;\delta\left(\frac{b}{c}\right) = \cos\varphi\;\delta\left(\frac{s}{c}\right) - a\;\delta\varphi$$

which gives finally

$$(1 + a\cos\varphi)\;\delta b = \{(1 + \alpha\cos\varphi)\;\beta - \sigma\cos\varphi\}\;\delta c - a\,c\;\delta\varphi \quad \ldots \quad (18)$$

The second equation (8) is

$$\frac{a}{c}\cos\varphi = \cosh\frac{b}{c} - 1 + \sin\varphi\sinh\frac{b}{c}$$

and hence

$$\cos\varphi\;\delta\left(\frac{a}{c}\right) - \frac{a}{c}\sin\varphi\;\delta\varphi$$

$$= \left(\sinh\frac{b}{c} + \sin\varphi\cosh\frac{b}{c}\right)\;\delta\left(\frac{b}{c}\right) + \cos\varphi\sinh\frac{b}{c}\;\delta\varphi$$

or by virtue of equations (9) and (10)

$$\cos\varphi\;\delta\left(\frac{a}{c}\right) = (\sin\varphi + \sigma\cos\varphi)\;\delta\left(\frac{b}{c}\right) + \sigma\;\delta\varphi$$

or again

$$\cos\varphi\;\delta a = (\sin\varphi + \sigma\cos\varphi)\;\delta b + \{a\cos\varphi - (\sin\varphi + \sigma\cos\varphi)\;\beta\}\;\delta c + \sigma\,c\;\delta\varphi$$

Substituting for δb from equation (18)

$$(1 + a\cos\varphi)\;\delta a = \{(a - \sigma\sin\varphi) - (\sigma^2 - a^2)\cos\varphi\}\,\delta c + (\sigma - a\sin\varphi)\sec\varphi\,c\;\delta\varphi$$

and then simplifying the coefficient of δc by means of equation (11)

$$(1 + a\cos\varphi)\;\delta a = -(a - \sigma\sin\varphi)\;\delta c + (\sigma - a\sin\varphi)\sec\varphi\,c\;\delta\varphi \quad \ldots \quad (19)$$

In order to determine the coefficients (a_1, b_1, a_2, b_2) from equations (17) it is necessary to express δc and $\delta \varphi$ in terms of δa and δb. These expressions are derived from the equations (18) and (19), and after some reduction

$$\left.\begin{aligned} \Delta\,\delta c &= a\cos\varphi\;\delta a + (\sigma - a\sin\varphi)\;\delta b \\ \Delta\, c\,\delta\varphi &= \{(1 + a\cos\varphi)\;\beta - \sigma\cos\varphi\}\cos\varphi\;\delta a \\ &\qquad + (\alpha - \sigma\sin\varphi)\cos\varphi\;\delta b \end{aligned}\right\} \quad (20)$$

where

$$\begin{aligned} \Delta &= (\sigma - a\sin\varphi)\;\beta - 2\,(\alpha - \sigma\sin\varphi) \\ &= \{\beta\sinh\beta - 2\,(\cosh\beta - 1)\}\cos\varphi \end{aligned} \quad \ldots \quad \ldots \quad (21)$$

and then from equations (17)

$$\left.\begin{aligned} \Delta\, a_1 &= a\sin\varphi\cos\varphi + \{(1 + a\cos\varphi)\;\beta - \sigma\cos\varphi\}\cos^2\varphi \\ \Delta\, b_1 &= a\cos^2\varphi - \{(1 + a\cos\varphi)\;\beta - \sigma\cos\varphi\}\sin\varphi\cos\varphi \\ \Delta\, a_2 &= (\sigma - a\sin\varphi)\sin\varphi + (\alpha - \sigma\sin\varphi)\cos^2\varphi \\ \Delta\, b_2 &= (\sigma - a\sin\varphi)\cos\varphi - (\alpha - \sigma\sin\varphi)\sin\varphi\cos\varphi \end{aligned}\right\} \quad \ldots \quad (22)$$

In order to obtain numerical solutions it is most convenient to start with any suitable values of φ and β. Equations (8) then determine the corresponding values of σ and a, and equations (21) and (22) determine the four coefficients (a_1, b_1, a_2, b_2). The necessary numerical work is rather lengthy and it is difficult to obtain high accuracy when σ is less than unity because the value of Δ is then very small. The results of these calculations are given in Table 3 at the end of the report for three values of the angle φ (0, 5, 10°). For convenience in the later work the table also includes the numerical values of

$$\left.\begin{aligned} S &= a_1 + b_2 \\ P &= a_1 b_2 - a_2 b_1 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

The coefficients (a_1, b_1, a_2, b_2) all decrease as σ increases, and when φ is zero b_1 is equal to a_2. For moderate values of σ all four coefficients are of the same order of magnitude, but for small values of σ the coefficient b_2 becomes very large and the coefficient a_1 falls below b_1 or a_2.

3. *Wire oscillations.*—Consider the motion of a body of weight W and drag D, ignoring any aerodynamic restoring or damping forces due to the displacement. The appropriate equations of motion for a displacement in the plane of symmetry are

$$m\,\frac{d^2 x}{d\,t^2} = -\,T_x = -\,R\,(a_1 x + a_2 y)$$

$$m\,\frac{d^2 y}{d\,t^2} = -\,T_y = -\,R\,(b_1 x + b_2\, y)$$

Now replace the time t by a non-dimensional time parameter τ defined by the equation

$$\frac{t}{\tau} = \sqrt{\frac{m}{R}} = \sqrt{\frac{cW}{gT}} = \sqrt{\frac{c}{g}\cos\varphi} \qquad \qquad (24)$$

and then the equations of motion become

$$\left.\begin{aligned} \frac{d^2 x}{d\tau^2} + a_1 x + a_2 y = 0 \\ \frac{d^2 y}{d\tau^2} + b_1 x + b_2 y = 0 \end{aligned}\right\} \qquad \qquad (25)$$

Assuming x and y to be proportional to an exponential factor $e^{\lambda\tau}$, the periods of the oscillations of the system are determined by the equation

$$\begin{vmatrix} \lambda^2 + a_1 & a_2 \\ b_1 & \lambda^2 + b_2 \end{vmatrix} = 0$$

or

$$\lambda^4 + S\lambda^2 + P = 0 \qquad \qquad (26)$$

The coefficients of this equation are both positive and there are always two negative roots. Writing the equation as

$$(\lambda^2 + A)(\lambda^2 + B) = 0$$

where

$$\left.\begin{aligned} A &= \frac{b_2 + a_1}{2} - \sqrt{\left(\frac{b_2 - a_1}{2}\right)^2 + a_2 b_1} \\ B &= \frac{b_2 + a_1}{2} + \sqrt{\left(\frac{b_2 - a_1}{2}\right)^2 + a_2 b_1} \end{aligned}\right\} \qquad \qquad (27)$$

the periods of the two oscillations are

$$\left.\begin{aligned} P_A &= \frac{2\pi}{\sqrt{A}}\sqrt{\frac{c}{g}\cos\varphi} \\ P_B &= \frac{2\pi}{\sqrt{B}}\sqrt{\frac{c}{g}\cos\varphi} \end{aligned}\right\} \qquad \qquad (28)$$

It is convenient, however, to calculate

$$\left.\begin{aligned} p_A &= 2\pi\sqrt{\frac{\cos\varphi}{A}} \\ p_B &= 2\pi\sqrt{\frac{\cos\varphi}{B}} \end{aligned}\right\} \qquad \qquad (29)$$

and the corresponding periods are then derived by multiplying p_A and p_B by $\sqrt{c/g}$.

In order to appreciate the significance of these two oscillations in the plane of symmetry, consider the purely hypothetical case when the cross-connecting term $a_2 b_1$ of equation (26) is zero. The two roots A and B then degenerate to a_1 and b_2, and hence the longer period (A) oscillation is a horizontal or longitudinal oscillation, and the shorter period (B) oscillation is a vertical or normal oscillation. More generally the (A) oscillation appears to be a pendulum oscillation of the whole system in the plane of symmetry, and the (B) oscillation to be a transverse oscillation due to bowing of the wire.

There are three oscillations in all of the wire system when the aerodynamic restoring and damping forces on the body are ignored, and these fundamental oscillations of the system may be specified as :—

(L) a lateral oscillation ;

(A) a pendulum oscillation in the plane of symmetry ;

(B) a bowing oscillation in the plane of symmetry.

The periods of these three oscillations are determined by equations (13) and (29), and numerical values are given in Table 4 at the end of the report. All three periods increase with the length of the wire, and decrease as φ increases. The periods of the pendulum and bowing oscillations are shown in Fig. 4, and the period of the lateral oscillation is slightly longer than that of the pendulum oscillation Assuming a typical value of 65 ft. for the fundamental length c, the periods of the lateral and pendulum oscillations are of the order of 6 to 10 sec. in the range of s/c considered, and the period of the bowing oscillation is of the order of 1 to 4 sec.

PART II.

Longitudinal Stability of the Body.

4. *Equations of Motion.*—Consider a body towed by a light inextensible wire of length s, attached to the centre of gravity B of the body and to an aeroplane A which is flying horizontally with the uniform velocity V. In equilibrium (Fig. 2) the body will hang at a distance a behind and at a depth b below the aeroplane. Take this point B_0 as origin and consider a displacement (x, y) of the body relative to the aeroplane and a rotation θ as shown in Fig. 3.

The change in shape and tension of the wire due to the displacement of the body causes a change in the force applied to the body, and if F_{x1} and F_{y1} are respectively the horizontal and vertical components of the force the analysis of para. 2 gives

$$\left.\begin{aligned} F_{x1} &= D - R\,(a_1 x + a_2 y) \\ F_{y1} &= mg - R\,(b_1 x + b_2 y) \end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

The numerical values of the coefficients (a_1, b_1, a_2, b_2) are given in Table 3.

Turning to the body itself and using axes (B X, Z) which are consistent with the axes generally used in developing the stability of an aeroplane, the velocity of the body relative to its own axes has the components.

$$u = V + \frac{dx}{dt}$$

$$w = V\theta - \frac{dy}{dt}$$

and hence the aerodynamic forces and moments are of the form

$$X = X_0 + \frac{dx}{dt} X_u + \left(V\theta - \frac{dy}{dt}\right) X_w + \frac{d\theta}{dt} X_q$$

Now $X_0 = -D$

and, assuming a symmetrical shape for the body, it can easily be seen that $(X_w, X_q, Z_o, Z_u, M_o, M_u)$ are all zero. Hence the forces and pitching moment affecting the longitudinal motion of the body are

$$\left.\begin{aligned} X &= -D + \frac{dx}{dt} X_u \\ Z &= \left(V\theta - \frac{dy}{dt}\right) Z_w + \frac{d\theta}{dt} Z_q \\ M &= \left(V\theta - \frac{dy}{dt}\right) M_w + \frac{d\theta}{dt} M_q \end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad (31)$$

and, provided that the body has weathercock stability, the five derivatives which occur in these expressions are all negative.

The horizontal and vertical components of the aerodynamic force are respectively

$$\begin{aligned} F_{x2} &= X + \theta Z \\ &= -D + \frac{dx}{dt} X_u \end{aligned}$$

and

$$F_{y2} = \theta X - Z$$
$$= -\theta D - \left(V\theta - \frac{dy}{dt}\right) Z_w - \frac{d\theta}{dt} Z_q$$

Hence the corresponding components of the resultant force on the body are

$$\left.\begin{aligned} F_x &= \frac{dx}{dt} X_u - R(a_1 x + a_2 y) \\ F_y &= \frac{dy}{dt} Z_w - \frac{d\theta}{dt} Z_q - \theta(D + VZ_w) - R(b_1 x + b_2 y) \end{aligned}\right\} \quad (32)$$

and the equations of motion of the body are

$$\left.\begin{aligned} m\frac{d^2x}{dt^2} &= F_x \\ m\frac{d^2y}{dt^2} &= F_y \\ B\frac{d^2\theta}{dt^2} &= M \end{aligned}\right\} \quad \ldots \quad (33)$$

where B is the moment of inertia of the body about its lateral axis.

In order to obtain a non-dimensional system similar to that used in developing the stability of an aeroplane, let l be some typical length of the system and let

$$\mu = \frac{m}{\varrho l^3}$$
$$t = \frac{\mu l}{V} \tau$$

where τ is now the non-dimensional time parameter. The length l may be chosen as some typical length either of the body or of the wire, but the second alternative appears to be the more convenient and the unit of time will therefore be chosen according to the previous definition (24). Thus

$$\frac{t}{\tau} = \sqrt{\frac{m}{R}} \quad \ldots \quad (34)$$

and then

$$\left.\begin{aligned} l^2 &= \frac{m}{\varrho V}\sqrt{\frac{R}{m}} \\ \mu^2 &= \frac{\varrho V^3}{R}\sqrt{\frac{m}{R}} \end{aligned}\right\} \quad \ldots \quad (35)$$

The non-dimensional coefficients for the aerodynamic forces and moments are now taken to be

$$
\left.\begin{aligned}
k_{\mathrm{D}} &= \frac{\mathrm{D}}{l^2\varrho \mathrm{V}^2} = \frac{\mathrm{D}}{m\mathrm{V}}\sqrt{\frac{m}{\mathrm{R}}} \\
x_{\mathrm{u}} &= -\frac{\mathrm{X_u}}{l^2\varrho \mathrm{V}} = -\frac{\mathrm{X_u}}{m}\sqrt{\frac{m}{\mathrm{R}}} \\
z_{\mathrm{w}} &= -\frac{\mathrm{Z_w}}{l^2\varrho \mathrm{V}} = -\frac{\mathrm{Z_w}}{m}\sqrt{\frac{m}{\mathrm{R}}} \\
z_{\mathrm{q}} &= -\frac{\mathrm{Z_q}}{l^3\varrho \mathrm{V}} = -\frac{\mu \mathrm{Z_q}}{m\mathrm{V}} \\
m_{\mathrm{w}} &= -\frac{m\,\mathrm{M_q}}{\mathrm{B}l\varrho \mathrm{V}} = -\frac{\mathrm{V}\mathrm{M_w}}{\mu \mathrm{B}}\cdot\frac{m}{\mathrm{R}} \\
m_{\mathrm{q}} &= -\frac{m\,\mathrm{M_q}}{\mathrm{B}l^2\varrho \mathrm{V}} = -\frac{\mathrm{M_q}}{\mathrm{B}}\sqrt{\frac{m}{\mathrm{R}}}
\end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (36)
$$

Writing also

$$\zeta = l\theta$$

the equations of motion (33) become

$$
\left.\begin{aligned}
&\frac{d^2x}{d\tau^2} + x_{\mathrm{u}}\frac{dx}{d\tau} + a_1 x + a_2 y = 0 \\
&\frac{d^2y}{d\tau^2} + z_{\mathrm{w}}\frac{dy}{d\tau} + b_1 x + b_2 y - z_{\mathrm{q}}\frac{d\zeta}{d\tau} - \mu\,(z_{\mathrm{w}} - k_{\mathrm{D}})\,\zeta = 0 \\
&\frac{d^2\zeta}{d\tau^2} + m_{\mathrm{q}}\frac{d\zeta}{d\tau} + \mu\, m_{\mathrm{w}}\,\zeta - m_{\mathrm{w}}\frac{dy}{d\tau} = 0
\end{aligned}\right\} \quad (37)
$$

and then, assuming (x, y, ζ) to be proportional to an exponential factor $e^{\lambda\tau}$, the fundamental equation for determining the stability of the motion is

$$
\begin{vmatrix}
\lambda^2 + x_{\mathrm{u}}\,\lambda + a_1 & a_2 & 0 \\
b_1 & \lambda^2 + z_{\mathrm{w}}\,\lambda + b_2 & -z_{\mathrm{q}}\lambda - \mu\,(z_{\mathrm{w}} - h_{\mathrm{D}}) \\
0 & -m_{\mathrm{w}}\lambda & \lambda^2 + m_{\mathrm{q}}\lambda + \mu\, m_{\mathrm{w}}
\end{vmatrix} = 0 \quad (38)
$$

On expanding this determinant it appears that z_{q} enters into the equation only in the form $(\mu - z_{\mathrm{q}})$, and in general z_{q} is only a small fraction of μ. In the subsequent work z_{q} will be neglected and then, after a small adjustment, the fundamental stability equation becomes

$$
\begin{vmatrix}
\lambda^2 + x_{\mathrm{u}}\,\lambda + a_1 & a_2 & 0 \\
b_1 & \lambda^2 + z_{\mathrm{w}}\,\lambda + b_2 & z_{\mathrm{w}} - k_{\mathrm{D}} \\
0 & \mu\, m_{\mathrm{w}}\,\lambda & \lambda^2 + m_{\mathrm{q}}\,\lambda + \mu m_{\mathrm{w}}
\end{vmatrix} = 0 \quad (39)
$$

This is an equation of the sixth degree and in general there are three oscillations of the system, the pendulum and bowing oscillations of the wire system and a pitching oscillation of the body. The constant term of the equation is

$$m_w (a_1 b_2 - a_2 b_1)$$

which is positive if the body has statical stability. If this condition is satisfied, instability of the system must arise first in the form of an increasing oscillation.

5. *The stability equation.*—If m_w is zero, as would occur with a spherical body, or if z_w is equal to k_D, the general stability equation (39) splits up into the factors

$$\lambda^2 + m_q \lambda + \mu m_w = 0$$

and

$$\begin{vmatrix} \lambda^2 + x_u \lambda + a_1 & a_2 \\ b_1 & \lambda^2 + z_w \lambda + b_2 \end{vmatrix} = 0$$

The first factor represents a stable or neutral motion. The second factor is

$$\lambda^4 + (x_u + z_w) \lambda^3 + (a_1 + b_2 + x_u z_w) \lambda^2 + (a_1 z_w + b_2 x_u) \lambda + (a_1 b_2 - a_2 b_1) = 0 \quad .. \quad .. \quad (40)$$

All the coefficients are positive and the discriminant of the equation is

$$\Delta = (x_u + z_w)(a_1 + b_2 + x_u z_w)(a_1 z_w + b_2 x_u) - (a_1 z_w + b_2 x_u)^2 - (x_u + z_w)^2 (a_1 b_2 - a_2 b_1)$$

$$= x_u z_w (x_u + z_w)(a_1 z_w + b_2 x_u) + (b_2 - a_1)^2 x_u z_w + a_2 b_1 (x_u + z_w)^2$$

which is essentially positive. Hence the whole motion is stable, and it follows that instability of the general equation (39) can arise only owing to the cross-connecting terms $\mu m_w (z_w - k_D)$.

Consider next the purely hypothetical condition that the cross-connecting term $a_2 b_1$ are zero. The general stability equation again splits up into factors

$$\lambda^2 + x_u \lambda + a_1 = 0$$

and

$$\begin{vmatrix} \lambda^2 + z_w \lambda + b_2 & z_w - k_D \\ m_w & \lambda^2 + m_q \lambda + \mu m_w \end{vmatrix} = 0$$

The first factor represents a stable motion, since x_u and a_1 are positive, and the second factor is

$$\lambda^4 + (z_w + m_q) \lambda^3 + (b_2 + \mu m_w + z_w m_q) \lambda^2 + (b_2 m_q + \mu m_w k_D) \lambda + b_2 \mu m_w = 0 \quad .. \quad (41)$$

whose coefficients are all positive if the body has statical stability. Neglecting the small term $z_w m_q$ in the coefficient of λ^2, the discriminant of this equation is

$$\begin{aligned}\Delta &= (z_w + m_q)(b_2 + \mu m_w)(b_2 m_q + \mu m_w k_D) \\ &\quad - (b_2 m_q + \mu m_w k_D)^2 - (z_w + m_q)^2 b_2 \mu m_w \\ &= (\mu m_w k_D - b_2 z_w)\{\mu m_w (z_w - k_D + m_q) - b_2 m_q\}\end{aligned}$$

and instability occurs if μm_w lies within the range defined by the inequality

$$\frac{z_w}{k_D} > \frac{\mu m_w}{b_2} > \frac{m_q}{z_w - k_D + m_q} \quad \ldots \quad \ldots \quad \ldots \quad (42)$$

In general k_D is a small fraction of z_w, and z_w is less than m_q; the upper limit of this inequality is of the order 5 to 10, and the lower limit is slightly less than unity. The value of b_2 increases rapidly as the length of the wire is reduced, and in practical applications probably lies between 10 and 30. Finally, the value of μm_w depends on the statical stability of the body, but is usually high and of the order of 100. From this discussion it would appear that the value of $\mu m_w/b_2$ will hardly ever lie below the lower limit of the inequality (42), but that there is a real danger that it may fail to excede the upper limit. Effectively therefore the condition for stability is that

$$\mu m k_D > b_2 z_w \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (43)$$

and the factors which tend towards instability may be specified as follows :—

(1) insufficient statical stability, μm_w small ;

(2) short body, which increases z_w for a given value of μm_w.

(3) short wire, which increases b_2 ;

(4) low drag, k_D small.

Any instability which occurs in practice can be cured by suitable modifications to any or all of these factors.

The stability criterion (43) is based on the impossible condition that $a_2 b_1$ is zero, but an examination of some typical numerical examples has shown that it is a fairly accurate approximation to the true condition. To examine the validity of the condition more closely it is necessary to revert to the general stability equation (39). On expansion this equation is

$$\begin{aligned}(\lambda^2 + m_q \lambda)\{\lambda^4 + (x_u + z_w)\lambda^3 + (S + x_u z_w)\lambda^2 \\ + (a_1 z_w + b_2 x_u)\lambda + P\} \\ + \mu m_w \{\lambda^4 + (x_u + k_D)\lambda^3 + (S + x_u k_D)\lambda^2 \\ + (a_1 k_D + b_2 x_u)\lambda + P\} = 0\end{aligned}$$

where, in accordance with the previous definitions (23)

$$S = a_1 + b_2$$
$$P = a_1 b_2 - a_2 b_1$$

Neglecting now $x_u z_w$ and $x_u k_D$ in comparison with S, the general stability equation becomes

$$\begin{aligned}\lambda^6 &+ (x_u + z_w + m_q)\lambda^5 + \{S + m_q(x_u + z_w) + \mu m_w\}\lambda^4 \\ &+ \{(a_1 z_w + b_2 x_u) + m_q S + \mu m_w (x_u + k_D)\}\lambda^3 \\ &+ \{P + m_q(a_1 z_w + b_2 x_u) + \mu m_w S\}\lambda^2 \\ &+ \{m_q P + \mu m_w (a_1 k_D + b_2 x_u)\}\lambda + \mu m_w P = 0 \qquad (44)\end{aligned}$$

Any attempt to obtain general expressions for the discriminants of this equation ends in hopeless complexity, but another method of attack is available for determining the conditions under which an undamped oscillation occurs. If such an oscillation occurs, two roots of the sextic equation (44) must be of the form

$$\lambda = \pm i\sqrt{\xi}$$

where ξ is essentially positive, and, on separating the real and imaginary parts of the equation, we then require simultaneously

$$\begin{aligned}\xi^3 &- \{S + m_q(x_u + z_w) + \mu m_w\}\xi^2 \\ &+ \{P + m_q(a_1 z_w + b_2 x_u) + \mu m_w S\}\xi \\ &- \mu m_w P = 0 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (45)\end{aligned}$$

and

$$\begin{aligned}(x_u + z_w + m_q)\xi^2 &- \{(a_1 z_w + b_2 x_u) + m_q S + \mu m_w (x_u + k_D)\}\xi \\ &+ \{m_q P + \mu m_w (a_1 k_D + b_2 x_u)\} = 0 \quad \ldots \quad \ldots \qquad (46)\end{aligned}$$

Equation (45) determines the periods of three oscillations and on substituting the appropriate value of ξ in equation (46) the necessary relationship between the various coefficients is derived.

If the damping derivatives are ignored, the equation (45) becomes

$$(\xi - \mu m_w)(\xi^2 - S\xi + P) = 0$$

and the three roots are μm_w, A and B, where the last two values are given by the formulæ (27). A second approximation to the equation (45), including now the damping derivatives, leads to corrections to these three roots which are negligibly small when μm_w is reasonably large.

The longest oscillation is usually represented by A and on substituting this value for ξ in equation (46) the coefficient of m_q vanishes and we obtain

$$(x_u + z_w) A^2 - \{ (a_1 z_w + b_2 x_u) + \mu m_w (x_u + k_D) \} A + \mu m_w (a_1 k_D + b_2 x_u) = 0$$

or

$$\frac{\mu m_w}{A} = \frac{(x_u + z_w) A - (a_1 z_w + b_2 x_u)}{(x_u + k_D) A - (a_1 k_D + b_2 x_u)}$$

Now let

$$\left.\begin{aligned} p &= \sqrt{\left(\frac{b_2 - a_1}{2}\right)^2 + a_2 b_1} + \frac{b_2 - a_1}{2} = b_2 - A \\ q &= \sqrt{\left(\frac{b_2 - a_1}{2}\right)^2 + a_2 b_1} - \frac{b_2 - a_1}{2} = B - b_2 \end{aligned}\right\} \quad .. \quad (47)$$

and note that

$$x_u = 2 k_D$$

Then, with these substitutions, the condition for zero damping of the pendulum (A) oscillation becomes

$$\frac{\mu m_w}{A} = 1 + \frac{q (z_w - k_D)}{(2p + q) k_D} \quad .. \quad .. \quad .. \quad .. \quad (48)$$

This value of μm_w is of the order of A and is much smaller than usually occurs in practice.

A similar analysis for the shorter bowing (B) oscillation leads to the condition

$$\frac{\mu m_w}{B} = 1 + \frac{p (z_w - k_D)}{(p + 2q) k_D} \quad .. \quad .. \quad .. \quad .. \quad (49)$$

and this represents a more critical condition, since B is larger than A and p is larger than q. This condition is in fact a more accurate expression of the approximate condition (43) and reduces to the same form if $a_2 b_1$ is zero, since q is then zero and B is equal to b_2. Thus the danger of instability arises in the bowing oscillation, which is the shorter of the two wire oscillations.

Finally susbtituting μm_w for ξ in equation (46) the condition that the pitching oscillation of the body shall have zero damping is

$$(\mu m_w)^2 (z_w - k_D + m_q) - \mu m_w \{ a_1 (z_w - k_D + m_q) + b_2 m_q \} + m_q P = 0 \quad .. \quad .. \quad (50)$$

and the insertion of typical numerical values for the coefficients shows that instability will not arise unless μm_w is unusually small or unless the wire is unduly short.

In order to illustrate these results numerically, calculations have been made using the following values of the aerodynamic coefficients :—

$$k_D = 0 \cdot 05,\ z_w = 0 \cdot 50,\ m_q = 1 \cdot 50.$$

The appropriate wire coefficients have been taken from Table 3 ($\varphi = 0$), and Table 1 gives the critical values of μm_w derived from the equations (48), (49) and (50). Since the approximations on which these formulæ were derived cease to be valid when μm_w is very small, too much importance must not be placed on the critical value of μm_w for the pendulum oscillation or on the lower critical value for the pitching oscillation. It does appear, however, that there is an unstable region of μm_w, and that when this region is entered by reducing the value of μm_w, instability arises first in the bowing oscillation. As a check on these approximate estimates, the complete series of discriminants has been examined for the one condition $\left(\frac{s}{c} = 1 \cdot 18\right)$, and this accurate analysis showed a range of instability from 15 to 122, which is reasonably consistent with the rough estimates contained in Table 1 if the range of instability is assumed to extend from the higher value of the pitching oscillation to the value of the bowing oscillation. The upper limit is the more important and corresponds to the condition (49). The criterion of stability may therefore be taken to be

TABLE 1.

Critical values of μm_w.

$\frac{s}{c}$	Pendulum.	Bowing.	Pitching Oscillation.
0·52	2·5	935	2·1 and 82
0·82	2·2	274	1·3 and 27
1·18	2·2	113	1·0 13
1·60	2·2	57	0·8 8

$$\mu m_w > B \left\{ 1 + \frac{p\,(z_w - k_D)}{(p + 2q)\,k_D} \right\} \quad \ldots \quad \ldots \quad \ldots \quad (51)$$

where B, p and q are wire coefficients denfied by the equations (47), and as a rough approximation this criterion may be taken in the simpler form

$$\mu m_w > \frac{b_2 z_w}{k_D} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (52)$$

PART III.

Lateral Stability of the Body.

6. *Equations of motion.*—With the assumptions of the preceding analysis the lateral displacement of the system is simply a rotation ε about the horizontal fore-and-aft axis through the point of attachment A. The lateral displacement of the body is therefore $b\,\varepsilon$ and the restoring sideforce due to the constraint of the wire is $T\,\varepsilon$. If the body also yaws through an angle ψ, the velocity of the body referred to its own axes will have the components

$$u = V$$

$$v = b\frac{d\,\varepsilon}{d\,t} - V\psi$$

and the aerodynamic sideforce and yawing moment will be

$$\left.\begin{aligned} Y &= \left(b\frac{d\,\varepsilon}{d\,t} - V\psi\right)Y_v + \frac{d\,\psi}{d\,t}Y_r \\ N &= \left(b\frac{d\,\varepsilon}{d\,t} - V\psi\right)N_v + \frac{d\,\psi}{d\,t}N_r \end{aligned}\right\} \quad \cdots \quad (53)$$

The resultant restoring sideforce at right angles to the original direction of motion is then

$$T\,\varepsilon - Y - X\,\psi$$

or

$$R\,c\,\varepsilon - \left(b\frac{d\,\varepsilon}{d\,t} - V\psi\right)Y_v - \frac{d\,\psi}{d\,t}Y_r + \psi D$$

and hence the equations of motion governing the lateral disturbance are

$$\left.\begin{aligned} m\,b\frac{d^2\varepsilon}{d\,t^2} - b\,Y_v\frac{d\,\varepsilon}{d\,t} + R\,c\,\varepsilon - Y_r\frac{d\,\psi}{d\,t} + (D + V\,Y_v)\,\psi = 0 \\ C\frac{d^2\psi}{d\,t^2} - b\,N_v\frac{d\,\varepsilon}{d\,t} - N_r\frac{d\,\psi}{d\,t} + V\,N_v\,\psi = 0 \end{aligned}\right\} \quad (54)$$

where C is the moment of inertia of the body about its normal axis.

Passing to the non-dimensional system, defined by the previous equations (34) and (35), the appropriate expressions for the aerodynamic coefficients are

$$\left.\begin{aligned} y_v &= -\frac{Y_v}{l^2\,\varrho\,V} = -\frac{Y_v}{m}\sqrt{\frac{m}{R}} \\ y_r &= -\frac{Y_r}{l^3\,\varrho\,V} = -\frac{\mu\,Y_r}{m\,V} \\ n_v &= -\frac{m\,N_v}{C\,l\,\varrho\,V} = -\frac{V\,N_v}{\mu\,C}\,\frac{m}{R} \\ n_r &= -\frac{m\,N_r}{C\,l^2\,\varrho\,V} = -\frac{N_r}{C}\sqrt{\frac{m}{R}} \end{aligned}\right\} \quad \cdots \quad (55)$$

In general, for a body with statical stability, y_v and n_r are positive, and y_r and n_v are negative. Also if the body is symmetrical about its longitudinal axis, these lateral derivatives can be replaced by corrseponding longitudinal derivatives according to equations

$$\left.\begin{aligned} y_v &= z_w, \quad y_r = -z_q \\ n_v &= -m_w, \quad n_r = m_q \end{aligned}\right\} \quad \ldots \qquad (56)$$

Transforming to the non-dimensional system and writing

$$\varepsilon' = \varepsilon \frac{b}{l}$$

the equations of motion (54) become

$$\left.\begin{aligned} &\frac{d^2 \varepsilon'}{d\tau^2} + y_v \frac{d\varepsilon'}{d\tau} + \frac{c}{b}\varepsilon' + y_r \frac{d\psi}{d\tau} - \mu(y_v - k_D)\psi = 0 \\ &\frac{d^2\psi}{d\tau^2} + n_r \frac{d\psi}{d\tau} - \mu n_v \psi + n_v \frac{d\varepsilon'}{d\tau} = 0 \end{aligned}\right\} \qquad (57)$$

and if ε' and ψ are proportional to an exponential factor $e^{\lambda\tau}$, the stability equation for the lateral motion becomes

$$\begin{vmatrix} \lambda^2 + y_v\lambda + \gamma & y_r\lambda - \mu(y_v - k_D) \\ n_v\lambda & \lambda^2 + n_r\lambda - \mu n_v \end{vmatrix} = 0 \quad \ldots \qquad (58)$$

where

$$\gamma = \frac{c}{b} \quad \ldots \qquad (59)$$

and may be derived from Table 3. Now y_r may be omitted from this stability equation for the same reason that z_q was omitted from the longitudinal equation, and then after a slight adjustment the stability equation becomes

$$\begin{vmatrix} \lambda^2 + y_v\lambda + \gamma & y_v - k_D \\ -\mu n_v\lambda & \lambda^2 + n_r\lambda - \mu n_v \end{vmatrix} = 0 \quad \ldots \qquad (60)$$

or in terms of the corresponding longitudinal derivatives

$$\begin{vmatrix} \lambda^2 + z_w\lambda + \gamma & z_w - k_D \\ \mu m_w\lambda & \lambda^2 + m_q\lambda + \mu m_w \end{vmatrix} = 0 \quad \ldots \qquad (61)$$

This stability equation is identical in form with the previous equation (41), the only difference being that the wire coefficient is γ instead of b_2. The previous approximate method of treatment is however no longer possible since γ is much smaller than b_2, and the application of the condition (42) would lead to values of μm_w so small that the term $z_w m_q$ in the coefficient of λ^2 is no longer negligible. On expansion the equation (61) becomes

$$\lambda^4 + (z_w + m_q)\lambda^3 + (\gamma + \mu m_w + z_w m_q)\gamma^2 + (\gamma m_q + \mu m_w k_D)\lambda + \gamma\mu m_w = 0$$

and the discriminant of this equation is

$$\Delta = (z_W + m_q)(\gamma + \mu m_W + z_W m_q)(\gamma m_q + \mu m_W k_D) - (\gamma m_q + \mu m_W k_D)^2 - \gamma \mu m_W (z_W + m_q)^2 \quad .. \quad (62)$$

This expression is of the form

$$\Delta = A_1 (\mu m_W)^2 - B_1 (\mu m_W) + C_1$$

and will be negative for a positive range of values of μm_W, if

$$B_1^2 - 4 A_1 C_1 > 0$$

By deriving these coefficients from the equation (62) this inequality can be reduced to the form

$$\gamma^2 (z_W - k_D)^2 - 2 \gamma k_D z_W m_q (z_W + 2 m_q - k_D) + k^2_D z^2_W m^2_q > 0$$

and is satisfied if γ lies outside the range defined by the two critical values

$$\gamma = \frac{k_D z_W m_q}{(z_W - k_D)^2} \left\{ \sqrt{z_W - k_D + m_q} \pm \sqrt{m_q} \right\}^2 \quad .. \quad (63)$$

This inequality determines a range of length of wire within which the lateral motion is stable for all values of μm_W, whilst outside these limits there is always a range of values of μm_W for which the motion becomes unstable.

Assuming the typical values

$$k_D = 0 \cdot 05, z_W = 0 \cdot 50, m_q = 1 \cdot 50,$$

the critical values of γ or c/b are $0 \cdot 005$ and $1 \cdot 27$, and hence instability of the lateral oscillation can arise only if b/c is less than $0 \cdot 79$ or greater than 200. The upper limit is absurdly high and effectively instability can arise only if b/c is unduly small.

When instability does arise, the dangerous range of values of μm_W can be derived from equation (62), and Table 2 gives the appropriate numerical values. Even when the wire is short instability arises only if the statical stability is rather poor, and instability with a reasonable degree of statical stability arises only when b/c is of the order of $0 \cdot 1$. When μm_W is reasonably large the critical length of wire at which lateral instability does arise may be deduced from the approximate condition (42) as

$$\frac{b}{c} = \frac{z_W}{\mu m_W k_D} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (64)$$

TABLE 2.

Critical values of μm_w.

b/c	Lateral oscillations.
0·75	5·5 and 8·1
0·50	4·8 16·0
0·25	5·9 26·3
0·10	10·4 96·6

TABLE 3.

Wire coefficients.

$$S = a_1 + b_2.$$

$$P = a_1 b_2 - a_2 b_1.$$

ϕ	$\frac{b}{c}$	$\frac{a}{c}$	$\frac{s}{c}$	a_1	b_1	a_2	b_2	S	P
0	0·50	0·128	0·521	8·06	24·1	24·1	98·3	106·4	212·8
	0·75	0·295	0·822	5·45	10·8	10·8	30·1	35·6	47·4
	1·00	0·543	1·175	4·13	6·10	6·10	13·2	17·3	17·3
	1·25	0·888	1·602	3·27	3·94	3·94	7·10	10·47	8·37
	1·50	1·352	2·129	2·86	2·77	2·77	4·35	7·21	4·81
	1·75	1·964	2·791	2·52	2·06	2·06	2·93	5·45	3·11
	2·00	2·762	3·627	2·25	1·60	1·60	2·10	4·35	2·18
5°	0·50	0·174	0·534	13·0	31·7	32·3	95·5	108·5	217·6
	0·75	0·368	0·851	7·68	12·8	13·3	29·1	36·8	53·3
	1·00	0·648	1·227	5·40	6·82	7·20	12·60	18·0	18·9
	1·25	1·032	1·686	4·17	4·22	4·53	6·75	10·92	9·01
	1·50	1·543	2·255	3·46	2·86	3·13	4·10	7·56	5·24
	1·75	2·215	2·974	2·97	2·06	2·30	2·73	5·70	3·37
	2·00	3·090	3·883	2·63	1·56	1·77	1·95	4·58	2·37
10°	0·50	0·222	0·552	19·6	39·2	40·6	93·0	112·6	231·3
	0·75	0·444	0·887	10·2	14·7	15·6	27·8	38·0	53·5
	1·00	0·759	1·289	6·78	7·46	8·22	11·95	18·7	19·7
	1·25	1·185	1·783	5·08	4·44	5·06	6·32	11·40	9·64
	1·50	1·749	2·400	4·08	2·91	3·44	3·81	7·89	5·54
	1·75	2·486	3·180	3·44	2·04	2·50	2·53	5·97	3·60
	2·00	3·445	4·170	3·01	14·9	1·91	1·79	4·80	2·54

TABLE 4.

ϕ	$\frac{s}{c}$	p_L	p_A	p_B
0	0·52	4·4	4·4	0·62
	0·82	5·4	5·3	1·07
	1·18	6·3	6·1	1·56
	1·60	7·0	6·7	2·03
	2·13	7·7	7·3	2·47
	2·79	8·3	7·8	2·86
	3·63	8·9	8·3	3·23
5°	0·53	4·4	4·4	0·61
	0·85	5·4	5·2	1·04
	1·23	6·3	6·0	1·52
	1·69	7·0	6·6	1·97
	2·26	7·7	7·1	2·40
	2·97	8·3	7·6	2·79
	3·88	8·9	8·1	3·14
10°	0·55	4·4	4·3	0·59
	0·89	5·4	5·1	1·02
	1·29	6·3	5·9	1·47
	1·78	7·0	6·5	1·91
	2·40	7·7	7·0	2·32
	3·13	8·3	7·5	2·69
	4·17	8·9	7·9	3·02

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FIG 1.

THE WIRE SYSTEM.

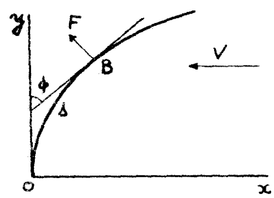


FIG. 2.

STEADY MOTION.

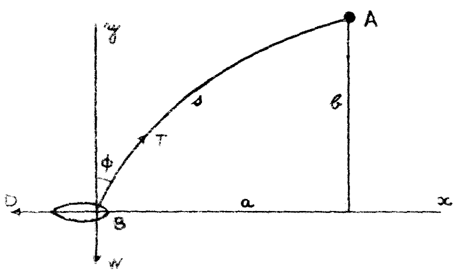
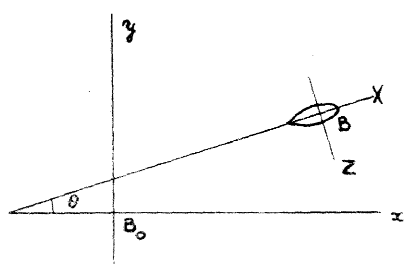


FIG. 3.

DISTURBED MOTION.



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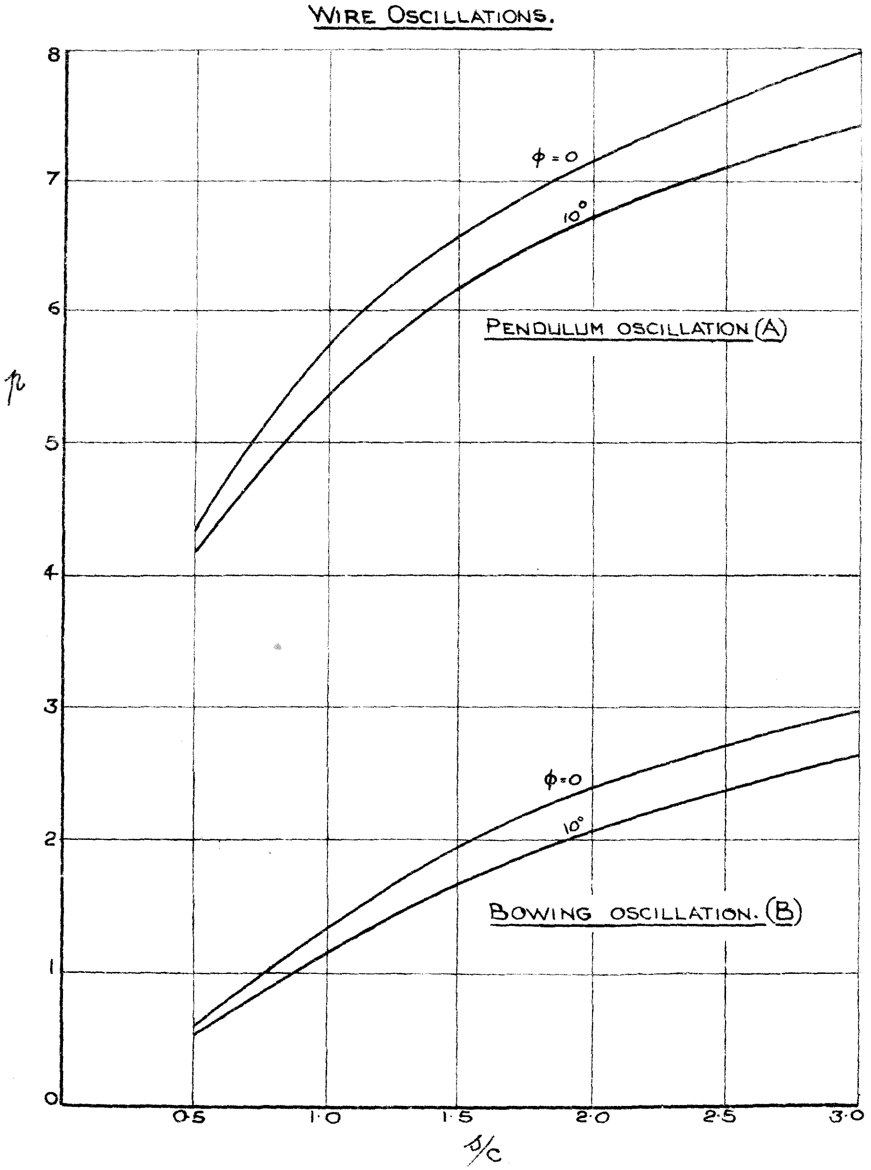
Malby & Sons, Photo-Lith.

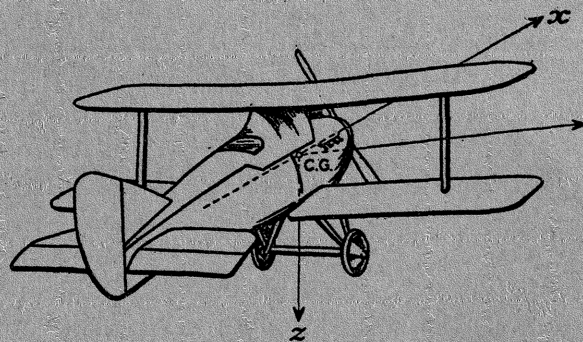
FIG. 4.



WIRE OSCILLATIONS.
8
7
6
5
4
3
2
1
0
p
φ = 0
10°
PENDULUM OSCILLATION (A)
φ=0
10°
BOWING OSCILLATION. (B)
0.5
1.0
1.5
2.0
2.5
3.0
s/c

P.3354.

SYSTEM OF AXES.



Axes	Symbol Designation. Positive direction	x longitudinal forward	y lateral starboard	z normal downward
Force	Symbol	X	Y	Z
Moment	Symbol Designation	L rolling	M pitching	N yawing
Angle of Rotation	Symbol	ϕ	θ	ψ
Velocity	Linear Angular	u p	v q	w r
Moment of Inertia		A	B	C

Components of linear velocity and force are positive in the positive direction of the corresponding axis. Components of angular velocity and moment are positive in the cyclic order y to z about the axis of x, z to x about the axis of y, and x to y about the axis of z.

The angular movement of a control surface (elevator or rudder) is governed by the same convention, the elevator angle being positive downwards and the rudder angle positive to port. The aileron angle is positive when the starboard aileron is down and the port aileron is up. A positive control angle normally gives rise to a negative moment about the corresponding axis. The symbols for the control angles are :–

ξ aileron angle
η elevator angle
η_T tail setting angle
ζ rudder angle

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