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THE ELASTICITY OF PINTSCH CRYSTALS OF TUNGSTEN.

BY S. J. WRIGHT, B.A.

WORK PERFORMED FOR THE
DEPARTMENT OF SCIENTIFIC AND INDUSTRIAL RESEARCH.

MARCH, 1929.

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AERODYNAMIC SYMBOLS.

1. GENERAL

m mass
t time
V resultant linear velocity
Ω resultant angular velocity
ρ density, σ relative density
ν kinematic coefficient of viscosity
R Reynolds number, $R = lV/\nu$ (where l is a suitable linear dimension), to be expressed as a numerical coefficient x 10^6

Normal temperature and pressure for aeronautical work are 15° C. and 760 mm.

For air under these conditions $\begin{cases} \rho = 0{\cdot}002378 \text{ slug/cu. ft.} \\ \nu = 1{\cdot}59 \times 10^{-4} \text{ sq. ft./sec.} \end{cases}$

The slug is taken to be 32·2 lb.-mass.

α angle of incidence
ϵ angle of downwash
S area
c chord
s semi-span
A aspect ratio, $A = 4s^2/S$
L lift, with coefficient $k_L = L/S\rho V^2$
D drag, with coefficient $k_D = D/S\rho V^2$
γ gliding angle, $\tan\gamma = D/L$
L rolling moment, with coefficient $k_l = L/sS\rho V^2$
M pitching moment, with coefficient $k_m = M/cS\rho V^2$
N yawing moment, with coefficient $k_n = N/sS\rho V^2$

2. AIRSCREWS

n revolutions per second
D diameter
J V/nD
P power
T thrust, with coeff
Q torque, with coeffi
η efficiency, $\eta = TV$

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Summary.—Purpose of Investigation.—Very little work has hitherto been done on the Elastic Properties of Single Crystals of Metals. In the case of Tungsten, which is the only cubic crystal whose elastic constants have been determined, the previous work of Bridgman based on static tests indicated that the constants satisfied the Isotropic relation. In the present investigation dynamical methods have been employed to redetermine these constants more accurately, and in particular to find out whether the crystals are truly isotropic.

Range of Investigation.—The moduli of Rigidity of six wires, five of which were single crystals of Tungsten of different orientations, and the remaining one of which contained three crystals, were determined by the method of Torsional Oscillations. From the variation of the modulus with orientation of the crystal the elastic symmetry could be determined. In addition the Young's modulus was determined by the method of Bending Oscillations, for two single crystal wires, and was directly compared with the Torsion Modulus.

Conclusions.—No change of Torsional Modulus of Rigidity with orientation could be observed. The order of accuracy of the results was such that the experiments would have disclosed any variation of as much as one part in 400 over the range of orientations employed. It is concluded that over the whole range of possible orientations the variation of the modulus of Rigidity cannot exceed 1 part in 200 and that the material is probably truly isotropic. The values of Young's Modulus, the Modulus of Rigidity, and Poisson's Ratio for the material, are given in the Report.

Further Work.—Since the only cubic crystal, which has been investigated, has been found to be isotropic within very close limits, it appears to be a matter of importance to find out whether isotropy is

(1) a property peculiar to Tungsten Crystals ;
or (2) a property of all body centred cubic crystals ;
or (3) a property of all cubic crystals.

It is proposed to carry out further experiments, on the same lines, using cubic crystals of other metals and to determine the type of Elastic Symmetry which exists, together with the necessary elastic constants.

Introductory.—The development of methods of preparing large single crystals of various metals has provided a new field for research into the Strength of Materials. While a great deal of work has been carried out on the behaviour of these crystals when strained plastically, very little has been done towards investigating their elastic properties.

Bridgman* has given values, based on static experiments, for the elastic constants of single crystals of seven metals including Tungsten. These values do not appear to have been confirmed by any other worker, nor has it been demonstrated that the elastic symmetry of these crystals is actually similar to their crystallographic symmetry.

In these circumstances it was considered that a systematic investigation of the elastic properties of crystals of one or more metals would be of value ; and it seemed probable that, provided suitable material could be obtained, dynamical methods would be more sensitive than static tests.

Pintsch crystals of Tungsten were chosen as the first material to be used because they were available in the form of wires of suitable diameter and further because they were known to have a large range of primitive elasticity.

Previous Work.—Tungsten crystals have a body-centred cubic lattice and therefore three constants are necessary in order to specify their elastic properties. With reference to the three cubic axes the stress strain relations can be written in the ordinary notation :—

$$\begin{aligned} e_{xx} &= S_{11}\,\widehat{xx} + S_{12}\,\widehat{yy} + S_{12}\,\widehat{zz} \\ e_{yy} &= S_{12}\,\widehat{xx} + S_{11}\,\widehat{yy} + S_{12}\,\widehat{zz} \\ e_{zz} &= S_{12}\,\widehat{xx} + S_{12}\,\widehat{yy} + S_{11}\,\widehat{zz} \\ e_{yz} &= S_{44}\,\widehat{yz} \\ e_{zx} &= S_{44}\,\widehat{zx} \\ e_{xy} &= S_{44}\,\widehat{xy} \end{aligned}$$

In these equations S_{11}, S_{12} and S_{44} are the three elastic constants. The specification of strain in terms of stress with reference to any other system of axes involves the usual 21 constants, all of which can, of course, be expressed in terms of the three primary constants when the orientation of the crystal with reference to the new axes is known. If the material considered is isotropic the only change necessary in the stress strain equations is that the relation

$$2\,(S_{11} - S_{12}) = S_{44}$$

must be satisfied.

* Proceedings, American Academy of Arts and Sciences, Oct., 1925, p. 305, Vol. 60.

In Bridgman's experiments five measurements were made on a single cylindrical bar of known crystal orientation :—

(1) The Longitudinal contraction under fluid pressure.

(2) The Longitudinal contraction under end compression.

(3) and (4) The Lateral expansions across two perpendicular diameters under the same stress as in (2).

(5) The twist due to torsional couples.

He thus had five equations from which to determine three unknowns and, being based on experiment, these equations were naturally not all consistent. Bridgman's procedure was to determine the constants S_{11} and S_{12} from the equations based on results (1) and (4) and to average the three values of S_{44} obtained by substitution of these constants in the remaining equations. The values obtained in this way were :—

$$\left.\begin{aligned} S_{11} &= 2\cdot534 \times 10^{-13} \\ S_{12} &= -0\cdot726 \times 10^{-13} \\ S_{44} &= 6\cdot55 \times 10^{-13} \end{aligned}\right\}\text{dyne centimetre units.}$$

Bridgman pointed out that these values very nearly satisfied the Isotropic relation $2(S_{11} - S_{12}) = S_{44}$.

An analysis of the actual equations given in his paper, however, made it appear doubtful whether they justified this conclusion. Thus, for example, the two lateral contractions measured, (3) and (4), differed from one another by about 17 per cent. The reason given for taking one of these values, rather than the other, in conjunction with the result (1) in his determination of S_{11} and S_{12} had no reference to the probability of it being the more accurate from an experimental standpoint and, in fact, there is no reason from his paper to conclude that either result is likely to be more accurate than the other.

In these circumstances it seemed more reasonable to discard the results (3) and (4) and to calculate the constants from the remaining equations (1), (2) and (5). When this was done the values obtained were :—

$$\left.\begin{aligned} S_{11} &= 2\cdot62 \times 10^{-13} \\ S_{12} &= -0\cdot77 \times 10^{-13} \\ S_{44} &= 6\cdot34 \times 10^{-13} \end{aligned}\right\}\text{dyne centimetre units}$$

and it was seen that the isotropic relation was not so nearly satisfied as before.

Range of Present Experiments.—For the reasons given in the previous paragraph there was, at the outset of the present work, no reason to believe that Tungsten crystals were truly isotropic ; and it seemed possible that careful measurements might disclose appreciable variation of the elastic properties with orientation of the test

(39062) Wt. 92/6003/1810 375 12/29 Harrow G 7/1

piece. It was intended in the present case to observe the periodic times of oscillations in torsion and in bending of wires of various orientations and thus to determine values of the Torsional Modulus of Rigidity and the longitudinal Young's Modulus for each wire. If the angles between the axis of the wire and the three cubic axes of the crystal are θ_1, θ_2, and θ_3, it can easily be shown that if the wire is of circular cross-section :—

The Torsional Modulus of Rigidity of the wire $= \dfrac{1}{S_{44} + 4\alpha S} = N$ say

The Longitudinal Young's Modulus of the wire $= \dfrac{1}{S_{11} - 2\alpha S} = E$ say

where $S = S_{11} - S_{12} - \frac{1}{2}S_{44}$

$$\alpha = \cos^2\theta_1 \cos^2\theta_2 + \cos^2\theta_2 \cos^2\theta_3 + \cos^2\theta_3 \cos^2\theta_1$$

If then we determine the values of N and E for a series of wires, we can plot their reciprocals against the various values of α and hence determine values of S_{11}, S_{44} and S.

The variation of these Moduli with orientation of the crystal axes of the wire may conveniently be exhibited on an Equilateral Triangular diagram. The three angles, θ_1, θ_2, and θ_3, are, of course, connected by the relation :—

$$\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3 = 1$$

If, then, the angular points of the equilateral triangle shown in Fig. 1a be supposed to represent the three cubic axes, the orientation of the axis of any wire can be represented by a point within the triangle by using the quantities $\cos^2\theta_1$, $\cos^2\theta_2$, and $\cos^2\theta_3$, as trilinear co-ordinates.

The scale of the diagram is such that the length of the perpendicular from the vertex to the base is equal to unity. The centroid of the triangle corresponds to the normal to the octahedral plane (111) while the mid-points of the sides represent the normals to three of the Rhombic dodecahedral (110) planes.

The advantage of this method in the present connection lies in the fact that in this diagram the family of curves $\alpha =$ const. is a series of concentric circles whose centre is the centroid of the triangle. It has been shown previously that lines for which α is constant will be lines of constant Young's Modulus, or of constant Torsional Modulus of Rigidity.

It will be seen that the six smaller triangles, into which the equilateral triangle of Fig. 1a has been divided by the perpendiculars from the vertices to the opposite sides, are all similar and are, in fact, crystallographically equivalent to one another. Only one of them is necessary in order to exhibit the relative orientations of a series of different wires and the corresponding points can all be brought into the particular triangle chosen by suitable rotation about the centroid.

Fig. 1b shows one of these triangles enlarged and some circles of constant α have been drawn in it. The points plotted refer to the wires used in these experiments, details of which will be given later.

Particulars of Material supplied.—All the Tungsten wires used in this work were supplied by the General Electric Company Research Laboratories, Wembley. The original wires were of 0·2 mm. diameter, but it soon became apparent that wires of larger diameter would be preferable. A second batch of twelve wires was then obtained, each approximately 10 cm. long and 0·9 mm. in diameter. It was necessary for the present purpose that the portion of the wire under test should consist wholly of one crystal. A preliminary X-ray examination at two points of each wire was therefore carried out by the Physics Department. Partly on account of the difficulty of putting reference marks on such small specimens and partly because the ψ-co-ordinates were not needed, only the θ-co-ordinates of the X-ray analysis were recorded in these preliminary determinations. This procedure led to an error which greatly delayed the work at its commencement.

It was found that a favourite mode of crystal arrangement in these wires was that two crystals of nearly the same orientation existed side by side with a longitudinal boundary between them, running often along the whole length of the wire. These crystals were so nearly of the same orientation that in the absence of the ψ–co-ordinates it was not at first recognised that more than one crystal was present. At a later stage one determination in which both co-ordinates were recorded, disclosed two crystals of this type in the wire concerned. This result was verified by microscopical examination, the crystal boundary being plainly visible.

Re-examination then showed that no less than nine out of the twelve wires suffered from this defect. The remaining three wires were very nearly of the same orientation, so that virtually only one specimen was available.

Seven more wires were then obtained and finally five wires, each consisting of one crystal only, over a sufficient portion of its length, and all of different orientations were found.

The diameters of these wires were measured in the Metrology Department in a machine capable of reading to 1/100,000th of an inch. The procedure adopted was to measure four diameters equally spaced around the circumference at about ten positions along the length of the wire. All the readings obtained for each wire were then averaged.

Table 1 gives particulars of these wires and also of a sixth wire consisting of three crystals whose torsional rigidity was also determined.

TABLE 1.

Particulars of Wires tested.

Number of Wire.	Diameter in Millimetres.	θ-co-ordinates of cubic axes.			Value of Constant α
		θ_1	θ_2	θ_3	
WD1	0·9334	52·2°	54·4°	57·8°	0·331
WD2	0·8958	44·2°	46·4°	84·9°	0·255
WD3	0·8466	44·2°	53·0°	69·4°	0·295
WA3	0·9128	28·3°	63·8°	80·0°	0·180
WA8	0·9207	27·7°	64·8°	77·2°	0·186
WA2	0·9271	—	—	—	—

The orientations of these wires have been plotted in Fig. 1b.

Experimental Methods.—Before describing the apparatus employed in this work it is necessary to point out that some modification of the usual methods may be necessary when the material under test is not isotropic. Examination of the equations given in a previous paper* will show that when a non-isotropic circular cylinder is subjected to torsional couples the twist which takes place is accompanied by a uniform bending of the axis. In the same way, when the cylinder is subjected to bending couples, a twist will occur. Consequently, in a static torsion test or in torsional oscillations the observed rigidity will depend on whether bending is permitted to take place or not and in the latter event a modified Modulus of Rigidity will be obtained. This modified modulus is analogous to the modified value of E which would be obtained in a compression test on isotropic material if lateral expansion were prevented.

Similar considerations will apply to bending tests. In this case, however, it can be shown that the tendency to twist will depend on the particular plane in which bending occurs. The tendency will vanish for a certain plane, depending on the orientation, and will reach a maximum value for a plane at right angles to this.

It was considered that the amounts of the secondary bending and twist would be small in the present case and it was decided to restrict them as much as possible in the experiments. The corrections which must be applied to the ordinary Moduli can easily be determined, but as it was found that these refinements were unnecessary in the case of Tungsten, the analysis will not be given here.

* "The Torsion of circular and elliptical cylinders of Homogeneous Aeolotropic Material." R. & M. 1031.

The length of wire available for test was not in general more than about 5 cm. The means of gripping the test piece therefore became of special importance as the end-effect which exists in the ordinary way would be sufficient to introduce considerable error in such short pieces. It was necessary to grip the wires in chucks which would permit different test lengths to be used so that the end-effect could be eliminated. Great difficulty was encountered in making chucks which would grip the wire in so constant a manner that the difference between the apparent length of wire and the true length would be the same after each adjustment. The difficulty was increased by the extreme hardness of Tungsten. Pin chucks of the ordinary type had to be screwed up so tightly in order to hold the wires firmly that their edges became deformed and their grip probably changed at each adjustment. A complete series of results originally obtained had to be discarded owing to the uncertainty introduced by progressive distortion of the chucks. Finally, small "Lorch" collet chucks were obtained and were found to work quite satisfactorily.

The apparatus used in the bending oscillations is shown in Figs. 2a and 2b. It is essentially the same as that originally devised by Searle* for this purpose. Two equal inertia bars AA are suspended parallel to one another by long fine threads of plaited silk. At the centre of their lengths two brass bosses BB are provided with holes in which the chucks can be clamped so that the wire is perpendicular to the two bars with its ends at their respective centres of gravity. To each bar two rods CC ground to points at their ends, are fixed so that the common axis of each pair is coincident with the axis of rotation of the corresponding bar during the experiment. While the bars are freely supported by the threads, the conical holes at the ends of four screws attached to the casting D can just be brought into engagement with the points of the rods CC. This feature was provided originally, in order to counteract the secondary twisting of the wire which, we have seen, might have occurred. Satisfactory tests were carried out with these pivots in operation. The damping was, of course, increased by the friction at the screws, but a sufficient number of oscillations could be timed to permit accurate determination of the Periodic Time.

It was, however, found that when the pivots were not in operation, no twisting of the wire took place, but this means of fixing the rods so that they could not swing about at the ends of their supporting threads, was found to be quite indispensable while inserting or removing the wire.

The method suggested by Searle of starting the oscillations by drawing the ends of the inertia bars together with thread, which is afterwards ignited so as to free the bars suddenly, was found to be

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* G. F. C. Searle. Phil. Mag., Feb., 1900, p. 197.

impracticable. In the present apparatus the oscillations were started and controlled by blowing gently down the fork-shaped glass tube E which was placed symmetrically with respect to one pair of ends of the bars.

A small mirror was attached to one of the rods C and the image in it of a distant scale was observed through a telescope. The distance of the scale from the mirror was generally a little over 50 ft., the exact distance being so chosen that an angular deflection of one minute gave a scale reading of one centimetre.

A series of reference marks on the chucks allowed the wire to be set so that the bending took place about any desired axis perpendicular to its length.

In the torsion tests one of the inertia bars from the above apparatus was employed, the upper rod C being removed and the reduced end of one chuck being inserted in its place. The bar was then suspended from the wire by clamping the other chuck to a rigid support with its axis vertical. The point of the lower rod C was generally allowed just to touch the surface of some vaseline. This had the effect of damping out any oscillations other than those due to twisting of the wire. It was hoped that if any secondary bending due to non-isotropy tended to occur, it would be prevented by the combined action of the vaseline and the weight of the inertia bar. No tendency to bend was, however, noticed in any of the Torsion tests.

In both tests very small oscillations of less than one minute of arc could be observed if necessary. This, of course, constitutes the principal advantage of dynamical methods over static tests, namely, that only extremely small stresses and strains need be used. This was, of course, not so important in the present case of Tungsten Crystals as it would have been if, for instance, aluminium crystals had been under test.

The oscillations were timed by means of a " Chronoscope " stop watch reading to 1/100 second. This watch was rated by the Metrology Department and was found to be in error on an hour's run by less than 1 part in 4,000.

In each test in both Bending and Torsion, four or five separate determinations of the time of a hundred oscillations were made generally with very close agreement. The error in any measured periodic time is of the same order as that of the stop watch. The inertia bars were carefully calibrated during the work ; the inertia of each was found to be 2987 Kg. cm.^{2} to within 1 part in 2,000.

The Results of the Experiments.—(a) *General.*—It has recently been found by Cox and Backhurst* that there is no change in the nature of the X-ray reflections from Tungsten under tensile stress

* R. & M. 1221. The effect of stress upon the X-ray reflections from tungsten wire at air temperature.

up to over 80 tons/in.^{2}. The extensive range of elasticity indicated by their experiment has been noted throughout the present work. In both Bending and Torsion tests permanent set was never encountered. Even when comparatively large accidental oscillations had been given to the inertia bars, no difference could be detected between the reading indicated when they had come to rest and that to which they had been adjusted initially.

The Periodic Time of the oscillations appeared to be constant for a given mean amplitude, however many oscillations of greater or less amplitude had previously taken place.

This fact is illustrated by the results given in Table 2 which refers to a test carried out on wire WA3 in the Bending apparatus. In this test it was arranged that the stop watch was started automatically, by means of electrical contact made by one of the inertia bars at the centre of its swing, and stopped in the same way after any desired number of oscillations. Ten separate determinations of the time of 20 oscillations were made at each of three different settings of the wire in the chucks. The amplitude in each run was about 15 minutes of arc and after each set of oscillations timed a number of larger oscillations were performed. The approximate time for 20 oscillations was 25 seconds ; the difference between the greatest and the least time recorded at any one setting was 0·02 seconds, so that the greatest variation from the mean value was only one part in 2,500. It is probable that even this small variation was due to imperfections in the timing arrangements.

TABLE 2.

Results of Bending Tests on WA3 showing Constancy of Period.

Number of Run.	Time of 20 Oscillations in Seconds.		
	1st Setting.	2nd Setting.	3rd Setting.
1	24·96	24·99	24·88
2	24·95	24·98	24·88
3	24·94	24·98	24·89
4	24·95	25·00	24·89
5	24·96	24·98	24·90
6	24·95	24·98	24·90
7	24·96	24·99	24·90
8	24·95	25·00	24·88
9	24·95	24·98	24·90
10	24·95	24·98	24·88

As is usual in such experiments, it was found that there was some variation of Period with Amplitude, the time of an oscillation increasing slightly with its amplitude. Fig. 3 shows the results of a test in Torsion on wire WD3 and indicates the order of the variation encountered. The periodic time for an oscillation of one degree semi-amplitude was about one part in 500 greater than that for an amplitude of one minute, the length of wire being about 4 cm.

The average semi-amplitude used in the determination of the elastic constants was about 5 minutes on a length of 4 cm., so that the recorded periodic times will not differ appreciably from those corresponding to an indefinitely small amplitude.

(b) *The Torsion Results.*—The modulus of Rigidity of a wire can be expressed in terms of T, the Periodic Time of Torsional oscillations by the equation :—

$$N = \frac{128\,\pi K\, l}{T^2\, d^4} \text{ dynes/cm.}^2$$

where K is the Moment of Inertia of the Inertia Bar, in Gm–cm.^{2}

d is the diameter of the wire in centimetres.
l is the length of the wire in centimetres.

In order to avoid the difficulty, previously referred to, of determining the exact length of wire gripped by the chucks, the Period of Torsional oscillations was determined for a number of different "nominal" lengths of each wire. The "nominal" length is the length between the faces of the chucks and was measured by means of a travelling microscope.

The results are given in Table 3 and in Fig. 4 the square of the Period has been plotted against the nominal length. It will be seen that the points corresponding to any one wire lie very nearly on a straight line which intersects the length axis at a small negative value. This indicates that the wires are gripped at points a little behind the faces of the chucks, so that the true length of wire under test is greater than the apparent length by a constant amount for each wire.

The difference between these two lengths was nearly the same for all the wires ; the actual value was determined by plotting to a large scale and is given in the table.

The slope of the lines in Fig. 4 gives the values of T^2/l for each wire. In order to compare directly the moduli of rigidity of the wires we require the values of T^2d^4/l. In the last two columns of Table 3 the values of T^2d^4 have been tabulated against the true length, that is the sum of the apparent length and the correction given in the table, and the results have been plotted in Fig. 5.

It will be seen that all the points (including those referring to wire WD2, which consisted of three crystals) now lie very nearly on one straight line. The slope of this line is the value of T^2d^4/l, and the results show that the value of this quantity is practically the same for all the wires.

TABLE 3.

Results of Torsion Experiments.

No. of Wire.	Length between Chucks. Cm.	Period in Seconds. (T)	T^2 (sec)^2	Difference between true and apparent Length. Cm.	True Length. Cm.	$T^2 \times d^4$ (sec)$^2 \times$ (cm)^4
WD1	3·869	6·525	42·58	0·180	4·050	0·003215
	3·433	6·164	37·99		3·615	0·002870
	2·952	5·741	32·96		3·130	0·002490
	2·276	5·077	25·78		2·455	0·001945
	1·396	4·066	16·53		1·575	0·001250
	0·690	3·024	9·14		0·870	0·000690
WD2	3·980	7·141	50·99	0·180	4·160	0·003290
	2·938	6·175	38·13		3·120	0·002460
	2·034	5·206	27·10		2·215	0·001750
	1·005	3·816	14·56		1·185	0·000939
WD3	2·237	6·085	37·03	0·180	2·415	0·001905
	1·549	5·163	26·66		1·730	0·001375
	1·373	4·877	23·79		1·555	0·001225
	1·132	4·476	20·03		1·310	0·001032
	0·543	3·348	11·21		0·725	0·000577
	0·314	2·761	7·62		0·495	0·000393
WA3	3·895	6·842	46·81	0·210	4·105	0·003255
	2·875	5·930	35·61		3·085	0·002445
	1·608	4·547	20·68		1·820	0·001435
	0·837	3·444	11·86		1·045	0·000824
WA8	3·464	6·333	40·11	0·180	3·644	0·002890
	3·155	6·067	36·81		3·335	0·002650
	2·476	5·411	29·28		2·656	0·002110
	1·868	4·762	22·68		2·050	0·001635
	1·506	4·316	18·63		1·686	0·001340
	0·987	3·595	12·92		1·165	0·000930
WA2	3·975	6·704	44·94	0·200	4·175	0·003315
	2·495	5·380	28·94		2·695	0·002135
	1·101	3·720	13·84		1·301	0·001022

We have seen that $T^2 d^4/l$ is proportional to $1/N$, and that the latter quantity is equal to $S_{44} + 4\,\alpha S$. The results, therefore, show that $S_{44} + 4\,\alpha S$ is practically independent of the value of α. This, of course, means that S must be very small in comparison with S_{44}. The condition $S = 0$ is the condition that the material is isotropic and it appears from the results of the Torsion Tests that Tungsten crystals are very nearly isotropic.

In order to show, more clearly, the actual experimental variation of the value of T^2d^4/l from wire to wire, Fig. 6 has been drawn.

In this figure the vertical scale has been greatly enlarged by plotting the differences between the experimental values of T^2d^4 at each length, and the value corresponding to the dotted line ($T^2d^4 = 0 \cdot 00075l$) in Fig. 5. Also, the points corresponding to each wire have been separated out by displacing each set through a certain horizontal distance.

Before considering this diagram it is necessary to discuss the limits of error of the work. By far the most serious error likely to occur is the error in measurement of the diameters of the wires which will be magnified four times in deducing Figs. 5 and 6 from Fig. 4.

The average diameter of the five wires concerned was 0·036 in. In each case between 30 and 40 measurements were taken in the manner previously described. The extreme limits of departure of the measured diameters from the mean for each wire were :—

WD1 $\pm$ 2 $\times$ 10^{-5} inch or $\pm$ 1 part in 1800
WD3 $\pm$ 2 $\times$ 10^{-5} inch or $\pm$ 1 part in 1800
WA8 $\pm$ 4 $\times$ 10^{-5} inch or $\pm$ 1 part in 900
WA3 $\pm$ 5 $\times$ 10^{-5} inch or $\pm$ 1 part in 700
WD2 $\pm$12 $\times$ 10^{-5} inch or $\pm$ 1 part in 300

These limits could be much reduced in each case by the rejection of a few of the measurements made. Thus, in the case of the first wire the limits are reduced to $\pm 1 \times 10^{-5}$ inch by the rejection of only two measurements out of 32. The third and fourth of the wires (WA8 and WA3) show rather larger departures than the first two, but these were due to the wires being slightly elliptical in cross-section, the cross-section being uniform along the length within very close limits. It is considered that for each of the four wires WD 1 and 3, and WA 8 and 3, the values adopted for d^4 are probably correct to 1 part in 500. It is difficult to estimate the accuracy of the value adopted for the remaining wire (WD2) because in this case the variations of the cross-section along the length were of the same order as the variations in diameter at any particular cross-section. It is probable, however, that the error will not be greater than about 1 part in 200.

In Fig. 6 the line corresponding to $T^2d^4/l = 0 \cdot 000793$ has been drawn for each set of points, while in the case of wire WD2 two dotted lines, representing departures of $\pm 0 \cdot 5$ per cent. from this value, have also been drawn.

It will be seen that, even when plotted to this scale, the points corresponding to the first three wires WD1, WA8, and WA3 show no appreciable variation from the mean line drawn.

In the case of the wire WD3 the points are so irregularly spaced in this diagram that it is difficult to estimate the slope of a mean line drawn through them. These irregularities are no doubt due to the fact that this wire was by no means straight when originally supplied, and when an attempt was made to straighten it, it fractured into four pieces. The experimental determinations were made on the two straightest of these pieces, but it could hardly be expected that the results would be as consistent as those obtained from the other wires.

The remaining wire WD2 shows a relatively large variation from the mean line. We cannot, however, accept this as genuine because the value of α for this wire is roughly half-way between the value corresponding to wire WD1 and the mean value for wires WA8 and WA3. If, therefore, the variation between this wire and WD1 were genuine we should expect to find a variation of twice the amount between, say, WD1 and WA3. We have, in any case, already seen that the error in the measured diameter is likely to be greater in the case of this wire.

From these considerations it is concluded that the variation of the modulus of rigidity for wires corresponding to the range of values of α considered is certainly not greater than 0·2 per cent.

The actual range of α experienced was from 0·180 to 0·331; while the extreme possible range is from 0 for a wire whose axis is perpendicular to a cube face to 1/3 for a wire perpendicular to an octahedral plane. The experimental range was therefore rather less than half the possible range.

Taking the extended range of α into account, the results of the experiments indicate that for wires of any orientation:—

$$\frac{T^2 d^4}{l} = 0{\cdot}000793 \pm 0{\cdot}000002 \text{ (cm.)}^3 \text{ (sec.)}^2$$

Bending Results.—In general, the Young's Modulus of a wire cannot be determined with the same accuracy as the Modulus of Rigidity because of the variations in diameter which exist. The Periodic Time of oscillation in Bending will vary with the plane of bending and although a number of different planes can be taken and the Periods averaged, the results will not be as accurate as those obtained in Torsion unless the wires are nearly uniform. Two of the wires, however, were uniform in diameter within about one part in 2,000 and the bending tests reported are restricted to these.

The procedure adopted was to show by means of tests on different lengths of one of these wires that the "end effect" was the same as in Torsion, and then to obtain a direct comparison between the Periodic Times in Bending and in Torsion for both wires without removing them from the chucks between the Tests.

In Table 4 the results of tests on different lengths of wire WD1 are given. It was found that the Period did not vary with the plane of bending by more than 1 part in 2,000.

The results are plotted in Fig. 7 together with the Torsion results on the same wire from Fig. 4. It will be seen that the difference between the true and apparent lengths is the same for the two tests.

TABLE 4.

Results of Tests in Bending on Wire WD3.

Length between Chucks.	Angular Setting in Apparatus.	Periodic Time (secs.) T.	T^2.
4·011	0°	4·134	17·09
	105°	4·135	
	30°	4·133	
	130°	4·135	
3·027	0°	3·610	13·03
2·226	0°	3·131	9·80
1·517	0°	2·629	6·91

The result of the comparison between the squares of the Periods in Bending and in Torsion is given in Table 5. For each wire the same length was used for the two tests without removal from the chucks in between.

TABLE 5.

Comparison between Flexural and Torsional Rigidities for Wires WD1 and WD3.

Wire No.	Bending Period T_b.	Torsion Period T_t.	$\frac{E}{N} = \frac{T_t^2}{T_b^2}$
WD1	4·134	6·625	2·568
WD3	4·026	6·449	2·566

General Conclusions.—The results of the Torsion Tests show that the Modulus of Rigidity, for crystals of any orientation, does not vary by more than 1 part in 200. This variation, even if it were shown to exist, is considerably less than that usually encountered in testing different samples of a statistically isotropic material. *We, therefore, conclude that Tungsten Crystals are truly Isotropic.*

Having reached this conclusion, we are in a position to calculate the values of the necessary elastic constants from the results previously given.

These values are :—

Young's Modulus $= 3\cdot886 \times 10^{12}$ dynes/cm.^{2} $=$ 25,150 tons/in.^{2}.
Modulus of Rigidity $= 1\cdot514 \times 10^{12}$ dynes/cm.^{2}$=$9,800 tons/in.^{2}.
Poisson's Ratio $= 0\cdot284$.

The values of the S– constants are, for comparison with Bridgman's results :—

$$\left.\begin{aligned} S_{11} &= 2\cdot573 \times 10^{-13} \\ S_{12} &= -0\cdot729 \times 10^{-13} \\ S_{44} &= 6\cdot604 \times 10^{-13} \end{aligned}\right\} \text{dyne/cm. units.}$$

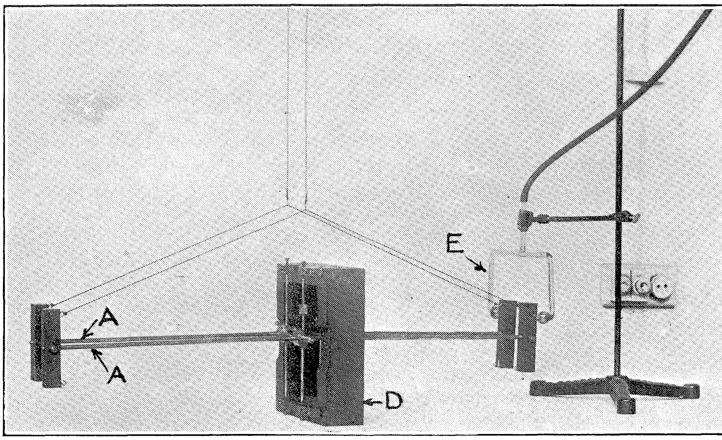

FIG. 2A. General view of Bending apparatus.

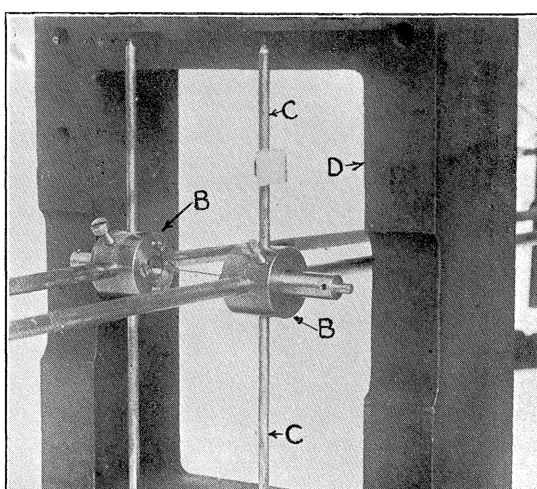

FIG. 2B. Enlarged view of centre of apparatus.

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FIG. 1.

DIAGRAM SHOWING:-

(1) Variation of α with orientation of wire.

(2) Relative orientations of wires used in tests.

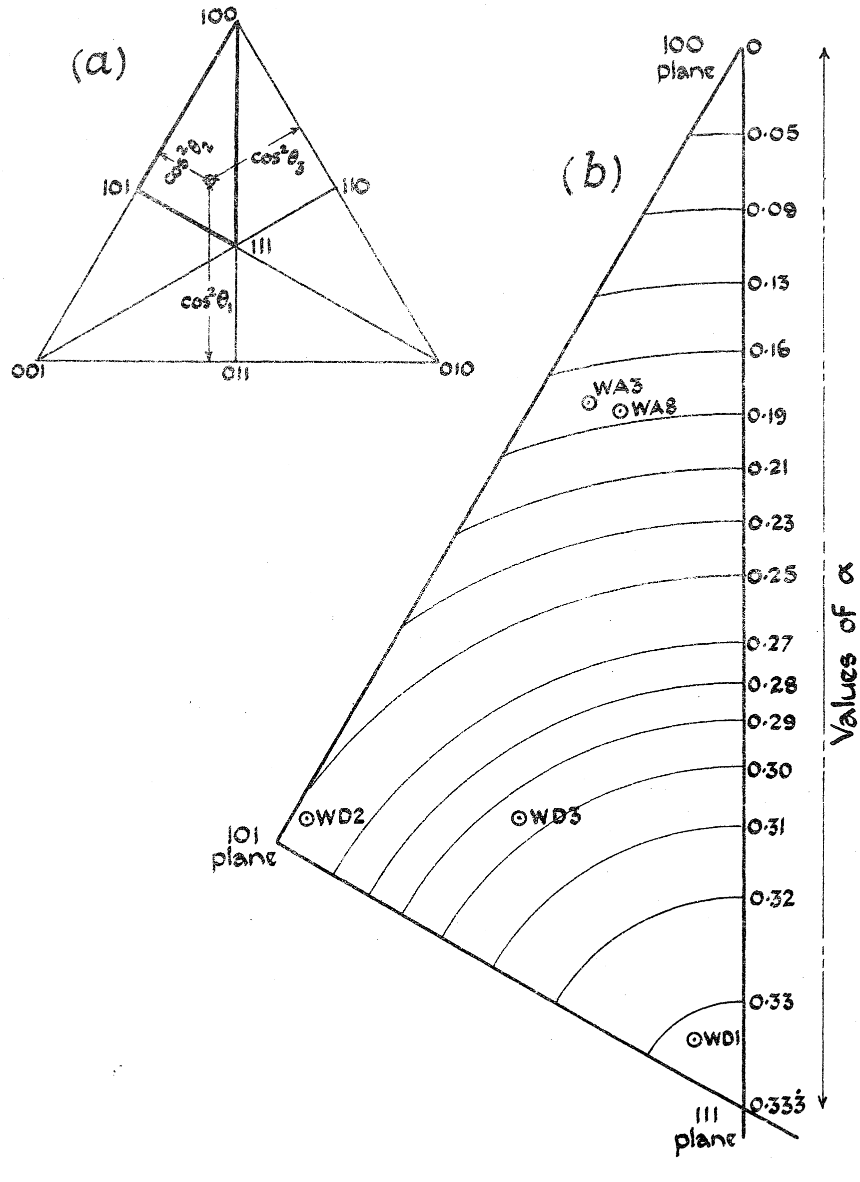

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Fig. 3.

TEST ON WIRE WD3 IN TORSION
SHEWING VARIATION OF PERIOD WITH AMPLITUDE

Length = 4 cm. approx.

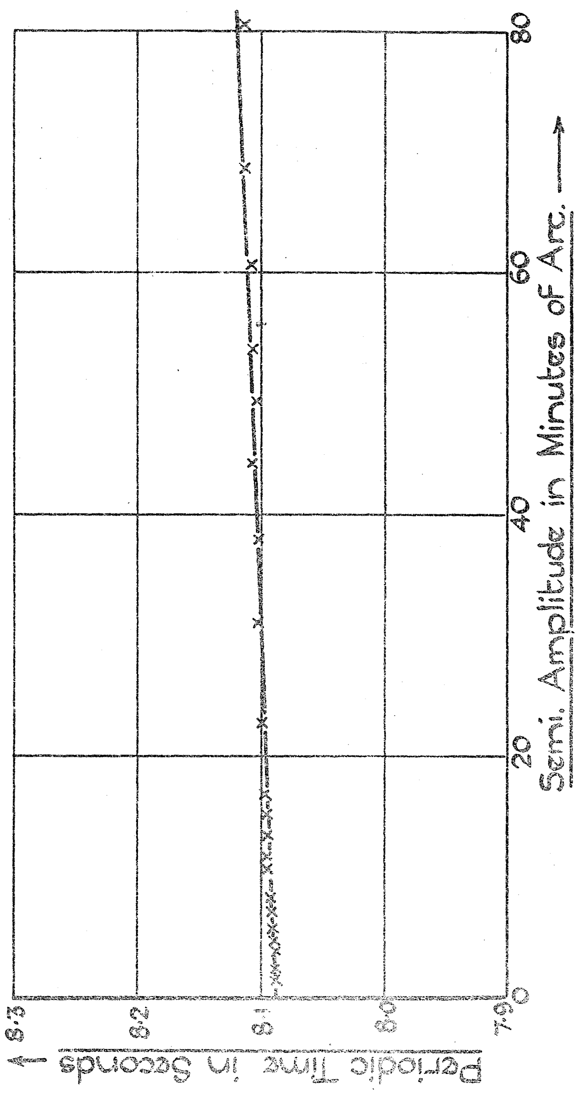

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FIG. 4.

RELATION BETWEEN PERIODIC TIME AND APPARENT LENGTHS.

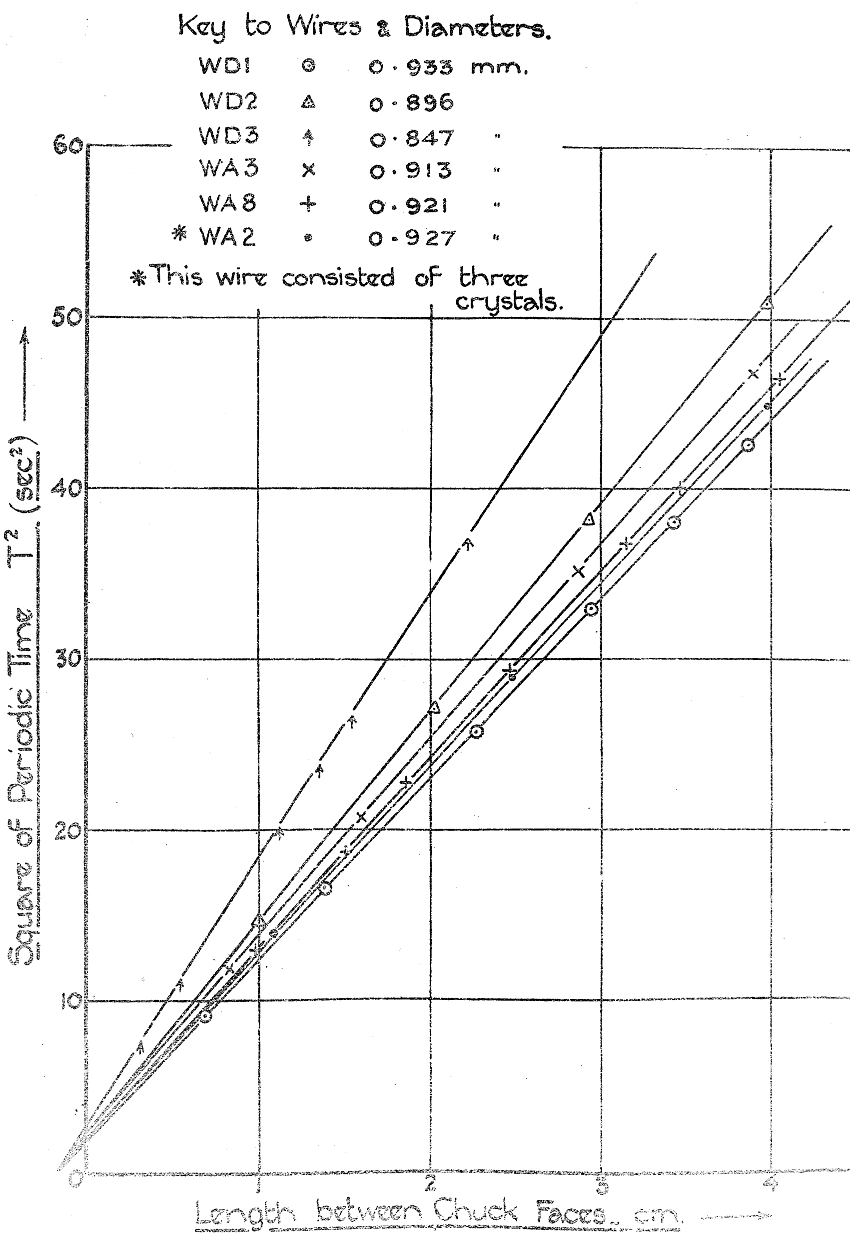

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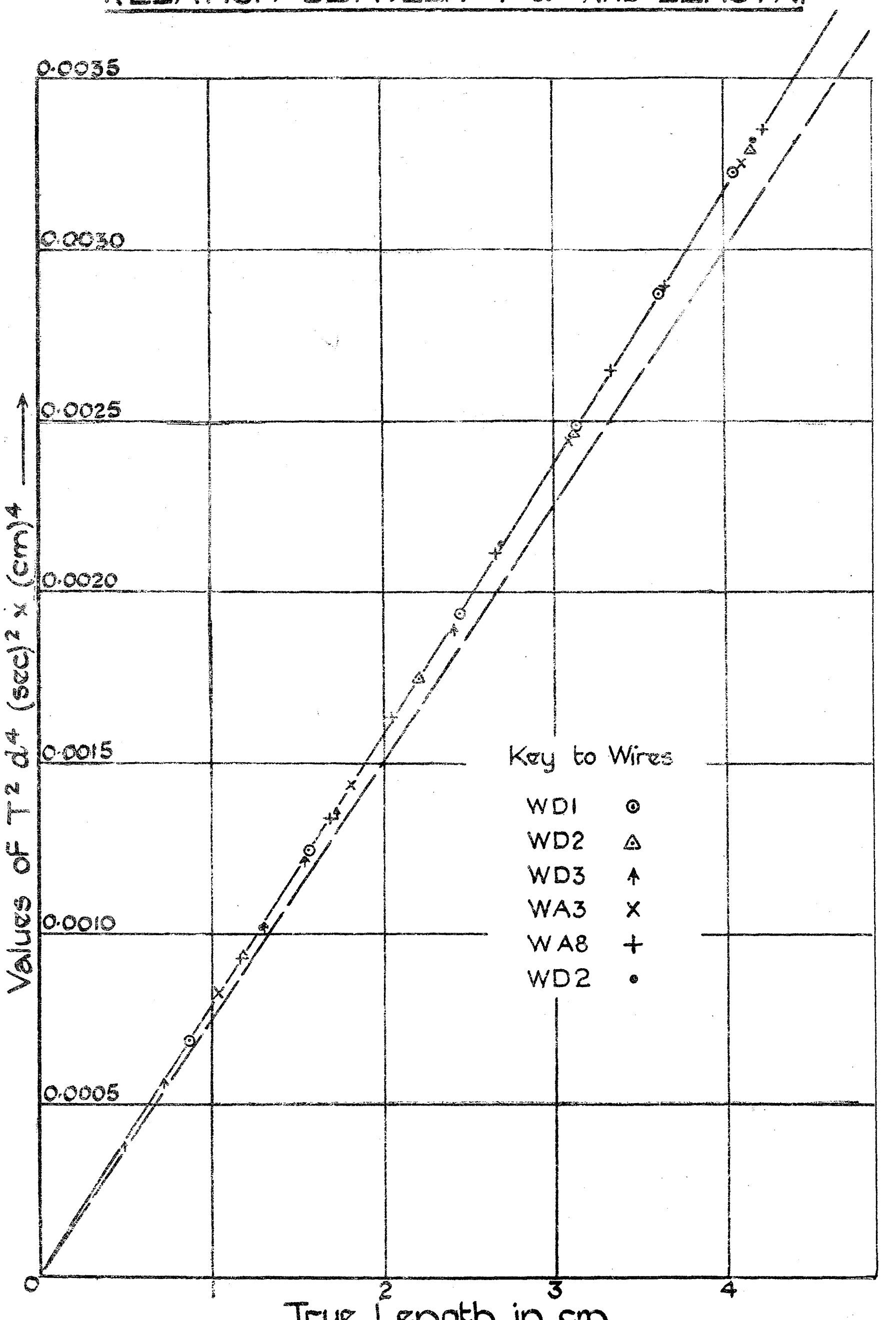
FIG. 5.

RELATION BETWEEN $T^2 d^4$ AND LENGTH.



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DIFFERENCES BETWEEN EXPERIMENTAL POINTS AND VALUES CORRESPONDING TO THE DOTTED LINE IN FIG. 5.

FIG. 6.

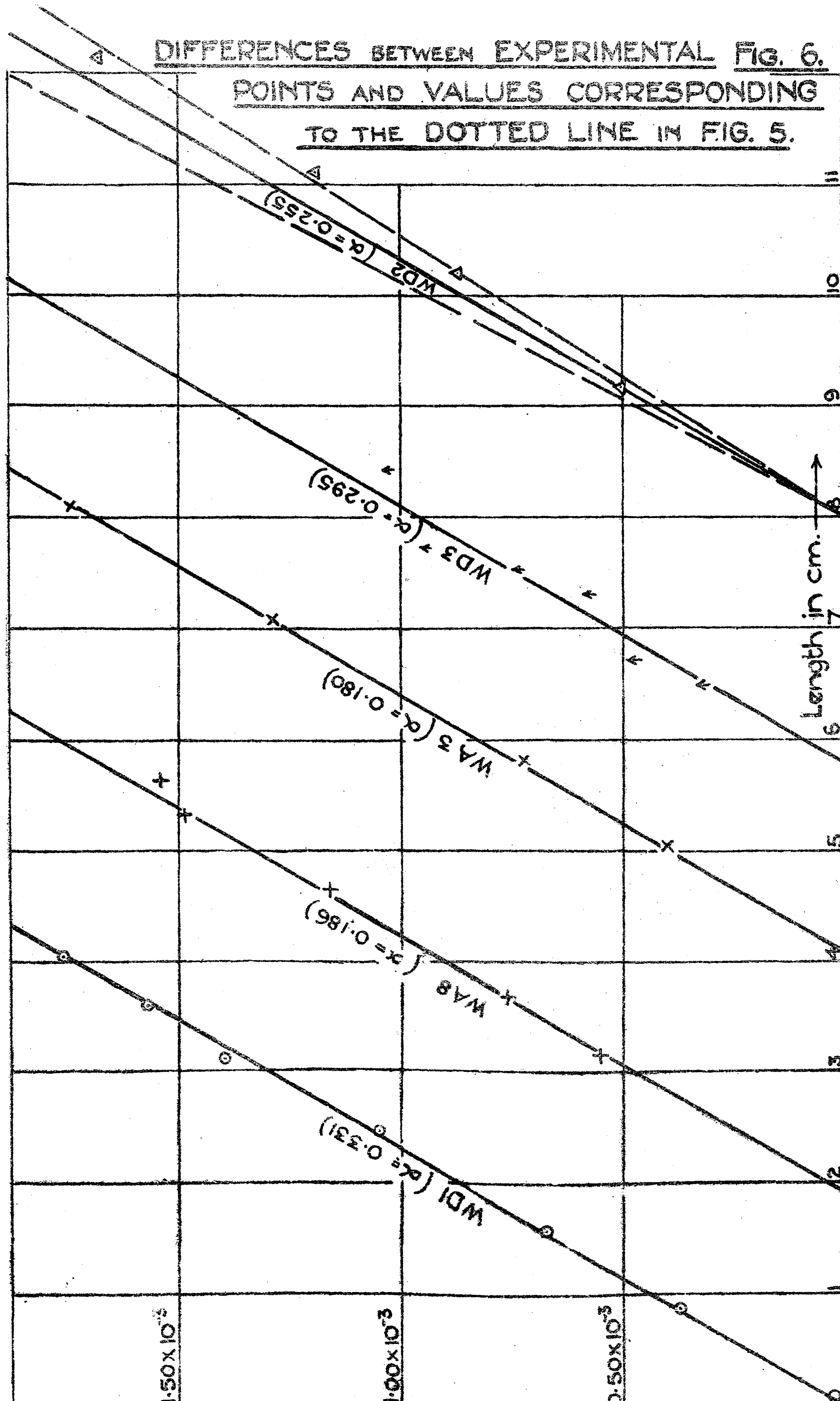

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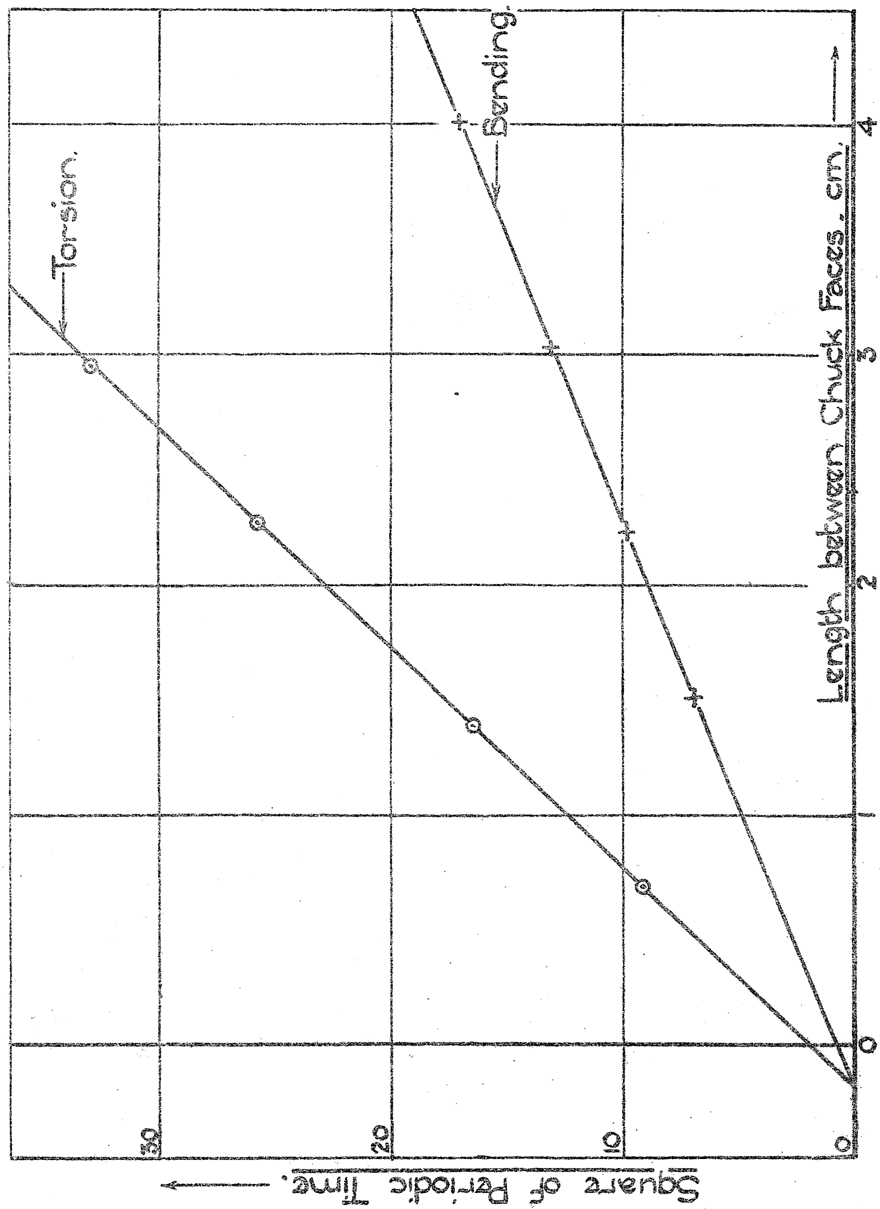
FIG. 7.

COMPARISON BETWEEN TORSION & BENDING RESULTS ON W D I SHEWING CONSTANCY OF END EFFECT.



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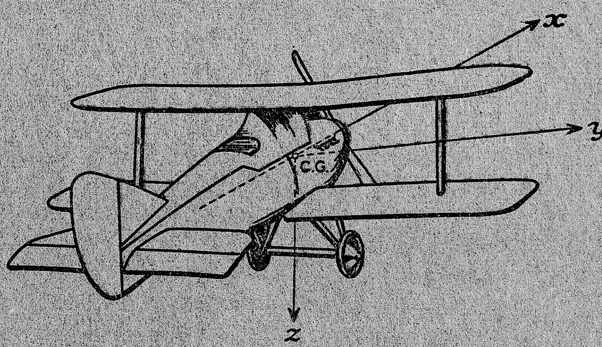
SYSTEM OF AXES.



Axes	Symbol Designation Positive direction	x longitudinal forward	y lateral starboard	z normal downward
Force	Symbol	X	Y	Z
Moment	Symbol Designation	L rolling	M pitching	N yawing
Angle of Rotation	Symbol	ϕ	θ	ψ
Velocity	Linear Angular	u p	v q	w r
Moment of Inertia		A	B	C

Components of linear velocity and force are positive in the positive direction of the corresponding axis. Components of angular velocity and moment are positive in the cyclic order y to z about the axis of x, z to x about the axis of y, and x to y about the axis of z.

The angular movement of a control surface (elevator or rudder) is governed by the same convention, the elevator angle being positive downwards and the rudder angle positive to port. The aileron angle is positive when the starboard aileron is down and the port aileron is up. A positive control angle normally gives rise to a negative moment about the corresponding axis. The symbols for the control angles are :–

ξ aileron angle
η elevator angle
η_T tail setting angle
ζ rudder angle