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(Ae. 274.)

THEORETICAL RELATIONSHIPS FOR AN AEROFOIL WITH HINGED FLAP—BY H. GLAUERT, M.A.

PRESENTED BY
THE DIRECTOR OF SCIENTIFIC RESEARCH.

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THEORETICAL RELATIONSHIPS FOR AN AEROFOIL WITH HINGED FLAP.

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Summary.—*Introductory.*—The use of an aerofoil with a hinged flap is of very general importance both for control surfaces and for main supporting surfaces, and in particular information is required as to the effect of varying the size of the flap. Hitherto it has been customary to rely wholly on experimental results, but theoretical expressions for the lift and pitching moment in two dimensional motion are given in report R. & M. 910.

Range of Investigation.—The analysis has now been extended, in an improved manner, to cover also the hinge moment and the case of rectangular aerofoils of finite span.

Conclusions.—The theoretical values appear to be in very satisfactory agreement with experimental results, and so the theoretical formulæ may be used with confidence to predict the effect of variation of the size of flap or of the aspect ratio of the aerofoil.

1. *Introduction.*—The aerodynamic behaviour of an aerofoil with a hinged flap is of very general importance, since this form of construction is used for all the control surfaces of an aeroplane and also occasionally for the main supporting surfaces. The aerodynamic characteristics which are susceptible to theoretical calculation are the lift, the pitching moment and the hinge moment of the aerofoil in the range where the lift varies linearly with the angle of incidence and flap angle. The drag of the aerofoil and the general behaviour for large angles of incidence and flap angles, when the aerofoil is stalled, are at present outside the range of calculation and must be determined empirically.

For the purpose of the present theoretical investigation it is proposed to consider the case of an aerofoil of symmetrical section. The configuration may then be illustrated as in Fig. 1, where AB is the front part of the aerofoil, BC is the flap, and AC is the base line joining the leading and trailing edges. The following notation* will be used :—

c = total chord, which is sensibly the same as AC.

$\mathrm{E}\,c$ = flap chord, BC or NC.

$\gamma\,c$ = height of the hinge B above the base line AC.

α = angle of incidence of base line AC.

α' = angle of incidence of front part AB.

η = flap angle, the acute angle between AB and BC,

from which it follows at once that

$$\left.\begin{aligned} \alpha' &= \alpha - \frac{\gamma}{1-\mathrm{E}} \\ \eta &= \frac{\gamma}{\mathrm{E}} + \frac{\gamma}{1-\mathrm{E}} \end{aligned}\right\} \quad \ldots \quad (1)$$

Considering first the case of two dimensional motion or infinite aspect ratio, the lift of the aerofoil is expressed in the form

$$\mathrm{L} = k_{\mathrm{L}}\, c\, \rho\, \mathrm{V}^2$$

and it is convenient to write

$$k_{\mathrm{L}} = a_1\,\alpha' + a_2\,\eta \quad \ldots \quad (2)$$

The pitching moment about the nose of the aerofoil, which is positive when it tends to increase the angle of incidence, is

$$\mathrm{M} = k_m\, c^2\, \rho\, \mathrm{V}^2$$

and the moment coefficient can be expressed in the form

$$k_m = -\tfrac{1}{4}\,k_{\mathrm{L}} - m\,\eta \quad \ldots \quad (3)$$

The hinge moment about the hinge B, due to the aerodynamic forces on the flap BC, is regarded as positive when it tends to increase the flap angle. The corresponding non-dimensional coefficient is defined by the equation

$$\mathrm{H} = k_{\mathrm{H}}\,(\mathrm{E}c)^2\,\rho\,\mathrm{V}^2$$

and the coefficient may be expressed in the alternative forms

$$\left.\begin{aligned} k_{\mathrm{H}} &= b_1\,\alpha' + b_2\,\eta \\ &= \frac{b_1}{a_1}\,k_{\mathrm{L}} - b\,\eta \\ \text{where} \quad b &= \frac{b_1 a_2 - b_2 a_1}{a_1} \end{aligned}\right\} \quad \ldots \quad (4)$$

These expressions for the lift coefficient, moment coefficient and hinge moment coefficient apply to the case of an aerofoil of symmetrical section. In the case of an aerofoil of more general shape the

* The notation is also summarised in an appendix.

only modification necessary is to add constant terms k_{L0}, k_{m0} and k_{H0} to the right hand sides of equations (2), (3), and (4) respectively.

The problem of the aerofoil with hinged flap has been investigated previously in report R. & M. 910* but this earlier analysis was not extended to the hinge moment and it related only to the case of two-dimensional motion. More recently an improved method of analysis† has been developed which makes it possible to treat the problem in a more complete manner. The analysis will be based, as before, on the assumption of a thin aerofoil, which justifies the device of replacing the actual aerofoil by its centre line for the purpose of calculation. It may be mentioned that experience has shown this assumption to be fully justified for aerofoils whose thickness-chord ratio is as high as 0·10 at least.

2. *Effect of aspect ratio.*—Before proceeding to the general analysis it is convenient to consider the modifications which are necessary to pass from the case of two-dimensional motion to that of a rectangular aerofoil of finite aspect ratio. The angle of no lift of an aerofoil is not altered by a change of aspect ratio but the slope of the curve of lift coefficient against angle of incidence falls with the aspect ratio. Hence the lift coefficient will remain of the form given in equation (2), and the ratio a_2/a_1 will be independent of the aspect ratio. The value of a_1, however, will vary according to the following table, which assumes a rectangular aerofoil of constant section and incidence.

TABLE 1.

Aspect ratio	∞	8	6	4	2
Value of a_1	3·14	2·42	2·27	2·01	1·52

It has been shown‡ also that if the moment coefficient for two dimensional flow is of the form (3), then the moment coefficient of the finite rectangular aerofoil will be of the same form. Hence the coefficient m is independent of the aspect ratio.

By the same argument it follows that if the hinge moment coefficient is expressed in the form

$$k_H = \frac{b_1}{a_1} k_L - b\,\eta$$

then b_1/a_1 and b will be independent of the aspect ratio.

These results may be summarised by stating that the four parameters

$$a_2/a_1,\ m,\ b_1/a_1,\ b$$

* R. & M. 910.—A theory of thin aerofoils.

† The elements of aerofoil and airscrew theory. H. Glauert, pp. 87–90. Published by Cambridge University Press.

‡ *Loc. cit.*, p. 150.

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are invariant as regards aspect ratio, and it will therefore be advantageous to express the results of the general analysis as the values of these four parameters.

The variation of a_1 with aspect ratio is given in Table 1 and the two coefficients a_2 and b_1 will vary in the same ratio. The variation of b_2 is of a more complex form.

3. *General analysis.*—To analyse the behaviour of an aerofoil in two dimensional motion the origin of co-ordinates is taken at the leading edge A of the aerofoil (Fig. 1), the axis of x is taken as the base line AC and the axis of y normal to it. The co-ordinate x, which passes from A to C along the aerofoil, is replaced by the angle θ defined by the equation

$$x = \tfrac{1}{2} c\,(1 - \cos\theta) \qquad \ldots \qquad (5)$$

so that θ passes from 0 to π along the aerofoil. The position of the hinge is then defined by the angle φ, where

$$\left.\begin{aligned} \cos\varphi &= -(1 - 2\,\mathrm{E}) \\ \sin\varphi &= 2\sqrt{\mathrm{E}\,(1 - \mathrm{E})} \end{aligned}\right\} \qquad \ldots \qquad (6)$$

and the shape of the aerofoil is defined by the equations

$$\left.\begin{aligned} \theta = 0 \text{ to } \varphi, &\qquad \frac{dy}{dx} = \frac{\gamma}{1-\mathrm{E}} \\ \theta = \varphi \text{ to } \pi, &\qquad \frac{dy}{dx} = -\frac{\gamma}{\mathrm{E}} \end{aligned}\right\} \qquad \ldots \qquad (7)$$

Assuming a distribution of vorticity along the aerofoil in the form of the series

$$k\,dx = c\,\mathrm{V}\left\{ \mathrm{A}_0(1 + \cos\theta) + \sum_1^{\infty} \mathrm{A}_n \sin n\theta \sin\theta \right\} d\theta \qquad (8)$$

it has been shown* that

$$\left.\begin{aligned} \alpha - \mathrm{A}_0 &= \frac{1}{\pi}\int_0^{\pi} \frac{dy}{dx}\,d\theta \\ \mathrm{A}_n &= \frac{2}{\pi}\int_0^{\pi} \frac{dy}{dx}\cos n\theta\,d\theta \end{aligned}\right\} \qquad \ldots \qquad (9)$$

and hence by virtue of equations (1) and (7) we obtain

$$\alpha - \mathrm{A}_0 = \frac{1}{\pi}\left\{ \frac{\gamma}{1-\mathrm{E}}\,\varphi - \frac{\gamma}{\mathrm{E}}\,(\pi - \varphi) \right\} = \frac{\varphi}{\pi}\,\eta - \frac{\gamma}{\mathrm{E}}$$

$$\mathrm{A}_n = \frac{2}{n\pi}\left\{ \frac{\gamma}{1-\mathrm{E}}\sin n\varphi + \frac{\gamma}{\mathrm{E}}\sin n\varphi \right\} = \frac{2\sin n\varphi}{n\pi}\,\eta$$

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* *Loc. cit.* p. 90.

or

$$\left.\begin{aligned} A_0 &= \alpha' + \frac{\pi - \varphi}{\pi}\eta \\ A_n &= \frac{2 \sin n\varphi}{n\pi}\eta \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (10)$$

The lift coefficient has been obtained as

$$\begin{aligned} k_L &= \pi (A_0 + \tfrac{1}{2} A_1) \\ &= \pi\left\{ \alpha' + \frac{\pi - \varphi}{\pi}\eta + \frac{\sin\varphi}{\pi}\eta \right\} \end{aligned}$$

and hence by comparison with equation (2)

$$\left.\begin{aligned} a_1 &= \pi \\ \frac{a_2}{a_1} &= \frac{\pi - \varphi}{\pi} + \frac{\sin\varphi}{\pi} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (11)$$

Also the moment coefficient has been obtained as

$$\begin{aligned} k_m &= -\tfrac{1}{4} k_L - \tfrac{\pi}{8}(A_1 - A_2) \\ &= -\tfrac{1}{4} k_L - \tfrac{1}{4}(\sin\varphi - \tfrac{1}{2}\sin 2\varphi) \end{aligned}$$

and hence by comparison with equation (3)

$$m = \frac{\sin\varphi}{4} - \frac{\sin 2\varphi}{8} = \tfrac{1}{4}\sin\varphi\,(1 - \cos\varphi) \quad \ldots \qquad (12)$$

4. *Hinge moment.*—To calculate the moment about the hinge B of the aerodynamic forces on the flap, we note that the lift force on the element dx of the aerofoil is simply $\rho\, V\, k\, d\, x$, and hence

$$H = -\int_B^C \rho\, V\, k \left\{ x - (1 - E)\, c \right\} dx$$

By virtue of equations (5), (6), and (8) this expression becomes

$$H = -\tfrac{1}{2} c^2 \rho\, V^2 \int_\varphi^\pi \left\{ A_0 (1 + \cos\theta) + \sum_1^\infty A_n \sin n\theta \sin\theta \right\} (\cos\varphi - \cos\theta)\, d\theta.$$

and then by comparison with equation (4) we obtain

$$\begin{aligned} &-8\, E^2 (b_1 \alpha' + b_2 \eta) \\ &= 2\cos\varphi \int_\varphi^\pi \left[2A_0 (1 + \cos\theta) + \sum_1^\infty A_n \left\{ \cos(n-1)\theta - \cos(n+1)\theta \right\} \right] d\theta \\ &- \int_\varphi^\pi \left[2A_0 (1 + 2\cos\theta + \cos 2\theta) + \sum_1^\infty A_n \left\{ \cos(n-2)\theta - \cos(n+2) \right\} \right] \\ &\quad d\theta \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (13) \end{aligned}$$

On integration, let C_n be the coefficient of A_n. Then in general, for $n > 2$, we have

$$\begin{aligned} C_n &= 2\cos\theta \left\{ \frac{\sin(n+1)\varphi}{n+1} - \frac{\sin(n-1)\varphi}{n-1} \right\} - \left\{ \frac{\sin(n+2)\varphi}{n+2} - \frac{\sin(n-1)\varphi}{n-2} \right\} \\ &= \frac{\sin(n-2)\varphi}{(n-2)(n-1)} - \frac{2\sin n\varphi}{(n-1)(n+1)} + \frac{\sin(n+2)\varphi}{(n+1)(n+2)} \end{aligned}$$

and for smaller values of n

$$C_2 = 2\cos\varphi\left\{\frac{\sin 3\varphi}{3} - \frac{\sin\varphi}{1}\right\} - \left\{\frac{\sin 4\varphi}{4} + (\pi - \varphi)\right\}$$

$$= -(\pi - \varphi) - \frac{2\sin 2\varphi}{1\cdot 3} + \frac{\sin 4\varphi}{3\cdot 4}$$

$$C_1 = 2\cos\varphi\left\{\frac{\sin 2\varphi}{2} + (\pi - \varphi)\right\} - \left\{\frac{\sin 3\varphi}{3} - \frac{\sin\varphi}{1}\right\}$$

$$= 2(\pi - \varphi)\cos\varphi + \frac{3}{2}\sin\varphi + \frac{\sin 3\varphi}{2\cdot 3}$$

$$Co = 4\cos\varphi\left\{(\pi - \varphi) - \sin\varphi\right\} - 2\left\{(\pi - \varphi) - 2\sin\varphi - \tfrac{1}{2}\sin 2\varphi\right\}$$

$$= -2(\pi - \varphi)(1 - 2\cos\varphi) + 4\sin\varphi - \sin 2\varphi$$

Then

$$-8E^2(b_1\alpha' + b_2\eta) = \sum_0^\infty C_n A_n$$

where, according to equation (10),

$$A_0 = \alpha' + \frac{\pi - \varphi}{\pi}\eta$$

$$A_n = \frac{2}{\pi}\frac{\sin n\varphi}{n}\eta$$

We obtain at once therefore

$$-8E^2 b_1 = Co$$

or

$$-E^2\frac{b_1}{a_1} = \frac{\sin\varphi}{2\pi} - \frac{\sin 2\varphi}{8\pi} - \frac{1}{2}\left(\frac{\pi - \varphi}{\pi}\right)\left(\frac{1}{2} - \cos\varphi\right) \quad (14)$$

The evaluation of b_2 is more complex. We have

$$-8\pi E^2 b_2 = (\pi - \varphi)Co + 2\sum_1^\infty \frac{\sin n\varphi}{n} Cn$$

and the general term of this expression ($n > 2$) is

$$\frac{\cos 2\varphi - \cos 2(n-1)\varphi}{(n-2)(n-1)n} - \frac{2(1 - \cos 2n\varphi)}{(n-1)n(n+1)} + \frac{\cos 2\varphi - \cos 2(n+1)\varphi}{n(n+1)(n+2)}$$

while for $n = 2$, $n = 1$, and $n = 0$, we obtain respectively

$$-(\pi - \varphi)\sin 2\varphi - \frac{2(1 - \cos 4\varphi)}{1.2.3} + \frac{\cos 2\varphi - \cos 6\varphi}{2.3.4}$$

$$2(\pi - \varphi)\sin 2\varphi + \frac{3}{2}(1 - \cos 2\varphi) + \frac{\cos 2\varphi - \cos 4\varphi}{1.2.3}$$

$$-2(\pi - \varphi)^2(1 - 2\cos\varphi) + 4(\pi - \varphi)\sin\varphi - (\pi - \varphi)\sin 2\varphi$$

Adding all these terms and denoting by S the sum of the series $\sum_1^\infty \frac{1}{n(n+1)(n+2)}$, which can easily be shown to be equal to 1/4, we obtain

$$- 8 \pi E^2 b_2 = -2 (\pi - \varphi)^2 (1 - 2 \cos \varphi) + 4 (\pi - \varphi) \sin \varphi + \left(\frac{3}{2} - 2 S\right) \left(1 - \cos 2 \varphi\right)$$

or

$$- E^2 \frac{b_2}{a_1} = \frac{\sin^2 \varphi}{4 \pi^2} + \left(\frac{\pi - \varphi}{\pi}\right) \frac{\sin \varphi}{2 \pi} - \frac{1}{2} \left(\frac{\pi - \varphi}{\pi}\right)^2 \left(\frac{1}{2} - \cos \varphi\right) \quad .. \quad (15)$$

Then finally, by virtue of equations (11) and (14) we obtain for b the equation

$$E^2 \frac{b}{a_1} = E^2 \left(\frac{a_2}{a_1} \frac{b_1}{a_1} - \frac{b_2}{a_1}\right)$$
$$= \frac{\sin \varphi (1 - \cos \varphi)}{4 \pi} \left\{\frac{\pi - \varphi}{\pi} - \frac{\sin \varphi}{\pi}\right\}$$

or

$$b = \frac{b_1 a_2 - b_2 a_1}{a_1} = \frac{\sin \varphi (1 - \cos \varphi)}{4 E^2} \left\{\frac{\pi - \varphi}{\pi} - \frac{\sin \varphi}{\pi}\right\} \quad .. \quad (16)$$

5. *Summary of results.*—The formulae which have been derived for the four quantities a_2/a_1, m, b_1/a_1 and b determine completely the lift, pitching moment, and hinge moment of any aerofoil with hinged flap, since they apply equally to two dimensional motion or to a finite rectangular aerofoil. The only other parameter is a_1 itself, whose value under different condition is given in Table 1.

The results may be summarised as follows :—

$$\left.\begin{aligned} \frac{a_2}{a_1} &= \frac{\pi - \varphi}{\pi} + \frac{\sin \varphi}{\pi} \\ m &= \tfrac{1}{4} \sin \varphi (1 - \cos \varphi) \\ - E^2 \frac{b_1}{a_2} &= \frac{\sin \varphi}{2 \pi} (1 - \tfrac{1}{2} \cos \varphi) - \frac{\pi - \varphi}{2 \pi} (\tfrac{1}{2} - \cos \varphi) \\ E^2 b &= \frac{1}{4} \sin \varphi (1 - \cos \varphi) \left(\frac{\pi - \varphi}{\pi} - \frac{\sin \varphi}{\pi}\right) \end{aligned}\right\} \quad .. \quad .. \quad (I)$$

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These formulae are expressed in terms of the angle φ since this is the form which is most useful for calculation. The size of flap corresponding to the angle φ is given by the equation

$$E = \tfrac{1}{2} (1 + \cos \varphi)$$

Alternatively, by means of equations (6), the formulae may be expressed in terms of E only, the appropriate substitution for φ being

$$\varphi = 2 \text{ arc} \cos \sqrt{E}$$

The expressions obtained are respectively

$$\left.\begin{aligned}
\frac{a_2}{a_1} &= 1 - \frac{2}{\pi}\left\{\text{arc cos}\sqrt{\text{E}} - \sqrt{\text{E}\,(1-\text{E})}\right\} \\
\text{m} &= (1-\text{E})\sqrt{\text{E}\,(1-\text{E})} \\
-\frac{b_1}{a_1} &= \frac{1}{\pi\,\text{E}^2}\left\{\left(\frac{3}{2}-\text{E}\right)\sqrt{\text{E}\,(1-\text{E})} - \left(\frac{3}{2}-2\text{E}\right)\left(\frac{\pi}{2}-\text{arc cos}\sqrt{\text{E}}\right)\right\} \\
b &= \frac{2\,(1-\text{E})\sqrt{\text{E}\,(1-\text{E})}}{\pi\,\text{E}^2}\left\{\frac{\pi}{2} - \text{arc cos}\sqrt{\text{E}} - \sqrt{\text{E}\,(1-\text{E})}\right\}
\end{aligned}\right\}\text{(II)}$$

6. *Expansion in Series.*—For small values of E it is possible to expand the expressions in ascending powers of E. This expansion is carried out most conveniently from the basis of equations (I) by writing $\varphi = \pi - \varepsilon$, where ε is small. Series in terms of ε are then obtained, which are subsequently converted to series in E by means of the relationship

$$\sin\varphi = 2\sqrt{\text{E}\,(1-\text{E})}$$

or

$$\varepsilon = 2\sqrt{\text{E}}\left(1 + \frac{1}{6}\,\text{E} + \ldots.\right)$$

We have

$$\begin{aligned}
\frac{a_2}{a_1} &= \frac{1}{\pi}\,(\varepsilon + \sin\,\varepsilon) \\
&= \frac{2}{\pi}\,\varepsilon\left(1 - \frac{1}{12}\,\varepsilon^2 + \ldots.\right) \\
\text{m} &= \frac{1}{4}\sin\,\varepsilon\,(1 + \cos\,\varepsilon) \\
&= \frac{1}{2}\,\varepsilon\left(1 - \frac{5}{12}\,\varepsilon^2 + \ldots.\right) \\
-\text{E}^2\,\frac{b_1}{a_2} &= \frac{1}{2\pi}\left\{\sin\,\varepsilon\left(1 + \frac{1}{2}\cos\,\varepsilon\right) - \varepsilon\left(\frac{1}{2} + \cos\,\varepsilon\right)\right\} \\
&= \frac{\varepsilon^5}{60\pi}\left(1 - \frac{13}{84}\,\varepsilon^2 + \ldots.\right) \\
\text{E}^2 b &= \frac{1}{4\pi}\sin\,\varepsilon\,(1 + \cos\,\varepsilon)\,(\varepsilon - \sin\,\varepsilon) \\
&= \frac{\varepsilon^4}{12\pi}\left(1 - \frac{7}{15}\,\varepsilon^2 + \ldots.\right)
\end{aligned}$$

and then converting to series in E, we obtain finally

$$\left.\begin{aligned} \frac{a_2}{a_1} &= \frac{4}{\pi}\sqrt{\mathrm{E}}\left(1 - \frac{1}{6}\mathrm{E} - \dots\right) \\ \mathrm{m} &= \sqrt{\mathrm{E}}\left(1 - \frac{3}{2}\mathrm{E} + \dots\right) \\ \frac{b_1}{a_1} &= -\frac{8}{15\pi}\sqrt{\mathrm{E}}\left(1 + \frac{3}{14}\mathrm{E} + \dots\right) \\ b &= \frac{4}{3\pi}\left(1 - \frac{6}{5}\mathrm{E} + \dots\right) \end{aligned}\right\} \quad \dots \quad \dots \quad \text{(III)}$$

As an indication of the accuracy of the expressions (III), carried to the second term of the series in each case, Table 2 gives the accurate and approximate values of the four parameters for the case $\mathrm{E} = 0{\cdot}25$. The agreement is good in all cases, the largest discrepancy occurring in the value of m.

TABLE 2.

$\mathrm{E} = 0{\cdot}25$

	a_2/a_1	m	$-b_1/a_1$	b
Accurate ..	0·609	0·325	0·090	0·300
Approximate	0·610	0·313	0·090	0·298

7. *Numerical results.*—Numerical values of the four principal parameters have been calculated by inserting in equations (I) a suitable series of values for $(\pi - \varphi)$. These values are given in Table 3 and are shown in Figures 3 and 4. For convenience also, values of the parameters at even values of E have been read from the curves drawn to a large scale and are given in Table 4. This table and the curves give the complete solution for the lift, pitching moment and hinge moment of any rectangular aerofoil with a hinged flap, for the appropriate values of a_1 are already known and are given in Table 1. As the aspect ratio changes, m and b remain unaltered, while a_1, a_2, and b_1 change in the same ratio. The coefficient b_2, however, varies in a rather complex manner and Table 5 has therefore been prepared, giving the values of the ratio b_1/b_2 for two dimensional motion and for the three aspect ratios 6, 4, and 2, which correspond roughly to the cases of a wing, a tailplane and elevator, and a fin and rudder respectively. These values are also shown in Fig. 2.

The results given in the tables and figures are the theoretical values of the respective parameters. It is well known, however, that an aerofoil frequently fails to realise the full theoretical lift, and it then becomes necessary to reduce the theoretical values by a certain efficiency factor. This loss of efficiency is particularly noticeable in the case of a control service, such as a tailplane, which operates in the wake of the wings and is also disturbed by the presence of the body. Moreover, a further deviation from the theoretical results may be anticipated when the elevators are divided at the centre or when there is a gap at the hinge between the tailplane and elevators.

The general efficiency factor should not modify the value of a ratio such as a_2/a_1, b_1/a_1, or b_1/b_2, and all the coefficients a_1, a_2, m, b_1, b_2, and b may be expected to fall in approximately the same ratio. On the other hand a gap at the hinge or a division of the elevator into two halves may be expected to react mainly on those coefficients which depend on the lift carried by the elevator. Thus changes would be anticipated in the values of a_2, m, b_1, b_2, and b, but not in a_1 or b_1/b_2. These changes cannot be estimated theoretically and their magnitude can be inferred only from a comparison with suitable experimental results.

8. *Comparison with Experimental Results.*—The ideal basis of comparison to check the accuracy of the theoretical results would be the systematic investigation of a symmetrical aerofoil with hinged flap, but in the absence of this information it is of interest to examine the expreimental results for a tailplane and elevator recorded in report R. & M. 761*. This report also contains an analysis of the loss of efficiency experienced by a tailplane mounted in its usual position at the rear of an aeroplane.

R. & M. 761 records the values of the various characteristics for tailplanes with varying proportion of elevators, mounted on the R.E.8 and S.E.5 aeroplanes. In general the values of a_2/a_1 lie below the theroetical values of Fig. 3, but in the case of the S.E.5 the investigation was extended to cover the case when the gap at the hinge was closed. The values then obtained in model and full scale tests were :—

$E = 0{\cdot}35$ .. .. $a_2/a_1 = 0{\cdot}68$ and $0{\cdot}72$.

$E = 0{\cdot}51$ .. .. $a_2/a_1 = 0{\cdot}77$, $0{\cdot}78$ and $0{\cdot}80$.

which agree quite well with the corresponding theoretical values 0·70 and 0·82.

Turning next to the values of $-b_1/a_1$, the comparison between theory and experiment is as follows

E	0·30	0·40	0·49	0·61	0·35	0·51
Experiment ..	0	0·116	0·133	0·137	0·112	0·125
Theory	0·100	0·117	0·134	0·157	0·109	0·138

The agreement is again quite good on the whole, though discrepancies occur with the smallest and largest elevators. The full scale value at E = 0·30 is clearly in error, and in general it must be

* R. & M. 761.—Experimental determination of tailplane characteristics.

remembered that the experimental determination of b_1 and b_2 depends on the measurement of small control forces and does not have a high degree of accuracy.

Finally a comparison may be made of the values of b_1/b_2. This ratio depends partly on the aspect of the tailplane and the actual plan form was not rectangular. The comparison has been based on a mean effective aspect ratio of $3\frac{1}{2}$.

E = ...	0·30	0·40	0·49	0·61	0·35	0·51
Experiment ..	0	0·46	0·50	0·67	0·52	0·66
Theory	0·47	0·56	0·65	0·76	0·52	0·67

The last two results, deduced from the experiments on S.E.5, are in exact agreement with the theoretical values, but the first four results, deduced from the earlier experiments on R.E.8, show a consistent difference. In view of the obvious error in the case E = 0·30, it is perhaps legitimate to regard the R.E.8 results as less reliable than the S.E.5 results and to admit a satisfactory agreement between theory and experiment.

In the light of these comparisons it appears that the theoretcial results represent the actual values with sufficient accuracy, but in the practical application there remains the question of the general loss of efficiency. For a wing alone this efficiency factor should not be less than 0·90 and is more probably 0·95, but for a tailplane it may fall as low as 0·6. An analysis of this efficiency factor is contained in report R. & M. 761, and when no gap occurs at the hinge a reasonable value of the efficiency factor appears to be 0·75 or 0·80.

In applying this efficiency factor to the theoretical results as given in Table 4 or Figures 3 and 4, the values of a_2/a_1 and b_1/a_1 are unaltered and the values of m and b are reduced by the efficiency factor. The value of a_1, for the appropriate aspect ratio, is deduced from Table 1 and is also reduced by the same factor. Alternatively, the theoretical values of a_1, a_2, m, b_1 and b_2 may be calculated and the efficiency factor then applied to all the quantities.

TABLE 3.

Calculated Values.

$\frac{\pi - \varphi}{\pi}$	E	$\frac{a_2}{a_1}$	m	$-\frac{b_1}{a_1}$	b
0·15	0·055	0·295	0·215	0·040	0·397
0·20	0·095	0·387	0·266	0·054	0·376
0·25	0·146	0·475	0·302	0·067	0·350
0·30	0·206	0·558	0·321	0·081	0·322
0·35	0·273	0·634	0·324	0·094	0·289
0·40	0·345	0·703	0·311	0·108	0·254
0·45	0·422	0·764	0·285	0·122	0·218
0·50	0·500	0·818	0·250	0·136	0·182
0·60	0·655	0·903	0·164	0·165	0·114
0·75	0·854	0·975	0·052	0·207	0·037
1·00	1	1	0	0·250	0

TABLE 4.

Values from Curves.

E	$\frac{a_2}{a_1}$	m	$-\frac{b_1}{a_1}$	b
0	0	0	0	0·425
0·05 ..	0·282	0·207	0·038	0·399
0·10 ..	0·396	0·270	0·055	0·373
0·15 ..	0·480	0·304	0·068	0·348
0·20 ..	0·550	0·320	0·080	0·324
0·30 ..	0·660	0·321	0·100	0·277
0·40 ..	0·746	0·294	0·117	0·230
0·50 ..	0·818	0·250	0·136	0·182
0·60 ..	0·876	0·196	0·155	0·137
0·80 ..	0·960	0·080	0·194	0·057
1·00 ..	1	0	0·250	0

TABLE 5.

Values of b_1/b_2 for Varying Aspect Ratio.

E	A = ∞	6	4	2
0	0	0	0	0
0·05 ..	0·275	0·203	0·181	0·140
0·10 ..	0·392	0·296	0·264	0·207
0·15 ..	0·475	0·365	0·332	0·259
0·20 ..	0·544	0·427	0·391	0·312
0·30 ..	0·649	0·531	0·490	0·403
0·40 ..	0·731	0·622	0·580	0·490
0·50 ..	0·803	0·713	0·675	0·589
0·60 ..	0·868	0·792	0·761	0·686
0·80 ..	0·950	0·920	0·903	0·868
1·00 ..	1	1	1	1

APPENDIX.

Notation.

In addition to the standard notation for speed, chord, lift coefficient, etc., the following symbols are used :—

E = ratio of flap chord to total chord.

α = incidence of line joining leading edge to trailing edge.

α' = incidence of front part of aerofoil.

η = flap angle.

θ = angle defining any point of aerofoil, equation (5).

φ = angle defining position of hinge, equation (6).

An = coefficient in series defining circulation, equation (8).

Cn = factor of An in series for hinge moment, equation (13).

Lift coefficient

$$k_L = a_1 \alpha' + a_2 \eta.$$

Pitching moment coefficient

$$k_m = -\tfrac{1}{4} k_L - m \eta$$

Hinge moment coefficient

$$k_H = b_1 \alpha' + b_2 \eta$$

$$= \frac{b_1}{a_1} k_L - b \eta$$

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FIG. 1.

AEROFOIL WITH HINGED FLAP.

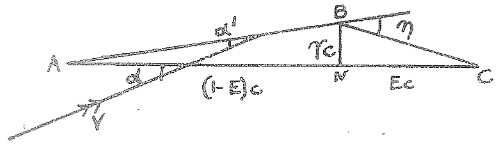

FIG. 2.

VALUES OF b_1/b_2 FOR VARYING ASPECT RATIO.

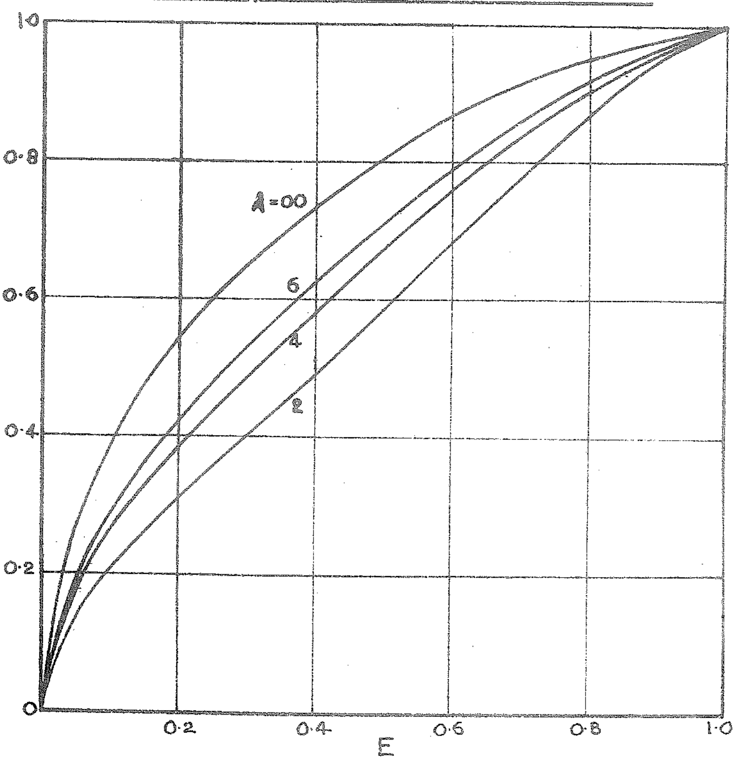

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FIG. 3.

AEROFOIL WITH HINGED FLAP.

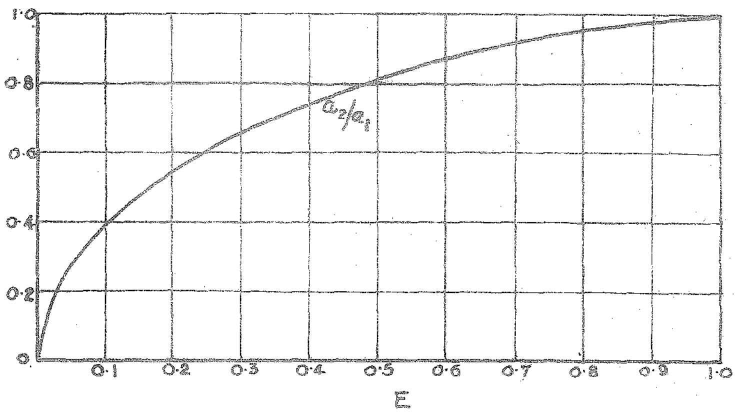
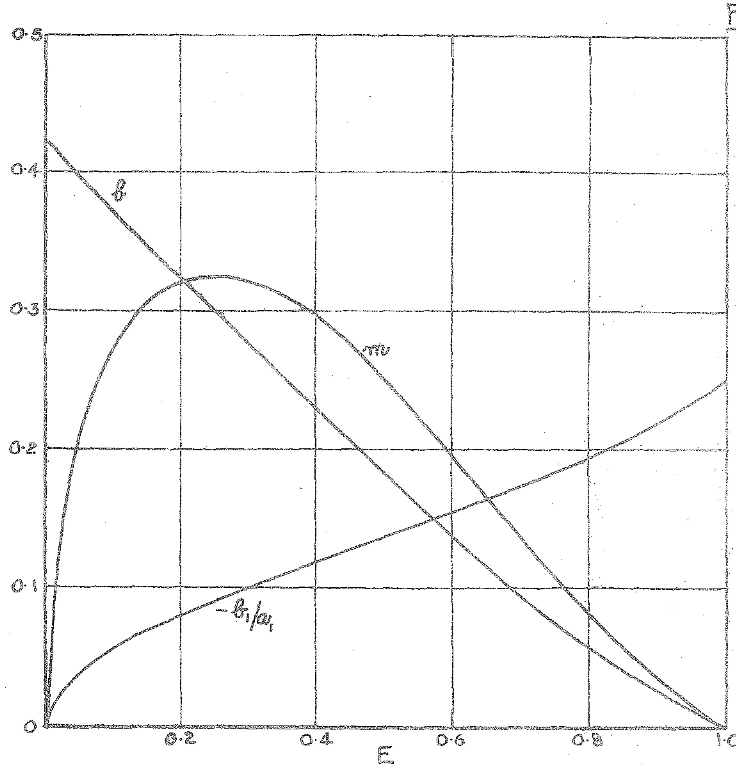


FIG. 4.



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