A.3.d. Airscrew, 55.
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SOME NOTES ON THE THEORY OF AN AIRSCREW WORKING IN A WIND CHANNEL.—By R. McK. WOOD and R. G. HARRIS, of The Royal Aircraft Establishment. Presented by THE DIRECTOR OF RESEARCH.

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SOME NOTES ON THE THEORY OF AN AIRSCREW WORKING IN A WIND CHANNEL.

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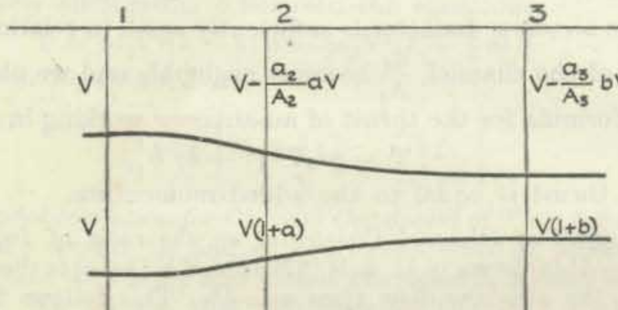
Summary.—(a) *Reasons for enquiry.*—(1) The Momentum Theory of R. E. Froude leads to equations which require some modification when the airscrew is working in a tunnel of dimensions comparable with the diameter of the screw.

(2) Some correction to the speed of the air in a wind channel must be made to obtain the equivalent speed of advance of the airscrew in free air.

(b) *Conclusions.*—(1) Formulæ for the thrust-momentum relation, inflow-outflow ratio and contraction ratio of the slipstream are deduced.

(2) A method of correcting the speed of advance for the effect of channel constraint of flow is obtained based upon the Momentum Theory, which may probably be relied upon so long as the correction required is reasonably small.

Froude's Theory applied to wind channel experiments.—It can be shown that Froude's assumptions applied to an airscrew surrounded by the constraint of channel walls lead to the conclusion that the thrust is greater than the added momentum measured at the narrowest part of the slipstream. We assume a



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perfect fluid and no rotation of any kind. The airscrew is represented by a disc over which the air receives a sudden uniform increase of total head (A). The channel is of circular section for simplicity. Sections 1 and 3 are sections of the channel before and behind the airscrew over which the flow is parallel and the pressure therefore uniform. Section 2 contains the airscrew disc. A and a denote respectively the sectional areas of the surrounding stream and of the stream which receives the increase of total head. The velocities are as shown.

$p_3 > p_1$ is obtained by applying Bernoulli's equation to the surrounding stream :—

$$p_3 - p_1 = \tfrac{1}{2}\rho V^2 \left[1 - \left(1 - \frac{a_3}{A_3} b\right)^2\right]$$
$$= \tfrac{1}{2}\rho \frac{a_3}{A_3} V^2 b \left(2 - \frac{a_3}{A_3} b\right)$$

The increase in momentum per sec. passing section 3 over that passing section 1 is

$$\Delta M = \rho V^2 \left[a_3 (1 + b) b - \frac{a_3}{A_3} A_3 \left(1 - \frac{a_3}{A_3} b\right) b\right]$$
$$= \rho a_3 V^2 \left(1 + \frac{a_3}{A_3}\right) b^2$$

Now this increase of momentum is equal to the airscrew thrust less the difference in total force over the sections 1 and 3.

$$\therefore T = \Delta M + (p_3 - p_1)(a_3 + A_3)$$
$$= \rho a_3 V^2 \left(1 + \frac{a_3}{A_3}\right) b^2 + \tfrac{1}{2}\rho a_3 b V^2 \left(2 - \frac{a_3}{A_3} b\right)\left(1 + \frac{a_3}{A_3}\right)$$
$$\therefore T = \rho a_3 V^2 b \left(1 + \frac{a_3}{A_3}\right)\left\{1 + b\left(1 - \frac{a_3}{2A_3}\right)\right\}$$
$$T = \rho a_3 V^2 b \left[1 + b - \frac{a_3}{A_3}\frac{b}{2} + \frac{a_3}{A_3} + \frac{a_3}{A_3} b - \left(\frac{a_3}{A_3}\right)^2 \frac{b}{2}\right]$$
$$\therefore T = \rho a_3 V^2 (1 + b) b \left[1 + \frac{a_3}{A_3}\frac{\left\{1 + \frac{b}{2}\left(1 - \frac{a_3}{A_3}\right)\right\}}{1 + b}\right] \quad (1)$$

If the airscrew diameter is sufficiently small in relation to the breadth of the channel, $\frac{a_3}{A_3}$ becomes negligible and we obtain the Froude formula for the thrust of an airscrew working in free air

$$T = \rho a_3 V^2 (1 + b)\, b$$

i.e., the thrust is equal to the added momentum.

The effect of Channel Constraint on the ratio of Inflow and Outflow.—It is shown in R. & M. 328 that if $V(1+a)$ is the velocity through the airscrew disc, then $a = \frac{1}{2}b$. This follows from the fact that the thrust is equal to the integral of the increase of total head over the airscrew disc. In practice it is found by

Fage & Howard (as previously by Dr. Stanton) that the difference in the integral of total head over sections of the stream immediately before and behind the airscrew is equal to the thrust, which is equally true in free air and in a channel. In a channel, however, the inflow a is not equal to one-half the outflow b.

The relation may be obtained as follows :—

The change of total head is

$$\frac{T}{a_2} = p_3 - p_1 + \tfrac{1}{2}\rho V^2 \left[(1+b)^2 - 1 \right]$$

$$= \tfrac{1}{2}\rho V^2 \left[\frac{a_3}{A_3} b \left(2 - \frac{a_3}{A_3} b \right) + (2+b)\, b \right]$$

$$\therefore T = \rho a_2 V^2 \left(1 + \frac{b}{2} \right) b \left[1 + \frac{a_3}{A_3} \frac{1 - \frac{a_3}{A_3}\frac{b}{2}}{1 + \frac{b}{2}} \right] \quad . \quad . \quad (2)$$

$$\text{Put } T = \rho\, a_2 V^2 (1+a)\, b \left[\frac{1 + \frac{a_3}{A_3} 1 + \frac{b}{2}\left(1 - \frac{a_3}{A_3}\right)}{1+b} \right]$$

from equation (1) since $a_3(1+b) = a_2(1+a)$.

After some reduction

$$1 - \frac{a}{\frac{1}{2}b} = \frac{a_3}{2A_3} \left[\frac{b}{1 + b\left(1 - \frac{a_3}{2A_3}\right)} \right]$$

Effect of Channel Constraint on slip stream contraction.—The contraction ratio $= \frac{a_3}{a_2} = \frac{1+a}{1+b}$

This is also affected by the constraint of the channel walls.

It is to be noted that the contraction ratio $\frac{a_3}{a_2}$ is a function of the thrust coefficient $T/\rho\, a_3 V^2$ and has limiting values 1 and $\frac{1}{2}$.

The relation in free air given by the simple Froude theory is obtained by eliminating b between the equations

$$T = \rho a_3 V^2 (1+b)\, b = \rho a_2 V^2 (1 + \tfrac{1}{2} b)\, b$$

from which we obtain

$$\frac{a_3}{a_2} = \tfrac{1}{2} \left\{ 1 + \frac{1}{\sqrt{1 + \frac{2T}{\rho a_2 V^2}}} \right\} \quad . \quad . \quad (4)$$

Suggested correction for Channel Constraint of Flow for Airscrew Tests.—We require to find an equivalent speed of advance (V′) which makes the thrust and torque the same in a channel and in free air. We assume that this requires that $V(1+a)$ (which determines the incidence of the blades) shall be the same in each case.

We have the four equations :—

(1) and (2) above for the channel case involving the quantities

$$\frac{T}{\rho a_3 V^2}, \frac{a_3}{A_3}, V(1+a), V, b,$$

and two similar equations for free air involving

$$\frac{T}{\rho a'_3 V'^2}, V'(1+a'), V', b'.$$

Eliminating b, b' and $V(1+a)$ (the last being equal to $V'(1+a')$) we obtain a relation between

$$\frac{V'}{V}, \frac{T}{\rho a_3 V^2}, \frac{a_3}{A_3},$$

assuming that $\frac{T}{\rho a_3 V^2}$ and $\frac{T}{\rho a'_3 V'^2}$ are equal, with sufficient accuracy for our purpose.

The relation we really require is that between

$$\frac{V'}{V}, \frac{T}{\rho a_2 V^2}, \frac{a_2}{A_2},$$

and this can be obtained although the work is rather less straightforward.

We shall use the following additional symbols :—

S = cross-section area of channel

$$x = \frac{a_3}{A_3}$$

$$y = \frac{T}{\rho a_2 V^2}$$

$$z = \frac{a_2}{S}$$

Equation (3) can be written in the form—

$$a = \tfrac{1}{2}b - \frac{\tfrac{1}{4}b^2 x}{1 + b - \frac{bx}{2}} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Under free air conditions, x is zero and equation (5) then becomes

$$a' = \tfrac{1}{2}b' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The condition that V and V′ are corresponding speeds is

$$V'(1+a') = V(1+a) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The substitution of (7) in (6) gives

$$\tfrac{1}{2}b' = \frac{V}{V'}(1+a) - 1 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The substitution of (5) in (8) gives :

$$\tfrac{1}{2}b' = \frac{V}{V'}\left[1 + \tfrac{1}{2}b - \frac{\tfrac{1}{4}b^2 x}{1 + b - \frac{bx}{2}}\right] - 1$$

i.e., $\frac{V'}{V}(1 + \frac{1}{2}b') = f(b, x)$ . . . . . . (9)

where $f(b, x)$ means $1 + \frac{b}{2} - \dfrac{\frac{1}{4}b^2x}{1 + b - \frac{bx}{2}}$

Equation (2) can be written as :

$$y = \frac{b^2}{2}(1 - x^2) + b(1 + x) \quad . \quad . \quad . \quad (10)$$

It follows that, in free air ($x = 0$) :

$$\frac{T}{\rho a_2 V'^2} = \frac{b'^2}{2} + b'$$

That is :

$$\left(\frac{V'}{V}\right)^2 \left(\frac{b'^2}{2} + b'\right) = y \quad . \quad . \quad . \quad . \quad (11)$$

Divide each side of equation (11) by the respective side of equation (9) :

$$\frac{V'}{V} b' = \frac{y}{f(b, x)}$$

Substitute this in equation (9) :

$$\frac{V'}{V} = f(b, x) - \frac{1}{2} \frac{y}{f(b, x)}$$

or

$$1 - \frac{V'}{V} = 1 + \frac{1}{2} \frac{y}{f(b, x)} - f(b, x) \quad . \quad . \quad (12)$$

The value of a_3 is $\frac{xS}{1 + x}$, therefore equation (1) can be written in the form :

$$yz\frac{x + 1}{x} = b\left[1 + b + x(1 + \frac{b}{2} - \frac{bx}{2})\right] = y + \frac{b^2}{2}(x + 1)$$

from equation (10). Hence

$$z = x\left\{\frac{1}{x + 1} + \frac{b^2}{2y}\right\} \quad . \quad . \quad . \quad . \quad (13)$$

Equation (10) gives :

$$\frac{2y}{b^2} = (1 + x)(1 - x + \frac{2}{b})$$

therefore z can be expressed as a function of x and b only, in the form :

$$z = \frac{x}{x + 1}\left\{1 + \frac{1}{1 - x + \frac{2}{b}}\right\} \quad . \quad . \quad . \quad (14)$$

Equations (10), (12) and (13) supply three relations between the five variables $\frac{V'}{V}$, y, z, x and b. The eliminant of x and b is an equation between $\frac{V'}{V}$, y, and z, *i.e.*, it determines $\frac{V'}{V}$ in terms of $\frac{T}{\rho a_2 V^2}$ and $\frac{a_2}{S}$.

Instead of direct elimination, use is made of the equivalent process of selecting two of the variables (x and b) as freedom variables and calculating the other three ($\frac{V'}{V}$, y and z) in terms of these. Equations (10), (12) and (13) give y, $\frac{V'}{V}$ and z in turn. This supplies the desired correspondence of values of $\frac{V'}{V}$, $\frac{a_2}{S}$, and $\frac{T}{\rho a_2 V^2}$; the values in Table 1 and Fig. 1 are obtained after some cross plotting of these results.

Appendix.

Approximation to $1 - \frac{V'}{V}$.— $f(b, x)$ will first be expanded in powers of x.

The solution of equation (10) as a quadratic in b is

$$b = \frac{-1 + \sqrt{1 + 2y\left(\frac{1-x}{1+x}\right)}}{1 - x}$$

The expansion of this with the aid of the Binomial Theorem is:

$$b = -1 + (1 + 2y)^{\frac{1}{2}} + x\left[(1 + 2y)^{-\frac{1}{2}} - 1\right] + x^2\left[\frac{1 + 4y + 2y^2}{(1 + 2y)^{3/2}} - 1\right] + \ldots$$

For shortness, let the above be written:

$$b = A + Bx + Cx^2 + \ldots \quad (15)$$

The last term of $f(b, x)$, viz., $\frac{-\frac{1}{4}b^2 x}{1 + b - \frac{bx}{2}}$ is then expansible as:

$$-\frac{1}{4(1 + A)}\left\{A^2x + x^2\left[2AB - \frac{A^2\left(B - \frac{A}{2}\right)}{1 + A}\right]\right\}$$

The first approximation to $f(b, x)$ is thus:

$$f(b, x) = 1 + \frac{A}{2} + x\left\{\frac{B}{2} - \frac{A^2}{4(1 + A)}\right\} \quad (16)$$

Let $(1 + 2y)^{\frac{1}{2}} = k$. Hence

$$A = k - 1$$

$$B = \frac{1 - k}{k}$$

Equation (16) then becomes:

$$f(b, x) = \frac{1 + k}{2} - \frac{xy}{2k}$$

Hence

$$\frac{y}{2f(b, x)} = \frac{y}{1 + k} + \frac{xy^2}{k(1 + k)^2}$$

Hence

$$\frac{V'}{V} = \frac{1 + k}{2} - \frac{y}{k + 1} - x\left\{\frac{y}{2k} + \frac{y^2}{k(1 + k)^2}\right\}$$

$$= 1 - \frac{x(k - 1)}{2}$$

since y is $\frac{k^2 - 1}{2}$. That is:

$$1 - \frac{V'}{V} = \frac{x}{2}\left[\sqrt{1 + 2y} - 1\right] \quad (17)$$

From equations (13) and (15) the first approximation to z is:

$$z = x\left(1 + \frac{A^2}{2y}\right) = x\,\frac{2k}{k + 1}$$

Hence

$$x = \frac{z}{2}\left\{1 + \frac{1}{\sqrt{1 + 2y}}\right\} \quad (18)$$

From equations (17) and (18) the first approximation to $1 - \frac{V'}{V}$ in terms of the two most useful independent variables y and z is :

$$1 - \frac{V'}{V} = \frac{z}{2} \frac{y}{\sqrt{1 + 2y}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

The relation between $1 - \frac{V'}{V}$, y and z is plotted in Fig. 1 with z as parameter, and a dotted line shows the satisfactory nature of the approximation of equation (19) for the case $z = 0{\cdot}2$ (*i.e.*, propeller diameter = 0·505 times the channel breadth, in the case of a square section channel).

Slip Stream.—The first approximation to the slip stream ratio b, from equations (15) and (18) is :

$$b = -1 + \sqrt{1 + 2y} - \frac{zy}{1 + 2y} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The slipstream ration b' at the corresponding speed V' in free air is given by putting $z = 0$, viz. :

$$b' = -1 + \sqrt{1 + 2y'}$$

In Fig. 2, b is plotted in terms of y and z, and the crosses indicate the nature of the approximation when z is 0·2.

Contraction Ratio.—The contraction ratio $\frac{a_3}{a_2}$ has the value $\frac{x}{z\,(1 + x)}$.

Hence from equation (14) :

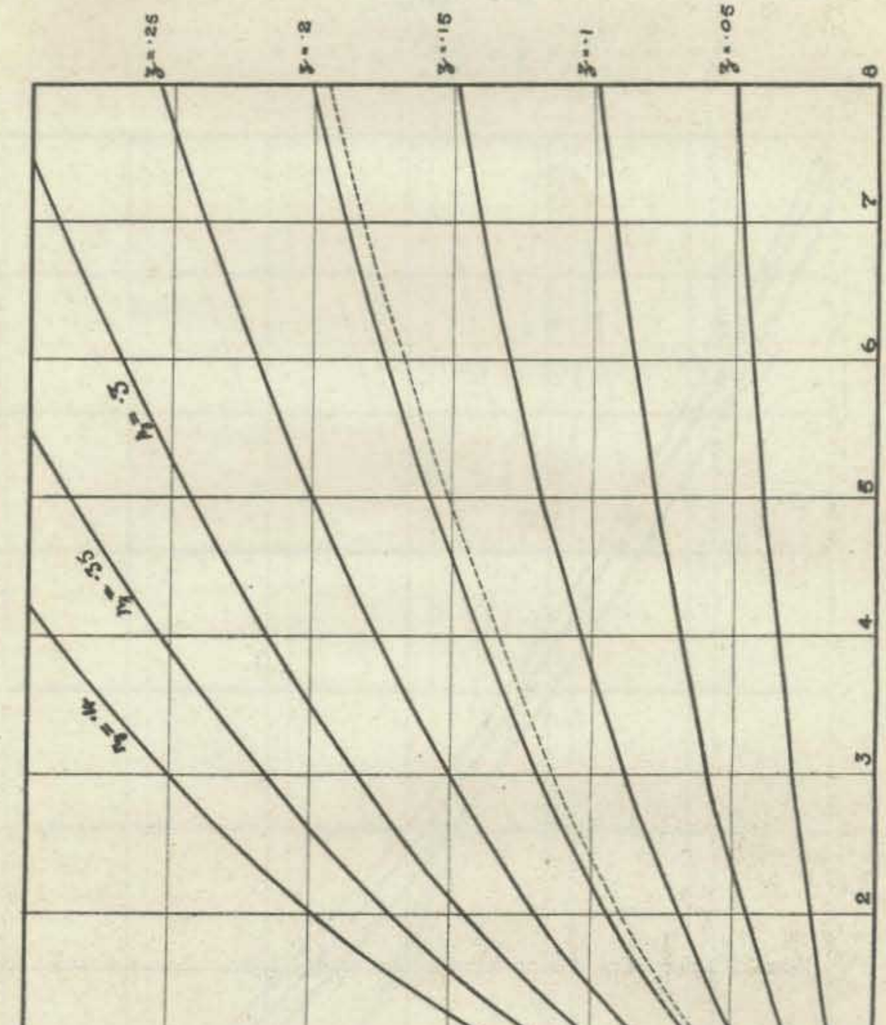
$$\frac{a_3}{a_2} = 1 - \frac{1}{2\left(1 + \frac{1}{b}\right) - x} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

When x is zero (the free air condition) this reduces to equation (4).

Table 2 gives the contraction ratio as a function of b and z ; in Fig. 3 it is plotted in terms of y and z.

CORRECTION TO WIND SPEED

THE DOTTED LINE REPRESENTS THE FIRST APPROXIMATION WHEN $z = \cdot 2$.



$$y = \frac{T}{\frac{\pi}{4}\rho D^2 V^2}$$

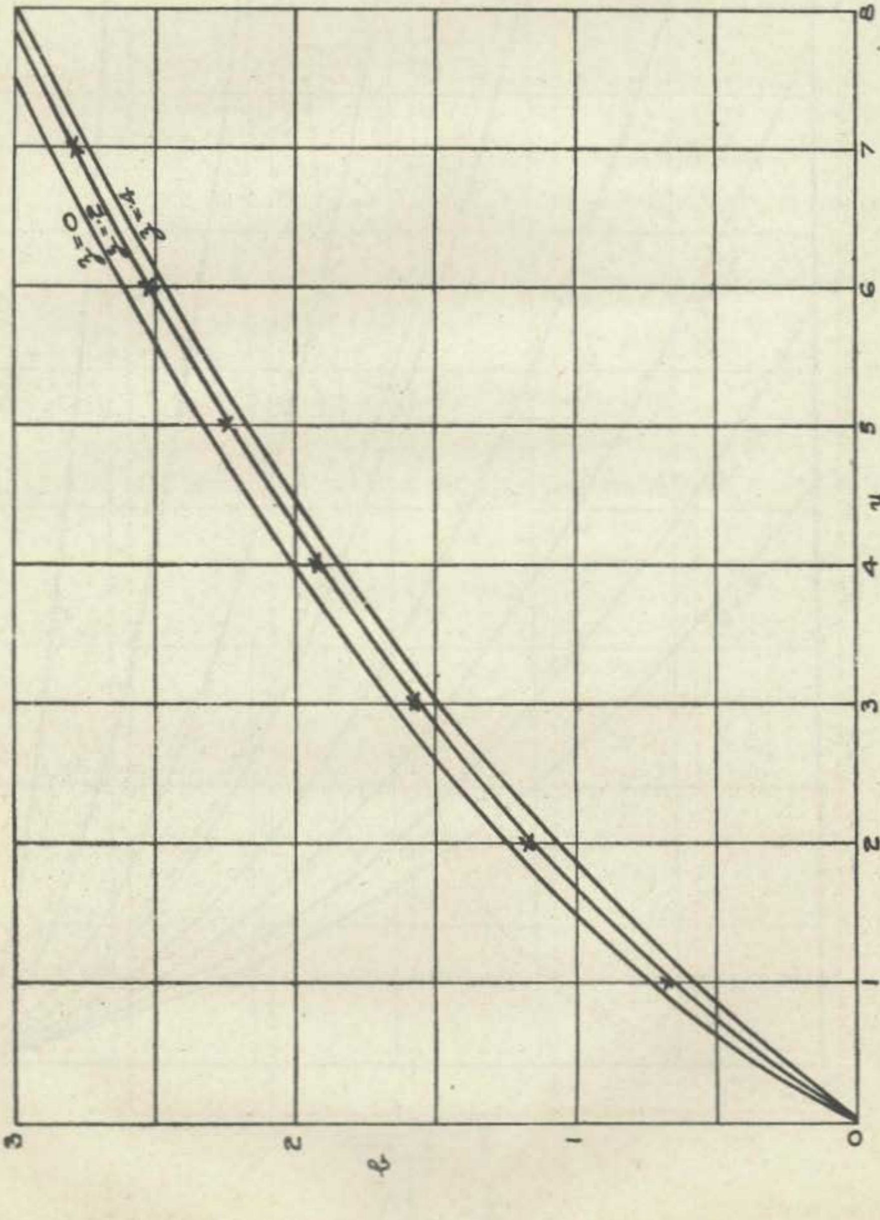
$$z = \frac{\frac{\pi}{4}D^2}{S}$$

THE SLIP STREAM RATIO (b)

THE CROSSES REPRESENT THE FIRST APPROXIMATION WHEN $z = \cdot 2$.

$$y = \frac{T}{\frac{\pi}{4}\rho D^2 V^2}$$

$$z = \frac{\frac{\pi}{4} D^2}{S}$$



THE CONTRACTION RATIO OF THE SLIP STREAM $\left(\frac{a_3}{a_2}\right)$

$$y = \frac{T}{\frac{\pi}{4}\rho D^2 V^2}$$

$$z = \frac{\frac{\pi}{4}D^2}{S}$$

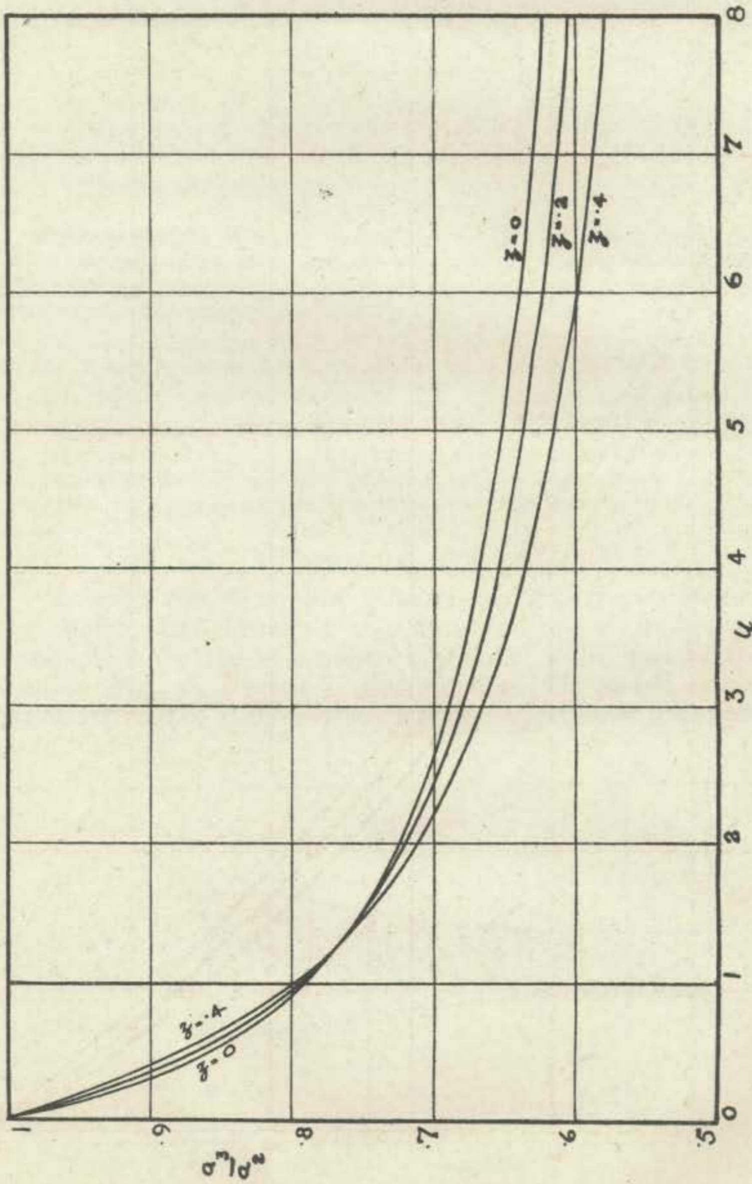


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Table 1.

Correspondence of values of y, z and $1 - \frac{V'}{V}$

b	$z = 0{\cdot}05.$			$z = 0{\cdot}10.$		
	x	y	$1 - \frac{V'}{V}$	x	y	$1 - \frac{V'}{V}$
0·2	0·048	0·23	0·005	0·101	0·24	0·010
0·5	0·044	0·65	0·010	0·092	0·67	0·022
1·0	0·039	1·54	0·019	0·080	1·57	0·038
1·5	0·036	2·66	0·026	0·075	2·72	0·055
2·0	0·035	4·06	0·034	0·071	4·12	0·069
2·5	0·034	5·70	0·042	0·068	5·77	0·083
3·0	0·033	7·58	0·049	0·066	7·66	0·096
	$z = 0{\cdot}15$			$z = 0{\cdot}20$		
0·2	0·160	0·25	0·015	0·224	0·27	0·022
0·5	0·143	0·70	0·035	0·199	0·72	0·048
1·0	0·125	1·62	0·061	0·174	1·65	0·084
1·5	0·115	2·77	0·083	0·159	2·84	0·115
2·0	0·110	4·19	0·107	0·149	4·25	0·144
2·5	0·105	5·85	0·127	0·143	5·92	0·172
3·0	0·101	7·75	0·146	0·138	7·83	0·199
	$z = 0{\cdot}25$			$z = 0{\cdot}30$		
0·2	0·295	0·28	0·028	0·377	0·30	0·037
0·5	0·260	0·74	0·063	0·329	0·78	0·078
1·0	0·225	1·70	0·108	0·281	1·75	0·133
1·5	0·205	2·90	0·147	0·254	2·94	0·180
2·0	0·192	4·31	0·183	0·237	4·36	0·225
2·5	0·183	5·99	0·219	0·225	6·03	0·266
3·0	0·176	7·89	0·253	0·217	7·94	0·310
	$z = 0{\cdot}35$			$z = 0{\cdot}40$		
0·2	0·471	0·32	0·045	0·575	0·34	0·055
0·5	0·403	0·81	0·095	0·488	0·85	0·113
1·0	0·341	1·80	0·160	0·406	1·83	0·188
1·5	0·306	3·00	0·215	0·361	3·02	0·251
2·0	0·285	4·40	0·267	0·333	4·45	0·308
2·5	0·270	6·07	0·316	0·315	6·10	0·367
3·0	0·258	7·97	0·364	0·300	8·00	0·422

Table 2.

Contraction Ratio $\frac{a_3}{a_2}$.

b	Contraction ratio when $z = 0$	$z = 0\cdot2$	$z = 0\cdot4$
0	1	1	1
0·2	0·917	0·915	0·913
0·5	0·833	0·828	0·819
1·0	0·750	0·739	0·722
1·5	0·700	0·685	0·664
2·0	0·667	0·649	0·625
2·5	0·643	0·624	0·598
3·0	0·625	0·605	0·578

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