SECOND REPORT ON THE TWISTING OF PROPELLER BLADES.

(SUPPLEMENTARY TO R. & M. 454.)

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SUMMARY.—The method of investigating the twist of propeller blades, which was developed in R. & M. 454, is interpreted mathematically by making a certain assumption as to the shape of the cross-sections. A general equation expressing the twist as a function of the radius is obtained, and an experimental method of solving it is evolved.

It is shown that blades of certain shapes may be peculiarly liable to torsional vibration, and that a plan form common in current practice possesses this property to an appreciable degree. It is further shown that the maximum stress due to torsion may determine fracture in this case.

A method of calculating the shape of plan form in any given case, in order that the blade shall not twist, is deduced, and it is shown that this leads to a nearly symmetrical form in one instance.

The effect of the large torsional hysteresis of timber in damping out vibrations is discussed, and it is suggested that herein may lie the reason for the comparative failure of metal propellers up to the present.

Finally, suggestions are made for the modification of current practice in accordance with the indications of the present theory.

1. *Statement of the Problem.*—In a previous report to the Committee (R. & M. 454) an approximate method of dealing with the problem of the twisting of propeller blades was discussed, and it was shown that it yielded results in fair agreement with those of actual experiments.

The application of this method involved a large amount of graphical work. In the present paper it will be shown that most of the latter can be eliminated and differential equations obtained, which express the torsion as a function of z only, by making certain assumptions regarding the shape of the cross-sections, which are substantially justified in the case of actual blades.

2. *Mathematical Analysis.*—The notation is the same as that used in R. & M. 454, save that γ is substituted for Y, the y-co-ordinate of the centroid of the curve of t^3, at any section.

In addition, however, to the relations

we shall put

$$\left.\begin{aligned} \int \frac{t^3}{12}\,dy &= \mathrm{I} \\ \int y\cdot\frac{t^3}{12}\,dy &= \mathrm{I}\,.\gamma \\ \int y^2\cdot\frac{t^3}{12}\,dy &= \mathrm{I}\,.k^2 \end{aligned}\right\} \quad . \quad . \quad . \quad (1)$$

the integrations being taken over a section, z = constant.

It will be assumed, besides the hypotheses made in T.1075, that the blade thicknesses can be represented in the form

$$\frac{t^3}{12} = \frac{\mathrm{I}}{b}\cdot\chi\left(\frac{y-\gamma}{b}\right) \quad . \quad . \quad . \quad (2)$$

where χ is any functional form independent of z, and it will also be assumed that the values of t^3 at the edges of the blade are sufficiently small to be neglected. These are fair assumptions in the case of propellers at present in use.

In its simplified form, the equation which represents the displacement, u, of the neutral surface of the blade, is

$$\mathrm{E}\,\frac{\partial^2}{\partial z^2}\left(\frac{t^3}{12}\cdot\frac{\partial^2 u}{\partial z^2}\right) - 4\,\mathrm{N}\,\frac{\partial^2}{\partial y\,\partial z}\left(\frac{t^3}{12}\cdot\frac{\partial^2 u}{\partial y\,\partial z}\right) \quad . \quad (3)$$

In deducing this equation, it was assumed that the stress couples and stress resultants acting on the elements of the blade could be expressed in terms of t and u by means of the relations which are valid in the case of plates of uniform thickness. It is known that this involves an approximation, though means are not yet available for finding accurately the extent of the deviation of equation (3) from the truth. An attempt was made in R. & M. 454 to estimate the deviation in the case of the torsional couples on the assumption that the distortion of the cross-sections was the same as in uniform torsion, when it was found that the error might reach 10 to 15 per cent. in important cases. These are probably outside figures, as it is almost certain that the actual distortion of a tapering blade must be less than that of a cylindrical one.

The assumption made in developing the graphical method of R. & M. 454, from this equation, was that $\frac{\partial^2 u}{\partial y\,\partial z}$ or τ was independent of y. The mathematical consequences of this assumption will now be examined.

We have

$$\frac{\partial^2 u}{\partial y \partial z} = \tau \quad . \quad . \quad . \quad . \quad (4)$$

hence

$$\frac{\partial u}{\partial z} = \tau y + \psi(z) \quad . \quad . \quad . \quad (5)$$

where ψ is independent of y, substituting in (3)

$$\mathrm{E}\frac{\partial^2}{\partial z^2} \cdot \left\{\frac{t^3}{12}\left(y\frac{d\tau}{dz} + \frac{d\psi}{dz}\right)\right\} - 4\,\mathrm{N}\frac{\partial^2}{\partial y \partial z}\left(\tau \cdot \frac{t^3}{12}\right) + \mathrm{P} = 0 \quad . \quad (6)$$

Integrating the terms of this equation over the surface of the blade, from the tip to the section $z = $ constant, we see that the term in P becomes the resultant shear force on the section, that is, it is equal to $-\frac{d\mathrm{M}}{dz}$ where M is the bending moment on the blade.

In performing the integrations it is to be noted that

$$\frac{\partial}{\partial z}\left(\frac{t^3}{12}\right) = \chi\left(\frac{y-\dot{\gamma}}{b}\right) \cdot \frac{d}{dz}\left(\frac{\mathrm{I}}{b}\right) + \mathrm{I}\,\frac{\partial}{\partial z}\left(\frac{y-\dot{\gamma}}{b}\right)\frac{d\chi}{dy} \quad . \quad (7)$$

and that, in the terms which are integrated by parts, the "integrated" terms vanish, since t^3 is zero at the edges.

We find

$$\mathrm{E}\frac{d}{dz}\left\{\mathrm{I}\left(\dot{\gamma}\frac{d\tau}{dz} + \frac{d\psi}{dz}\right)\right\} = \frac{d\mathrm{M}}{dz}$$

or

$$\frac{d\psi}{dz} = \frac{\mathrm{M}}{\mathrm{E\,I}} - \dot{\gamma}\frac{d\tau}{dz} \quad . \quad . \quad . \quad (8)$$

whence ψ is known.

Now multiply (3) by y and integrate as before. The term containing P is now the moment of the forces acting on the section, about oz. In addition to the points referred to above, it will be observed that $k^2 - \dot{\gamma}^2$ is independent of the origin of y and that $(k^2 - \dot{\gamma}^2)/b^2$ is a constant depending on χ only.

Calling T the couple resultant due to the stresses on the section, we have, as the final result,

$$\frac{d}{dz}\left(\mathrm{M}\dot{\gamma} + \mathrm{E\,I}(k^2 - \dot{\gamma}^2)\frac{d\tau}{dz}\right) + 4\,\mathrm{NI}\tau = \mathrm{T} \quad . \quad (9)$$

The quantity

$$\mathrm{T} - \frac{d}{dz}(\mathrm{M}\dot{\gamma}) \quad . \quad . \quad . \quad (10)$$

depends only on the applied load and the shape of the blade. It may be found directly by multiplying the resultant shear force on the section by the distance between its line of action and the principal flexure point (*see* T.1075). The distance of the latter point from oz is, of course,

$$\frac{d}{dz}(\mathrm{M}\,\dot{\gamma})\cdot\frac{d\mathrm{M}}{dz} \quad . \quad . \quad . \quad . \quad (11)$$

Putting

$$\left.\begin{aligned} \mathrm{T} - \frac{d}{dz}(\mathrm{M}\,\dot{\gamma}) &= b \\ \mathrm{E\,I}\,(k^2 - \dot{\gamma}^2) &= \mathrm{Q}^2 \\ 4\,\mathrm{NI} &= \mathrm{R} \end{aligned}\right\} \quad . \quad . \quad (12)$$

we see that (9) becomes

$$\mathrm{Q}^2\frac{d^2\tau}{dz^2} + 2\mathrm{Q}\frac{d\mathrm{Q}}{dz}\cdot\frac{d\tau}{dz} + \mathrm{R}\tau = \mathrm{G} \quad . \quad . \quad (13)$$

Put $\mathrm{U} = \tau\,\mathrm{Q}$ then (13) reduces to

$$\mathrm{Q}\frac{d^2\mathrm{U}}{dz^2} + \left(\frac{\mathrm{R}}{\mathrm{Q}} - \frac{d^2\mathrm{Q}}{dz^2}\right)\mathrm{U} = \mathrm{G} \quad . \quad . \quad (14)$$

Since the coefficients of this equation are all functions of z, its direct analytical solution presents considerable difficulties, but a certain amount of useful general information may be drawn from it. This will be referred to later.

It will be noted, however, that the coefficients depend only on the shape and size of the cross-sections of the blade, and that the independent term, G, depends only on the shape of the plan form and the nature of the load. Hence, if we have one plan form, A, given, such that the couples on it are G when it is subjected to the air load, then we can find another plan form, B, on which the couples are also G when it is bent by a single load, W, applied at the tip. Further, we can adjust the cross-sections of blade B so that its coefficients in (14) are the same as those of blade A. It follows that the twist curves of the two blades in these circumstances are identical. It will be shown subsequently that this can be made the basis of a simple experimental method of finding the twist of propellers under their air-load.

If the values of Q and R are given, it is possible to find the shape of the plan form which the blade must have in order that it shall not twist.

Let the centre of pressure of the air-load acting on a portion of the blade bounded by planes $z = \text{constant}$, separated by a small distance δz, be at a distance c from the centroid of the t^3 curve of this portion, that is, let the y-co-ordinate of the centre of pressure be $c + \dot{\gamma}$. Then, from (13) the condition for zero, τ, is

$$\frac{d^2}{dz^2}(\mathrm{M}\,\dot{\gamma}) = (\dot{\gamma} + c)\frac{d^2\mathrm{M}}{dz^2} \quad . \quad . \quad . \quad (15)$$

where c is independent of $\dot{\gamma}$, if it be assumed that the air-load distribution is not altered by changing the shape of the plan form.

Integrating this equation, we find

$$\dot{\gamma} = \int \frac{1}{M^2} \int c\, M \frac{d^2M}{dz^2}\, dzdz \quad . \quad . \quad . \quad (16)$$

the first constant of integration being zero if the integration be started from the tip of the blade, and the second merely additive and therefore of no importance.

From (16) the curve of $\dot{\gamma}$, and hence the developed plan form, may be set out in its proper relation to the direction of the grain (axis of z).

3. *Application of the Mathematical Work.*—In equation (14) the terms in U and $\frac{d^2U}{dz^2}$ may be either of the same or opposite sign. In the former case the torsion due to the couple G is less than would have been obtained on the old theory, that is to say, the effective torsional stiffness is increased by the term in $\frac{d^2U}{dz^2}$. In the latter case the reverse is true, and if at any point the two terms should happen to be nearly equal numerically, then the effective stiffness may be very small indeed and hence a local region of very high stress is to be expected. It is also evident that a blade containing regions of this sort must be very much more susceptible to torsional vibrations than one which does not contain such regions.

Although it is difficult to solve (14) directly, yet the interest attaching to the shape of the torsion curve, more especially in view of the above observation, is so great that some method of finding it would appear to be essential to progress in design. This might be done experimentally by loading an actual blade with sand or shot in such a way as to approximate to the air-load. Naturally, this method presents considerable difficulties. We may, however, find the same twist curve experimentally by constructing another blade, whose plan form is determined from the original one by the method explained in the analytical section, and loading it with a single load at the tip. The method consists simply in adjusting the shapes of the two blades so that the equations (14) representing their torsions are identical in these different conditions of loading.

It will be seen that the technical details may be further simplified by making the sections of the auxiliary blade rectangular, since their shape is immaterial, provided that the quantities Q and R are fixed. Again, the auxiliary blade may be made on any convenient scale, and one piece of wood only need be used in its construction. Hence, from every point of view, it is far more probable that the indirect experiment will give an average result, representative of a uniform blade, than it is that such a result would be obtained by the direct method.

Another point worthy of note is that the true values of Q and R which should be used in equation (14) are greater than those given by relations (12) by an unknown amount, on account of the tapering of the blades. It is probable, in view of the nature of this factor, that these quantities are also greater in the case of the auxiliary blade by approximately the same amount.

Fig. 1 shows the twist and torsion curves of the S.E. 5 two-blader propeller dealt with in R. & M. 454, obtained by means of an auxiliary blade in the manner described above. The maximum twist is about 1·5°, and it occurs at the tip of the blade. As was generally anticipated, the twist is only important in the part between the tip and a point halfway between the tip and the centre of the boss.

On the assumption that the shear stresses due to torsion and bending are the same as they would be in a cylindrical bar under uniform torsion and a concentrated load respectively, the maximum total shear stress, in the directions yz, works out at 380 lbs. per sq. in. at a point 13 inches from the tip.

The effect on the torsion of variations in the value of E, such as are known to be possible, has been estimated, and it has been found that if the modulus vary uniformly with y, in such a way that the leading lamination is 1·8 times as stiff as the trailing one, then the maximum shear stress may be as much as 500 lbs. per sq. in.

These figures are certainly below the truth on account of the neglect of the unknown factors due to taper. It is improbable that these involve an additional stress of less than 100 lbs. per sq. in., and it is quite possible that their effect may be much greater.

Reliable figures relating to the shear strength of walnut under various types of loading have yet to be obtained. Some simple torsion experiments have been made which seem to indicate that the ultimate strength of walnut in this respect is not much greater than 2,000 lbs. per sq. in. under a unidirectional torque applied for about fifteen minutes only. Under a steady torque, applied for an indefinite period, it may be little more than half as much. Regarding the effect of vibration on the ultimate strength nothing whatever is known.

In the foregoing investigation no account is taken of the possible effect of vibration in raising the maximum stress. When this is allowed for it becomes highly probable that blades of the form under discussion are not over-designed even on the basis of ultimate strength.

At present it is usual to design propellers on the basis of the greatest longitudinal tension. It is known, however, that in propeller bursts tension failures are comparatively rare. In view of the present figures this is not surprising, and it is evident that the method of designing for maximum tension, without reference to the shape of the plan form, is inadmissible.

Fig. 2 shows the curve of U (equation 14), obtained from the experimental torsion curve of Fig. 1, for the important part of the blade. It will be seen from the form of (14) that the value of U at any place where the curve has a point of inflection should be equal to $QG/\left(R - Q \cdot \frac{d^2Q}{dz^2}\right)$. The dotted curve of Fig. 2 shows the latter quantity for all points along the blade. It will be seen that it cuts the curve of U very nearly at the points of inflexion of the latter. In calculating R, allowance has been made for the probable value of the factor due to taper. This only amounts to 5 to 10 per cent. at most parts of the curve, and therefore does not affect the general nature of the result.

At points where the U curve lies above the dotted one $\frac{d^2U}{dz^2}$ and U are of opposite signs, and hence these are portions where undue susceptibility to torsional vibrations exists. It is unfortunate that there is, in this type of blade, a well marked region of this sort in the neighbourhood of the section where the maximum stress occurs. This is evident from the diagram; in fact the torsion at the section of maximum stress is about 1·8 times as great as the value which would have been obtained if $\frac{d^2U}{dz^2}$ had been neglected.

The most satisfactory way of avoiding these sources of trouble is to choose the plan form in such a way as to make the twist as small as possible. If U is small and of the first order, then $\frac{d^2U}{dz^2}$ is a small quantity of the third order, that is, it is of the second order compared with U and hence can never be important.

Equation (16) of the mathematical discussion determines the shape of plan form of any blade in order that it shall not twist.

This equation has been applied to the blade examined above, and the resultant developed plan form is shown in Fig. 3. The dotted curve shows the present form of blade. It will be seen that the shape given by (16) is much more nearly symmetrical than the original blade. The axis of z represents, of course, the general direction of the grain.

The effect of possible variations in the value of E has been investigated in this case also, and it has been found that with the same variation as before the maximum shear stress could only be about one-fifth of its value in the previous case.

4. *The Damping of Torsional Vibrations in Propeller Blades.*—Another report to the Committee (R. & M. 528) gives an account of some torsion experiments on wood which indicate that a very large amount of hysteresis must accompany any torsional vibrations of wooden members. It is suggested that this hysteresis may be responsible for most of the damping of such

TORSION OF A PROPELLER BLADE UNDER AIR LOAD.

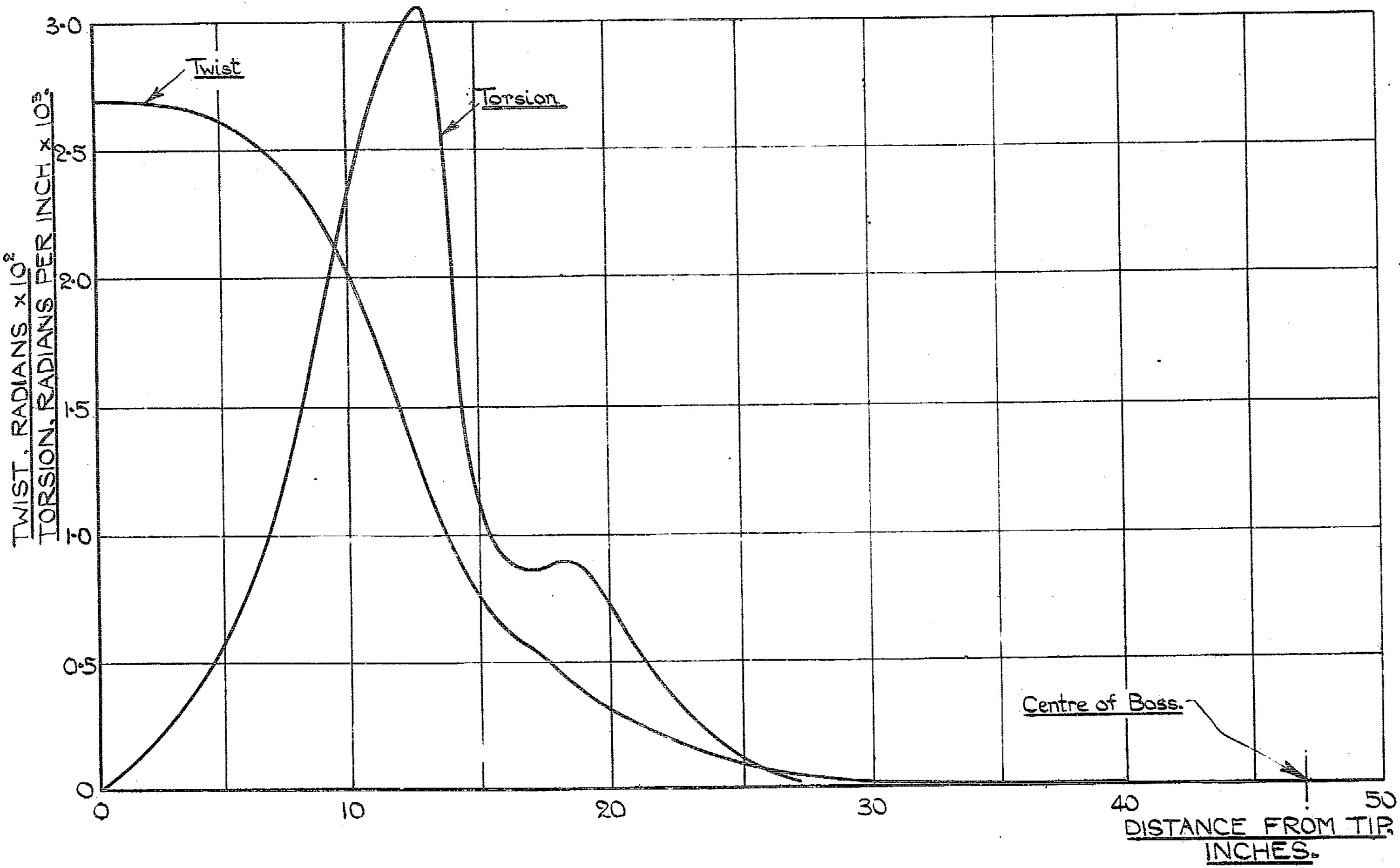

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FIG 1.

TORSION OF A PROPELLER BLADE UNDER AIR LOAD.

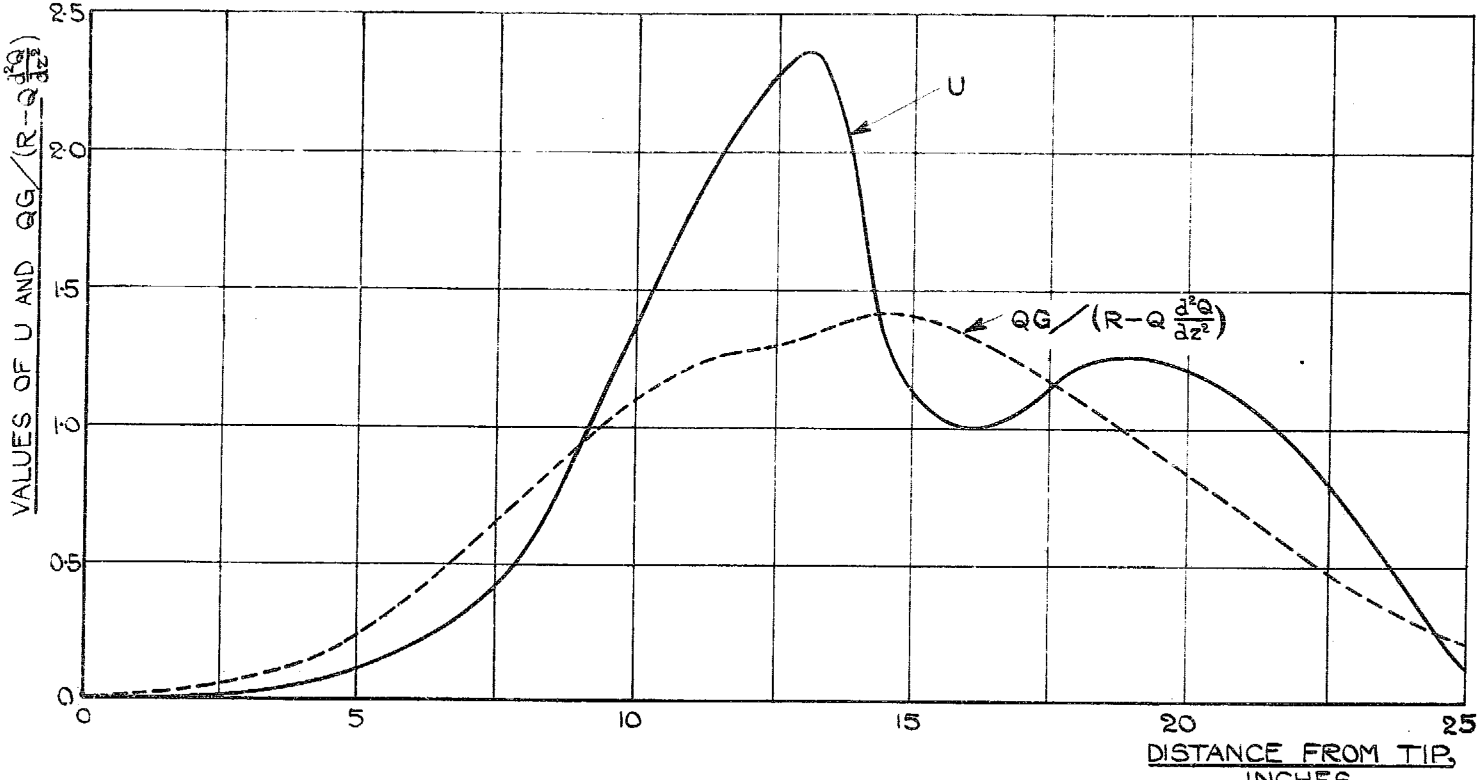

FIG 2.

DEVELOPED PLAN FORM OF BLADE HAVING ZERO TORSION.

THE DOTTED CURVE SHOWS THE PRESENT PLAN FORM.

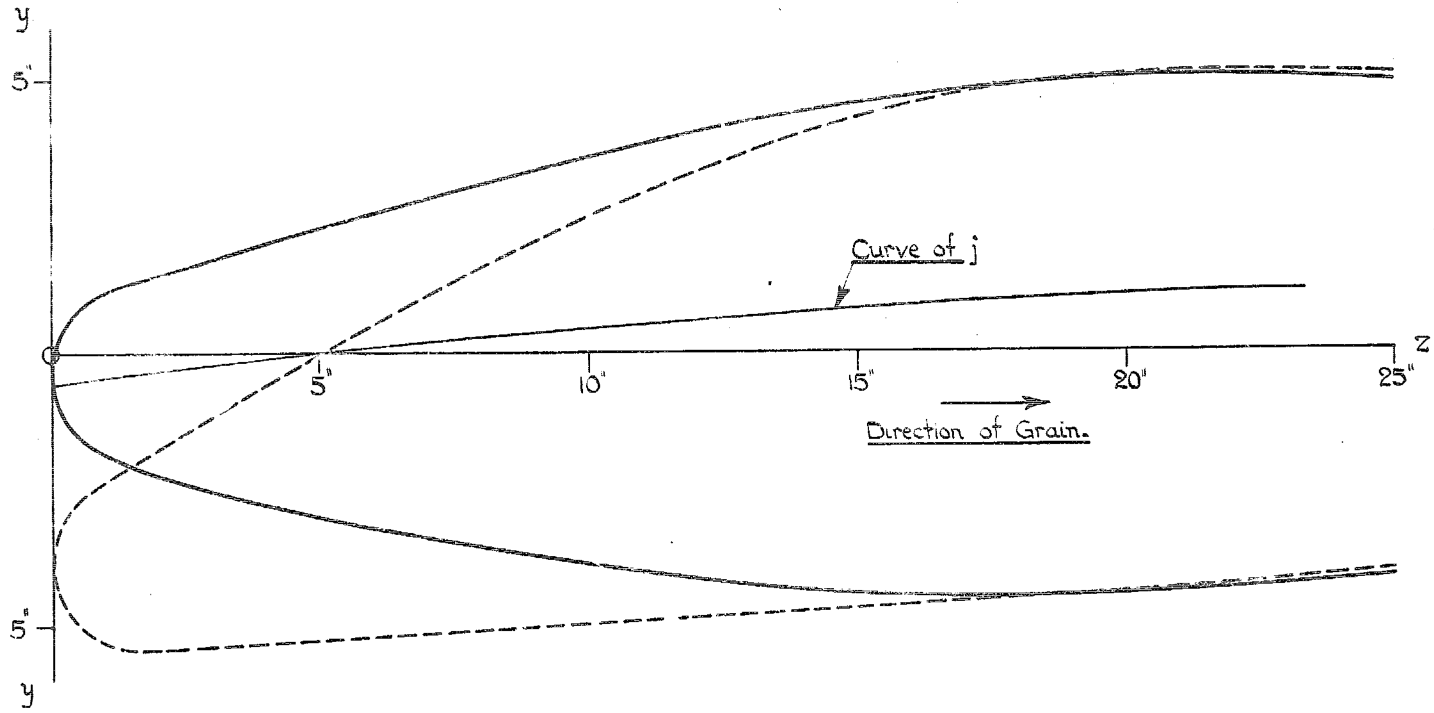

FIG 3.

vibrations in the case of air propellers, and therefore may be one of the most valuable properties of propeller timber.

There is, in fact, a certain amount of evidence to show that the damping due to the air forces alone is insufficient in the very severe vibration conditions which subsist on aircraft. If a blade twist so as to reduce the angle of incidence (which is the direction usual with present plan forms) two things happen. In the first place the air-load is decreased, and in the second the centres of pressure move towards the trailing edge. These effects tend respectively to decrease and increase the couple G, and hence the damping moment due to them may be either positive or negative according to the shape of the plan form, or it may be positive in some places and negative in others.

More direct evidence is provided by the numerous attempts to make metal propellers. So far, none of these have been successful from the point of view of strength, although the metal construction is theoretically as strong as the wooden one on the basis of direct tension, and almost certainly considerably stiffer torsionally.

This would appear to indicate that in these cases, where damping due to elastic hysteresis must be practically negligible until the elastic limit is reached, the air-damping is insufficient to keep the torsional vibrations within safe limits.

As further evidence on this point, it may be noted that all-metal propellers have been known to fail at the boss, whereas propellers with wooden blades and metal bosses of the variable pitch type have stood up successfully.

If this should prove to be the true explanation, it implies that metal is an unsuitable material for propeller blades unless the air-damping can be largely increased by a suitable modification of the plan form and cross-sections, or unless the cyclic variations of engine torque can be largely reduced.

5. *Application to Design.*—It is suggested that the matter dealt with in this report and in R. & M. 454 points to the following as being desirable modifications in propeller practice, in the order of their probable importance :—

(1) Adoption of the shape of developed plan form given by equation (16) above.

(2) Adoption of some method of grading timber according to the value of its Young's modulus, as detailed as practical considerations will permit.

(3) Use of machine shaping in preference to hand work wherever possible in order to secure uniformity in the blade sections, since it is impossible to predict the twist curve of a blade unless the shapes of its sections are known accurately.

(4) Replacement of the electric drive usual in bursting and other propeller tests by an arrangement more nearly representative of the vibration conditions which obtain on aircraft engines. It may be noted, in this connection, that little information can be obtained, regarding the strength of propellers under flying conditions, from a test in which the motor is almost ideally free from vibration, since actual bursts on service, in the great majority of cases, occur on engines in which vibration is particularly noticeable.

(5) Introduction of some simple torsional vibration experiment into propeller timber inspection routine in order to obtain a figure for its damping efficiency.

(6) Use of two-bladers in preference to four-bladers whenever other circumstances permit.

The advisability of this course arises from the circumstance that it is much easier to get elastic balance, and the consequent comparative freedom from noise and vibration, in two-bladers than in four-bladers.

6. *Values of Section, Moduli, &c.*—The following figures, representing average values for the quantities used in the mathematical work, may be found useful in applying it to actual blades :—

b = blade width.

t_m = max. thickness of section.

Distance of centroid of t^3 curve from trailing edge = $0{\cdot}59\ b$.

$I = 0{\cdot}0434\ b^3 \,.\, t_m^3$.

$R = 4\ NI$.

$Q = 0{\cdot}189\ b\sqrt{EI}$.

(E = Young's Modulus. N = Modulus of rigidity.)