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Experience of the Denton Blade-to-Blade Time Marching Computer Programs

- by -

J.D. Bryce and M.R. Litchfield

EXPERIENCE OF THE DENTON BLADE-TO-BLADE
TIME MARCHING COMPUTER PROGRAMS

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SUMMARY

The Denton two-dimensional and three-dimensional time marching blade-to-blade computer programs have been used at NGTE to analyse the flow past transonic turbine blading. In this work experience has been gained in operating the programs and this paper describes the major findings in respect of how the programs have performed. Solutions for several blade sections, giving Mach number distribution, are included.

CONTENTS

1.   Introduction

The flow in a single stage, high work capacity turbine is complex by nature
of the flow Mach numbers at exit from the nozzle guide vane and rotor being
supersonic.   In designing such a turbine the problem arises of how to select
the blade profile shapes to meet the mixed flow conditions.   One possible
approach would be to base the aerodynamic design on the use of streamline
curvature methods, suitably modified to work for transonic flow, for assessing
the blade surface Mach numbers.   Alternatively it might be possible to use the
method of characteristics for designing the supersonic sections of the blades.
However, in 1974 the Denton time marching, blade-to-blade computer programs were
published[1] and these appeared to offer the best theoretical solution for
transonic flows in turbine blade passages.

At NGTE the Denton programs have been used to analyse the transonic flow
past a variety of turbine blade configurations and this paper describes the
experience gained on the operation of the programs.   Five numerical examples
are presented to illustrate the use of the programmes both as an analysis and
design tool.

2.   Factors Affecting Operation of the Computer Programs

In the work described here three time marching programs have been used:

(a)   The original two-dimensional (2D) program written by Denton.

(b)   A modified 2D program which has the facility of allowing for a
       variation of streamtube thickness thus enabling some 3D effects
       (for example end wall flare) to be simulated.

(c)   The Denton three-dimensional (3D) program.

Many different blade profiles have been investigated using these programs and
Figs.1 and 2 illustrate typical Nozzle Guide Vane (NGV) and rotor blade
profiles and associated channel shapes on which the technique has been used.
The majority of the prediction work has been done using programs (a) and (b).
The three-dimensional program has been employed on only a few occasions to
check that three-dimensional effects do not invalidate predictions obtained
using the two-dimensional programs.

2.1 Grid size

In analysing the flow past transonic turbine blades it was considered
advisable, (because of the complex shape and associated complicated flow) to
employ in the majority of cases the full complement of grid points allowed by
the two-dimensional programs.   Such a grid has 40 axial × 10 pitchwise grid
points and typical spacing and location employed is shown in Figs.1 and 2a for
a nozzle and rotor respectively.   The first calculating plane was located
about one-third to one-half an axial chord upstream of the leading edge of the
blade.   The final plane was positioned a similar axial distance downstream of
the trailing edge.   As shown on the figures, cusps were positioned at the
leading and trailing edges, formed by drawing tangents to the upper and lower
surfaces of the blade.

In an attempt to reduce computing time a coarse grid spacing was tried
but this gave rise in some instances to significant differences in blade surface
Mach number distribution when compared with results of a fine grid solution.
Figure 2 shows a comparison of fine mesh (40 × 10) and coarse mesh (24 × 6)
spacing used on a transonic rotor blade and Fig.3a shows the blade surface
Mach number distribution for a rotor blade investigated in some preliminary

work. It will be noted that significant differences occur between the Mach number distributions for the two grid spacings - particularly regarding the peak Mach numbers on the suction surface and the minimum values on the pressure surface. For this particular blade therefore (where channel shape had not been optimised so as to give an even surface Mach number distribution) it would appear that the less precise definition of channel geometry resulting from a coarse mesh was having some significant effect. On the other hand, however, later calculations done on a profile whose channel shape had been carefully modified to produce an even distribution of surface Mach number, far less discrepancy resulted between the fine and coarse mesh solutions as Fig.3b illustrates. From the limited evidence available it would seem to be advisable to adopt as fine a mesh as possible for computing the flow in transonic blade passages.

Although it has not been checked to any degree, a similar argument on grid size possibly applies to the use of the three-dimensional program. Here computer storage considerations limited the number of grid points to 24 axial × 7 pitchwise × 7 radial - ie, a number of axial and pitchwise grid points equivalent to the above coarse grid two-dimensional prediction.

2.2 Time step

Smoothing factor, relaxation factor and time step were parameters whose magnitude could be altered in the input data of the programs. The smoothing factor is used to smooth the flow properties in the pitchwise direction at each time step, and eliminates any 'waviness' in this direction. The relaxation factor, RF, is defined directly as in Denton's original paper[1]. The time step is defined as the given fraction of the maximum permissible time step for the procedure to remain stable. For all cases completed to date smoothing factor was maintained at 0.02 and relaxation factor at 0.1. The magnitude of the time step had an important influence on the rate of convergence and stability of the calculation. For two-dimensional programs used on NGVs of the type shown in Fig.1 featuring large nose radii and supersonic outlet Mach numbers ($M \simeq 1.4$) and for transonic rotors as in Fig.2 having large deflections (125°) and high subsonic inlet Mach number ($M \simeq 0.7$) it was generally found necessary to decrease the time step from 0.7 to 0.5. Some NGVs of more modest nose radius and rotors having lower deflections and inlet Mach numbers, permitted the use of higher time steps. For the limited number of cases done on the 3D program, it has been found necessary to use a time step of 0.4 to achieve satisfactory convergence.

2.3 Rate of convergence

The calculation is said to have converged when the change in axial velocity at all grid points is less than 0.01% between successive iterations ie, $(\Delta V_x/V_x)_{max} < 0.0001$. The change in this parameter with time step number is shown on Fig.4 for three cases.

(a)   Two-dimensional solution on a transonic NGV of the type shown in Fig.1 having a 40 × 10 grid and time step = 0.5.

(b)   Two-dimensional solution on a transonic rotor of the type shown in Fig.2 with 40 × 10 grid and time step 0.5 and 24 × 6 grid with time step = 0.5.

(c)   Three-dimensional solution on a transonic rotor of the type shown in Fig.2 having a 24 × 7 × 3 grid and time step 0.4.

The results show the rates of convergence which were generally achieved for a large number of nozzle and rotor cases investigated. For NGVs the convergence settles down after about 200 to 300 time steps for the fine grid and after about 200 time steps for rotors. Generally the solution for NGVs did not satisfy the convergence

criterion throughout the flow field after 1000 time steps, the 'errors' chiefly
occurring around the nose and in the subsonic section of the channel. On rotors
the convergence was relatively slow after 200 time steps although in a significant
number of cases complete convergence was attained before 1000 time steps expired.
It will be noted from Fig.4b that the rate of convergence on the coarse grid is
significantly better than the fine grid. Similarly the three-dimensional
solution (Fig.4c) is also relatively rapid in its convergence, although it takes
much longer to perform each time step as will be seen later. As illustrated in
Fig.5, differences between the two-dimensional solutions after 500 and 1000 time
steps are relatively minor in the case of NGVs but a little more significant in
the case of transonic rotors. This finding was borne out in a large number of
cases.

2.4 Total pressure variation within the blade channel

A steady-state inviscid solution of the flow within a blade channel
requires that the total pressure at every point in the channel is equal. Where
gradual convergence of the steady-state solution is obtained by integrating the
time-dependent equations of motion with time, then there must be some variation
of total pressure up to the point where complete convergence is obtained. A
measure of this variation with time is shown on Fig.6 for both NGV and a rotor
where it can be seen that the oscillations in total pressure over the surface of
the blade and also upstream and downstream of the blade, decrease as the
calculation progresses. However, for the NGV and the rotor blade cases
illustrated, fairly significant changes of total pressure still exist after the
solution has converged within the stipulated velocity criterion. The
discrepancies occur chiefly at the nose and trailing edge region but significant
errors of about ±5% also exist within the channel. It is probable that these
would further decrease if the calculation proceeded to more rigorous convergence
limits, but it is also probable that the imprecise definition of the leading and
trailing edge geometries is also contributing to the variation. However, this
variation in total pressure does not appear to influence the quality of the
overall prediction of blade surface Mach numbers which, as will be shown in a
later section, is in good agreement with transonic cascade experimental results.

2.5 Computation time

For the solutions presented in this paper the time marching programs were
operated on ICL 1904A computer. A comparison of the effect of grid size on
computational speed, and the difference in speed between the two-dimensional
and three-dimensional programs can be obtained from the figures listed in the
following table which apply to a test case on a transonic rotor blade.

Program	Two-dimensional	Two-dimensional	Three-dimensional
No of axial planes	40	24	24
No of pitchwise stations	10	7	7
No of radial stations	-	-	3
Time step	0.5	0.5	0.4
No of time steps for convergence	930	510	630
Convergence. % Change in V	-0.008	-0.01	0.01
CPU time MILL SECS	5691	1300	12869

From the above table it can be seen that on the two-dimensional program,
a case with a full complement of grid points (40 × 10) takes more time steps and

more than four times the time to reach the required convergence criterion than a case with 24×7 grid points. This computation time is in rough agreement with the relationship: total computation time $\propto$ (number of axial grid stations)$^2 \times$ No of pitchwise stations. The three-dimensional program with only three radial stations takes roughly ten times longer than the two-dimensional program having the same number of axial and pitchwise grid stations, although as has already been noted in the previous section, its rate of convergence with time step is very similar. For an increased number of radial stations it can be assumed that the computation time would increase roughly proportionately.

3. Numerical Examples

Five examples are given to illustrate the use of the programs. The first four are for transonic NGV and transonic rotor blades for which cascade test data were available to assess the accuracy of the numerical solutions of blade surface Mach numbers. The fifth example is of a transonic NGV and shows the effect of changes in channel shape on the surface Mach number, so demonstrating the use of the time marching program as a design tool. The five cases were:

(a) A transonic NGV tested in a two-dimensional cascade[2].

(b) A transonic rotor blade tested in a two-dimensional cascade[2].

(c) The same transonic rotor blade as case (b), tested in a cascade tunnel with wall flare[3].

(d) A convergent-divergent NGV - having a concave back tested in a VKI cascade tunnel[4].

(e) Two-dimensional flow past a NGV.

3.1 Transonic NGV and transonic rotor blade in a two-dimensional cascade

The NGV was tested over a range of positive and negative incidence (-10 to $+10°$) at different outlet Mach numbers (0.8 to 1.4). At each test condition detailed outlet traverses were made (at approximately $\frac{1}{2}$ axial chord length downstream) to obtain downstream total pressure and outlet angle. Also detailed blade surface static pressure measurements allowed surface Mach number distributions to be determined for direct comparison with predicted values. The case chosen for comparison was the approximate stator design condition at zero incidence and outlet Mach number of 1.23, and is illustrated on Fig.7. Agreement is generally good on both suction and pressure surfaces with the magnitude of peak suction surface Mach number being correctly predicted but occurring slightly downstream of the measured value. A repeat calculation on the two-dimensional program using a variation of streamtube thickness to simulate blockage effect arising from viscous losses through the cascade (determined from the cascade measurements of total pressure loss) did not appear to have any significant effect on altering the position of the aforementioned peak, nor did modifying the NGV profile to allow for boundary layer displacement thickness. It is suggested that this shift is caused by viscous effects difficult to simulate in the program - such as a shock wave emanating from the blade trailing edge which interacts with the boundary layer on the section surface of the adjacent blade so modifying the effective shape of the blade in this region. Although not shown here a case was also computed at positive incidence ($+10°$) and again good agreement was obtained.

The transonic rotor blade was tested in the same cascade over an incidence range of -20 to $+10°$ at varying outlet Mach numbers. The first case selected for comparison with the two-dimensional time marching prediction is the rotor design case at zero incidence and outlet Mach number of 1.235 and this is shown in Fig.8. Generally agreement between prediction and measurement is good with

the same small shift in position of peak Mach number on the suction surface
occurring as for the stator. Again attempts to simulate blockage effect
failed to alter the position of the peak significantly and it is surmised
that this small discrepancy again arises from shock boundary layer interaction
effects. With the second case selected for comparison - positive incidence of
10° and outlet Mach number of 1.21 - the predicted distribution shown on Fig.9
reaches a significantly higher peak Mach number around the noise region on the
suction surface compared with measurement. The discrepancy possibly arises
because in practice strong local shocks would be formed on this region which
would interact with the boundary layer and 'spread' the interaction effect
over a wider region of the suction surface. Elsewhere there is reasonable
agreement between theory and experiment, with the same remarks made earlier in
connection with the viscous effects downstream of the throat applying here as
well. As an illustration that the program will handle very high values of
incidence a case was run at -20° at an outlet Mach number of 1.295. Here,
agreement is generally good (Fig.10). Some discrepancy occurs around the nose
on the pressure surface and possibly the geometry of the cusp assumed in the
prediction could be having some effect in this instance.

Turning now to the comparison between mean outlet angle as measured in the
cascade and the value predicted by the time marching program, results are shown
on Fig.11 for both stator and rotor. At the design outlet Mach number of
1.23 and zero incidence on the stator the predicted inviscid angle is about 2°
higher than the mean measured value, agreeing closely with the design outlet
angle and about 1.5° less than $\cos^{-1}$ throat opening/pitch. For the rotor at
zero incidence, three cases have been predicted, one at low outlet Mach number
(0.95) one at the approximate design Mach number of 1.23 and one at a higher
Mach number 1.34, and the predicted angles are 2 to 3° higher than the
measured outlet angles. The absolute level of outlet angle falls with
increasing supersonic outlet Mach number in the same way as the measured
results. At the design outlet Mach number of 1.23 the predicted inviscid angle
is about 1° higher than design and 2° less than $\cos^{-1}$ O/S. The difference of
approximately 2° between the predicted inviscid angle and the measured one on
both NGV and rotor is to be expected insofar as that under test conditions
deviation of the flow occurs in the complex shock environment downstream of the
blade trailing edge, and this reduces the magnitude of the measured angle at
the downstream survey plane. The predicted inviscid value does not include such
effects.

3.2 Transonic rotor blade cascade with wall flare

This rotor blade had the same profile as the one described in 3.1 but it
was tested in a cascade incorporating wall flare, to simulate the annulus wall
flare which occurs in a turbine. The prediction of surface Mach numbers for
zero incidence and outlet Mach number of 1.17 as obtained from the two-dimensional
program with streamtube thickness variation is shown on Fig.12 and is compared
with the distribution derived from the blade surface static tappings. Agreement
between theory and experiment is quite good apart from the magnitude of the peak
Mach number downstream of the throat on the suction surface which is
underpredicted. It is of interest to note that compared with the two-dimensional
solution shown on Fig.8 the effect of wall flare has been to increase the inlet
Mach number from 0.45 to 0.6 as a result of the throat of the blade being able to
pass more flow. Also it will be noted that the Mach number distribution
occurring immediately downstream of the nose on the suction surface is more
'peaky' than the two-dimensional solution.

3.3 Convergent-divergent nozzle

The VKI nozzle blade tested in a supersonic cascade features a convergent-
divergent section downstream of the throat and concavity on the suction surface

downstream of the throat. The test and predicted surface Mach number distributions associated with an outlet Mach number of 1.33 and zero incidence are also shown on the Fig.13. Although the shape and level of the Mach number distributions are generally similar over the suction and pressure surface marked, local discrepancies occur in the throat region and further downstream on the suction surface, where in both locations the experimental results show distinctive peaks. Examination of Schlieren pictures taken of flow in the cascade shows that shock and viscous effects are present in both situations. Firstly, in the throat region, a train of lamda shocks local to the suction surface would appear to thicken the boundary layer and modify the effective shape of the blade, such that a local expansion and recompression occurs. Secondly, further downstream, a shock emanating from the trailing edge of the blade interacts with the already thickened boundary layer on the suction surface of the adjacent blade so altering the channel shape in this critical region. These effects are difficult to simulate in the inviscid time marching method.

3.4 Two-dimensional flow past a NGV

A study was made using the two-dimensional time marching program to investigate the effect of passage or channel shape on the Mach number distribution for NGV designed for supersonic exit Mach number. Four basic shapes of profile were investigated:

(a) Convergent blade passage with a convex or straight suction surface downstream of the throat.

(b) Convergent blade passage with a concave suction surface downstream of the throat.

(c) Convergent-divergent blade passage with a straight suction surface downstream of the throat.

(d) Convergent-divergent passage with a concave suction surface downstream of the throat.

The main findings of this study are illustrated in Fig.14 which shows that the passage (d) employing internal divergence and concavity on the downstream suction surface has a superior blade surface Mach number distribution. This clearly illustrates the value of the time marching technique as a design tool.

4. Conclusions

In the experience of NGTE, the Denton two-dimensional and three-dimensional blade-to-blade time marching programs have proved reliable and easy to use in the analysis of the flow past transonic turbine blades. From the analysis a number of main conclusions can be drawn.

(i) On turbine blading involving high Mach numbers and deflections it would appear advisable to use the full complement of grid points allowed by the programs. Coarse grid spacings could result in some 'smoothing' or over emphasis of peaks in the velocity distribution over the blades.

(ii) For the transonic turbine blade profiles investigated a time step of 0.5 was generally found necessary for satisfactory convergence of the two-dimensional programs. Rate of convergence tended to be more rapid on rotors than NGVs but differences in blade surface Mach number were generally small for both rotors and NGVs after about 500 time steps.

(iii) There appeared to be a significant variation in total pressure remaining within the blade channel after the specified convergence criterion had been achieved. This did not appear to have a detrimental influence on the solution.

(iv) Generally good agreement was achieved between the two-dimensional prediction and the results of transonic NGV and transonic rotor blade cascade tests. It appears that in some cases viscous effects can have a significant effect on both the magnitude of peak Mach numbers on the suction surface and the location in which they occurred.

Acknowledgement

References

No	Author(s)	Title, Date, etc.
1	J.D. Denton	A time marching method for two- and three-dimensional blade-to-blade flows. ARC R&M No.3775. 1975.
2	-	Rolls-Royce private communication.
3	-	Rolls-Royce private communication.
4	C. Sieverding	Base pressure measurements in transonic turbine cascades. Transonic Flows in Axial Turbomachinery Lecture Series 84, Von Karman Institute for Fluid Dynamics. 2-6 February 1976.

FIG. 1

N.G.V.
40 AXIAL X 10 PITCHWISE
GRID POINTS

FIRST
AXIAL
PLANE

NOSE CUSP

PITCHWISE

AXIAL

FINAL
CALCULATION
PLANE

TRAILING EDGE CUSP
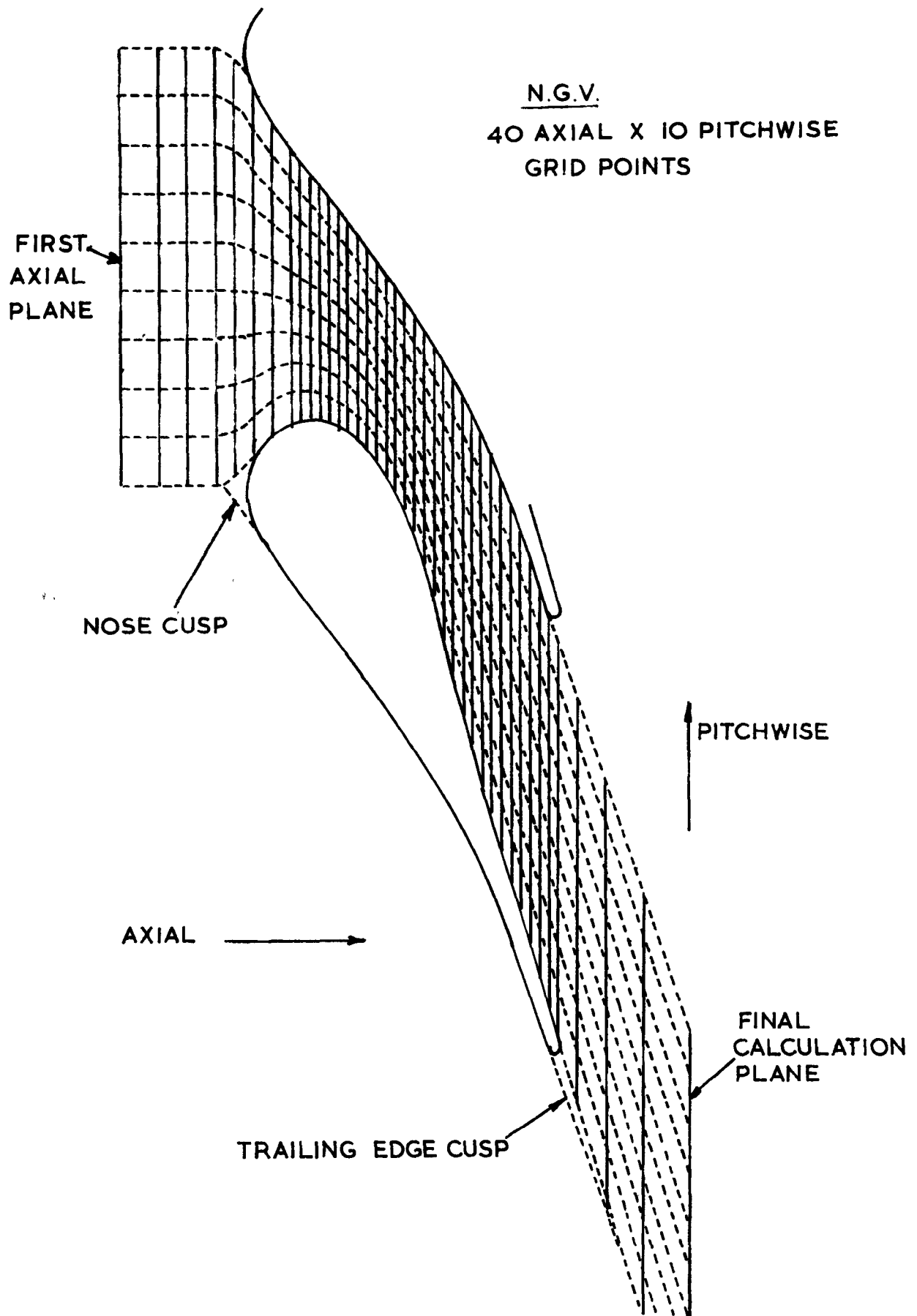
FIG. 1   TYPICAL GRID SPACING FOR A TRANSONIC NGV

FIG. 2

(a) FINE GRID (40 AXIAL
X 10 PITCHWISE POINTS)

NOSE
CUSP

PITCHWISE

TRAILING EDGE
CUSP

FIRST
CALCULATION
PLANE

FINAL
CALCULATION
PLANE

(b) COARSE GRID
(24 AXIAL X 6 PITCHWISE
POINTS)

AXIAL
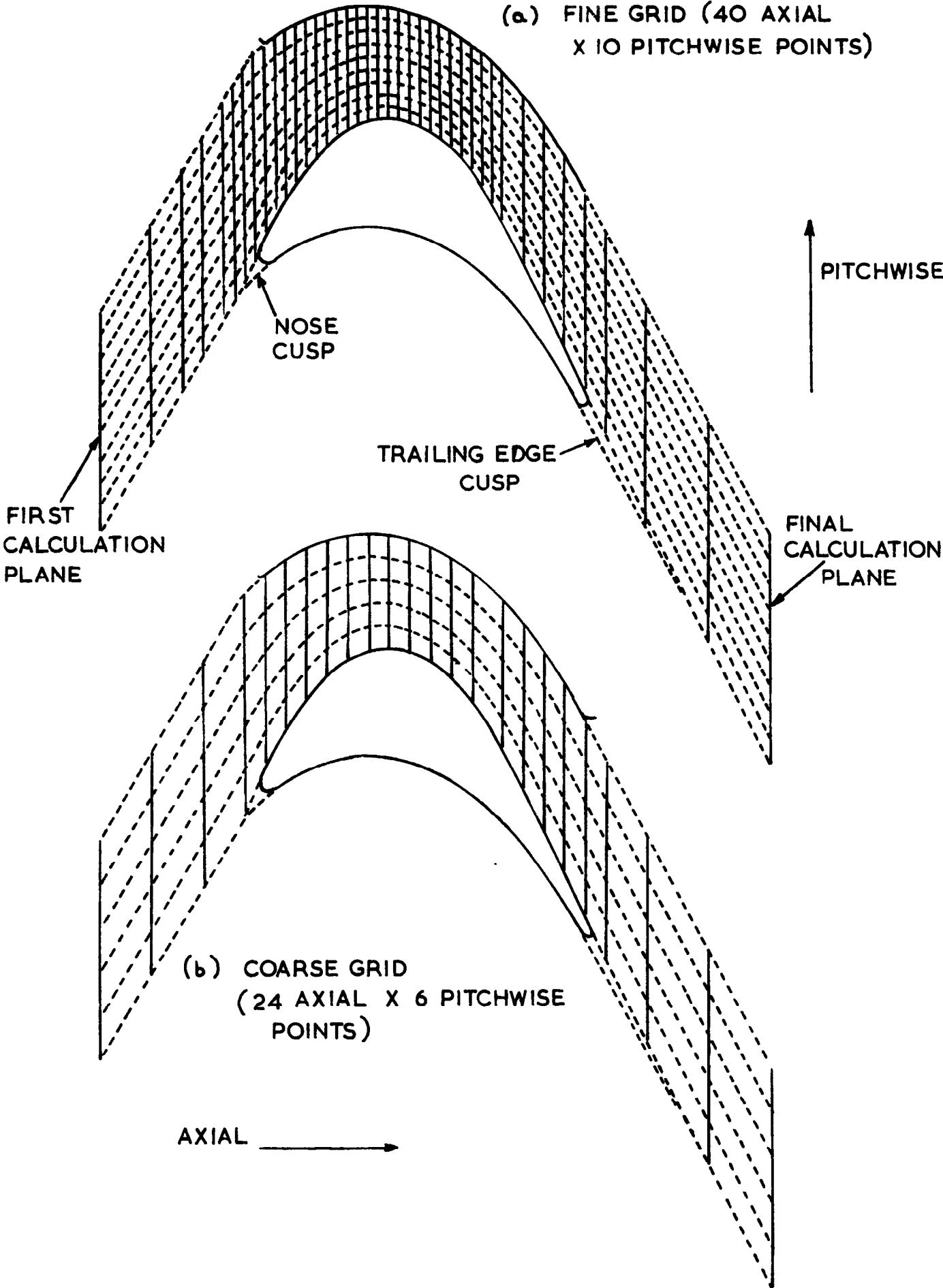
FIG. 2   TYPICAL GRID SPACING ON A TRANSONIC ROTOR BLADE
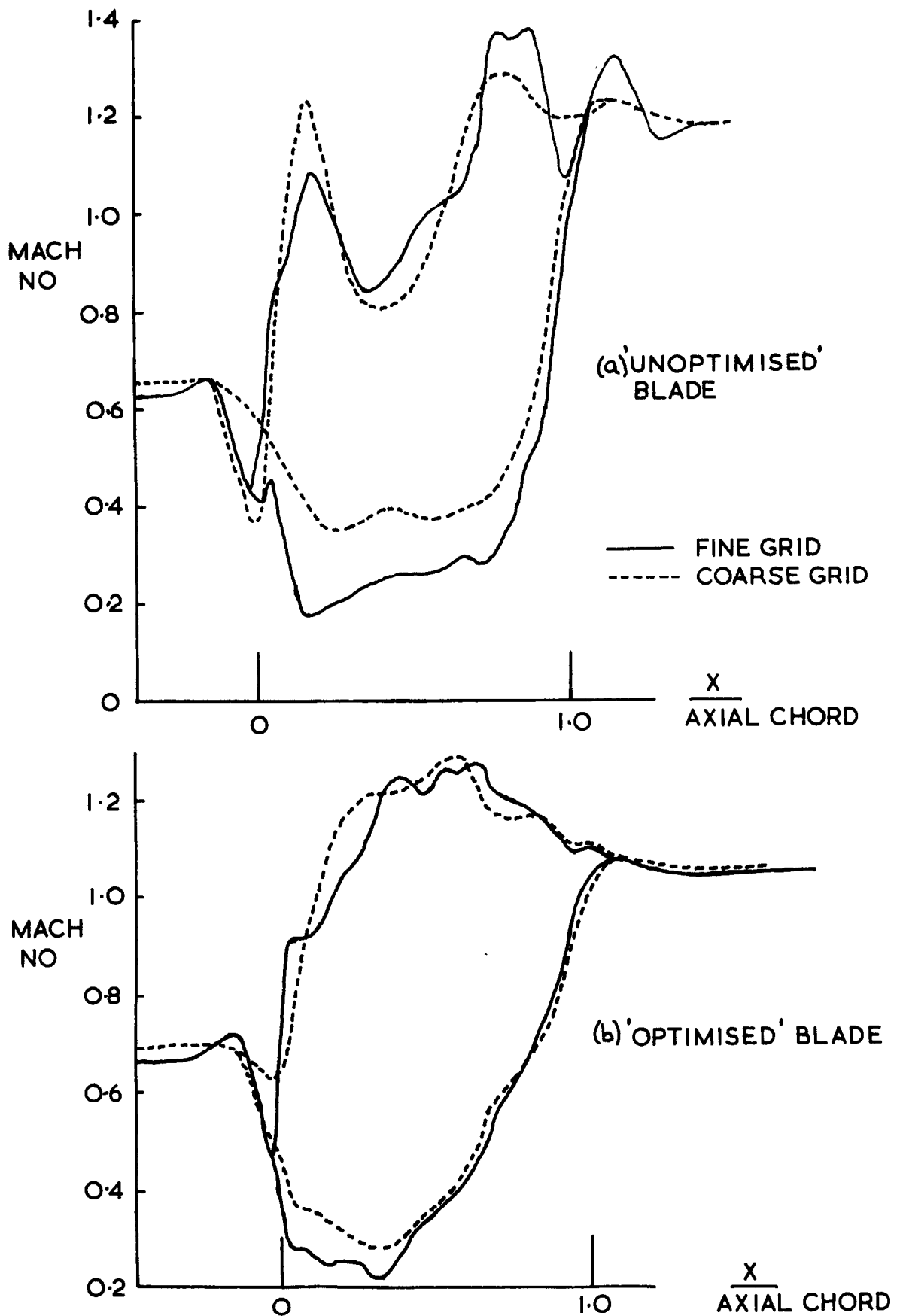
FIG. 3 TRANSONIC ROTOR BLADE SURFACE MACH NUMBER
DISTRIBUTIONS FOR FINE AND COARSE GRIDS

FIG. 4

(a) NGV ON 2D PROGRAM
40 X 10 GRID
TIME STEP 0.5

CONVERGENCE AFTER 1000 TIME STEPS .013 PER CENT

(b) TRANSONIC ROTOR ON 2D PROGRAM
TIME STEP 0.5

——— FINE GRID 40 X 10
--------- COARSE GRID 24 X 6

CONVERGENCE ON COARSE GRID .009 PER CENT

CONVERGENCE WITH FINE GRID AFTER 1000 TIME STEPS .01 PER CENT

(c) TRANSONIC ROTOR ON 3D PROGRAM
24 X 7 X 3 GRID
TIME STEP = 0.4

CONVERGENCE AFTER 575 TIME STEPS .01 PER CENT

CHANGE IN AXIAL VELOCITY V_x PERCENTAGE

TIME STEPS

FIG. 4   RATE OF CONVERGENCE

FIG. 5

SURFACE
MACH NO

1·4

1·2

1·0

0·8

0·6

0·4

0·2

0

N G V

- - - - AFTER 500
TIME STEPS

———— AFTER 1000
TIME STEPS

40 X 10 GRID

SURFACE
MACH NO

1·4

1·2

1·0

0·8

0·6

0·4

0·2

40 X 10 GRID

TRANSONIC ROTOR
BLADE

- - - - - - AFTER 500
TIME STEPS

———— AFTER 1000
TIME STEPS

BLADE AXIAL DISTANCE ⟶
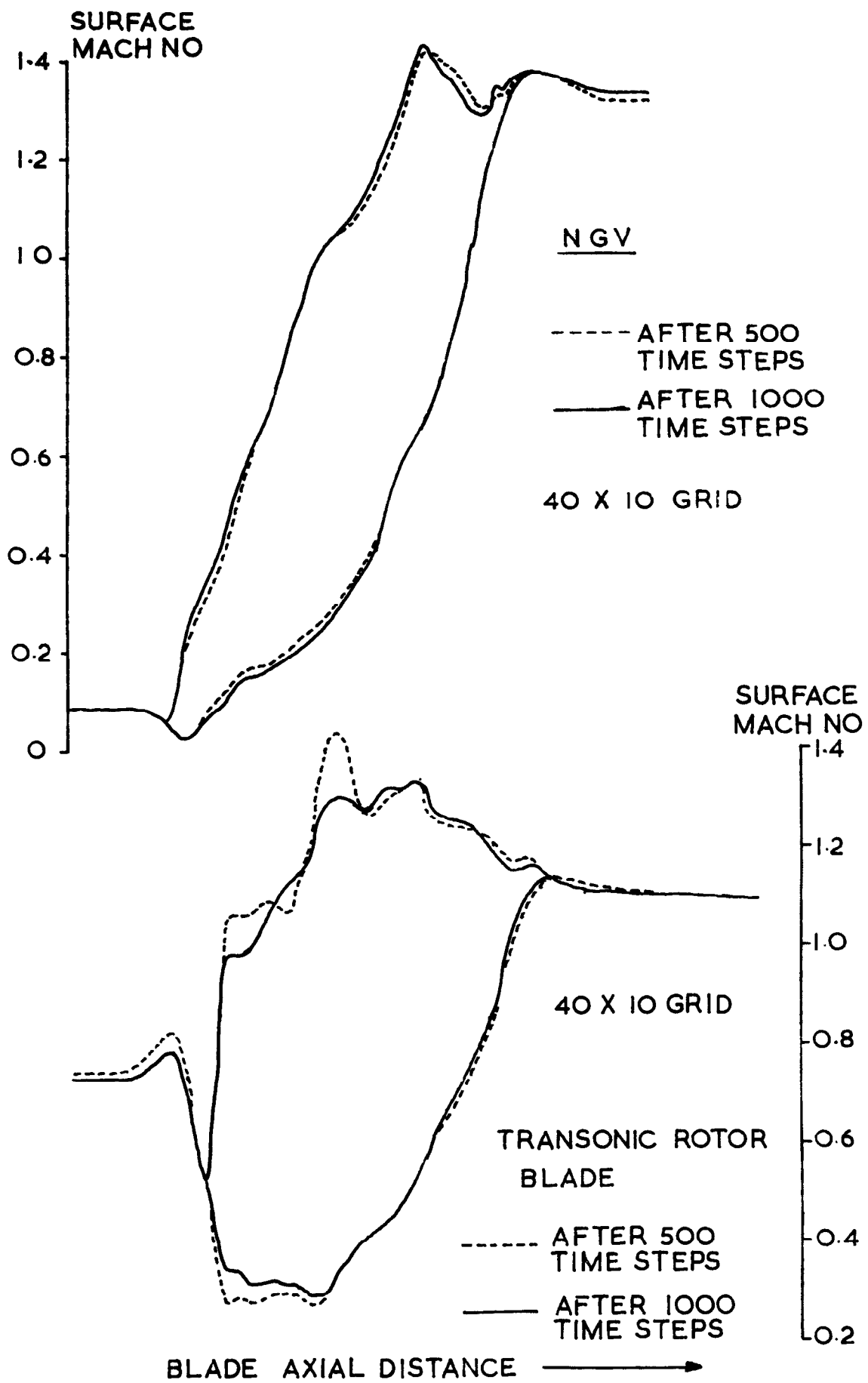
FIG. 5   BLADE MACH NUMBER DISTRIBUTIONS AFTER 500 AND
1000 TIME STEPS — 40 x 10 GRID
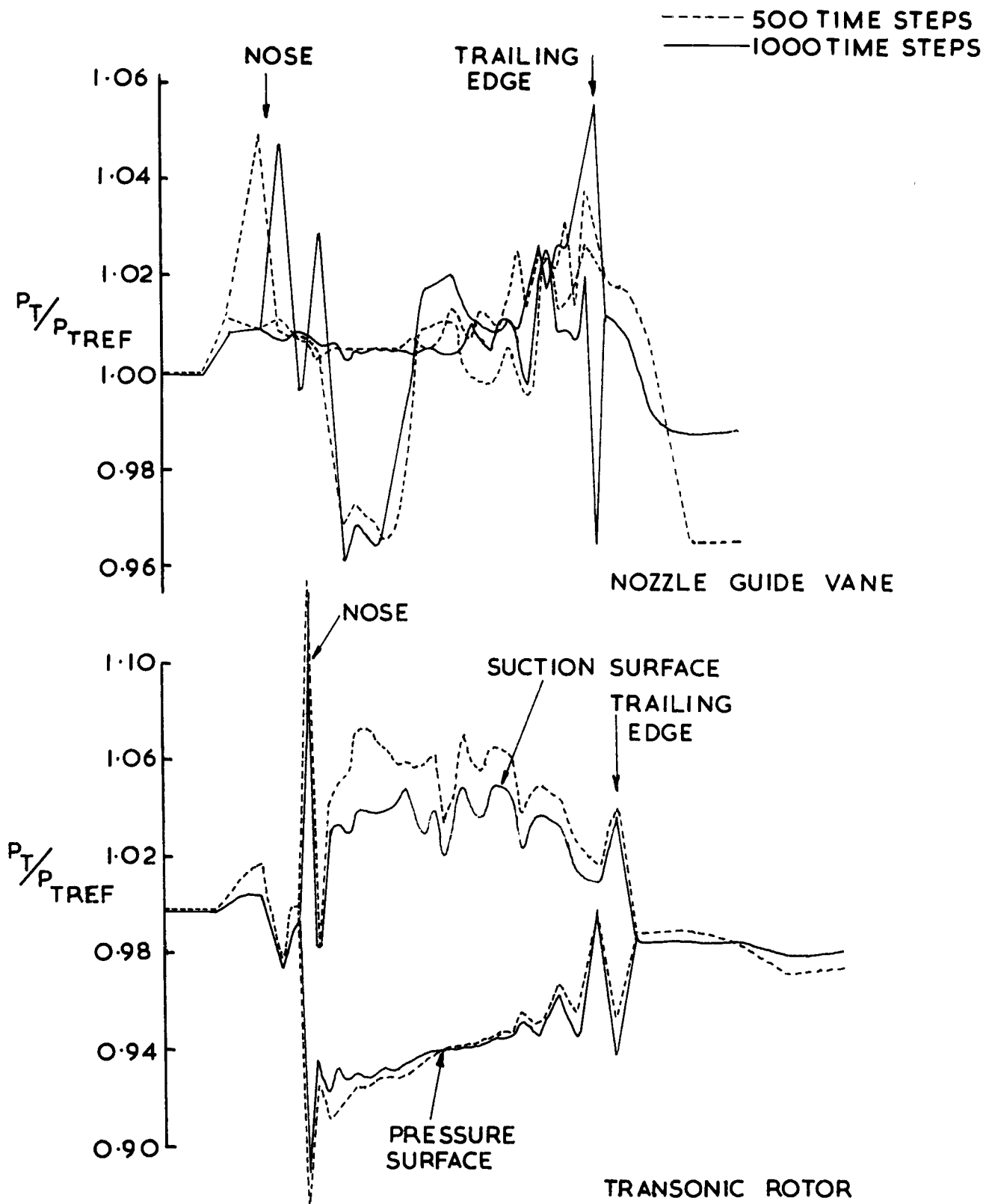
FIG. 6   TOTAL PRESSURE VARIATION IN THE BLADE CHANNEL — 2D
SOLUTION 40 x 10 GRID

FIG. 7
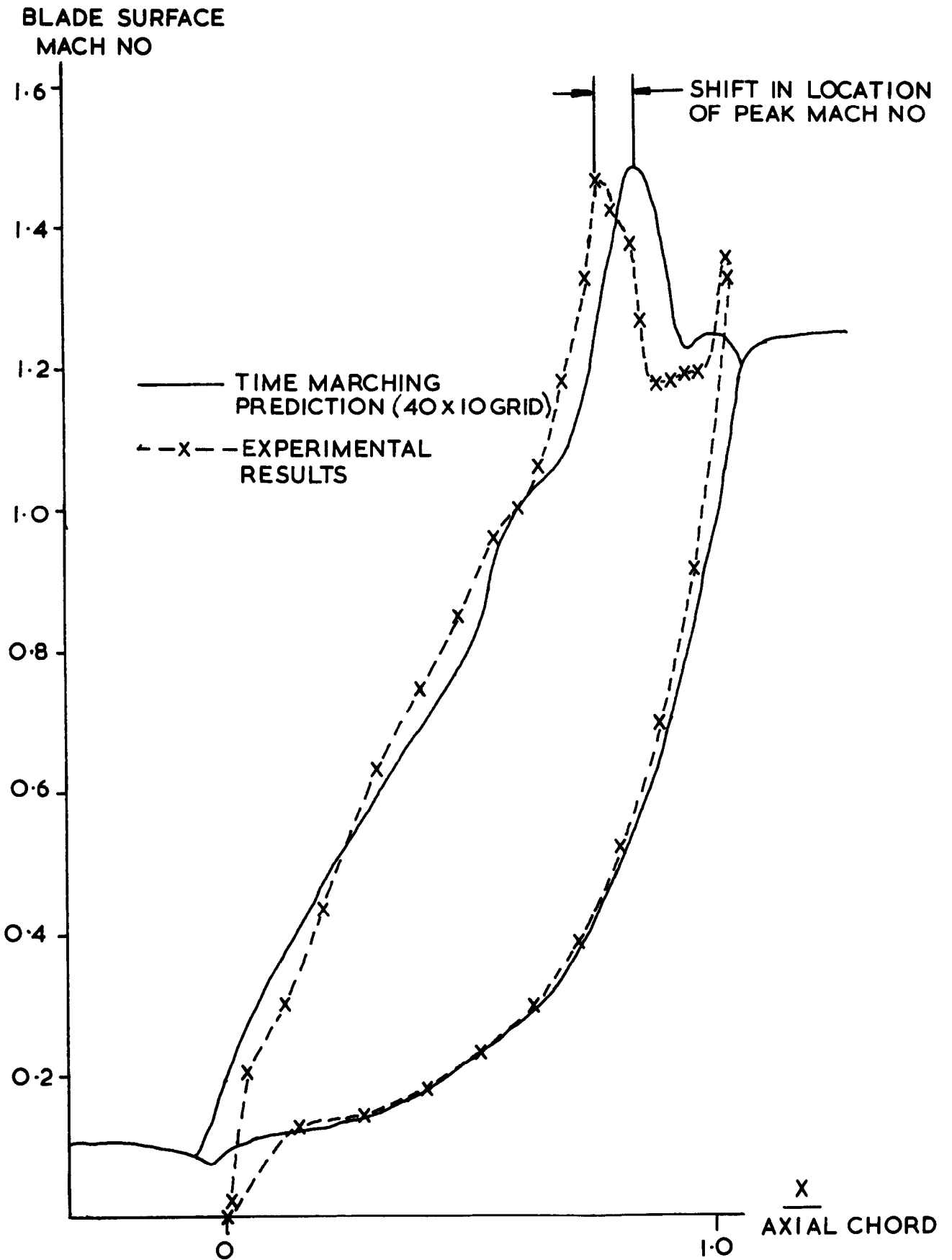
FIG. 7   TRANSONIC NGV IN A 2D CASCADE AT ZERO INCIDENCE
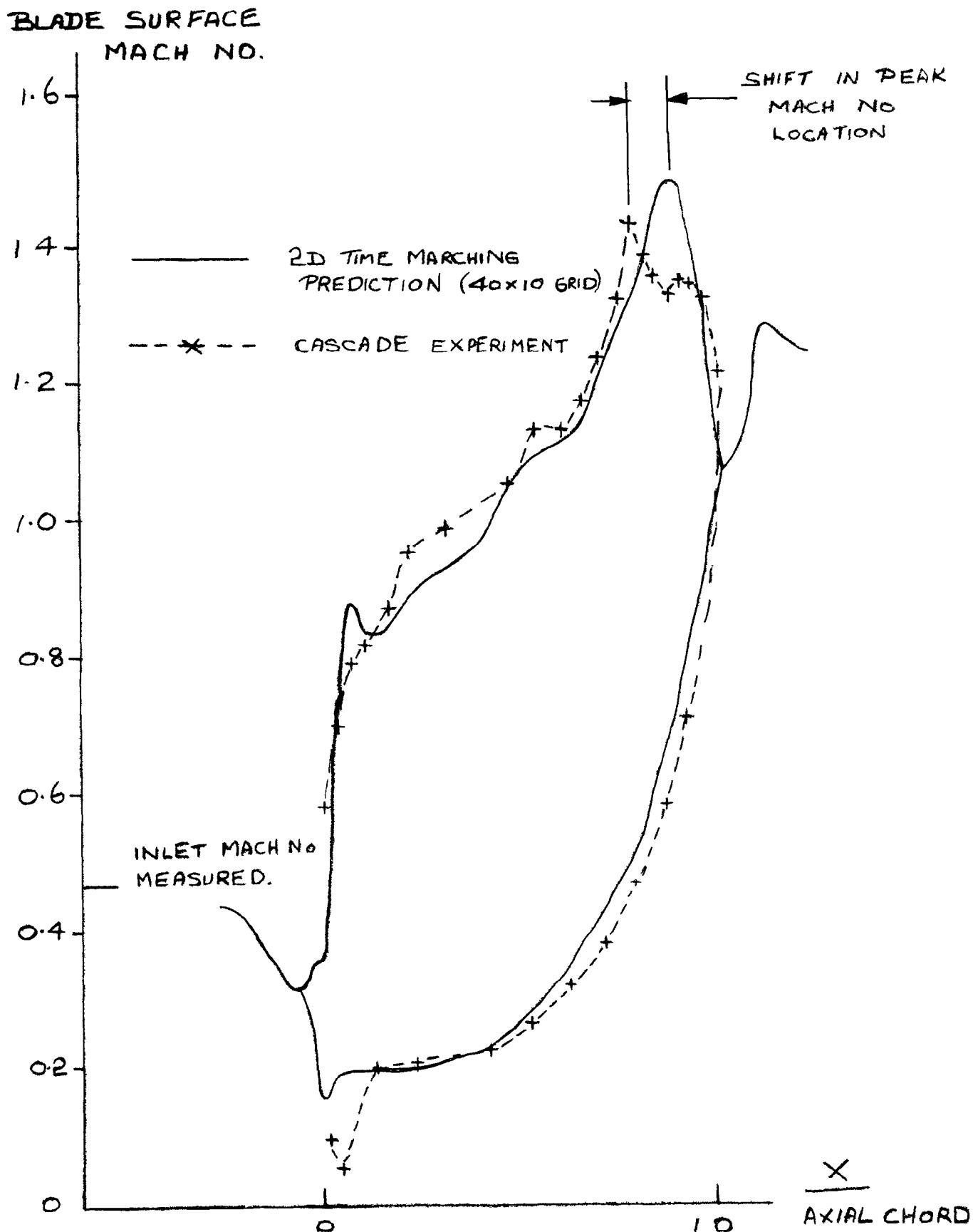
FIG. 8 TRANSONIC ROTOR BLADE IN A 2D CASCADE AT ZERO INCIDENCE

FIG. 9
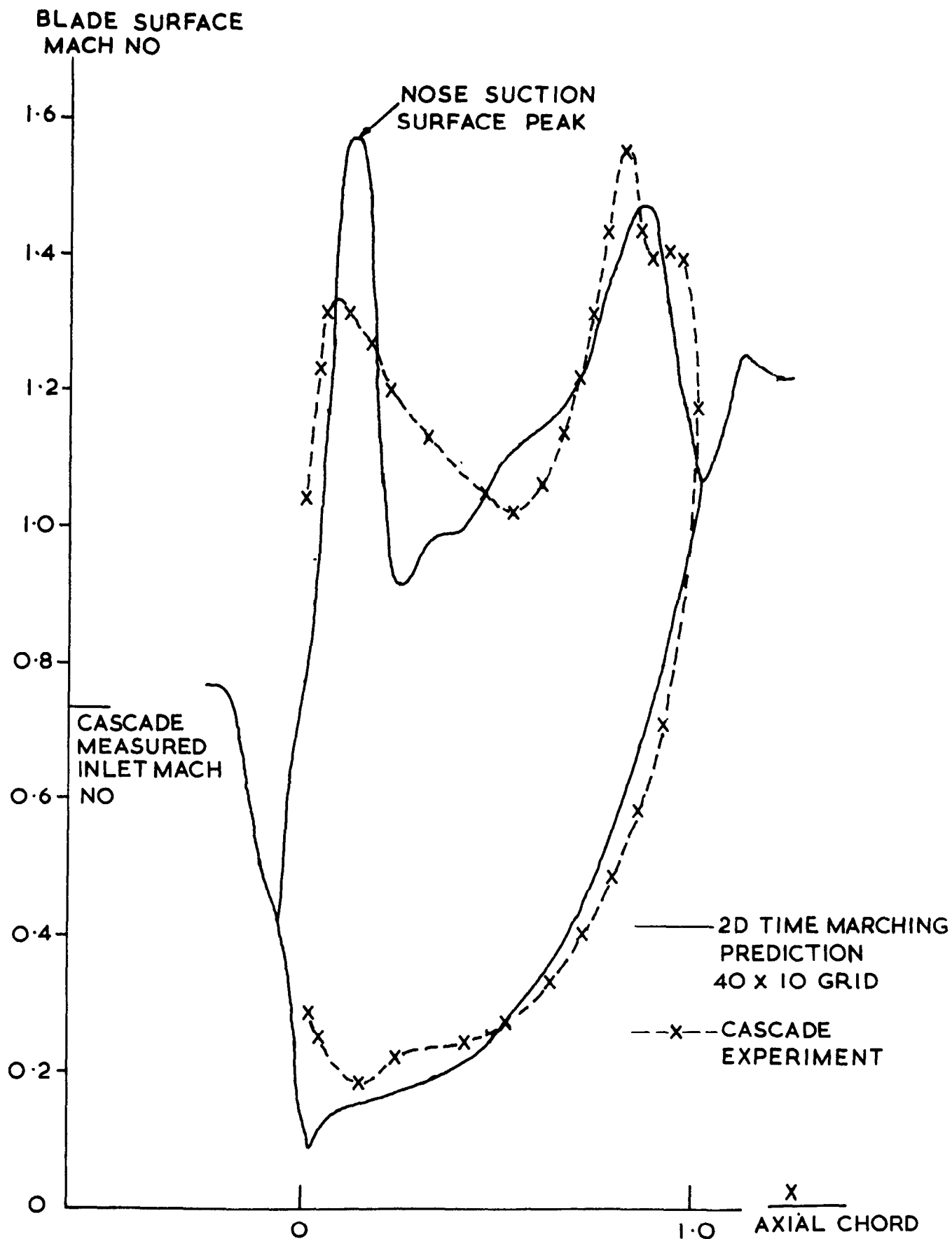
FIG. 9   TRANSONIC ROTOR BLADE IN A 2D CASCADE AT +10°
INCIDENCE

FIG. 10
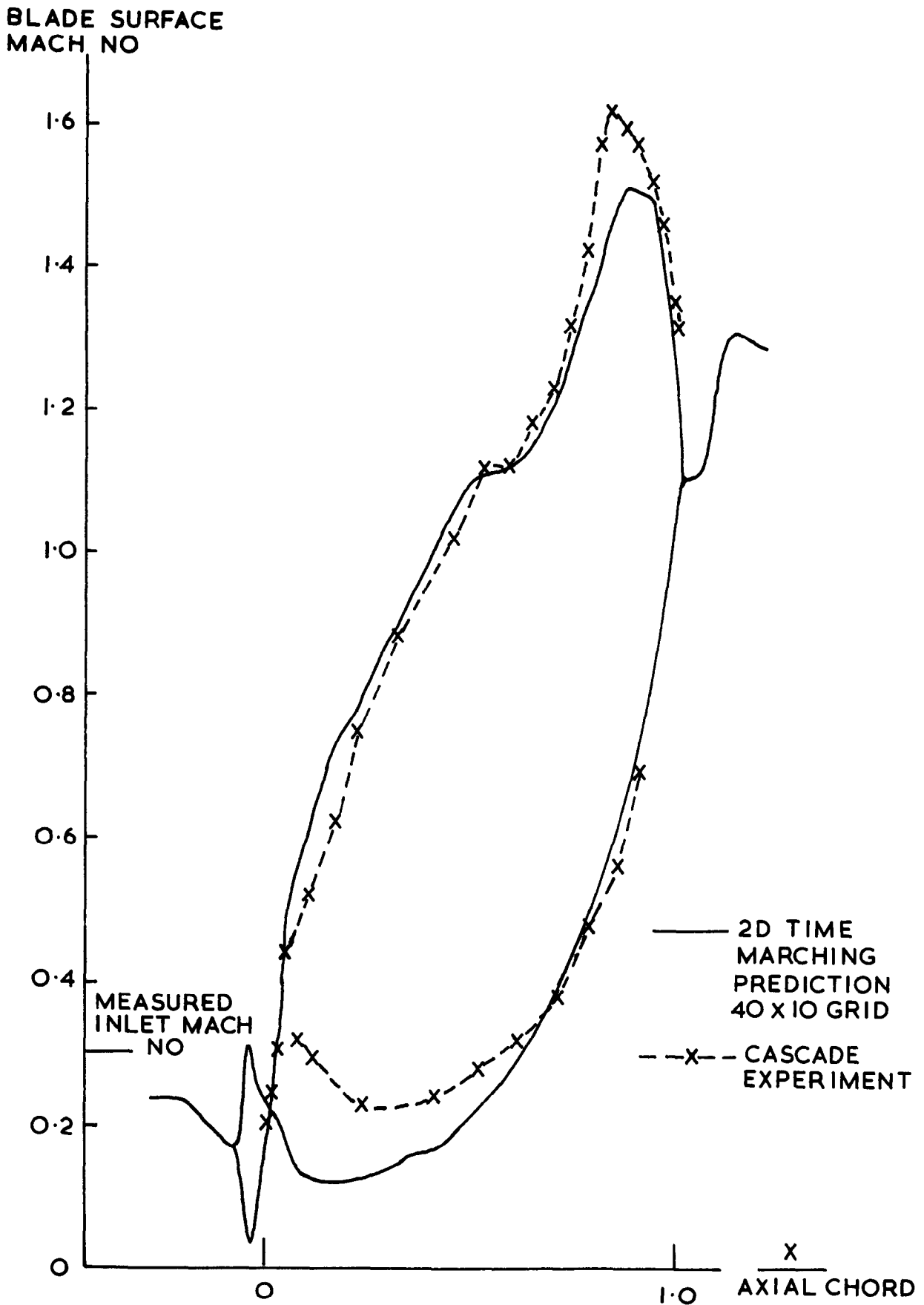
FIG. 10   TRANSONIC ROTOR BLADE IN A 2D CASCADE AT -20°
INCIDENCE

FIG. 1
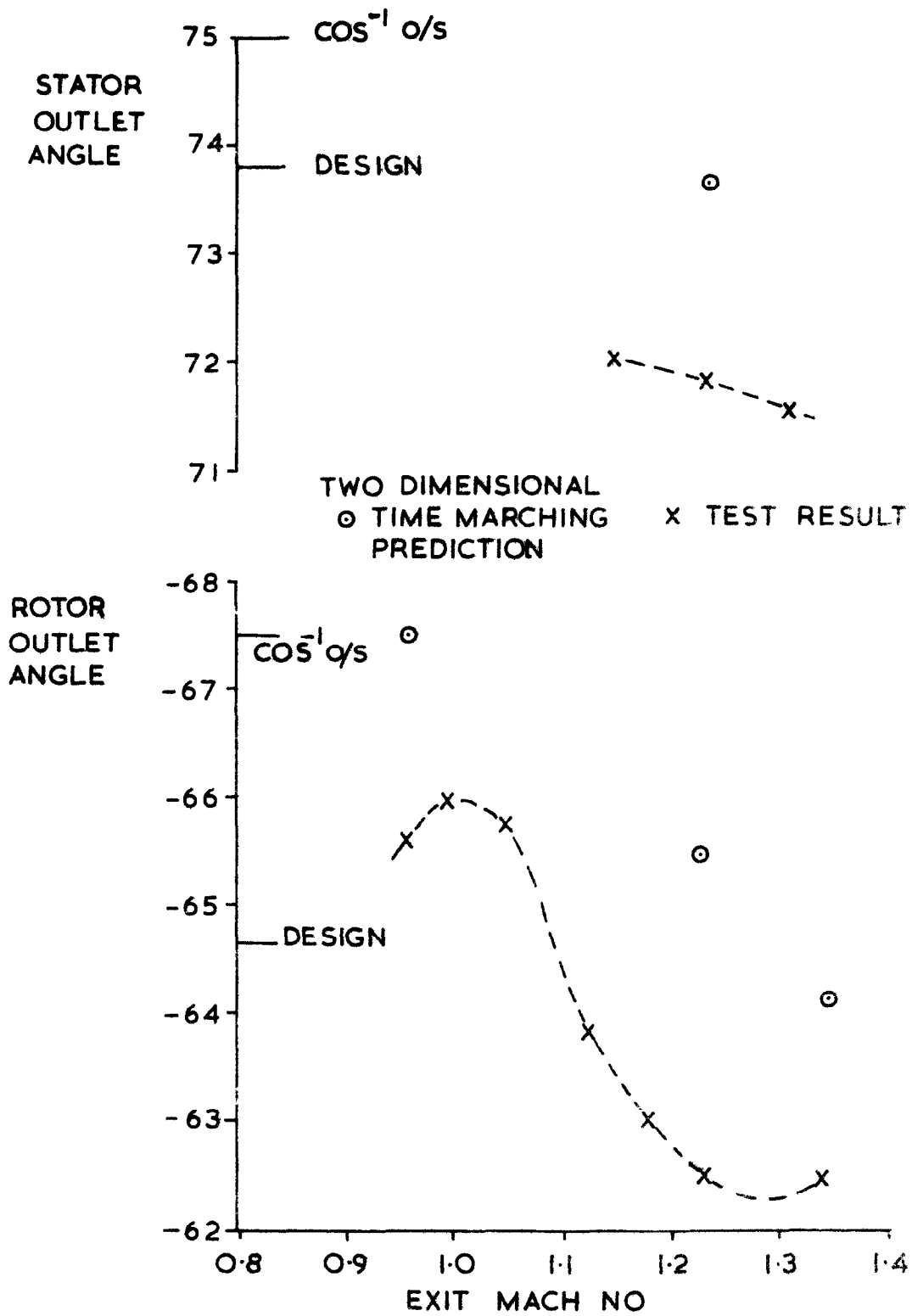
FIG. 11   COMPARISON OF PREDICTED AND EXPERIMENTAL EXIT
GAS ANGLE AT ZERO INCIDENCE

FIG. 12

BLADE
SURFACE
MACH NO

TIME MARCHING
PREDICTION
● } CASCADE
× } EXPERIMENT

LEADING EDGE   TRAILING EDGE

O   1 O   AXIAL CHORD
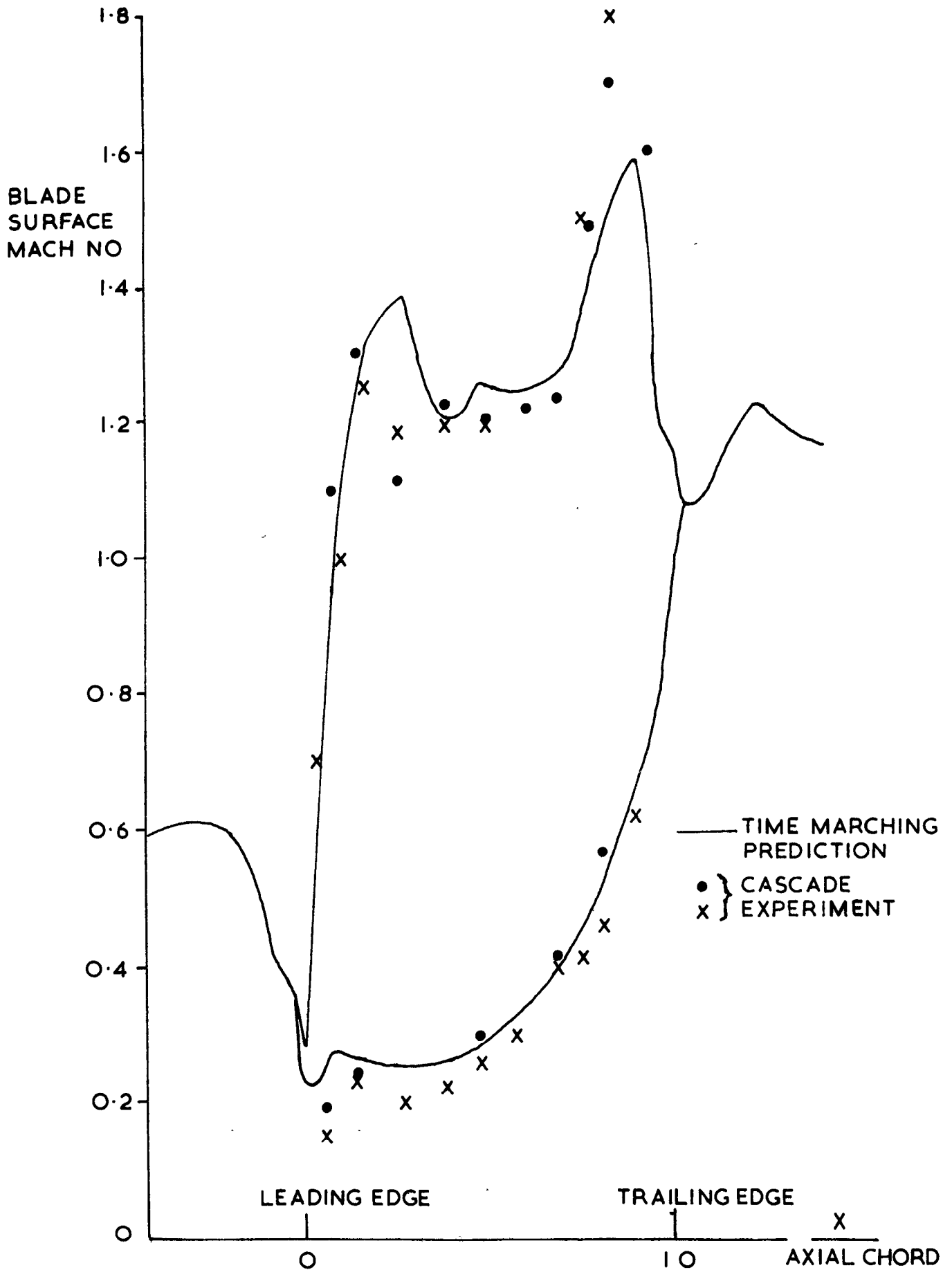
FIG. 12   TRANSONIC ROTOR BLADE CASCADE WITH WALL FLARE
           AT ZERO INCIDENCE

FIG. 13
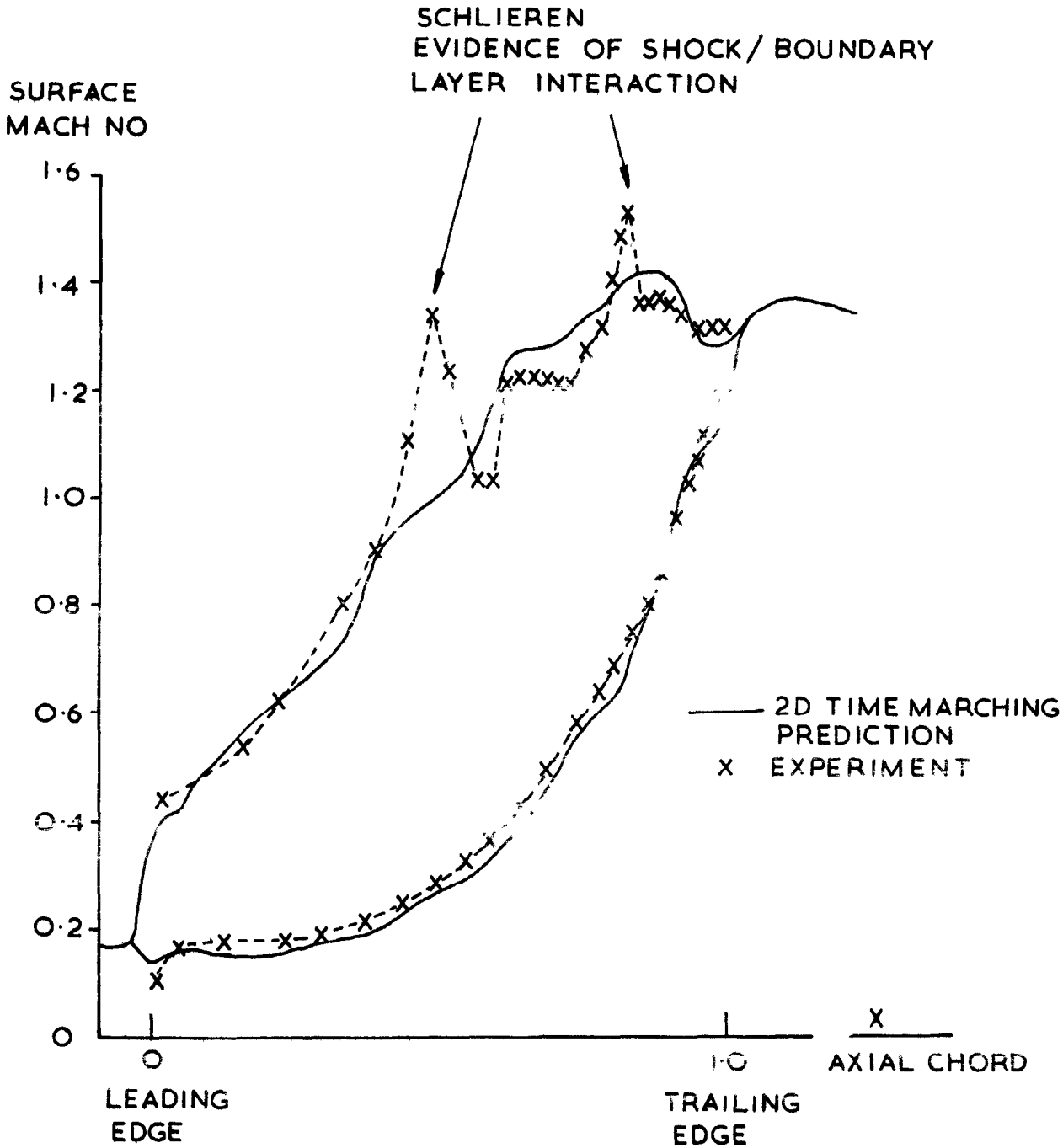
FIG. 13   CONVERGENT/DIVERGENT NGV IN A 2D CASCADE

FIG. 14

THROAT

CONVERGENT/
DIVERGENT BLADE
PASSAGE

BLADE WITH
CONVERGENT PASSAGE

STRAIGHT

CONCAVE

SURFACE
MACH NO

1·5

1·0

0·5

0

.... CONVERGENT PASSAGE
CONVEX BACK
— — — CONVERGENT PASSAGE
CONCAVE BACK
—··— CONVERGENT/DIVERGENT
PASSAGE, STRAIGHT
BACK
———— CONVERGENT/DIVERGENT
PASSAGE, CONCAVE
BACK
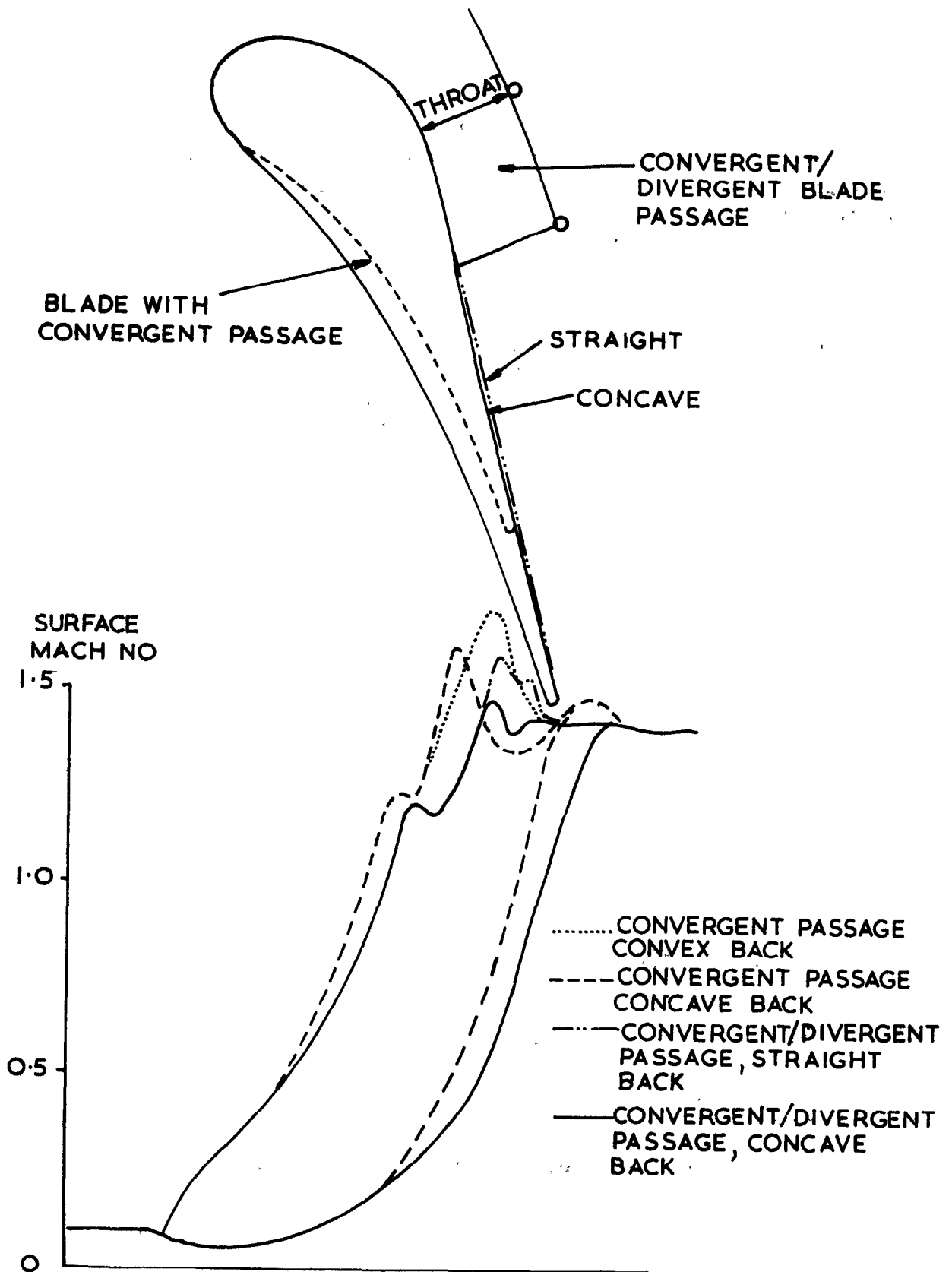
FIG. 14   NGV SURFACE MACH NUMBER DISTRIBUTIONS

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Solutions for several blade sections, giving Mach number
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DETACHABLE ABSTRACT CARDS