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The Definitions of the Angles of Incidence and of Sideslip

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The Definitions of the Angles of Incidence and of Sideslip

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SUMMARY

The use of large angles of incidence and of sideslip in missile work, and recent changes in wind tunnel testing techniques, have shown the need for clear and precise definitions of the angles of incidence and of sideslip. The suitability of different definitions for both experimental and theoretical work in both the aircraft and missile field is considered, and it is concluded that, as no single definition is universally acceptable, care should be taken in theoretical and experimental reports to define precisely the angles used.

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1 Introduction

Recent discussions on nomenclature for aerodynamic coefficients, both for aircraft[1] and for missiles, and recent changes in wind tunnel testing techniques, have shown the need for precision in the definitions of the commonly used quantities, angle of incidence and angle of sideslip. In this memorandum the merits of the two main alternative definitions are investigated, and suggestions are made as to the most suitable definitions to adopt for both experimental and theoretical work in both the aircraft and missile fields.

2 Some Definitions

To assist in making the definitions precise it is convenient to define a system of rectangular axes.

For aircraft, the x-axis will be defined as forward in the direction of some representative line, which for the sake of simplicity will be called the "chord", and which could be either the wing root chord, or wing mean chord, or wing no-lift line, or even the body datum line. In all cases the y-axis is to starboard normal to the plane of symmetry, and the z-axis is downward in the plane of symmetry, and normal to the appropriate x-axis.

For missiles, the x-axis will be defined as forward in the direction of the axis of symmetry, and the y- and z-axes as normal to the axis of symmetry is some convenient mutually perpendicular directions, such as in the planes of the "horizontal" and "vertical" wings for a cruciform missile.

We shall denote the velocity of the aircraft or missile by V, and the component velocities in the directions of the x-, y-, and z-axes by u, v, w respectively, adding some appropriate suffix to indicate particular axes where necessary.

Two definitions of incidence (or of sideslip) merit consideration. We shall use usual aircraft parlance, the phrase "plane of the wing" being defined as a plane normal to the plane of symmetry, and containing the chord (defined above). The missile worker must interpret the phrase "plane of the wing" by "plane of the horizontal wings", and the phrase "plane of symmetry" by "plane of the vertical wings": to him the chord will, of course, be the axis of symmetry.

The definitions are phrased in terms of incidence, but the corresponding phrases appropriate to sideslip are given in parentheses.

<u>Tangent Definition.</u> The following four expressions of this definition are equivalent:

(1) the angle between the chord and the projection of the line of flight on the plane of symmetry (plane of the wing).

(2) the angle between the chord and the projection of the chord on the plane containing the line of flight and the y-axis (z-axis).

(3) $\tan\alpha = \frac{w}{u}$ $\left(\tan\beta = \frac{v}{u}\right)$

(4) $\sin\alpha = \frac{w}{\sqrt{V^2-v^2}}$ $\left(\sin\beta = \frac{v}{\sqrt{V^2-w^2}}\right)$

Sine Definition. The following four expressions of this definition are equivalent:

(1) the angle between the line of flight and the projection of the line of flight on the plane of the wing (plane of symmetry).

(2) the angle between the line of flight and the projection of the chord on the plane containing the line of flight and the z-axis (y-axis).

(3) $\tan\alpha = \frac{w}{\sqrt{u^2+v^2}}$ $\qquad \left(\tan\beta = \frac{v}{\sqrt{u^2+w^2}}\right)$

(4) $\sin\alpha = \frac{w}{V}$ $\qquad \left(\sin\beta = \frac{v}{V}\right)$

The present practice with aircraft is to use the tangent definition for incidence and the sine definition for sideslip, although the British Standard Glossary of Aeronautical Terms[2] is not precise on this point. It defines the angle of incidence as:

the angle between the chord line of an aerofoil and the relative airflow.

This definition is obviously applicable only if the aerofoil is not sideslipping, and in the case of no sideslip this definition and the tangent and sine definitions are all equivalent. The angle of sideslip is defined as:

the angle between the plane of symmetry and the direction of motion of an aircraft.

This definition is equivalent to the sine definition.

3 Suitability of the Two Definitions in Various Fields of Work

3.1 Aircraft Problems

In work on the stability, control, and response of aircraft it is usual, following Bryant and Gates[3], to use wind axes and to make the assumption that the squares of the quantities v, w can be neglected, and that u is equivalent to V. The tangent and sine definitions are therefore equivalent to this order of accuracy, and either definition would, presumably, be equally acceptable in these circumstances.

3.2 Missile Problems

In missile problems one is usually concerned with larger values of the quantities v, w than with aircraft, and in this case the two definitions are different. Moreover the worker in this field is adamant that the symmetry inherent in the missile be preserved by adopting the same definition, be it the tangent or the sine, for defining the incidences of the two sets of wings.

3.3 Wind Tunnel Experiments

In this work it is convenient to choose angles that are easily applied and measured in the wind tunnel. There are two cases to be considered, according to whether the rig consists of a strut or wire support, or the newer sting support.

3.31 Strut or Wire Support Rig

The technique with the usual strut or wire support rig is to pitch the model through an angle θ about its y-axis, and then to yaw the model and balance through an angle ψ about the original z-axis. As pointed out by J.S. Thompson, the clearest way of showing the incidence and sideslip that are produced by such an operation is to visualise the spherical triangles described by the chord, or x-axis of the model. Thus, in figure 1, if O is the position of the chord initially, pitching the model through an angle θ will move the chord to P, the plane of the wing moving to E P W. If the model is now yawed about the original z-axis through an angle ψ, the chord will move to P', and the plane of the wing to E'P'W'. If OQ is a perpendicular from O to E'P'W', then the definitions of section 2 show that the angles of incidence and of sideslip are as follows:

tangent definition	-	incidence sideslip	$\alpha_t = P'O'$, $-\beta_t = P'Q$,
sine definition	-	incidence sideslip	$\alpha_s = OQ$ $-\beta_s = OO'$,

where the usual conventions have been adopted regarding signs. Elementary spherical trigonometry enables one to write down the relations connecting the angles through which the model has to be pitched and yawed with the corresponding angles of incidence and sideslip. The relations are as follows:

(i) both incidence and sideslip defined by the tangent definition

$$\theta = \alpha_t$$

$$\tan\psi = -\tan\beta_t \cdot \cos\alpha_t$$

(ii) both incidence and sideslip defined by the sine definition

$$\sin\theta = \sin\alpha_s \cdot \sec\beta_s$$

$$\psi = -\beta_s$$

(iii) incidence defined by the tangent definition; sideslip by the sine definition

$$\theta = \alpha_t$$

$$\psi = -\beta_s$$

The simplicity of the relations in case (iii) is no doubt one of the reasons why incidence and sideslip have been defined in this manner with aircraft up to now. However, if it is essential that both incidence and sideslip be defined by the same definition, as it is in missile problems (see section 3.2), then to have them defined by the tangent definition would appear to be preferable to the sine definition. The use of the sine definition (case (ii) above) would mean that the pitch of the model would have to be adjusted at every sideslip in the usual lateral test of an aircraft model at constant incidence; but with the tangent definition (case (i) above) no such adjustment would be necessary, although there would be the less troublesome complication that the angle through which the model must be yawed to produce a given angle of sideslip would vary with incidence.

So far we have investigated what combinations of definitions lead to angles of incidence and of sideslip that can easily be applied in the wind tunnel. But it is also of importance to investigate what angles can easily be measured, for the incidence and sideslip of a wind tunnel model alter under load, and it is usual to measure them, and adjust them if necessary, at each angle of incidence and of sideslip. A common technique is to have a line on the model parallel to the chord, and to measure the angles of "pitch" θ', and of "yaw" ψ', of this line as seen by telescopes situated at the side of, and above, the tunnel. In terms of the true angles of pitch θ, and of yaw ψ, the angles θ' and ψ' are given by

$$\tan\theta' = \tan\theta \cdot \sec\psi,$$

$$\psi' = \psi .$$

We note that the angle θ' is not equivalent to either α_t or α_s, and thus neither the tangent nor the sine definition leads to an angle of incidence that is readily measurable. The sine definition, however, leads to an angle of sideslip that can be measured directly.

3.32 Sting Support Rig

The technique with the usual sting support rig is to pitch the model through an angle θ about its y-axis, and then to roll the model through an angle ϕ about the new position of the x-axis. Alternatively the model may be rolled through an angle ϕ about its x-axis, and then pitched through an angle θ about the original y-axis. Following Thompson we consider the spherical triangles described by the chord, or x-axis, of the model. Thus, in figure 2, if O is the position of the chord initially, pitching the model through an angle θ will move the chord to P, the plane of the wing moving to E P W. If the model is now rolled about the x-axis through an angle ϕ, the plane of the wing will move to E'P W' and the plane of symmetry to N'P S'. If OQ and OR are perpendiculars from O to N'P S' and E'P W' respectively, then the definitions of section 2 show that the angles of incidence and of sideslip are as follows:

tangent definition	-	incidence	$\alpha_t = PQ$,
		sideslip	$\beta_t = PR$,
sine definition	-	incidence	$\alpha_s = OR$,
		sideslip	$\beta_s = OQ$.

Elementary spherical trigonometry enables one to write down the relations connecting the angles through which the model has to be pitched and rolled with the corresponding angles of incidence and sideslip. The relations are as follows:

(i) both incidence and sideslip defined by the tangent definition

$$\tan^2\theta = \tan^2\alpha_t + \tan^2\beta_t$$

$$\tan\phi = \frac{\tan\beta_t}{\tan\alpha_t}$$

(ii) both incidence and sideslip defined by the sine definition

$$\sin^2\theta = \sin^2\alpha_s + \sin^2\beta_s$$

$$\tan\phi = \frac{\sin\beta_s}{\sin\alpha_s}$$

(iii) incidence defined by the tangent definition; sideslip by the sine definition

$$\cos\theta = \cos\alpha_t \cdot \cos\beta_s$$

$$\tan\phi = \frac{\tan\beta_s}{\sin\alpha_t}$$

Although none of these relations is as simple as those in case (iii) of section 3.31 the relations in case (iii) above are again somewhat simpler than in the other cases, although there is a symmetry about those in which the incidence and sideslip are defined by the same definition.

3.4 Theoretical Work

In theoretical work symbols for the non-dimensional values of the translatory velocity components $\frac{u}{V}, \frac{v}{V}, \frac{w}{V}$, are required. If the sine definition is adopted for both incidence and sideslip $\sin\beta$ and $\sin\alpha$ can be used to represent the second and third of these quantities. On the other hand, in most linear theory work, it is usual to consider first the flow field due to a uniform stream along the direction of the x-axis, u say, and then to superimpose in turn flow fields corresponding to uniform cross streams such as w and v. In such work the quantities $\frac{w}{u}$ and $\frac{v}{u}$ assume significance, and as in the tangent definition these are equivalent to $\tan\alpha$ and $\tan\beta$ respectively, there appears to be a case for this definition. However, in the linear theory the squares of the quantities v, w are usually neglected, and in these circumstances the two definitions are of course, equivalent.

3.5 General Considerations regarding Change of Axes

It often happens that one knows the forces and moments on an aircraft or missile in terms of, say, body axes (body datum line axes), and one wishes to find the forces and moments in terms of wing axes (wing root chord axes, say). It will clearly be convenient if the difference in incidence in these two cases is equal to the angle between the wing root chord and the body datum line. Let this angle be i, and let suffix B denote quantities referred to body datum line axes and suffix W quantities referred to wing root chord axes. Then we have

$$\left.\begin{aligned} u_W &= u_B \cos i - w_B \sin i \\ v_W &= v_B \\ w_W &= w_B \cos i + u_B \sin i \end{aligned}\right\} \qquad (1)$$

With the tangent definition we have

$$\tan \alpha_B = \frac{w_B}{u_B} ,$$

$$\tan \alpha_W = \frac{w_W}{u_W} ,$$

and therefore

$$\tan (\alpha_B + i) = \frac{w_B \cos i + u_B \sin i}{u_B \cos i - w_B \sin i} ,$$

which using the relations (1) gives

$$\alpha_B + i = \alpha_W .$$

With the sine definition, however, we have

$$\sin \alpha_B = \frac{w_B}{V} ,$$

$$\sin \alpha_W = \frac{w_W}{V} ,$$

and therefore

$$\sin (\alpha_B + i) = \frac{w_B}{V} \cos i + \frac{\sqrt{u_B^2 + v_B^2}}{V} \sin i,$$

which, from the relations (1), only yields the result

$$\alpha_B + i = \alpha_W,$$

when

$$v_B = 0.$$

In this respect therefore the tangent definition is preferable.

4 Discussion and Conclusions

From the arguments put forward in section 3 it would appear that either definition would be suitable for aircraft or missile work, but that the missile worker insists that the same definition be adopted for both incidence and sideslip. In wind tunnel work there is a lot to be said for a combination in which incidence is defined by the tangent definition, and sideslip by the sine definition, as is current aircraft practice. However, for missiles, where the same definition for both incidence and sideslip is essential, the tangent definition offers advantages over the sine definition. In theoretical work a case can be made for either the tangent definition or the sine definition for both incidence and sideslip, but the tangent definition leads to particularly neat formulae for a change of axes.

As neither definition appears to be universally acceptable, and as each has its field of application, it is recommended that the definitions most suitable to the problem be adopted in any context, but that care be taken to define precisely the angles used.

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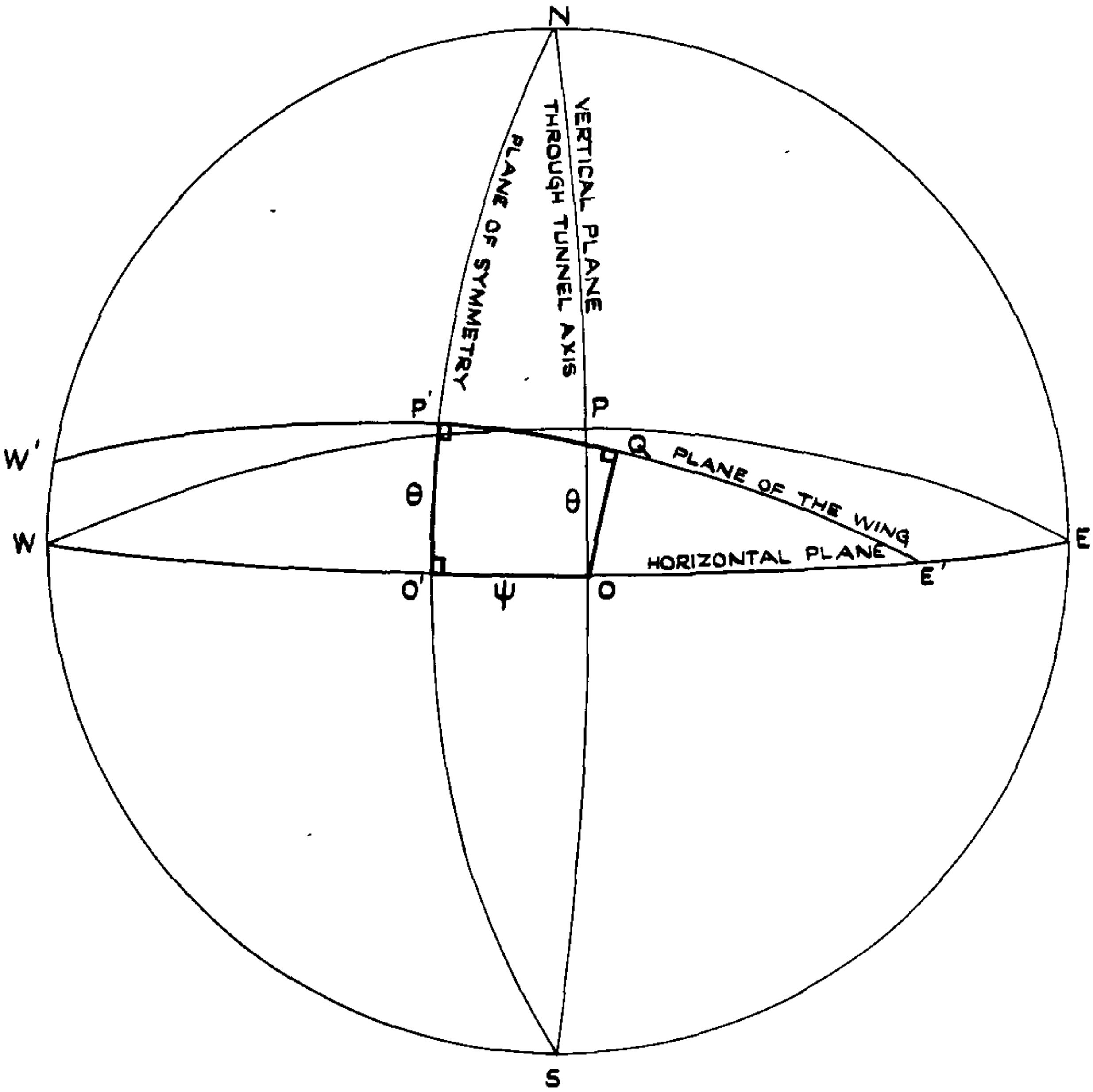
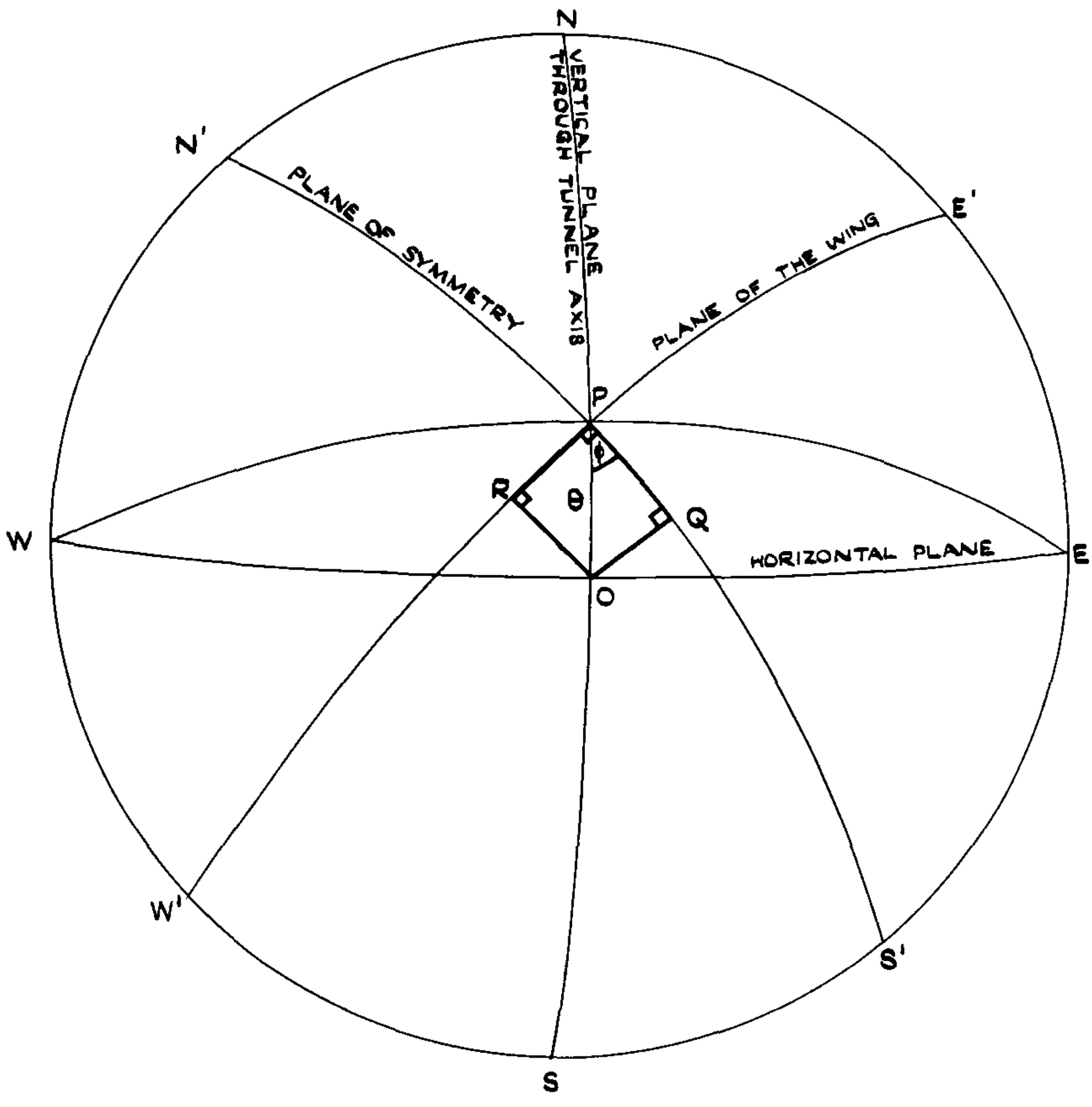


TANGENT DEFINITIONS. { INCIDENCE α_t = P'O' ; SIDESLIP $-\beta_t$ = P'Q }

SINE DEFINITIONS. { INCIDENCE α_s = OQ ; SIDESLIP $-\beta_s$ = OO' }

FIG. 1. ANGLES WITH A STRUT OR WIRE SUPPORT RIG.

FIG. 2.



TANGENT DEFINITIONS. { INCIDENCE α_t = PQ
SIDESLIP β_t = PR.

SINE DEFINITIONS. { INCIDENCE α_s = OR.
SIDESLIP β_s = OQ.

FIG. 2. ANGLES WITH A STING SUPPORT RIG.

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