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by

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NUMERICAL METHODS FOR CALCULATING UNSTEADY FLOWS IN SUBSONIC AND SUPERSONIC TURBOMACHINERY CASCADES

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SUMMARY

Two numerical methods are proposed for the prediction of unsteady fluid flows in turbomachinery cascades; they both assume that the unsteady effects are linear and sinusoidal in time, and are superimposed on a previously determined, non-uniform steady flow-field. In the first method an unsteady velocity potential is used, whilst in the second the unsteady velocity components and pressure and density perturbations are used in either a differential or integral representation of the unsteady Euler equations of motion.

The methods may be used to predict blade flutter effects and the acoustic field generated by interacting rows of blades; they may be generalised to deal with three-dimensional flows although only two-dimensional flows are considered here.

Some preliminary results for the case of a flat plate cascade operating at zero mean incidence at subsonic and supersonic inlet Mach numbers are presented and compared with existing analytic and numerical solutions. In general the agreement is very good for subsonic cascades; for supersonic difficulty was encountered when trying to enforce the cascade repeat condition.

*Replaces A.R.C.36 372

CONTENTS

FIGURES

NOTATION

A	Area
c	Velocity of sound; blade chord
E	Energy $p^{\gamma-1} + \frac{1}{2}\rho(u^2+v^2)$
F	Vector in equation (11)
G	Vector in equation (11)
h	Cascade dimension (see Figure 1); displacement normal to surface
j	$\sqrt{-1}$
L	Lift on blade (per unit span)
M	Mach number W/c; moment about mid chord (per unit span)
n	Direction normal to a streamline
o	Cascade dimension (see Figure 1)
p	Static pressure
ΔQ	Inter blade phase angle
r	Acoustic mode index; mesh size ratio
s	Blade spacing
t	Time
u	Velocity in the x direction
U	Steady uniform velocity in the x direction
v	Velocity in the y direction
V	Steady uniform velocity in the y direction
w	Wake direction
W	Velocity in the wake direction; vector in equation (11)
x,y	Cartesian co-ordinates
X	x direction mass flux ρu
Y	y direction mass flux ρv
α,β	Wave numbers - equation (27)
α	Relaxation factor; unsteady incidence angle
γ	Ratio of specific heats
ρ	Density
ξ	Vorticity
ϕ	Velocity potential
ω	Circular frequency
C_L	Lift coefficient $L/\frac{1}{2}\rho_o U\overline{v}_n c$
C_M	Moment coefficient $M/\frac{1}{2}\rho_o U\overline{v}_n c^2$
C_P	Pressure coefficient $\Delta p'/\frac{1}{2}\rho_o U\overline{v}_n$

Subscripts

i	Imaginary
n	Normal to blade surface
o	Steady state
r	Real
s	Along blade surface

NOTATION (Continued)

Superscripts

$\bar{\ }$	Amplitude
$'$	Unsteady part

1. INTRODUCTION

Early methods of computing the steady flow through cascades of aerofoils were based on replacing the blades by distributions of source, doublet or vortex singularities and solving an integral relationship to satisfy the prescribed boundary conditions. These methods have been superseded by field methods based on finite difference or finite element techniques and solutions to three-dimensional and transonic steady flows are readily obtainable. Only recently has the equivalent step from singularity to field methods been taken in the case of unsteady flow. Previously it was possible to calculate flows with only a very simple geometry and, for example, the effect of steady blade loading in an unsteady, compressible flow was unknown. Field methods, based on finite difference techniques, overcome all these difficulties, though often at the expense of increased computing time.

The methods proposed here are based on established steady flow solution procedures and it is expected that their development will be along similar lines to recent work on steady flows. In each case the disturbances are assumed to be harmonic and small compared with the steady flow, and to possess a magnitude and phase with respect to some datum. The linearisation may not be valid near the sonic speed. Complex notation is used to describe the disturbances, and it is found that the equations relating the unsteady perturbations are, in general, complex. These may be separated into real and imaginary parts (or, in engineering terms, in-phase and out-of-phase components). In the sections that follow the governing equations are derived and methods of solution suggested; the boundary conditions, both within the blade rows and far upstream and downstream are discussed, and finally some examples are given and the results compared with existing analytic and numerical solutions.

2. COMPLEX REPRESENTATION OF THE UNSTEADY PERTURBATIONS

In the work that follows the dependent variables will be assumed to consist of a steady part (subscript o) and a small, harmonic, unsteady perturbation (superscript $'$) oscillating with fixed angular frequency ω, thus

$$\phi(x,y) = \phi_o(x,y) + \phi'(x,y)\, e^{j\omega t} \quad \text{etc.} \tag{1}$$

The quantity ϕ' represents both the magnitude and phase of the disturbance and may be regarded as complex. Equation (1) may be substituted into the appropriate, non-linearised equations of motion; after elimination of second order terms in $(\phi')^2$ etc, and the known steady state terms which separately satisfy the steady form of those equations, the term $e^{j\omega t}$ cancels everywhere. The remaining terms then represent a linear complex set of equations for the (complex) perturbation quantities; it may be shown that satisfying both real and imaginary parts of these equations separately is identical to satisfying the relationships between in- and out-of-phase quantities for all t.

3. VELOCITY POTENTIAL METHOD

3.1 Governing Equations

The unsteady compressible flow of a perfect gas may be described by the equation for velocity potential ϕ (see Shapiro, 1953)

$$\phi_{xx}\{1-\frac{\phi_x^2}{c^2}\} + \phi_{yy}\{1-\frac{\phi_y^2}{c^2}\} - 2\{\frac{\phi_{xt}\phi_x+\phi_{yt}\phi_y}{c^2}\} - 2\frac{\phi_x\phi_y\phi_{xy}}{c^2} - \frac{\phi_{tt}}{c^2} = 0 \qquad (2)$$

where the velocities in the x and y directions are given by

$$u = \frac{\partial\phi}{\partial x} \equiv \phi_x \qquad (3)$$

$$v = \frac{\partial\phi}{\partial y} \equiv \phi_y \qquad (4)$$

and c, the velocity of sound is related to the stagnation velocity of sound, c_t, by

$$c^2 = c_t{}^2 - \frac{\gamma-1}{2}\{\phi_x^2+\phi_y^2\} - (\gamma-1)\frac{\partial\phi}{\partial t} \quad . \qquad (5)$$

Linearising equations (2) and (5) by substituting equation (1), a linear, complex equation in the perturbation velocity potential $\phi'(x,y)$ is obtained

$$\begin{aligned}
&\phi'_{xx}\{1-M_x^2\} + \phi'_{yy}\{1-M_y^2\} + \frac{\omega^2}{c_o{}^2}\phi' \\
&- \frac{\phi'_x}{c_o{}^2}\{(\gamma+1)\ u\frac{\partial u}{\partial x} + (\gamma-1)\ u\frac{\partial v}{\partial y} + 2v\frac{\partial v}{\partial x}\} \\
&- \frac{\phi'_y}{c_o{}^2}\{(\gamma+1)\ v\frac{\partial v}{\partial y} + (\gamma-1)\ v\frac{\partial u}{\partial x} + 2u\frac{\partial u}{\partial x}\} \qquad (6) \\
&- \frac{\phi'_{xy}}{c_o{}^2}\{2uv\} - j\omega\frac{\phi'_x}{c_o{}^2}\{2u\} - j\omega\frac{\phi'_y}{c_o{}^2}\{2v\} \\
&- j\omega\frac{\phi'}{c_o{}^2}\{(\gamma-1)(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})\} = 0
\end{aligned}$$

where u and v are obtained from a known steady state solution for ϕ_o, and M_x and M_y are the local Mach numbers in the x and y directions.

3.2 Method of Solution

Solutions to equation (6) may be obtained by replacing the derivatives ϕ'_x, ϕ'_{xx} etc. by finite difference approximations, care being taken near blade boundaries to ensure high order approximations to the normal derivative of ϕ'. In subsonic flow, the resulting set of linear equations may then be solved by any of a number of techniques including Gaussian elimination, over-relaxation and dynamic relaxation (Rushton (1973)), provided the reduced frequency $\omega c/U$ is low. At high reduced frequencies, or more specifically when the physical length of the computational region exceeds half a wavelength of the disturbance, the latter two iterative techniques will not converge and Gaussian elimination must be used. Provided care is taken in making full use of the banded nature of the set of linear equations this may be done economically and is, in general, very reliable.

In supersonic problems, the hyperbolic nature of the equations means that an approach closer to the traditional method of characteristics is necessary to ensure that the domain of dependence of the numerical schemes does not exceed the domain of dependence of the real problem. This is ensured by basing the finite difference grid on the characteristic lines

$$\frac{dy}{dx} = \pm(M^2 - 1)^{-\frac{1}{2}} \tag{7}$$

and marching the solution downstream (Figure 1). When the flow is uniform, equation (6) becomes

$$(M^2-1)\ \phi'_{xx} - \phi'_{yy} - \frac{\omega^2\phi'}{c_o^2} + \frac{2Uj\omega}{c_o^2}\phi'_x = 0 \tag{8}$$

and the absence of the cross derivative ϕ_{xy} simplifies the method since the equations may be solved explicitly. In general, and for reasons associated with setting up a suitable grid in the entrance region of the cascade, the x-direction grid spacing Δx may not be chosen independently of Δy. Provided the ratio

$$r = \frac{\Delta x}{\Delta y . (M^2-1)^{\frac{1}{2}}} \tag{9}$$

is greater than about 0.9, however, the effect on the lift and moment coefficients of the blades is small, although some smoothing of wavefronts does occur, and slight numerical instabilities may be present. If r exceeds 1.0, of course, the domain of dependence condition is violated and no solution may be obtained.

Finally, the blade pressure distribution may be obtained from

$$p' + u\frac{\partial\phi'}{\partial x} + v\frac{\partial\phi'}{\partial y} + j\omega\,\phi' = 0 \tag{10}$$

whereupon the blade lift and moment coefficients may be calculated.

4. HARMONIC TIME MARCHING METHOD

4.1 Governing Equations

The Euler equations of motion may be written in conservation form

$$\frac{\partial W}{\partial t} = \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} \tag{11}$$

where

$$W = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ E \end{bmatrix} \qquad F = \begin{bmatrix} -\rho u \\ -\rho u^2 - p \\ -\rho uv \\ -u(E+p) \end{bmatrix} \qquad G = \begin{bmatrix} -\rho v \\ -\rho uv \\ -\rho v^2 - p \\ -v(E+p) \end{bmatrix} \tag{12}$$

and $E = p^{\gamma-1} + \frac{1}{2}\rho(u^2+v^2)$

where an energy equation has been included so that unsteady flows caused by entropy variations may be dealt with.

Several approaches are now possible. In the first the dependent variables are assumed to be of the form

$$\rho(x,y) = \rho_o(x,y) + \rho'(x,y)\,e^{j\omega t} \quad \text{etc.} \tag{13}$$

Introducing, for convenience, mass flux variables X′ and Y′

$$X' = (\rho u)' = \rho u' + u\rho' \tag{14}$$

$$Y' = (\rho v)' = \rho v' + v\rho' \tag{15}$$

Equations (11) and (12) may be linearised, so that

$$W' = \frac{1}{j\omega}\left[\frac{\partial F'}{\partial x} + \frac{\partial G'}{\partial y}\right] \tag{16}$$

where

$$W' = \begin{bmatrix} \rho' \\ X' \\ Y' \\ E' \end{bmatrix} \quad F' = \begin{bmatrix} -X' \\ -2uX'+u^2\rho'-p' \\ -uY'-vX'+uv\rho' \\ -u(E'+p')-u'(E+p) \end{bmatrix} \quad G' = \begin{bmatrix} -Y' \\ -uY'-vX'+uv\rho' \\ -2vY'+v^2\rho'-p' \\ -v(E'+p')-v'(E+p) \end{bmatrix} \tag{17}$$

with

$$E' = (\gamma-1)p^{\gamma-2}p' + \frac{1}{2}\{2uX' - u^2\rho' + 2uY' - v^2\rho'\} \quad . \tag{18}$$

4.2 Method of Solution - Differential Form

Solutions may be obtained by expressing the derivatives $\partial F'/\partial x$, $\partial G'/\partial y$ in finite difference form and solving the resulting set of linear equations by Gaussian elimination or relaxation. Using the latter method, for example, at iteration n

$$W'_{n+1} = W'_n (1-\alpha) + \frac{\alpha}{j\omega}\left[\frac{\partial F'}{\partial x} + \frac{\partial G'}{\partial y}\right]_n \tag{19}$$

with α a relaxation factor. This may now be compared with an alternative approach proposed by previous authors, e.g. Ni & Sisto (1975),

$$\begin{aligned} W &= W_o + W''(x,y,t)\, e^{j\omega t} \\ F &= F_o + F''(x,y,t)\, e^{j\omega t} \quad \text{etc.} \end{aligned} \tag{20}$$

Substitution into equation (11) then gives, after rearranging,

$$\frac{\partial W''}{\partial t} = -j\omega W'' + \frac{\partial F''}{\partial x} + \frac{\partial G''}{\partial y} \tag{21}$$

or, in a simple one-step time marching method, at time $n\Delta t$

$$\begin{aligned} W''_{n+1} &= W''_n + \Delta t\left(-j\omega W'' + \frac{\partial F''}{\partial x} + \frac{\partial G''}{\partial y}\right)_n \\ &= W''_n (1 - j\omega\Delta t) + \Delta t\left[\frac{\partial F''}{\partial x} + \frac{\partial G''}{\partial y}\right]_n . \end{aligned} \tag{22}$$

which may be compared with equation (19). Thus, time marching, in this case, may be regarded as a relaxation scheme provided

$$\alpha \equiv j\omega\Delta t \quad . \tag{23}$$

The advantage of deriving the relaxation scheme using the second method is that a condition for numerical stability is obtained. Applying the Courant-Friedrichs-Lewy criterion, which is necessary but not sufficent,

$$\Delta t < \frac{\min(\Delta x,\Delta y)}{c(1+M)} \quad . \tag{24}$$

4.3 Method of Solution - Integral Form

In some problems where the computational mesh is not orthogonal, some advantage may be obtained by writing equation (12) in integral form and repeating the derivation of the linear equations on that basis. Integrating equation (11) over an area A, bounded by a closed curve C

$$\iint_A \frac{\partial W}{\partial t}\, dxdy = \iint_A F_x + G_y \; dxdy \quad . \tag{25}$$

Applying Green's theorem to the right hand side, and simplifying the integral on the left hand side,

$$\frac{\partial W}{\partial t} .A = \int_C -Gdx + Fdy \quad . \tag{26}$$

Approximations for the line integrals and time derivatives may then be derived and the analysis then follows closely the method based on the differential form of the equations.

5. BOUNDARY CONDITIONS

5.1 Far Upstream

There may exist, in any problem, a variety of waves crossing the upstream boundary. These may be downstream travelling acoustic or vorticity waves which may be regarded as the exciting influence on the problem, or upstream travelling waves obtained by reflection or transmission of other incident waves. Velocity potential may only be used in the absence of vorticity waves or where the streamline, and hence vorticity distributions, are known a priori (this dual velocity potential/stream function method has been proposed by Whitehead, 1974), a condition which is satisfied when the steady flow is uniform. The harmonic time marching method, however, has no such restriction.

The steady conditions far upstream (at least for axial flow machines) may be regarded as uniform, having a velocity U in the axial (x) direction and V in the tangential (y) direction. Now, the velocity and pressure perturbations, far upstream, may be written (Smith, 1971)

$$\begin{bmatrix} u' \\ v' \\ p' \end{bmatrix} = \begin{bmatrix} \bar{u} \\ \bar{v} \\ \bar{p} \end{bmatrix} e^{j(\omega t + \alpha x + \beta y)} \tag{27}$$

where α and β are wave numbers, and β is related to the inter-blade phase angle ΔQ by

$$\beta = \frac{\Delta Q - 2\pi r}{s} \tag{28}$$

r is the acoustic mode index and varies between $\pm\infty$. Any general disturbance may be regarded as an infinite series of terms such as those in equation (27). The conditions for the perturbation to be irrotational (acoustic) are

$$\bar{\xi} = j(\alpha\bar{v} - \beta\bar{u}) = 0 \tag{29}$$

$$\frac{\bar{p}}{\bar{v}} = \frac{(\omega + U\alpha + V\beta)}{\beta} \tag{30}$$

$$\frac{\bar{u}}{\bar{v}} = \frac{\alpha}{\beta} \tag{31}$$

and

$$(\omega + U\alpha + V\beta)^2 - c^2(\alpha^2 + \beta^2) = 0 \quad . \tag{32}$$

When the disturbance is simply a spatial distribution of vorticity caused by wakes and convecting with the fluid the conditions to be satisfied are

$$\bar{p} = 0 \tag{33}$$

$$\frac{\bar{u}}{\bar{v}} = -\alpha/\beta \tag{34}$$

$$\bar{\xi} = i(\alpha + \frac{\beta^2}{\alpha})\bar{v} \neq 0 \tag{35}$$

and

$$\alpha = -\frac{(\omega + V\beta)}{U} \quad . \tag{36}$$

Now, in some cases, numerical problems may arise when trying to provide, at an upstream location, an input velocity perturbation in the form of a downstream travelling wave. The method of specifying the boundary condition must allow upstream travelling waves, of known wave-numbers α, β, but unknown strengths, to pass through this boundary. It is entirely wrong, for example, to specify $\bar{u} = o$ since this implies that the boundary is solid to acoustic waves and upstream going waves will be reflected back into the flow. This difficulty may be overcome by eliminating the unwanted waves in turn. For example, if an incoming wave, amplitude $\bar{u}_o$

produces a single reflected wave of unknown amplitude $\bar{u}_1$ so that

$$u' = \bar{u}_o \exp j(\omega t + \alpha_o x + \beta_o y) + \bar{u}_1 \exp j(\omega t + \alpha_1 x + \beta_1 y) \qquad (37)$$

then $\bar{u}_1$ may be eliminated by use of an operator B

$$B_n \equiv [1 - \frac{1}{j\alpha_n} \frac{\partial}{\partial x}] \qquad (38)$$

so that the boundary condition becomes

$$B(u') = \bar{u}_o \{1 - \frac{\alpha_o}{\alpha_1}\} \exp j(\omega t + \alpha_o x + \beta_o y) \qquad (39)$$

where the right hand side is known, and finite difference approximation to $B(u')$ may be deduced from equation (38). Repeated use of the operator B may be used to filter out several waves but the order of the boundary condition increases with each filtered wave.

5.2 Far Downstream

Far downstream of a blade row the effect of the unsteady vorticity shed by the blade and convected down the steady state streamline is dominant. However, since the pressure perturbations produced by the wake are zero (equation (33)) a suitable far downstream condition may be simply

$$p' = 0 \qquad (40)$$

where p' may be expressed in terms of the velocity potential at points 1 and 2 (Figure 2) using equation (10).

In circumstances where acoustic waves are present at the downstream boundary, however, an approach similar to that at the upstream boundary is necessary.

5.3 Repeat Condition Upstream

In cascade problems the solution must repeat from blade to blade with a phase difference ΔQ. The simplest way to impose this condition in the case of the velocity potential method is to relate points 3 and 4 one blade spacing apart (Figure 2) using

$$\phi_3' = \phi_4' . e^{j\Delta Q} \qquad (41)$$

or, using velocity and pressure perturbations,

$$\begin{bmatrix} u' \\ v' \\ p' \end{bmatrix}_3 = \begin{bmatrix} u' \\ v' \\ p' \end{bmatrix}_4 . e^{j\Delta Q} \qquad (42)$$

5.4 Blade Surface Conditions

In all cases, where the steady flow is uniform, velocities normal to the blade surface must be equal to that of the blade itself; a further effect which must be considered arises when the blade has a steady loading; as the blade vibrates it moves to a region where the steady state normal velocity v_n is different. If the blade surface at any point vibrates with a normal amplitude h, velocity jωh and angular amplitude α, the normal velocity condition becomes

$$v_n' = j\omega h + \frac{\partial v_n}{\partial n}.h + \alpha.v_s \qquad (43)$$

on the reference blade. For the flat plate cascade v_n, the steady state normal velocity, is zero everywhere and the condition is simplified.

Great care must be taken when applying the normal boundary condition so as to preserve a high order difference scheme on the boundary. This may be achieved by placing dummy points across the boundary and eliminating them using the original full differential equations. Figure 2 shows the six points (numbered 5 to 10) of a skewed mesh used to calculate the normal derivative of velocity potential at the central point on the boundary; the three dummy points shown may be eliminated simultaneously when the steady flow field is uniform.

5.5 Wake Conditions

Vorticity is shed from the blades as the lift varies and is convected down the stagnation streamline. It is possible to equate this shed vorticity with a velocity or velocity potential jump across the wake and, together with the condition that the velocities normal to the wake must be the same on either side, to solve the problem for a guessed (complex) value of the shed vorticity. The velocity at the trailing edge must remain finite and the shed vorticity must be adjusted and the solution repeated until this is so.

A better scheme eliminates the last iterative step by using the fact that there is no pressure jump at the wake. It may be shown by reference to the energy equation that this scheme is entirely equivalent to the shed vorticity model and automatically satisfies the Kutta-Joukowski condition of zero trailing edge loading.

5.6 Supersonic Cascade Boundary Conditions

In cases where the axial Mach number is greater than 1.0 and the blade spacing such that aerofoils may be treated as isolated, the far upstream condition becomes one of zero disturbance and the far downstream condition is unimportant since acoustic waves cannot propagate upstream. When the axial Mach number is subsonic, however, neither of these conditions holds since, at all frequencies and phase differences, some waves propagate both upstream and downstream. Two approaches are then possible. In the first, the cascade is treated as a long linear one (Figure 1a), the end blade being replaced by, for example, a solid wall, or better, a boundary which will not reflect, numerically, the downward running disturbances back into the cascade. The solution proceeds from upstream to downstream in a single pass, sweeping over successive blades until the results from two adjacent blades are similar; whether these results are then representative of an annular cascade is debatable. The second approach is to use a long strip, one blade spacing in height (Figure 1b) and to sweep repeatedly from upstream to downstream, updating ϕ' values on the edges of the strip by utilising the cascade condition.

$$\phi'_{m+1} = \phi'_m \cdot \exp\ (jQ) \tag{44}$$

linking corresponding points on the top and bottom edges. Unfortunately, this method requires more sophisticated treatment of the far upstream and downstream boundary conditions and convergence is not guaranteed.

Problems also arise near blade boundaries when the normal velocity condition has to be satisfied; in this case dummy points may be set up across the wall so that, for example, (see Figure 7)

$$\phi'_4 = \phi'_2 \mp 2\Delta y\ v'_o \tag{45}$$

where v'_o is the normal velocity of the blade at O. This second order treatment is inaccurate near leading edges and other discontinuities and may be replaced there by a first order approximation

$$\phi'_4 = \phi'_o - \Delta y . v_a . \exp M(x_a - x_o)/l \tag{46}$$

where

$$l = \frac{(M^2-1)W}{\omega M}, \tag{47}$$

W is the (assumed uniform) free stream velocity and point a, on the blade, is on a characteristic

$$- dy/dx = (M^2-1)^{-\frac{1}{2}} \tag{48}$$

drawn through the point $(x_o, -\Delta y/2)$. This approximation was derived by reference to Stewartson's (1950) exact solution

$$\phi'(x,o) = -\frac{\text{sgn } y}{\sqrt{M^2-1}} \int_o^x v_n'(\xi) \exp\left[-j\frac{M(x-\xi)}{\ell}\right] J_o\left(\frac{x-\xi}{\ell}\right) d\xi \quad . \qquad (49)$$

6. RESULTS

6.1 Subsonic Cascades

Some in- and out-of-phase pressure distributions have been calculated for a cascade of bending flat plates at zero stagger, with zero and approximately $\pi/4$ phase difference, and at .0 and .5 Mach numbers. These were obtained using the velocity potential method and an iterative solution procedure and compared with results obtained using Smith's (1971) integral method.

Figure 3 shows the case where the stagger is zero, the reduced frequency, $\omega c/W$, unity and inter-blade phase angle zero. Agreement is excellent when the coarseness of the computational mesh is considered. The method fails to predict the behaviour near the leading edge singularity and is inaccurate near the trailing edge where the velocity gradients are, theoretically, infinite. For this latter reason an examination of the strength of the shed vorticity is not an accurate guide to the unsteady lift.

Figure 4 shows the effect of increasing the Mach number from zero to .5; the effect is small but is accurately predicted.

Figure 5 shows the results for an inter-blade phase angle of .7071. Convergence was poor in this case but the agreement with Smith's results was excellent.

Other comparisons for cases of torsion and non-zero stagger have been made and agreement with Smith's results has again been very good.

6.2 Supersonic Isolated Aerofoils and Cascades

The velocity potential method has been programmed to predict lift and pressure coefficients of several isolated aerofoils and cascades rotating and flapping in a uniform supersonic stream. Figure 6 compares the lift coefficient for a flapping isolated aerofoil with those obtained analytically by Shade (1946); the agreement is very good everywhere. Figure 7 shows the in- and out-of-phase distribution of lift on an isolated aerofoil and is compared with the distribution obtained by numerical integration of Stewartson's exact solution - equation (49). This problem was repeated with a mesh spacing chosen so that $r = \Delta x/\Delta y\sqrt{M^2-1} = .96$. The solution oscillates about its true value close to the leading edge discontinuity but, when it is properly integrated to give the total lift, the errors involved are very small.

Finally, the cascade problem was examined and results obtained for the case h/c = .4, o/c = .68 (see Figure 1) using the linear cascade method. The variation of lift and moment coefficients with blade number are shown in Figure 8; it may be seen that convergence is rather poor although the levels approach those calculated by Verdon (1973) using a related method. No convergence was obtainable using the single strip method.

7. CONCLUSIONS

Several examples have been given of how a combination of the complex representation of unsteady flows and modern numerical methods for solving steady state flow problems may be combined to predict unsteady effects in turbomachinery cascades. As a preliminary examination of the accuracy of the methods some numerical examples have been evaluated for a variety of simple cases and compared with existing methods; agreement was, in general, very good.

The methods described may be used to predict unsteady flows in regions where the steady flow is non-uniform, although in this report no numerical examples of this are given.

A simpler approach to the far upstream and downstream boundary conditions is needed to deal with cases where several acoustic modes propagate away from the blade rows.

8. ACKNOWLEDGEMENTS

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REFERENCES

No.	Author(s)	Title, Date, etc.
1	R.H. Ni and F. Sisto	Numerical computation of non-stationary aerodynamics of flat plate cascades in compressible flow. ASME Journal of Engineering for Power, Paper 75-GT-5. 1975.
2	K.R. Rushton	Dynamic relaxation solution of three-dymensional subsonic compressible oscillatory flow. Numerical Methods in Fluid Dynamics, Pentech Press, London. 1973.
3	T.H. Shade	Aeronautical Research Council, R&M No.9506. 1946.
4	A.H. Shapiro	The dynamics and thermodynamics of compressible fluid flow. Ronald Press Co., New York. 1953.
5	S.N. Smith	Discrete frequency sound generation in axial flow turbomachines. Cambridge University Report CUED/A-Turbo/TR29. 1971.
6	K. Stewartson	On the linearised potential theory of unsteady supersonic motion. Quart. J. Mech. Applied Maths, 3. 182-99. 1950.
7	J.M. Verdon	The unsteady aerodynamics of a finite supersonic cascade with subsonic axial flow. Trans. ASME, Ser. E, 40, 3, 667-671. 1973.
8	D.S. Whitehead	The basis of a field method for the calculation of unsteady flow in cascades. Cambridge University Report CUED/A-Turbo/TR60. 1974.

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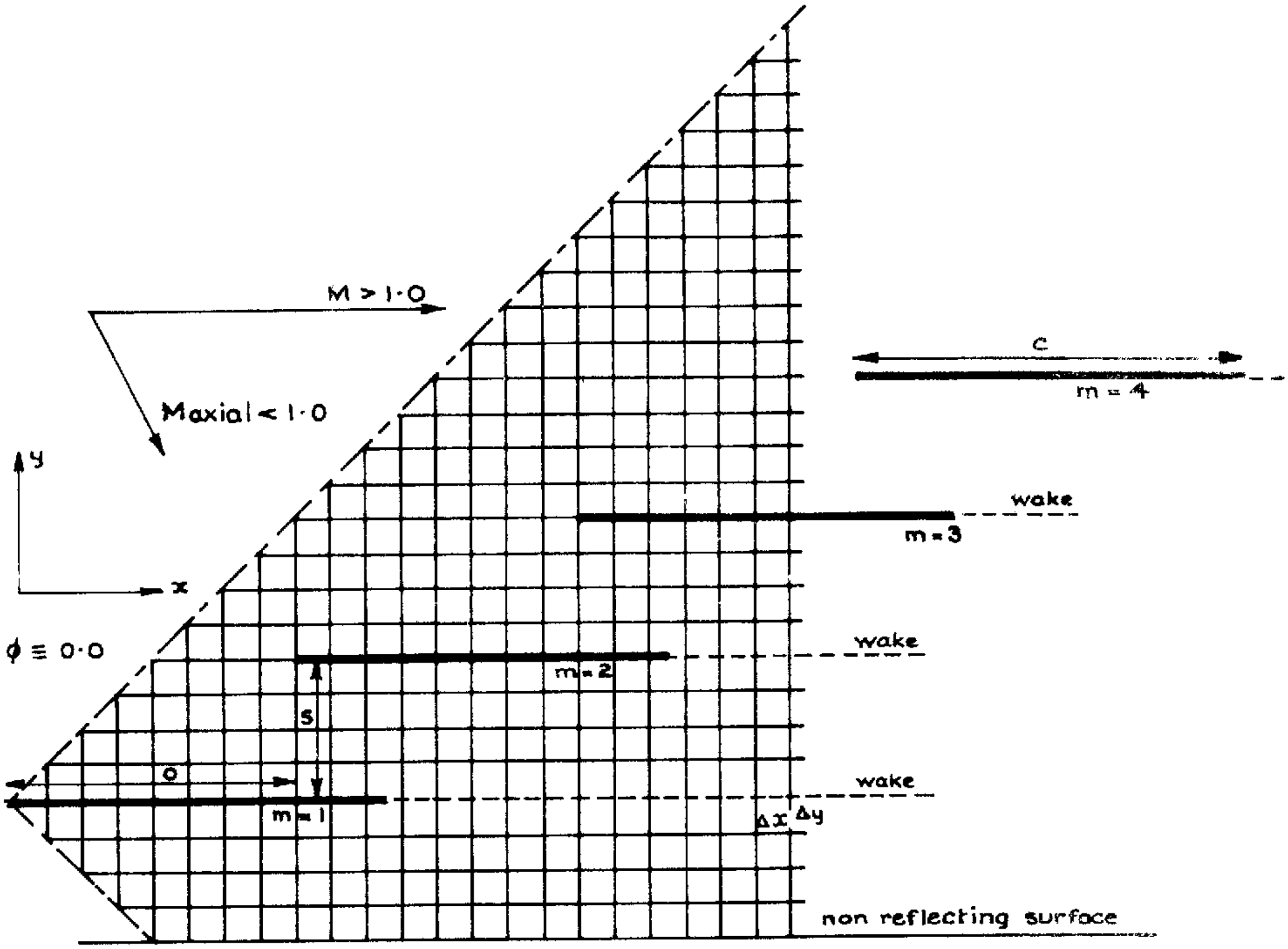
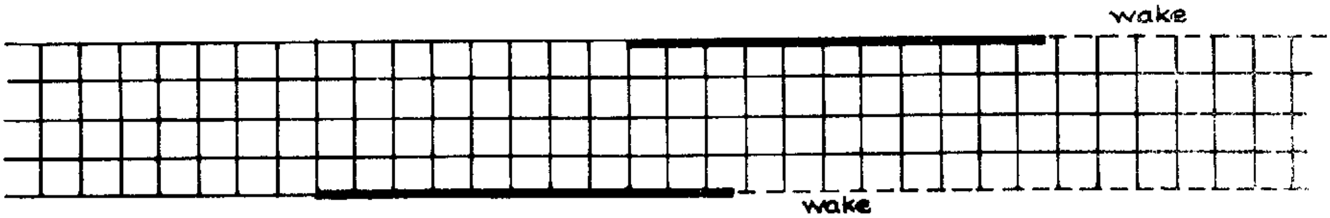


Finite difference mesh - linear cascade model Figure 1a



Finite difference mesh - single strip model Figure 1b

FIGURE 1: Supersonic Cascade. Finite difference mesh.

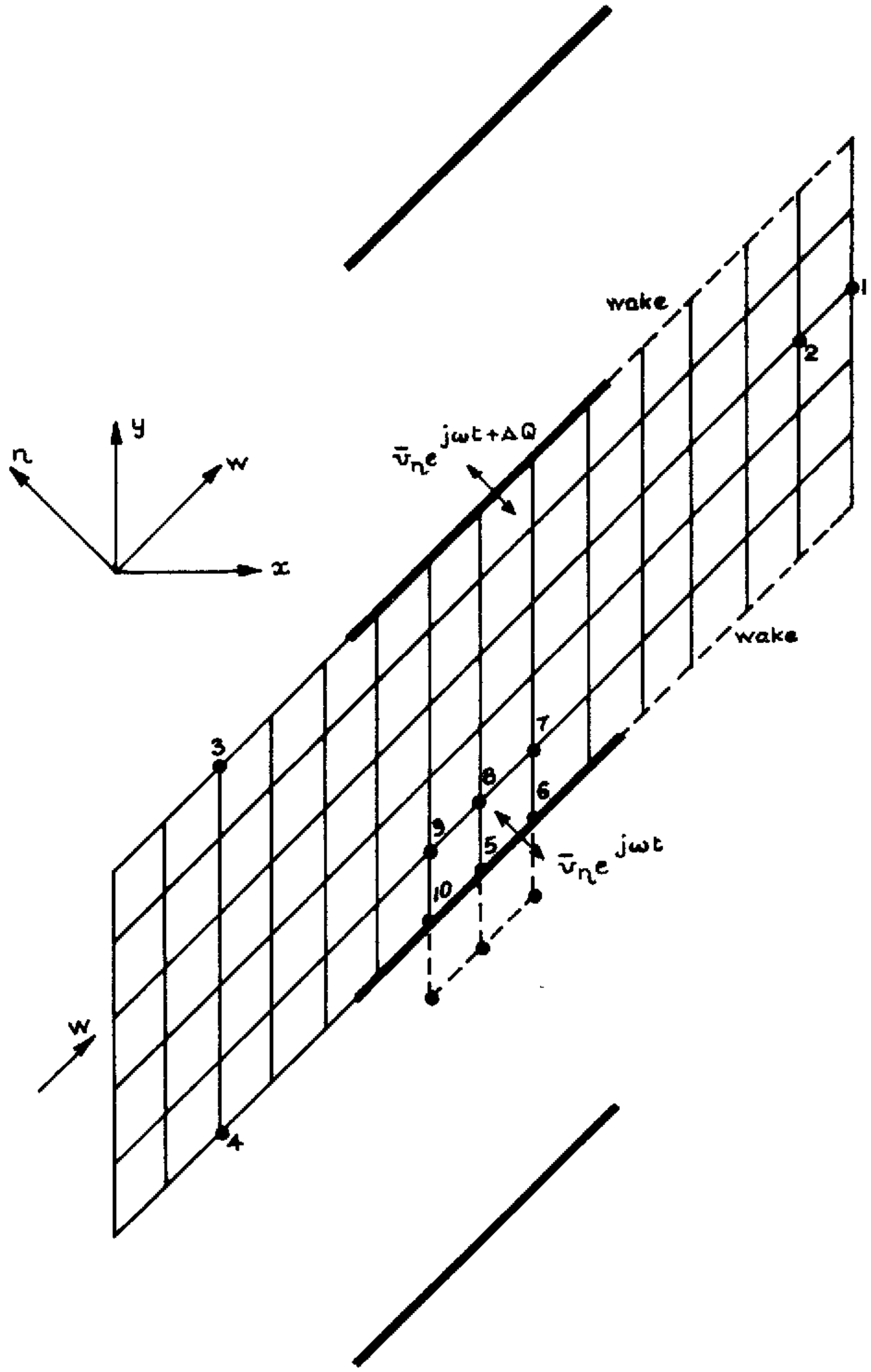


FIGURE 2: Subsonic Cascade. Finite difference mesh.

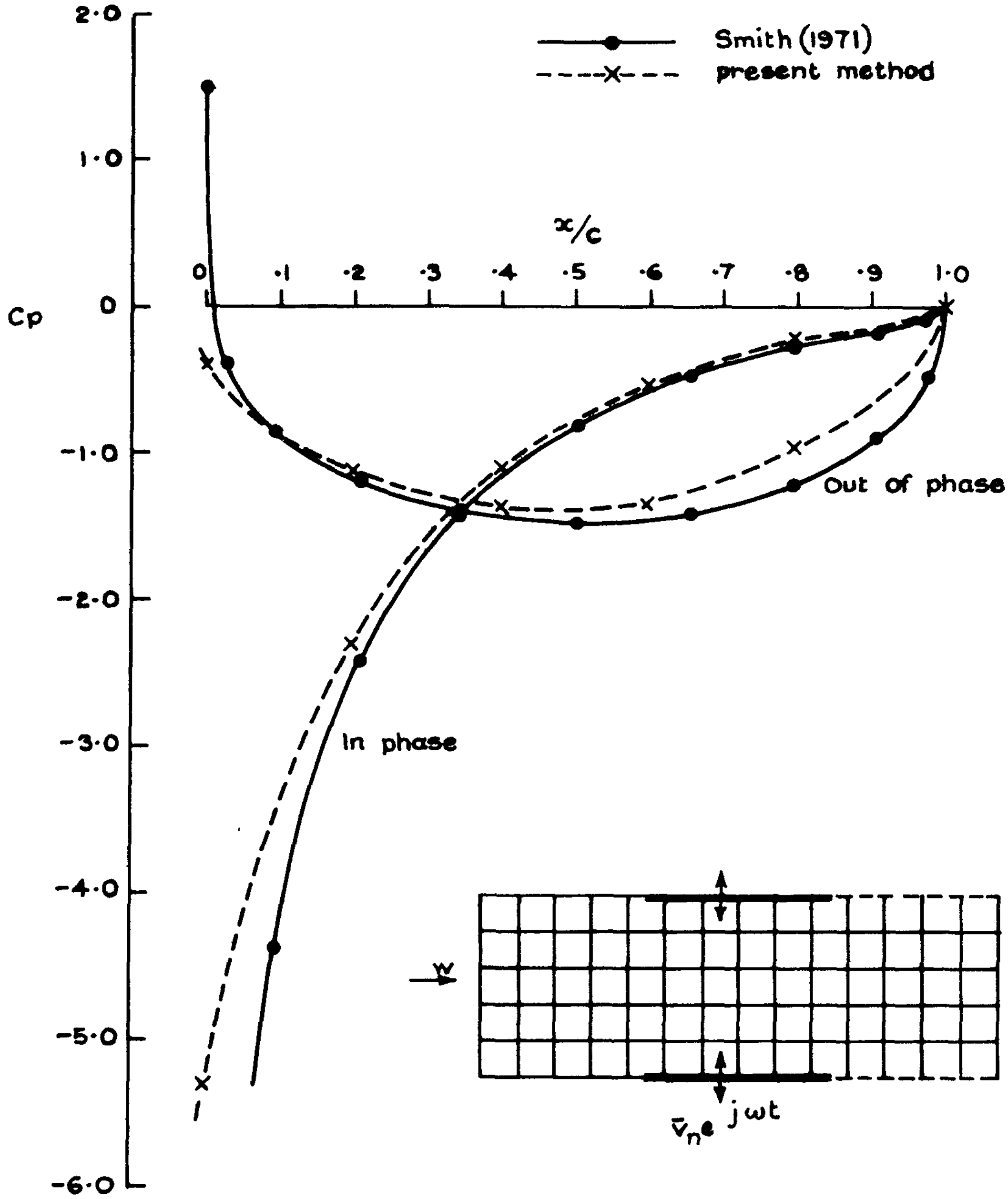


FIGURE 3: Subsonic Cascade with zero stagger and zero phase difference.

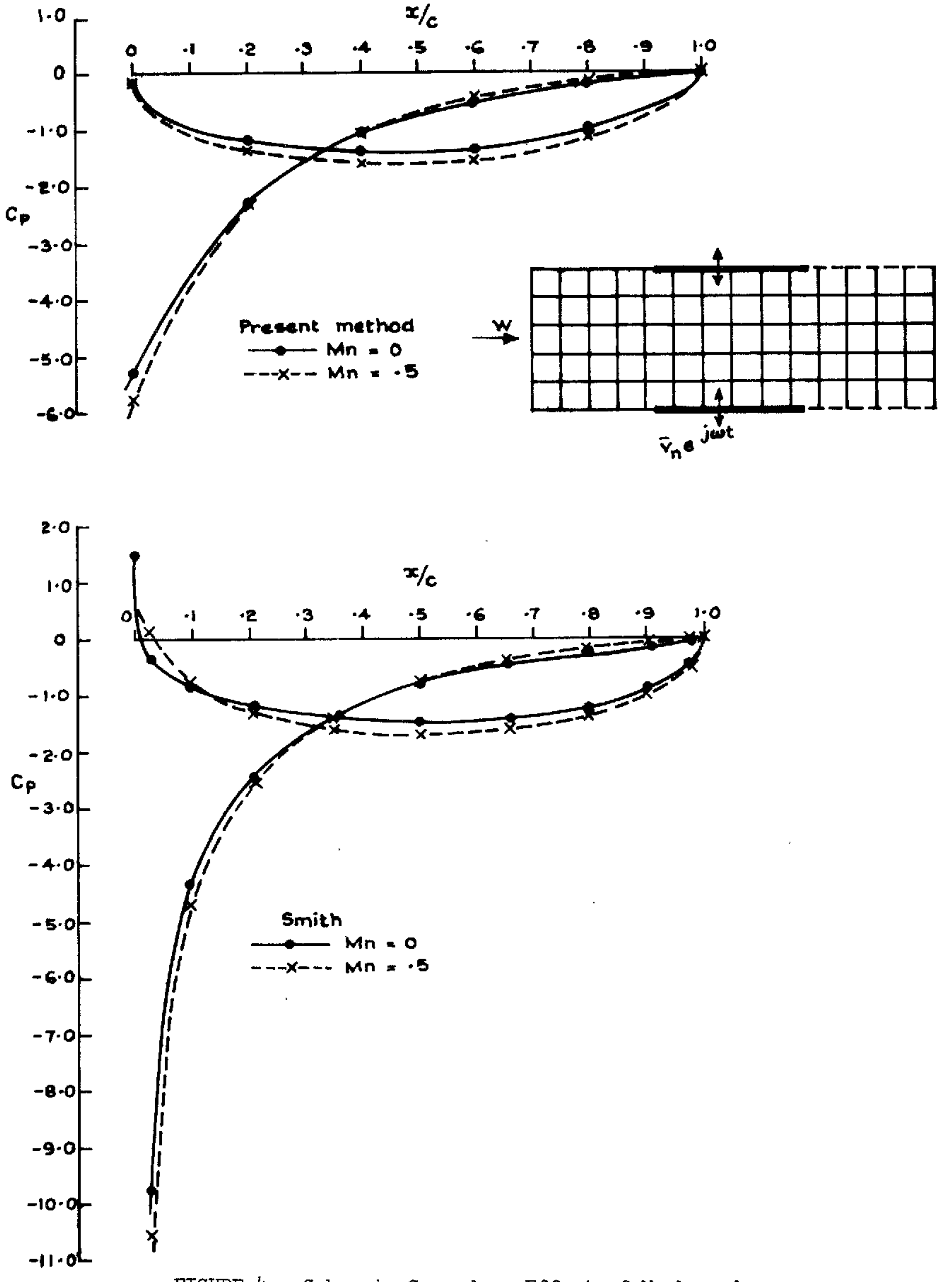


FIGURE 4: Subsonic Cascade. Effect of Mach number.

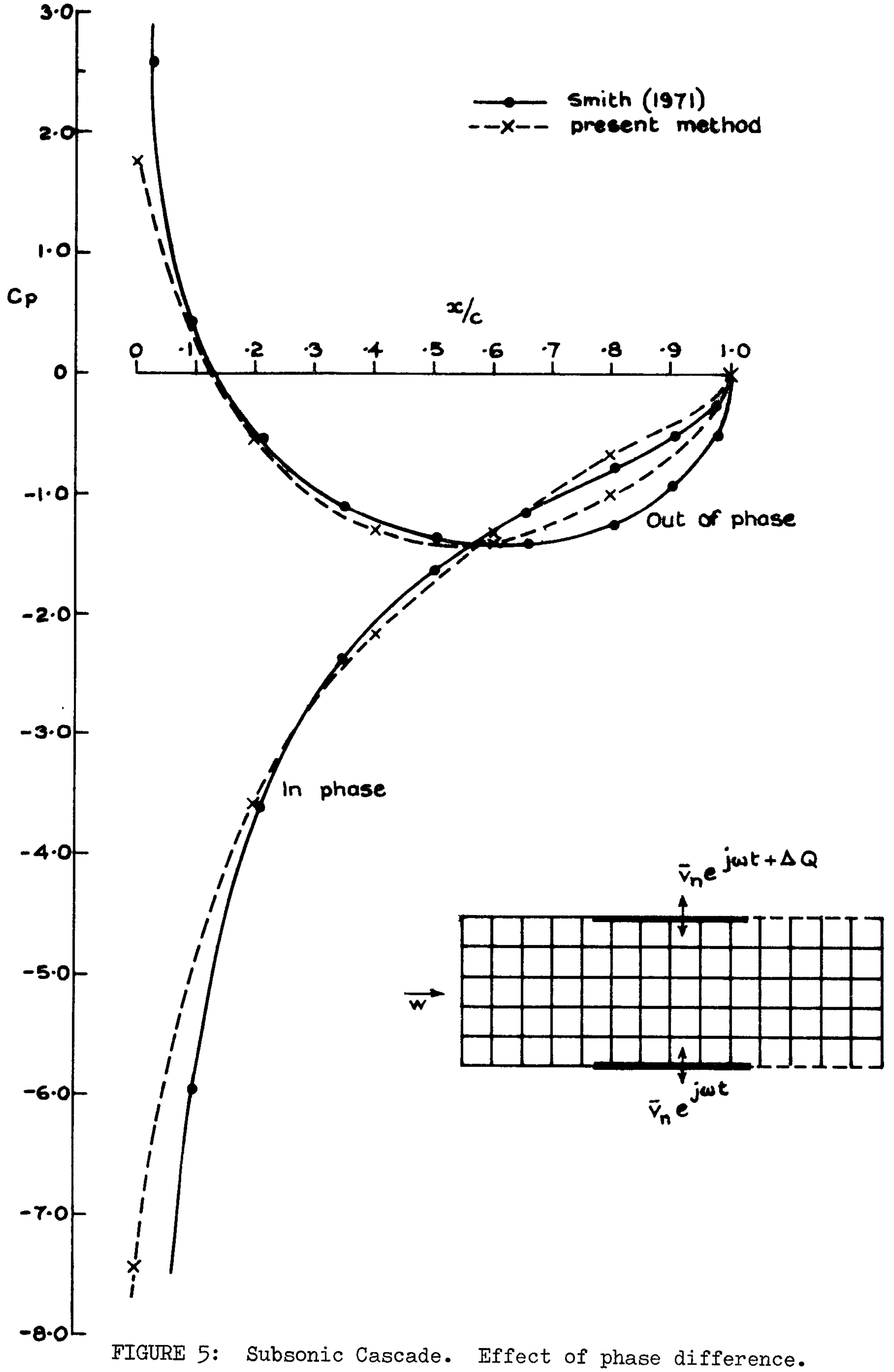


FIGURE 5: Subsonic Cascade. Effect of phase difference.

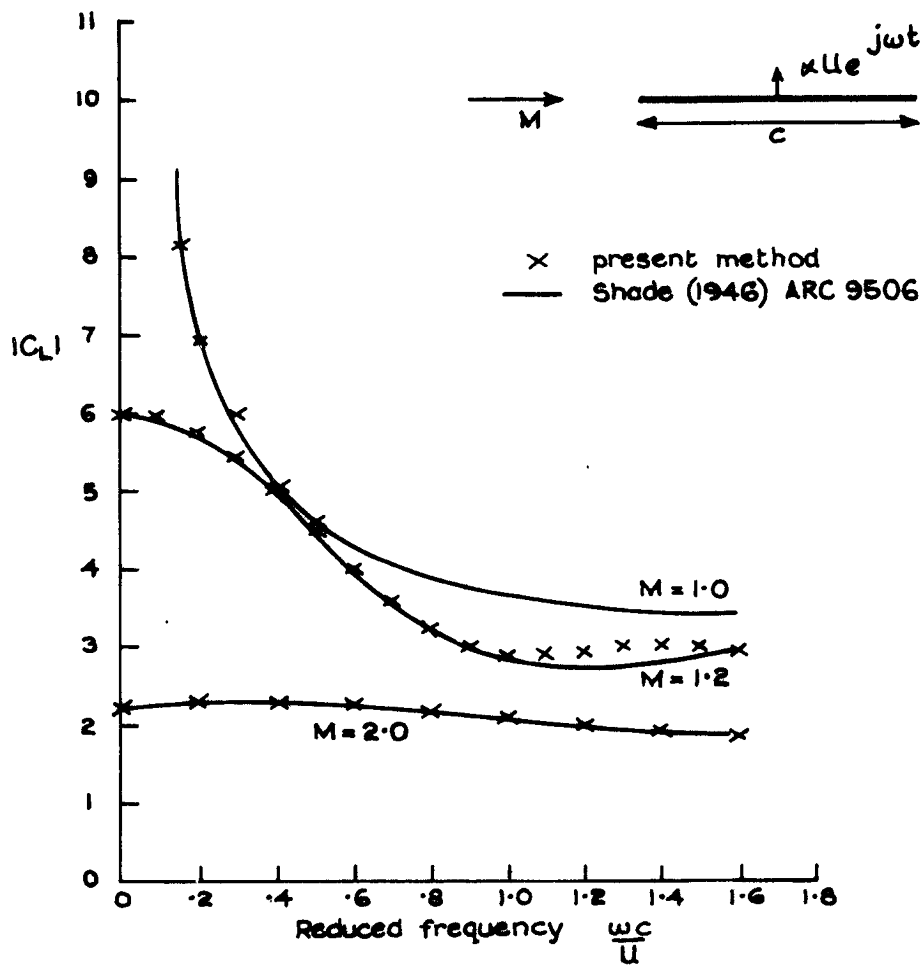


FIGURE 6: Supersonic isolated aerodoil. Effect of Mach number and reduced frequency on lift coefficient.

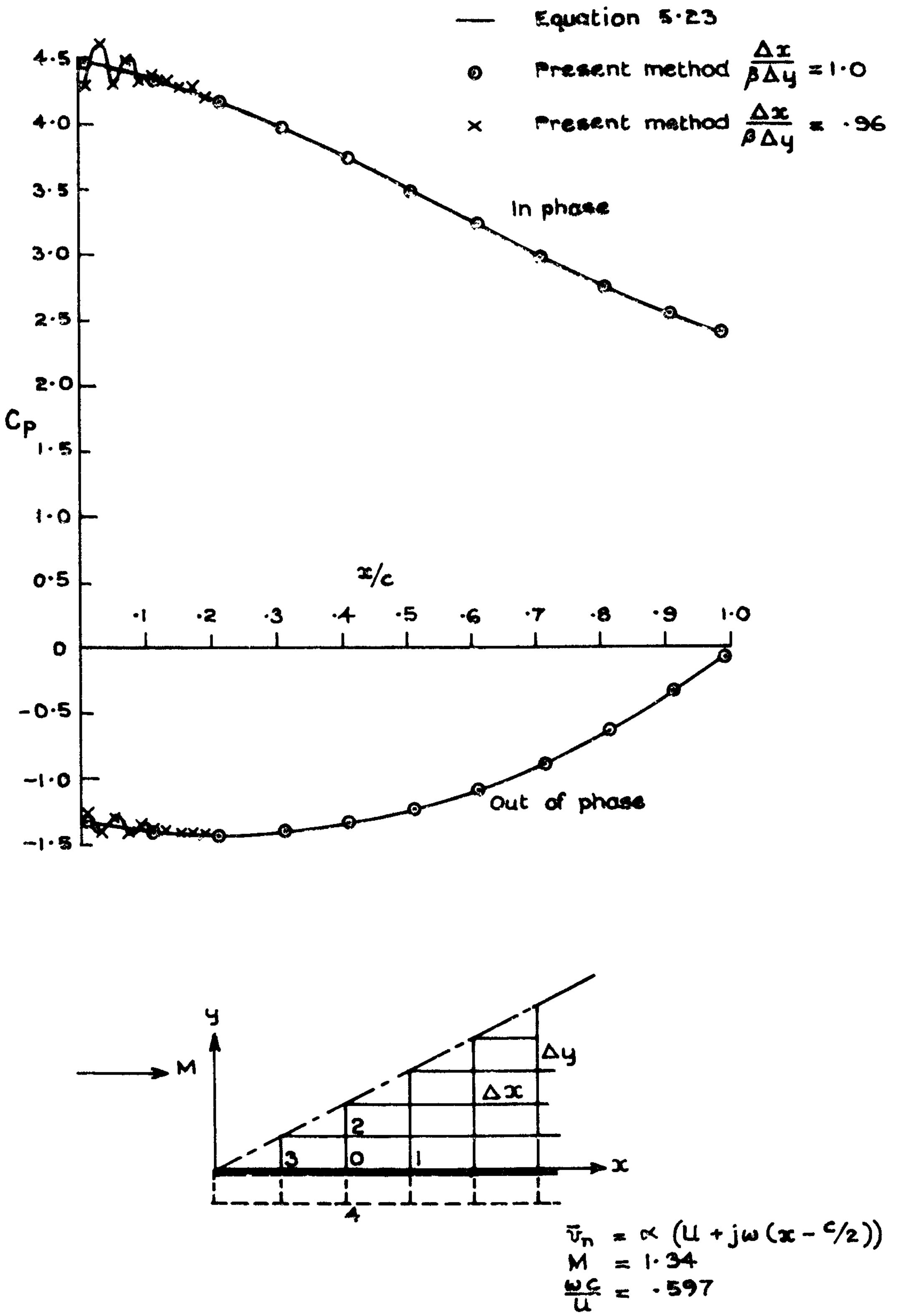


FIGURE 7: Supersonic isolated aerofoil. Pressure distribution.

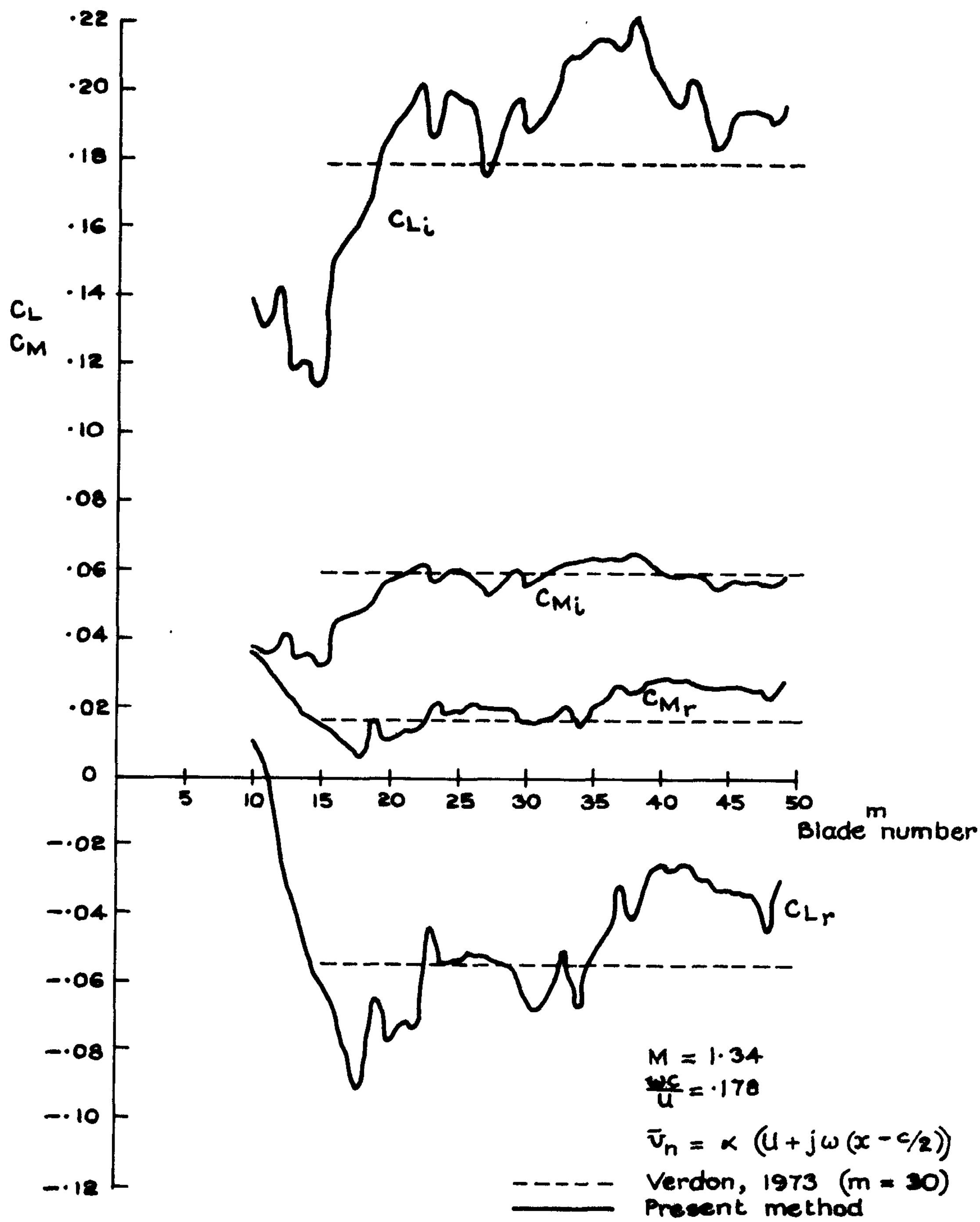


FIGURE 8: Supersonic Cascade. Variation of lift and moment coefficients with blade number.

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