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The Low Speed Performance of a Helicopter

By

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The low speed performance of a helicopter

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A. L. Oliver, B.Sc.(Eng.)

Summary

The analysis and estimation of helicopter performance is dependent upon the accurate assessment of rotor induced velocity. An empirical curve relating the flow through the rotor to the flight speed is used for vertical flight and the momentum theory is sufficiently accurate for a tip speed ratio greater than about 0.1, but no simple method has been generally available for the intermediary speed range.

Sets of empirical curves covering this speed range and based on the analysis of low speed flight performance are given in this Report. The charts give values of the rotor induced velocity varying smoothly from the vertical flight state to the forward flight region in which the momentum theory becomes accurate. The charts are presented in forms suitable for determining steady flight performance and also for estimating rotor thrust during accelerated motion in, for example, take-off flight.

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1. Introduction

The analysis and estimation of helicopter performance depends upon accurate assessment of the induced flow at the rotor. For flight speeds where the tip speed ratio, μ, is greater than about 0.1 the momentum theory (Ref.1) is fairly accurate; for vertical flight an empirical curve of rotor coefficients (Ref.2) is now commonly used. There is, however, no generally accepted simple method of estimation for the intermediary range of airspeed.

Theoretical curves relating rotor induced velocity, flight velocity and rotor disc incidence in a form derived from the momentum theory but modified to fit the accepted empirical data for vertical flight, were proposed for this intermediary speed range by Hafner (Ref.3). These curves, however, are inaccurate in the low speed range and it was therefore decided to establish mean empirical curves, of similar form, which would fair into the momentum theory curves at the speeds at which this theory becomes valid.

A list of symbols is given at the end of the report.

2. Theory of analysis

2.1 General. The basic helicopter theory (Ref.1) is developed for a single rotor machine with some form of torque reaction; it is assumed that the main rotor blades are untapered and untwisted and that the induced velocity is normal to and constant across the disc. A formula for the rotor thrust is adopted from the momentum theory in the form

$$T = 2\pi \rho_o R^2 v_i V_i' \qquad \text{..........(1)}$$

where v_i is the induced velocity and V_i' the resultant airflow at the rotor.* The induced velocity is thus related to the main airstream velocity and the rotor disc incidence, and may be used in conjunction with the equations of motion to determine helicopter flight performance.

The momentum theory, however, gives optimistic results for vertical flight conditions, and in its place an empirical curve of rotor coefficients, (Fig.1) relating the induced velocity to the main airstream velocity, is now generally used.

It is apparent that the momentum theory cannot produce results in the low forward speed range which are consistent with those obtained for vertical flight from the empirical curve. A way of avoiding this discontinuity has been suggested by Hafner (Ref.3). He points out that the momentum thrust equation may be written in the form

$$\left[\frac{V_i \cos i}{v_T}\right]^2 + \left[\frac{V_i \sin i}{v_T} + \frac{v_i}{v_T}\right]^2 = 1\Big/\left[\frac{v_i}{v_T}\right]^2 \qquad \text{..........(2)}$$

$$\text{where } v_T = \sqrt{\frac{T}{2\pi \rho_o R^2}}$$

Thus, for any given value of v_i/v_T the plot of $V_i \sin i/v_T$ against $V_i \cos i/v_T$ is a circle. Hafner, however, proposes that the centres and radii of the family of curves be derived from the established data for vertical flight, and not from the theoretical basis. The curves are therefore accurate in vertical flight and they approach the momentum theory form at high speed but they have been found to be unreliable in the low speed region. Accurate assessment of performance is particularly important in this region - for example, for the case of vertical take-off at different wind speeds - and an attempt has been made, therefore, to establish mean empirical curves which are of the type suggested by Hafner and which, in order to achieve continuity over the whole speed range, are faired into curves derived from the momentum theory at the speeds at which this theory becomes valid.

/The..

*In order that the theory may be applied to any atmospheric conditions it is convenient, as in performance reduction (Ref.4), to work in terms of equivalent airspeeds; these speeds are denoted throughout the report by the suffix i.

The analysis of measured steady low speed performance data is made using the rotor power and energy equations (Ref.4). The equations may be written

$$u_i = \frac{1}{T}\left\{EP\sqrt{\sigma} - \frac{\pi}{8}\rho_o C_D s \Omega_i^3 R^5 (1 + \mu^2)\right\} \quad ....(3)$$

and
$$EP\sqrt{\sigma} = WV_c\sqrt{\sigma} + Tv_i + \frac{\pi}{8}\rho_o C_D s\Omega_i^3 R^5 (1 + 3\mu^2) + \frac{D}{10^4}\cdot V_i'^3 \quad .....(4)$$

Subtracting (3) from (4) we get

$$V_i \sin i = \frac{1}{T}\left\{WV_c\sqrt{\sigma} + \frac{\pi}{8}\rho_o C_D s \Omega_i^3 R^5 (2\mu^2) + \frac{D}{10^4}\cdot V_i'^3\right\} \quad ....(5)$$

For practical steady flight conditions it is reasonable to assume that the rotor thrust is equal to the aircraft weight.

Knowing the performance and design characteristics of the helicopter, the airflow at the rotor can be calculated and empirical charts constructed. It has been found convenient (for reasons discussed in para.2.2) to establish curves relating u_i, V_i and i, in addition to the basic chart relating v_i, V_i and i. For the presentation in terms of u_i the momentum theory curves into which the empirical curves are faired have the family form

$$\frac{1}{e^4\left(\frac{u_i}{v_T} - \frac{V_i \sin i}{v_T}\right)^2} = \left(\frac{V_i \cos i}{v_T}\right)^2 + \left(\frac{u_i}{v_T}\right)^2 \quad ....(6)$$

The factor e is introduced to account for rotor tip losses, and is assumed to be equal to 0.95.

2.2 <u>Analysis of experimental results.</u> The flight data used in the analysis were obtained from test flights on various helicopters, including the Hoverfly, Dragonfly, and Sycamore types. Allowance was made for rotor blade taper by using a mean value of the blade chord; none of the rotors had twisted blades. The effects of compressibility, blade stalling, reversed flow and radial flow were neglected because estimates showed that these effects were small over the low speed range.

The results of tests made on any one aircraft in various operating conditions were divided into groups in each of which the atmospheric conditions were approximately the same, and small corrections for temperature made using the methods of Ref.4. Mean performance curves were then drawn because definition of the empirical curves depends on determining the relationship between speed and disc incidence at particular values of v_i/v_T and u_i/v_T, and isolated performance points will not, in general, give one of these values. The tests covered only climb at low forward speeds and autorotative glides; no results were obtained in the power-on descent flight condition.

Analysis of the performance curves for u_i and i depends on knowledge of P, E and C_D. P was obtained from the maker's power curves, corrected where necessary for temperature variations by the methods of Ref.5; E was determined from estimates of the power to the tail rotor, gear losses, cooling etc; C_D was taken from blade design data. Both E and C_D were checked by correlation with flight test data in vertical flight and in flight at medium speeds, and values were obtained for the low speed range by interpolating on curves of E and C_D plotted against speed. Equations (3) and (5) were then solved by successive approximation since initially μ was not known, and u_i/v_T, $V_i \sin i/v_T$, $V_i \cos i/v_T$ and v_i/v_T were calculated and plotted against speed. The values of v_i/v_T or u_i/v_T were thus obtained and are plotted in Figs.2 and 3.

Fig.2 gives the fundamental information concerning rotor induced velocity, but the form of presentation of Fig.3, using the axial velocity at the rotor, has been found to make fairing into the momentum equation curves easier than does the form shown in Fig.2. On the other hand, many more experimental points are obtained in terms of v_i/v_T than are obtained in terms of u_i/v_T. The shape of the curves was therefore established more firmly at the lower end of the speed range using the presentation in Fig.2 and at the upper end of the range by Fig.3; cross-plotting was necessary to ensure that both forms of presentation were consistent over the whole speed range. Establishing the curves was assisted by the fact that there is no discontinuity in airflow conditions at the rotor from forward to backward flight, so that all curves of both v_i/v_T and u_i/v_T are normal to the line $V_i \cos i/v_T = 0$. Both charts have been extended to cover the vortex ring and low speed autorotation regions, the extrapolations being shown by dotted lines.

3. Application of empirical curves

3.1 General. The charts shown in Figs.2 and 3 present information concerning the rotor airflow velocities in fundamental forms. These forms are not, however, the most suitable for direct application to performance estimation or analysis, and other forms of presentation have accordingly been considered.

It should be noted that the curves represent the case in which there are no effects on the rotor from blade taper or twist, compressibility or blade stalling and reversed or radial flow. In applying the curves to performance estimation separate allowance should be made for any of these effects which is thought likely to be significant.

3.2 Estimation of steady performance. In steady flight conditions the performance at a given airspeed for known rotor speed and engine conditions is normally required; it is therefore necessary to determine the rotor disc incidence, i. Fig.4 presents a chart for the estimation of steady low speed power-on performance; it has been derived from Fig.3 and shows the variation of u_i/v_T with V_i/v_T for specific values of i. In using the chart, u_i/v_T is calculated for selected values of V_i/v_T from equation (3); as a first approximation the effect of μ^2 on u_i may be neglected when V_i/v_T is small. i follows from Fig.4, and the true rate of climb V_c is then determined from equation (5).

3.3 Estimation of rotor thrust. In steady flight the rotor thrust is approximately equal to the aircraft weight, but in accelerated motions - during take-off, for example - the rotor thrust may be appreciably greater than the weight. The empirical curves may be presented in a form from which the rotor thrust at any stage of such a motion can be determined (neglecting unsteady aerodynamic effects) provided that the power conditions, flight speed and disc incidence are known. The disc incidence depends on the disc attitude to the horizontal as well as the flight path angle and is therefore a function of the flying technique.

The form of presentation follows from the fact that the rotor power equation (3) may be written in the form

$$\frac{v_T}{u_i} \cdot \left[\frac{V_i}{v_T}\right]^3 = V_i^3 \left[2\pi \rho_0 R^2 \Big/ \left\{EP\sqrt{\sigma} - \frac{\pi}{8}\rho_0 C_D s \Omega_i^3 R^5 (1+\mu^2)\right\}\right] \quad \ldots(7)$$

The empirical curves in Fig.4 give u_i/v_T as a function of V_i/v_T and i; the left-hand side of equation (7) may therefore be considered to be a function of V_i/v_T and i. Hence, if $V_i\sqrt{2\pi\rho_0 R^2} \big/ \sqrt{T}$ $(= V_i/v_T)$ is charted as a function of i and $\left[V_i \,.\, 2\pi\rho_0 R^2 \Big/ \left\{EP\sqrt{\sigma} - \frac{\pi}{8}\rho_0 C_D s \Omega_i^3 R^5 (1+\mu^2)\right\}\right]$, it is possible, since P, Ω_i, V_i and i are known, to find T. A chart of this form is given in Fig.5 in two parts, the scale for values of the speed function from 0 to 2.0 being four times that used for the range from 2.0 to 20.0

4. Further developments

The curves given in this report will be used to develop performance reduction methods for the low speed region of flight.

The analysis of flight data will be continued when torquemeters are available to measure the effective power at the main rotor in flight, and it is also proposed to analyse any rotor tower test data which may become available.

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3	Hafner	Rotor systems and control problems in the helicopter. Aeronautical Conference, London. September, 1947
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5	Cameron	British performance reduction methods for modern aircraft. R.& M.2447 November, 1941

List of symbols

b - number of main rotor blades

c - rotor blade chord at $r = 0.7R$

C_D - blade profile drag coefficient

D - helicopter fuselage drag at 100 f.p.s.

e - tip loss factor ; $e = 0.95$

E - ratio of effective power at rotor to total power

i - rotor disc incidence to the airstream

P - engine power

R - rotor radius

s - rotor solidity ; $s = bc/\pi R$

T - rotor thrust

u - total flow normal to the rotor disc

u_i - $u\sqrt{\sigma}$

v - induced velocity at rotor

v_i - $v\sqrt{\sigma}$

V - airspeed of helicopter

V_i - $V\sqrt{\sigma}$

V_c - true rate of climb of helicopter

V' - resultant airflow at rotor; $V' = (u^2 + V^2 \cos^2 i)^{\frac{1}{2}}$

V_i' - $V'\sqrt{\sigma}$

v_T - $\sqrt{\dfrac{T}{2\pi \rho_o R^2}}$

W - helicopter all up weight

μ - tip speed ratio ; $\mu = V_i \cos i/\Omega_i R$

ρ - air density

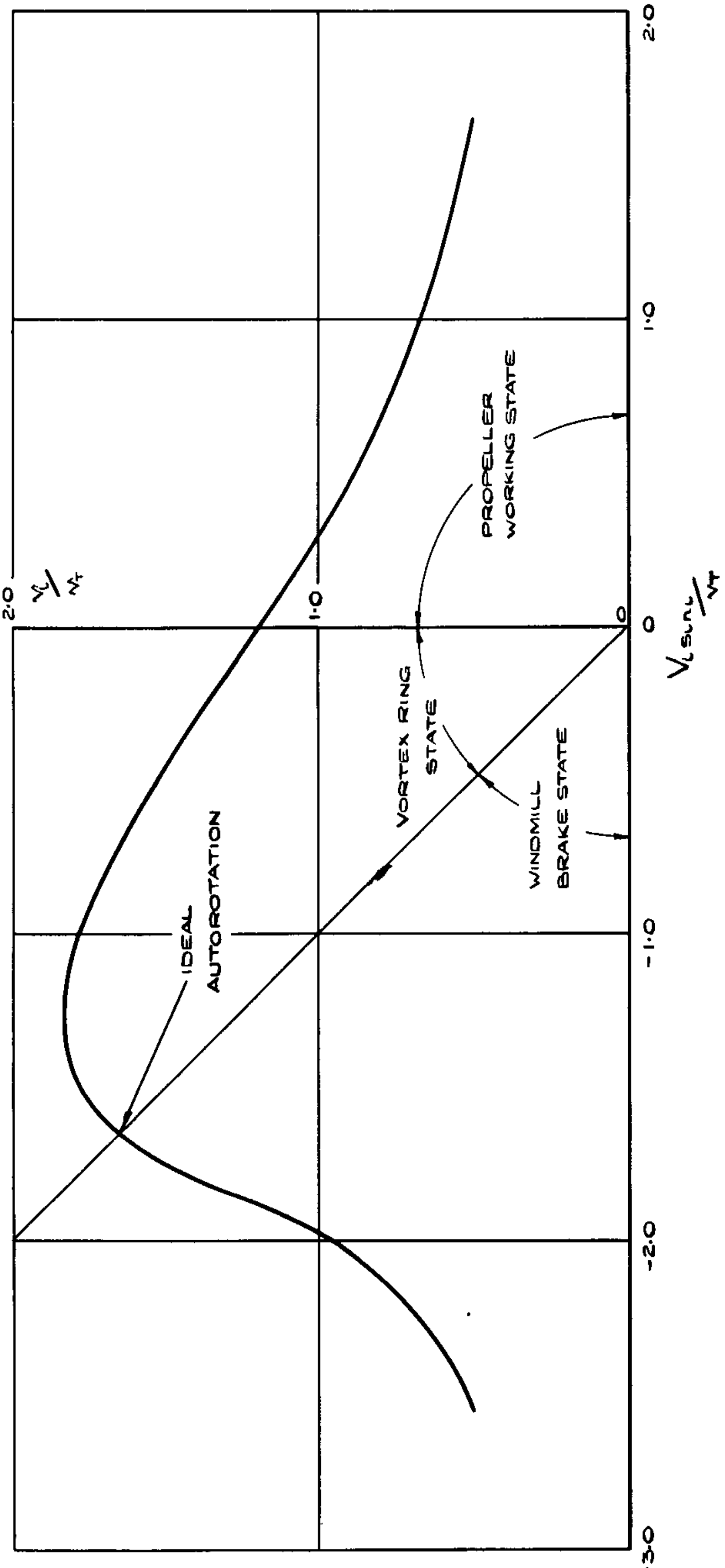
ρ_o - air density at sea level I.C.A.N.

σ - relative air density

Ω - rotor angular velocity

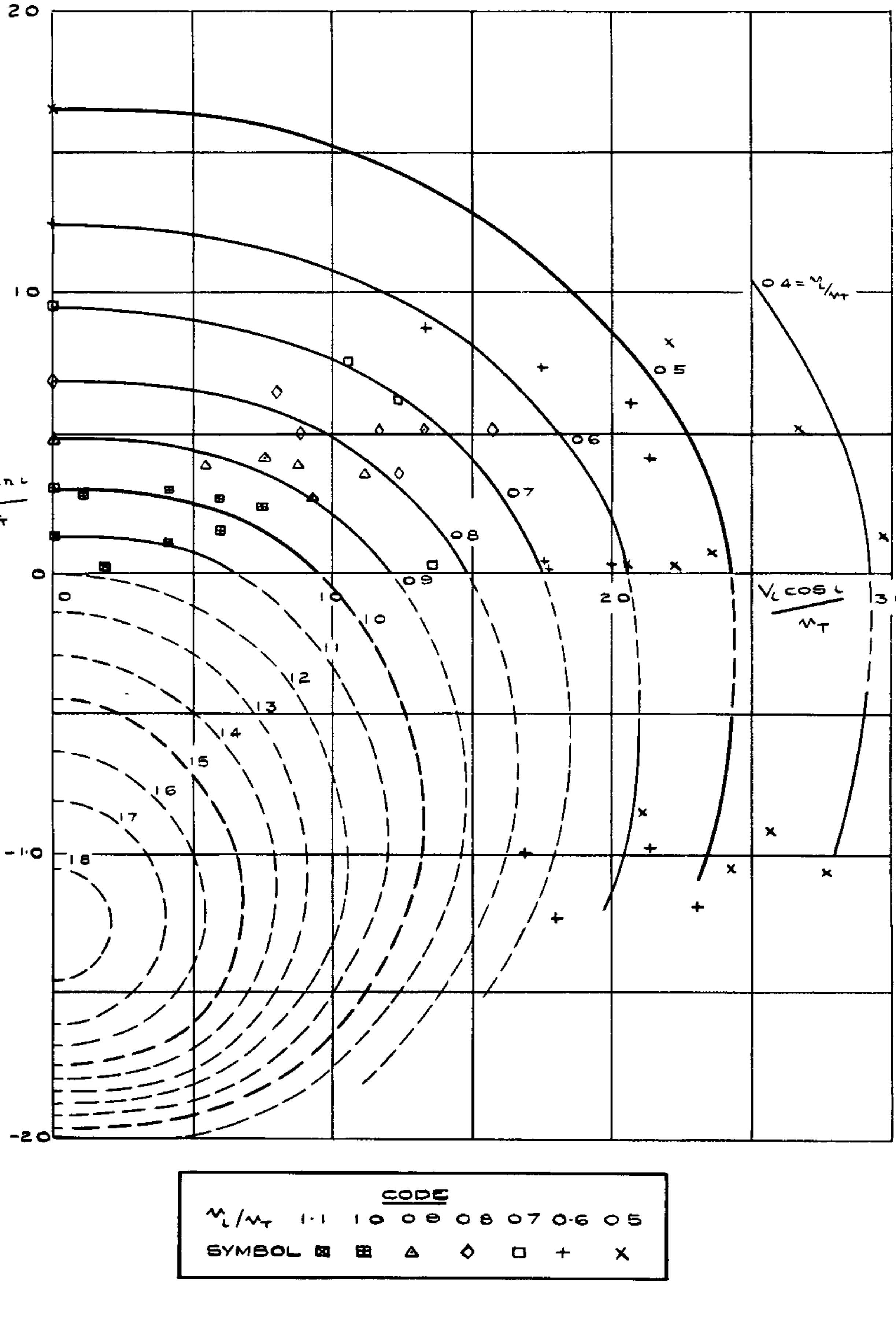
Ω_i - $\Omega\sqrt{\sigma}$

FIG.1.

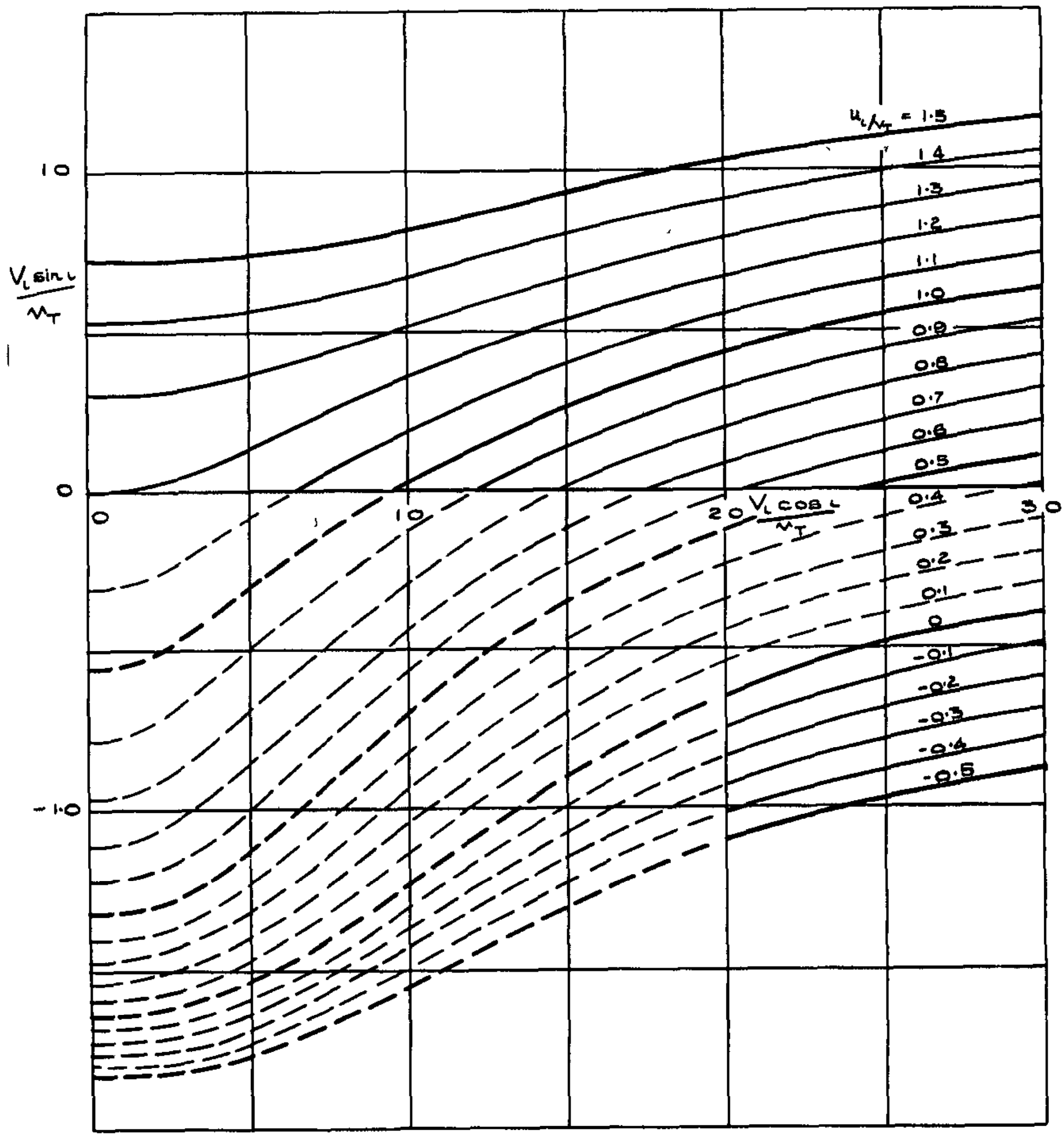


INDUCED FLOW COEFFICIENTS FOR VERTICAL FLIGHT.

FIG.2.



COEFFICIENTS OF INDUCED FLOW AT A HELICOPTER ROTOR.



COEFFICIENTS OF AXIAL FLOW THROUGH A HELICOPTER ROTOR.

FIG.4.

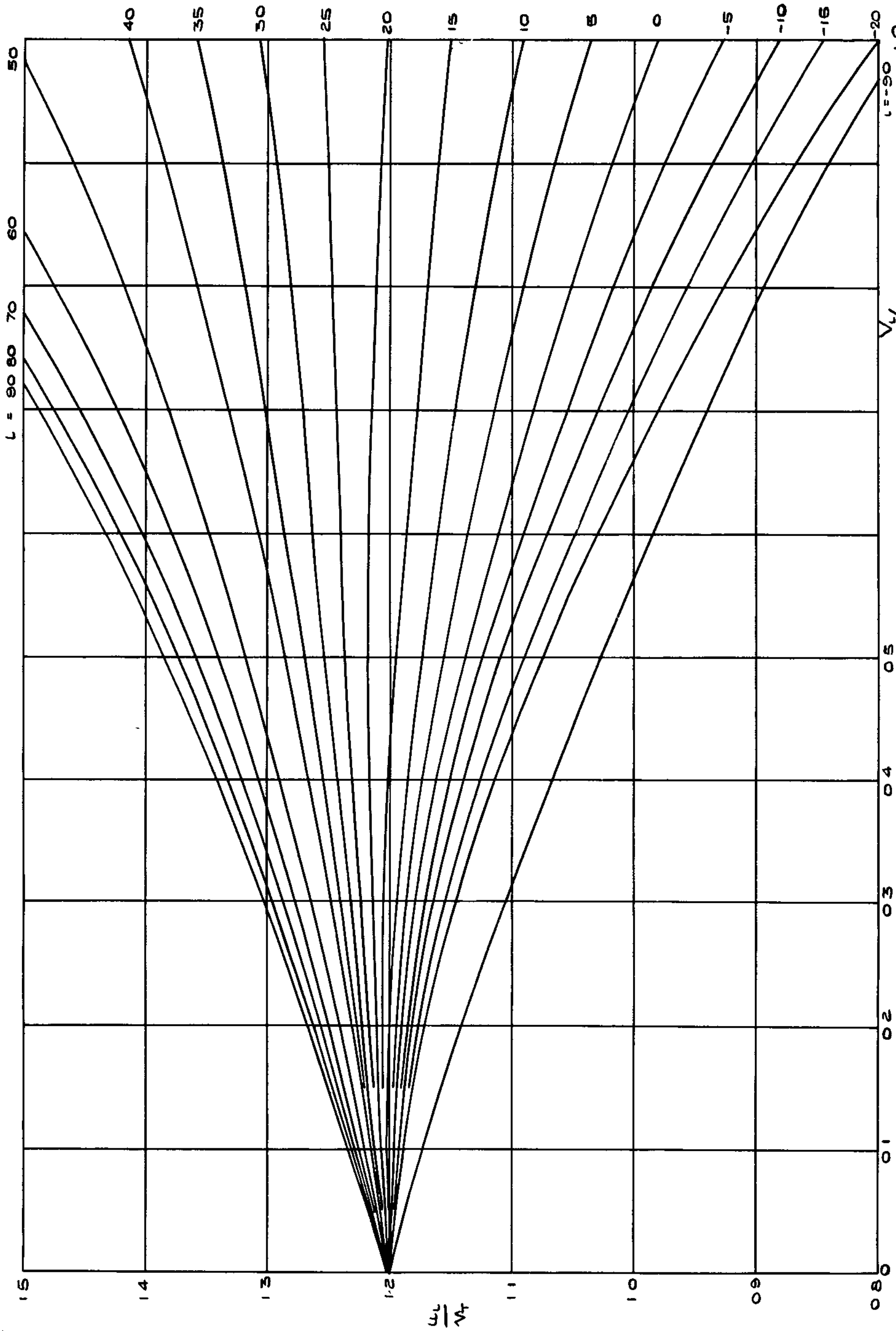


CHART FOR ESTIMATING STEADY LOW SPEED POWER-ON PERFORMANCE.

FIG. 5.

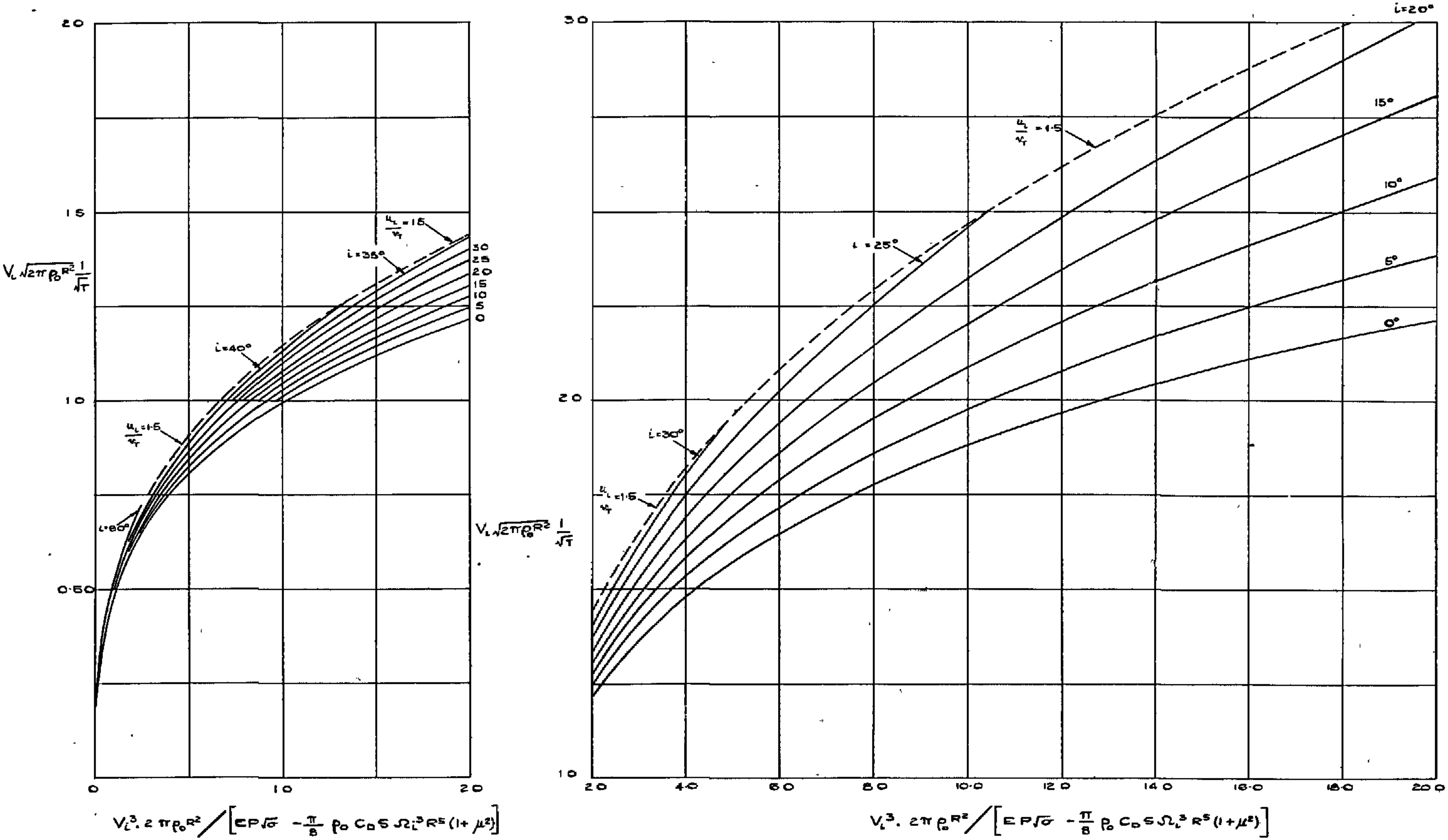


CHART FOR ESTIMATING ROTOR THRUST

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