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by

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Aerodynamics Dept., R.A.E., Bedford

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PLANE TURBULENT JET FLOW IN A FAVOURABLE PRESSURE GRADIENT**

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SUMMARY

An experimental study has been made of an incompressible turbulent plane jet exhausting downstream into an external flow having a favourable longitudinal pressure gradient approximating that of two-dimensional sink flow. Four flows were studied, for which the ratios of mainstream and jet velocities at the exit plane of the jet were in the range from 0.1 to 0.5. For each case velocity profiles were measured at four streamwise stations and a traverse was made along the jet centre line. The mean velocity profiles are shown to have the same shape as previously reported profiles for jets in a uniform stream. The streamwise development of the width of the jet and the velocity on its centre line are presented nondimensionally and values of the lateral shear stress gradient, $\frac{\partial \tau}{\partial y}$, are evaluated on the jet centre line to give some indication of the development of the turbulence structure.

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* Replaces RAE Technical Report 71047 - ARC 33086

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1 INTRODUCTION

An interesting feature of turbulent jet flows is that there appears to be no simple model of the turbulence structure which adequately describes the whole range of flows. Dimensional arguments lead to 'self-preserving' or 'similar' solutions for a jet issuing into still air, as a limiting case for a jet with a small velocity increment in a uniform external stream, and for certain cases having a particular streamwise distribution of velocity in the external stream. But, since these self-preserving flows cannot be reconciled with each other by postulating the same simple structure underlying each of them, it is difficult to proceed from a knowledge of self-preserving flows to the prediction of flows of a more general nature. This paper represents one further small addition to the body of knowledge on turbulent jets to which Dr. Reichardt has made such a valuable contribution. It presents the results of measurements made to determine the mean velocity distribution in an incompressible turbulent plane jet exhausting downstream into an external flow having a favourable longitudinal pressure gradient. Hot wire measurements of turbulence intensity and shear stress were also made in the same series of experiments, but these are not included in the present paper.

2 DESCRIPTION OF EXPERIMENTS

The experiments were performed in the 4ft × 3ft (1.22m × 0.915m) wind tunnel at Queen Mary College, London University. A model constructed for a previous investigation of a jet in a uniform external stream[1] was used to provide a plane jet exhausting from a slot 460mm long and 3.2mm deep in the trailing edge of a thin uncambered wing. Details of the flow turning and smoothing installations within this wing are given in Ref.1. A general view of the apparatus is shown in Fig.1 and a cross-sectional view is given in Fig.2. The wing was mounted horizontally at zero incidence across the centre of the wind tunnel working section, and the jet and its external stream confined by vertical walls to maintain two-dimensionality of the flow. Plane walls above and below the jet formed a convergent channel such that in the region of interest the stream external to the jet approximated a sink flow. Inlet conditions at the entry to the contracting passage were controlled so as to avoid any separations; this was by careful adjustment of a nozzle formed by flexible flaps fitted to the trailing edges of the inclined walls.

The mean flow measurements were made using miniature pitot and static tubes which could be traversed both longitudinally along the centre line of the plane of the jet, and laterally across the jet. The traverse gear, shown in Fig.1 enabled measurements to be made up to 685 mm from the nozzle. Pressures were read on a Betz manometer. Preliminary measurements confirmed that the stream velocity was uniform in planes normal to the flow direction and that its longitudinal variation corresponded closely (to within ½%) to two-dimensional flow into a sink; with small variations between the four cases studied, the sink was situated at approximately 2.8 m downstream of the exit plane of the jet. It was also confirmed that the jet itself remained in the plane of symmetry of the apparatus and that it was free from significant variations along lines parallel to the line of the nozzle. For ratios of stream velocity to jet velocity at the exit plane of the jet of 0.138, 0.196, 0.342 and 0.430 velocity profiles were measured at mid span at distances of 0.25, 0.4, 0.5 and 0.65 m from the exit plane and detailed traverses were made along the jet centre line.

3 ANALYSIS OF RESULTS

The discharge of a turbulent plane jet into a sink flow is shown schematically in Fig.3. If the nozzle and its fairing are sufficiently small, the influence of conditions at the nozzle exit (jet velocity, nozzle width and fairing boundary layers) will disappear and a typical turbulent jet flow will develop within a distance which is still small compared to the distance, s, between the virtual origin and the apparent sink. Then, well away from the nozzle the jet development will be a function of one parameter only. Defining an initial momentum thickness Θ for the jet by

$$-\rho u_s^2 \Theta = \text{momentum excess of the jet at its virtual origin}$$

$$= \lim_{x \to x_0} \int_{-\infty}^{\infty} \rho u(u_1 - u)dy ,$$

we may choose for the parameter, $\frac{s}{\Theta}$. If the velocity profile at any particular station is then characterised by the excess velocity u_0 on the centre line of the jet and the 'half width', $\delta\left(u = u_1 + \frac{u_0}{2} \text{ at } y = \delta\right)$ we find by dimensional reasoning that

$$\frac{u_0}{u_s} = F\left(\frac{x - x_0}{\Theta}, \frac{s}{\Theta}\right) ,$$

$$\frac{\delta}{\Theta} = G\left(\frac{x - x_0}{\Theta}, \frac{s}{\Theta}\right) .$$

The variation of $\left(\frac{u_s}{u_0}\right)^2$ and $\frac{\delta}{\Theta}$ with $\frac{x - x_0}{\Theta}$, for values of $\frac{s}{\Theta}$ corresponding to the flows with the three strongest jets in the present experiments is shown schematically in Figs.4 and 5.

The limiting curve $\frac{s}{\Theta} = -\infty$ represents the jet in a uniform stream. At the origin, all the other curves approach this one asymptotically. At the opposite limit, as the jet approaches the sink position and pressure forces dominate the turbulent mixing processes, u_0 and δ vary in proportion to the distance from the sink. Hence, at $\frac{x - x_0}{\Theta} = \frac{s}{\Theta}$, $\frac{\delta}{\Theta}$ intersects the axis and $\left(\frac{u_s}{u_0}\right)^2$ has a vertical asymptote. Fig.6 shows the profiles of time mean excess velocity $u - u_1$ scaled by u_0 and δ. They are seen to fit closely to the exponential function found previously[1] to describe profiles in a free jet, although there is a tendency for the scatter to increase as the jet velocity becomes very small. The possible influence of pressure gradients on the shape of the profile above was estimated by calculating the inviscid development of a stream with an initial velocity profile given by the expression in Fig.6. For overall pressure changes of the magnitude imposed in the present experiments, this influence was found to be negligible.

To determine the initial momentum thickness Θ of the jet, the boundary layer momentum integral equation was integrated from each measuring station back to the virtual origin. If the profiles of excess velocity are geometrically similar, i.e.

$$\frac{u - u_1}{u_0} = f\left(\frac{y}{\delta}\right) = f(\eta) ,$$

with f given by the expression in Fig.6, the momentum integral equation can be expressed:

$$\Theta = - \left[\delta\left(\frac{u_0 u_1}{u_s^2} I_1 + \frac{u_0^2}{u_s^2} I_2\right)\right]_x - \frac{I_1 s}{u_s} \int_{x_0}^{x} \frac{\delta u_0}{[s - (x - x_0)]^2} dx ,$$

where $I_1 = \int_{-\infty}^{\infty} f(\eta)d\eta = 2.025$; $I_2 = \int_{-\infty}^{\infty} f^2(\eta)d\eta = 1.467$.

To determine Θ from this equation, the position of the virtual origin must be known. In the present work the origin was determined, by a process of trial and error, as that value of x_0 for which the initial development of the jet in the coordinates of Fig.4 was asymptotic, at its origin, to the simple expression for a jet in a uniform stream given in Ref.2. Fig.7 shows values of Θ that are calculated from the profiles measured at each of the four traverse stations. For each jet, the fairly good agreement between the four values of Θ indicates that the flow along the central plane of the tunnel was closely two-dimensional.

In Figs.8 to 11, the measured values of u_0 and δ are presented in the nondimensional form of Figs.4 and 5. The values for the weakest jet, $\frac{s}{\Theta} = -560$, showed considerable scatter and therefore only a dashed curve is given to indicate the approximate trend in this case. Table 1 defines the mean curves through the experimental results.

From the development of the mean velocity field some insight into the turbulence structure of the jet can be obtained by applying the equation of mean motion along the jet centre line. Thus on the centre line

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{1}{\rho} \frac{dp}{dx} = \frac{1}{\rho} \frac{\partial \tau}{\partial y}$$

may be written

$$\frac{u_1}{u_0}\left(\frac{\delta}{u_1}\frac{du_1}{dx} + \frac{\delta}{u_0}\frac{du_0}{dx}\right) + \frac{\delta}{u_0}\frac{du_0}{dx} = -\left(\frac{\partial g}{\partial \eta}\right)_{CL} ,$$

where $g = \frac{-\tau}{\rho u_0^2}$

and so, knowing δ and the streamwise gradients of u_0 and u_1 we can evaluate $\left(\frac{\partial g}{\partial \eta}\right)_{CL}$, the nondimensional turbulent shear stress gradient on the jet centre line. In self preserving jets it is a constant throughout the flow. It is also implied to be a constant in many theoretical treatments based on hypotheses about the mixing length or eddy viscosity of the turbulence. For both a jet in a uniform stream, and the accelerating stream of the present experiments, the shear stress profiles retain almost exact geometric similarity. Consequently the general level of shear stress across the jet is essentially proportional to $\left(\frac{\partial g}{\partial \eta}\right)_{CL}$. Measured values of $\left(\frac{\partial g}{\partial \eta}\right)_{CL}$ are shown in Figs.12 and 13, along with values for a plane jet in a uniform stream, and the self preserving cases of a jet in still air and a small decrement wake. Because the determination of $\left(\frac{\partial g}{\partial \eta}\right)_{CL}$ involves the differentiation of experimental data, the results were subject to scatter, and are therefore presented only as broad bands.

4 DISCUSSION

For the three strongest jets, the measurements appear to be sufficiently reliable to define the first quarter, approximately, of the development of a plane turbulent jet in an external sink flow. There is evidence[3] that for the weaker jets produced with this apparatus, the initial turbulent intensity is insufficient to break down the turbulence structure generated by the nozzle, before the start of the measuring region.

Figs.12 and 13 showing the lateral shear stress gradient along the jet centre line illustrate the changes to the shear stress levels associated with the acceleration of the external flow. Generally $\left(\frac{\partial g}{\partial \eta}\right)_{CL}$ is increased from the level in a jet in a uniform stream, with the exception of the values at small $\frac{x_0 - x}{\Theta}$ for $\frac{s}{\Theta} = -196$, and for the weak jet where $\frac{s}{\Theta} = -560$. The turbulence structure here is thought to be influenced by the nozzle conditions, and these regions are not representative of ideal plane turbulent jet flows.

These experiments therefore, invite two main conclusions. First the presence of pressure gradients does not perceptibly influence the shape of the profiles of excess velocity, which are virtually identical with those of a jet in a uniform stream. Secondly, although there is some indication that, in relation to u_0^2, shear stress levels are higher than in a flow at constant pressure, the difference is fractional only. Like the jet in a uniform stream, the jets in these experiments appear to have a structure intermediate between those of a jet in still air and the small defect wake.

Acknowledgments

The measurements presented in this paper were made while the author was a research student in the Aeronautical Engineering Department at Queen Mary College, London University. The author would like to thank Professor A.D. Young and his staff for their help and encouragement during the course of this research, and Dr. J.E. Green of RAE Bedford for his helpful criticism in the preparation of this paper.

Table 1

MEAN CURVES THROUGH EXPERIMENTAL RESULTS

$\frac{s}{\Theta} = -15.9$		
$\frac{x_0 - x}{\Theta}$	$-\frac{\delta}{\Theta}$	$\left(\frac{u_s}{u_0}\right)^2$
0	0	0
0.5	0.036	0.07
1.0	0.068	0.16
1.5	0.096	0.26
2.0	0.122	0.38
2.5	0.144	0.50
3.0	0.164	0.64
3.5	0.183	0.78
4.0	0.199	0.94
4.5	0.213	1.10

$\frac{s}{\Theta} = -38$		
$\frac{x_0 - x}{\Theta}$	$-\frac{\delta}{\Theta}$	$\left(\frac{u_s}{u_0}\right)^2$
0	0	0
1	0.068	0.15
2	0.127	0.33
3	0.175	0.55
4	0.217	0.80
5	0.253	1.06
6	0.283	1.36
7	0.310	1.70
8	0.334	2.08
9	0.356	2.50
10	0.374	3.00

$\frac{s}{\Theta} = -193$		
$\frac{x_0 - x}{\Theta}$	$-\frac{\delta}{\Theta}$	$\left(\frac{u_s}{u_0}\right)^2$
0	0	0
5	0.28	1.0
10	0.46	2.1
15	0.59	3.3
20	0.70	4.6
25	0.79	6.0
30	0.87	7.6
35	0.94	9.4
40	0.99	11.4
45	1.04	13.7
50	1.08	16.5
55	1.11	20.0

$\frac{s}{\Theta} = -\infty$ (Ref.3)		
$\frac{x_0 - x}{\Theta}$	$-\frac{\delta}{\Theta}$	$\left(\frac{u_s}{u_0}\right)^2$
0	0	0
2	0.127	0.335
4	0.221	0.696
6	0.302	1.077
8	0.375	1.474
10	0.444	1.885
12	0.508	2.306
20	0.732	4.060
30	0.970	6.390
40	1.181	8.845
50	1.374	11.41
60	1.553	14.08
70	1.723	16.85
80	1.884	19.70
90	2.039	22.63
100	2.187	25.62

SYMBOLS

g	nondimensional shear stress $(-\tau/\rho u_0^2)$
h	jet nozzle width
I_1, I_2	mean velocity profile integrals
p	static pressure
s	distance of sink from virtual origin
u	velocity in x direction
u_1	local stream velocity
u_0	centre line excess velocity
u_s	stream velocity at virtual origin
x, y	co-ordinate system with origin in jet nozzle
x_0	virtual origin position
δ	jet 'half width'
η	y/δ
Θ	initial jet momentum thickness
ρ	air density
τ	shear stress

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No.	Author(s)	Title, etc.
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2	L.J.S. Bradbury	Simple expressions for the spread of turbulent jets. The Aeronautical Quarterly Vol.XVIII, (1967)
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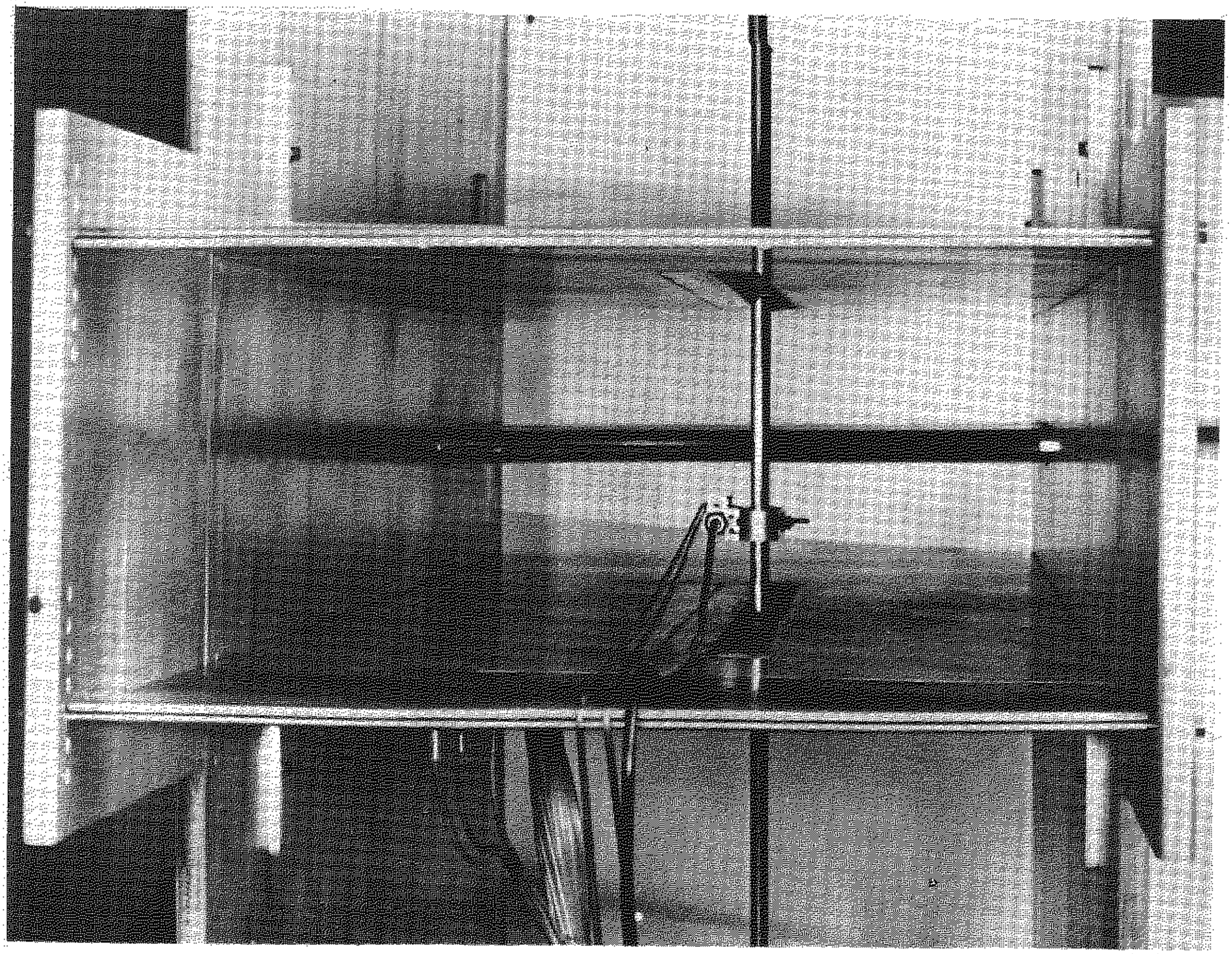


Fig.1. General view of apparatus

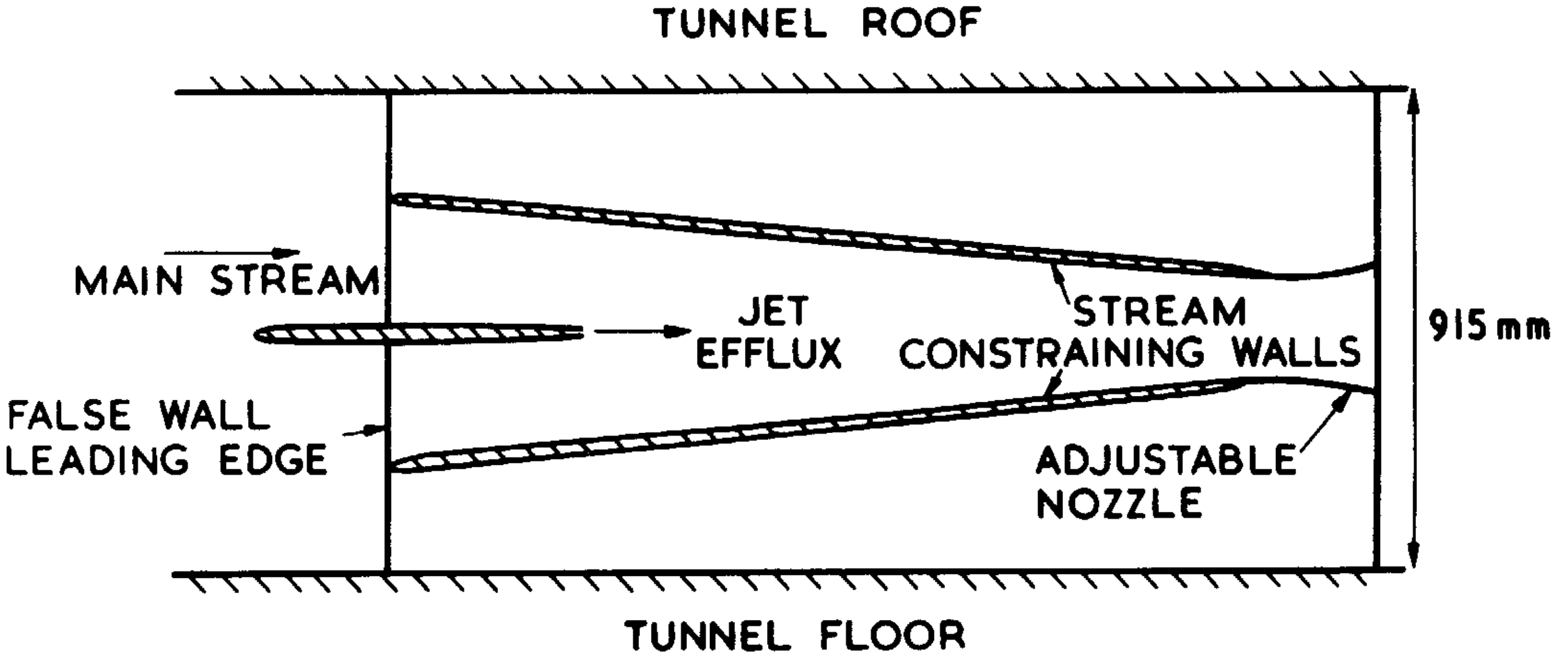
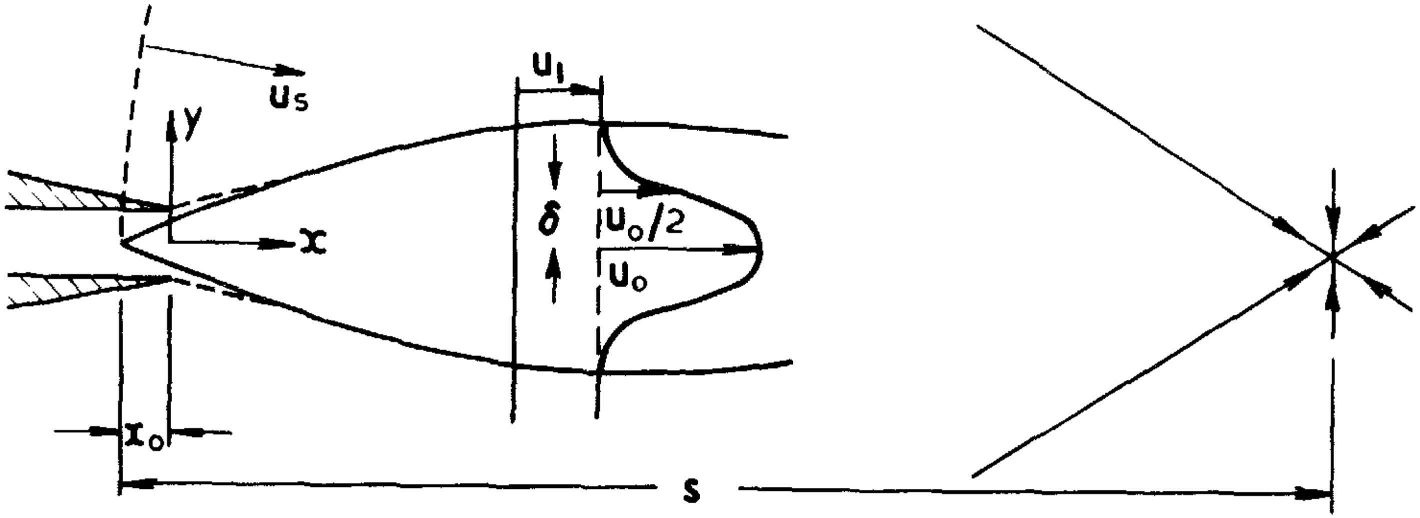


FIG. 2 CROSS SECTION THROUGH APPARATUS IN PLANE OF MEASUREMENTS



$$\frac{u_0}{u_s} = F\left(\frac{x-x_0}{\Theta}, \frac{s}{\Theta}\right) \qquad \frac{\delta}{\Theta} = G\left(\frac{x-x_0}{\Theta}, \frac{s}{\Theta}\right)$$

FIG. 3 NOTATION AND IDEALISED FLOW FIELD

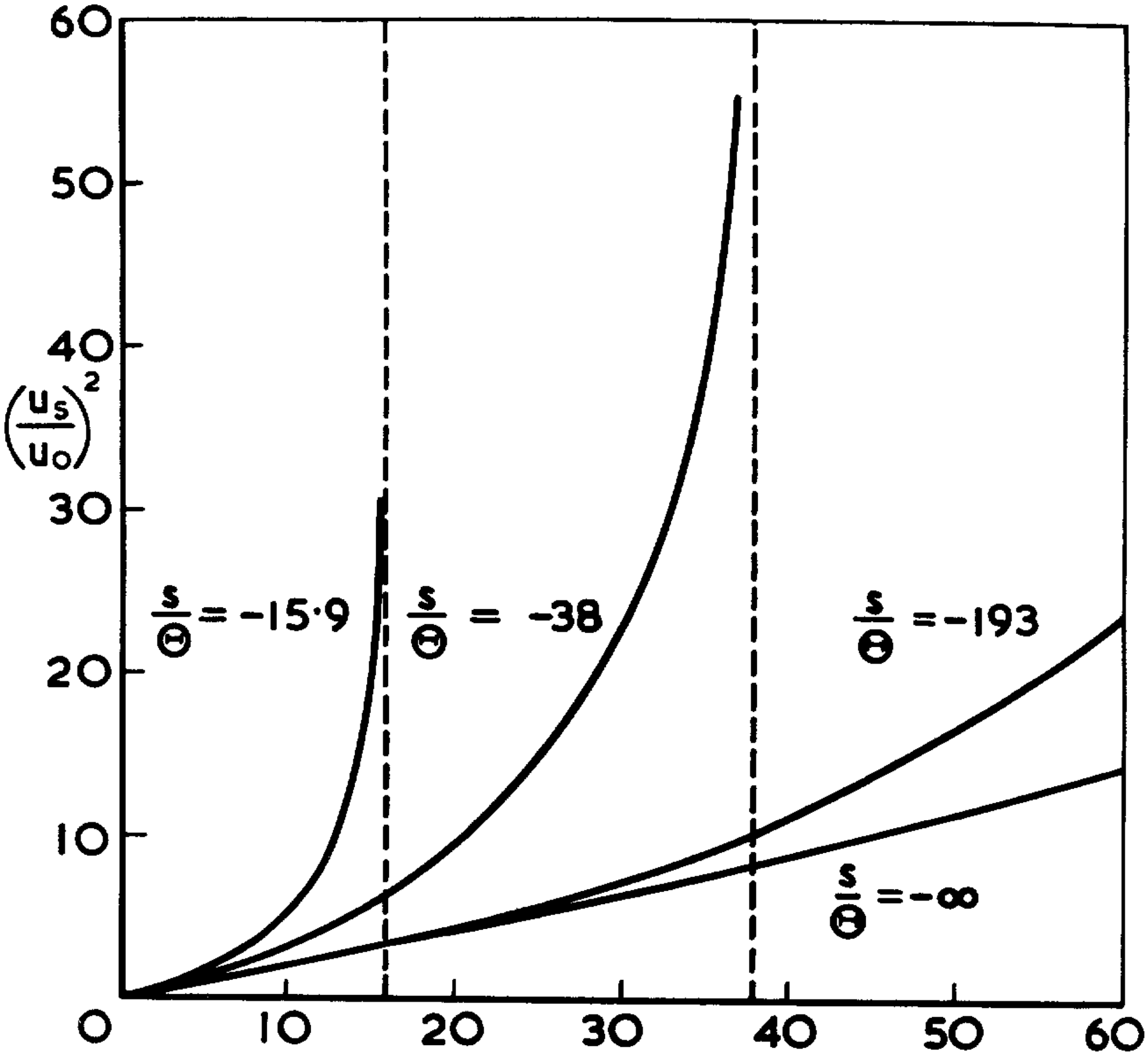


FIG. 4 SKETCH OF OVERALL VARIATION OF CENTRE-LINE VELOCITY

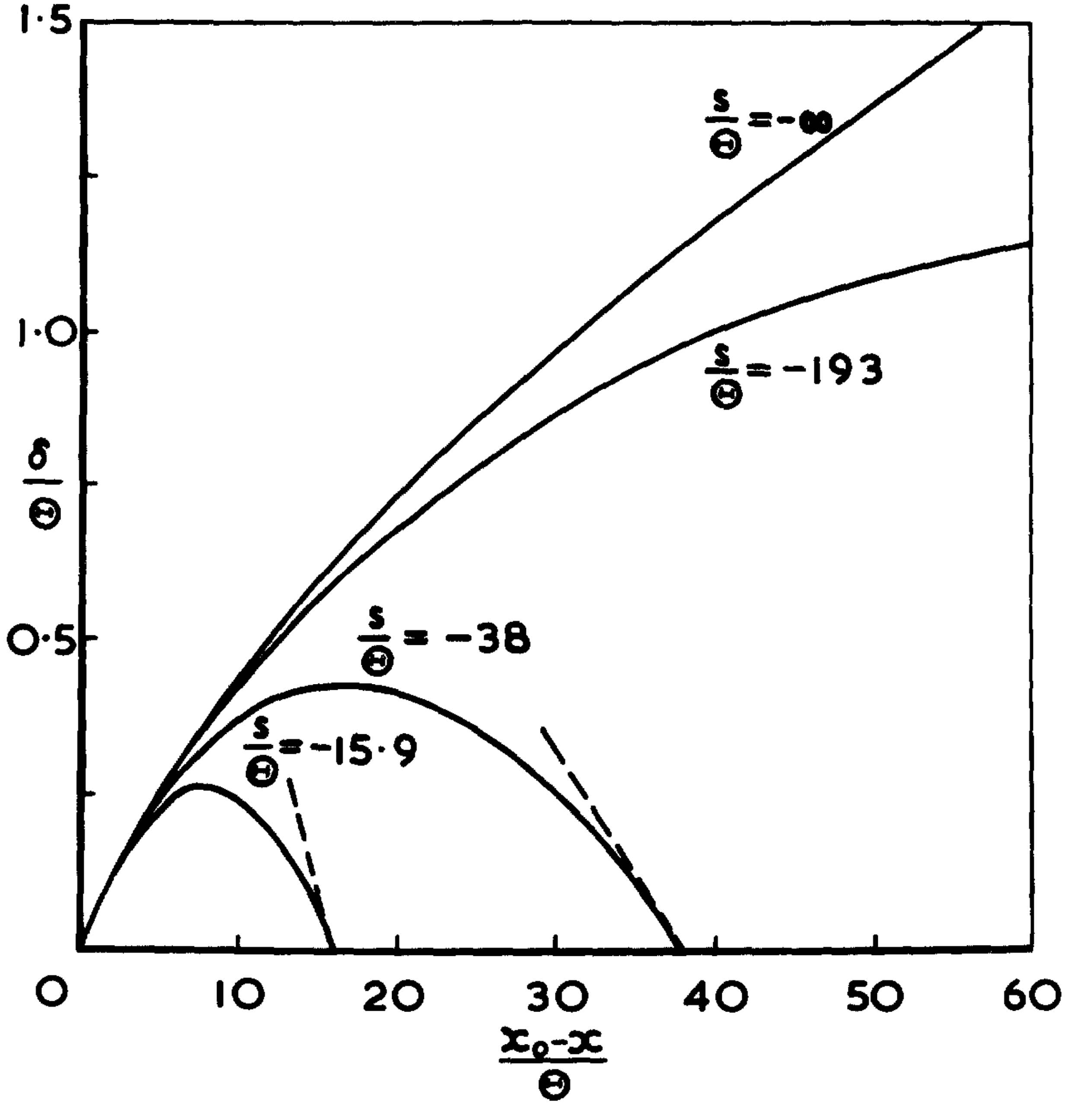
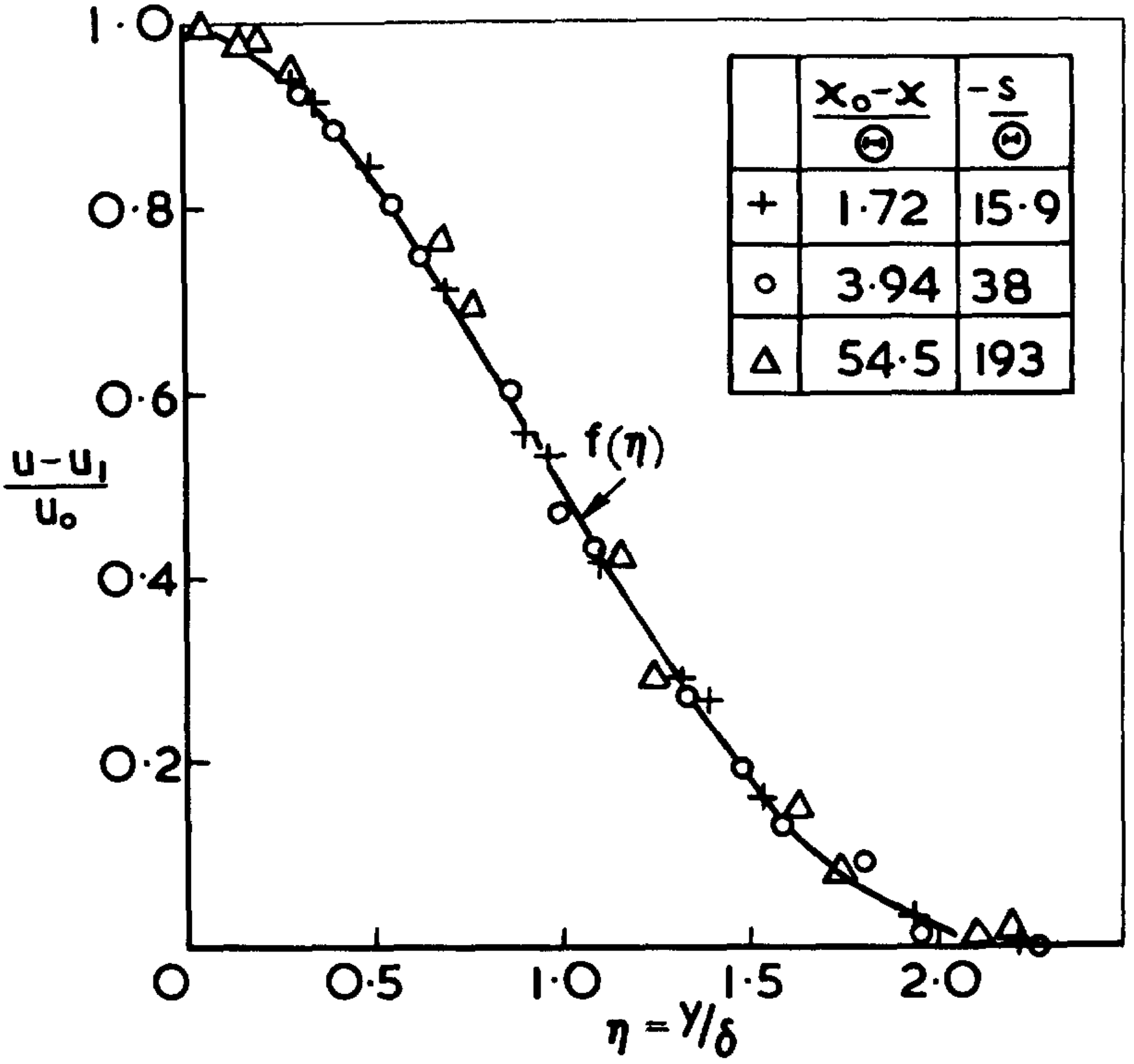


FIG. 5 SKETCH OF OVERALL VARIATION OF JET WIDTH



$$f(\eta)=\exp\left[-0{\cdot}6749\eta^2\left(1+0{\cdot}027\eta^4\right)\right]$$

FIG. 6 MEAN VELOCITY PROFILES

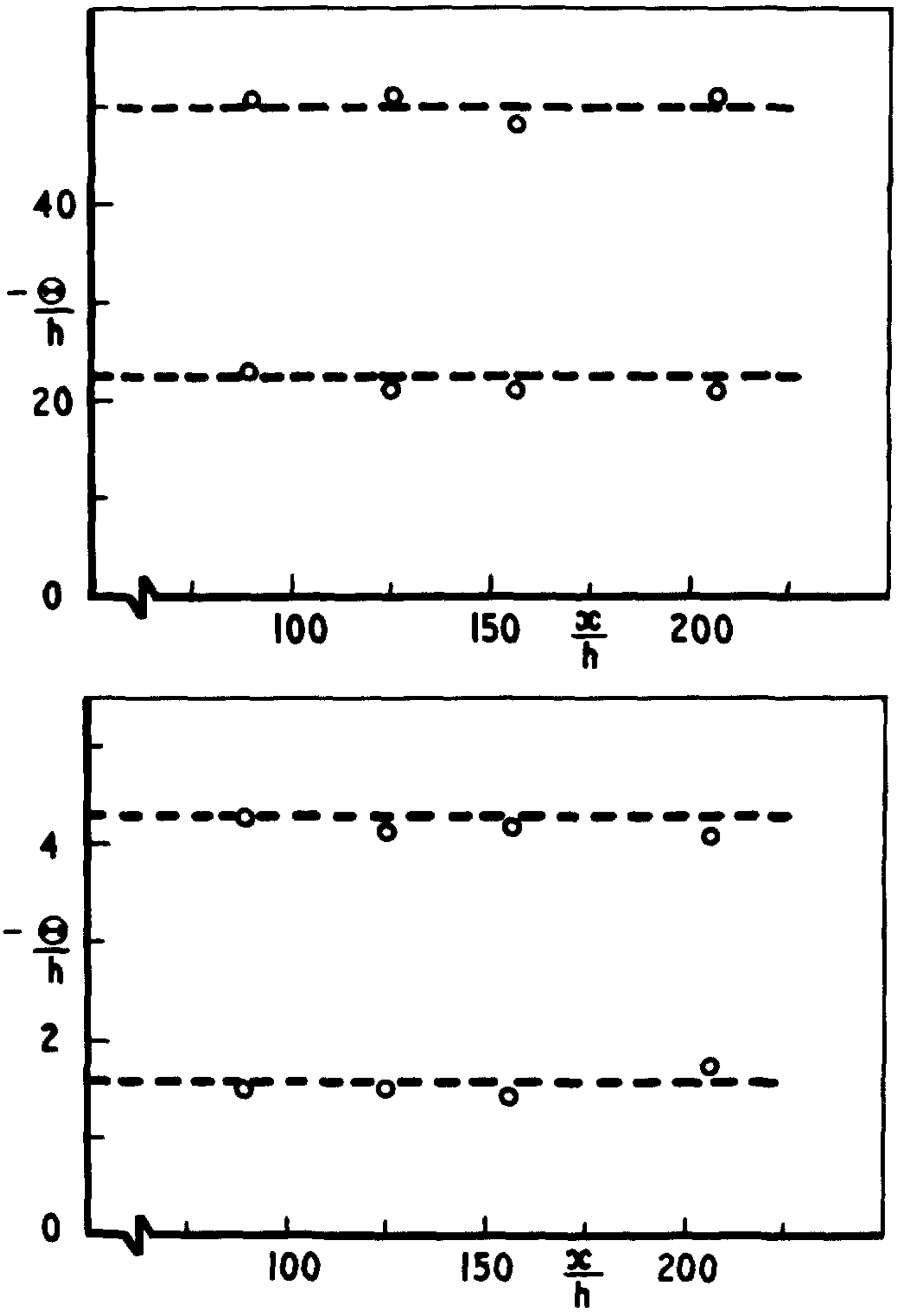


FIG. 7 INITIAL MOMENTUM THICKNESS EVALUATED AT FOUR STREAMWISE STATIONS

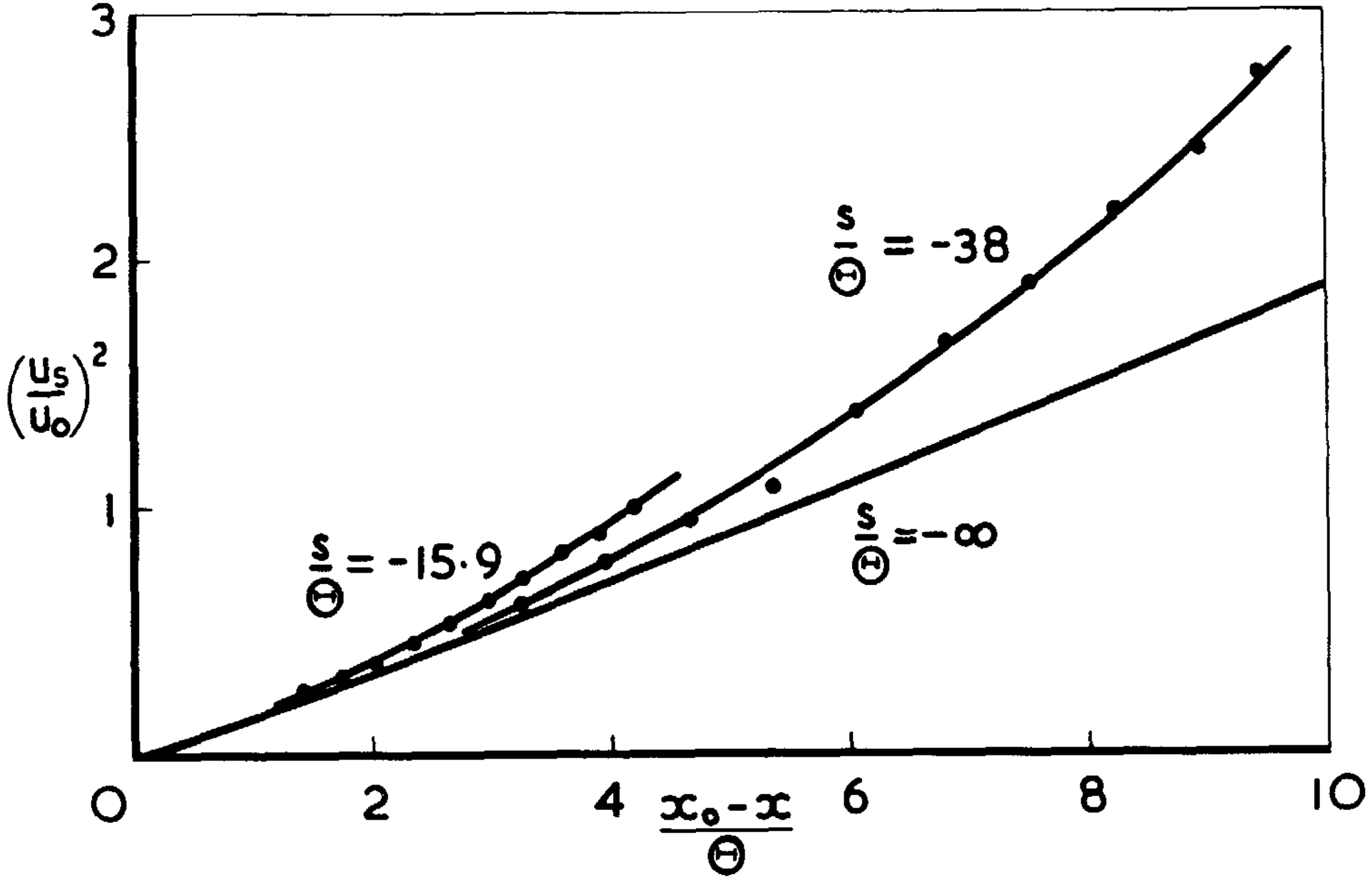


FIG. 8 DECAY OF CENTRE LINE VELOCITY, $\frac{x_0-x}{\Theta} < 10$

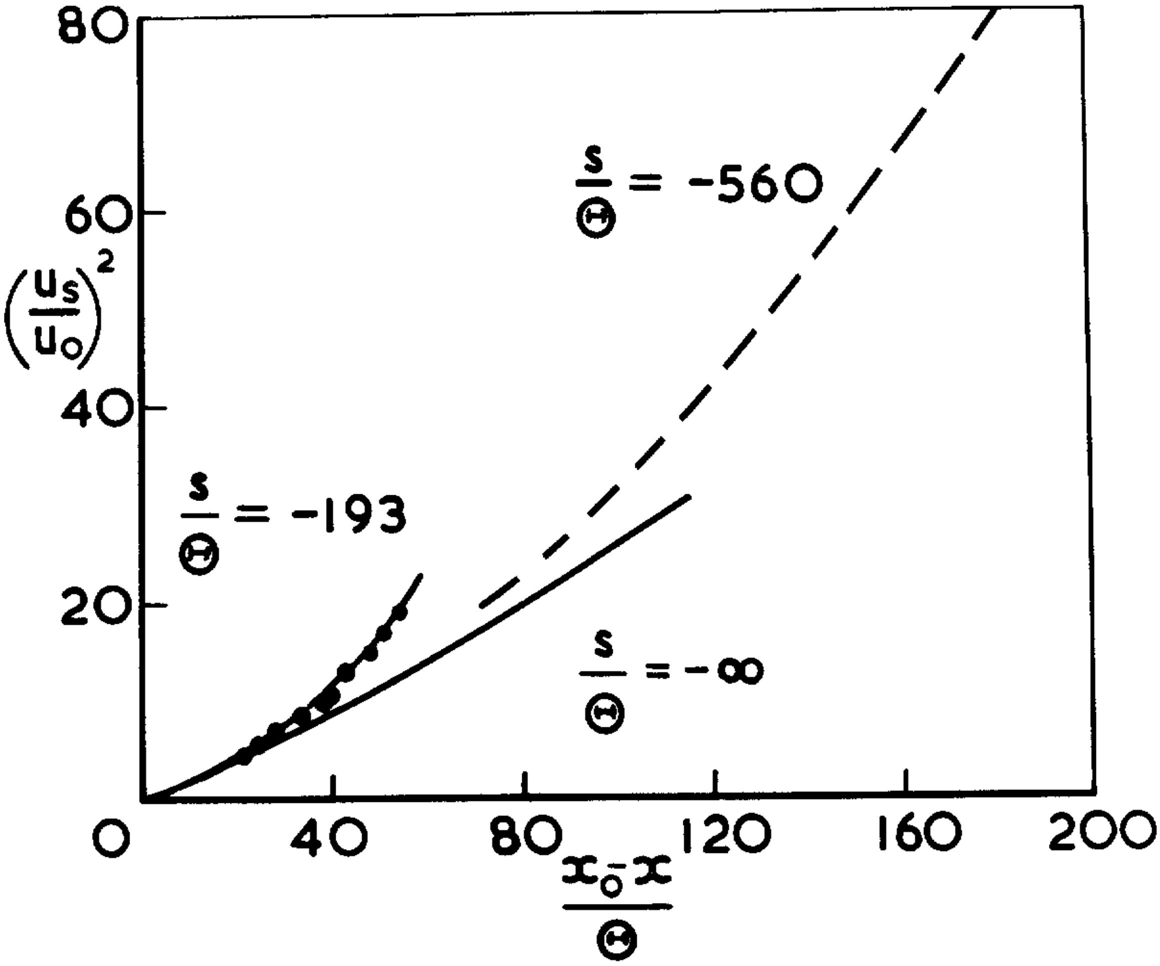


FIG. 9 DECAY OF CENTRE LINE VELOCITY, $\frac{x_0 - x}{\Theta} < 200$

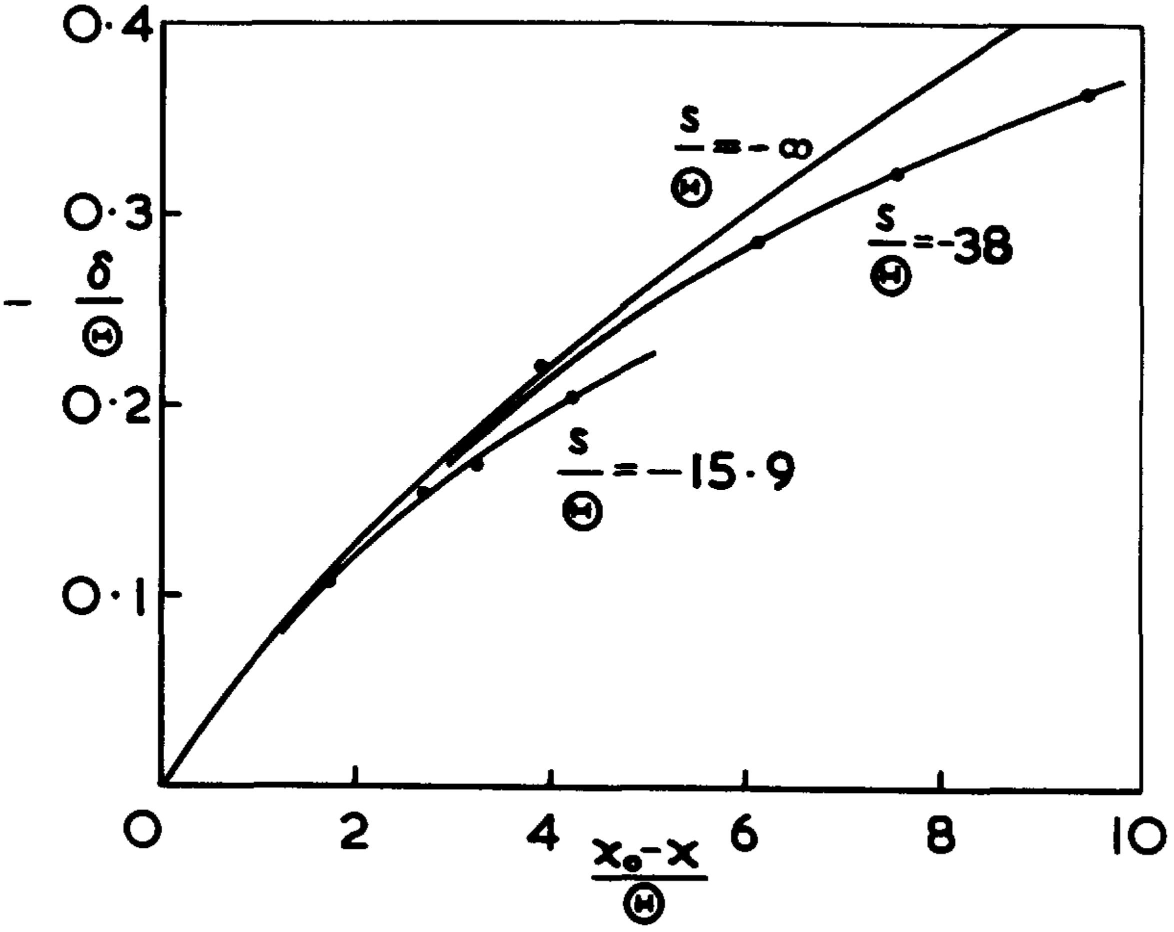


FIG. 10 SPREAD OF JET, $\frac{x_0 - x}{\Theta} < 10$

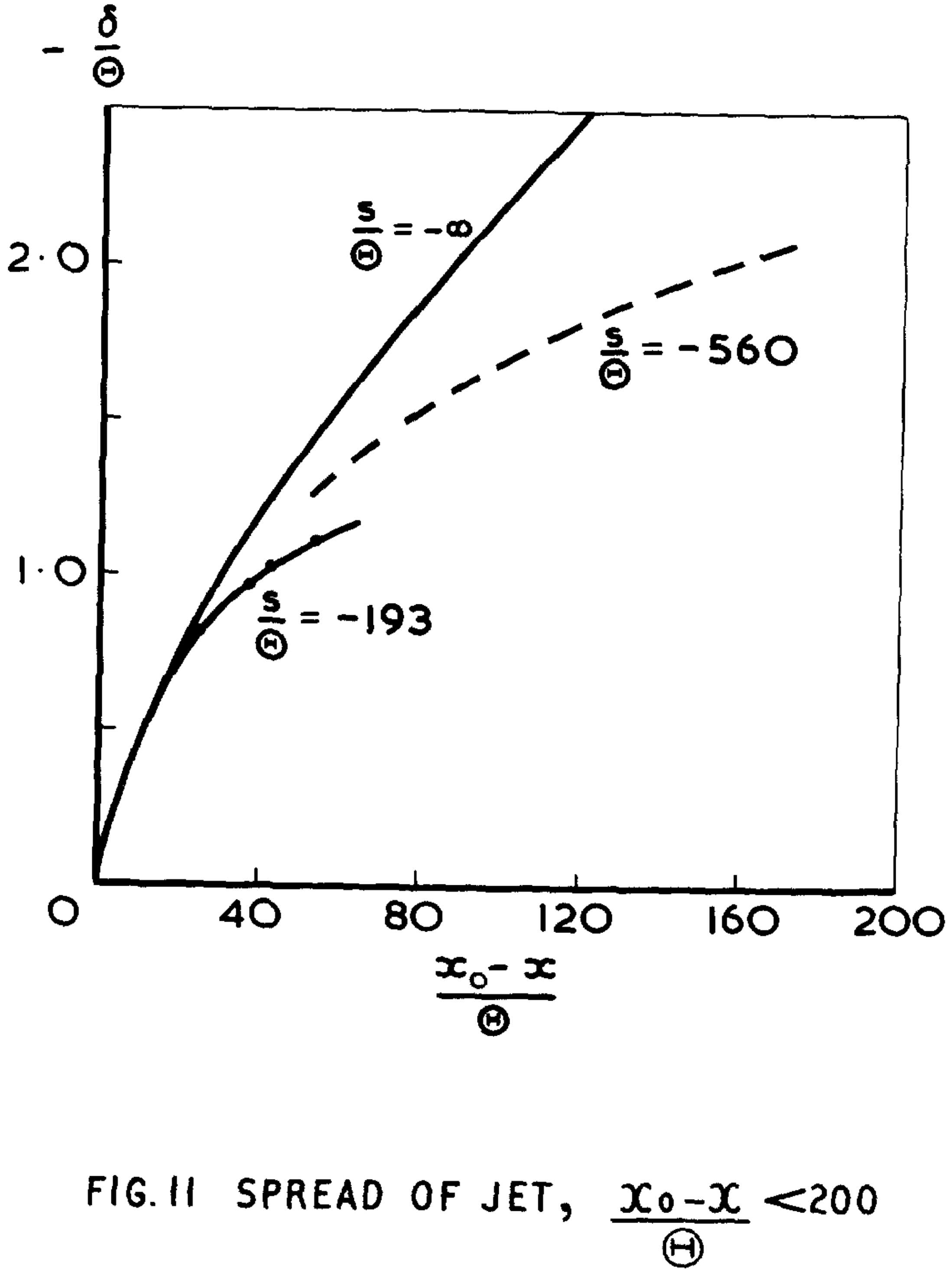


FIG. 11 SPREAD OF JET, $\frac{x_0 - x}{\Theta} < 200$

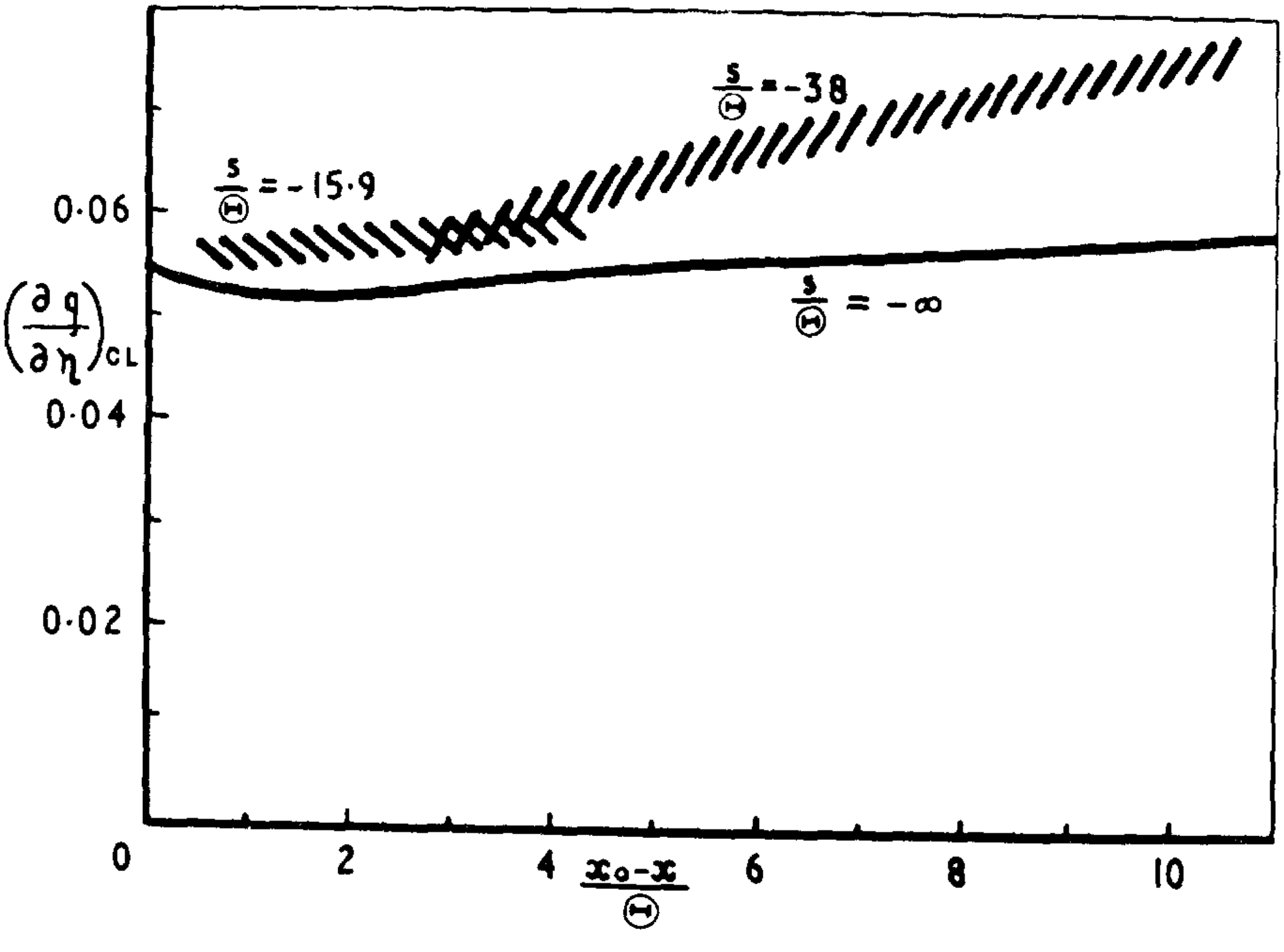


FIG 12 CENTRE LINE VALUES OF TRANSVERSE SHEAR-STRESS GRADIENT

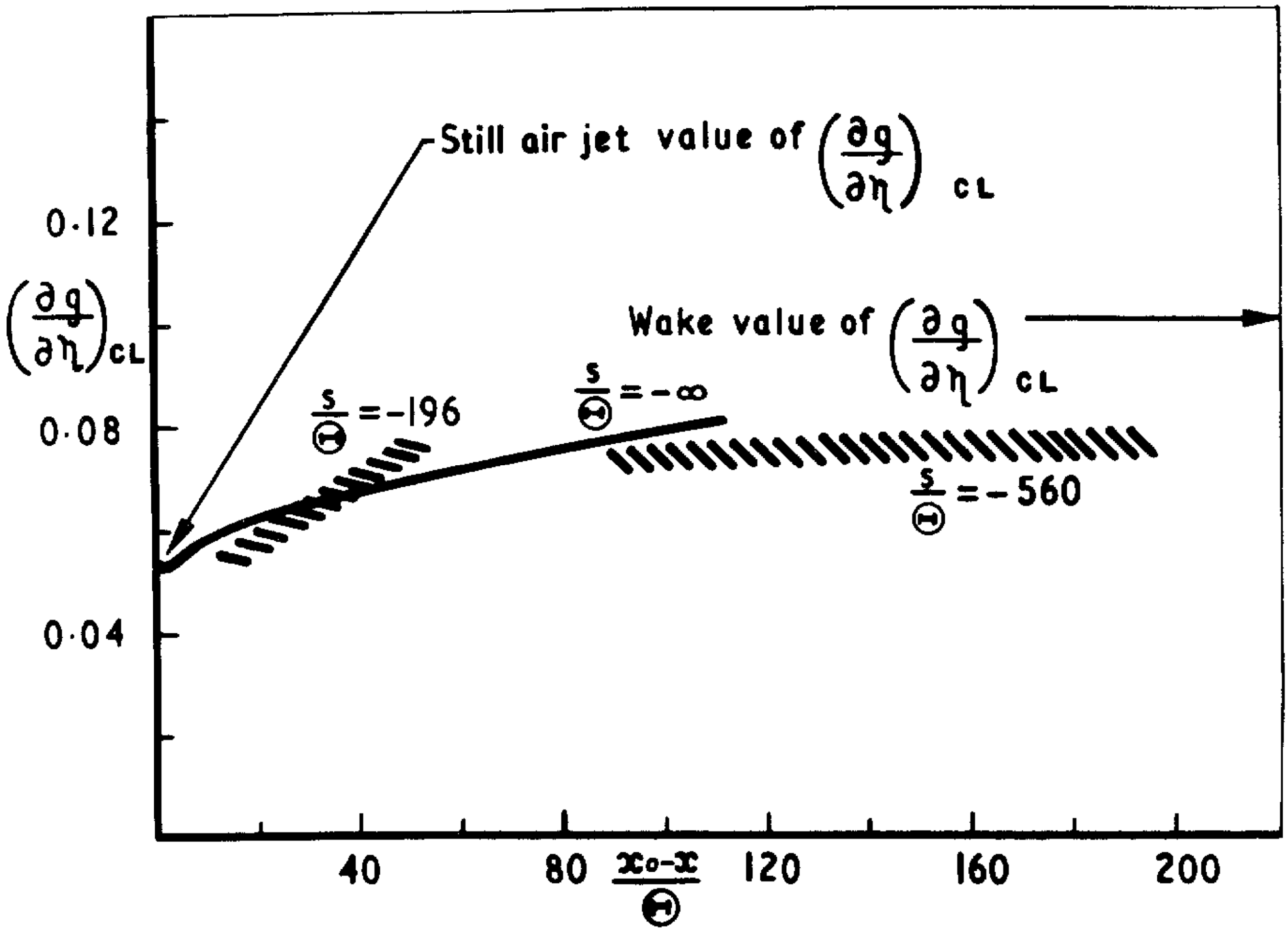


FIG. 13 CENTRE LINE VALUES OF TRANSVERSE SHEAR - STRESS GRADIENT

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