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Tailplane Loads and Normal Accelerations after an Automatic Control Failure

By

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Tail Plane Loads and Normal Accelerations After an Automatic Control Failure

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SUMMARY

A method is given for the calculation of aircraft behaviour when failure of the autopilot in pitch produces sudden elevator movements. Expressions for the changes in aircraft normal acceleration, tail unit accelerations and aerodynamic tail loads are derived. These expressions together with their maxima and the times taken to reach these maxima are tabulated. A calculation on a specific aircraft shows the use of the table.

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1 Introduction

The safety aspects of aircraft with automatic pilots and powered controls have been considered at a meeting held at the R.A.E.[6]. One of the findings of this meeting was that calculations of the probable aircraft behaviour after autopilot failure would be of great use in the design stages for ensuring safety in new aircraft types, and that to this end publication of a note defining a standard approach to the calculations would be of considerable value. The aim of this report is to provide a method by which the calculations can be made when considering autopilot failure in pitch.

From the stressman's point of view that most important things about the aircraft behaviour after failure of the autopilot in pitch are the normal accelerations at the c.g., the tail unit accelerations (different from those at the c.g. due to angular acceleration) and the aerodynamic tail loads. Thus this report concentrates on the derivation of expressions for these accelerations and loads in terms of the time from the start of the uncontrolled motion. As the chief interest is in the maximum values, these have been calculated together with the times required to reach them from the moment of autopilot failure.

2 Basic Assumptions

To simplify the analytical approach to the problem the following assumptions are made.

(i) The forward speed is constant during the disturbed motion of the aircraft induced by the failure of the autopilot, up to the time of occurrence of maxima of the quantities considered.

(ii) The component of the aircraft weight normal to the flight path remains constant during that time.

(iii) The lift and the pitching moment on the tailplane produced by changes in elevator deflection are negligible.

These assumptions are identical with those made by Czaykowski[1] and thus the equations of motion of the aircraft, equations (5) and (6) of Appendix I, are also identical.

The following assumption concerning the elevator movement is intended to cover the general requirements[5] relating to malfunction of the autopilot. It is assumed that, after the failure of the autopilot in pitch, there is an initial instantaneous movement of the elevator which is limited either by the control hitting the stop or by the aerodynamic hinge moment equalling the stalling torque of the motor. In addition, if the control reaches the stop, either instantaneously or after a time interval (due to aircraft response), then it is assumed to stay there.

The assumption of an initial instantaneous elevator movement gives conservative values for the estimation of tail loads for design since the control movement will not be instantaneous in practice. On the other hand the effect of the rate of elevator application on the maximum normal acceleration realised is small, though the time to reach the peak may be altered appreciably.

With the above assumption the following three types of elevator movement are possible:—

Type A : The elevator is deflected instantaneously right to the stop and remains there throughout the forced manoeuvre.

Type B : The elevator is deflected instantaneously without reaching the stop. Owing to the ensuing response of the aircraft a further elevator movement takes place during which the elevator never reaches the stop.

Type C : The elevator is deflected instantaneously without reaching the stop. Because of the aircraft response it is moved farther, reaches the stop, and from then on stays there throughout the manoeuvre.

For any one aircraft and autopilot combination only one of the three types of elevator motion will occur in any given flight condition.

3 Method

The analytical treatment of the problem, presented in Appendix I, consists in finding the response of the aircraft expressed in general terms for each of the three types of elevator movement, the initial attitude of the aircraft corresponding with steady level flight conditions. Expressions are derived for the changes in the aircraft normal accelerations, tail unit accelerations and aerodynamic tail loads. These expressions are analysed and maximum values found for all the types of elevator motion referred to above together with the times taken to reach these maxima.

General formulae for the coefficients of aircraft normal acceleration n , tail unit acceleration n_t , and the aerodynamic load P together with their maximum values and times of occurrence of these maxima are collected in Table I. These expressions give incremental values which must be added to the values realised in the initial steady flight condition.

4 Specimen Calculation

Consider a particular aircraft flying in level flight at a given speed and altitude. Then, with the assumptions of this report, failure of the autopilot control will result in one, and only one, of the types of motion suggested in Section 3. Once the type of motion has been determined, the design accelerations and tail loads follow readily from Tables I and II. The initial instantaneous elevator deflection (assuming no stops) due to application of full torque from the control motor $\left(\frac{C_H}{b_2}\right)$ must be calculated. Where this is greater than the deflection permitted by the elevator stops (i.e. $\frac{C_H}{b_2} > \eta_F$) the stops are reached instantaneously with control failure and the motion conforms to Type A; where it is less, the motion will be either of Type B or C. The maximum elevator angle which would be reached because of the aircraft response (again assuming no stops) can now be calculated from equations (63) or (64) of Appendix I. Where this is greater than the deflection to the stops the motion conforms to Type C and where less to Type B. Inspection of equations (63) and (64) shows that with $\frac{b_1}{b_2}$ $(\equiv \bar{b})$ negative the response of the aircraft will always diminish the elevator angle and thus, in this case, the maximum elevator angle will be the one reached instantaneously with failure and, if the stops are not reached then, the motion will be of Type B.

Take as an example an aircraft with the following characteristics:-

W	$= 63{,}000$ lb	$\left(\dfrac{\partial C_M}{\partial \alpha}\right)$ less tail	$= 0.344$ per radian
S	$= 1408\ ft^2$	$\dfrac{\partial \varepsilon}{\partial \alpha}$	$= 0.35$
c	$= 13.42$ ft	a_1	$= 3.84$ per radian
S'	$= 401\ ft^2$	a_2	$= 1.81$ per radian
ℓ	$= 42.25$ ft	b_1	$= 0.86$ per radian
$\bar{\bar{V}}$	$= 0.8966$	b_2	$= -0.109$ per radian
k_B	$= 13.4$ ft	$\bar{b}$	$= -0.789$
a	$= 4.53$ per radian	η_F	$= \pm 10^\circ$
$\dfrac{\partial C_M}{\partial \alpha}$	$= -1.894$ per radian		

With these basic data ν, χ and R have the following values,

$$\nu = 5.44 \quad ; \quad \chi = 1.90 \quad \text{and} \quad R = 4.802$$

Knowing the speed and height of flight it is possible to calculate the other necessary coefficients. Thus with

V	= 260 m.p.h. E.A.S. at sea level	$\mu =$	13.83
V	= 381 ft/sec (true)	$\hat{t} =$	1.53 seconds
ρ	$= 0.002378\ slug/ft^3$	$B =$	3.126
$\frac{1}{2}\rho V^2$	$= 173\ lb/ft^2$	$C =$	0.375
Mach No.	= low	$\delta =$	35.44
ω	= 41.36	$C_H =$	$\pm$ 0.004
J	= 5.533		

Now $\dfrac{C_H}{b_2} = \pm 0.0372$ radians $= \pm 2.13^\circ$ i.e. less than η_F and this eliminates Type A motion.

$\bar{\chi}$, $\bar{\omega}$, $\bar{\nu}$ must now be evaluated and the following results are obtained

$$\bar{\chi} = 2.608 \quad ; \quad \bar{\omega} = 59.56 \quad ; \quad \bar{\nu} = 7.46$$

With these values the roots of equation (9) (the stability quadratic) are complex and this eliminates the type of motion B(i) i.e. the formulae containing λ's in Table I can be ignored. $\bar{R}$ and $\bar{J}$ may now be computed (had the roots of equation (9) been real λ_1 and λ_2 would have been calculated). Thus,

$$\bar{J} = 6.20 \quad ; \quad \bar{R} = 6.166 \quad ; \quad \bar{J}^2 + \bar{R}^2 = 76.46$$

Normally the maximum elevator angle which would be reached as a result of the aircraft response would have to be computed from equation (64). However, as noted above, with $\bar{b}$ negative the maximum elevator angle is reached at the moment of autopilot failure. Thus the stops are never reached and the only formulae to be considered are those of Type B(ii) Table I.

Note that 'η max' computed from equation (64) is $0.65\,\bar{\eta}$ showing that the minimum elevator angle reached is, in this case, about two thirds of the initial angle.

Should the angle found from equation (64) be greater than η_F then only the formulae of Type C in Table I need be considered.

Finally compute the following

D	$= 17.52$	$n_{1_{max}}$	$= 1.32$
$\bar{H}$	$= 2.45$	$n_{t_{max}}$	$= 1.39$
A	$= 69.3 \times 10^3$ lb	P_{max}	$= 1,310$ lb

5 Further Work

Further work is required to investigate the effects of:-

(a) Finite rates of elevator movement after autopilot failure as compatible with the characteristics of the autopilot-servo unit-elevator combination.

(b) Pilot's corrective action.

NOTATION

$A \quad = \frac{1}{2}\rho V^2 S'$

$a \quad = \frac{\partial C_L}{\partial \alpha}$ whole aircraft

$\bar{a} \quad = \frac{a_1}{a_2}$

$a_1 \quad =$ slope of lift-incidence curve of tailplane

$a_2 \quad =$ slope of lift-elevator deflection curve

$B \quad = \left(1 - \frac{d\varepsilon}{d\alpha} + \frac{a}{2\mu}\right) a_1$

$\bar{b} \quad = \frac{b_1}{b_2}$

$b_1 = \dfrac{\partial C_{H_e}}{\partial \alpha'}$

$b_2 = \dfrac{\partial C_{H_e}}{\partial \zeta}$

$C = \left(1 + \dfrac{d\varepsilon}{d\alpha}\right) \dfrac{a_1}{\mu}$

C_M = pitching moment coefficient of aircraft

C_{H_o} = elevator hinge moment coefficient in steady flight (equation (1))

$\dfrac{\partial C_M}{\partial \alpha} = \left(\dfrac{\partial C_M}{\partial \alpha}\right)_{\text{less tail}} - \dfrac{S'\ell}{Sc}\left(1 - \dfrac{d\varepsilon}{d\alpha}\right) a_1$

C_{H_t}, C_H = elevator hinge moment coefficient corresponding to the constant hinge moment applied by the autopilot servo motor

C_{H_e} = elevator hinge moment coefficient

c = wing mean chord

$D = \frac{1}{2}\rho V^2 S \frac{a}{W}$ (equation (34))

E = arbitrary constant, equation (10)

F = arbitrary constant, equation (10)

G, $\bar{G}$ = see Table II

H, $\bar{H}$ = see Table II

$J = \sqrt{\omega + \frac{1}{2} a\nu - R^2}$

$\bar{J} = \sqrt{\bar{\omega} + \frac{1}{2} a\bar{\nu} - \bar{R}^2}$

K = see Table II

k_B = radius of gyration of aircraft about lateral axis

L = arbitrary constant, equation (13)

ℓ = distance from c.g. of aircraft to mean quarter chord point of tailplane

M, $\bar{M}$ = see Table II

$N = 1 - \dfrac{\bar{b}}{\bar{a}}$

n_1 = coefficient of aircraft normal acceleration at c.g.

n_t = coefficient of normal acceleration at the tailplane $\frac{1}{4}$ chord point

P = aerodynamic tail load

Q, $\overline{Q}$ = see Table II

q = angular velocity of aircraft in pitch

$\hat{q}$ = $\hat{t}q$ = non-dimensional angular velocity in pitch

R = $\frac{1}{2}(\nu + \chi + \frac{1}{2}a)$

$\overline{R}$ = $\frac{1}{2}(\overline{\nu} + \overline{\chi} + \frac{1}{2}a)$

S = wing area

S' = tailplane and elevator area

t = $\hat{t}.\tau$ = time in seconds

$\hat{t}$ = $\mu \frac{\ell}{V}$ = aerodynamic time unit, seconds

V = aircraft forward speed

$\overline{V}$ = tail volume coefficient = $\frac{S'\ell}{Sc}$

W = aircraft weight

w = velocity component in a vertical plane perpendicular to initial flight path (positive down)

$\hat{w}$ = $\frac{w}{V}$ (= incremental incidence, α)

$\hat{w}_o$ = see Table II

$\left(\frac{d\hat{w}}{d\tau}\right)_o$ = see Table II

α = wing incidence

α'_o = see equations (1) and (2)

α'_t = see equations (1) and (2)

α' = effective angle of incidence at the tail plane

γ = see Table II

γ' = see Table II

δ = $-\frac{Wc}{2g\rho S k_B^2} \cdot \frac{\partial C_M}{\partial \eta}$ = elevator effect coefficient

ε = angle of downwash at the tail

ζ, ζ' = see Table II

η = elevator deflection from initial steady flight conditions

$\bar{\eta}$ = $\frac{C_H}{b_2}$ = elevator deflection corresponding to application of full torque from the autopilot servo motor

η_F = elevator angle from trimmed condition to elevator stop

Λ = see Table II

λ_1, λ_2 = roots of stability quadratic

μ = $\frac{W}{\rho g S \ell}$

ν = $\frac{1}{2} \frac{S' \ell^2}{S k_B^2} \cdot a_1$

$\bar{\nu}$ = $\nu - \frac{\delta \bar{b}}{\mu}$

ρ = air density

τ = aerodynamic time = $\frac{t}{\hat{t}}$

ϕ, ϕ' = see Table II

χ = $\frac{d\varepsilon}{d\alpha} \cdot \nu$

ω = $- \frac{Wc}{2 g \rho S k_B^2} \cdot \frac{\partial C_M}{\partial \alpha}$

REFERENCES

No.	Author	Title, etc.
1	T. Czaykowski	Estimation of tail-plane loads in Pull Out Manoeuvres. In course of preparation.
2	B. Melvill-Jones	Aerodynamic Theory. Vol.5 N.II Sect.41. Edited by Durand.
3	L.W. Bryant and S.B. Gates	Nomenclature for Stability Coefficients. R & M.1801, 1937.
4	S. Neumark	The disturbed longitudinal motion of an uncontrolled aircraft and of an aircraft with automatic control. R & M.2078, 1943.
5	-	Design Requirements for Aeroplanes. (A.P.970).
6	E. Finn and E.A. Poulton	Aircraft with automatic pilots and powered controls: safety aspects. R.A.E. Technical Note No. Structures 58 A.R.C.13,609

APPENDIX I

Mathematical analysis of aircraft response to control failure in pitch

A.1 Elevator deflections under constant hinge moment

One of the assumptions made in deriving the equations of motion is that the aircraft forward speed remains constant during the short time interval in which we are interested. It is further assumed that failure of the automatic pilot results in an immediate application of a constant torque equal to the stalling torque of the control motor. Thus a constant hinge moment is applied to the control and, with the assumption of constant aircraft speed and small change in air density, the hinge moment coefficient will remain constant.

At any time before failure the aircraft is flying trimmed and the elevator hinge moment coefficient will be zero.

Thus

$$C_{H_o} = b_1\,\alpha'_o + b_2\,\eta_o = 0 \qquad (1)$$

At any other time, t after failure, the hinge moment coefficient will be connected to the tail incidence and elevator angle by the equation,

$$C_{H_t} = b_1\,\alpha'_t + b_2\,\eta_t \qquad (2)$$

Subtracting (1) from (2) gives

$$C_{H_t} = b_1\,(\alpha'_t - \alpha'_o) + b_2\,(\eta_t - \eta_o)$$

Thus if we write

$\alpha' \equiv \alpha'_t - \alpha'_o$ = change of incidence at the tail

$\eta \equiv \eta_t - \eta_o$ = change of elevator angle from trimmed position

$C_H \equiv C_{H_t}$ = hinge moment coefficient corresponding to the constant hinge moment applied by the autopilot

Then

$$C_H = b_1\,\alpha' + b_2\,\eta \qquad (3)$$

The change of incidence at the tail is given by,

$$\alpha' = \frac{w}{V}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{\ell}{V}\cdot q + \frac{\ell}{V^2}\cdot\frac{dw}{dt}\cdot\frac{d\varepsilon}{d\alpha}$$

The first and second terms in this expression are readily appreciated as being due to the change in incidence of the whole aircraft less the effect of downwash and the rotation of the aircraft respectively. The third term arises due to the time lag ℓ/V seconds between the creation of the downwash by the wings and its action on the tail[2].

The elevator angle, η , is governed then by the following equation,

$$\eta = \frac{C_H}{b_2} - \frac{b_1}{b_2}\left\{\frac{w}{V}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{\ell}{V} \cdot q + \frac{\ell}{V^2} \cdot \frac{dw}{dt} \cdot \frac{d\varepsilon}{d\alpha}\right\}$$

At the instant of control failure $w = q = 0$ and we shall see later from the equations of motion that this results in $\frac{dw}{dt} = 0$. Thus the initial change in elevator angle will be equal to $\frac{C_H}{b_2}$ and we may write this equal to $\bar{\eta}$.

It is convenient to use non-dimensional notation as established in Ref.3 with modifications introduced in Ref.4 together with the additional use of $\hat{q}$ instead of $\hat{t}q$.

Thus

$$\eta = \bar{\eta} - \bar{b}\left\{\hat{w}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{\hat{q}}{\mu} + \frac{1}{\mu} \cdot \frac{d\varepsilon}{d\alpha} \cdot \frac{d\hat{w}}{d\tau}\right\} \qquad (4)$$

where

$$\bar{b} = \frac{b_1}{b_2}$$

A.2 The equations of motion

The equations of motion in non-dimensional form appear as,

$$\frac{d\hat{w}}{d\tau} + \frac{a}{2}\hat{w} - \hat{q} = 0 \qquad (5)$$

$$\chi\frac{d\hat{w}}{d\tau} + \omega\hat{w} + \frac{d\hat{q}}{d\tau} + \nu\hat{q} = -\delta \cdot \eta \qquad (6)$$

The reader to whom these equations are not familiar may consult such works as Refs.2 and 4.

A.3 The equations of motion rewritten to include the effect of elevator movement under constant hinge moment

By putting the value of η established under (4) into equation (6) we arrive at the following for the second equation of motion,

$$\left(\chi - \frac{\delta}{\mu} \cdot \bar{b} \cdot \frac{d\varepsilon}{d\alpha}\right)\frac{d\hat{w}}{d\tau} + \left[\omega - \delta \cdot \bar{b}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right]\hat{w} + \frac{d\hat{q}}{d\tau} + \left(\nu - \frac{\delta\bar{b}}{\mu}\right)\hat{q} = -\delta\bar{\eta}$$

or

$$\bar{\chi}\frac{d\hat{w}}{d\tau} + \bar{\omega}\hat{w} + \frac{d\hat{q}}{d\tau} + \bar{\nu} \cdot \hat{q} = -\delta \cdot \bar{\eta} \qquad (7)$$

(Note: this is of the same form as (6))

where

$$\overline{\chi} = \chi - \frac{\delta}{\mu} \cdot \overline{b} \cdot \frac{d\varepsilon}{d\alpha}$$

$$\overline{\omega} = \omega - \delta \cdot \overline{b}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$\overline{\nu} = \nu - \frac{\delta \overline{b}}{\mu}$$

A.4 The solution of the equations

The equations to be solved are,

$$\frac{d\hat{w}}{d\tau} + \frac{a}{2}\hat{w} - \hat{q} = 0 \qquad (5)$$

$$\overline{\chi} \cdot \frac{d\hat{w}}{d\tau} + \overline{\omega} \cdot \hat{w} + \frac{d\hat{q}}{d\tau} + \overline{\nu}\,\hat{q} = -\delta\overline{\eta} \qquad (7)$$

and $\overline{\eta}$ is a constant $= \frac{C_H}{b_2}$.

$\hat{q}$ may be eliminated from equation (7) by substituting from (5) and $\frac{d\hat{q}}{d\tau}$ may be eliminated by differentiating (5) and substituting the value so obtained in (7). Thus we arrive at the differential equation for $\hat{w}$.

$$\frac{d^2\hat{w}}{d\tau^2} + (\overline{\chi} + \overline{\nu} + \frac{a}{2})\frac{d\hat{w}}{d\tau} + (\overline{\omega} + \frac{a}{2}\,\overline{\nu})\,\hat{w} = -\delta \cdot \overline{\eta} \qquad (8)$$

The solution may take either of two forms depending on whether the roots of the following equation are real or complex,

$$\lambda^2 + (\overline{\chi} + \overline{\nu} + \frac{a}{2})\lambda + (\overline{\omega} + \frac{a}{2}\,\overline{\nu}) = 0 \qquad (9)$$

With the normal range of values of χ, ν and ω the roots of the stability equation with fixed elevator movement are usually complex. However, with the modification of χ, ν and ω to $\overline{\chi}$, $\overline{\nu}$ and $\overline{\omega}$ due to the peculiar form of elevator movement the roots may become real and we give both solutions.

A.4.1 Real roots

Complete solution is

$$\hat{w} = F e^{\lambda_1 \tau} + E e^{\lambda_2 \tau} - \frac{\delta\overline{\eta}}{\lambda_1 \lambda_2} \qquad (10)$$

where

$$\lambda_1 = -\tfrac{1}{2}\left(\overline{\chi} + \overline{\nu} + \frac{a}{2}\right) - \sqrt{\tfrac{1}{4}\left(\overline{\chi} + \overline{\nu} + \frac{a}{2}\right)^2 - \left(\overline{\omega} + \frac{a}{2}\overline{\nu}\right)} \qquad (11)$$

$$\lambda_2 = -\tfrac{1}{2}\left(\overline{\chi} + \overline{\nu} + \frac{a}{2}\right) + \sqrt{\tfrac{1}{4}\left(\overline{\chi} + \overline{\nu} + \frac{a}{2}\right)^2 - \left(\overline{\omega} + \frac{a}{2}\overline{\nu}\right)} \qquad (12)$$

i.e.

$$|\lambda_1| > |\lambda_2|$$

and F and E are arbitrary constants depending on the initial conditions.

A.4.2 Complex roots

Complete solution is

$$\hat{w} = L\, e^{-\overline{R}\tau} \cdot \cos(\overline{J}\tau - \gamma) - \frac{\delta\,\overline{\eta}}{\overline{J}^2 + \overline{R}^2} \qquad (13)$$

where

$$\overline{R} = \tfrac{1}{2}\left(\overline{\chi} + \overline{\nu} + \frac{a}{2}\right) \qquad (14)$$

and

$$\overline{J} = \sqrt{\overline{\omega} + \frac{a}{2}\overline{\nu} - \overline{R}^2} \qquad (15)$$

and L and γ are again arbitrary constants depending on the initial conditions.

A.5 The initial conditions

The aircraft is considered initially to be in straight and level flight and this means that both $\hat{w}$ and $\hat{q}$ are zero. From the first equation of motion (5) it is then apparent that $\frac{d\hat{w}}{d\tau}$ is also zero and we shall take as our initial condition,

$$\hat{w} = \frac{d\hat{w}}{d\tau} = 0 \qquad \text{when} \qquad \tau = 0$$

If the aircraft response is such that the aerodynamic hinge moment of the control falls to a value which allows the control to reach the stop, a new phase of motion will take place. This phase will be governed by equations (5) and (6) the solutions of which are exactly similar to equations (10) and (13) with χ, ν and ω substituted for $\overline{\chi}$, $\overline{\nu}$ and $\overline{\omega}$; and $\overline{\eta}$ is replaced by η_F, the elevator angle at the stop. The values of the constants in the solutions are again derived from the initial conditions for this phase. The condition here is that $\hat{w}$ and $\hat{q}$ must be the same as at the end of the first phase and as the first equation of motion (5) does not contain the coefficients χ, ν, ω or η then $\frac{d\hat{w}}{d\tau}$ is also continuous. Thus in the general case we shall take as our initial condition,

A.5.2 Complete solutions - 2nd phase of motion

For aircraft with a normal range of values for χ, ω and ν it is only in rare cases that there will be real roots of equation (9) in the second phase and this case will not be considered.

Complex roots

$$\hat{w} = - \frac{\delta \eta_F}{J^2 + R^2} \left\{ 1 - \Lambda e^{-R\tau} . \cos (J\tau - \gamma) \right\} \qquad (24)$$

$$\frac{d\hat{w}}{d\tau} = - \frac{\delta \eta_F}{(J^2 + R^2)^{\frac{1}{2}}} . \Lambda e^{-R\tau} . \cos (J\tau - \gamma - \xi) \qquad (25)$$

$$\frac{d^2\hat{w}}{d\tau^2} = \delta . \eta_F . \Lambda e^{-R\tau} . \cos (J\tau - \gamma - 2\xi) \qquad (26)$$

where

$$\Lambda = \left\{ \left(1 + \frac{\hat{w}_o}{K}\right)^2 + \left[\left(1 + \frac{\hat{w}_o}{K}\right) \frac{R}{J} + \frac{1}{JK} \cdot \left(\frac{d\hat{w}}{d\tau}\right)_o \right]^2 \right\}^{\frac{1}{2}} \qquad (27)$$

$$K = \frac{\delta \eta_F}{J^2 + R^2} \qquad (28)$$

$$\gamma = \tan^{-1} \left\{ \frac{R}{J} + \frac{1}{JK} \cdot \frac{\left(\frac{d\hat{w}}{d\tau}\right)_o}{\left(1 + \frac{\hat{w}_o}{K}\right)} \right\} \qquad (29)$$

$$\xi = \tan^{-1} . \frac{J}{R} \qquad (30)$$

$$R = \tfrac{1}{2} (\chi + \nu + \frac{a}{2}) \qquad (31)$$

$$J = (\omega + \frac{a}{2} \nu - R^2)^{\frac{1}{2}} \qquad (32)$$

*A.6 Aircraft normal accelerations at the c.g.

Tail unit accelerations

Aerodynamic tail loads

Expressions for these three quantities have been derived in Ref.1 and they are merely quoted here.

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* See Addenda pp. 32 to 35

A.6.1 Aircraft normal acceleration at the c.g.

$$n = 1 + D\hat{w} \tag{33}$$

where

$$D = \tfrac{1}{2}\rho V^2 S \frac{a}{mg} \tag{34}$$

and n is the coefficient of normal acceleration at the c.g.

A.6.2 Tail unit accelerations

$$n_t = 1 + D\left[\hat{w} - \frac{1}{\mu}\left(\frac{2}{a}\cdot\frac{d^2\hat{w}}{d\tau^2} + \frac{d\hat{w}}{d\tau}\right)\right] \tag{35}$$

A.6.3 Aerodynamic tail load

The incremental value of the aerodynamic tail load due to the disturbance is given as

$$P = A\left(B\hat{w} + C\frac{d\hat{w}}{d\tau} + a_2\eta\right)$$

where

$$A = \tfrac{1}{2}\rho V^2 S' \tag{36}$$

$$B = \left(1 - \frac{d\varepsilon}{d\alpha} + \frac{a}{2\mu}\right)a_1 \tag{37}$$

$$C = \left(1 + \frac{d\varepsilon}{d\alpha}\right)\frac{a_1}{\mu} \tag{38}$$

Thus in the 2nd phase of motion if the control has reached the stop,

$$P = A\left(B\hat{w} + C\,.\,\frac{d\hat{w}}{d\tau} + a_2\eta_F\right) \tag{39}$$

and in the 1st phase of motion before the control reaches the stop the value for P is obtained by using the value of η given in equation (4), using equation (5) to eliminate $\hat{q}$.

i.e.

$$\eta = \bar{\eta} - \bar{b}\left\{\hat{w}\left(1 - \frac{d\varepsilon}{d\alpha} + \frac{a}{2\mu}\right) + \frac{1}{\mu}\frac{d\hat{w}}{d\tau}\left(1 + \frac{d\varepsilon}{d\alpha}\right)\right\}$$

$$= \bar{\eta} - \frac{\bar{b}}{a_1}\left\{B\hat{w} + C\frac{d\hat{w}}{d\tau}\right\} \tag{40}$$

Thus

$$P = A\left\{\left[B\hat{w} + C\frac{d\hat{w}}{d\tau}\right]\left(1 - \frac{\bar{b}}{\bar{a}}\right) + a_2\bar{\eta}\right\} \qquad (41)$$

where

$$\bar{a} = \frac{a_1}{a_2}$$

A.7 Three types of motion considered

Three types of motion are considered under the following heads,

Type A

The elevator stops are reached instantaneously with failure of the automatic control.

Type B

The aerodynamic hinge moment is such as to prevent the control from ever reaching the stop.

Type C

The aerodynamic hinge moment is such as to prevent the control from reaching the stop instantaneously with failure but, due to aircraft response, the control eventually reaches the stop.

Types B and C can be subdivided into two further types which, although not different physically, produce different mathematical solutions depending whether the roots of the equation (9) are real (subdivision (i)) or complex (subdivision (ii)).

A.7.1 Type A

Expressions for the normal acceleration, tail unit accelerations and aerodynamic tail loads are obtained by substituting the appropriate values of $\hat{w}$, $\frac{d\hat{w}}{d\tau}$ and $\frac{d^2\hat{w}}{d\tau^2}$ in the equations (33), (35) and (39). In this case the appropriate values are those as given for the 2nd phase of motion with $\hat{w}_0$ and $\left(\frac{d\hat{w}}{d\tau}\right)_0$ equated to zero.

Thus

$$n = 1 - D \cdot \frac{\delta\eta_F}{J^2 + R^2} \cdot \left[1 - \frac{(J^2 + R^2)^{\frac{1}{2}}}{J} \cdot e^{-R\tau} \cdot \cos(J\tau - \gamma)\right] \qquad (42)$$

$$n_t = 1 - D \cdot \frac{\delta\eta_F}{J^2 + R^2}\left[1 - \frac{(J^2 + R^2)^{\frac{1}{2}}}{J}\left\{\left(1 + \frac{R}{\mu} - \frac{2(R^2 - J^2)}{\mu a}\right)^2 + \right.\right.$$

$$\left.\left. + \left(\frac{J}{\mu} - \frac{4JR}{\mu a}\right)^2\right\}^{\frac{1}{2}} e^{-R\tau} \cdot \cos(J\tau - \gamma - \Phi)\right] \qquad (43)$$

where

$$\Phi = \tan^{-1} \frac{Ja - 4JR}{\mu a + aR - 2(R^2 - J^2)} \tag{44}$$

$$P = A\,\eta_F \left\{ a_2 - \frac{\delta}{J^2+R^2} \left[B - \frac{(J^2+R^2)^{\frac{1}{2}}}{J} \sqrt{(B-CR)^2+(CJ)^2} . e^{-R\tau} . \cos(J\tau - \gamma + \zeta) \right] \right\} \tag{45}$$

where

$$\zeta = \tan^{-1} \frac{CJ}{B - CR} \tag{46}$$

Maximum values together with the time taken to reach them are given as follows:-

$$(n)_{max} = 1 - D . \frac{\delta \eta_F}{J^2 + R^2} \left(1 + e^{-\frac{R}{J} . \pi} \right) \tag{47}$$

$$J\tau = \pi$$

$$(n_t)_{max} = 1 - D . \frac{\delta \eta_F}{J^2 + R^2} \left[1 + \left\{ \left(1 + \frac{R}{\mu} - \frac{2(R^2 - J^2)}{\mu a} \right)^2 + \left(\frac{J}{\mu} - \frac{4JR}{\mu a} \right)^2 \right\}^{\frac{1}{2}} . e^{-\frac{R}{J}(\pi + \Phi)} \right] \tag{48}$$

$$J\tau = \pi + \Phi$$

$$P_{max} = A\,\eta_F \left\{ a_2 - \frac{\delta}{J^2 + R^2} \left[B + [(B-CR)^2 + (CJ)^2]^{\frac{1}{2}} . e^{-\frac{R}{J}(\pi - \zeta)} \right] \right\} \tag{49}$$

$$J\tau = \pi - \zeta$$

Note that there may be another minimum or maximum tail load for $J\tau + \zeta = 0$; but this can only be so if ζ can be negative.

A.7.2 <u>Type B(i)</u>

The following are obtained by substituting equations (16), (17) and (18) in (33), (35) and (41).

$$n = 1 - D \cdot \frac{\delta \bar{\eta}}{\lambda_1 \lambda_2} \left\{ 1 + \frac{\lambda_2}{\lambda_1 - \lambda_2} \cdot e^{\lambda_1 \tau} - \frac{\lambda_1}{\lambda_1 - \lambda_2} \cdot e^{\lambda_2 \tau} \right\} \qquad (50)$$

$$n_t = 1 - D \cdot \frac{\delta \bar{\eta}}{\lambda_1 \lambda_2} \left\{ 1 + \frac{1}{\lambda_1 - \lambda_2} \left[\lambda_2 \left(1 - \frac{2\lambda_1^2}{\mu a} - \frac{\lambda_1}{\mu} \right) e^{\lambda_1 \tau} - \right. \right.$$

$$\left. \left. - \lambda_1 \left(1 - \frac{2\lambda_2^2}{\mu a} - \frac{\lambda_2}{\mu} \right) e^{\lambda_2 \tau} \right] \right\} \qquad (51)$$

$$P = A \bar{\eta} \left\{ a_2 - \frac{\delta}{\lambda_1 \lambda_2} \left(1 - \frac{\bar{b}}{\bar{a}} \right) \left[B + \frac{1}{\lambda_1 - \lambda_2} \left[\lambda_2 (B + \lambda_1 C) e^{\lambda_1 \tau} - \lambda_1 (B + \lambda_2 C) e^{\lambda_2 \tau} \right] \right] \right\}$$

$$(52)$$

Maximum values are obtained by putting $e^{\lambda_1 \tau} = e^{\lambda_2 \tau} = 0$ or by giving τ the value appropriate to the pilots reaction time. (Note that, although mathematically the time to reach the maximum is infinite, in practical cases the motion is usually so damped that the maximum is reached within one or two seconds).

Type B(ii)

The following are obtained by substituting equations (19), (20) and (21) in (33), (35) and (41).

$$n' = 1 - D \cdot \frac{\delta \bar{\eta}}{\bar{J}^2 + \bar{R}^2} \left[1 - \frac{(\bar{J}^2 + \bar{R}^2)^{\frac{1}{2}}}{\bar{J}} \cdot e^{-\bar{R}\tau} \cdot \cos(\bar{J}\tau - \gamma') \right] \qquad (53)$$

$$n_t = 1 - D \cdot \frac{\delta \bar{\eta}}{\bar{J}^2 + \bar{R}^2} \left[1 - \frac{(\bar{J}^2 + \bar{R}^2)^{\frac{1}{2}}}{\bar{J}} \left\{ \left(1 + \frac{\bar{R}}{\mu} - \frac{2(\bar{R}^2 - \bar{J}^2)}{\mu a} \right)^2 + \right. \right.$$

$$\left. \left. + \left(\frac{\bar{J}}{\mu} - \frac{4\bar{J}\bar{R}}{\mu a} \right)^2 \right\}^{\frac{1}{2}} e^{-\bar{R}\tau} \cdot \cos(\bar{J}\tau - \gamma' - \phi') \right] \qquad (54)$$

where

$$\phi' = \tan^{-1} \frac{\bar{J}a - 4\bar{J}\bar{R}}{\mu a + a\bar{R} - 2(\bar{R}^2 - \bar{J}^2)} \qquad (55)$$

$$P = A \bar{\eta} \left\{ a_2 - \frac{\delta}{\bar{J}^2 + \bar{R}^2} \left(1 - \frac{\bar{b}}{\bar{a}} \right) \left[B - \frac{(\bar{J}^2 + \bar{R}^2)^{\frac{1}{2}}}{\bar{J}} \right. \right.$$

$$\left. \left. \sqrt{(B - C\bar{R})^2 + (C\bar{J})^2} \cdot e^{-\bar{R}\tau} \cdot \cos(\bar{J}\tau - \gamma' + \zeta') \right] \right\} \qquad (56)$$

where

$$\zeta' = \tan^{-1} \frac{C\bar{J}}{B - C\bar{R}} \tag{57}$$

Maximum values together with the time taken to reach them are given as follows,

$$(n)_{max} = 1 - D \cdot \frac{\delta \bar{\eta}}{\bar{J}^2 + \bar{R}^2} \left(1 + e^{-\frac{\bar{R}}{\bar{J}} \cdot \pi}\right) \tag{58}$$

$$\bar{J}\tau = \pi$$

$$(n_t)_{max} = 1 - D \cdot \frac{\delta \bar{\eta}}{\bar{J}^2 + \bar{R}^2} \left[1 + \left\{\left(1 + \frac{\bar{R}}{\mu} - \frac{2(\bar{R}^2 - \bar{J}^2)}{\mu a}\right)^2 + \left(\frac{\bar{J}}{\mu} - \frac{4\bar{J}\bar{R}}{\mu a}\right)^2\right\}^{\frac{1}{2}} e^{-\frac{\bar{R}}{\bar{J}}(\pi + \Phi')}\right] \tag{59}$$

$$\bar{J}\tau = \pi + \Phi'$$

$$P_{max} = A \bar{\eta} \left\{a_2 - \frac{\delta}{\bar{J}^2 + \bar{R}^2} \cdot \left(1 - \frac{\bar{b}}{\bar{a}}\right)\left[B + [(B - C\bar{R})^2 + (C\bar{J})^2]^{\frac{1}{2}} e^{-\frac{\bar{R}}{\bar{J}}(\pi - \zeta')}\right]\right\} \tag{60}$$

$$\bar{J}\tau = \pi - \zeta'$$

All these values are identical with those obtained in Case A, with values of $\bar{J}$, $\bar{R}$ etc. substituted for J, R etc. except for the inclusion of the factor $\left(1 - \frac{\bar{b}}{\bar{a}}\right)$ in the aerodynamic tail load expression.

A.7.3 <u>Type C</u>

Before solutions for this case can be obtained the value of τ taken to reach the stop must be calculated and this is then used to find the values of $\hat{w}_0$ and $\left(\frac{d\hat{w}}{d\tau}\right)_0$ so that the second phase of motion can be calculated. The time taken to reach the stops is calculated from equation (4), or more conveniently, as expressed in equation (40), by putting $\eta \equiv \eta_F$ and substituting for $\hat{w}$ and $\frac{d\hat{w}}{d\tau}$ either from equations (16) and (17) or (19) and (20).

Thus if the roots of (9) are real then we have to solve,

$$\frac{\eta_F}{\bar{\eta}} = 1 + \frac{\delta \bar{b}}{a_1}\left[\frac{B}{\lambda_1 \lambda_2} + \frac{B + C\lambda_1}{\lambda_1(\lambda_1 - \lambda_2)} \cdot e^{\lambda_1 \tau} - \frac{B + C\lambda_2}{\lambda_2(\lambda_1 - \lambda_2)} \cdot e^{\lambda_2 \tau}\right] \tag{61}$$

for τ .

This can usually be solved by neglecting the term in $e^{\lambda_1 \tau}$ (assuming $|\lambda_1| > |\lambda_2|$), solving and then repeating the calculation treating $e^{\lambda_1 \tau}$ as a correction term. The value of τ so found is then used to calculate $\hat{w}_o$ and $\left(\frac{d\hat{w}}{d\tau}\right)_o$ from equations (16) and (17).

When (9) has complex roots then the equation for the derivation of τ is,

$$\frac{\eta_F}{\bar{\eta}} = 1 + \frac{\delta \bar{b}}{a_1} \left[\frac{B}{\bar{J}^2 + \bar{R}^2} - \frac{\left[(B-C\bar{R})^2 + (C\bar{J})^2\right]^{\frac{1}{2}}}{\bar{J}(\bar{J}^2 + \bar{R}^2)^{\frac{1}{2}}} e^{-\bar{R}\tau} . \cos(\bar{J}\tau - \gamma' + \zeta') \right] \quad (62)$$

These two equations for τ are best solved graphically and a method is suggested in Appendix II. Again this value of τ is replaced in equations (19) and (20) to find values of $\hat{w}_o$ and $\left(\frac{d\hat{w}}{d\tau}\right)_o$.

It is worth quoting, at this stage, maximum values of η which would be reached were there no stops. These values can then be used to ascertain whether the stops will be reached or not.

Thus

$$\eta_{max} = \bar{\eta} \left\{ 1 + \frac{\delta \bar{b}}{a_1} . \frac{B}{\lambda_1 \lambda_2} \right\} \quad \text{for real roots of equation (9)} \quad (63)$$

or

$$\eta_{max} = \bar{\eta} \left\{ 1 + \frac{\delta \bar{b}}{a_1} \left(\frac{B}{\bar{J}^2 + \bar{R}^2} + \frac{\left[(B-C\bar{R})^2 + (C\bar{J})^2\right]^{\frac{1}{2}}}{\bar{J}^2 + \bar{R}^2} . e^{-\frac{\bar{R}}{\bar{J}}(\pi - \zeta')} \right) \right\}$$

with complex roots of equation (9) (64)

Having obtained values for $\hat{w}_o$ and $\left(\frac{d\hat{w}}{d\tau}\right)_o$ the motion after the stops have been reached is governed by equations (24) to (36) irrespective of whether the roots of equation (9) were real or complex before the stops were reached. Thus we arrive at one set of solutions.

$$n = 1 - D . \frac{\delta \eta_F}{J^2 + R^2} \left\{ 1 - A e^{-R\tau} . \cos(J\tau - \gamma) \right\} \quad (65)$$

$$n_t = 1 - D . \frac{\delta \eta_F}{J^2 + R^2} \left\{ 1 - A \left(\left[1 + \frac{R}{\mu} - \frac{2(R^2 - J^2)}{\mu a} \right]^2 + \left[\frac{J}{\mu} - \frac{4JR}{\mu a} \right]^2 \right)^{\frac{1}{2}} e^{-R\tau} . \cos(J\tau - \gamma - \Phi) \right\} \quad (66)$$

where

$$\Phi = \tan^{-1} \frac{Ja - 4JR}{\mu a + aR - 2(R^2 - J^2)}$$

$$P = A\eta_F \left\{ a_2 - \frac{\delta}{J^2 + R^2} \left[B - \Lambda \left(\{B-CR\}^2 + \{CJ\}^2 \right)^{\frac{1}{2}} e^{-R\tau} . \cos(J\tau - \Upsilon + \zeta) \right] \right\} \tag{67}$$

$$\zeta = \tan^{-1} \frac{CJ}{B - CR}$$

The maximum values together with the time taken to reach them are given as follows. Note that the total time to reach the maximum will be the time quoted here, which is the time taken to reach the maximum once the control has reached the stop, with the addition of the time taken to reach the stop as found from equation (61) or (62).

$$(n)_{max} = 1 - D . \frac{\delta\eta_F}{J^2 + R^2} \left\{ 1 + \frac{J}{(J^2+R^2)^{\frac{1}{2}}} \Lambda e^{-R\tau} \right\} \tag{68}$$

$$J\tau - \alpha = \tan^{-1} \left(- \frac{R}{J} \right)$$

$$(n_t)_{max} = 1 - D . \frac{\delta\eta_F}{J^2+R^2} \left\{ 1 + \frac{J}{(J^2+R^2)^{\frac{1}{2}}} . \Lambda \left(\left\{ 1 + \frac{R}{\mu} - \frac{2(R^2-J^2)}{\mu a} \right\}^2 + \left\{ \frac{J}{\mu} - \frac{4JR}{\mu a} \right\}^2 \right)^{\frac{1}{2}} . e^{-R\tau} \right\} \tag{69}$$

$$J\tau - \alpha - \Phi = \tan^{-1} \left(- \frac{R}{J} \right)$$

$$P_{max} = A\eta_F \left\{ a_2 - \frac{\delta}{J^2+R^2} \left[B + \frac{J}{(J^2+R^2)^{\frac{1}{2}}} . \Lambda \left(\{B-CR\}^2 + \{CJ\}^2 \right)^{\frac{1}{2}} e^{-R\tau} \right] \right\} \tag{70}$$

$$J\tau - \alpha + \zeta = \tan^{-1} \left(- \frac{R}{J} \right)$$

A.8 Summary of results

A summary of the results is given in Table I and additional formulae necessary for a solution are given in Table II.

APPENDIX II

The solution of the transcendental equations of Appendix I

In Appendix I solutions of the following equations are required,

$$\frac{\eta_F}{\bar{\eta}} = 1 + \frac{\delta\bar{b}}{a_1}\left[\frac{B}{\lambda_1\lambda_2} + \frac{B + C\lambda_1}{\lambda_1(\lambda_1-\lambda_2)} \cdot e^{\lambda_1\tau} - \frac{B + C\lambda_2}{\lambda_2(\lambda_1-\lambda_2)} \cdot e^{\lambda_2\tau}\right] \qquad (61)$$

and

$$\frac{\eta_F}{\bar{\eta}} = 1 + \frac{\delta\bar{b}}{a_1}\left[\frac{B}{\bar{J}^2+\bar{R}^2} - \frac{[(B-C\bar{R})^2+(C\bar{J})^2]^{\frac{1}{2}}}{\bar{J}(\bar{J}^2+\bar{R}^2)^{\frac{1}{2}}} \cdot e^{-\bar{R}\tau} \cdot \cos(\bar{J}\tau-\Upsilon+\zeta')\right] \qquad (62)$$

In both these equations everything is known except τ .

Solution of equation (61)

Two methods of solution are considered here. The first is of use when $|\lambda_2|$ is rather less than $|\lambda_1|$ and the second method should be used when $|\lambda_2|$ approaches the value of $|\lambda_1|$.

Method 1

Equation (61) can be rearranged to read

$$\left\{\left[\left(\frac{\eta_F}{\bar{\eta}} - 1\right)\frac{a_1}{\delta\bar{b}} - \frac{B}{\lambda_1\lambda_2}\right]\frac{\lambda_2(\lambda_2-\lambda_1)}{B + C\lambda_2} + \frac{\lambda_2}{\lambda_1} \cdot \frac{B+C\lambda_1}{B+C\lambda_2} \cdot e^{\lambda_1\tau}\right\}^{-1} = e^{-\lambda_2\tau}$$

Equations (11) and (12) specify $|\lambda_1| > |\lambda_2|$ or $\frac{\lambda_2}{\lambda_1}$ to be less than 1 and as $\frac{B + C\lambda_1}{B + C\lambda_2}$ is approximately 1 then the coefficient of $e^{\lambda_1\tau}$ is less than one and $\frac{\lambda_2}{\lambda_1} \cdot \frac{B + C\lambda_1}{B + C\lambda_2} \cdot e^{\lambda_1\tau}$ is small compared with $e^{\lambda_2\tau}$.

For a first approximation to τ (say τ_1) we may neglect this and find,

$$\tau_1 = -\frac{1}{\lambda_2} \cdot \log_e\left\{\left[\left(\frac{\eta_F}{\bar{\eta}} - 1\right)\frac{a_1}{\delta\bar{b}} - \frac{B}{\lambda_1\lambda_2}\right]\frac{\lambda_2(\lambda_2-\lambda_1)}{B + C\lambda_2}\right\}^{-1}$$

This approximation can then be used to find a more accurate value of τ (say τ_2) thus,

$$\tau_2 = -\frac{1}{\lambda_2} \cdot \log_e\left\{\left[\left(\frac{\eta_F}{\bar{\eta}} - 1\right)\frac{a_1}{\delta\bar{b}} - \frac{B}{\lambda_1\lambda_2}\right]\frac{\lambda_2(\lambda_2-\lambda_1)}{B + C\lambda_2} + \frac{\lambda_2}{\lambda_1} \cdot \frac{B+C\lambda_1}{B+C\lambda_2} \cdot e^{\lambda_2\tau_1}\right\}^{-1}$$

By repeated approximation, i.e. at one stage

$$\tau_n = -\frac{1}{\lambda_2} \cdot \log_e \left\{ \left[\left(\frac{\eta_F}{\bar{\eta}} - 1 \right) \frac{a_1}{\delta \bar{b}} - \frac{B}{\lambda_1 \lambda_2} \right] \frac{\lambda_2(\lambda_2 - \lambda_1)}{B + C\lambda_2} + \frac{\lambda_2}{\lambda_1} \cdot \frac{B + C\lambda_1}{B + C\lambda_2} e^{\lambda_1 \tau_{n-1}} \right\}^{-1}$$

we may obtain any desired accuracy for τ. When $|\lambda_2|$ is a lot less than $|\lambda_1|$ the process will be rapidly convergent but when λ_1 approaches λ_2 it will be preferable to use the following method.

Method 2

To understand the basis of this method let us assume the two roots λ_1 and λ_2 of the stability quadratic (9) to be equal. This is equivalent to assuming

$$\bar{\omega} + \frac{a}{2} \bar{\nu} = \frac{1}{4} (\bar{\chi} + \bar{\nu} + \frac{a}{2})^2$$

The complete solution of equation (8) now becomes,

$$\hat{w} = -\frac{\delta \bar{\eta}}{\lambda^2} \left[1 - (1 - \lambda \tau) e^{\lambda \tau} \right] \quad \text{cf. equation (16)}$$

and

$$\frac{d\hat{w}}{d\tau} = -\delta \bar{\eta} \tau \cdot e^{\lambda \tau}$$

The equation for τ, the time to reach the stop in equation (40) with $\eta \equiv \eta_F$

i.e.

$$\eta_F = \bar{\eta} - \frac{\bar{b}}{a_1} \left\{ B\hat{w} + C \frac{d\hat{w}}{d\tau} \right\}$$

Thus

$$\frac{\eta_F}{\bar{\eta}} = 1 + \frac{\delta \bar{b}}{a_1} \cdot \frac{B}{\lambda^2} \left\{ 1 - e^{\lambda \tau} \left[1 - \left(1 + \frac{C\lambda}{B} \right) \lambda \tau \right] \right\} \quad \text{cf. equation (61)}$$

This equation may be arranged thus,

$$e^{\lambda \tau} \left[1 - \lambda \tau \left(1 + \frac{C}{B} \cdot \lambda \right) \right] = 1 - \left(\frac{\eta_F}{\bar{\eta}} - 1 \right) \frac{a_1 \lambda^2}{\delta \bar{b} B}$$

Consider first the constant on the right hand side of the equation; this will lie somewhere between 0 and 1 because, if

$$1 - \left(\frac{\eta_F}{\bar{\eta}} - 1\right) \frac{a_1 \lambda^2}{\delta \bar{b}\, B} \quad \text{were greater than 1}$$

then

$$\left(\frac{\eta_F}{\bar{\eta}} - 1\right) \frac{a_1 \lambda^2}{\delta \bar{b}\, B} \quad \text{would have to be negative.}$$

Now a_1, λ^2, B, and δ must be positive; we are only considering positive values of $\bar{b}$ for otherwise the stops will never be reached unless they are reached instantaneously with failure; $\frac{\eta_F}{\bar{\eta}}$ must be greater than 1. Therefore $1 - \left(\frac{\eta_F}{\bar{\eta}} - 1\right) \frac{a_1 \lambda^2}{\delta \bar{b}\, B}$ must be less than 1.

Furthermore, if

$$1 - \left(\frac{\eta_F}{\bar{\eta}} - 1\right) \frac{a_1 \lambda^2}{\delta \bar{b}\, B} < 0$$

Then

$$\frac{\delta \bar{b}\, B}{a_1 \lambda^2} < \frac{\eta_F}{\bar{\eta}} - 1$$

or

$$\eta_F > \bar{\eta} \left\{1 + \frac{\delta \bar{b}}{a_1} \cdot \frac{B}{\lambda^2}\right\}$$

which is the condition for the stops never to be reached (see Equation (63)).

Now consider the L.H.S. of the equation. $\left(1 + \frac{C}{B}\lambda\right)$ will probably lie somewhere between 0 and 1. We shall consider the structure of $e^{\lambda\tau}\left[1 - \lambda\tau\left(1 + \frac{C}{B}\lambda\right)\right]$ for a mean value of $\left(1 + \frac{C}{B}\lambda\right) = 0.5$ (as an example) and for values of $-\lambda\tau$ up to 4. Thus

$\lambda\tau$	$e^{\lambda\tau}$	$\left[1-\lambda\tau\left(1+\frac{C}{B}\lambda\right)\right]$	$e^{\lambda\tau}\left[1-\lambda\tau\left(1+\frac{C}{B}\lambda\right)\right]$
0	1	1	1
-0.2	0.82	1.1	0.90
-0.5	0.61	1.25	0.76
-1.0	0.37	1.50	0.55
-2.0	0.13	2.0	0.26
-4.0	0.02	3.0	0.06

The function has been plotted in Fig. 1 and the value of τ for which $e^{\lambda\tau}\left[1 - \lambda\tau\left(1 + \frac{C}{B}\lambda\right)\right] = 1 - \left(\frac{\eta_F}{\bar{\eta}} - 1\right)\frac{a_1\lambda^2}{\delta\,\bar{b}\,B}$ can be readily ascertained.

Thus it is suggested that when λ_1 and λ_2 are relatively close the solution of equation (61) is best obtained by taking a mean value λ, plotting a rough curve of $e^{\lambda\tau}\left[1 - \lambda\tau\left(1 + \frac{C}{B}\lambda\right)\right]$ similar to Fig. 1 (note this curve only required 5 points) and determining the value of τ where this function equals $1 - \left(\frac{\eta_F}{\bar{\eta}} - 1\right)\frac{a_1\lambda_1\lambda_2}{\delta\,\bar{b}\,B}$.

If λ_1 and λ_2 are very close this value will be sufficient. If not, a mean value from plots of both $e^{\lambda_1\tau}\left[1 - \lambda_1\tau\left(1 + \frac{C}{B}\lambda_1\right)\right]$ and $e^{\lambda_2\tau}\left[1 - \lambda_2\tau\left(1 + \frac{C}{B}\lambda_2\right)\right]$ should give sufficient accuracy. Otherwise a small plot of the function,

$$1 + \frac{1}{\lambda_1 - \lambda_2}\left\{\lambda_2\left(1 + \frac{C}{B}\lambda_1\right)e^{\lambda_1\tau} - \lambda_1\left(1 + \frac{C}{B}\lambda_2\right)e^{\lambda_2\tau}\right\}$$

in the region of τ (now known approximately) will give the exact answer, if we find τ where the function equals $1 - \left(\frac{\eta_F}{\bar{\eta}} - 1\right)\frac{a_1\lambda_1\lambda_2}{\delta\,\bar{b}\,B}$.

Solution of equation (62)

This equation may be expressed in the form,

$$e^{-\bar{R}\tau}\left[\cos\bar{J}\tau + \frac{\bar{R} - \frac{C}{B}(\bar{J}^2+\bar{R}^2)}{\bar{J}}\,.\,\sin\bar{J}\tau\right] = 1 - \left(\frac{\eta_F}{\bar{\eta}} - 1\right)\frac{a_1(\bar{J}^2+\bar{R}^2)}{\delta\,\bar{b}\,B}$$

and in this form it may be solved in a graphical manner presented and explained in Ref. 1. It cannot be directly solved from the curves given there as this equation will lead to expanding spiral curves.

TABLE I

Aircraft Normal Accelerations and Tail Loads

	Type A	Type B ((i) real roots equation (9); (ii) complex roots equation (9))	Type C
n	$1 - D.G[1-Qe^{-R\tau}.\cos(J\tau-\gamma)]$	(i) $1 - D\frac{\delta\bar\eta}{\lambda_1\lambda_2}\left[1+\frac{\lambda_2}{\lambda_1-\lambda_2}e^{\lambda_1\tau}-\frac{\lambda_1}{\lambda_1-\lambda_2}e^{\lambda_2\tau}\right]$ (ii) $1 - D.\bar G[1-\bar Q e^{-\bar R\tau}.\cos(\bar J\tau-\gamma')]$	$1 - DG[1-\Lambda e^{-R\tau}\cos(J\tau-\gamma)]$
n_t	$1 - D.G.[1-Q.H.e^{-R\tau}.\cos(J\tau-\gamma-\phi)]$	(i) $1 - D\frac{\delta\bar\eta}{\lambda_1\lambda_2}\left\{1+\frac{1}{\lambda_1-\lambda_2}\left[\lambda_2\left(1-\frac{2\lambda_1^2}{\mu a}-\frac{\lambda_1}{\mu}\right)e^{\lambda_1\tau}-\lambda_1\left(1-\frac{2\lambda_2^2}{\mu a}-\frac{\lambda_2}{\mu}\right)e^{\lambda_2\tau}\right]\right\}$ (ii) $1 - D.\bar G[1-\bar Q\bar H e^{-\bar R\tau}.\cos(\bar J\tau-\gamma'-\phi')]$	$1 - DG[1-\Lambda He^{-R\tau}.\cos(J\tau-\gamma-\phi)]$
P	$A\{a_2\eta_F - B.G[1-QMe^{-R\tau}.\cos(J\tau-\gamma+\zeta)]\}$	(i) $A\bar\eta\left\{a_2-\frac{\delta}{\lambda_1\lambda_2}\left(1-\frac{\bar b}{a}\right)\left[B+\frac{1}{\lambda_1-\lambda_2}\left(\lambda_2[B+\lambda_1C]e^{\lambda_1\tau}-\lambda_1[B+\lambda_2C]e^{\lambda_2\tau}\right)\right]\right\}$ (ii) $A\{a_2\bar\eta-\bar G N B[1-\bar Q\bar M e^{-\bar R\tau}.\cos(\bar J\tau-\gamma'+\zeta')]\}$	$A\{a_2\eta_F - BG[1-\Lambda Me^{-R\tau}.\cos(J\tau-\gamma+\zeta)]\}$
n_{max}	$1 - D.G(1+e^{-R/J\,\pi})$	(i) $1 - D\frac{\delta\bar\eta}{\lambda_1\lambda_2}$ * $\tau\to\infty$ (ii) $1 - D\bar G[1+e^{-\bar R/\bar J\,\pi}]$	$1 - DG\left[1+\frac{\Lambda}{Q}e^{-R\tau}\right]$
	$J\tau=\pi$	$\bar J\tau=\pi$	$J\tau-\alpha=\tan^{-1}-R/J$
$n_{t_{max}}$	$1-DG(1+He^{-R/J(\pi+\phi)})$	(i) $1 - D\frac{\delta\bar\eta}{\lambda_1\lambda_2}$ * $\tau\to\infty$ (ii) $1 - D.\bar G[1+\bar H e^{-\bar R/\bar J(\pi+\phi')}]$	$1 - DG\left[1+\frac{\Lambda}{Q}.He^{-R\tau}\right]$
	$J\tau=\pi+\phi$	$\bar J\tau=\pi+\phi'$	$J\tau-\alpha-\phi=\tan^{-1}{-R/J}$
P_{max}	$A(a_2\eta_F-BG[1+Me^{-R/J(\pi-\zeta)}])$	(i) $A\bar\eta\left\{a_2-\frac{\delta}{\lambda_1\lambda_2}\left(1-\frac{\bar b}{\bar a}\right)B\right\}$ * $\tau\to\infty$ (ii) $A(a_2\bar\eta-NB\bar G[1+\bar M e^{-\bar R/\bar J(\pi-\zeta')}])$	$A\left\{a_2\eta_F-BG\left[1+\frac{\Lambda}{Q}.Me^{-R\tau}\right]\right\}$
	$J\tau=\pi-\zeta$	$\bar J\tau=\pi-\zeta'$	$J\tau-\alpha+\zeta=\tan^{-1}-R/J$

* See note of paragraph A.7.2 of Appendix I.

TABLE II

Subsidiary Formulae

$$\gamma = \tan^{-1}\left\{ \frac{R}{J} + \frac{\left(\frac{d\hat{w}}{d\tau}\right)_o}{JK\left(1 - \frac{\hat{w}_o}{K}\right)} \right\} \quad \text{for case A} \left(\frac{d\hat{w}}{d\tau}\right)_o = \hat{w}_o = 0 \text{ i.e. } \gamma = \tan^{-1}\left(\frac{R}{J}\right)$$

$$K = \frac{\delta \cdot \eta_F}{J^2 + R^2}$$

$$\phi = \tan^{-1}\left\{ \frac{Ja - 4JR}{\mu a + aR - 2(R^2 - J^2)} \right\}$$

$$\zeta = \tan^{-1}\left\{ \frac{CJ}{B - CR} \right\}$$

$$\gamma' = \tan^{-1}\left\{ \frac{\bar{R}}{\bar{J}} \right\}$$

$$\phi' = \tan^{-1}\left\{ \frac{\bar{J}a - 4\overline{JR}}{\mu a + a\bar{R} - 2(\bar{R}^2 - \bar{J}^2)} \right\}$$

$$\zeta' = \tan^{-1}\left\{ \frac{C\bar{J}}{B - C\bar{R}} \right\}$$

$$\Lambda = \left\{ \left(1 + \frac{\hat{w}_o}{K}\right)^2 + \left[\left(1 + \frac{\hat{w}_o}{K}\right) \cdot \frac{R}{J} + \frac{1}{J.K} \cdot \left(\frac{d\hat{w}}{d\tau}\right)_o\right]^2 \right\}^{\frac{1}{2}}$$

$$\hat{w}_o = -\frac{\delta\,\bar{\eta}}{\bar{J}^2 + \bar{R}^2}\left\{1 - \frac{(\bar{J}^2 + \bar{R}^2)^{\frac{1}{2}}}{\bar{J}} \cdot e^{-\bar{R}\tau} \cdot \cos(J\tau - \gamma')\right\}$$

or

$$-\frac{\delta\,\bar{\eta}}{\lambda_1\lambda_2}\left\{1 + \frac{1}{\lambda_1 - \lambda_2}(\lambda_2 e^{\lambda_1\tau} - \lambda_1 e^{\lambda_2\tau})\right\}$$

$$\left(\frac{d\hat{w}}{d\tau}\right)_o = -\frac{\delta\bar{\eta}}{\bar{J}} \cdot e^{-R\tau} \cdot \cos\left(J\tau - \frac{\pi}{2}\right)$$

or

$$-\frac{\delta\,\bar{\eta}}{\lambda_1 - \lambda_2}\left\{e^{\lambda_1\tau} - e^{\lambda_2\tau}\right\}$$

where τ is found from

$$\frac{\eta_F}{\bar{\eta}} = 1 + \frac{\delta\bar{b}}{a_1}\left\{\frac{B}{\bar{J}^2 + \bar{R}^2} - \frac{\left[(B-C\bar{R})^2 + (C\bar{J})^2\right]}{\bar{J}(\bar{J}^2 + \bar{R}^2)^{\frac{1}{2}}} \cdot e^{-R\tau} \cdot \cos\left(\bar{J}\tau - \gamma' + \zeta'\right)\right\}$$

or

$$1 + \frac{\delta\bar{b}}{a_1}\left\{\frac{B}{\lambda_1\,\lambda_2} + \frac{B + C\lambda_1}{\lambda_1(\lambda_1 - \lambda_2)} \cdot e^{\lambda_1\tau} - \frac{B + C\lambda_2}{\lambda_2(\lambda_1 - \lambda_2)} \cdot e^{\lambda_2\tau}\right\}$$

$$Q = \left[1 + \left(\frac{R}{J}\right)^2\right]^{\frac{1}{2}} \quad ; \quad \bar{Q} = \left[1 + \left(\frac{\bar{R}}{\bar{J}}\right)^2\right]^{\frac{1}{2}}$$

$$G = \frac{\delta\,\eta_F}{J^2 + R^2} \quad ; \quad \bar{G} = \frac{\delta\,\bar{\eta}}{\bar{J}^2 + \bar{R}^2}$$

$$N = 1 - \frac{\bar{b}}{\bar{a}}$$

$$H = \left[\left(1 + \frac{R}{\mu} - \frac{2(R^2 - J^2)}{\mu a}\right)^2 + \left(\frac{J}{\mu} - \frac{4JR}{\mu a}\right)^2\right]^{\frac{1}{2}} \quad ;$$

$$\bar{H} = \left[\left(1 + \frac{\bar{R}}{\mu} - \frac{2(\bar{R}^2 - \bar{J}^2)}{\mu a}\right)^2 + \left(\frac{\bar{J}}{\mu} - \frac{4\bar{J}\bar{R}}{\mu a}\right)^2\right]^{\frac{1}{2}}$$

$$M = \left[\left(1 - \frac{C}{B} \cdot R\right)^2 + \left(\frac{C}{B} \cdot J\right)^2\right]^{\frac{1}{2}} \quad ;$$

$$\bar{M} = \left[\left(1 - \frac{C}{B} \cdot \bar{R}\right)^2 + \left(\frac{C}{B} \cdot \bar{J}\right)^2\right]^{\frac{1}{2}}$$

Some typical values of some of these parameters on 13 aircraft are given below:

A/C No.	Q	$\frac{G}{\eta_F}$	H	M	ζ°	ϕ°
1	2.355	2.57	0.942	0.50	33.7	16.5
2	1.508	3.76	3.04	0.71	27.2	12.1
3	1.325	0.66	2.03	0.80	55.2	39.2
4	1.628	1.18	1.26	0.64	36.4	34.2
5	1.289	0.61	1.96	0.80	54.0	33.6
6	1.418	0.95	1.46	0.71	39.1	29.5
7	1.222	1.49	1.34	0.58	22.3	9.50
8	1.09	1.11	1.57	0.92	27.5	8.65
9	1.29	1.49	1.49	0.78	41.2	19.9
10	1.535	2.94	1.11	0.71	23.1	12.6
11	1.318	1.43	1.27	0.77	48.6	16.0
12	1.530	1.02	1.59	0.85	57.1	16.0
13	1.520	2.56	0.98	0.52	29.5	18.1

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ADDENDA

In paragraph A.6 of Appendix I, the reader is referred to a paper by Czaykowski (Ref.1) for the derivation of aircraft normal accelerations at the c.g., tail unit accelerations and aerodynamic tail loads. The paper by Czaykowski is in the course of preparation and the following brief derivations are given here for ease of reference until Ref.1 is generally available. The notation used below is incorporated in the main text of this report. Equation numbers refer to the equations in these Addenda only, except where noted otherwise.

1 Aircraft normal accelerations at the c.g. (see para. A.6.1. Appendix I)

The lift force on the whole aircraft $= \frac{1}{2}\rho V^2 Sa \times$ wing incidence. The wing incidence can be regarded as the incidence in steady level flight plus the incremental incidence, $\alpha \equiv \frac{w}{V} \equiv \hat{w}$ (see notation), due to aircraft response. Now the lift force on the whole aircraft in the steady level flight condition equals the weight W. Thus, the lift force on the whole aircraft

$$= W + \tfrac{1}{2}\rho V^2 Sa.\hat{w} \qquad (1)$$

and, as Force = Mass × Acceleration $= \frac{W}{g} \cdot (ng)$ (2)

then $n = 1 + D\hat{w}$ (equation (33) of Appendix I)

where $D = \frac{1}{2}\rho V^2 S \frac{a}{W}$ (equation (34) of Appendix I)

Note: This takes into account the standard practice of counting the coefficient of normal acceleration (i.e. acceleration divided by g) as 1 when the aircraft is in steady level flight. It would perhaps be more accurate to state, above, that the net vertical force on the aircraft equals the lift minus the weight (which is the same as saying that the net force is produced by the incremental incidence, $\hat{w}$) and equating this to the mass × acceleration. This leads to a value of n equal to $D\hat{w}$ and 1 would have to be added to this to comply with standard practice.

2 Tail unit accelerations (see para. A.6.2 of Appendix I)

The tail unit acceleration is equal to the acceleration at the c.g. plus an addition due to the angular acceleration of the aircraft, in pitch, about the c.g. The addition is equal to the angular acceleration of the aircraft multiplied by the distance of the tail plane from the c.g. (i.e. the distance of the tail plane ¼ chord point from the c.g.).

Thus the coefficient of normal acceleration at the tail plane, n_t (i.e. acceleration divided by g) is given by,

$$n_t = 1 + D\hat{w} - \frac{\ell}{g} \cdot \frac{dq}{dt} \qquad (3)$$

where the first two terms are the aircraft normal acceleration at the c.g. (equation 33) and the third term is the addition due to angular acceleration in pitch. A negative sign appears with the latter term as the angular velocity in pitch, q, is reckoned positive when the nose of the aircraft comes up, i.e. as the tail goes down.

Now from the Notation,

$$q = \frac{\hat{q}}{\hat{t}} \quad \text{and} \quad t = \hat{t}\tau$$

thus

$$dq = \frac{1}{\hat{t}} \cdot d\hat{q} \quad \text{and} \quad dt = \hat{t}\, d\tau$$

and ∴

$$\frac{dq}{dt} = \frac{1}{\hat{t}^2} \cdot \frac{d\hat{q}}{d\tau} \qquad (4)$$

∴ from (3)

$$n_t = 1 + D\hat{w} - \frac{\ell}{g} \cdot \frac{1}{\hat{t}^2} \cdot \frac{d\hat{q}}{d\tau} \qquad (5)$$

But from one of the basic equations of motion in non-dimensional terms

$$\hat{q} = \frac{d\dot{w}}{d\tau} + \frac{a}{2} \cdot \hat{w} \quad \text{(see equation (5) of Appendix I)}$$

and differentiating

$$\frac{d\hat{q}}{d\tau} = \frac{d^2\hat{w}}{d\tau^2} + \frac{a}{2}\,\frac{d\hat{w}}{d\tau} \qquad (6)$$

Thus

$$n_t = 1 + D\left[\hat{w} - \frac{\ell}{Dg\hat{t}^2} \cdot \frac{a}{2}\left(\frac{2}{a}\,\frac{d^2\hat{w}}{d\tau^2} + \frac{d\hat{w}}{d\tau}\right)\right] \quad \text{from (5) and (6)}$$

By referring to the Notation alone it is easy to show that by definition

$$\frac{\ell a}{Dg\hat{t}^2 2} \equiv \frac{1}{\mu}$$

Hence finally

$$n_t = 1 + D\left[\hat{w} - \frac{1}{\mu}\left(\frac{2}{a} \cdot \frac{d^2\hat{w}}{d\tau^2} + \frac{d\hat{w}}{d\tau}\right)\right] \quad \text{(equation (35) of Appendix I)}$$

3 Aerodynamic tail load
(see para. A.6.3 Appendix I)

By definition, the incremental value of the aerodynamic tail load P is given by,

$$P = \tfrac{1}{2}\rho V^2 S' \; (a_1 \alpha' + a_2 \eta) \qquad (7)$$

$$\equiv A \; (a_1 \alpha' + a_2 \eta) \qquad (8)$$

where α' and η are incremental values, of the tailplane incidence and elevator angle respectively, from the steady flight condition and

$$A = \tfrac{1}{2}\rho V^2 S'$$

On the first page of Appendix I, α' is given as

$$\alpha' = \frac{w}{V}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{\ell}{V} \cdot q + \frac{\ell}{V^2} \cdot \frac{dw}{dt} \cdot \frac{d\varepsilon}{d\alpha}$$

together with a brief derivation of this equation.

By referring to the Notation we can transform this equation into non-dimensional notation, thus,

$$\alpha' = \hat{w}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{\ell}{V\hat{t}} \cdot \hat{q} + \frac{\ell}{V^2}\frac{V}{\hat{t}}\frac{d\hat{w}}{d\tau} \cdot \frac{d\varepsilon}{d\alpha} \qquad (9)$$

or $$\alpha' = \hat{w}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{1}{\mu} \cdot \hat{q} + \frac{1}{\mu} \cdot \frac{d\hat{w}}{d\tau} \cdot \frac{d\varepsilon}{d\alpha} \qquad (10)$$

Again from one of the basic equations of motion in non-dimensional notation

$$\hat{q} = \frac{d\hat{w}}{d\tau} + \frac{a}{2} \cdot \hat{w}$$ (see equation (5) of Appendix I)

and substituting this in (10),

$$\alpha' = \hat{w}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{1}{\mu}\left(\frac{d\hat{w}}{d\tau} + \frac{a}{2}\hat{w}\right) + \frac{1}{\mu}\frac{d\hat{w}}{d\tau} \cdot \frac{d\varepsilon}{d\alpha}$$

$$= \left(1 - \frac{d\varepsilon}{d\alpha} + \frac{a}{2\mu}\right)\hat{w} + \left(1 + \frac{d\varepsilon}{d\alpha}\right)\frac{1}{\mu} \cdot \frac{d\hat{w}}{d\tau}$$

and therefore we may write,

$$a_1 \alpha' = B\hat{w} + C\frac{d\hat{w}}{d\tau} \qquad (11)$$

where

$$B = \left(1 - \frac{d\varepsilon}{d\alpha} + \frac{a}{2\mu}\right) a_1$$

and

$$C = \left(1 + \frac{d\varepsilon}{d\alpha}\right) \frac{a_1}{\mu}$$

Returning to our initial equation,

$$P = A\,(a_1\alpha' + a_2\eta) \qquad (8)$$

and filling in our derived equation for $a_1\alpha'$ from (11) then the incremental value of the aerodynamic tail load is given by,

$$P = A\left(B\hat{w} + C\,\frac{d\hat{w}}{d\tau} + a_2\,\eta\right) \qquad (12)$$

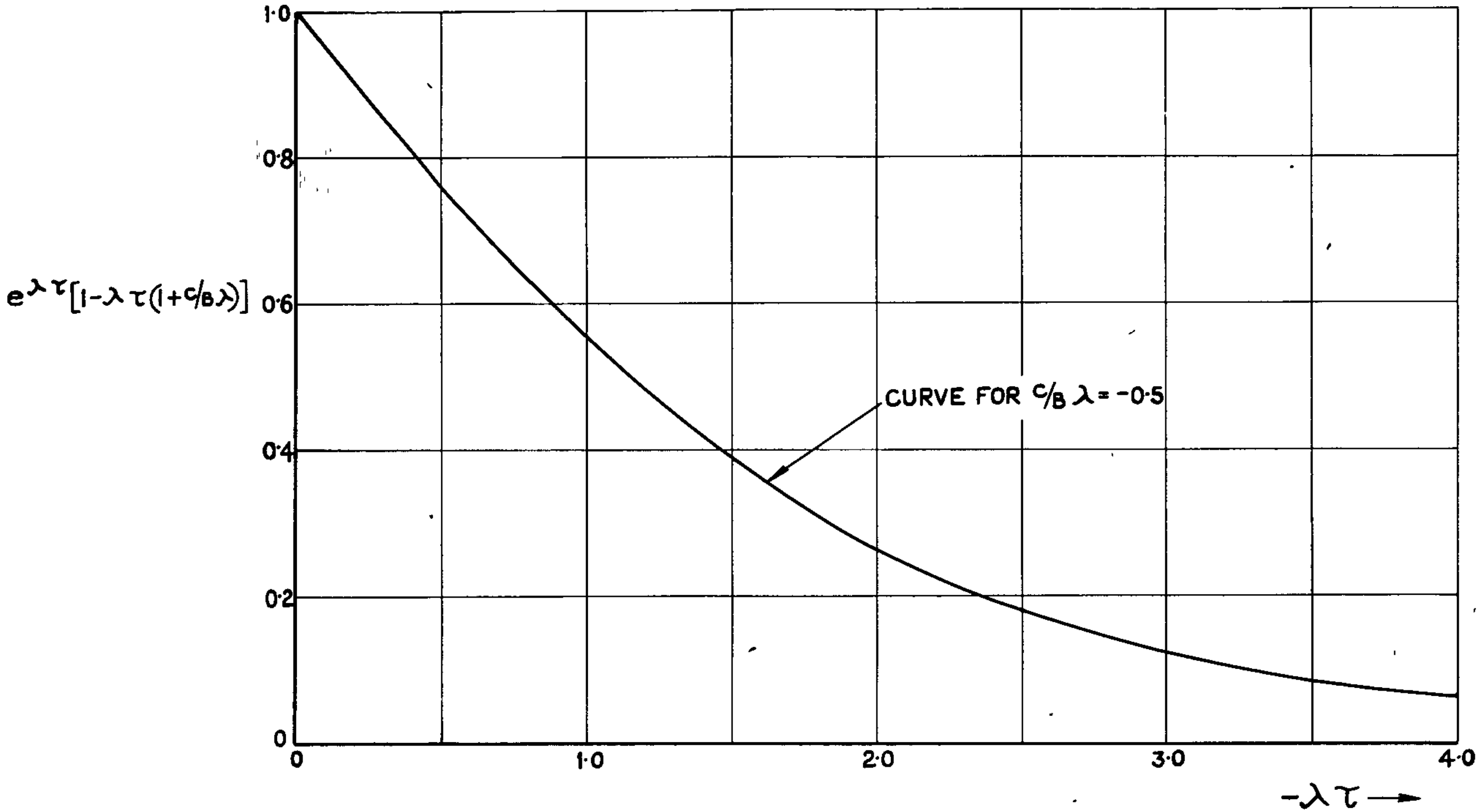

FIG.1. CURVE OF $e^{\lambda\tau}[1-\lambda\tau\,(1+c/B\,\lambda)]$ AGAINST $\lambda\tau$ FOR $c/B\,\lambda = -0{\cdot}5$.

FIG.1.

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