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SUMMARY

In this paper data obtained in a range of experimental investigations of cooling problems in the gas turbine are compared with theoretical predictions made by using a number of hypotheses concerning the analysis of turbulent boundary layers. Integral and differential theories have been used, and applied to flows over convection- and transpiration-cooled turbine blades and simplified representations of combustion systems and turbine discs.

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Introduction/

## Introduction

Although turbulent flows occur frequently in numerous engineering systems, particularly power-producing plant, the prediction of the important flow features remains one of the most difficult computational tasks facing the engineer. As a consequence much effort has been and is being directed towards the development of reliable calculation techniques which ideally should be capable of dealing with a wide range of flow situations. Because our understanding of the basic physics of turbulence is currently far from complete most prediction methods require, in varying degrees, some empirical input. This is obtained either from prototype or model testing of the systems being designed, often made necessary by lack of confidence in prediction methods, or from fundamental experiments to furnish data, in areas of uncertainty, made by those developing the prediction techniques. It is apparent nevertheless that much careful experimentation remains urgently necessary in each area to keep abreast of modern numerical procedures for the solution of the equations which govern the flow.

The present paper reports results and experiences gained in the application of a number of modern boundary layer calculation techniques to problems either directly or indirectly concerned with cooling gas turbine components in and downstream of the combustion system. Where possible the results obtained are compared with our own or other workers' data. Because the techniques used are well documented in the literature, details of the basic strategies are not included. Expedient modifications to the methods found necessary are however discussed and reasons for our choices given. In this way it is hoped that experiences which we have gained may be usefully passed on to other workers in this and related areas.

## Flows over Turbine Blades

Modern gas turbine blades are heavily loaded, with turning angles up to 120 degrees, and with severe pressure gradients. Although modern potential flow theories appear to make possible the accurate design of blade geometries which reduce or even eliminate regions of adverse pressure gradient (the so-called 'prescribed velocity distribution' procedures) large favourable pressure gradients are inevitable, of the order of 50 lb/in<sup>2</sup> per inch of blade chord. These operating conditions should be borne in mind in planning laboratory experimental programmes to yield turbulent flow data of relevance to the turbine designer. In our research programme experimental data has been obtained on the rates of heat transfer over internally convection-cooled, impermeable blades, and from transpiration-cooled porous blades, and on the corresponding irreversible pressure losses. These data are compared with predictions made from a number of published theories of turbulent, and where necessary, laminar boundary layers.

## Head's Entrainment Method

In our early work<sup>1</sup> to determine the pressure loss over sets of turbine nozzle blades, use was made of a modification of the method of predicting the development of a turbulent boundary layer proposed by Head<sup>2</sup>. This integral technique of analysis is based upon the assumption that there is a functional relationship between the shape factor and mass rate of entrainment into a boundary layer. It seemed, however, to have severe limitations in its application to the practical turbine situation in which, as has been explained above, severe pressure gradients exist. By coupling Head's functional entrainment relations with an equivalent energy dissipation

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term within the boundary layer we were able to make predictions of the loss factor associated with a moderately loaded impermeable turbine nozzle blade. The results of the predictions are compared in Fig.1a with experimental measurements. The agreement is not very satisfactory although the loss factors calculated with the two extreme assumptions, that no mixing of the flows through each throat had occurred, (i.e., the condition at the cascade outlet) and, on the other hand, that complete mixing of wakes had taken place, can be seen to span the measured values. Many further assumptions were however necessary to obtain the predicted values, and seem to us to show very clearly some of the disadvantages of integral methods in general. In particular, an essential auxiliary equation in the method is an empirical relation for the friction factor,  $c_f$ . There is relatively little data in the literature which is applicable to the (by modern turbine standards) modest adverse and favourable pressure gradients which were involved in our tests. When the calculation technique is extended to deal with the problems of transpiration-cooled blades, in which coolant effuses through the surface of the blade to mix with the turbulent boundary layer, the approximations and uncertainties become correspondingly more severe. In this situation there is no published data of any real relevance to the practical problem, so that in our attempts to determine the values of the parameters in the auxiliary equations there was a good deal of latitude. As Fig.1b shows, although in general the quantitative level of agreement can be made satisfactory with a suitable choice of the parameters in the auxiliary equations, the trends between theory and experiment are not very closely in line. Certainly in these circumstances there would be grave reservations about extrapolating from the simple experimental situation to the harsh conditions of turbine practice.

#### The Method of Kutateladze and Leont'ev

Unlike Head's method of boundary layer analysis, that of Kutateladze and Leont'ev<sup>3</sup> is readily applicable to the calculation of heat transfer. Their method is mainly an explicit integral technique although its auxiliary functions are in part derived from the mixing length hypothesis of turbulent transport.

With a Reynolds number based on the enthalpy deficit thickness  $R_{h,2}$  the integral energy equation is written in the form

$$\frac{d R_{h,2}}{dx} + \frac{R_{h,2}}{\Delta T} \cdot \frac{d \Delta T}{dx} = R_L S_o [\psi + b_h] \quad \dots (1)$$

$R_{h,2}$  is a basic criterion of the flow; this and other symbols are defined in the nomenclature, page 17.

Before this equation can be solved several empirical relationships are required. The reduction of the Stanton number with increasing coolant injection on a transpiration-cooled blade, is represented as a function of the ratio of the effusion rate to a critical rate which lifts off the boundary layer, so that convective heat transfer goes to zero. Expressions for the blowing parameters and the semi-empirical basic relationship between  $R_{h,2}$  and  $S_o$ , the Stanton number unperturbed by non-isothermality or transpiration are given in Ref.3. Comparison of the heat transfer rates predicted by this method of analysis with experimental measurements made in porous blades is made later in this paper.

## The Method of Spalding and Patankar<sup>4</sup>

This method of analysing boundary layers is of the differential form, in that it allows numerical integration of the differential equations of the boundary layer. It used the Prandtl mixing length hypothesis to relate the turbulent transport properties to the flow conditions, linking this via an exponential decay law to the so-called 'Couette-flow' relationships in the laminar regions near the bounding wall of the flow. To predict heat transfer rates over both transpiration-cooled and impermeable turbine blades some modifications have been made to the standard procedure of Ref.4 for the numerical solution of the boundary layer equations. In particular the exponential decay law has been eliminated, and the mixing-length relationship allowed to extend into the laminar sub-layer where the turbulent viscosity becomes negligible of itself. This procedure has been found particularly justifiable for the porous blades, where the effusion is known to reduce the decay of turbulence in the 'buffer' layer.

Some trouble was initially experienced with the entrainment calculation and the present procedure is to take the maximum value obtained for the velocity and thermal boundary layers. The number of cross-stream steps in the numerical integration mesh was found to have little effect on the results provided the node near the wall was placed sufficiently into the "viscous sublayer". It was found necessary to place maximum and minimum restrictions on the downstream step size in order to attain economy when the boundary layer is thin and accuracy when the boundary layer is thick.

### Comparison with Experiment - Transpiration-Cooled Blades

The experimental blade section is shown in Fig.2 together with a typical experimental surface velocity profile<sup>1</sup>. The transpiration-cooled blade was divided into 18 internal passages each having its own supply of coolant and temperature instrumentation. This enabled the local heat transfer coefficient to be measured at 19 separate positions for various coolant and mainstream flow rates.

Fig.3 shows a comparison between the experimental results and the two prediction methods of Refs.3 and 4 for 3 surface effusion rates maintained constant over the whole blade surface. An important difference between the two theoretical predictions is that as the injection rate is increased Spalding and Patankar<sup>4</sup> gives a reduction in heat transfer coefficient which is in better agreement with the observed reduction. Kutateladze and Leont'ev<sup>3</sup> clearly overestimate the heat transfer over the greater part of the suction surface and it is also clear that the parameter  $R_{h,2}$  does not adequately represent the flow, especially when there are significant pressure gradients, Ref.5. Taken overall the procedure of Spalding and Patankar appears to be in better agreement with the observed heat transfer coefficients.

It should be noted, in considering these comparisons, that a turbulent boundary layer appeared not to be obtained over the whole surface for all effusion rates and exit Reynolds numbers. In Fig.4, it is shown that, with a mainstream turbulence level of 0.5%, as the exit Reynolds number of the main flow increases, the pressure surface shows evidence of a moving laminar-turbulent transition point. This figure shows, however, that this transition region was completely removed by raising the mainstream inlet turbulence level to 2.2 per cent.

### Convection-cooled Blades

Here the cooling is achieved by means of internal cooling passages machined within the blade section. With a low level of mainstream turbulence laminar boundary layers would almost certainly develop on both surfaces of the blade in the favourable pressure gradient regions. Transition on the suction surface would also occur at or slightly beyond the start of the adverse pressure gradient region. The major uncertainty is caused by the unknown effect of the mainstream turbulence level. The favourable pressure gradients present are in fact higher than those which would normally re-laminarize a turbulent boundary layer<sup>6,7</sup> and it would seem therefore that there is little likelihood of a clear transition point, irrespective of turbulent level, on the pressure surface on which the flow accelerates from the leading to the trailing edge. The flow acceleration can be characterised by the velocity

gradient parameter  $K$ , defined as  $\frac{v}{Vg^2} \frac{dV}{dx}$ , and Launder and Jones<sup>6</sup> suggest

$2 \times 10^{-6}$  laminarization effects will become significant. For the present experiments some values of  $K$  are shown in Fig.2. Approximate values of  $K$  for a first stage turbine rotor blade of a modern more heavily loaded bypass engine are  $30 \times 10^{-6}$ ,  $8 \times 10^{-6}$  and  $4 \times 10^{-6}$  for positions at 25%, 50% and 75% chord respectively on the pressure surface.

An experiment<sup>8</sup> on the same blade section as was used in the transpiration-cooling investigation showed that the level of free-stream turbulence had a marked effect on the local and the mean level of heat transfer to the blade. It is shown in Fig.5 that increasing the free-stream turbulence intensity from 0.5% to 6% increases the mean Nusselt number over the range of Reynolds number tested. Also shown in this figure is the correlation given by Halls<sup>35</sup>.

The local values of heat transfer coefficient deduced from detailed measurements of the blade surface and air coolant temperatures are shown in Fig.6 for three different turbulence intensities<sup>8</sup>. A clear indication of transition is seen on the suction surface approximately 10% chord beyond the point of minimum pressure, but on the pressure surface the effect of free-stream turbulence is apparently very similar to that obtained on the leading edge of cylinders in cross-flow described by Kestin<sup>9</sup>.

For the solid blades the integral method of predicting heat transfer due to Kutateladze and Leont'ev<sup>3</sup> consistently overestimated the local heat transfer coefficients by an unacceptable amount on both pressure and suction surfaces.

The method of Spalding and Patankar<sup>4</sup> has been found to be more satisfactory for both laminar and turbulent boundary layer predictions. With low free-stream turbulence the laminar prediction is in satisfactory agreement on both surfaces of the blade and at high free-stream turbulence the turbulent theory agrees on the pressure surface but overestimates on the leading region of the suction surface, due, presumably, to the very severe favourable pressure gradient (see Fig.2).

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Work is continuing<sup>10</sup> with the Spalding and Patankar calculation method and encouraging results are being obtained in predicting the increase in heat transfer through a laminar boundary layer due to the presence of free-stream turbulence. It was suggested by Spalding that the effects of free-stream turbulence could be accounted for by using an effective viscosity given by

$$\mu_{\text{eff}} = \rho \ell u'_{\text{rms}}$$

where  $u'_{\text{rms}}$  is the free-stream R.M.S. turbulent fluctuation velocity and  $\ell$  is the usual turbulent mixing length. Some preliminary results with this procedure are given in Fig.6.

#### Flow in a Porous Walled Annular Duct

Because of its axisymmetric geometry the concentric annulus is eminently suitable for a fundamental study of the influence of suction or blowing on flow and heat transfer in confined flows; such a study is also very relevant to the problems of transpiration cooled turbine blades and flame tubes.

If it is assumed that the boundary layer approximations are applicable, the axial momentum equation may be truncated to the usual parabolic form and solved, in principle, by the numerical method proposed by Spalding and Patankar<sup>4</sup>. In its original form the method was mainly concerned with unconfined flows where the pressure distribution was known. In the present case the fluid is constrained to flow between the inner and outer tubes which comprise the annular duct and the pressure distribution is a dependent variable which must be evaluated as the computation proceeds downstream. A method for achieving this tactical feature must be decided.

The usual assumptions concerning the inter-nodal velocity variations necessary for the construction of the finite difference representation of the differential equations (viz: linear in the cross stream and stepwise in the axial directions respectively) lack sufficient accuracy in the vicinity of a wall. To overcome this Spalding and Patankar ignored axial convection and curvature near a wall so that the boundary layer equation could be further truncated to an ordinary differential equation which was integrated for a number of flow conditions. The solutions of this equation and its heat transfer counterpart were used to form wall flux relationships which the flow near a wall was constrained to satisfy. When the flow is laminar a closed form solution of the Couette-type equation exists which accounts for the simultaneous effect of pressure gradient and suction or blowing. By assuming a Van Driest effective viscosity model near a wall the equations were solved numerically and "once for all" relationships constructed. When dealing with turbulent flow a further decision concerning the choice of a mathematical representation of the turbulence must be taken. From the viewpoint of the design engineer, who requires a quick answer, a model having ease of application together with an acceptable level of accuracy has obvious advantages. To this end the mixing length concept of Prandtl or the Van Driest exponentially damped effective viscosity model have some attraction even though some physical drawbacks are apparent.

Initially it was decided in the present analysis to treat the case of turbulent flow in a solid-walled annulus and compare the fully developed solutions with known experimental results. To this end the turbulent wall

flux/



flux relationships of Spalding and Patankar were used together with the "short and correct" technique for the pressure field evaluation. Also a simple mixing length distribution involving a linear variation near the walls with a uniform distribution in the central flow region was taken. Difficulty was encountered in controlling an oscillation on the calculated pressure gradient despite attempts to dilute the correction used for the fictitious area difference between the flow area and the true cross sectional area of the annulus. Uncertainty was also felt in the location of the grid points immediately adjacent to the annulus walls to ensure that implied assumptions used in the formation of the turbulent wall flux relationships were not contravened. This problem was accentuated for annuli having small radius ratio values (say less than 0.2) because a grid point seemingly very close to the inner wall could imply a physical distance which was large in relation to the inner radius of the duct with consequential breakdown of the wall flux relationships.

To overcome these difficulties it was decided to arrange the finite difference network so that a number of points were located in the laminar sublayer and to use the closed form solution of the Couette flow equation obtained for laminar flow. The use of a mesh having constant steps in the cross stream direction with a number of points in the laminar sublayer would be excessively fine in the central region of the flow. Consequently, a mesh was selected having steps which expanded in geometric progression from the inner wall up to the centre line of the duct flow channel (based on the normalised stream function used as the cross stream variable) and which then compressed in a similar fashion to the outer wall. To determine the unknown pressure field a rapid iterative procedure was used which ensured that the pressure gradient calculated at a given downstream location gave a resulting velocity profile which satisfied continuity to a predetermined level of acceptability. The method is an adaptation to account for curvature and wall porosity of that used by Bayley and Owen<sup>11</sup> for the investigation of flow near a rotating disc. In principle the method has also been used by Worsoe-Schmidt and Leppart<sup>12</sup>.

It was felt necessary to ensure that the overall technique, together with the tactical decisions made, was functioning correctly by checking the predictions given against those of other solution techniques for three laminar flow test cases. These were:-

- (1) flow development between impervious parallel plates with an initially flat velocity profile;
- (2) flow development along an impervious annular duct with an initially flat velocity profile;
- (3) flow development between parallel plates with Poiseuille flow at entry and uniform suction at both walls.

For the first test case good agreement with the numerical solution of Bodoia and Osterle<sup>13</sup> was found. For  $Z = 0.015$  (see Nomenclature) comparison of the predicted velocity profile with that for developed flow showed excellent agreement and at this location the computed pressure gradient was already within 99.2% of its established value.

For/

For the second test case the present predictions were compared with those of Sparrow and Lin<sup>14</sup> who obtained an analytic solution after having linearized the convective terms in the momentum equation. Agreement was generally good and is typified by Fig.7, where the predicted profile development is shown for a radius ratio of 0.8. The development of the pressure field was also in good agreement with that predicted by these authors.

For the third test case the analytic solution of Berman<sup>15</sup> was compared with the present predictions. Once more agreement was good as may be seen by reference to Fig.8 which shows the development of the pressure parameter,  $P$ , for various levels of suction.

Having demonstrated the validity of the procedure, predictions of the influence of suction and blowing at the inner wall of the annulus were made for the case where the initial velocity profile was uniform. The manner in which the pressure typically develops is shown in Fig.9 where the effect of the porous walls is shown for an annulus having a radius ratio of 0.8. For a solid-walled annulus the location of the point of maximum velocity occurs nearer the inner wall. Fig.10 shows the effect of suction or blowing on the velocity profile at a typical downstream location ( $Z = 0.03$ ) for a number of radius ratio values. It is seen that suction moves the point of maximum velocity even more towards the inner wall whereas the converse is true for the blowing case. Further details for the laminar flow case are given by Morris<sup>16</sup>. For all laminar flow calculations 100 cross stream steps were used with a stepsize ratio of 1.1.

Figs.11 to 14 inclusive show some comparisons of predicted and measured developed turbulent velocity profiles for a solid-walled annulus assuming that a Van Driest effective viscosity model may be used from the walls to the location of the point of maximum velocity. This assumption has previously been made by Wilson and Medwell<sup>17</sup> and shown to give reasonable results. The experimental data has been taken from the work of Brighton<sup>18</sup> and Quarmby<sup>19</sup>. Generally the predictions were near the experimental data even with the simple assumption made concerning the nature of the turbulence model. Plotted on a  $Y^+ - W^+$  basis the velocity profiles at the inner and outer walls show a dependency on Reynolds number. At a given  $Y^+$  the value of  $W^+$  increases with increases in the Reynolds number according to the predictions. This feature is not in accord with the experimental data of Brighton but does qualitatively agree with the data of Quarmby. This feature highlights a common problem when dealing with turbulent flows; namely the acquisition of reliable experimental data.

Experimentally it is known that for developed turbulent flow the friction-factor-Reynolds number variation is apparently independent of the radius ratio. Fig.15a shows an experimental band which spans the data of Brighton and also shows the values predicted from the present analysis. Generally the predictions are higher than the experimental data but similar trends are evident. Likewise Fig.15b illustrates the effect of Reynolds number on the ratio of outer wall shear/inner wall shear,  $\tau_o/\tau_i$ , for two values of radius ratio. The solid lines were obtained from the experimental results of Quarmby<sup>20</sup> and the predictions of the present analysis are in excellent agreement. The turbulent predictions were achieved with a simple Van Driest hypothesis for the effective viscosity where the empirical constants  $k$  and  $A^+$  were taken to be 0.4 and 26 respectively. Adjustment of these constants on a trial and error basis could easily give a better friction factor prediction. Broadly speaking about twenty equivalent diameters were necessary in the downstream direction for an initially flat velocity profile to achieve an established form.

The work reported in this section is continuing, and currently the effect of suction and blowing on the development of turbulent flow is being computed using the same technique as above.

### Flow over a Rotating Turbine Disc

Whilst the complex geometry of a gas turbine rotor and its stationary casing is not conducive to a fundamental study of convective heat transfer, the model of a disc rotating close to a stator - shown in Fig.16a - allows considerable information to be elicited by both theoretical and experimental techniques. By means of this model it is possible to show the effect of the axial gap,  $s$ , the coolant mass flow rate,  $\dot{W}$ , and the rotational speed,  $\omega$  on the fluid dynamics and heat transfer of the system. With the addition of a peripheral shroud - shown in Fig.16b - the model can be made to resemble the actual turbine situation, where seals are used to control the egress of coolant and to prevent the ingress of hot gases over the rotor faces.

Prior to the commencement of the current research programme at the University of Sussex, integral boundary layer equations for steady, incompressible, turbulent flow had been applied successfully to the case of the free disc<sup>21</sup> where no stator is present - and to the case of the enclosed disc<sup>22,23</sup> - where the stator completely encloses the rotor to prevent any fluid efflux. The technique used for turbulent flow in these cases was to employ  $1/7$ th power law profiles for the radial and tangential velocity components, and to solve the radial and tangential momentum equations using the Blasius relationships for wall shear stresses. For the enclosed disc, where separate boundary layers could be considered to exist on the rotor and the stator, the rotating core was used to prescribe the radial pressure gradient.

At first sight it would appear that the problem of the cooled turbine disc should be amenable to analysis by the above techniques. However, as with all integral methods, it is necessary to know in advance, or to presume, the likely behaviour of the velocity profiles. It is also necessary to postulate the type of model to be studied, i.e., separate or merged boundary layers on the rotor and stator. If the model is applicable, and the velocity profiles are 'well behaved' the analysis is acceptable: if not, the analysis fails. Unfortunately, for the cases of an outflow of coolant, the radial velocity profiles are difficult to characterise by one or two parameters (as is illustrated by looking ahead to Fig.19). The assumption of separate boundary layers requires the simultaneous solutions of four first-order differential equations, the differential coefficients of which can form an ill-conditioned matrix, and several attempts to solve the system met with a lack of success. In fairness to integral methods, it should be mentioned that recent unpublished attempts assuming fully-developed velocity profiles have proved useful in predicting the asymptotic moment coefficients for large and small coolant outflows.

After the initial failures with integral boundary layer methods,<sup>4</sup> it was decided to use the differential technique of Spalding and Patankar. The space between the rotor and stator was treated as an internal boundary layer (which is valid for  $r \gg s$ ), and the fluid dynamics of the system was studied by the solution of the radial and tangential momentum equations and the continuity equation<sup>11</sup>. Initial values of the velocity profiles were assumed, and the predictions of the frictional moment coefficient, the radial pressure distribution and the outlet velocity profiles were not significantly affected by the initial values chosen.

It was necessary to assume an effective viscosity distribution for the radial and tangential shear stresses, and the following model was used,

$$\tau_r = \mu_{r,eff} \frac{\partial V_r}{\partial z} = \left\{ \mu + \rho \ell^2 \left| \frac{\partial V_r}{\partial z} \right| \right\} \frac{\partial V_r}{\partial z}$$

and

$$\tau_\phi = \mu_{\phi,eff} \frac{\partial V_\phi}{\partial z} = \left\{ \mu + \rho \ell^2 \left| \frac{\partial V_\phi}{\partial z} \right| \right\} \frac{\partial V_\phi}{\partial z}.$$

The mixing-length,  $\ell$ , involved three empirical constants, which were evaluated once-for-all from experimental data, and  $\ell$  was modified near the walls according to the van Driest hypothesis. Employing the above hypothesis it was possible to use the Couette-flow relationships of Spalding and Patankar to evaluate the wall stresses, although it was necessary to modify these relationships near the rotating disc.

The radial pressure gradient was calculated by the simultaneous iterative solution of the radial momentum equation and the integrated continuity equation, and a typical comparison of calculated and experimental values is shown in Fig.17. The calculated moment coefficients were, in general, lower than the measured values, as can be seen in Fig.18, but this is attributed to the effective viscosity model chosen. Fig.19 shows the predicted outlet radial and tangential velocity profiles, and whilst the latter are in good agreement with the experimental data the former tend to be less satisfactory at higher rotational speeds. Owing to the complexity of the flow system, and the simplicity of the turbulence model used, the fluid dynamics results are considered to be encouraging.

For the case of heat transfer from a disc rotating near a stator with a radial outflow of coolant, it is possible, under certain conditions, to apply the Reynolds analogy<sup>24</sup> to produce Nusselt numbers from the moment coefficients. In general, however, heat transfer can only be satisfactorily predicted by solution of the energy equation, and the Spalding-Patankar method was also used for this purpose<sup>25</sup>. Again, semi-empirical relationships for the turbulent terms are necessary, and an 'effective conductivity' was used where the heat flux,  $q$ , is given by

$$q = - \lambda_{eff} \frac{\partial T}{\partial z} = - \left[ \frac{\mu}{Pr} + \frac{\mu_{\phi,t}}{Pr_t} \right] c \frac{\partial T}{\partial z}$$

where  $Pr$  and  $Pr_t$  are laminar and 'turbulent' Prandtl numbers, assumed

constant in this analysis. The choice of  $\mu_{\phi,t}$   $\left( \mu_{\phi,t} = \rho \ell^2 \left| \frac{\partial V_\phi}{\partial z} \right| \right)$  for

the appropriate turbulent viscosity is considered justified owing to the

analogy/

analogy between the tangential momentum and energy equations. By solution of the differential boundary layer equations it is possible to take into account the effect of axial gap mass flow rate, rotational speed, frictional heating, Prandtl number and the thermal boundary conditions on the local and overall Nusselt numbers.

The calculated effect of Prandtl number on the overall Nusselt number compared with the free disc result of Dorfman<sup>26</sup>, is shown in Fig.20 for a range of flow parameters, whilst Fig.21 shows a comparison of calculated local Nusselt numbers for an air-cooled disc with the free disc results. The available experimental data for air-cooled discs is limited and contradictory: Krieth et al<sup>27</sup> have shown that rotation has only a small effect on heat transfer, whilst Kapinos<sup>28</sup> has obtained experimental results showing that the heat transfer is more affected by rotation than by mass flow rate. Nusselt numbers calculated by the Spalding-Patankar method form a transition between these extreme views (as do the Nusselt numbers calculated in Ref.24), as is shown in Fig.22. It is apparent that more experimental data are necessary before the quantitative accuracy of the heat transfer predictions can be determined, but the qualitative trends are certainly consistent with free disc results. The current research programme at Sussex is aimed at providing heat transfer data, with which to compare the predicted results, and it is intended to extend the analysis to include other turbulence hypotheses.

In conclusion it can be stated that solution of the differential boundary layer equations for a disc rotating near a stator with a radial outflow of coolant has, despite the continuous turbulence models used, provided fluid dynamics results that are in satisfactory agreement with experimental data and heat transfer results that are in accord with free disc data. Whilst integral techniques proved to be unsuccessful for handling the case of separate boundary layers, subsequent application of integral techniques - armed with the hindsight of experimental data and numerical predictions - have been employed to calculate the asymptotic fluid behaviour for high and low mass flow rates. These integral solutions employing very simple velocity profiles, are regarded as bounding the problems rather than of providing detailed knowledge of the flow system. In order to use integral methods to describe the flow in detail it would be necessary to introduce more parameters into the velocity profiles, necessitating more auxiliary relationships and increasing the risk of ill-conditioning.

### Conclusions

The following principal conclusions may be drawn from the work reported in this paper:-

- (1) Although the integral entrainment method<sup>2</sup> of calculation appears to be soundly based, and can be made to yield predictions of aerodynamic performance at least, in fair accord with experimental observation, the shortage of data for the necessary auxiliary equations which are relevant to practical conditions leads to grave reservations about the reliability of the method in the design of turbine blading.
- (2) The semi-integral method employing the enthalpy deficit Reynolds number as a criterion of the flow conditions<sup>3</sup> is convenient to use for heat transfer calculations over both convection- and transpiration-cooled turbine blades. It yields acceptably accurate results, however, only for the pressure surfaces of porous blades and in all other cases appreciably over-predicts the heat transfer rates.

- (3) The numerical procedure for solving the differential equations of the boundary layer, and employing the mixing length hypothesis for turbulent viscosity and conductivity<sup>4</sup>, has proved the most effective of the methods used in the present work. Where the boundary layers are clearly laminar or turbulent, predictions of heat transfer rates are satisfactory on both impermeable and porous blades, although uncertainties arise near the laminar-turbulent transition point, even where this can be accurately placed. The method shows some promise for dealing with the effects of mainstream turbulence upon heat transfer rates, a condition of critical importance in the practical turbine situation.
- (4) For confined axisymmetric flows with suction at the walls the differential method of calculation<sup>4</sup> has also shown its versatility in predicting the development of the boundary layer. It has been demonstrated that acceptable results may be obtained for the porous walled annulus, with its transpiration-cooled rotor blade and combustion chamber connotations. However, there remains a scarcity of reliable experimental data for these cases.
- (5) Although integral methods of analysis have shown their value in dealing with special bounding cases of the flows over turbine disc-stator systems, the differential procedures of Ref.4 have been very satisfactory in predicting the detailed hydrodynamic behaviour of a wide range of flows, despite the clear oversimplification of the two-component shear system. There is little reliable experimental data available with which to compare predictions of heat transfer rates over cooled turbine discs, although calculated results are consistent with data from 'free' discs.

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Nomenclature/

Nomenclature

	A+	empirical constant in van Driest effective viscosity model
	$b_h$	dimensionless blowing parameter
	C	specific heat at constant pressure
	$C_m \equiv M_o / (\frac{1}{2} \rho \omega^2 r_o^5)$	moment coefficient
	$C_p \equiv (p_A - p) \rho r_o^2 / \mu^2$	pressure coefficient
	$C_w \equiv \dot{W} / (\mu r_o)$	mass flow coefficient
	d	duct hydraulic diameter
	$G \equiv s / r_o$	disc gap ratio
	k	empirical constant in van Driest effective viscosity model
	$K \equiv \frac{\nu}{V} \sum \frac{dV}{g} \frac{dV}{dx}$	velocity gradient parameter
	$\ell$	mixing length
	$\dot{m}$	effusing mass flow rate
	$M \equiv \frac{u}{w_m} Re$	duct wall mass flow parameter
	$M_i$	inner duct wall mass flow parameter
	$M_o$	disc friction moment
	$Nu, \overline{Nu}$	local, overall Nusselt numbers
	$p, p_A, p_o$	pressure, atmospheric pressure, entry pressure
	$P \equiv \frac{p_o - p}{\frac{1}{2} \rho W_m^2}$	pressure parameter
	$Pr, Pr_t$	laminar, turbulent Prandtl number

	q	heat flux
	r	radial co-ordinate
	$r_o$	outer radius
Re	$\equiv \frac{\rho d w_m}{\mu}$	duct Reynolds number
	$\equiv \frac{\rho \omega r_o^2}{\mu}$	disc Reynolds number
	$R_{h,2}$	enthalpy deficit thickness Reynolds number
	$R_L$	Reynolds number using overall length of blade
R	$\equiv r/r_o$	radius ratio
	S, $S_c$	axial gap between disc and stator, and disc and shroud
	$S_o$	Stanton number at datum condition
	T, $\Delta T$	temperature, temperature difference between mainstream and blade
	$u'_{rms}$	r.m.s. turbulent velocity in mainstream gas
	u	radial velocity in duct
	$V_g$	local free stream velocity for blade
	$V_r, V_\phi$	radial, tangential velocity between disc and stator
	w	axial velocity in duct
	$w_m$	mean axial velocity at entry to duct
W	$\equiv \frac{w}{w_m}$	dimensionless axial velocity in duct
	$\dot{W}$	disc coolant mass flow rate
W +	$\equiv \frac{w}{\sqrt{\tau/\rho}}$	dimensionless velocity

x	distance along blade in downstream direction
y	distance across duct measured from inner wall
$Y \equiv \frac{y}{d}$	dimensionless distance across duct
$Y^+ \equiv \frac{y \sqrt{\tau/\rho}}{\mu}$	dimensionless distance
z	axial co-ordinate
$Z \equiv \frac{z}{dRe}$	dimensionless axial co-ordinate in duct
$\lambda_{eff}$	effective thermal conductivity
$\mu$	absolute viscosity
$\mu_{eff}, \mu_{r,eff}, \mu_{d,eff}$	effective viscosities
$\nu$	kinematic viscosity
$\rho$	density
$\tau$	shear stress in duct
$\tau_r, \tau_\phi$	radial, tangential shear stress
$\Gamma \equiv \frac{\tau Re}{\rho w_m^2}$	components between disc and stator dimensionless duct shear stress
$\psi$	ratio of actual Stanton number to $S_0$
$\omega$	angular velocity of rotating disc

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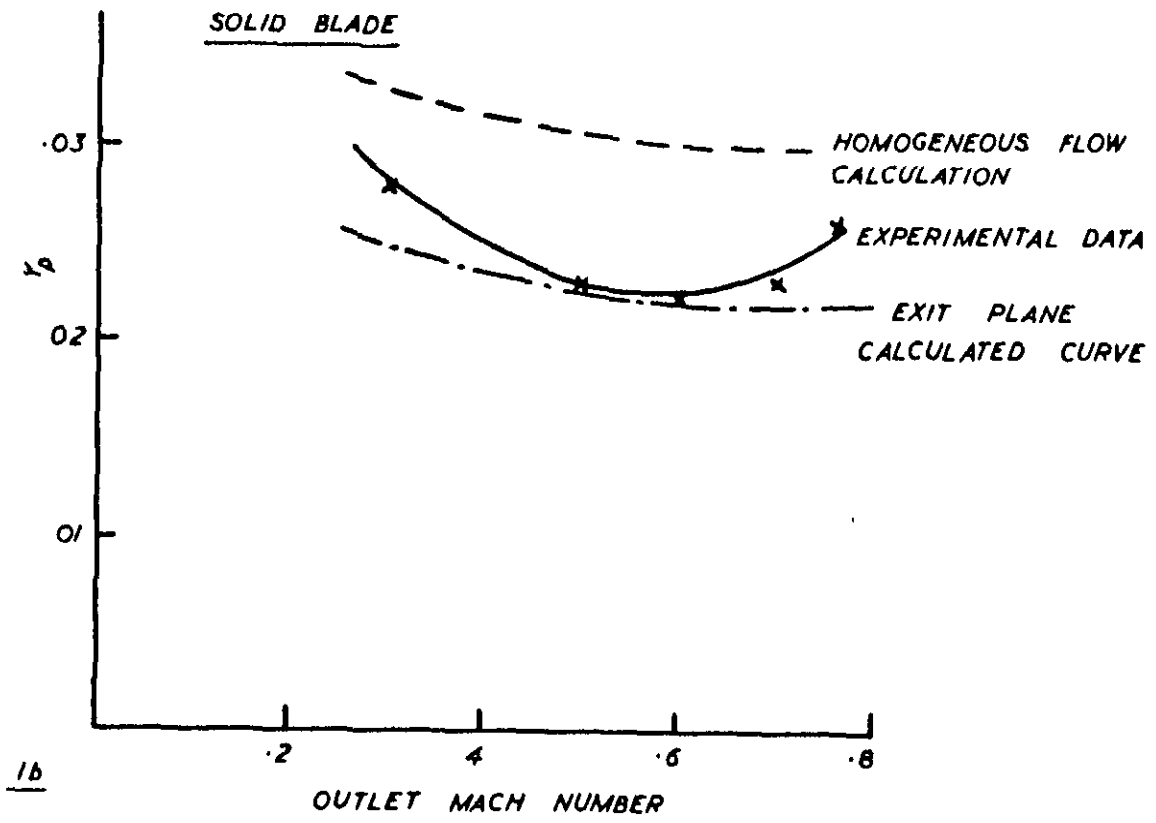
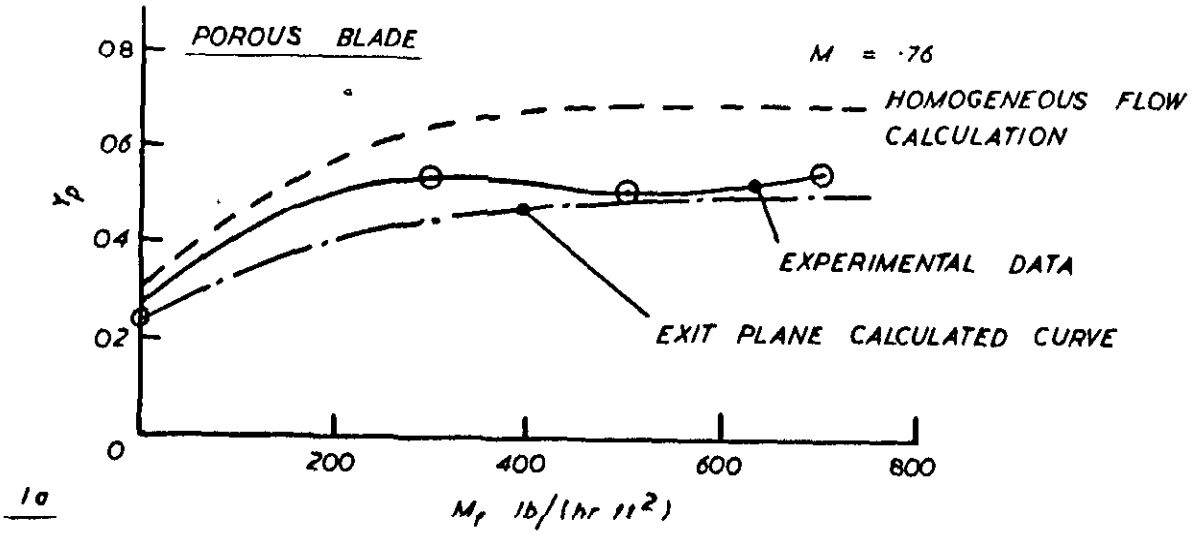
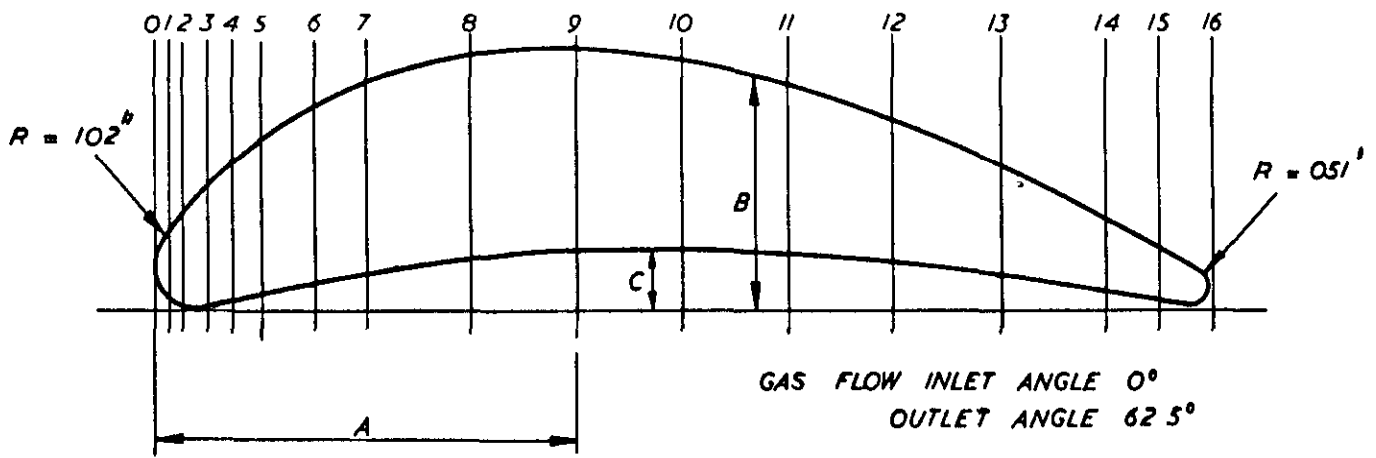


FIG 1. COMPARISON OF MEASURED AND CALCULATED LOSS COEFFICIENTS



STATION	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A"	0	0.35	0.69	1.38	2.08	2.77	4.15	5.54	8.31	11.08	13.85	16.62	19.39	22.16	24.93	26.32	2.77
B"	102	193	245	326	386	435	511	564	633	647	627	573	484	369	236	159	0.51
C"	102	0.26	0.06	0.05	0.16	0.28	0.61	0.89	1.24	1.38	1.35	1.23	1.05	0.76	0.38	0.16	0.51

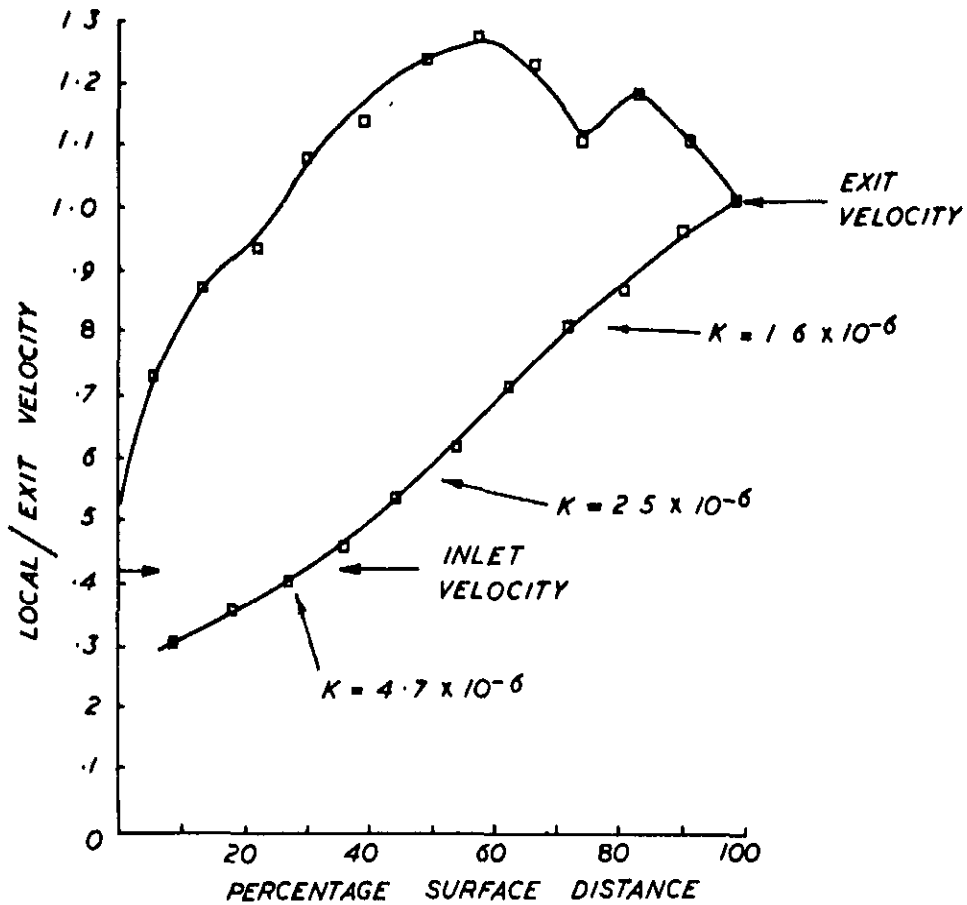


FIG 2 BLADE SECTION DETAILS AND EXPERIMENTAL VELOCITY DISTRIBUTION (M O.77)



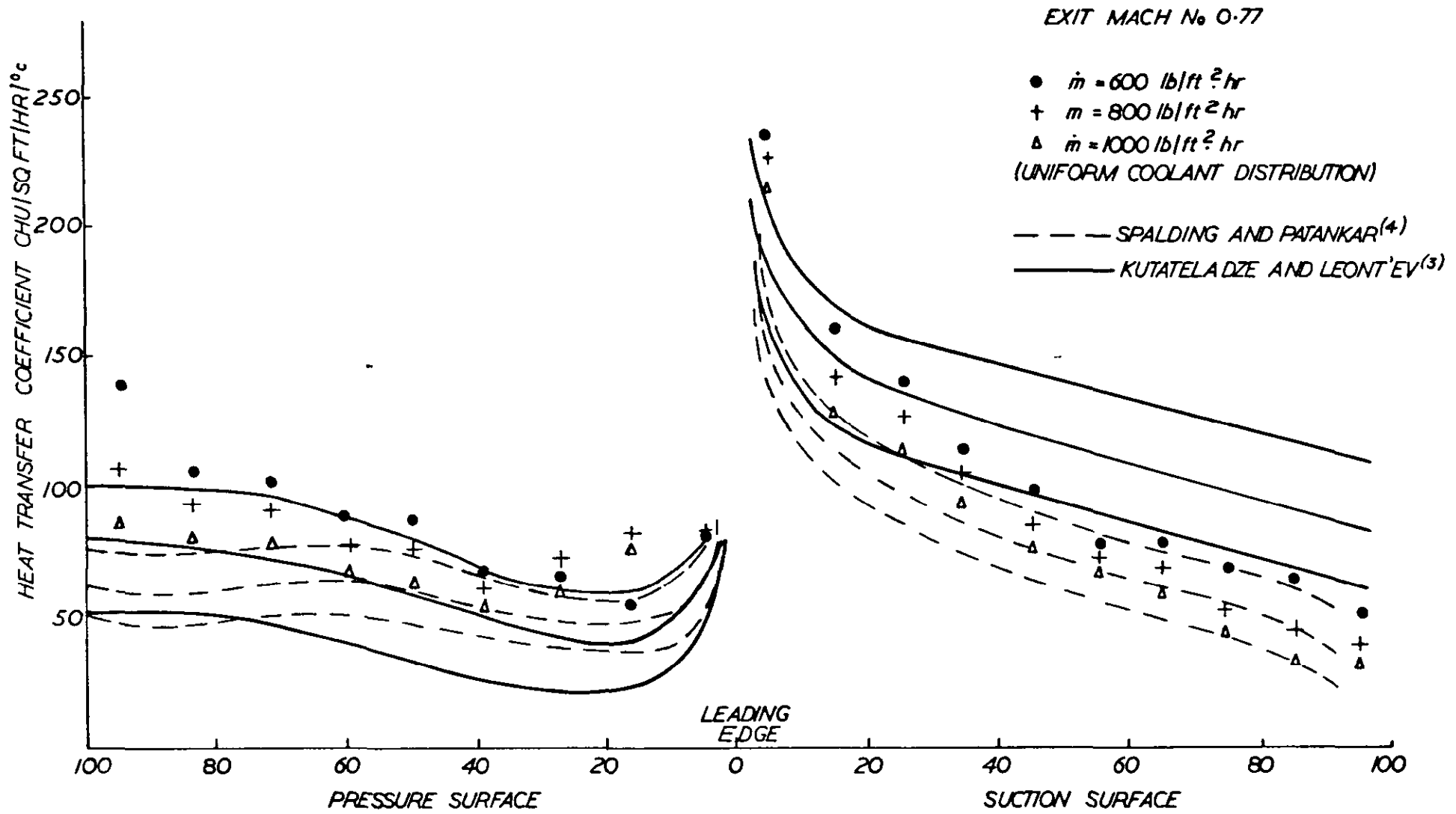


FIG. 3 TRANSPIRATION COOLED BLADE HEAT TRANSFER

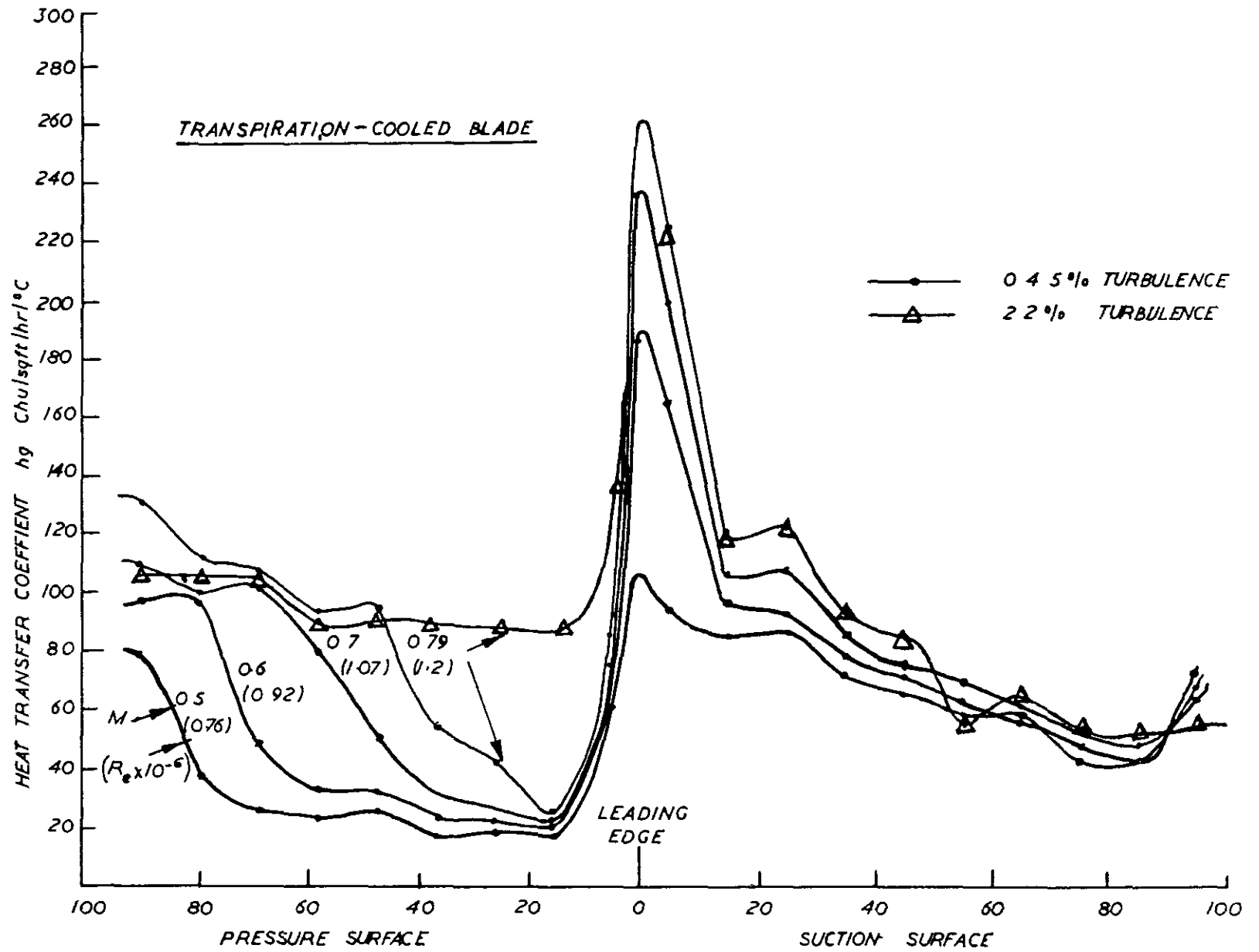


FIG 4 VARIATION OF ISOTHERMAL BLADE HEAT TRANSFER COEFFICIENT WITH MACH (AND REYNOLDS) NUMBER  
TEMPERATURE RATIO 0.37

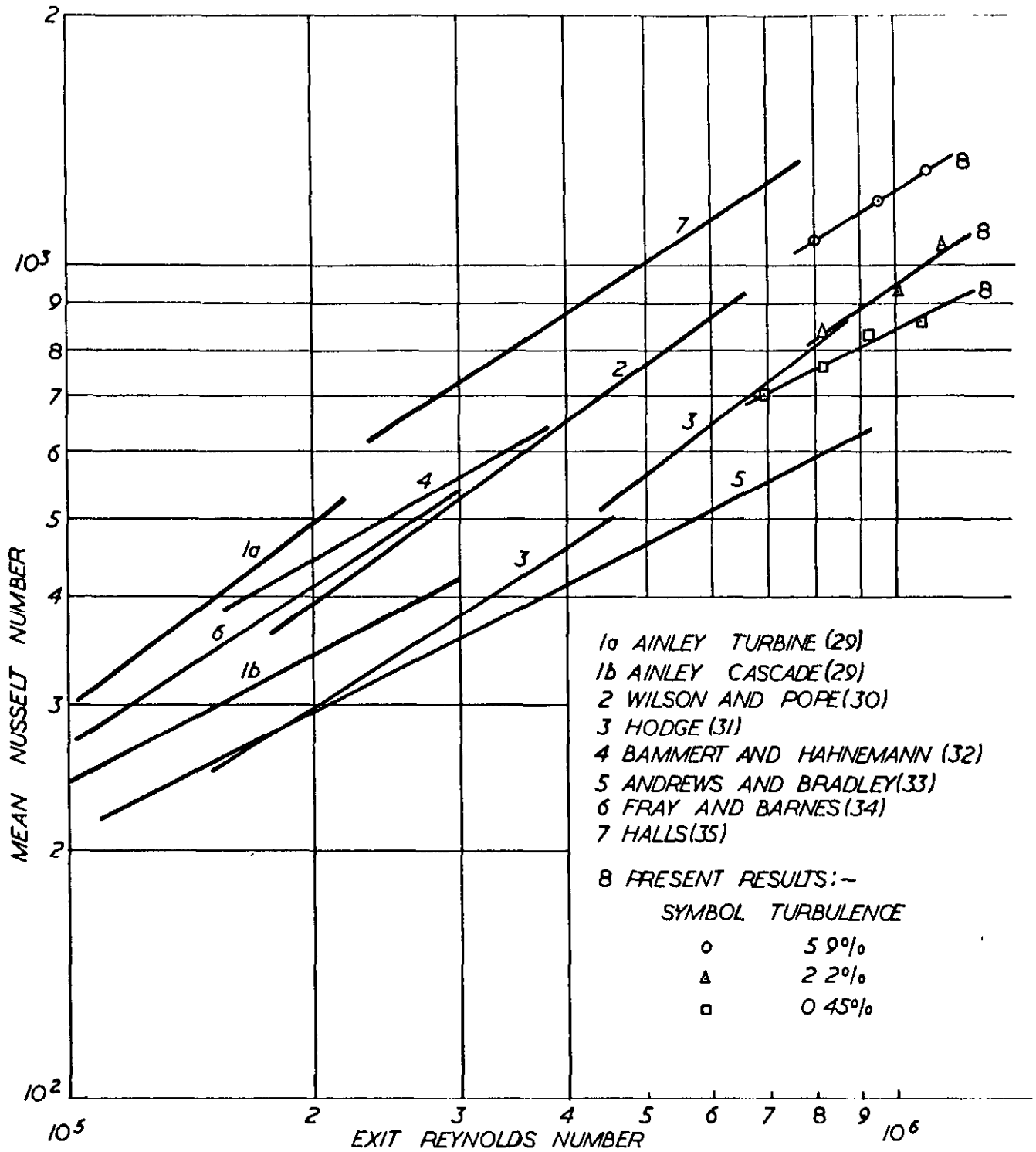


FIG 5 MEAN CONVECTION - COOLED BLADE HEAT TRANSFER RESULTS COMPARED WITH OTHER DATA

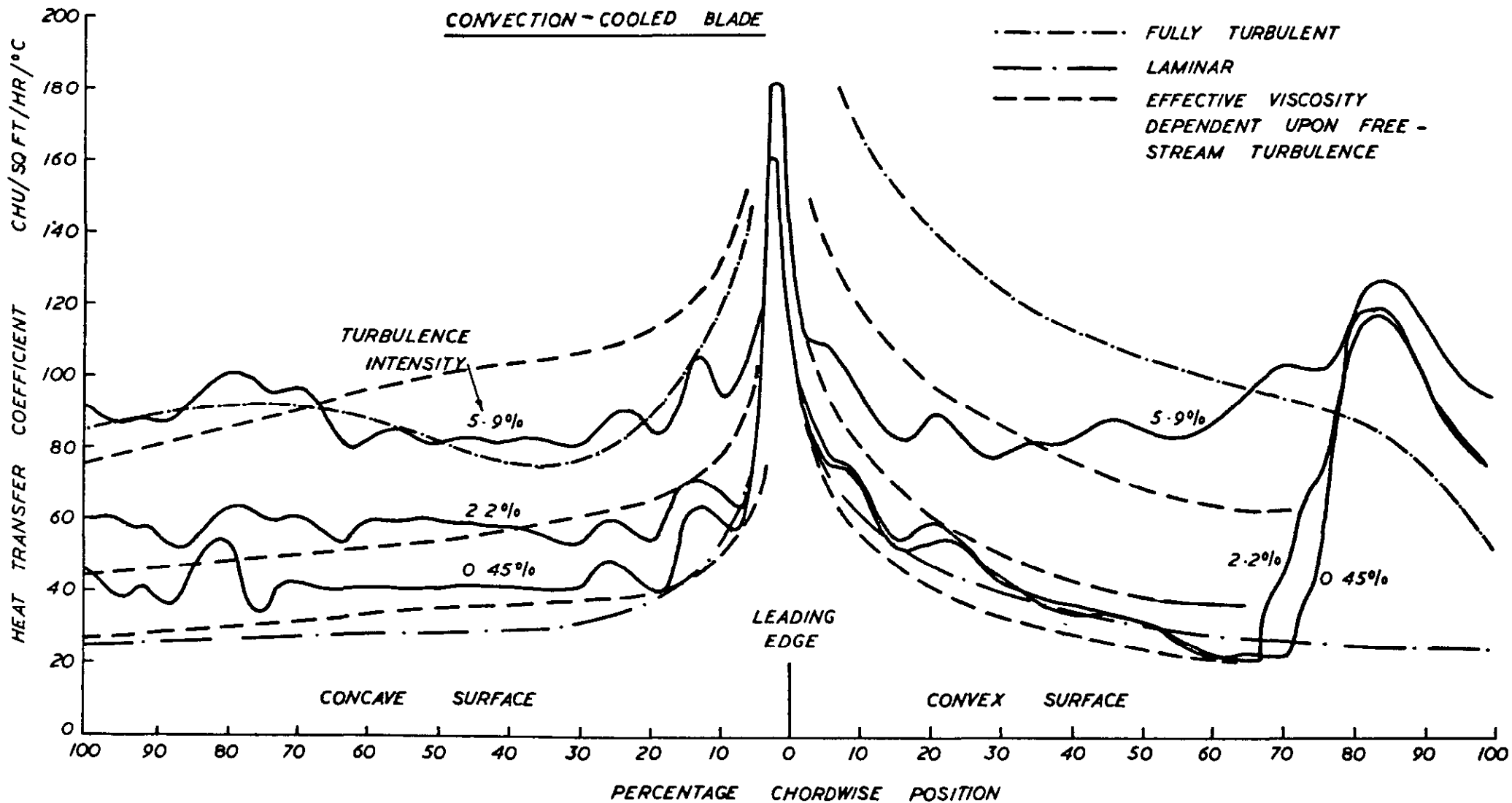
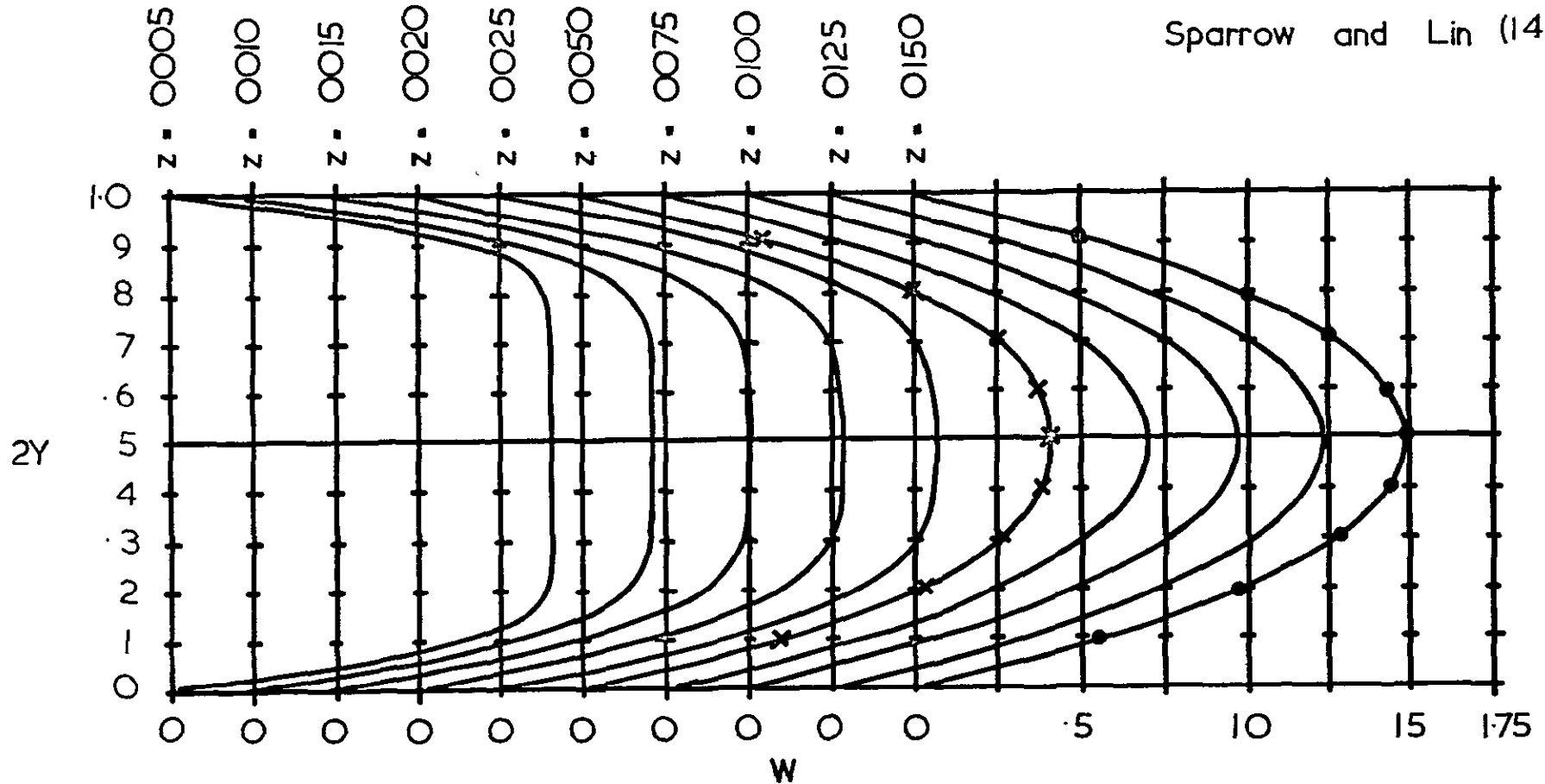


FIG 6 EXPERIMENTAL DISTRIBUTION OF HEAT TRANSFER COEFFICIENT FOR AN EXIT MACH NUMBER OF 0.75.  
 COMPARISON WITH THEORY OF SPALDING AND PATANKAR (4)

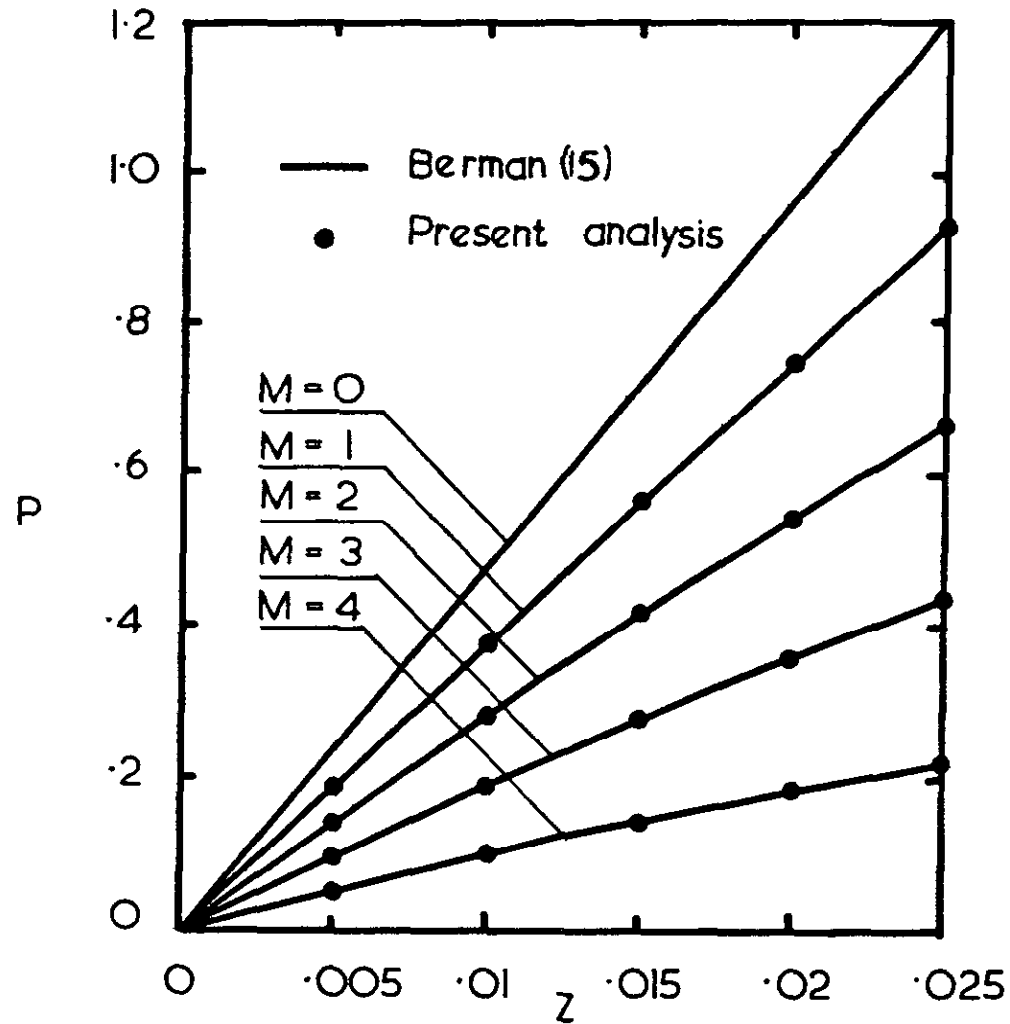
- Fully developed solution
- ✕ Typical comparison with Sparrow and Lin (14)



DEVELOPMENT OF LAMINAR FLOW IN A CONCENTRIC ANNULUS  
WITH IMPERVIOUS WALLS AND UNIFORM ENTRY VELOCITY PROFILE

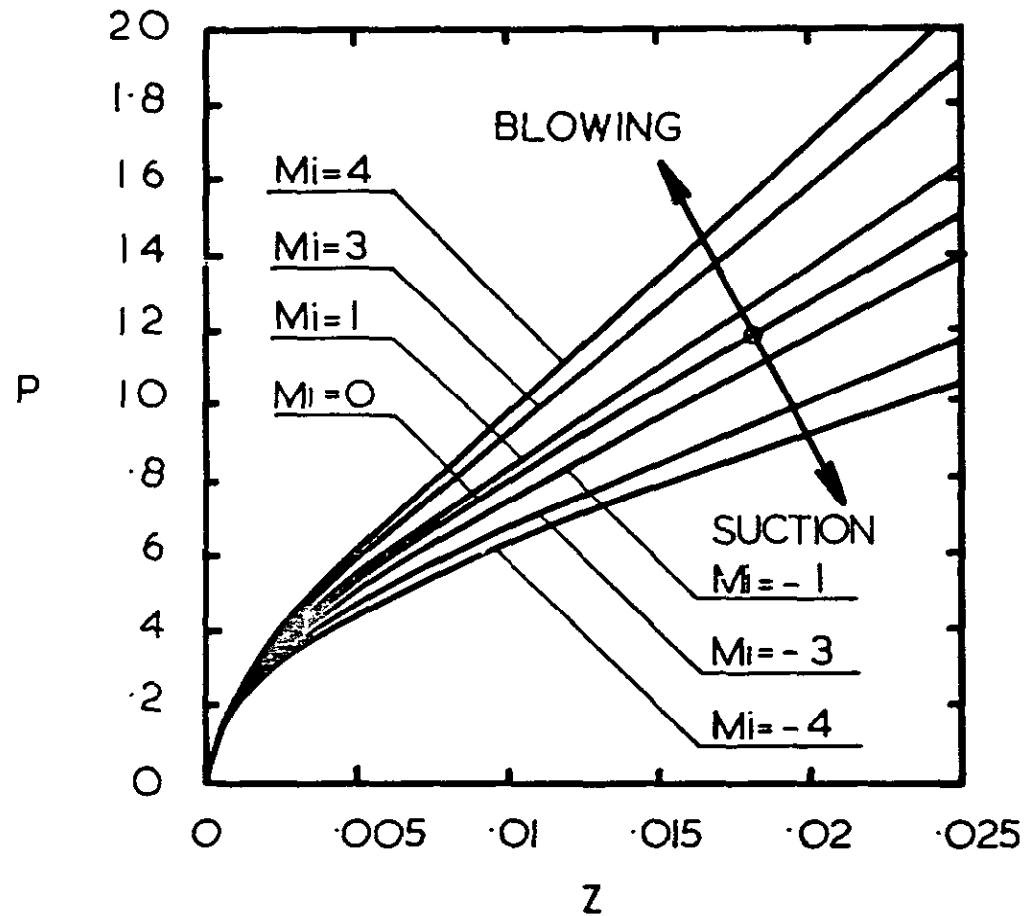
RADIUS RATIO = 0.80

FIGURE 7



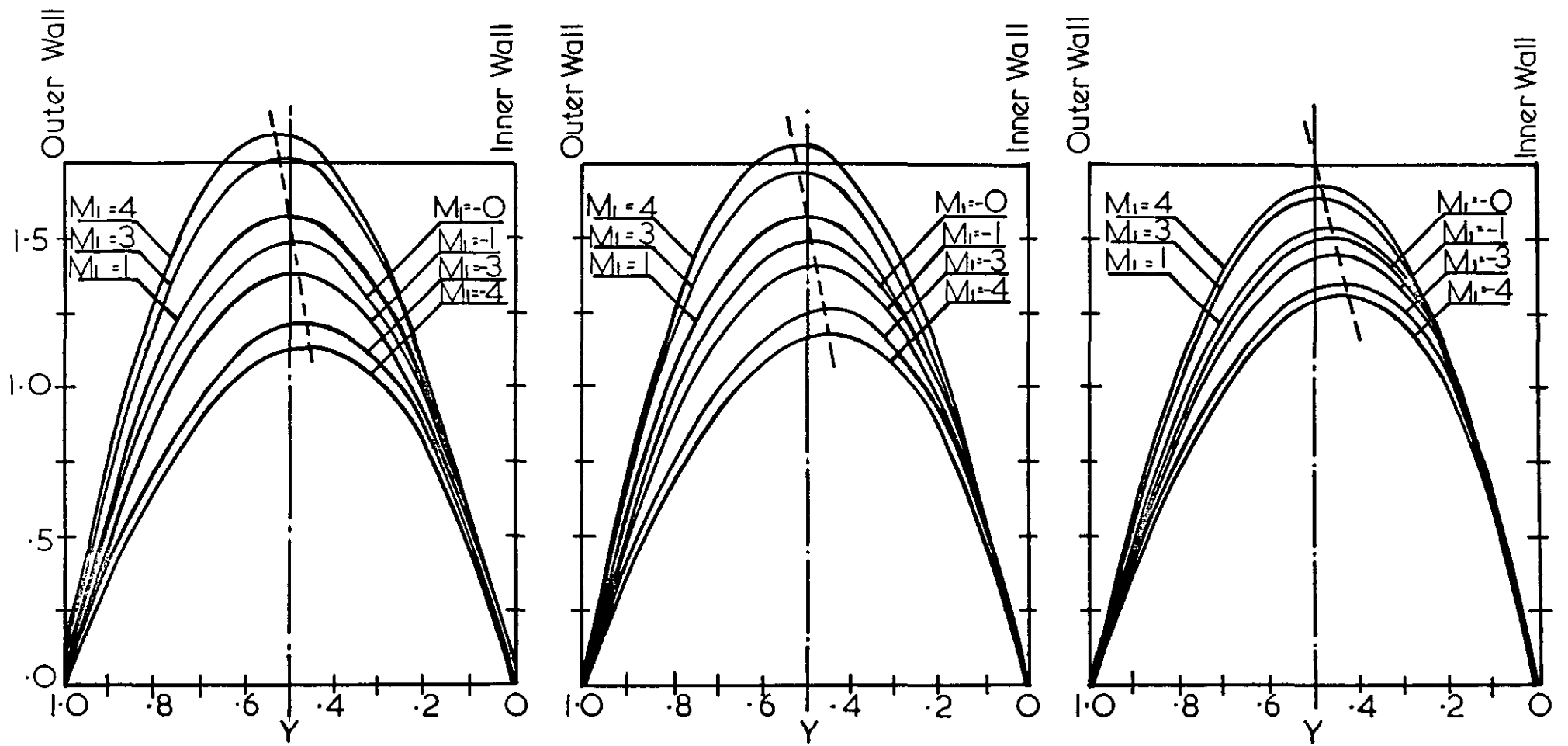
THE INFLUENCE OF UNIFORM SUCTION AT BOTH WALLS OF A PARALLEL SIDED DUCT ON THE PRESSURE DISTRIBUTION WHEN FLOW IS ESTABLISHED AT ENTRY.

FIGURE 8



INFLUENCE OF UNIFORM SUCTION OR BLOWING AT THE INNER  
 WALL OF AN ANNULAR DUCT OF RADIUS RATIO = 0.8  
 ( Velocity profile is uniform at entry )

FIGURE 9



(a) Radius ratio = 1.0

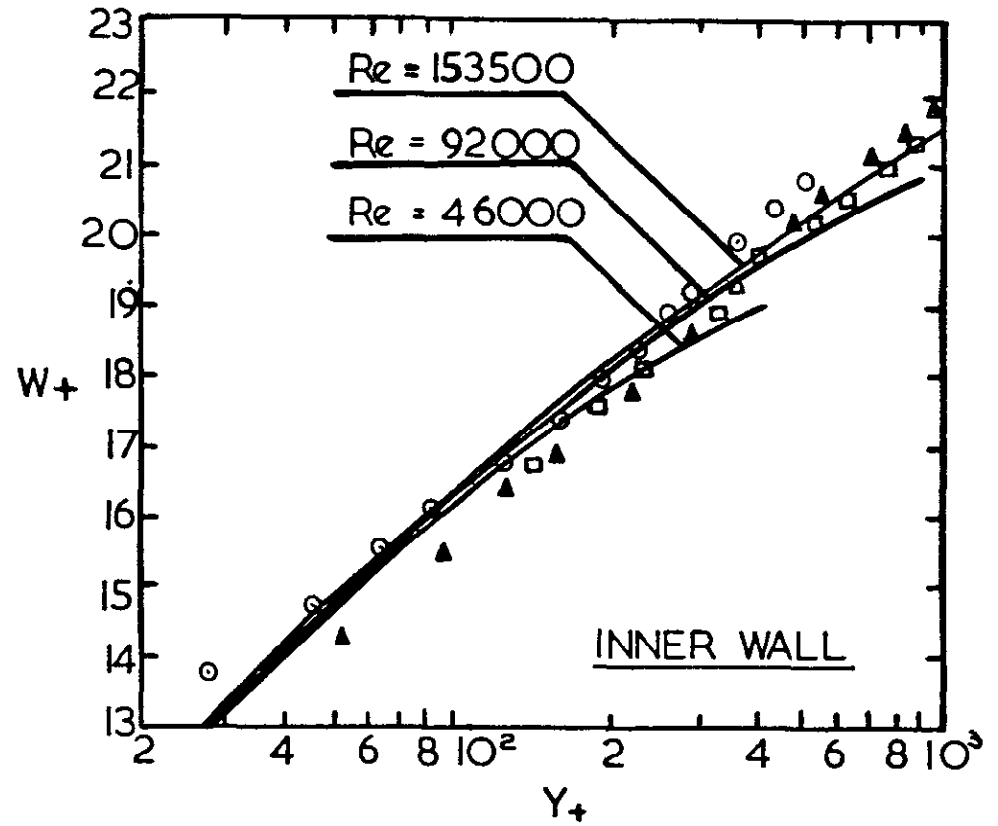
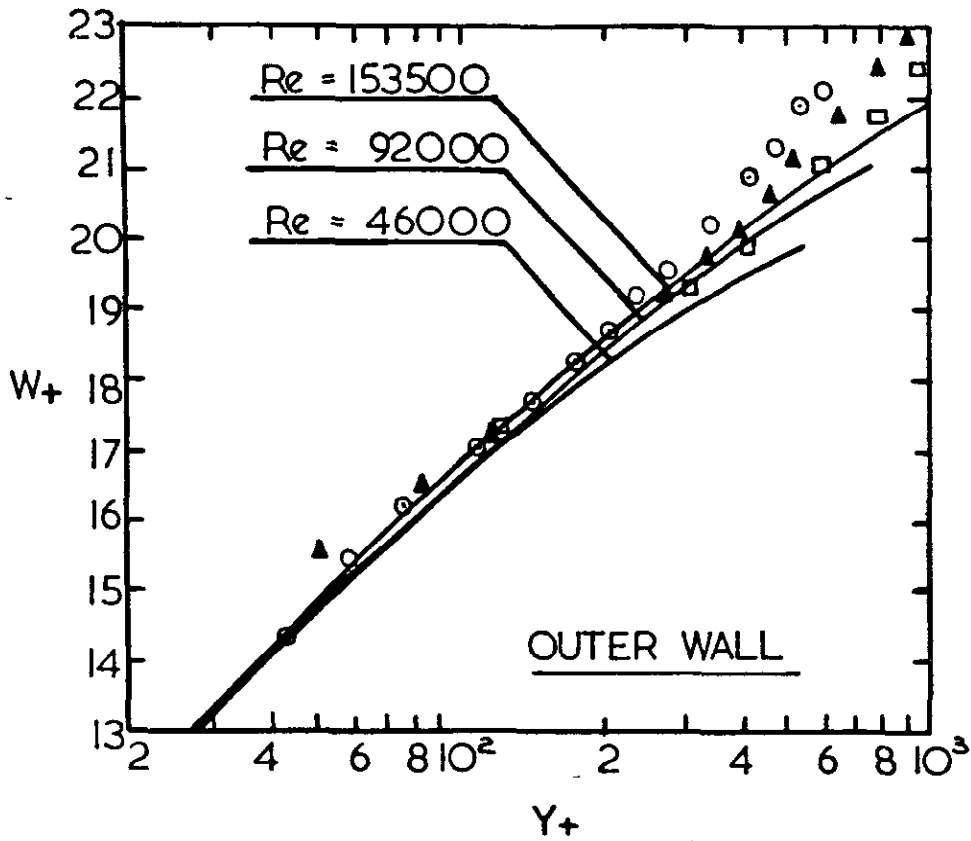
(b) Radius ratio = 0.8

(c) Radius ratio = 0.4

INFLUENCE OF UNIFORM SUCTION OR BLOWING AT THE INNER WALL  
OF A CONCENTRIC ANNULUS WITH UNIFORM ENTRY VELOCITY PROFILE. ( $z = 0.03$ )

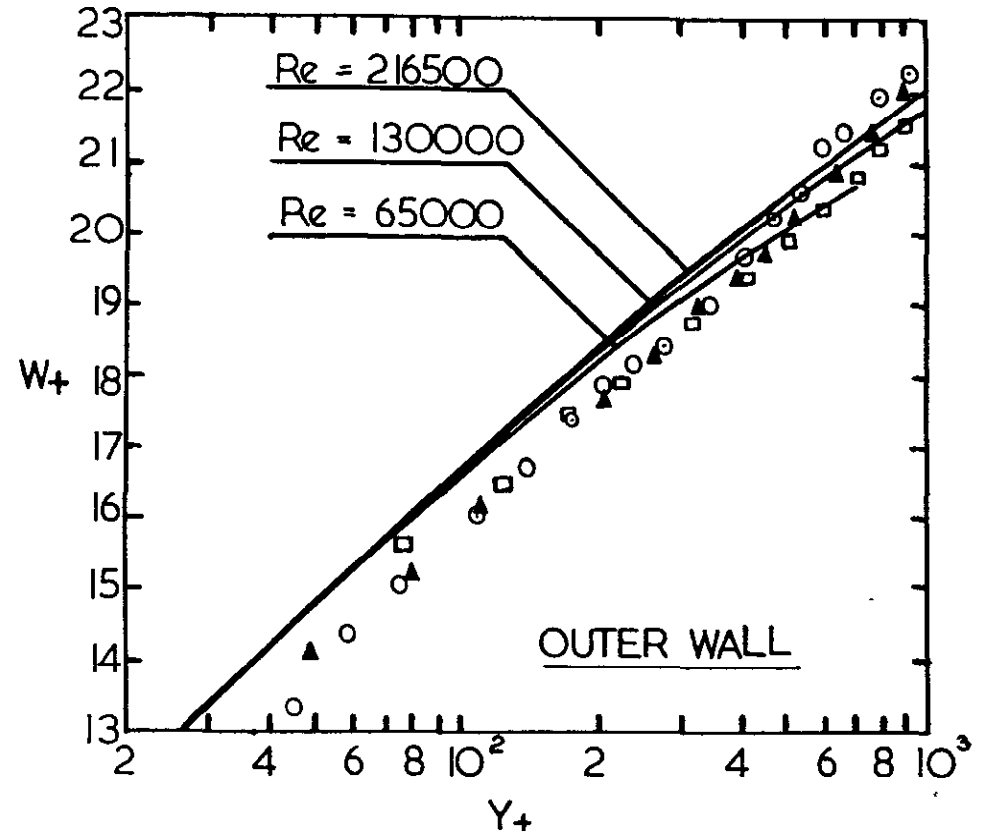
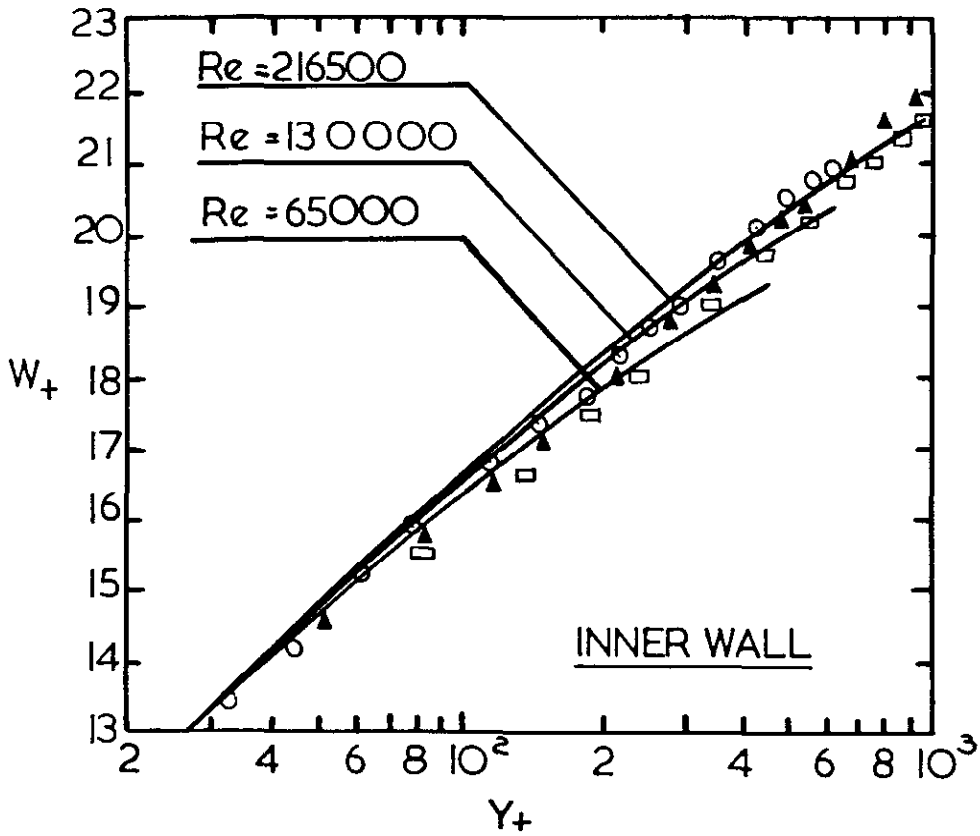
FIGURE 10





Legend :-  $\circ$  Re = 46000,  $\blacktriangle$  Re = 92000,  $\square$  Re = 153500  
Brighton(8)

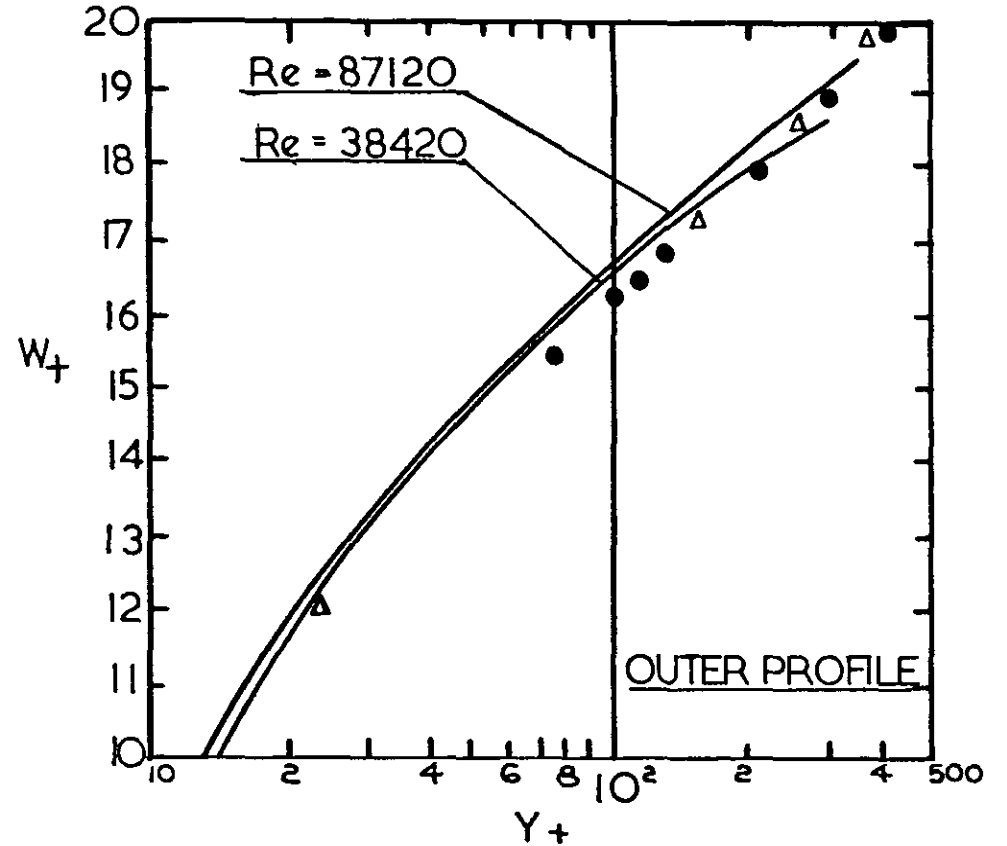
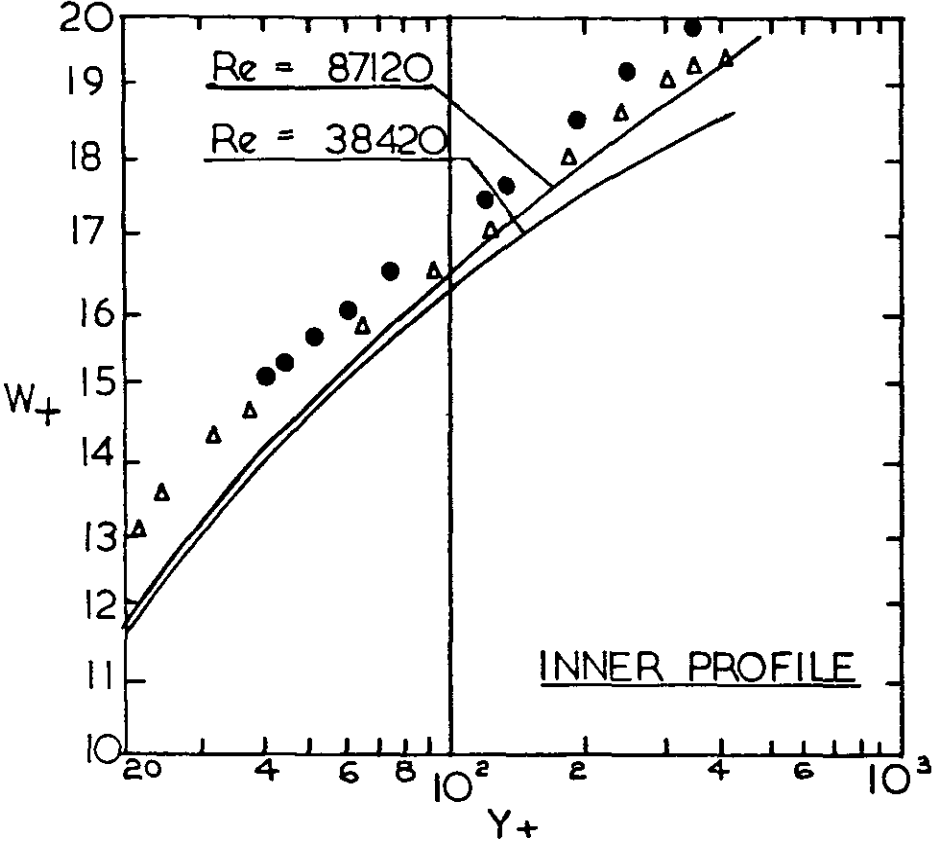
COMPARISON OF PREDICTED AND EXPERIMENTAL VELOCITY PROFILES  
FOR  $R_i/R_o = 0.562$   
FIGURE II



Legend -  $\circ$  Re = 65000,  $\blacktriangle$  Re = 130000,  $\square$  Re = 216500  
Brighton (18)

COMPARISON OF PREDICTED AND EXPERIMENTAL VELOCITY PROFILES  
FOR  $R_1/R_0 = 0.375$

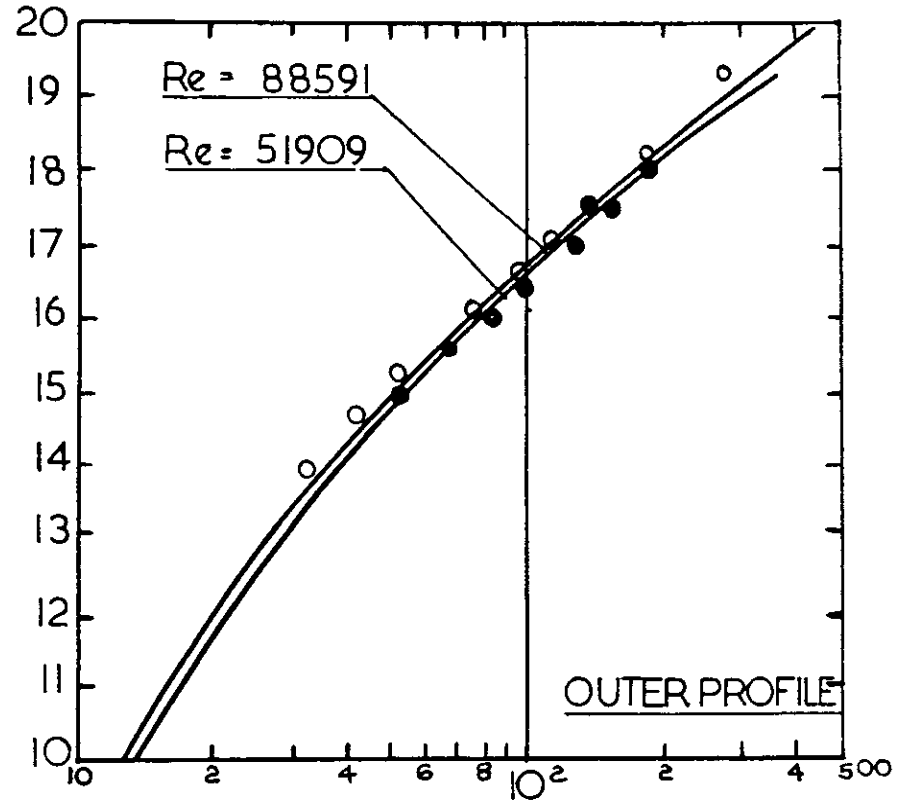
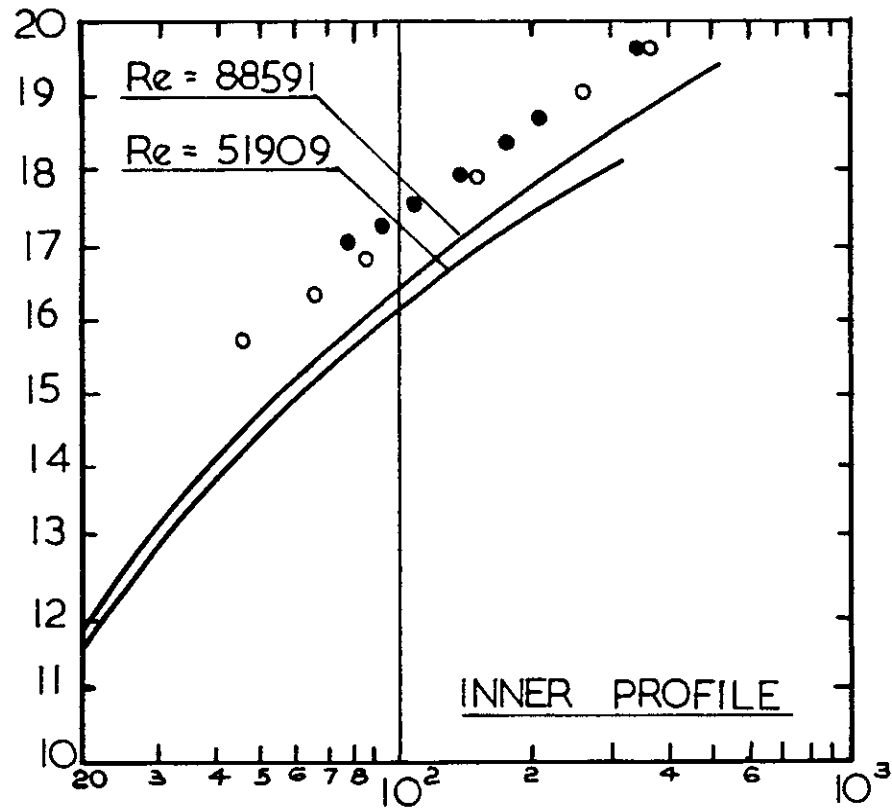
FIGURE 12



Legend      $\Delta$     $Re = 38420$  ,                      $\bullet$     $Re = 87120$  (Quarmby (19) )

COMPARISON OF PREDICTED AND EXPERIMENTAL RESULTS FOR  $R_i/R_o = 0.347$

FIGURE 13



Legend

○ Re = 51909

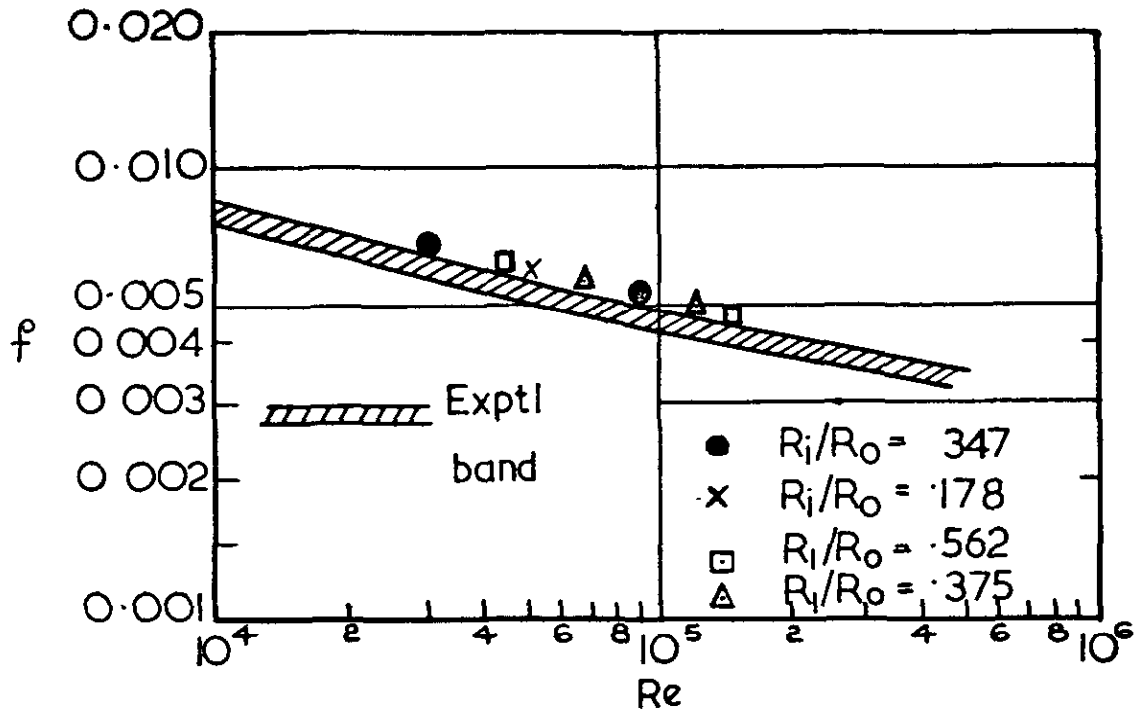
● Re = 88591

(Quarmby (19) )

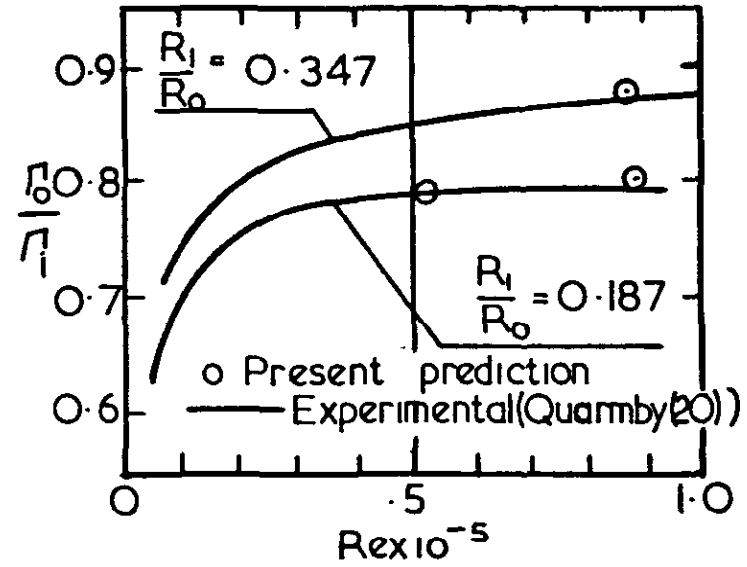
COMPARISON OF PREDICTED AND EXPERIMENTAL VELOCITY PROFILES

FOR  $R_i/R_o = 0.178$

FIGURE 14



(a)



(b)

COMPARISON OF FRICTION FACTORS AND WALL SHEAR STRESSES WITH EXPERIMENTAL DATA OF BRIGHTON (18) AND QUARMBY (20)

FIGURE 15

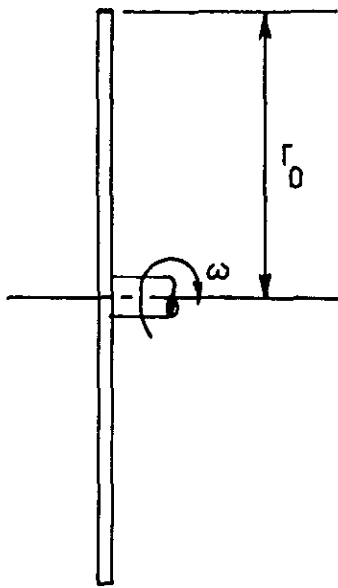


Fig16a Free disc

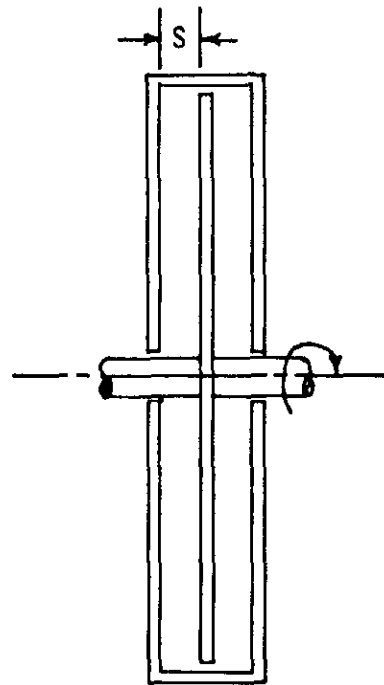


Fig16b Enclosed disc

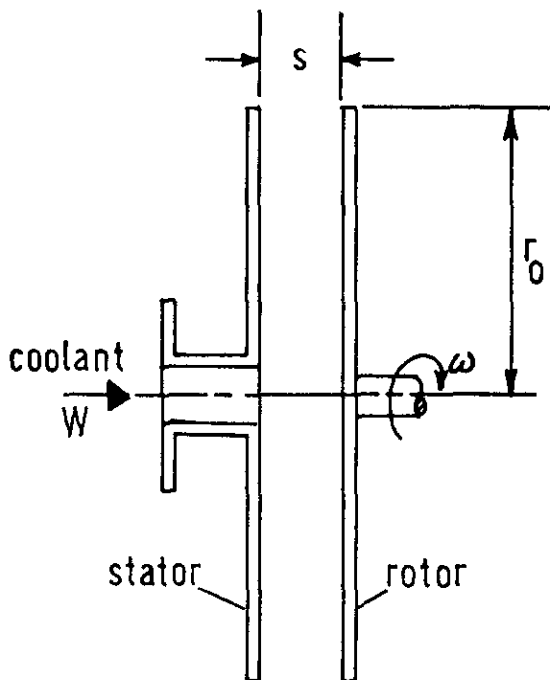


Fig16c Disc rotating near an unshrouded stator

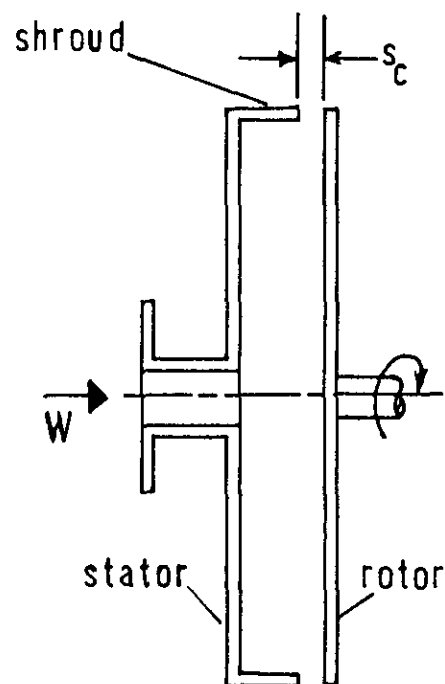


Fig16d Disc rotating near a shrouded stator

$$C_p \equiv \rho r_0^2 (\rho - \rho_A) / \mu^2$$

$$C_p \cdot 10^{10}$$

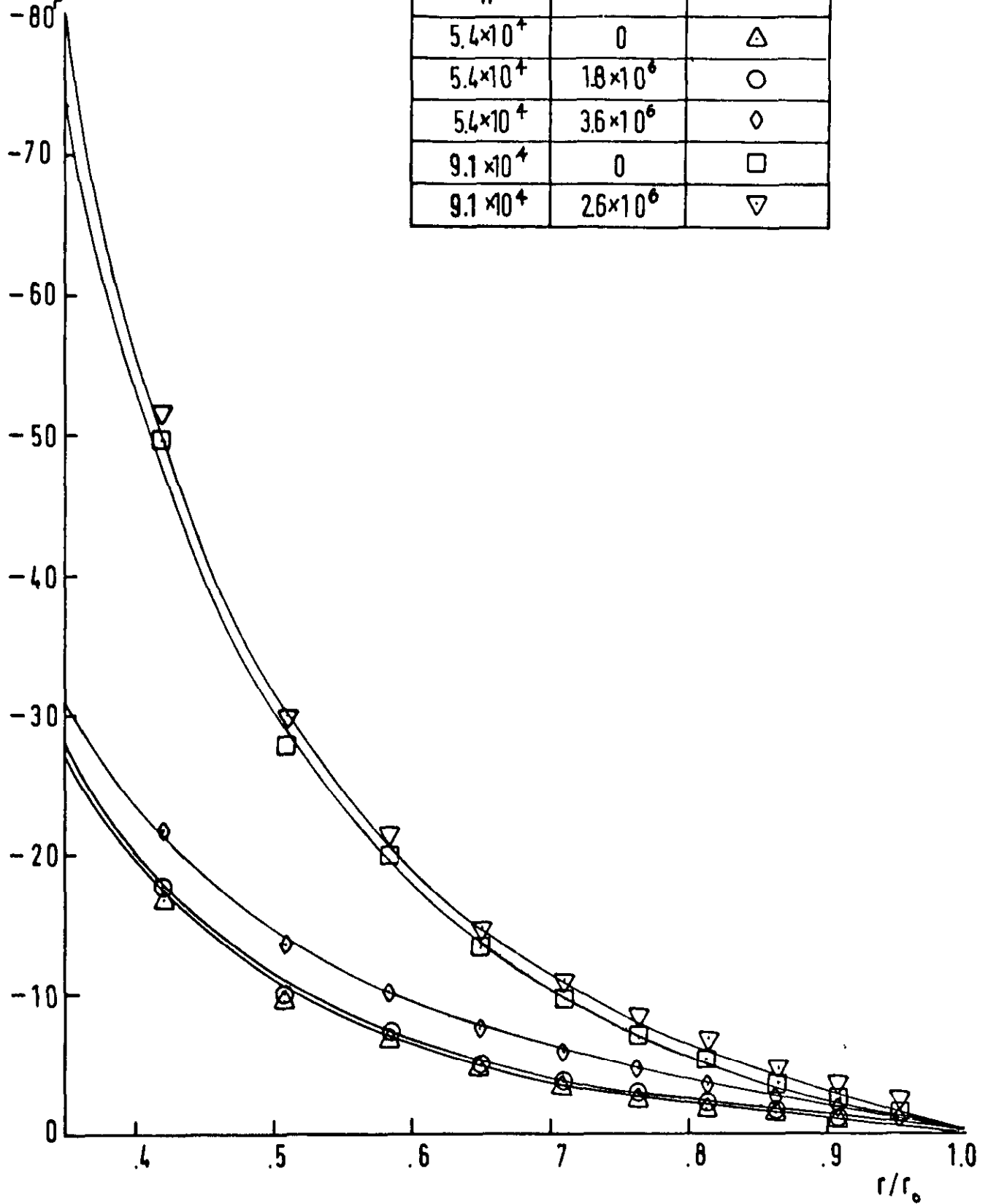


FIG 17 Calculated pressure coefficients for  $G=0.03$

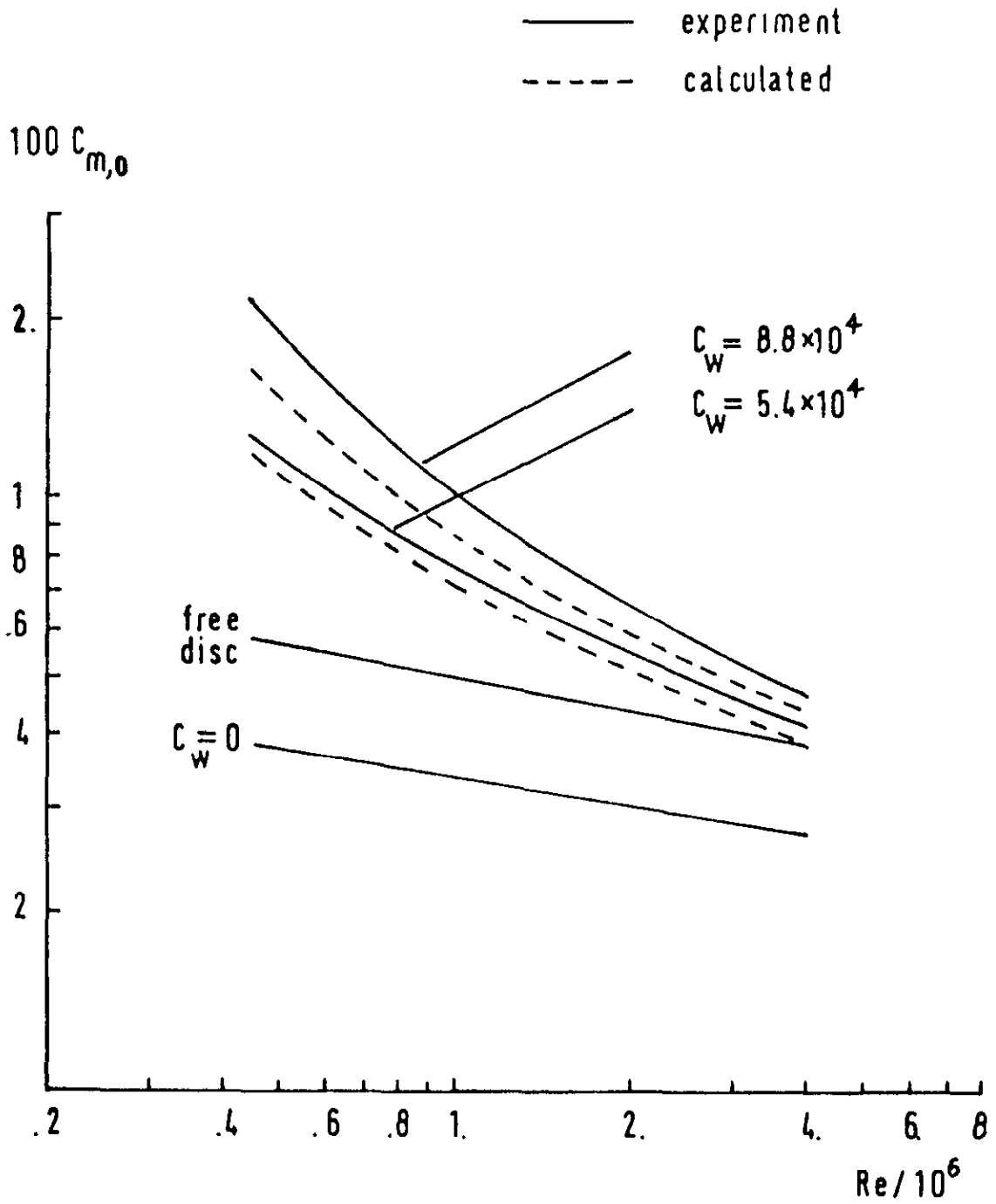


FIG18 Calculated moment coefficients for  $G = 0.03$



experiment	Re
○	0
□	$1.7 \times 10^6$
△	$3.4 \times 10^6$

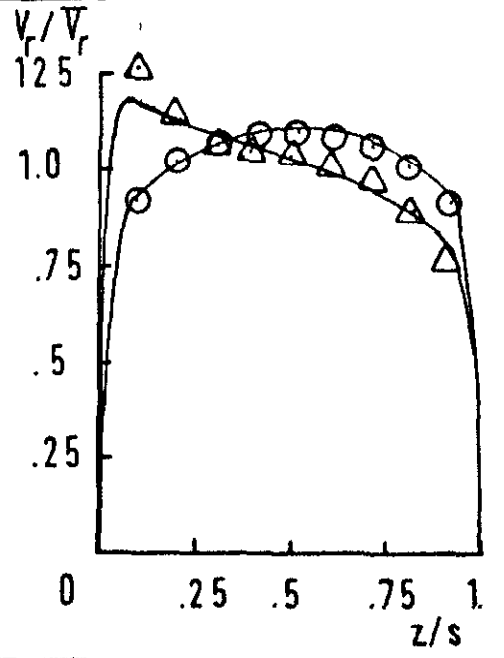
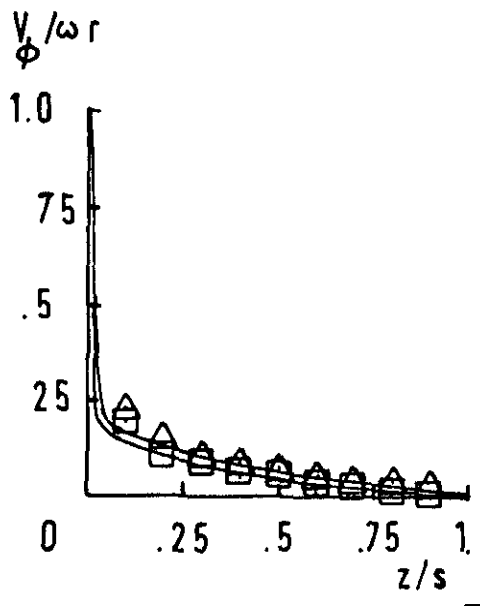
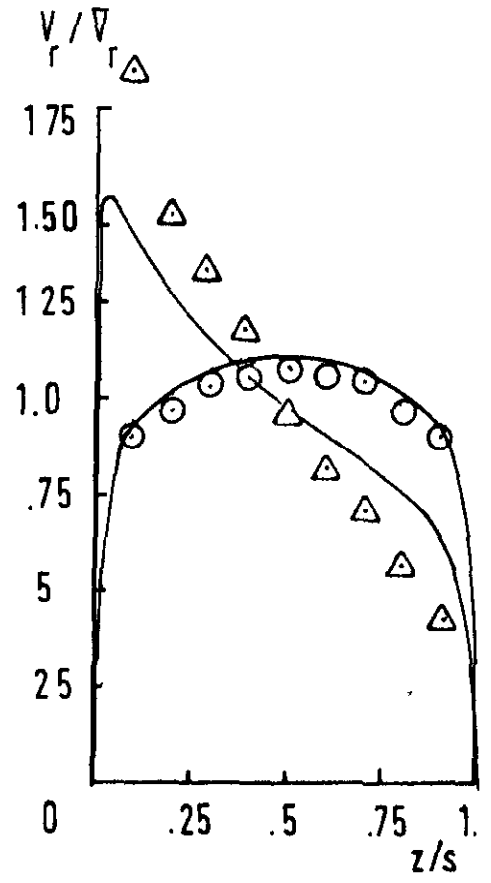
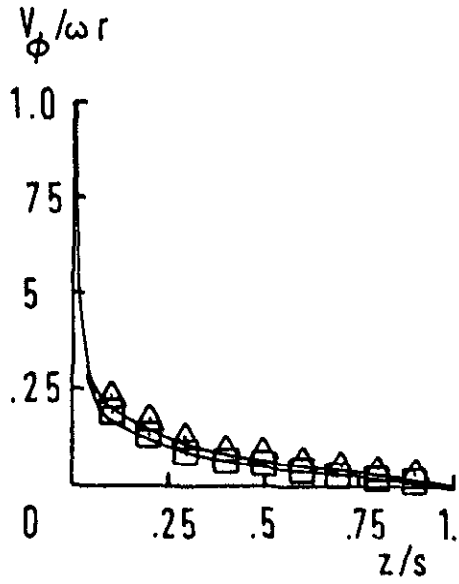


FIG19 Calculated velocities for  $G=0.03, r/r_0=1$

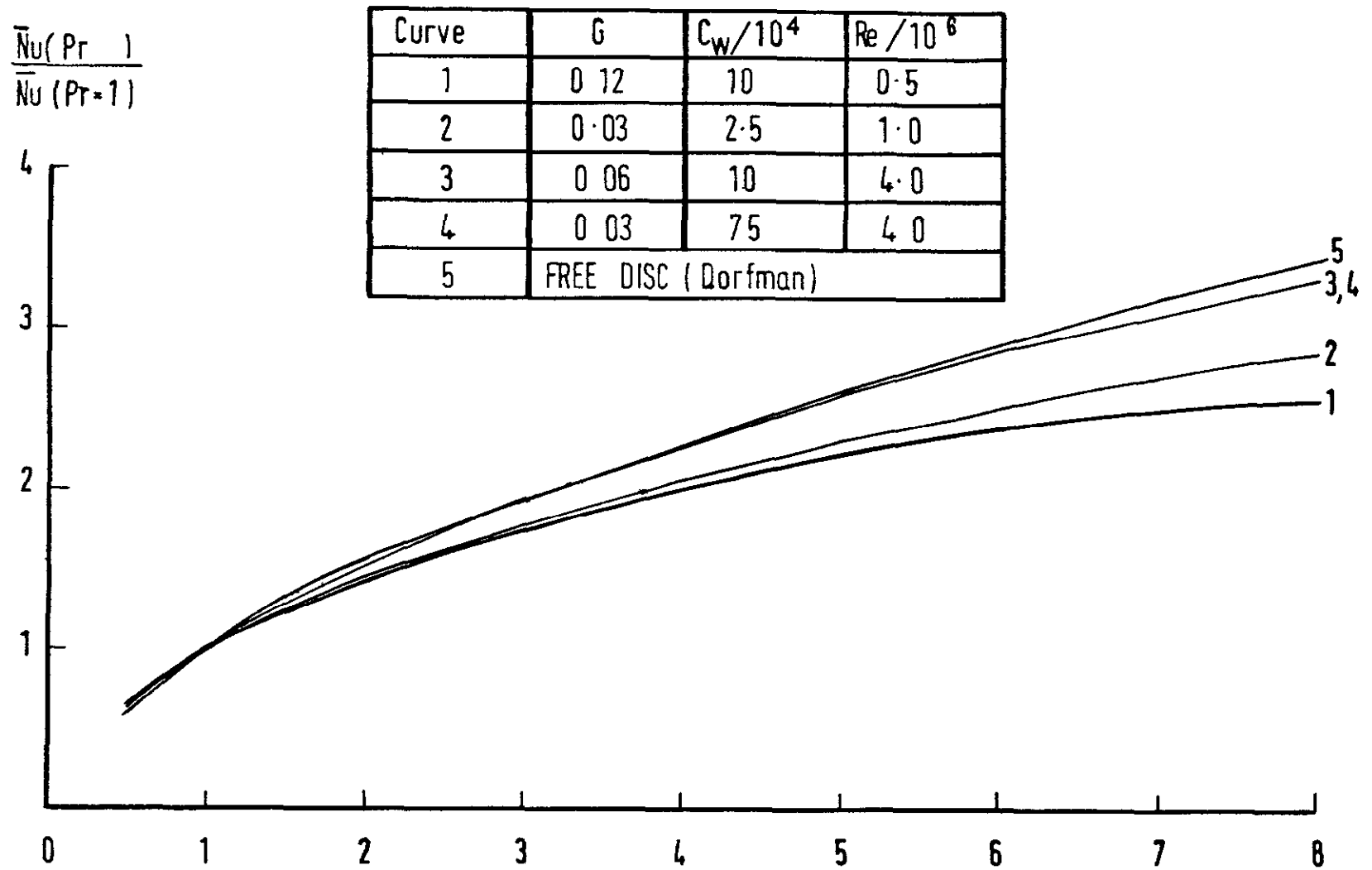


FIG 20 Calculated effect of Prandtl number on the mean Nusselt number for a range of flow parameters

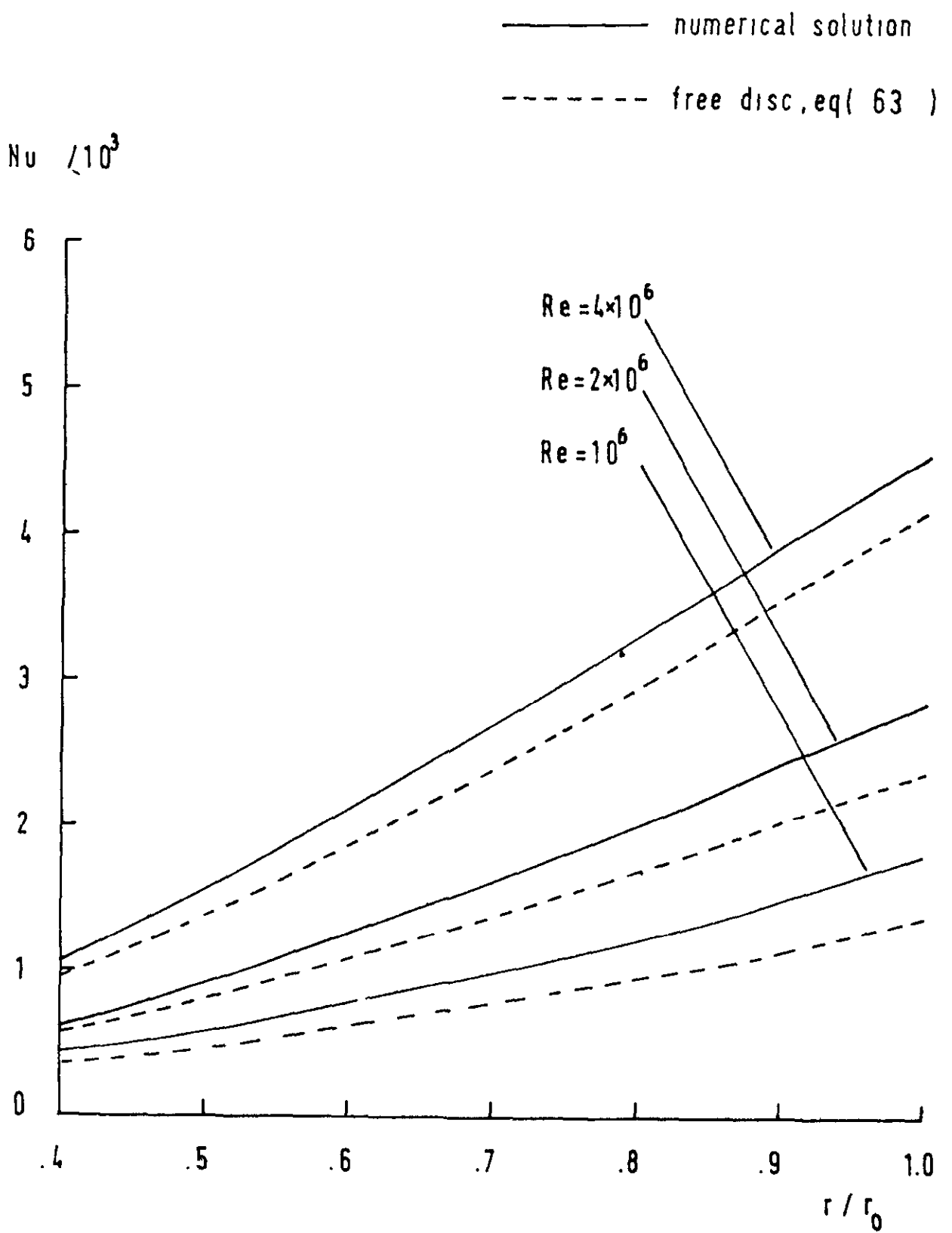


FIG 21 Local Nusselt numbers for  $G = 0.06$ ,  $C_w = 5 \times 10^{-4}$ ,  $Pr = 0.72$

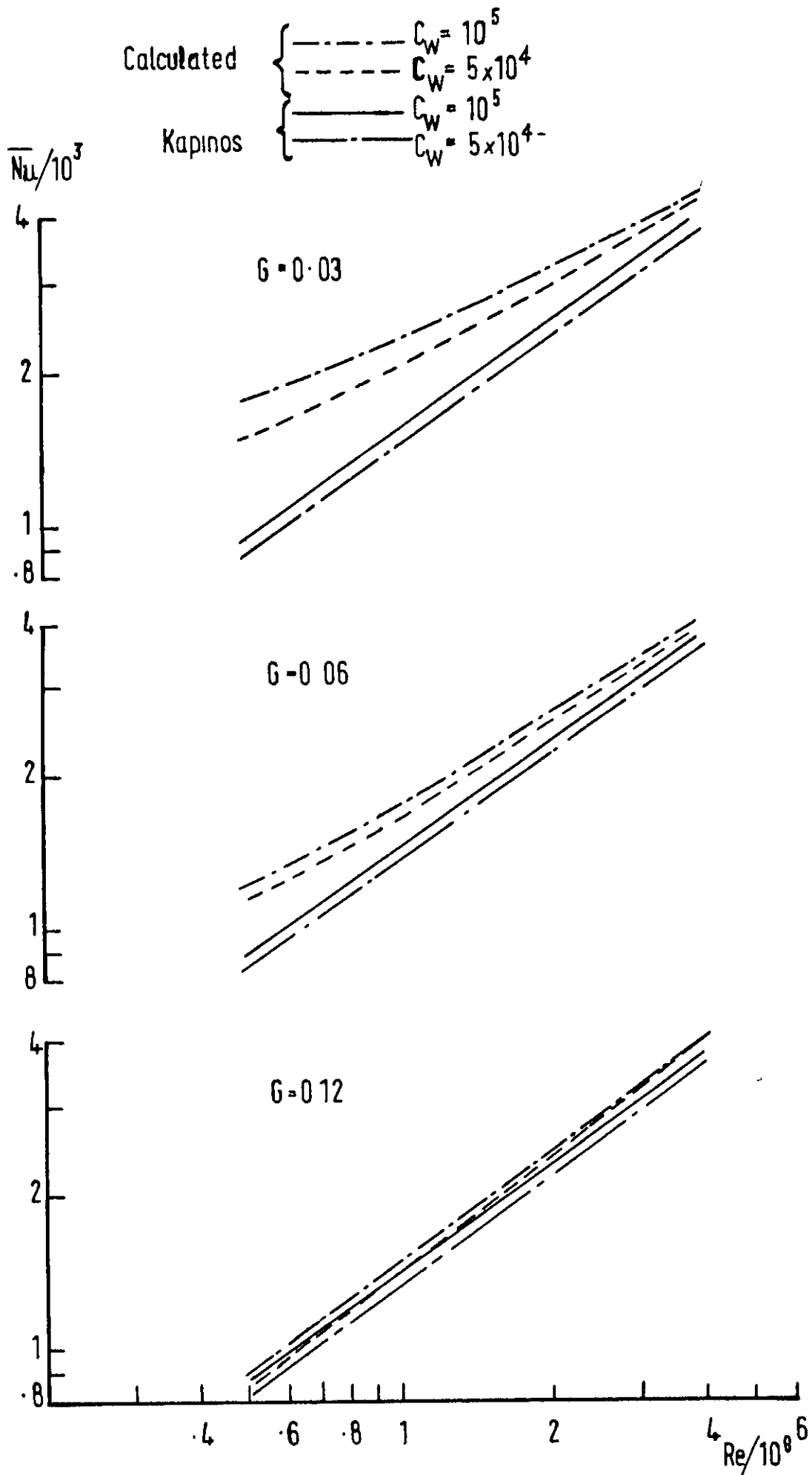


FIG 22 Comparison of calculated mean Nusselt numbers with the empirical correlation of Kapinos



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